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GRAVITATION AND COSMOLOGY

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1. CLASSICAL BLACK HOLES AS DISSIPATIVE BRANES
2. EXPERIMENTAL TESTS OF RELATIVISTIC GRAVITY
3. GRAVITATIONAL WAVES FROM COSMIC (SUPER)STRINGS
4. GRAVITATIONAL WAVES FROM COALESCING BLACK HOLES
5. CHAOS AND SYMMETRY IN 'STRING COSMOLOGY'

1. CLASSICAL BLACK HOLES AS DISSIPATIVE BRANES

- AIM: DERIVE THE VALUE OF THE (SURFACE) SHEAR VISCOSITY OF BLACK HOLES:

$$\boxed{\eta_{BH} = \frac{1}{16\pi G}} \quad (\text{WITH} \quad c=1)$$

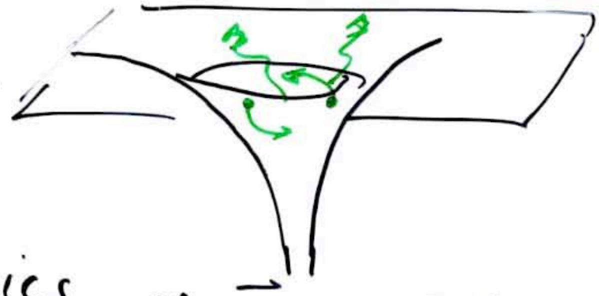
$$\Rightarrow \quad \frac{\eta_{BH}}{s_{BH}} = \frac{1/(16\pi G)}{1/(4\hbar G)} = \frac{\hbar}{4\pi} \quad \leftarrow \text{ OF RECENT INTEREST IN CONNECTION WITH AdS/CFT}$$

(Kovtun, Son, Starinets)

ENTROPY DENSITY $= \frac{1}{4}\frac{1}{\hbar G}$

- EVOLVING VIEWS OF BLACK HOLES

PASSIVE GRAVITATIONAL WELLS



PHYSICAL OBJECTS: GLOBAL DYNAMICS: $M, \vec{J}, Q, \delta M, M_{irr} \ldots$

LOCAL DYNAMICS OF HORIZON: $\sim$ 'MEMBRANE' WITH DISSIPATIVE PROPERTIES

RESISTIVITY = 377 OHM = 4π

SHEAR VISCOSITY $= \frac{1}{16\pi G}$

QUANTUM OBJECTS: QUANTUM INSTABILITIES, PAIR CREATION

MICROSCOPIC ORIGIN OF BH ENTROPY

SPHERICALLY SYMMETRIC BLACK HOLES $(D=3+1)$

$$ds^2 = -A(r)\,dT^2 + B(r)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$A(r) = \frac{1}{B(r)} = 1 - \frac{2GM}{r} + G\frac{Q^2}{r^2} \qquad (c=1)$$

Reissner-Nordstrøm

TOTAL MASS M TOTAL ELECTRIC CHARGE

SOLUTION OF $\quad R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu{}^\sigma F_{\nu\sigma} - \text{trace}\right)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$:

$$-A_0 = +V = +\frac{Q}{r}$$

$$G=1 \qquad A(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{r^2 - 2Mr + Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2}$$

(OUTER) HORIZON $\quad \boxed{r_+ = M + \sqrt{M^2 - Q^2}} \qquad r_- = M - \sqrt{M^2 - Q^2}$

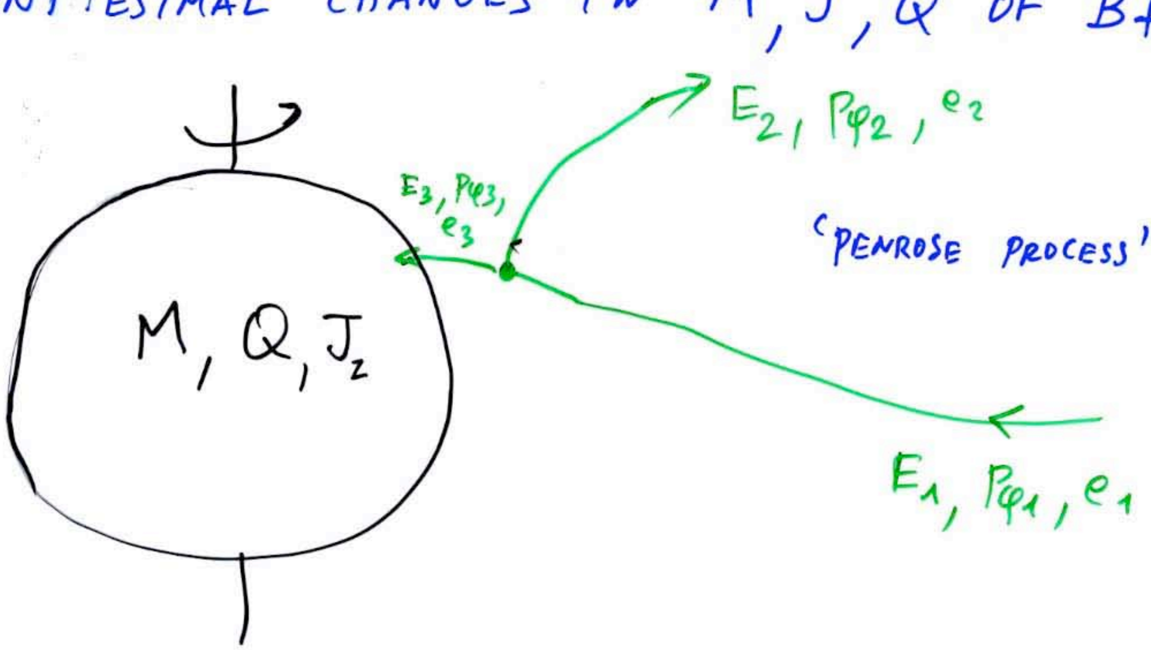
TOTAL ANGULAR MOMENTUM

Kerr - Newman BH: $\quad M, J, Q$

$$r_+ = M + \sqrt{M^2 - Q^2 - a^2}$$

$$a \equiv J/M$$

INFINITESIMAL CHANGES IN M, J, Q OF BH



TEST-PARTICLE OF REST-MASS μ IN BH BACKGROUND

GEODESIC DYNAMICS : $S = -\int \mu \, ds$

HAMILTON-JACOBI EQ FOR $p_\mu = \dfrac{\partial S}{\partial x^\mu}$:

$$g^{\mu\nu}(p_\mu - e A_\mu)(p_\nu - e A_\nu) = -\mu^2$$

e: ELECTRIC CHARGE OF TEST PARTICLE

SYMMETRIES OF BH Background $\rightarrow$ CONSERVED QUANTITIES

CONSERVED ENERGY (OF TEST PARTICLE) : $E = -P_T = -P_0$

CONSERVED Z-COMPONENT OF ANGULAR MOMENTUM : P_φ

+ CONSERVED ELECTRIC CHARGE : e

PENROSE PROCESS:

IN and OUT at ∞ — AT and AFTER SPLITTING

CHANGE IN TOTAL ENERGY OF SPACE-TIME

$$\delta M = E_1 - E_2 = E_3$$

$$\delta J = p_{\varphi 1} - p_{\varphi 2} = p_{\varphi 3}$$

$$\delta Q = e_1 - e_2 = e_3$$

SOLVE THE MASS-SHELL CONDITION

$$A_0 = -\frac{Q}{r}$$

$$-\mu^2 = g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = -\frac{(p_0 - eA_0(r))^2}{A(r)} + A(r)\, p_r^2 + \frac{1}{r^2}\left(p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}\right)$$

$$p^r \equiv g^{rr} p_r = A(r)\, p_r$$

CONSERVED $L^2 = p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}$

$$\boxed{-p_0 = E = \frac{eQ}{r} + \sqrt{(p^r)^2 + A(r)\left(\mu^2 + \frac{L^2}{r^2}\right)}}$$

CHRISTODOULOU, CHRISTODOULOU-RUFFINI '71

INFINITESIMAL VARIATION OF BH MASS

APPLY TO E_3 ON THE HORIZON

$E = E_3 = \delta M$; $e = e_3 = \delta Q$

$$\boxed{\delta M = \frac{Q\,\delta Q}{r_+(M,Q)} + |p^r|_{r_+} \geqslant \frac{Q\,\delta Q}{r_+(M,Q)}}$$

INEQUALITY ⟹ IRREVERSIBILITY IN BH PHYSICS

CHRISTODOULOU-RUFFINI

WHEN ADDING ANGULAR MOMENTUM:

$$\delta M - \frac{a\,\delta J + r_+ Q\,\delta Q}{r_+^2 + a^2} = \frac{r_+^2 + a^2\cos^2\theta}{r_+^2 + a^2}\,|p^r| \geq 0$$

→ CHRISTODOULOU-RUFFINI MASS FORMULA

$$\boxed{M^2 = \left(M_{IRR} + \frac{Q^2}{4M_{IRR}}\right)^2 + \frac{J^2}{4M_{IRR}^2}}$$

$$\boxed{\delta M_{IRR} \geq 0}$$

M_{IRR} = INTEGRATION CONSTANT ALONG 'REVERSIBLE' TRANSFORMATIONS:

$$\delta M = \frac{a\,\delta J + r_+ Q\,\delta Q}{r_+^2 + a^2}$$

→ CLASSICALLY EXTRACTABLE ENERGY OF A BH : $M - M_{IRR}$

- UP TO 29% M IN ROTATIONAL ENERGY
- UP TO 50% M IN COULOMB EN.

GIVEN INEQUALITY

$$a^2 + Q^2 \leq M^2$$

GEOMETRICAL MEANING OF M_{IRR} : $\boxed{A \equiv \underset{\text{HORIZON}}{\text{AREA}} = 16\pi M_{IRR}^2}$

GENERAL RESULT OF HAWKING '71

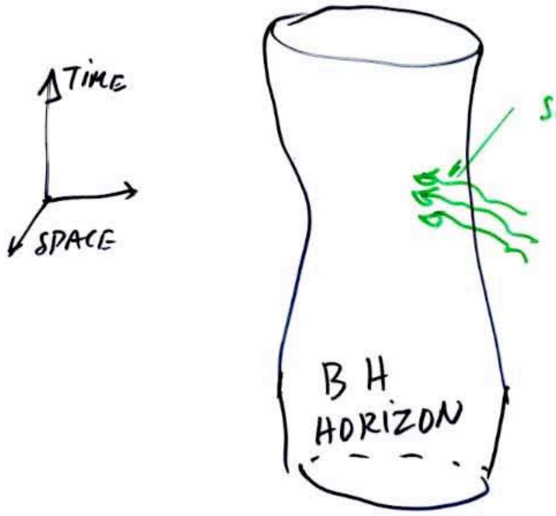
$$\boxed{\delta A \geq 0}$$

VARIOUS FACETS OF BH IRREVERSIBILITY:

CLASSICALLY $\delta A \nearrow$ CAN BE INTERPRETED AS : BH ~ DISSIPATIVE MEMBRANE

QUANTUM MECHANICALLY BEKENSTEIN SUGGESTED $A \propto$ BH ENTROPY

LET US STUDY MORE IN DETAIL WHAT HAPPENS WHEN ONE IS 'THROWING STUFF IN A BLACK HOLE':



$\longrightarrow$ DERIVE AN EQUATION FOR THE DYNAMICS OF THE BH 'SURFACE' WHICH IS CLOSELY ANALOGOUS TO THE NAVIER-STOKES EQ. OF A VISCOUS FLUID (Damour '79)

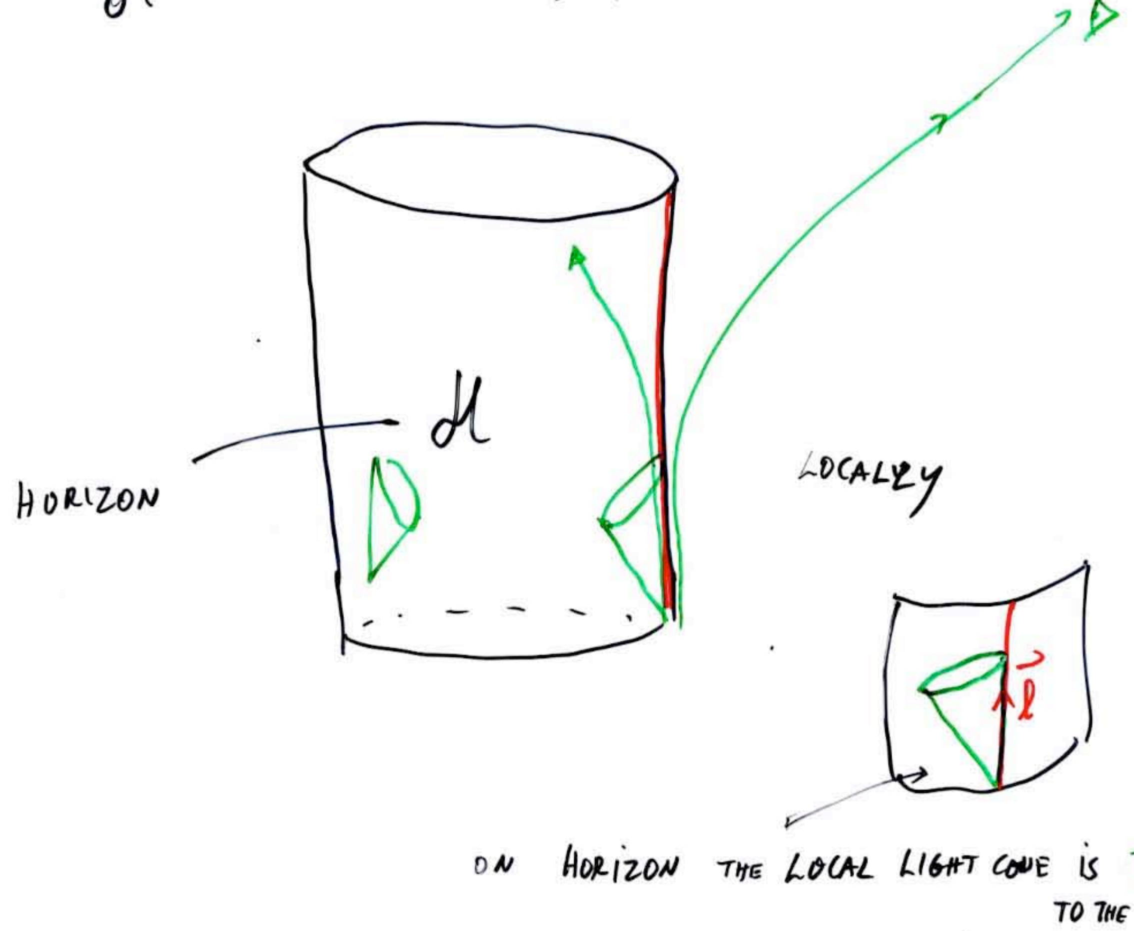
(1) DEFINITION OF BH HORIZON (OR BH 'SURFACE')

IN A GENERAL TIME-DEPENDENT SITUATION

PENROSE, HAWKING, HAWKING-ELLIS

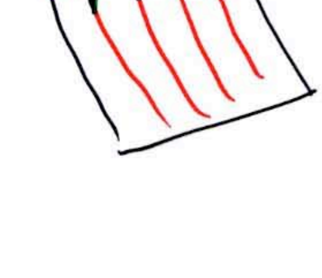
BH HORIZON = BOUNDARY BETWEEN LIGHT SIGNALS THAT ESCAPE TO ∞ AND LIGHT SIGNALS THAT DO NOT ESCAPE

$\mathcal{H}$



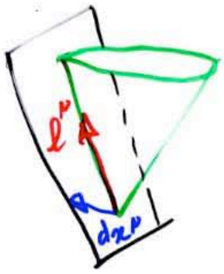
ON HORIZON THE LOCAL LIGHT CONE IS TANGENT TO THE HORIZON

$\rightarrow$ HORIZON = NULL HYPERSURFACE



$\vec{\ell}$ = PARTICULAR LIGHT-LIKE DIRECTION ALONG WHICH $\mathcal{H}$ IS TANGENT TO LIGHTCONE

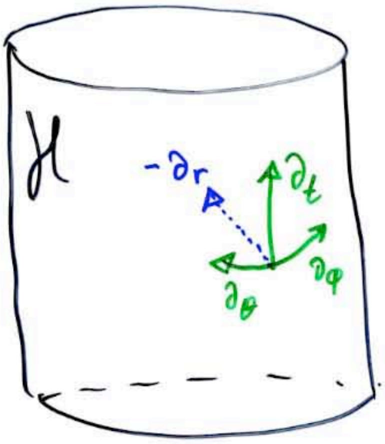
NB $\vec{l} = l^\mu \frac{\partial}{\partial x^\mu}$ IS BOTH TANGENT AND NORMAL TO $\mathcal{H}$



$$l_\mu l^\mu = 0 \qquad l_\mu \underset{\uparrow \text{ WITHIN } \mathcal{H}}{dx^\mu} = 0$$

NULL

USE AN ADAPTED COORDINATE SYSTEM WHICH IS REGULAR ON $\mathcal{H}$



i.e. SIMILAR TO USUAL 'INGOING EDDINGTON-FINKELSTEIN COORDINATES' OF SCHWARZSCHILD (OR REISSNER-NORDSTRØM)

SINGULAR ON $\mathcal{H}$: $A(r_+) = 0$

$$ds^2 = -A(r)\,dT^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

INTRODUCE 'TORTOISE' r_*

$$= -A(r)\left[dT^2 - \left(\frac{dr}{A(r)}\right)^2\right] + r^2 d\Omega^2$$

$$\boxed{r_* \equiv \int \frac{dr}{A(r)}}$$

$$= -A(r)\,(dT - dr_*)(dT + dr_*) + r^2 d\Omega^2$$

AND THE NEW TIME

$$\boxed{t = T + r_*}$$

$$= -A(r)\,(dt - 2dr_*)\,dt + r^2 d\Omega^2$$

$$\boxed{ds^2 = -A(r)\,dt^2 + 2\,dt\,dr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)}$$

$A(r) = \frac{(r-r_+)(r-r_-)}{r^2}$ VANISHES ON $\mathcal{H}$ BUT ds^2 REMAINS REGULAR IN COORDS (t, r, θ, φ)

$\mathcal{H}$: $r =$ const: $\frac{\partial}{\partial r}$ IS TRANSVERSE, WHILE $\frac{\partial}{\partial t}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \varphi}$ ARE TANGENT TO $\mathcal{H}$

FOR ARBITRARY, TIME-DEPENDENT HORIZON LET US CHOOSE COORDINATES S.T.

- $\mathcal{H}$ IS LOCATED AT $x^1 = 0$ (i.e. $x^1 \sim r - r_+$)

- THE 3-d HYPERSURFACE $\mathcal{H}$ IS PARAMETRIZED BY x^0, x^2, x^3 ($\sim t, \theta, \varphi$)

$\partial_1 \equiv \frac{\partial}{\partial x^1}$ IS TRANSVERSE TO $\mathcal{H}$, WHILE $\partial_0 = \frac{\partial}{\partial x^0}, \partial_2, \partial_3$ ARE TANGENT

- NORMALIZE $\vec{l} = l^\mu \partial_\mu$ S.T. $\boxed{\vec{l} = \partial_0 + v^A \partial_A}$ ON $\mathcal{H}$, $A = 2,3$

EXPLOIT $g_{\mu\nu} l^\mu l^\nu = \vec{l} \cdot \vec{l} = 0$ (ON $\mathcal{H}$)

$\rightarrow$ FOR ANY TANGENT VECTOR $\vec{t}$ TO $\mathcal{H}$ $0 = \nabla_{\vec{t}} (\vec{l} \cdot \vec{l}) = 2 \vec{l} \cdot \underbrace{\nabla_{\vec{t}} \vec{l}}$

NORMAL TO $\mathcal{H}$ MUST BE TANGENT TO $\mathcal{H}$

ONE DEFINES THE FOLLOWING SPECIAL QUANTITIES

$\vec{e}_A \equiv \partial_A \equiv \frac{\partial}{\partial x^A}$

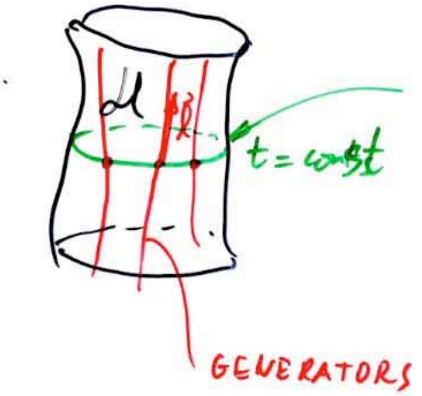
$$\boxed{\begin{aligned} \nabla_{\vec{l}} \vec{l} &= g \, \vec{l} \\ \nabla_{\vec{e}_A} \vec{l} &= -8\pi G \pi_A \, \vec{l} + D_A^{\ B} \vec{e}_B \end{aligned}}$$

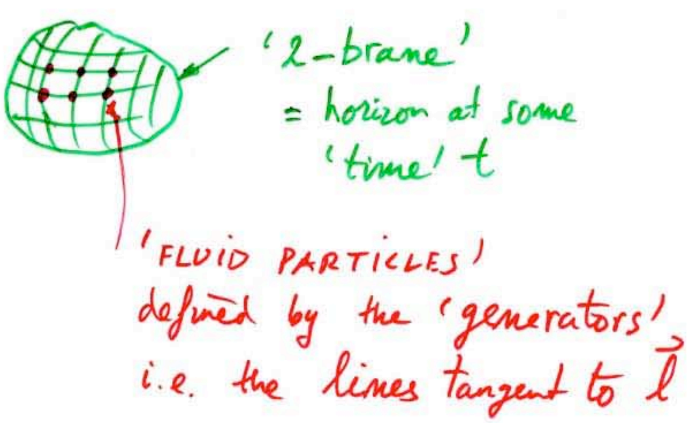
g is called SURFACE GRAVITY $\quad g^{\text{Schwarzschild}} = \dfrac{GM}{r_+^2}$

$\mathcal{D}_A{}^B$ = DEFORMATION TENSOR OF the horizon geometry:

$$ds^2|_{\mathcal{H}} = \gamma_{AB}(t, x^A)(dx^A - v^A dt)(dx^B - v^B dt)$$

METRIC TENSOR OF A $t = \text{const}$ SLICE OF $\mathcal{H}$





'CONVECTIVE DERIVATIVE' OF γ_{AB}:

$$\mathcal{D}_{AB} \equiv \frac{1}{2}\frac{D}{dt}\gamma_{AB} = \frac{1}{2}\mathcal{L}_{\vec{\ell}}\gamma_{AB} = \frac{1}{2}\left(\partial_t \gamma_{AB} + v^C \partial_C \gamma_{AB} + \partial_A v^C \gamma_{CB} + \partial_B v^C \gamma_{AC}\right)$$

$$\mathcal{D}_{AB} = \gamma_{BC}\,\mathcal{D}_A^C \qquad = \frac{1}{2}\left(\partial_t \gamma_{AB} + v_{A|B} + v_{B|A}\right)$$

AS IN USUAL FLUID: RATE OF DEFORMATION OF FLUID ELEMENTS

AS USUAL, DECOMPOSE

$$\boxed{\mathcal{D}_{AB} = \sigma_{AB} + \theta\,\gamma_{AB}}$$

SHEAR TENSOR: trace-free part

EXPANSION RATE

$$\theta = \mathcal{D}_A^A = \frac{1}{2}\gamma^{AB}\partial_t\gamma_{AB} + v^A{}_{|A}$$

? MEANING OF π_A AND EXPLANATION OF COEFFICIENT $\chi = 8\pi G$

TOTAL ANGULAR MOMENTUM OF ANY AXISYMMETRIC SPACETIME:

KILLING VECTOR $m^\mu \partial_\mu = \partial/\partial\varphi$

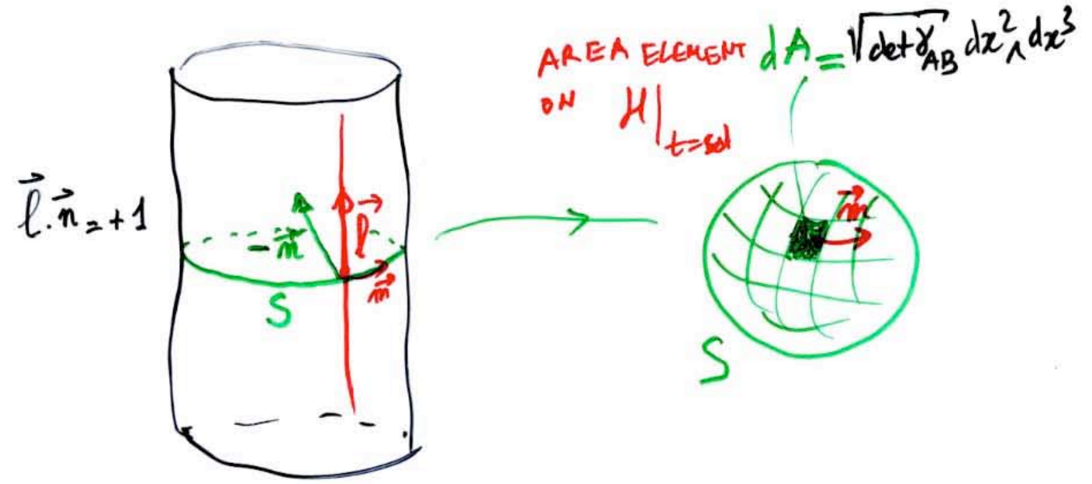
$$J_\infty = -\frac{1}{\chi}\oint_{S_\infty} \frac{1}{2}\nabla^\nu m^\mu d^2S_{\mu\nu} = -\frac{1}{\chi}\oint_{\mathcal{H}} \frac{1}{2}\nabla^\nu m^\mu d^2S_{\mu\nu} + \int m^\mu T^\nu{}_\mu d^3\Sigma_\nu$$

DEFINES THE ANG. MOMENTUM OF B.H.

ANG. MOMENTUM OF MATTER, AFTER USING $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi T_{\mu\nu}$

USING $d^2S^{\mathcal{H}}_{\mu\nu} = (n_\mu \ell_\nu - n_\nu \ell_\mu)\, dA$ AND $\ell^\nu \nabla_\nu m^\mu = m^\nu \nabla_\nu \ell^\mu$

$$\boxed{J_{BH} = -\frac{1}{8\pi G}\oint_{\mathcal{H}} n_\mu m^\nu \nabla_\nu \ell^\mu\, dA = \oint_S m^A \pi_A\, dA = \oint_S \pi_\varphi\, dA}$$



$\rightarrow$ $\boxed{\pi_A = \text{'SURFACE DENSITY OF LINEAR MOMENTUM OF B.H.'}}$

NAVIER-STOKES-LIKE EQUATION OF BH (Damour '79)

BY PROJECTING EINSTEIN'S EQS $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ ALONG $\ell^\mu e_A^\nu$ (USING THE FACT THAT (g, π_A, D_A^B) GENERALIZES THE 2^{ND} FUNDAMENTAL FORM (Weingarten map), AND USING A NULL generalization of the Gauss-Codazzi eqs) ONE FINDS

$$\boxed{\frac{D}{dt}\pi_A = -\frac{\partial}{\partial x^A}\left(\frac{g}{8\pi G}\right) + \frac{1}{8\pi G}\sigma_{A|B}^{B} - \frac{1}{16\pi G}\partial_A \theta - \ell^\mu T_{\mu A}}$$

CONVECTIVE ∂, i.e. Lie derivative

$\frac{D\pi_A}{dt} = (\partial_t + \theta)\pi_A + v^B \pi_{A|B} + v^B_{|A}\pi_B$

SHEAR RATE

EXPANSION RATE $\theta = \frac{\partial_t \sqrt{\gamma}}{\sqrt{\gamma}} + v^A_{|A}$

RATE OF FLUX OF MOMENTUM THROUGH THE HORIZON

NEARLY IDENTICAL TO USUAL, NON-RELATIVISTIC NAVIER-STOKES EQ. FOR A (2-d) VISCOUS FLUID

$$\boxed{\frac{D'}{dt}\pi_i = -\frac{\partial}{\partial x^i} p + 2\eta\, \sigma_{i,k}^{k} + \zeta\, \partial_i \theta + f_i}$$

Newtonian convective ∂

$(\partial_t + \theta)\pi_i + v^k \pi_{i,k}$

PRESSURE

SHEAR VISCOSITY

BULK VISCOSITY

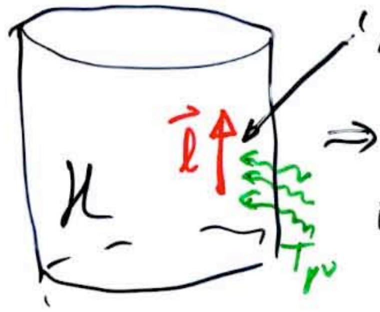
FORCE DENSITY

$$\Rightarrow \quad \boxed{p_{BH} = \frac{g}{8\pi G}} \quad \boxed{\eta_{BH} = \frac{1}{16\pi G}} \quad \boxed{\zeta_{BH} = -\frac{1}{16\pi G}}$$

gives $\frac{\eta_{BH}}{s_{BH}} = \frac{(16\pi G)^{-1}}{(4\hbar G)^{-1}} = \frac{\hbar}{4\pi}$

Bekenstein-Hawking

IRREVERSIBLE THERMODYNAMICS OF BH's



'NORMAL' THROUGH HORIZON IS $\vec{l}$

$\Rightarrow$ FLUX OF ENERGY-MOMENTUM THROUGH $\mathcal{H}$

is $l^\mu T_{\mu\nu}$

NAVIER-STOKES FOLLOWS FROM $\nu = A$: "SPATIAL AND TANGENT TO $\mathcal{H}$"

ANOTHER USEFUL RESULT FOLLOWS FROM "$\nu = 0$" 'TIME AND TANGENT'

NAMELY $l^\mu l^\nu T_{\mu\nu}$ ('Raychauduri eq.')

FOLLOWING BEKENSTEIN - HAWKING ATTRIBUTE AN ENTROPY $s = \hat{\alpha}\, dA$ TO EACH BH SURFACE ELEMENT

$$\boxed{\frac{d}{dt} s - \tau \frac{d^2}{dt^2} s = \frac{1}{T_{BH}}\Big[2\eta_{BH}\sigma_{AB}\sigma^{AB} + \zeta_{BH}\,\theta^2 + \rho_{BH}\underbrace{(\vec{K} - \sigma_H \vec{v})^2}_{\text{"JOULE'S LAW"}}\Big] dA}$$

$\tau = \frac{1}{g}$

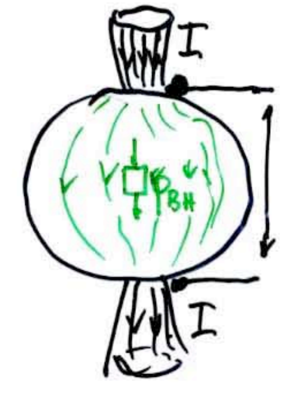
$T_{BH} = \frac{g}{8\pi G \hat{\alpha}}$

ρ_{BH}: BH SURFACE RESISTIVITY $4\pi = 377$ OHM

& ALSO, THERE $\exists$ BH OHM'S LAW (Damour, Znajek '78)

$$\boxed{\vec{E} + \vec{v} \times \vec{B} = \rho_{BH}(\vec{K} - \sigma_H \vec{v})}$$

THIS RESULT CONFIRMS THE CONSISTENCY OF INTERPRETING A BH AS A VISCOUS MEMBRANE, ENDOWED WITH ELECTRICAL RESISTIVITY ρ_{BH}

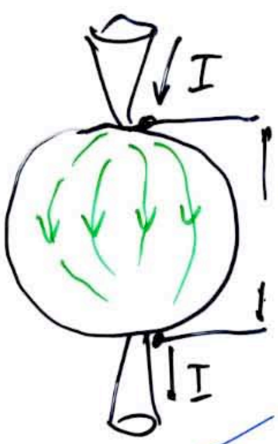


$V = R_{BH}\, I$

WITH R_{BH} COMPUTED FROM ρ_{BH}

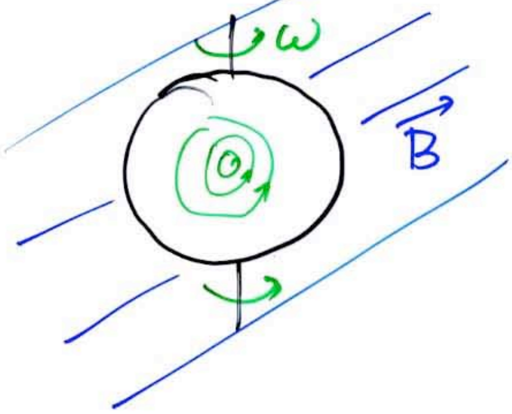
EXAMPLES OF DISSIPATIVE PROCESSES

CONSEQUENCES OF $\rho_H = 4\pi = 377\Omega \neq 0$



$V = R I$ AND $\frac{dQ}{dt} = T\frac{dS}{dt} = R I^2$

$R \sim 30\,\Omega$

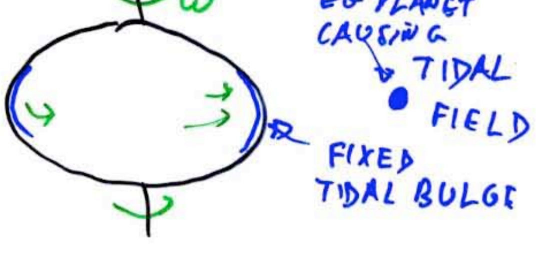


$\rightarrow$ EDDY CURRENTS $\rightarrow$ DISSIPATION AND TORQUE ALIGNING $\vec{\omega}$ TOWARD $\vec{B}$

CONSEQUENCES OF $\eta_{BH} = \frac{1}{16\pi G} \neq 0$

B.H. EQUILIBRIUM STATES : $\sigma_{AB} = \theta = 0$, $\partial_t = 0 \rightarrow$ UNIFORM 'PRESSURE' $\Rightarrow g$ UNIFORM ON $\mathcal{H}$

TIDAL BULGE $\rightarrow$ DISSIPATION (Hartle)



$\rightarrow$ DISSIPATION $\rightarrow$ $\frac{d\omega}{dt} < 0$

+ VALIDITY OF 'MINIMUM ENTROPY PRODUCTION PRINCIPLE'

à la Prigogine

BEKENSTEIN's VIEW OF WHY $T_{BH} = \frac{g}{8\pi G \hat{\alpha}}$ SURFACE GRAVITY

RECALL CHRISTODOULOU-RUFFINI (SPHERICAL SYMM.)

$$\delta M = \frac{Q\delta Q}{r_+(M,Q)} + |g^{rr} p_r|_{r_+}$$

REVERSIBLE (WORK) IRREVERSIBLE (HEAT)

$$\delta E = \delta W + T\delta S$$

TO REACH REVERSIBILITY ONE WOULD NEED

TO FIX BOTH $r = r_+$ AND $p_r = 0$ SIMULTANEOUSLY

$\not\,$ CONTRARY TO HEISENBERG's $\delta r\, \delta p_r \geqslant \frac{1}{2}\hbar$

$$\Rightarrow (r - r_+)\, p_r \geqslant \frac{1}{2}\hbar$$

USING $g^{rr} = A(r) = \frac{(r-r_+)(r-r_-)}{r^2} \simeq \left(\frac{\partial A}{\partial r}\right)_{r_+}(r-r_+)$

$$g^{rr} p_r = A(r)\, p_r \simeq \left(\frac{\partial A}{\partial r}\right)_{r_+} \underbrace{(r-r_+)\, p_r}_{\text{Heisenberg}} \geqslant \frac{1}{2}\hbar \left(\frac{\partial A}{\partial r}\right)_{r_+}$$

$$\Rightarrow \quad T\delta S \geqslant \frac{1}{2}\hbar \left(\frac{\partial A}{\partial r}\right)_{r_+}$$

WHEN ABSORBING ONE PARTICLE

ONE CHECKS $$\boxed{\left.\frac{\partial A(r)}{\partial r}\right|_{r_+} = 2g}$$

EG FOR SCHWARZSCHILD $A(r) = 1 - 2\frac{GM}{r}$

$$\Rightarrow \frac{\partial A}{\partial r} = 2\frac{GM}{r^2}$$

SO THAT

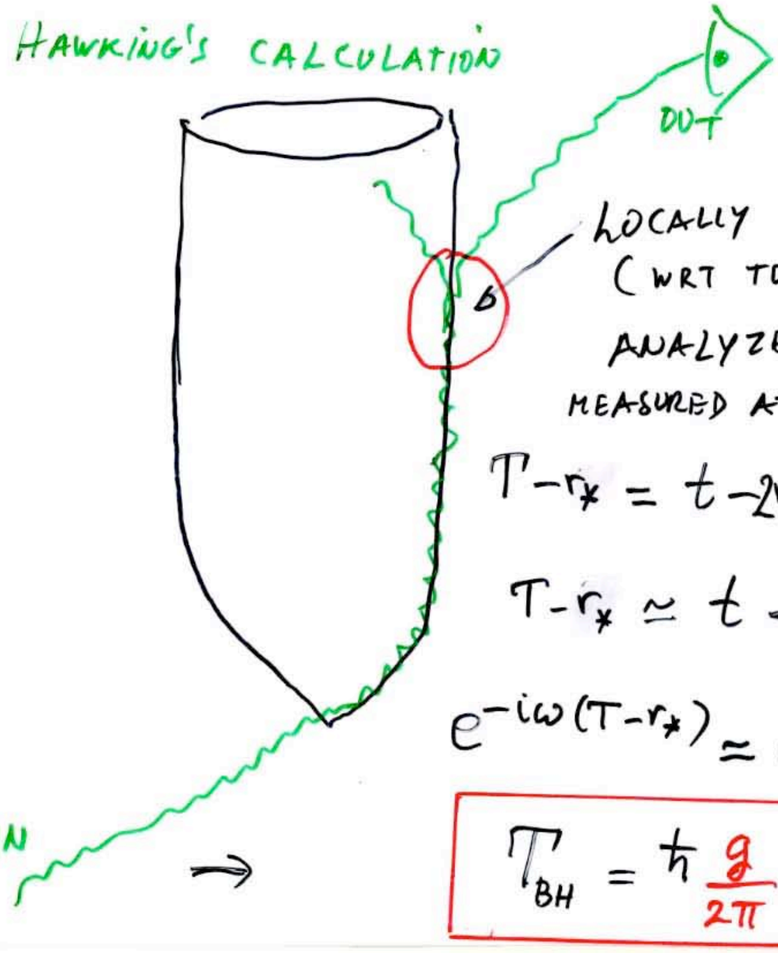
$$T\,\delta S \geq \hbar g$$

UPON ABSORBING ONE PARTICLE

BEKENSTEIN ARGUED $\delta S \gtrsim 1$ ('ONE BIT OF INFORMATION LOST')

$$\Rightarrow \quad T_{BH} \sim \hbar g$$

HAWKING'S CALCULATION



LOCALLY NEGATIVE-FREQUENCY MODE (WRT TO LOCALLY REGULAR COORDS t, r, θ, φ) ANALYZED WRT GLOBAL TIME MEASURED AT ∞,

$A(r) \simeq 2g(r - r_+)$

$$T - r_* = t - 2r_* = t - 2\int \frac{dr}{A(r)}$$

$$T - r_* \simeq t - \frac{1}{g}\ln(r - r_+)$$

$$e^{-i\omega(T - r_*)} \simeq e^{-i\omega t} \exp\left(i\frac{\omega}{g}\ln(r - r_+)\right)$$

$$\rightarrow \quad \boxed{T_{BH} = \hbar\frac{g}{2\pi}}$$