3

GRAVITATIONAL WAVES

FROM

COSMIC (SUPER) STRINGS

VARIOUS COSMOLOGICAL SCENARIOS

STANDARD COSMOLOGICAL
- "SCENARIO" TO EXPLAIN WHY UNIVERSE SO LARGE, SO HOMOGENEOUS, + $\frac{\delta\rho}{\rho} \sim 5\times10^{-5}$

GR IS VALID

$\exists$ 'INFLATON' φ WITH $V(\varphi)$ VERY FLAT

$\epsilon \sim M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1$ AND $\eta \sim M_P^2 \frac{V''}{V} \ll 1$

QUANTUM FLUCT. $\hat{\delta\varphi} \Rightarrow$ ADIABATIC GAUSSIAN $\frac{\delta\rho}{\rho}$

$\frac{\delta\rho}{\rho} \sim 5\times10^{-5} \Leftrightarrow \exists$ SMALL PARAMETER

$V(\varphi) = \lambda\varphi^4$ or $\frac{1}{2}m^2\varphi^2$ $\quad \lambda \sim 10^{-13}$, $m^2 \sim 10^{-12}\, m_P^2$

- STRING THEORY CHALLENGES

- FIND A NATURAL CANDIDATE FOR THE INFLATON FIELD φ

E.G. DILATON (Veneziano, Gasperini ...)

SEPARATION OF D-BRANES (Dvali, Tye; Burgess...Quevedo; KKLT, KKLMMT, ...)

- GET GR WITHOUT DILATON-MODULI EFFECTS WHICH "KILL" INFLATION BY INTRODUCING "STEEP" DIRECTIONS IN $V(\varphi, \Phi, \dots)$

E.G. WARPED FLUX COMPACTIFICATIONS (Giddings, Kachru, Polchinski; Kachru et al...

- ARRANGE EXISTENCE OF SLOW-ROLL REGIONS OF $V(\varphi)$

E.G. LARGE BRANE SEPARATION: (Dvali, Tye; ...; KKLMMT

OR SYMMETRIC CONFIGURATIONS (Trivedi, ...

OR HIGH-DERIVATIVE TERMS $\sim -g(\phi)\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} - V(\phi)$ (Silverstein, Tong)

à la k-inflation (Armendariz-Picon, Damour, Mukhanov; Garriga, Mukhanov)

- TUNE-IN SOME SMALL PARAMETER ($\lambda \sim 10^{-13}$) TO ARRANGE $\frac{\delta\rho}{\rho} \sim 5\times10^{-5}$
- INCORPORATE STANDARD MODEL, AND ARRANGE FOR REHEATING
- ? ARRANGE INITIAL CONDITIONS, OR USE "ANTHROPIC-LIKE" ARGUMENTS

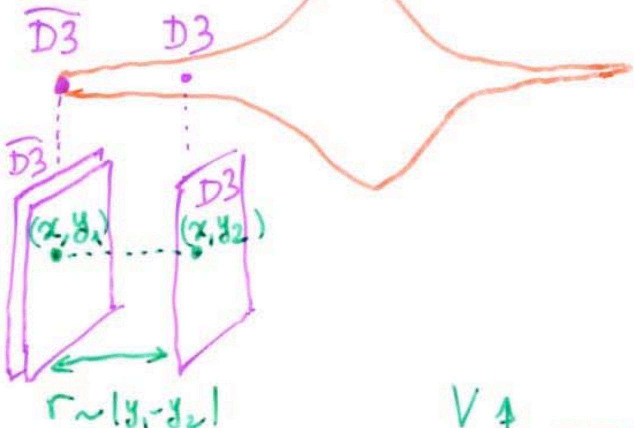
COSMIC SUPERSTRINGS?

Witten '85; ... Dvali, Tye; Tye, ...; KKLMMT; Copeland, Myers, Polchinski; Dvali, Vilenkin

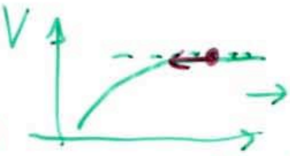
10 dim spacetime:

$$X^M = (x^\mu, y^a)$$

4, 6 COMPACT



$$\downarrow \quad V(r) = A - \frac{B}{r^4}$$

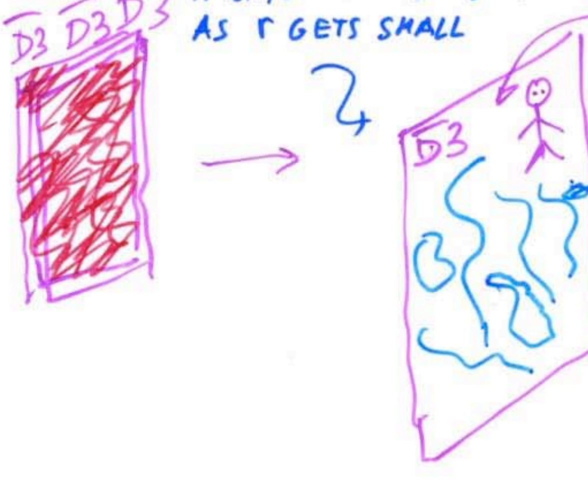


SLOW ROLL INFLATION

$\varphi(x) = r(x)$

TACHYONIC MODE $-\partial T \partial \bar{T} + |m^2| \bar{T} T$
AS r GETS SMALL

HEAT OF HOT BIG BANG



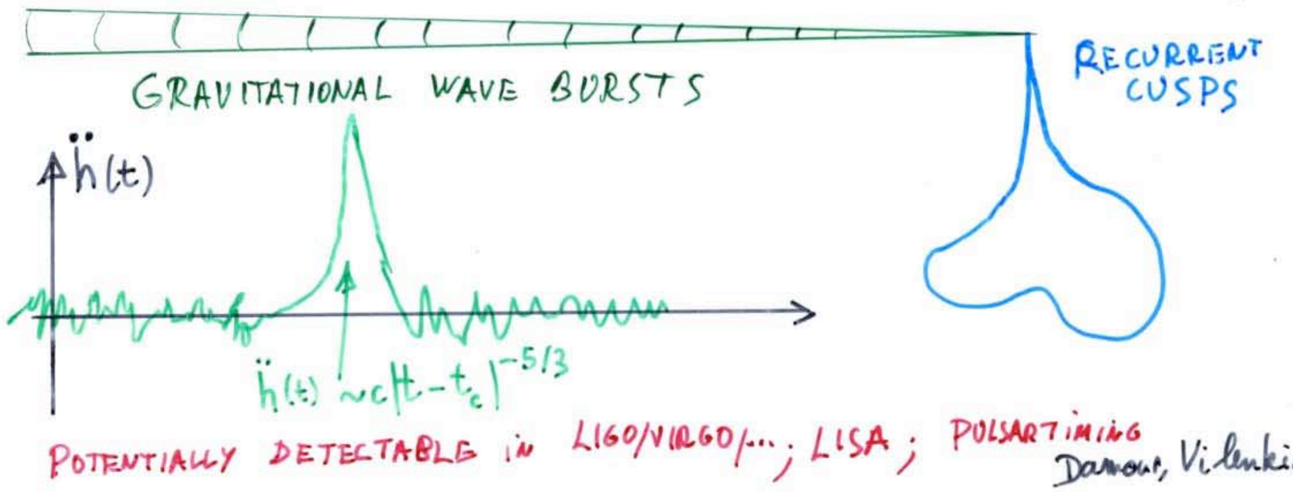
OUR WORLD

COSMOLOGICAL NETWORK OF MASSIVE (F OR D) STRINGS

WITH STRING TENSION

$$10^{-11} \lesssim G\mu \lesssim 10^{-6} \quad \text{Tye}$$

$$G\mu \sim 10^{-8} - 10^{-9} \quad \text{KKLMMT, Copeland MP}$$



POTENTIALLY DETECTABLE IN LIGO/VIRGO/...; LISA; PULSAR TIMING

Damour, Vilenkin

STRING DYNAMICS: $X^\mu(\tau,\sigma)$

$$S_{\text{NAMBU}} = -\mu \int d\tau\, d\sigma\, \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2 (X')^2}$$

$\partial_\tau X^\mu$ $\partial_\sigma X^\mu$ $\dot{X}\cdot X' \equiv g_{\mu\nu}(X)\dot{X}^\mu X'^\nu$

$$S_{\text{POLYAKOV}} = -\frac{1}{2}\mu \int d\tau\, d\sigma\, \sqrt{h}\, h^{ab} g_{\mu\nu}(X)\, \partial_a X^\mu \partial_b X^\nu$$

CONFORMAL GAUGE: $\sqrt{h}\, h^{ab} = \eta^{ab}$

$\Rightarrow$ CONSTRAINTS $\dot{X}^2 + X'^2 = 0$; $\dot{X}\cdot X' = 0$

EQ. OF MOTION: $\boxed{\ddot{X}^\mu - X''^\mu + \Gamma^\mu_{\alpha\beta}(X)\,(\dot{X}^\alpha \dot{X}^\beta - X'^\alpha X'^\beta) = 0}$

IN FLAT SPACE: $\ddot{X}^\mu - X''^\mu = 0$

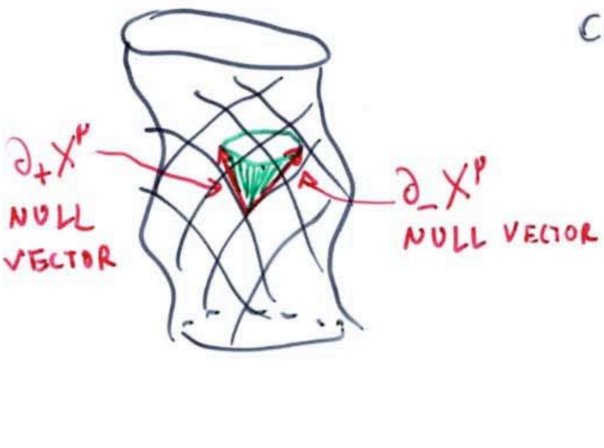
NULL WORLD-SHEET COORDS:

$$\sigma_\pm = \tau \pm \sigma$$

EOM: $\frac{\partial}{\partial\sigma_+}\frac{\partial}{\partial\sigma_-} X^\mu = 0$

LEFT-MOVING RIGHT-MOVING

$$\boxed{X^\mu(\tau,\sigma) = \frac{1}{2}\left[X^\mu_+(\sigma_+) + X^\mu_-(\sigma_-)\right]}$$



CONSTRAINTS: $\boxed{\begin{aligned}(\partial_+ X^\mu_+)^2 &= 0\\ (\partial_- X^\mu_-)^2 &= 0\end{aligned}}$

CUSPS

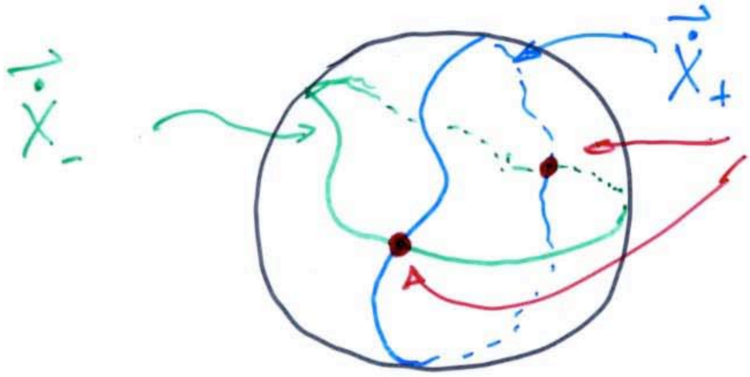
TIME GAUGE: $X^0(\tau,\sigma) = \tau = \frac{1}{2}[\sigma_+ + \sigma_-]$

IN CENTER-OF-MASS FRAME: $X^i_\pm(\sigma_\pm) = \text{PERIODIC} \Rightarrow \langle \dot{X}^i_\pm(\sigma_\pm) \rangle = 0$

CONSTRAINTS:
$$(\partial_\pm X^\mu_\pm)^2 = -(\partial_\pm \underbrace{X^0_\pm}_{\sigma_\pm})^2 + (\partial_\pm X^i_\pm)^2 = 0$$
$$-1 + (\dot{X}^i_\pm)^2 = 0$$
$$(\vec{\dot{X}}_+)^2 = 1 = (\vec{\dot{X}}_-)^2$$

$\vec{\dot{X}}_+$ AND $\vec{\dot{X}}_-$ ARE PERIODIC (WITH ZERO AVERAGE) ON UNIT SPHERE

Kibble, Turok '82

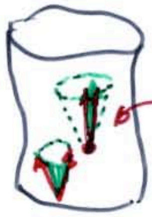


GENERICALLY EXPECT

$\exists$ 2 INTERSECTIONS

Turok '84

INTERSECTION: $\partial_+ X^\mu_+ = \partial_- X^\mu_- = \ell^\mu$



LIGHT-CONE TANGENT TO WORLD-SHEET

IN SPACE:

GW BURSTS FROM CUSPY STRINGS

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \qquad \bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \tfrac{1}{2} h \eta_{\mu\nu}$$

$$\partial^\nu \bar{h}_{\mu\nu} = 0$$

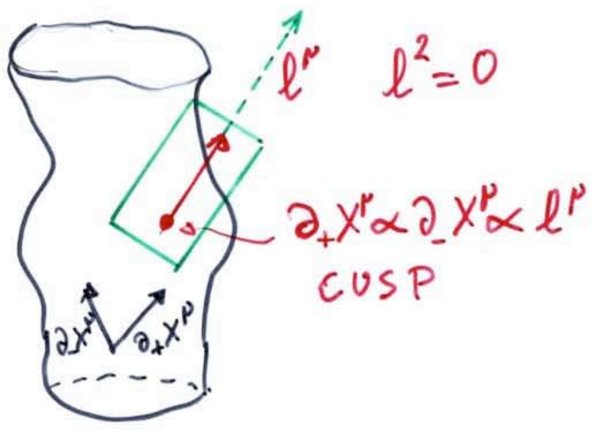
HARMONIC GAUGE

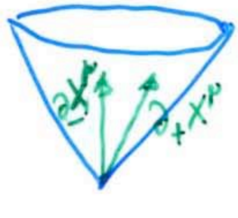
STRING STRESS-ENERGY TENSOR

$$T^{\mu\nu}(x^\lambda) = \mu \int d\tau d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta^{(4)}\left(x^\lambda - X^\lambda(\tau,\sigma)\right)$$

- USE LEFT – RIGHT DECOMPOSITION : $\boxed{\sigma_\pm \equiv \tau \pm \sigma}$

$$\boxed{X^\mu(\tau,\sigma) = \tfrac{1}{2}\left[X^\mu_+(\sigma_+) + X^\mu_-(\sigma_-) \right] \qquad (\partial_+ X^\mu_+)^2 = 0 \quad (\partial_- X^\mu_-)^2 = 0}$$





- USE FOURIER TRANSFORM

$$\boxed{T^{\mu\nu}(k^\lambda) = \frac{\mu}{T_\ell} \int_{\Sigma_\ell} d\tau d\sigma\, \dot{X}_+^{(\mu} \dot{X}_-^{\nu)} e^{-\frac{i}{2} k \cdot (X_+ + X_-)}}$$

- STAY POINCARÉ COVARIANT

GW AMPLITUDE FROM STRINGS

$$\boxed{\bar{h}_{\mu\nu}(t,\vec{x}) = \frac{\kappa_{\mu\nu}(t-r,\vec{n})}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)}$$

$$\kappa_{\mu\nu}(t-r,\vec{n}) = \sum_{\omega = \pm m \frac{2\pi}{T_\ell}} 4G\, e^{-i\omega(t-r)}\, T_{\mu\nu}(\omega, \vec{k}=\omega\vec{n})$$

$T_\ell = \frac{\ell}{2}$ ∧ invariant length $\ell = \frac{E_0}{\mu}$

$$\boxed{T^{\mu\nu}(\vec{k}_m, \omega_m) = \frac{\mu}{\ell}\, I_+^{(\mu}\, I_-^{\nu)}}$$

$$\boxed{I_\pm^\mu \equiv \int_0^\ell d\sigma_\pm\, \dot{X}_\pm^\mu(\sigma_\pm)\, e^{-\frac{i}{2} k_m \cdot X_\pm}}$$

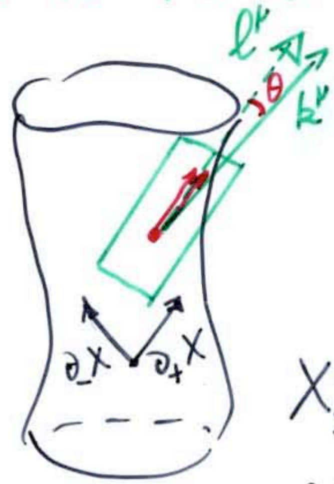
LOGARITHMIC FOURIER TRANSFORM OF WAVEFORM (HIGH-FREQ PART)

$$\kappa^{\mu\nu}(f,\vec{n}) \equiv |f| \int dt\, e^{2\pi i f t}\, \kappa^{\mu\nu}(t-r,\vec{n})$$

$$\boxed{\kappa^{\mu\nu}(f,\vec{n}) = 2G\mu\, |f|\, I_+^{(\mu}(\omega,\omega\vec{n})\, I_-^{\nu)}(\omega,\omega\vec{n})}$$

LEFT-RIGHT FACTORIZATION OF GW AMPLITUDE $\kappa^{\mu\nu}(k)$

GW AMPLITUDE FROM CUSPS



AT CUSP

$$\partial_+ X^\mu \propto \partial_- X^\mu \propto \ell^\mu$$ NULL VECTOR

NEAR CUSP

$$X^\mu_\pm(\sigma_\pm) = X^\mu_c + \ell^\mu \sigma_\pm + \frac{1}{2}\ddot{X}^\mu_\pm \sigma_\pm^2 + \frac{1}{6}\dddot{X}^\mu_\pm \sigma_\pm^3 + \dots$$

$$\dot{X}^\mu_\pm(\sigma_\pm) = \ell^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \frac{1}{2}\dddot{X}^\mu_\pm \sigma_\pm^2 + \dots$$

$$\kappa^{\mu\nu}(f) \propto I_+^{(\mu} I_-^{\nu)}$$

$$\omega_\ell = \frac{2\pi}{T_\ell} = \frac{4\pi}{\ell}$$

$$I^\mu_\pm = \int_{\sigma_0}^{\sigma_0+\ell} d\sigma_\pm \left(\ell^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \dots\right) e^{+\frac{i}{12} m \omega_\ell \ddot{X}_\pm^2 \sigma_\pm^3 + \dots}$$

CAN BE GAUGED AWAY (WHEN $\theta = 0$)

$m \to \pm\infty$

$\sim \theta \sigma_\pm^2 + \theta^2 \sigma_\pm$ WHEN $\theta \neq 0$

$$\boxed{\kappa^{\mu\nu}(f, \vec{n}_{cusp}) = -C \frac{G\mu}{(2\pi |f|)^{1/3}} e^{2\pi i f t_c} A_+^{(\mu} A_-^{\nu)} + \text{GAUGE}}$$

i.e. FOR $\theta = 0$

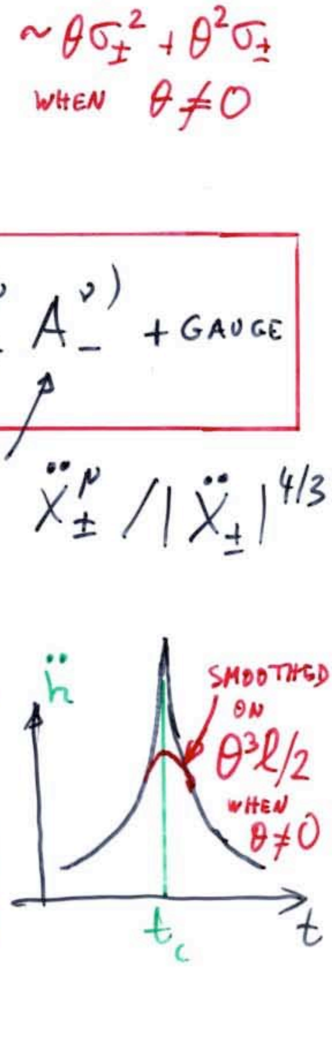
$$C = 4\pi (12)^{4/3} / [3\Gamma(\tfrac{1}{3})]^2$$

$$A^\mu_\pm = \ddot{X}^\mu_\pm / |\ddot{X}_\pm|^{4/3}$$

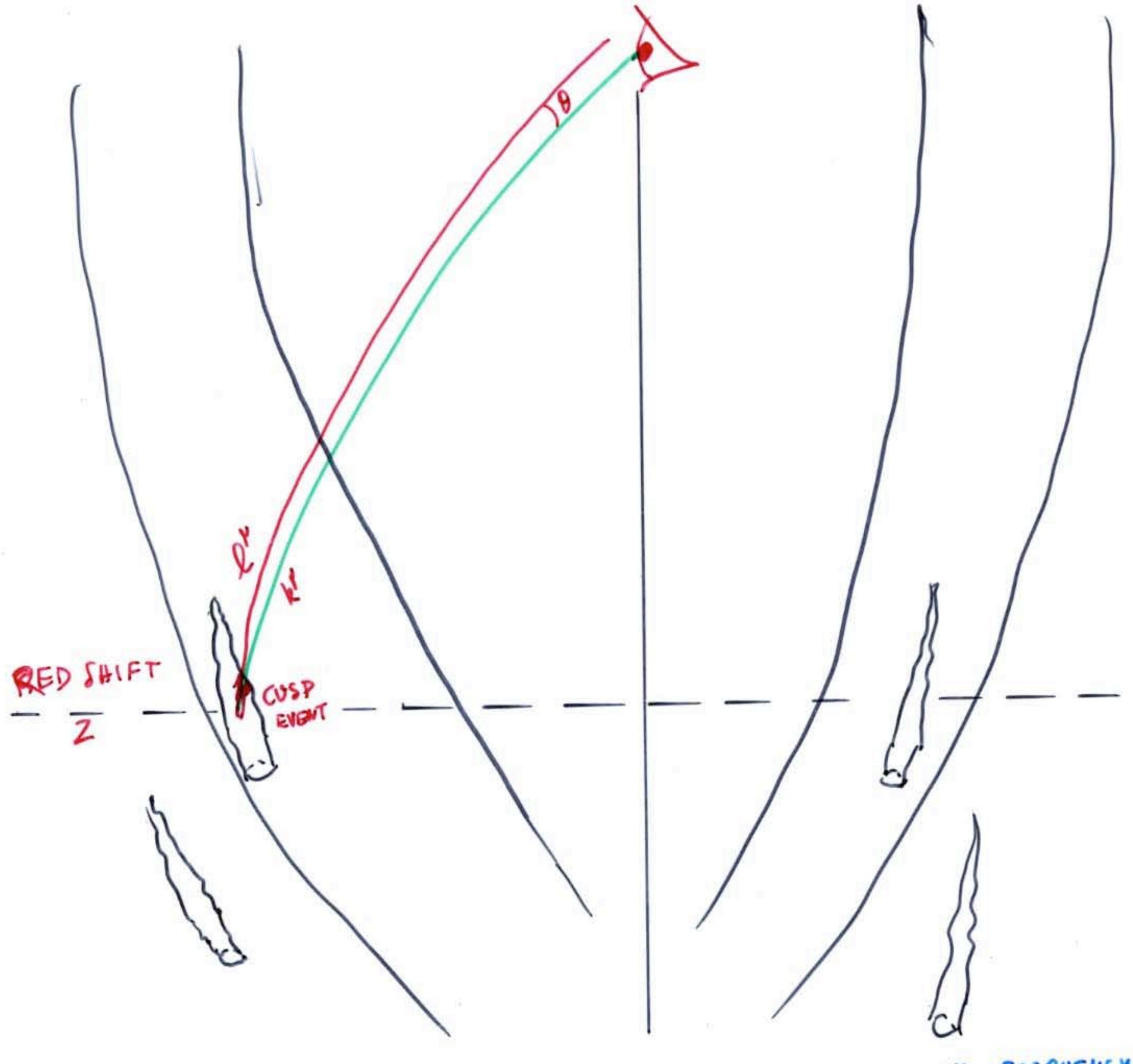
TIME DOMAIN

SIGNAL ROBUST UNDER $\exists$ SMALL-SCALE WIGGLES (Siemens, Olum '03)

$$\boxed{\begin{aligned} \kappa(t) &\propto |t - t_c|^{1/3} \\ \ddot{\kappa}(t) &\propto |t - t_c|^{-5/3} \end{aligned}}$$



GW BURSTS FROM COSMOLOGICAL STRING NETWORK



- EFFECT OF COSMOLOGICAL EXPANSION ON $\bar{h}_{\mu\nu}(f)$ ON FREQUENCY / ON AMPLITUDE
- NUMBER OF CUSP EVENTS PER UNIT SPACE-TIME VOLUME

$$\boxed{\nu(t) \sim c\, n_\ell(t)/(\ell/2)}$$

$c \equiv$ # cusp events per loop period

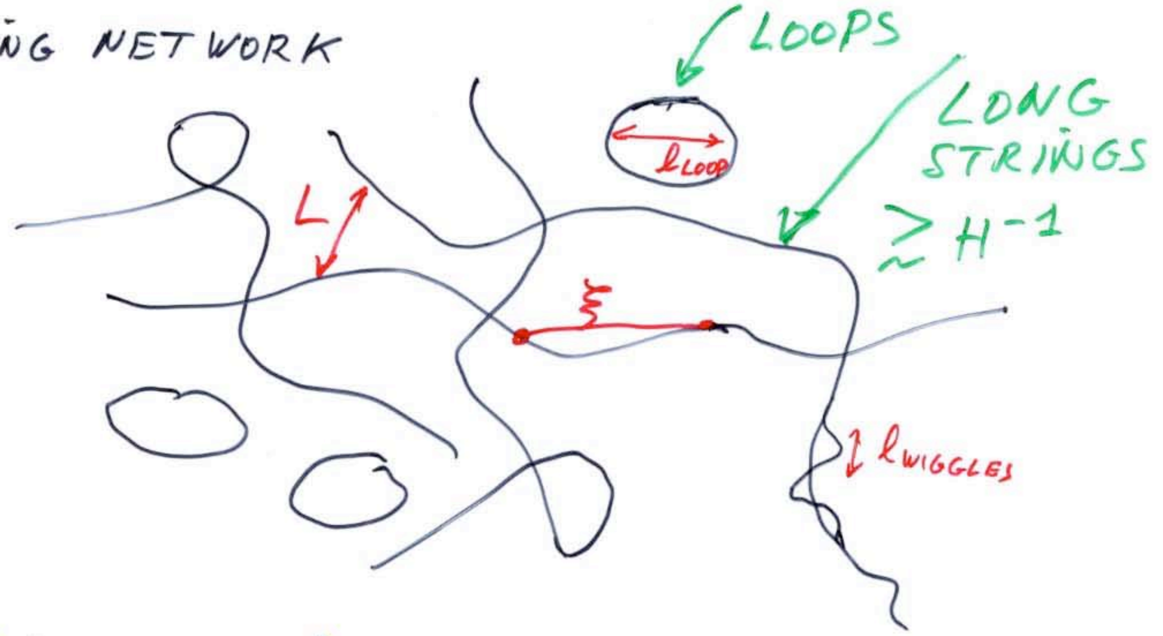
$n_\ell(t)$ = loop density

- BEAMING FRACTION WITHIN $\boxed{\theta_m \equiv \left[(1+z) f \ell/2\right]^{-1/3}}$

LOOP NUMBER DENSITY $n_\ell(t)$?

SEVERAL COMPETING PHENOMENA AT WORK:

- STRING NETWORK



COSMOLOGICAL EXPANSION : $\rightarrow$ TENDS TO STRAIGHTEN OUT STRINGS

STRING INTERACTIONS : INTERSECTIONS, RECONNECTIONS (WITH PROBA. p), SELF-RECONNECTIONS
$\rightarrow$ CREATES LOOPS AND SMALL-SCALE STRUCTURE

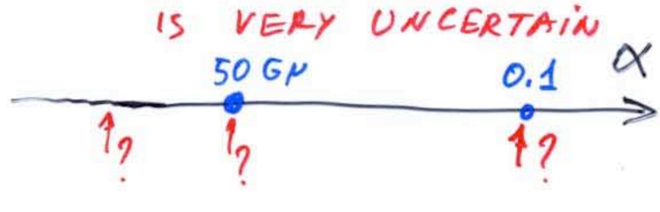
- VARIOUS SCALES : ξ, L, ℓ_{LOOP}, $\ell_{WIGGLES}$

- NUMERICAL SIMULATIONS INDICATE SCALING BUT
$\exists$ LARGE UNCERTAINTY IN FINAL SCALING STATE AND PARAMETERS

PROBABLY : $\xi(t) \sim t$, $\boxed{L(t) \sim p^{1/2}\, t}$, $\boxed{\ell_{WIGGLES} \sim \underset{\text{JUST FORMED}}{\ell_{LOOP}} \sim \alpha\, t}$

GOOD NEWS $p \ll 1$
INCREASES THE DENSITY OF LONG STRINGS AND LOOPS

CRUCIAL DIMENSIONLESS PARAMETER α IS VERY UNCERTAIN



α DETERMINES $\ell_{LOOP} \sim \alpha t \rightarrow$ LOOP LIFE-TIME $\tau \sim \frac{\alpha}{50G\mu} t$

JUST FORMED

$\Rightarrow$ LOOP DENSITY

redshifts $z \lesssim 1$

high-redshift

$$\boxed{n_\ell \sim \frac{1}{p\, 50 G\mu t^3} + n_\ell^{z>1}}$$

DOMINATES IF $\alpha \lesssim 50\, G\mu$ (considered by Damour Vilenkin 05)

DOMINATES IF $\alpha \gg 50\, G\mu$ (considered by Hogan06)

$\exists$ LARGE RANGE OF VALUES OF $G\mu$ WHERE GW BURSTS WOULD BE OBSERVABLE BY LIGO OR LISA. IN ADDITION, AT LOW GW FREQUENCIES CONFUSION NOISE $\longrightarrow$ STOCHASTIC GW BACKGROUND OBSERVABLE BY PULSAR TIMING

OBSERVABLE RANGE OF $G\mu$ VALUES IS LARGER BUT THE POPULATION OF HIGH z LOOPS CREATE A LARGER CONFUSION NOISE (i.e. STOCHASTIC BACKGRND) WHICH MAKES MORE DIFFICULT TO SEE INDIVIDUAL BURST EVENTS

LIGO COULD DETECT $G\mu \gtrsim 10^{-12}$

LISA COULD DETECT $G\mu \gtrsim 10^{-14}$

LISA COULD DETECT $G\mu \gtrsim 10^{-15}$

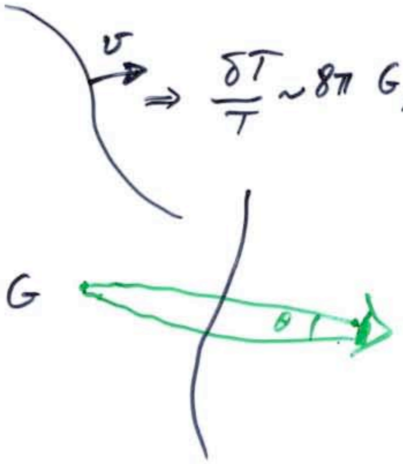
BUT, PULSAR TIMING GIVE ALREADY STRINGENT LIMITS ON THE EXISTENCE OF A STOCHASTIC BACKGRND OF GW's (Jenet et al. 06) WHICH (PROBABLY) ALREADY SETS SEVERE LIMITS ON COSMIC (SUPER) STRINGS

? $G\mu \lesssim 10^{-9}$ OR EVEN $G\mu \lesssim 10^{-10}$?

! WHICH IS ALREADY $\sim$ THE KKLMMT LEVEL !

NOTE THE VARIOUS POSSIBLE OBSERVABLE SIGNALS FROM COSMIC (SUPER)-STRINGS

CMB



$\Rightarrow \frac{\delta T}{T} \sim 8\pi G\mu v \gamma$ CURRENT BOUND $G\mu \lesssim 5\times 10^{-7}$

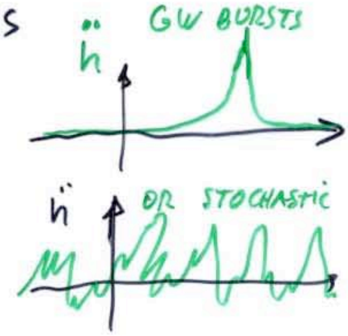
LENSING ONLY SENSITIVE TO $G\mu \gtrsim 10^{-7}$

SEVERAL GW DETECTORS

LIGO / VIRGO / GEO

ADVANCED LIGO

LISA



POTENTIALLY SENSITIVE DOWN TO $G\mu \gtrsim 10^{-12}$

$G\mu \gtrsim 10^{-14}$ OR 10^{-15}

PULSAR TIMING

STOCHASTIC SUPERPOSITION OF GW's

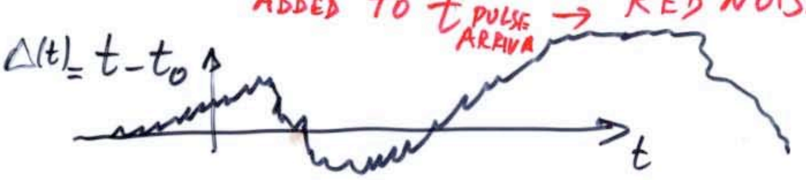
$$H_{ij}(t) = \int dt\, h_{ij}(t)$$

$\vec{m}$ = DIRECTION OF GW

$$\Rightarrow t_{\substack{\text{PULSAR}\\ \rightarrow \text{EARTH}}} = t_0 + \frac{1}{2}\frac{1}{1-\vec{m}\cdot\vec{e}}\, e^i e^j \left[H_{ij}(t) - H_{ij}\left(t - (1-\vec{m}\cdot\vec{e})t_0\right)\right]$$

$\vec{e}$ = DIRECTION OF ELM WAVE

STOCHASTIC FLUCTUATION ADDED TO $t_{\text{PULSE ARRIVAL}}$ → RED NOISE



FREQUENCY ANALYSIS OF $\Delta(t)$: $$\Delta(f) = \frac{H_0^2}{8\pi^4} \frac{\Omega_{GW}(f)}{f^4}$$

ENERGY DENSITY OF GW

- EXCITING POSSIBILITY OF DETECTING COSMIC (SUPER)STRINGS
- FRUSTRATING UNCERTAINTIES IN NETWORK PROPERTIES (see Polchinski)