Parma

2

EXPERIMENTAL TESTS

OF

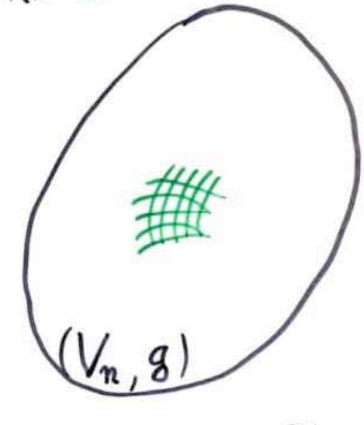
RELATIVISTIC GRAVITY

EINSTEIN'S VISION

BERN 1907

$$\vec{x}' = \vec{x} - \frac{1}{2}\vec{g}\,t^2$$

RIEMANN ~ 1856



LOCAL EFFACEMENT OF $\vec{g}$

$$\exists\, x'^{\mu};\ g'_{\mu\nu}(x'^{\lambda}) = f_{\mu\nu} + O[(x'-x'_0)^2]$$

LOCAL EFFACEMENT OF $\Gamma^{\lambda}_{\mu\nu} \sim \partial g$

UNIVERSALITY OF FREE FALL $\dashrightarrow$ UNIVERSAL COUPLING OF MATTER
$\longleftarrow$ TO ONE $g_{\mu\nu}(x^{\lambda})$

('HYPOTHESIS OF EQUIVALENCE')

['EQUIVALENCE PRINCIPLE']

$$\boxed{S_{TOT} = \frac{c^4}{16\pi G}\int \sqrt{g}\,\frac{d^4x}{c}\,R(g) \quad + S_{MATTER}[\psi, A, H;\ g_{\mu\nu}]}$$

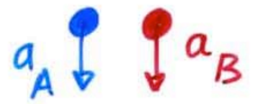
TWO SORTS OF EXPERIMENTAL TESTS

- MATTER-COUPLING TESTS (RHS)
- TESTS OF THE DYNAMICS OF $g_{\mu\nu}$ (LHS)

TESTS OF THE COUPLING MATTER ∩ GRAVITY

"EQUIVALENCE PRINCIPLE" $S_{MATTER}[\psi, A, H; g_{\mu\nu}]$

- UNIVERSALITY OF FREE FALL



Adelberger's group $\left(\frac{\Delta a}{a}\right)_{Fe\sim Si} = (3.6 \pm 5.0_{STAT} \pm 0.7_{SYST}) \times 10^{-13}$

Lunar Laser Ranging's group $\left(\frac{\Delta a}{a}\right)_{\oplus ☽} = (-1.0 \pm 1.4) \times 10^{-13}$

- CONSTANCY OF "CONSTANTS" $\alpha_{EM} \equiv \frac{e^2}{\hbar c}$

Atomic Clock Tests
Marion...'03; Bize...'03; Fischer'05

$$\frac{\dot{\alpha}_{EM}}{\alpha_{EM}} = (-0.9 \pm 2.9) \times 10^{-15}\, yr^{-1}$$

Oklo's natural fission reactor
Shlyakhter'76, Damour Dyson'96, Fujii...'00

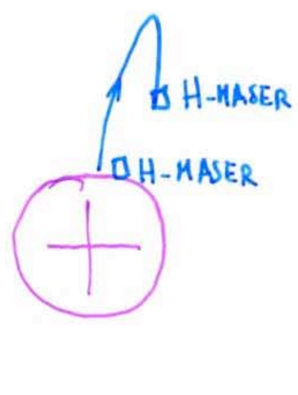
$$\frac{\langle \dot{\alpha}_{EM} \rangle}{\alpha_{EM}} = (-0.9 \pm 5.9) \times 10^{-17}\, yr^{-1}$$

Quasar absorption lines
Quast...'04; Srianand...'04

$$\frac{\langle \dot{\alpha}_{EM} \rangle}{\alpha_{EM}} = (-0.7 \pm 1.9) \times 10^{-16}\, yr^{-1}$$

- GRAVITATIONAL REDSHIFT

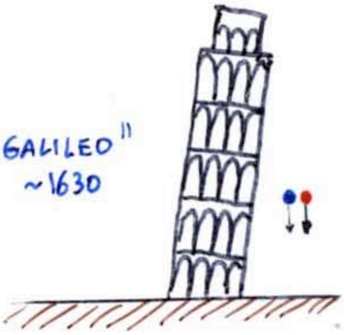
Vessot, Levine '79



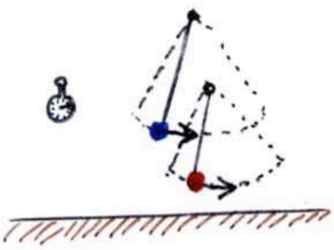
$$\frac{\Delta \nu}{\nu} = (1 \pm 10^{-4}) \frac{\Delta U}{c^2}$$

UNIVERSALITY OF FREE FALL

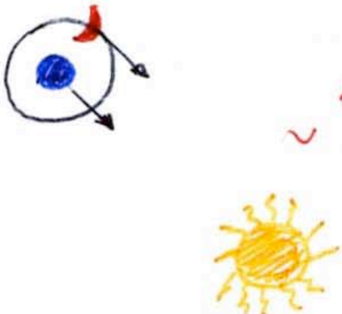
"GALILEO"
~1630



NEWTON
≤1686



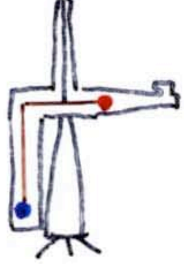
10^{-3}



$\sim 10^{-7}$

LAPLACE
≤1805

[POINCARÉ 1906 $\sim 10^{-8}$]



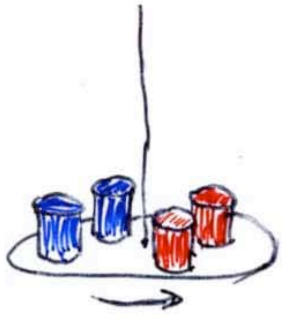
$\sim 10^{-8}$

EÖTVÖS
1896

DICKE ET AL.
1962

$\sim 3\times 10^{-11}$

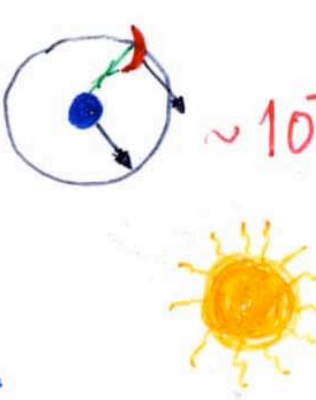
BRAGINSKY AND PANOV 1972



$\sim 10^{-12}$

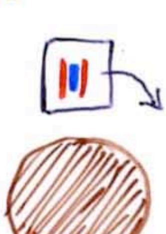
ADELBERGER ET AL.
1989–1994, 2001–

LUNAR
LASER
RANGING



$\sim 10^{-13}$

1969 – NOW
DICKEY ET AL. '94
WILLIAMS ET AL. '96



SPACE
TESTS
OF THE
EQUIVALENCE PRINCIPLE

MICROSCOPE 2009 $\sim 10^{-15}$?
STEP 200? $\sim 10^{-18}$?

EXPLICIT FORM OF EINSTEIN'S FIELD EQUATIONS

(IN HARMONIC COORDINATES)

$$\boxed{\begin{aligned} & -g^{\alpha\beta}\partial_{\alpha\beta}g_{\mu\nu} + g^{\alpha\beta}g^{\gamma\delta}\Big(\partial_\gamma g_{\mu\alpha}\,\partial_\delta g_{\nu\beta} - \partial_\gamma g_{\mu\alpha}\,\partial_\beta g_{\nu\delta} \\ & + \partial_\gamma g_{\mu\alpha}\,\partial_\nu g_{\beta\delta} + \partial_\gamma g_{\nu\alpha}\,\partial_\mu g_{\beta\delta} - \frac{1}{2}\partial_\mu g_{\alpha\gamma}\,\partial_\nu g_{\beta\delta}\Big) \\ & \qquad = \frac{16\pi G}{c^4}\Big(T_{\mu\nu} - \frac{1}{D-2}\,T g_{\mu\nu}\Big) \end{aligned}}$$

- WEAK-FIELD REGIME : $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$; $h_{\mu\nu} \ll 1$

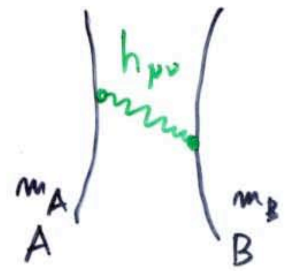
$$\longrightarrow \qquad -\Box\, h_{\mu\nu} = \frac{16\pi G}{c^4}\Big(T_{\mu\nu} - \frac{1}{D-2}\,\eta_{\mu\nu} T\Big)$$

- SOLAR-SYSTEM :

$(D=4)$
$$-\Box\Big(h_{00} + \frac{1}{2}h_{00}^2\Big) = \frac{8\pi G}{c^4}\big(T^{00} + T^{ii}\big)$$

- STRONG-FIELD REGIME : NEED EXACT FORM

ONE-GRAVITON EXCHANGE INTERACTION



$$S = -m_A \int \underbrace{ds_A}_{\sqrt{-g_{\mu\nu}(x_A)\,dx_A^\mu dx_A^\nu}} - m_B \int ds_B + \int \frac{\sqrt{g}\,R}{16\pi G}$$

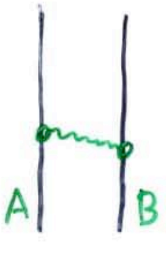
EXPAND $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$S \simeq -m_A \int \sqrt{-\eta_{\mu\nu} dx_A^\mu dx_A^\nu} - m_B \int \sqrt{-\eta_{\mu\nu} dx_A^\mu dx_A^\nu} + \frac{1}{32\pi G} \int h^{\mu\nu} \Box \left(h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}\right)$$
$$+ \frac{1}{2} \int h_{\mu\nu} T_A^{\mu\nu} + \frac{1}{2} \int h_{\mu\nu} T_B^{\mu\nu}$$

$$T_{A(x)}^{\mu\nu} = \int ds_o\, m_A u_A^\mu u_A^\nu\, \delta(x^\mu - x_A^\mu) \qquad u_A^\mu = \frac{dx_A^\mu}{ds_o}$$

ELIMINATE $h_{\mu\nu}$ BY SOLVING $\frac{\delta S}{\delta h_{\mu\nu}} = 0$, i.e.

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{D-2} T \eta_{\mu\nu}\right)$$

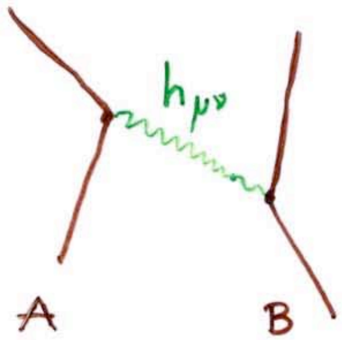


$$\boxed{S^{int} = -\frac{8\pi G}{c^4} \int d^D x\, T_A^{\mu\nu} \underbrace{\Box^{-1} \left(T_{\mu\nu}^B - \frac{1}{D-2} T^B \eta_{\mu\nu}\right)}_{\Box^{-1} P_{\mu\nu}^{\rho\sigma} T_{\rho\sigma}^B}}$$

$$\boxed{S^{int} = 2G \iint ds_A^o ds_B^o\, m_A u_A^\mu u_A^\nu P_{\mu\nu}^{\rho\sigma} G(x_A(s_A) - x_B(s_B)) m_B u_\rho^B u_\sigma^B}$$

Green's function $\Box G(x) = -4\pi \delta^D(x)$

DYNAMICS OF THE GRAVITATIONAL FIELD : WEAK FIELD REGIME



"ONE - GRAVITON EXCHANGE"

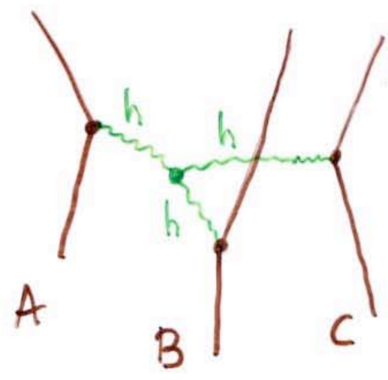
$\Updownarrow$

LINEARIZED EINSTEIN'S EQUATIONS

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu}\right)$$

$$S_{INT} = 2G \iint ds_A \, ds_B \; m_A u_A^\mu u_A^\nu \; P_{2\,\mu\nu}^{\;\rho\sigma} \, D[x_A(s_A) - x_B(s_B)] \, m_B u_{B\rho} u_{B\sigma}$$

$$L^{2\text{-BODY}} = \frac{1}{2}\sum_{A\neq B} \frac{G m_A m_B}{r_{AB}}\left[1 + \frac{3}{2c^2}(\vec{v}_A^{\,2} + \vec{v}_B^{\,2}) - \frac{7}{2c^2}\,\vec{v}_A\cdot\vec{v}_B - \frac{1}{2c^2}(\vec{n}_{AB}\cdot\vec{v}_A)(\vec{n}_{AB}\cdot\vec{v}_B) + O\left(\frac{1}{c^4}\right)\right]$$



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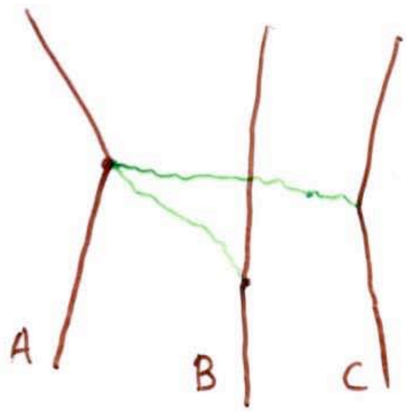
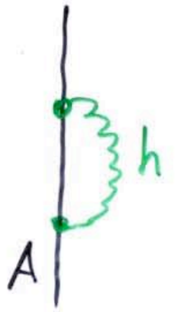


$$L^{3\text{-BODY}} = -\frac{1}{2}\sum_{B\neq A\neq C} \frac{G^2 m_A m_B m_C}{r_{AB}\, r_{AC}\, c^2} + O\left(\frac{1}{c^4}\right)$$

'CLASSICAL LOOPS'

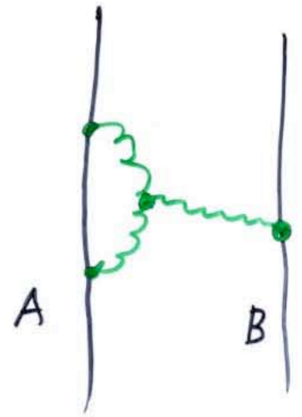
CLASSICAL CALCULATION ALSO CONTAINS (SELF) DIAGRAMS

ONE LOOP



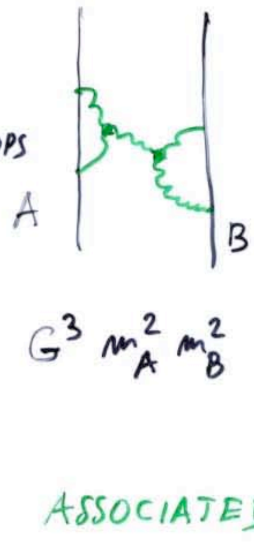
i.e. $T_A \Box^{-1} P T_A \quad \propto G m_A^2$

AND, AT HIGHER ORDERS,



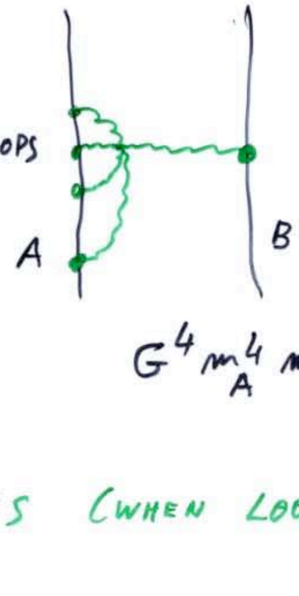
$\propto \quad G^2 m_A^2 m_B$

TWO LOOPS



$G^3 m_A^2 m_B^2$

.... THREE LOOPS



$G^4 m_A^4 m_B$

→ ASSOCIATED DIVERGENCES (WHEN LOOPS SHRINK)

→ BECOME NASTY AT THREE LOOPS

→ RENORMALIZABLE OR FINITE IN DIMENSIONAL REGULARIZATION

EXPECT NON-RENORMALIZABLE AMBIGUITIES ONLY AT $\geq$ 5 LOOPS

TESTS OF THE DYNAMICS OF THE GRAV. FIELD

SOLAR-SYSTEM TESTS :

WEAK ($h_{\mu\nu} < 10^{-6}$) AND QUASI-STATIC ($\frac{\partial_t h}{c\,\partial_x h} \sim \frac{v}{c} \lesssim 10^{-4}$) FIELDS

- PERIHELIÓN ADVANCE OF MERCURY
  I. Shapiro '90; ASSUMING $J_{2\odot} \simeq 2 \times 10^{-7}$

$$\Delta\dot{\varpi} = \underset{\dot{\varpi}^{GR}}{\underset{\parallel}{42''\!.98}}(1.000 \pm 0.001)$$

- LIGHT DEFLECTIÓN (VLBI)
  S. S. Shapiro... '04

$$\Delta\theta = \Delta\theta^{GR}\left(1 + (-0.9 \pm 2.2) \times 10^{-4}\right)$$

- ORBITAL MOTIÓN OF THE MOON
  (Nordtvedt '68) LUNAR LASER RANGING
  Williams... '04

$$(\Delta r_{\oplus D})_{SYNODIC} = (3 \pm 4)\ \mathrm{mm}\ \cos D$$

- VARYING FREQUENCY SHIFT OF RADIO LINKS: CASSINI SPACECRAFT
  (Bertotti, Iess, Tortora '03)

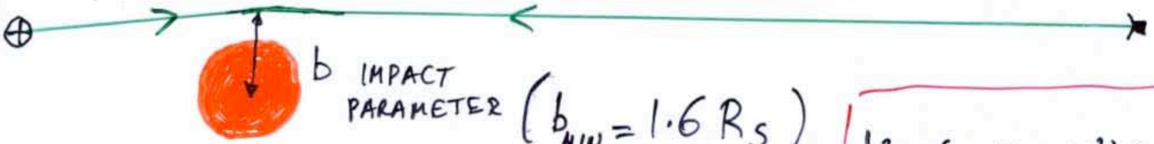
$$\frac{\Delta\nu/\nu}{(\Delta\nu/\nu)^{GR}} = 1 + (1.0 \pm 1.1) \times 10^{-5}$$

QUASI-STATIC, WEAK-FIELD EINSTEIN GRAVITY OK AT

10^{-5} LEVEL

VARYING FREQUENCY SHIFT OF RADIO LINKS WITH THE CASSINI SPACECRAFT

(Bertotti, Iess, Tortora '03)



b IMPACT PARAMETER $\left(b_{MIN} = 1.6\, R_S\right)$

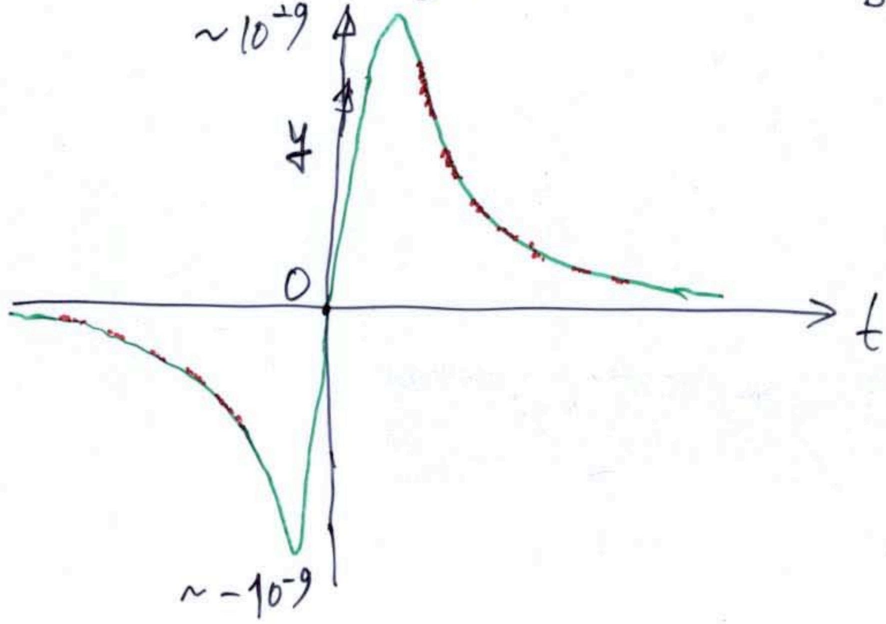
$$\boxed{ds^2 = -\left(1 - \frac{2U}{c^2} + 2\beta\frac{U^2}{c^4}\right)dt^2 + \left(1 + 2\gamma\frac{U}{c^2}\right)d\vec{x}^2}$$

TWO-WAY RELATIVISTIC FREQUENCY SHIFT

$$\gamma_{GR} \equiv 1$$

$$y \equiv \left(\frac{\Delta\nu}{\nu}\right)^{\text{TWO-WAY}}_{\text{GRAV.}} = -4(1+\gamma)\frac{GM_{SUN}}{c^3 b}\frac{db}{dt} = -\,0.2\,\mu\text{sec}\,(1+\gamma)\frac{1}{b}\frac{db}{dt}$$

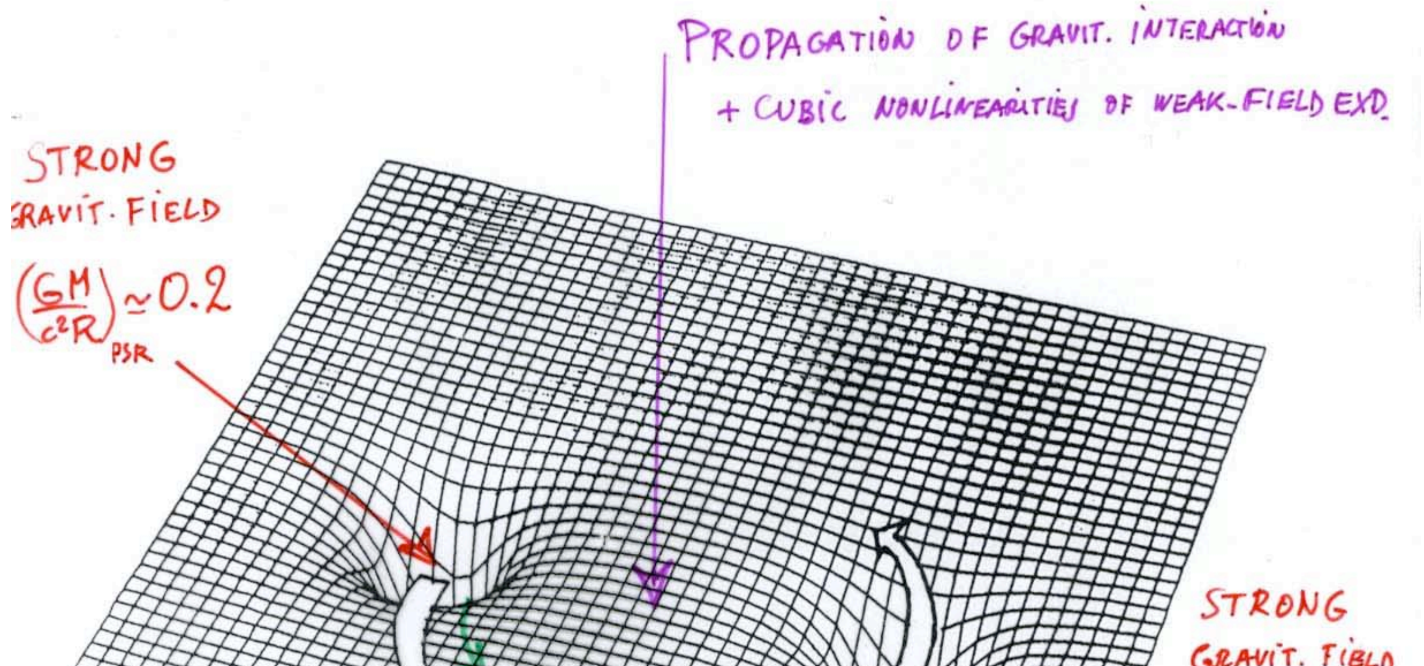
AS EARTH MOVES



$$\boxed{\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}}$$

BINARY PULSARS: FIRST POSSIBILITY OF PROBING THE FULL STRUCTURE OF RELATIVISTIC GRAVITY

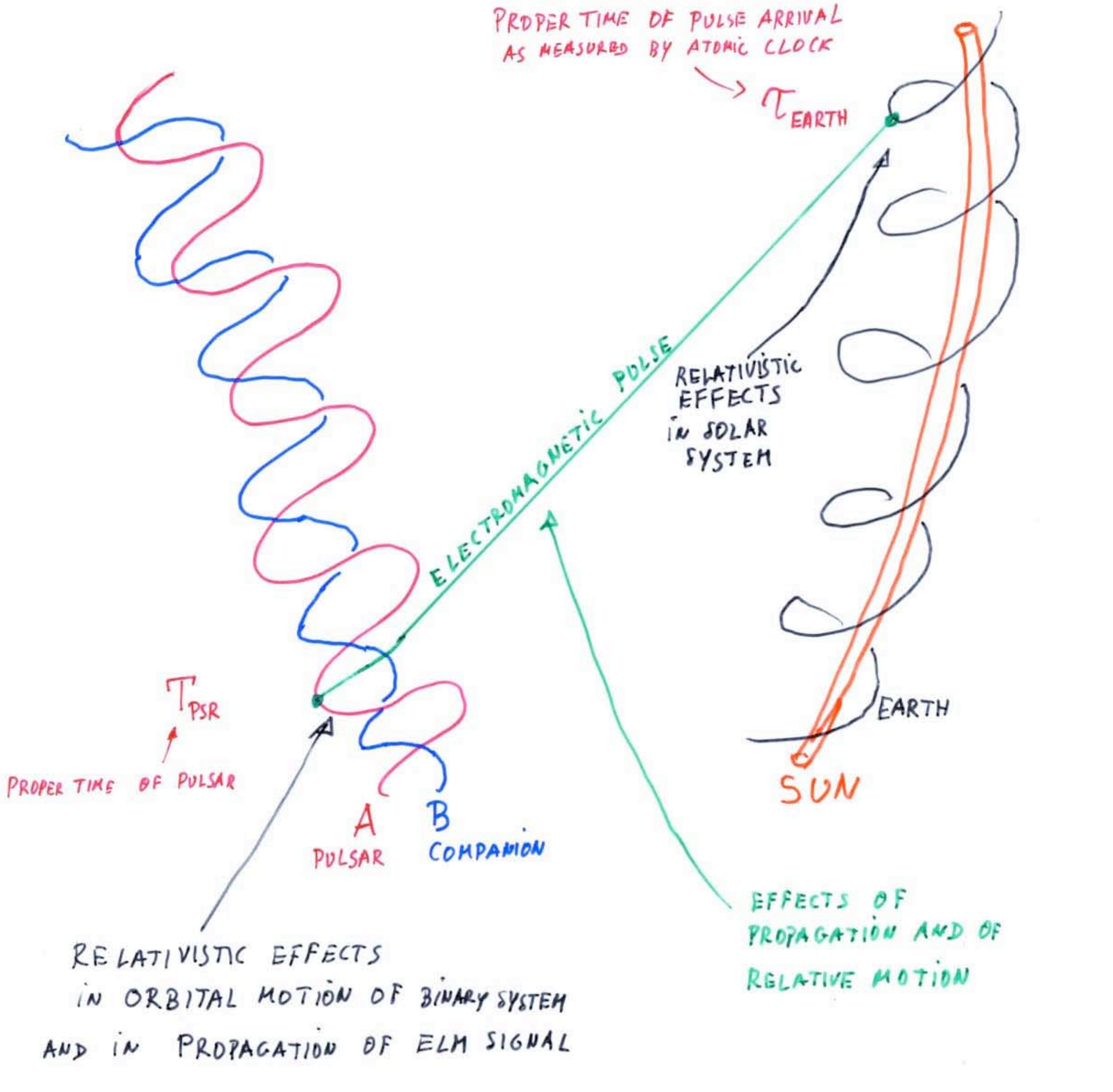
- RADIATIVE EFFECTS [FIELD PROPAGATION]
- HIGHLY NON-LINEAR EFFECTS [SRONG FIELDS]

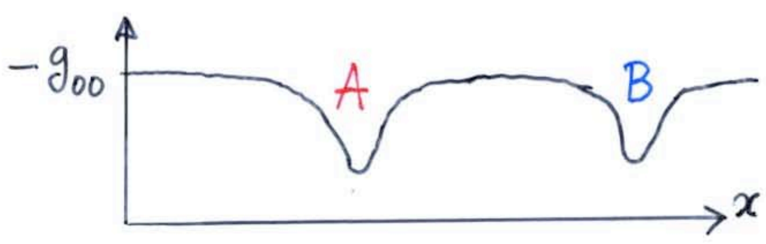


THEORETICAL ASPECTS OF BINARY PULSARS:

1. MOTION OF TWO STRONGLY SELF-GRAVITATING BODIES
   (T.D. & DERUELLE '81, T.D. 82, 83)
2. RELATIVISTIC TIMING OF A BINARY PULSAR
   (BLANDFORD, TEUKOLSKY '76, T.D. & DERUELLE '85, 86)
3. USE OF BINARY PULSARS AS PROBES OF RELATIVISTIC GRAVITY
   (EARDLEY '75, WILL, EARDLEY '77, T.D. '88, T.D. & TAYLOR '92)

RELATIVISTIC TIMING OF A BINARY PULSAR





$(V/c)^5$ EQUATIONS OF MOTION IN GENERAL RELATIVITY

G6
OL16

accelerations. Then each body must satisfy the following equation of motion (Damour and Deruelle, 1981*a*; Damour, 1982):

$$a^i = A_0^i(\vec{z}-\vec{z}') + c^{-2}A_2^i(\vec{z}-\vec{z}', \vec{v}, \vec{v}') + c^{-4}A_4^i(\vec{z}-\vec{z}', \vec{v}, \vec{v}', \vec{S}, \vec{S}')$$
$$+ \underline{c^{-5}A_5^i(\vec{z}-\vec{z}', \vec{v}-\vec{v}')} + O(c^{-6}), \tag{154}$$

with

$$A_0^i = -Gm'R^{-2}N^i, \tag{155}$$

$$A_2^i = Gm'R^{-2}\{N^i[-v^2 - 2v'^2 + 4(vv') + \tfrac{3}{2}(Nv')^2 + 5(Gm/R) + 4(Gm'/R)]$$
$$+ (v^i - v'^i)[4(Nv) - 3(Nv')]\}, \tag{156}$$

"DRESSED MASSES" INCORPORATING STRONG-SELF-FIELD EFFECTS

$$A_4^i = B_4^i + C_4^i + D_4^i, \tag{157}$$

$$B_4^i = Gm'R^{-2}\{N^i[-2v'^4 + 4v'^2(vv') - 2(vv')^2 + \tfrac{3}{2}v^2(Nv')^2 + \tfrac{9}{2}v'^2(Nv')^2$$
$$- 6(vv')(Nv')^2 - \tfrac{15}{8}(Nv')^4 + (Gm/R)(-\tfrac{15}{4}v^2 + \tfrac{5}{4}v'^2 - \tfrac{5}{2}(vv') + \tfrac{39}{2}(Nv)^2$$
$$- 39(Nv)(Nv') + \tfrac{17}{2}(Nv')^2) + (Gm'/R)(4v'^2 - 8(vv') + 2(Nv)^2$$
$$- 4(Nv)(Nv') - 6(Nv')^2)]$$
$$+ (v^i - v'^i)[v^2(Nv') + 4v'^2(Nv) - 5v'^2(Nv') - 4(vv')(Nv)$$
$$+ 4(vv')(Nv') - 6(Nv)(Nv')^2 + \tfrac{9}{2}(Nv')^3$$
$$+ (Gm/R)(-\tfrac{63}{4}(Nv) + \tfrac{55}{4}(Nv')) + (Gm'/R)(-2(Nv) - 2(Nv'))]\}, \tag{158}$$

$$C_4^i = G^3m'R^{-4}N^i[-\tfrac{57}{4}m^2 - 9m'^2 - \tfrac{69}{2}mm'], \tag{159}$$

$$D_4^i = \left(\frac{S^{ik}}{m} + 2\frac{S'^{ik}}{m'}\right)(v^l - v'^l)\left(\frac{Gm'}{R}\right)_{,kl} + \left(2\frac{S^{kl}}{m} + 2\frac{S'^{kl}}{m'}\right)(v^l - v'^l)\left(\frac{Gm'}{R}\right)_{,ik}, \tag{160}$$

and

$$A_5^i = \tfrac{4}{5}G^2mm'R^{-3}\{V^i[-V^2 + 2(Gm/R) - 8(Gm'/R)]$$
$$+ N^i(NV)[3V^2 - 6(Gm/R) + \tfrac{52}{3}(Gm'/R)]\}. \tag{161}$$

GRAVITATIONAL RADIATION DAMPING

The two parameters m and m' appearing in eqs. (154)–(161) are the 'Schwarzschild masses' of the condensed bodies. They are two constants which appear in the external gravitational field, in which are hidden many internal structure effects (see the discussion of the 'effacement of internal structure' in Section 6.14). On the other hand, the spin tensors undergo a slow evolution (on the post-Newtonian time scale, i.e. β_e^{-2} times the orbital period) which is also obtained in the Einstein–Infeld–Hoffmann–Kerr-type approach (Damour, 1982, and references therein). Introducing, *à la* Schiff, a suitable spin-vector, $\vec{S}$, associated with $S_{\mu\nu}$, the law of evolution ('spin precession') reads for the first body (see also references in Section 6.13.2)

$$\frac{d\vec{S}}{dt} = \left[\frac{Gm'}{c^2R^2}\vec{N} \times \left(\frac{3}{2}\vec{v} - 2\vec{v}'\right)\right] \times \vec{S} + O\left(\frac{1}{c^4}\right). \tag{162}$$

DIRECT EFFECT OF PROPAGATION OF GRAVITY AT SPEED C

RELATIVISTIC TIMING FORMULA

Damour and Deruelle [36, 47] proved that it is possible to describe all of the independent $O(v^2/c^2)$ timing effects in a simple mathematical way common to a wide class of alternative theories. This made it possible to revert to a theory-independent analysis of timing data, and led to the possibility of working within a strong-field analog of the PPN formalism, the so-called [37] "parametrized post-Keplerian" approach. The part of the Damour–Deruelle phenomenological timing model describing orbital effects reads

$$\boxed{t_b - t_0 = F[T; \{p^{\mathrm{K}}\}; \{p^{\mathrm{PK}}\}; \{q^{\mathrm{PK}}\}] ,} \tag{2.1a}$$

where t_b denotes the solar-system barycentric (infinite frequency) arrival time, T the pulsar proper time (corrected for aberration, see below),

$$\{p^{\mathrm{K}}\} = \{P_b, T_0, e_0, \omega_0, x_0\} \tag{2.1b}$$

is the set of Keplerian parameters,

$$\boxed{\{p^{\mathrm{PK}}\} = \{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}} \tag{2.1c}$$

the set of *separately measurable* post-Keplerian parameters, and

$$\{q^{\mathrm{PK}}\} = \{\delta_r, A, B, D\} \tag{2.1d}$$

the set of *not separately measurable* post-Keplerian parameters. The right hand side of Eq. (2.1a) is given by

$$F(T) = D^{-1}[T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)] , \tag{2.2a}$$

$$\Delta_R = x \sin\omega[\cos u - e(1+\delta_r)] + x[1 - e^2(1+\delta_\theta)^2]^{1/2} \cos\omega \sin u , \tag{2.2b}$$

$$\Delta_E = \gamma \sin u , \tag{2.2c}$$

$$\Delta_S = -2r \ln\{1 - e\cos u - s[\sin\omega(\cos u - e) + (1-e^2)^{1/2} \cos\omega \sin u]\} , \tag{2.2d}$$

$$\Delta_A = A\{\sin[\omega + A_e(u)] + e\sin\omega\} + B\{\cos[\omega + A_e(u)] + e\cos\omega\} , \tag{2.2e}$$

where

$$x = x_0 + \dot{x}(T - T_0) , \tag{2.3a}$$

$$e = e_0 + \dot{e}(T - T_0) , \tag{2.3b}$$

and where $A_e(u)$ and ω are the following functions of u,

$$A_e(u) = 2 \arctan \left[\left(\frac{1+e}{1-e} \right)^{1/2} \tan\frac{u}{2} \right] , \tag{2.3c}$$

$$\omega = \omega_0 + k\, A_e(u) , \tag{2.3d}$$

and u is the function of T defined by solving the Kepler equation

$$u - e\sin u = 2\pi \left[\left(\frac{T - T_0}{P_b} \right) - \frac{1}{2}\dot{P}_b \left(\frac{T - T_0}{P_b} \right)^2 \right] . \tag{2.3e}$$

TESTING RELATIVISTIC GRAVITY WITH BINARY PULSAR DATA

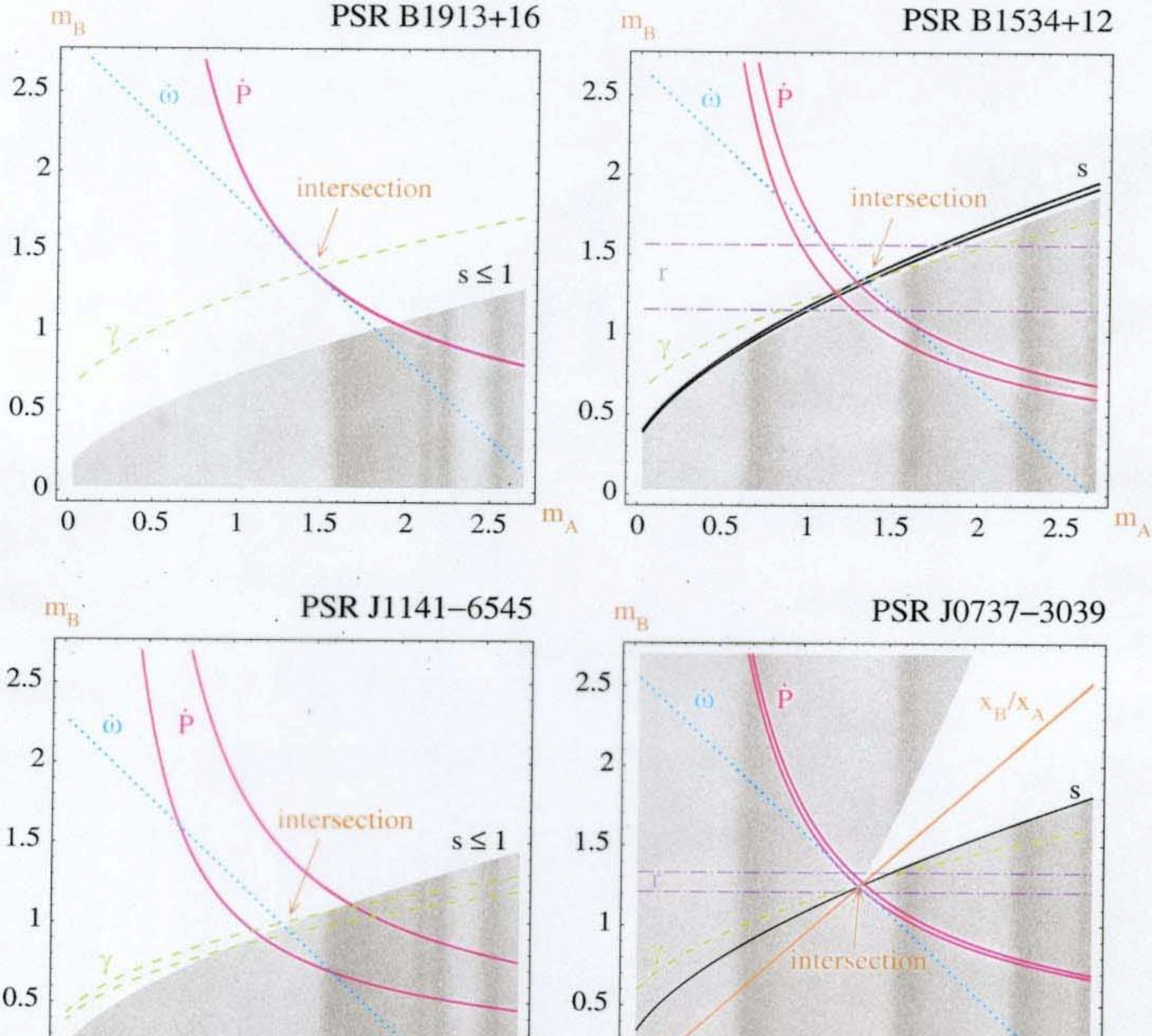
TWO APPROACHES

- "THEORY-INDEPENDENT" OR "PHENOMENOLOGICAL"

  PARAMETRIZED POST-KEPLERIAN

- "THEORY-DEPENDENT"

  - BEYOND USUAL POST-NEWTONIAN PARAMETERS
  - CLASSES OF TENSOR-SCALAR THEORIES


PSR B1913+16
PSR B1534+12
PSR J1141−6545
PSR J0737−3039
m_B
m_A
$\dot{\omega}$
$\dot{P}$
γ
intersection
s ≤ 1
s
r
x_B/x_A
0
0.5
1
1.5
2
2.5

SOME HIGH-PRECISION BINARY-PULSAR TESTS

1913+16:

Weisberg, Taylor '04

Damour Taylor '91 14σ ^{OBS} CORRECTION

$$\frac{\dot{P}_b^{OBS} - \dot{P}_b^{GALACTIC}}{\dot{P}_b^{GR}\left[k^{OBS}, \gamma^{OBS}_{TIMING}\right]} = 1.0013 \pm 0.0021$$

1534+12:

Taylor, Wolszczan, Damour, Weisberg '92
Stairs et al. '02

$$\frac{s^{OBS}}{s^{GR}\left[k^{OBS}, \gamma^{OBS}_{TIMING}\right]} = 1.000 \pm 0.007$$

0737-3039

Lyne et al. '04, Kramer et al '04, '06

$$\frac{s^{OBS}}{s^{GR}\left[k^{OBS}, R^{OBS}\right]} = 0.99987 \pm 0.00050$$

RADIATIVE AND STRONG-FIELD EINSTEIN GRAVITY OK AT

10^{-3} LEVEL

A CLASS OF ALTERNATIVES TO GR:

"MATTER METRIC"

TENSOR-SCALAR GRAVITY $S = \frac{1}{16\pi G_*}\int \sqrt{g}\left[R(g) - 2(\partial\varphi)^2\right] + S_{MATTER}\left[\text{matter}; \tilde{g}_{\mu\nu} = e^{2a(\varphi)} g_{\mu\nu}\right]$

"EINSTEIN METRIC"

SCALAR FIELD φ

e.g. $-\int \tilde{m}_A d\tilde{s} = -\int \tilde{m}_A e^{a(\varphi)} ds$

HERE, ONE HAS KEPT THE 'UNIVERSAL COUPLING' MATTER $\wedge$ GRAVITY

[CAN BE AN APPROXIMATION OF EP-VIOLATING DILATON-LIKE COUPLINGS]

EXAMPLES:

- IN KALUZA-KLEIN, AND JORDAN-FIERZ-BRANS-DICKE ONE NATURALLY GETS

$$\tilde{g}_{\mu\nu}^{matter} = e^{\alpha\varphi} g_{\mu\nu}^{Einstein}$$

- NON-MINIMAL COUPLING $(\Box + \xi R)\varphi = 0$

$$S = \int -(\partial\varphi)^2 + \xi\varphi^2 R + \int \frac{\sqrt{g}R}{16\pi G_0} + S_{matter}[g_{\mu\nu}]$$

NATURALLY LEADS TO

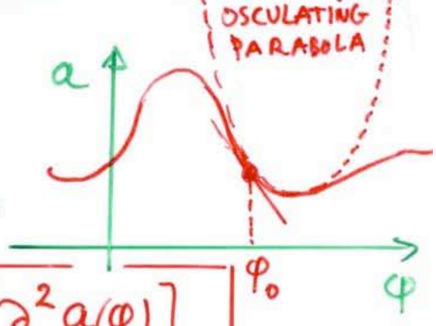
$$g_{\mu\nu}^{matter} = f(\phi)\, g_{\mu\nu}^{Einstein}$$

THE TWO BASIC COUPLING PARAMETERS OF WEAK-FIELD TENSOR-SCALAR GRAVITY

$$m_A(\varphi) = \exp\left[a(\varphi)\right] m_A^{(0)}$$

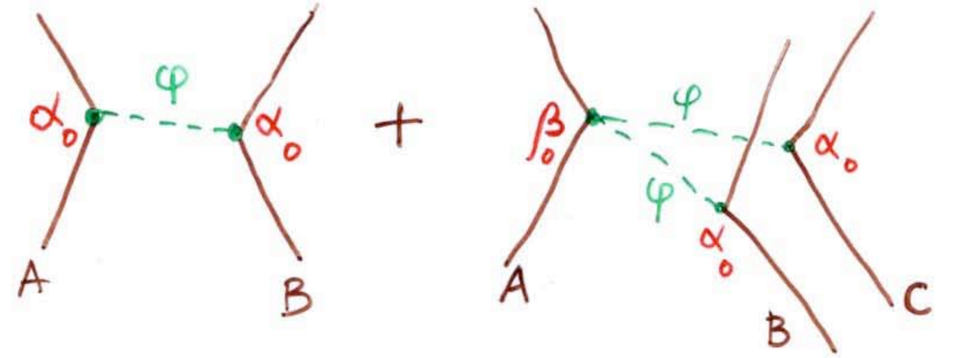
COUPLING FUNCTION

$$\varphi = \varphi_0 + \delta^{local}\varphi$$



$$\boxed{\alpha_0 \equiv \left[\frac{\partial\, a(\varphi)}{\partial \varphi}\right]_{\varphi_0} \qquad \beta_0 \equiv \left[\frac{\partial^2 a(\varphi)}{\partial \varphi^2}\right]_{\varphi_0}}$$

NEW INTERACTIONS:



$$\Rightarrow L^{2\text{-BODY}} = \frac{1}{2}\sum_{A\neq B} G_{AB}\frac{m_A m_B}{r_{AB}}\left[1 + \frac{\text{GReffects}}{c^2} + \bar{\gamma}\frac{(\vec{v}_A - \vec{v}_B)^2}{c^2}\right]$$

$$L^{3\text{-BODY}} = -\frac{1}{2}\sum_{B\neq A\neq C}(1+2\bar{\beta})\,\frac{G_{AB}\,G_{AC}\,m_A m_B m_C}{r_{AB}\, r_{AC}\, c^2}$$

MORE GENERALLY

$$\bar{\gamma} = \frac{\langle \text{spin}^2\rangle - 4}{2}$$

LIGHT-DEFLECTION EXPTS MEASURE DIRECTLY THE BASIC α_0

OBSERVABLE "POST-EINSTEIN" PARAMETERS

$$\boxed{\begin{aligned} \bar{\gamma} &\equiv \gamma_{PPN} - 1 = -2\frac{\alpha_0^2}{1+\alpha_0^2} \\ \bar{\beta} &\equiv \beta_{PPN} - 1 = +\frac{1}{2}\,\beta_0\frac{\alpha_0^2}{(1+\alpha_0^2)^2} \end{aligned}}$$

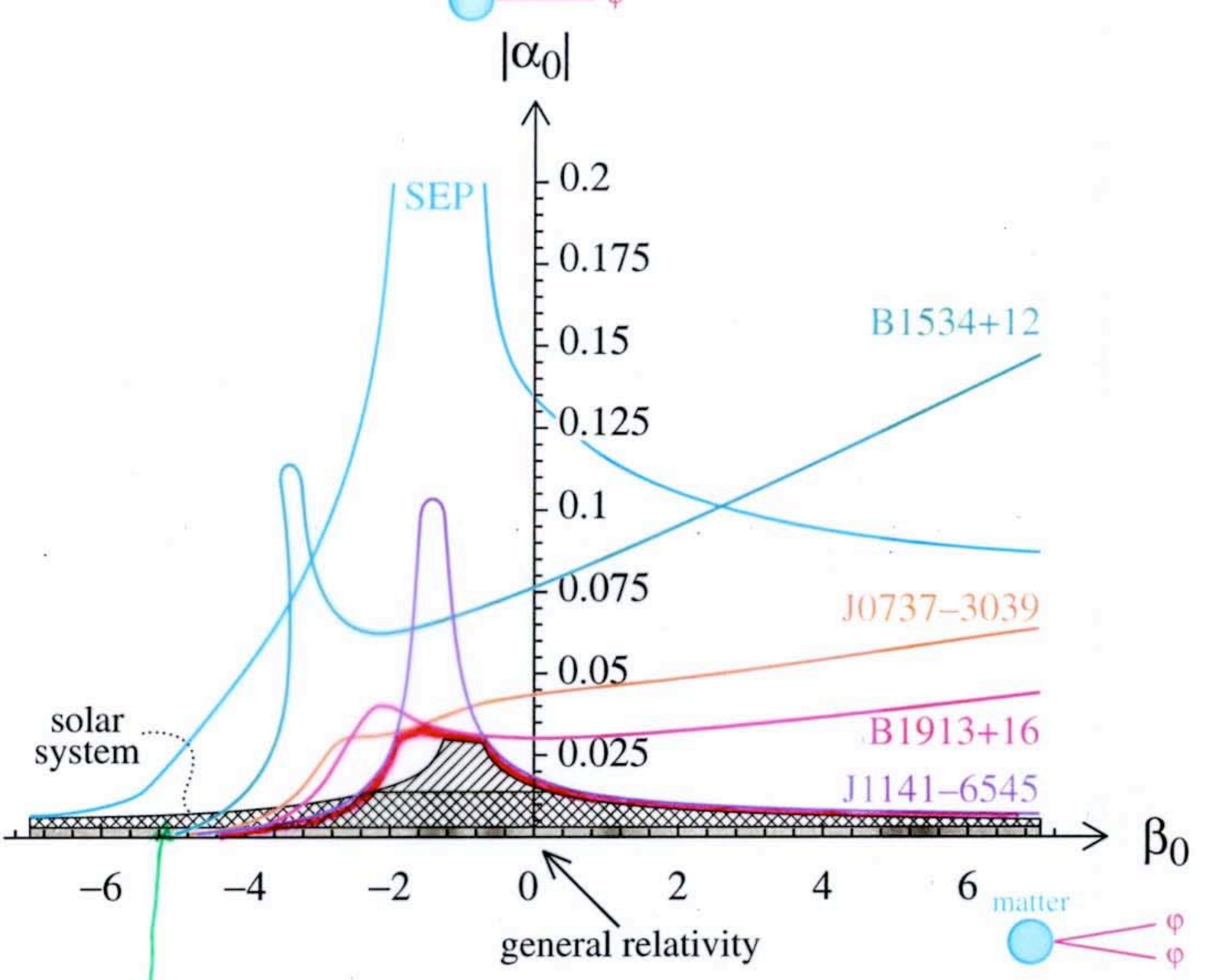
PPN PARAMETERS OF EDDINGTON-NORDTVEDT-WILL

$$G_{AB} = G\, e^{2a(\varphi_0)}\left[1+\alpha_0^2\right]\left[1 + (4\bar{\beta}-\bar{\gamma})\left(\frac{E_A^{grav}}{m_A c^2} + \frac{E_B^{grav}}{m_B c^2}\right)\right]$$

THEORY-SPACE ANALYSIS OF PULSAR AND SOLAR-SYSTEM DATA

$$T(\alpha_0, \beta_0): \quad S = \frac{1}{16\pi G_*}\int \sqrt{g}\left[R(g) - 2(\partial\varphi)^2\right] + S_{MATTER}\left[\text{matter}; \tilde{g}_{\mu\nu} = e^{2a(\varphi)} g_{\mu\nu}\right]$$

$$a(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2$$

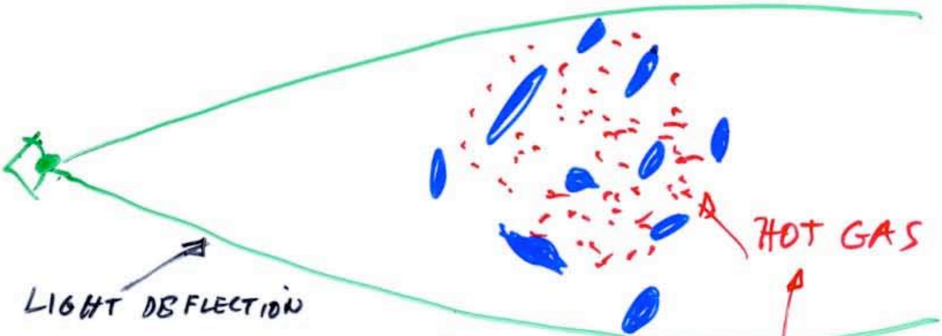


DOMAIN OF THEORY-SPACE
ALLOWED BY SOLAR-SYSTEM EXPTS
BUT EXCLUDED BY BINARY PSR ONES

(Damour Esposito-Farèse '06)

TESTS OF GRAVITY ON VERY LARGE SCALES

GRAVITATIONAL LENSING BY GALAXY CLUSTERS



LIGHT DEFLECTION IN $ds^2 \simeq -(1-2U)dt^2 + (1+2U)d\vec{x}^2$

HOT GAS distribution depends on $U(x)$

→ INDEPENDENTLY OF 'DARK MATTER ISSUE' ONE CHECKS CONSISTENCY OF GR ON LENGTH SCALES ~ 100 kpc

PRIMORDIAL NUCLEOSYNTHESIS OF LIGHT ELEMENTS (He, Li, D)

CHECK THAT

$$\frac{G_{BBN}}{G^{NOW}} \simeq 1 + O(10\%)$$

ALSO: TESTS OF GRAVITY ON SMALL SCALES

SEVERAL EXPERIMENTS HAVE RECENTLY PROBED NEWTON'S $1/r^2$ LAW ON SMALL DISTANCES.

NO DEVIATION HAS BEEN FOUND DOWN TO ~ 0.2 mm