Event Generator Physics

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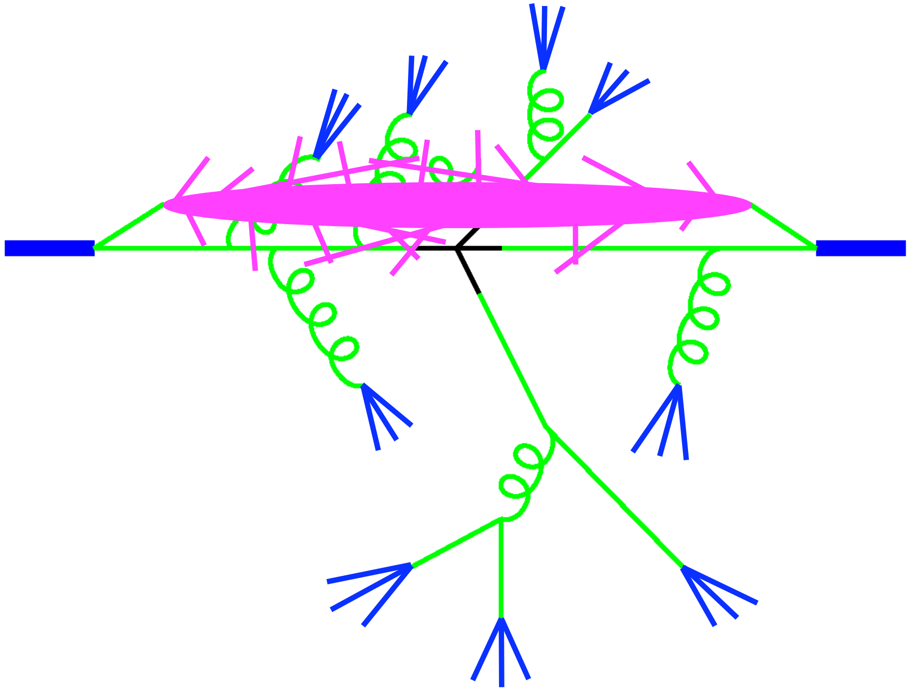
Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event



Lecture 2: Parton Showers

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.

gluons also charged.

→ cascade of partons.

= parton shower.

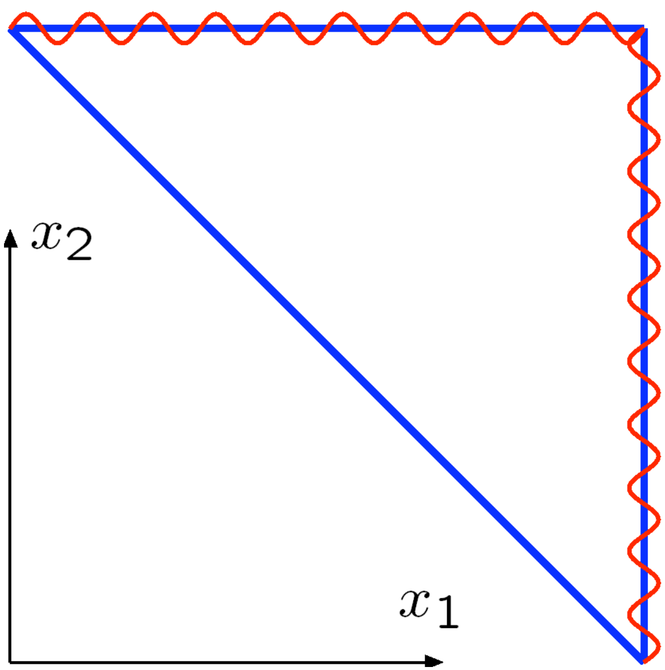
1. e^+e^-annihilation to jets.
2. Universality of collinear emission.
3. Sudakov form factors.
4. Universality of soft emission.
5. Angular ordering.
6. Initial-state radiation.
7. Hard scattering.
8. Heavy quarks.
9. The Colour Dipole Model.

e^+e^- annihilation to jets

Three-jet cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0\, C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

singular as $x_{1,2} \to 1$



Rewrite in terms of quark-gluon opening angle θ and gluon energy fraction x_3 :

$$\frac{d\sigma}{d\cos\theta\, dx_3} = \sigma_0\, C_F \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right\}$$

Singular as $\sin\theta \to 0$ and $x_3 \to 0$.

can separate into two independent jets:

$$\begin{aligned}
\frac{2\, d\cos\theta}{\sin^2\theta} &= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta} \\
&= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar{\theta}}{1-\cos\bar{\theta}} \\
&\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}
\end{aligned}$$

jets evolve independently

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

Exactly same form for anything $\propto \theta^2$

eg transverse momentum: $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$

invariant mass: $q^2 = z(1-z)\,\theta^2\,E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}$$

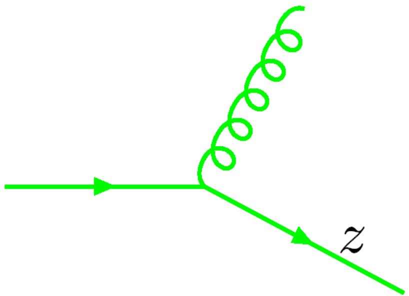
Collinear Limit

Universal:

$$d\sigma = \sigma_0 \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz\, P(z, \phi) d\phi$$

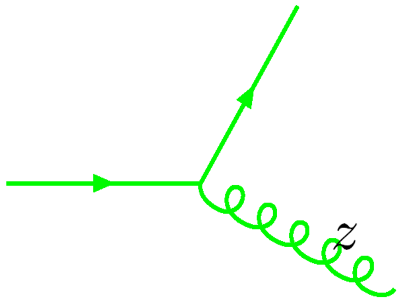
$$P(z, \phi) =$$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour and spin



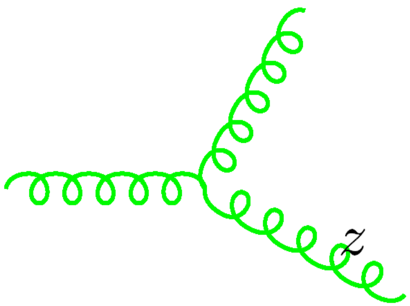
$q \to qg$

$$C_F \frac{1+z^2}{1-z}$$



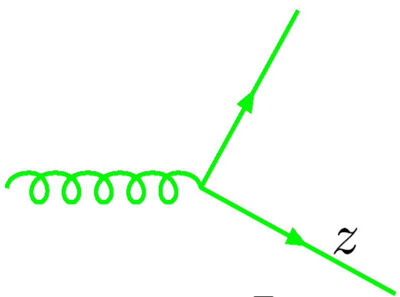
$q \to gq$

$$C_F \frac{1+(1-z)^2}{z}$$



$g \to gg$

$$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$$



$g \to q\bar{q}$

$$T_R \left(z^2 + (1-z)^2\right)$$

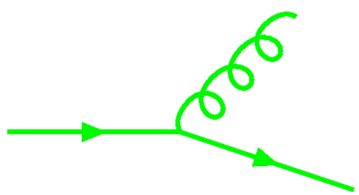
Resolvable partons

What is a parton?

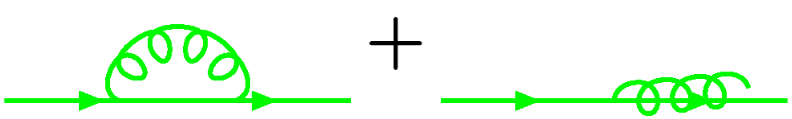
Collinear parton pair $\longleftrightarrow$ single parton

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable real emission



Resolvable emission: Finite



Virtual + Unresolvable emission: Finite

Unitarity: P(resolved) + P(unresolved) = 1

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Gives evolution equation

$$-\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2)\frac{d\mathcal{P}}{dq^2}$$

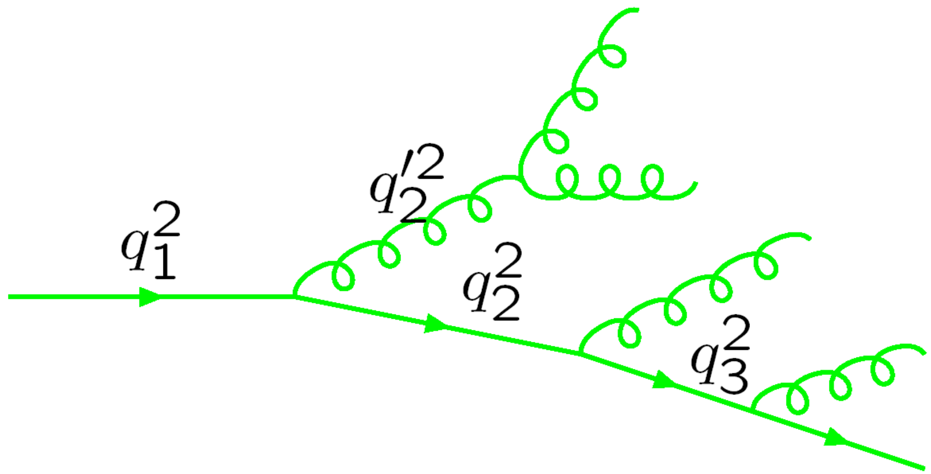
$$\Rightarrow \Delta(Q^2, q^2) = \exp - \int_{q^2}^{Q^2} \frac{dk^2}{k^2}\bar{P}(k^2).$$

$\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor factor

=Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp - C_F\frac{\alpha_s}{2\pi}\log^2\frac{Q^2}{Q_0^2}$$

Multiple emission



$$q_1^2 > q_2^2 > q_3^2 > \ldots$$
$$q_1^2 > q_2'^2 \ldots$$

But initial condition? $q_1^2 < ???$

Process dependent

Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2, q^2)$$

By choosing $0 < \rho < 1$ uniformly:

If $\rho < \Delta(Q^2)$ no resolvable radiation, evolution stops.

Otherwise, solve $\rho = \Delta(Q^2, q^2)$

for q^2 = emission scale

Considerable freedom:

Evolution scale: $q^2/k_\perp^2/\theta^2$?

z: Energy? Light-cone momentum?

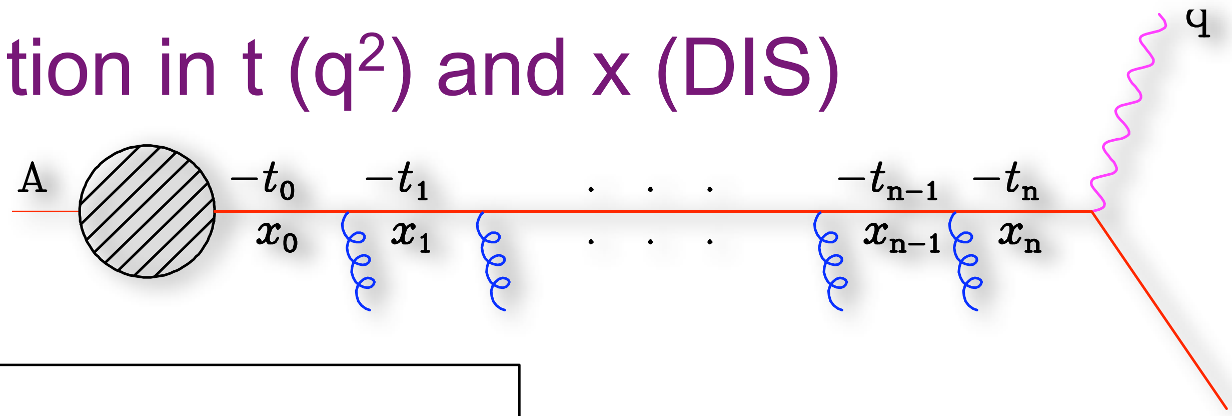
Massless partons become massive. How?

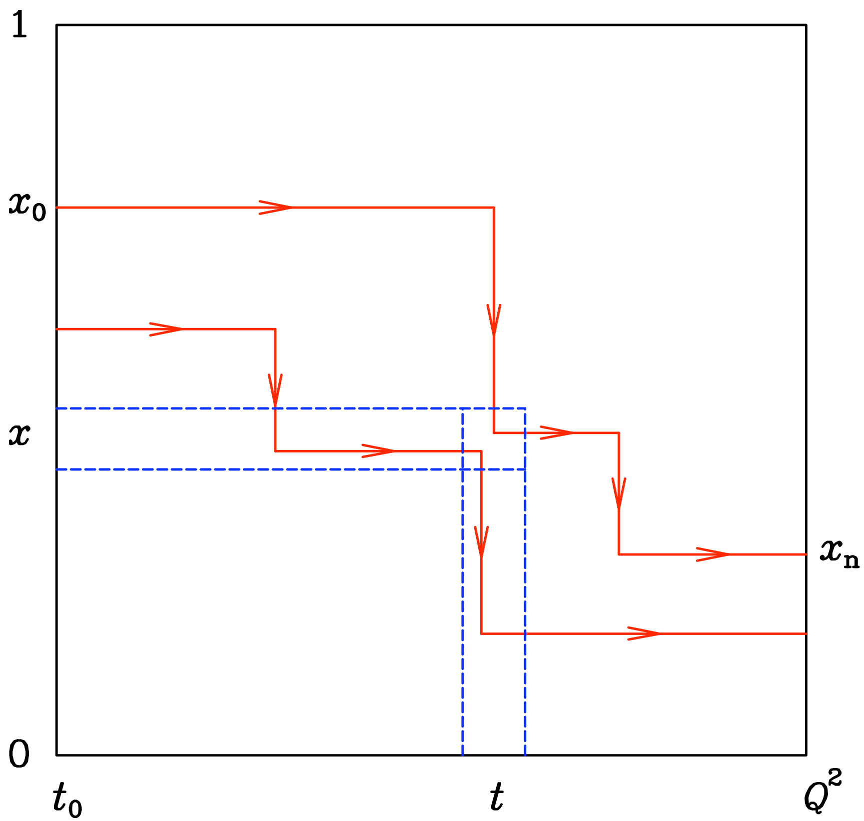
Upper limit for q^2?

} Equivalent at this stage, but can be very important numerically

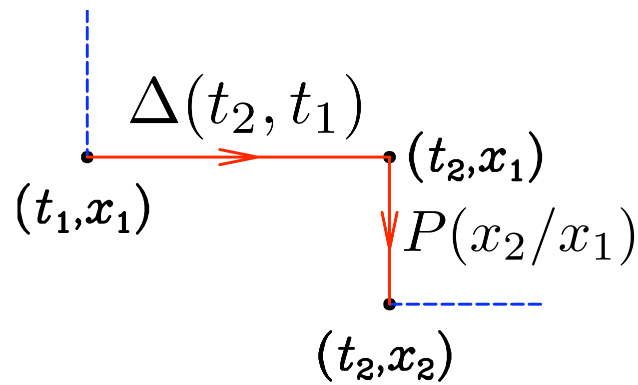
Parton Shower

- Evolution in t (q^2) and x (DIS)





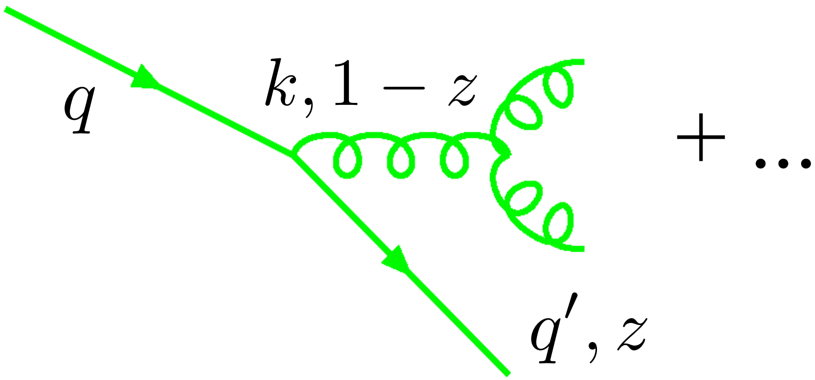
Basic 2-step:



e^+e^-: same formula, opposite direction!

Running coupling

Effect of summing up higher orders:



Scale is set by maximum virtuality of emitted gluon

$$k^2_{\rm max} = (1-z)q^2$$

Similarly in $g \to gg'$, scale is set by

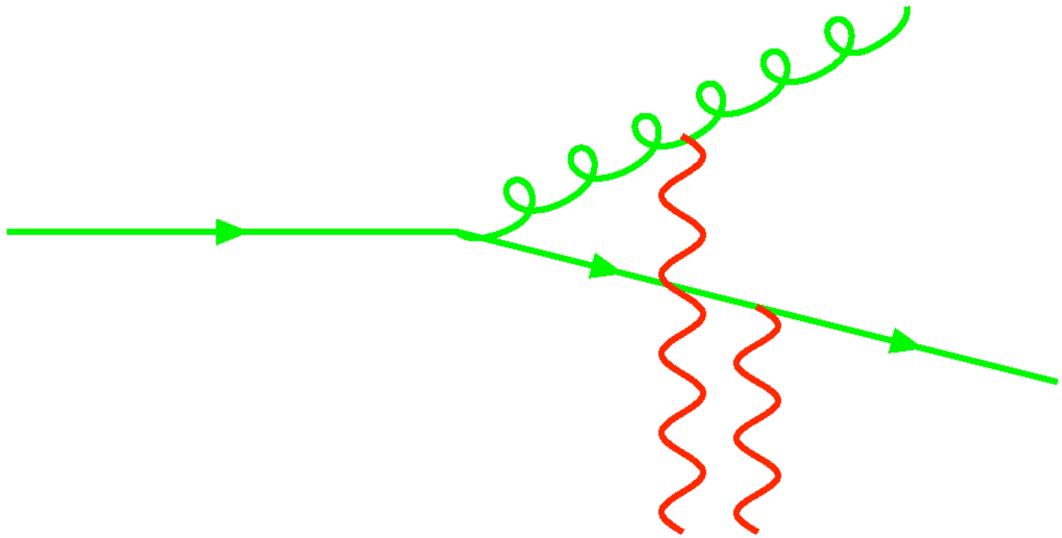
$$\min\{k^2_{\rm max}, k'^2_{\rm max}\} = \min\{z,(1-z)\}q^2 \simeq z(1-z)q^2 \equiv k_T^2$$

Scale change absorbed by replacing $\alpha_{\rm S}(q^2)$ by $\alpha_{\rm S}(k_T^2)$

➡ Faster parton multiplication

Soft limit

Also universal. But at amplitude level…



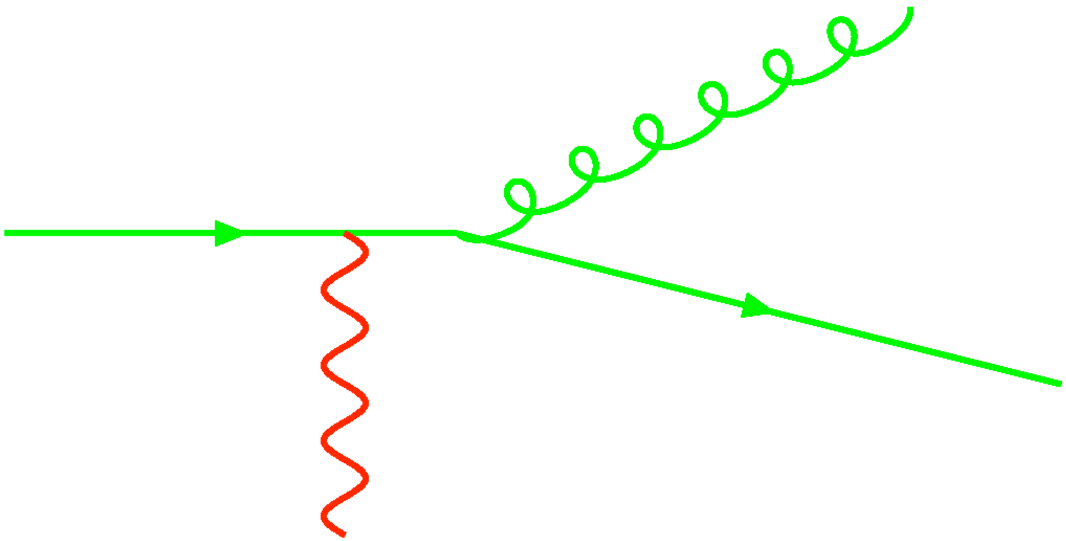
soft gluon comes from everywhere in event.

→ Quantum interference.

Spoils independent evolution picture?

Angular Ordering

NO:



outside angular ordered cones, soft gluons sum coherently: only see colour charge of whole jet.

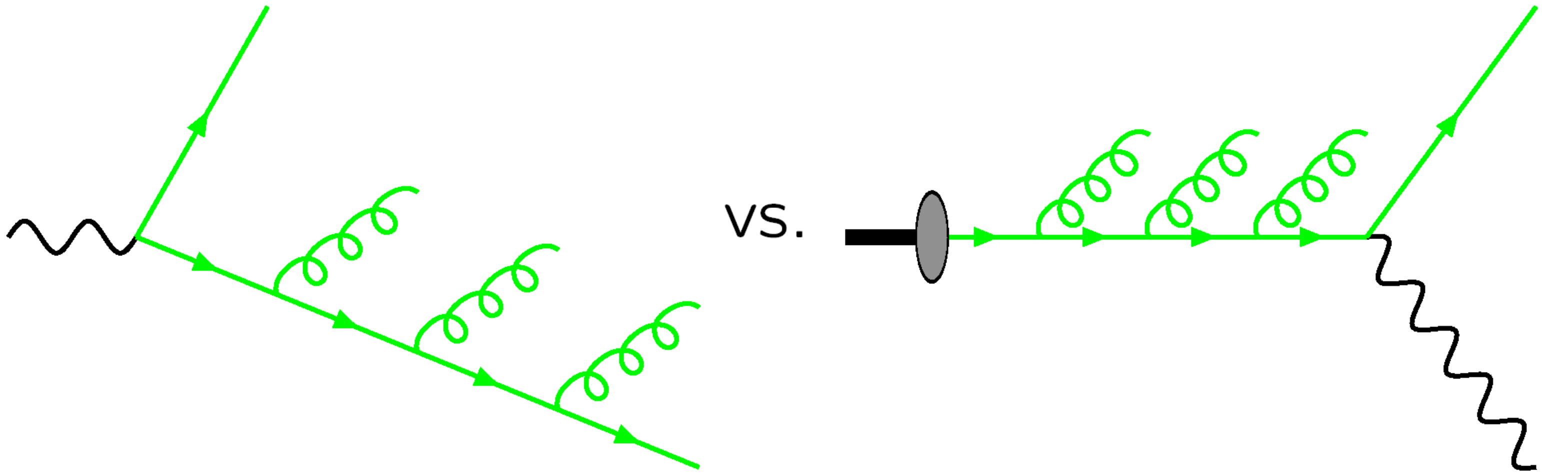
Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering

First gluon not necessarily hardest!

Initial state radiation

In principle identical to final state (for not too small x)

In practice different because both ends of evolution fixed:



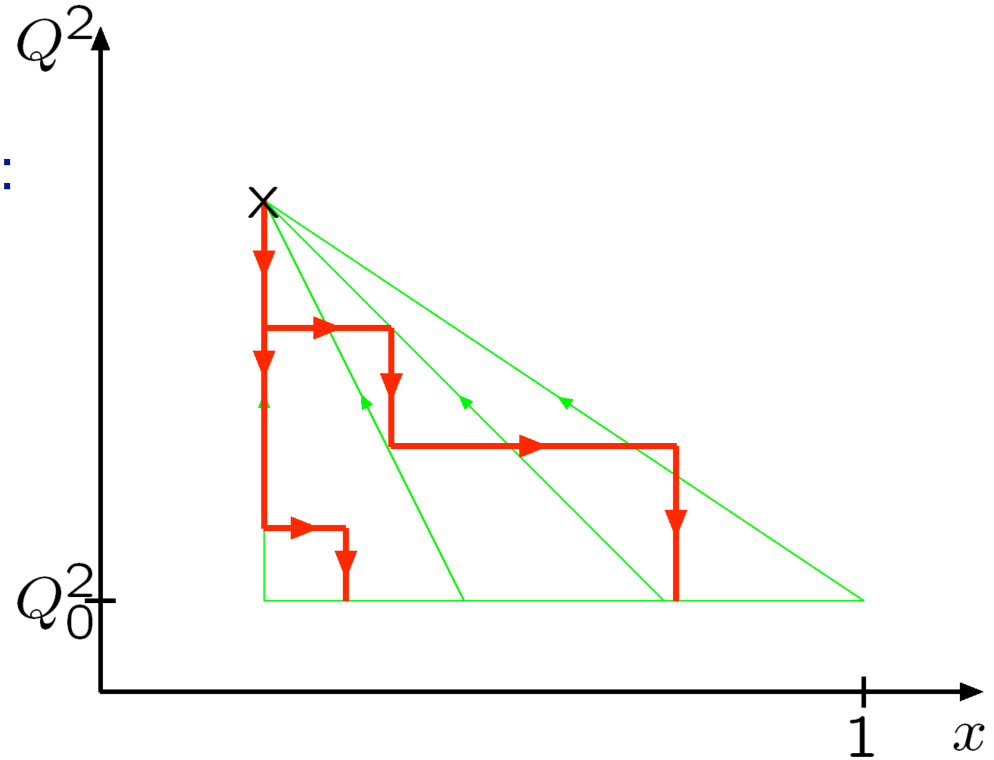
Use approach based on evolution equations…

Backward Evolution

DGLAP evolution: pdfs at (x, Q^2) as function of pdfs at $(> x, Q_0^2)$:

Evolution paths sum over all possible events.

Formulate as backward evolution: start from hard scattering and work down in q^2, up in x towards incoming hadron.



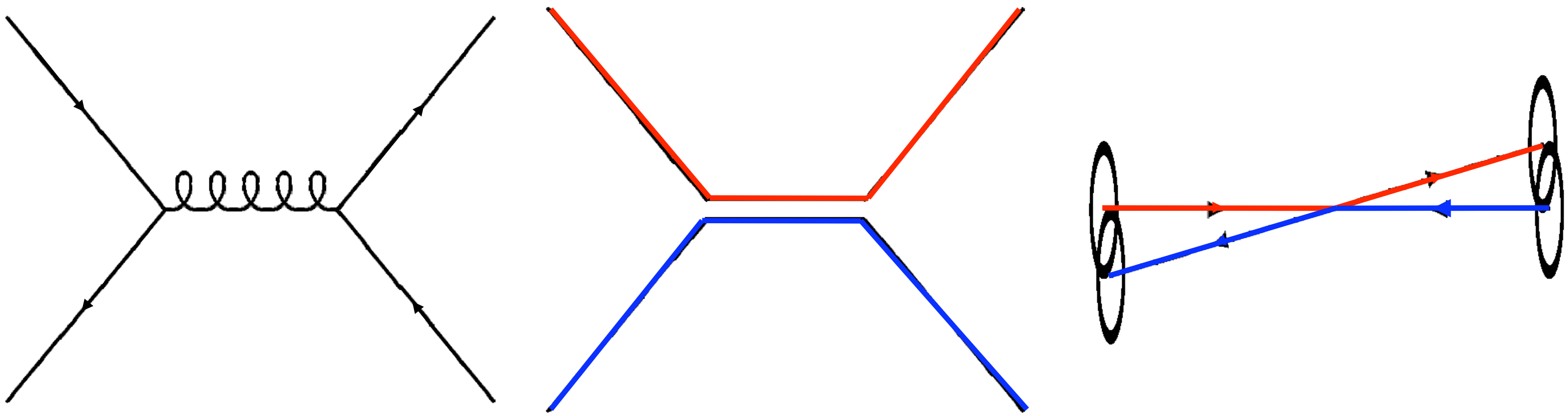
Algorithm identical to final state with $\Delta_i(Q^2, q^2)$ replaced by

$$\Delta_i(Q^2, q^2)/f_i(x, q^2).$$

Hard Scattering

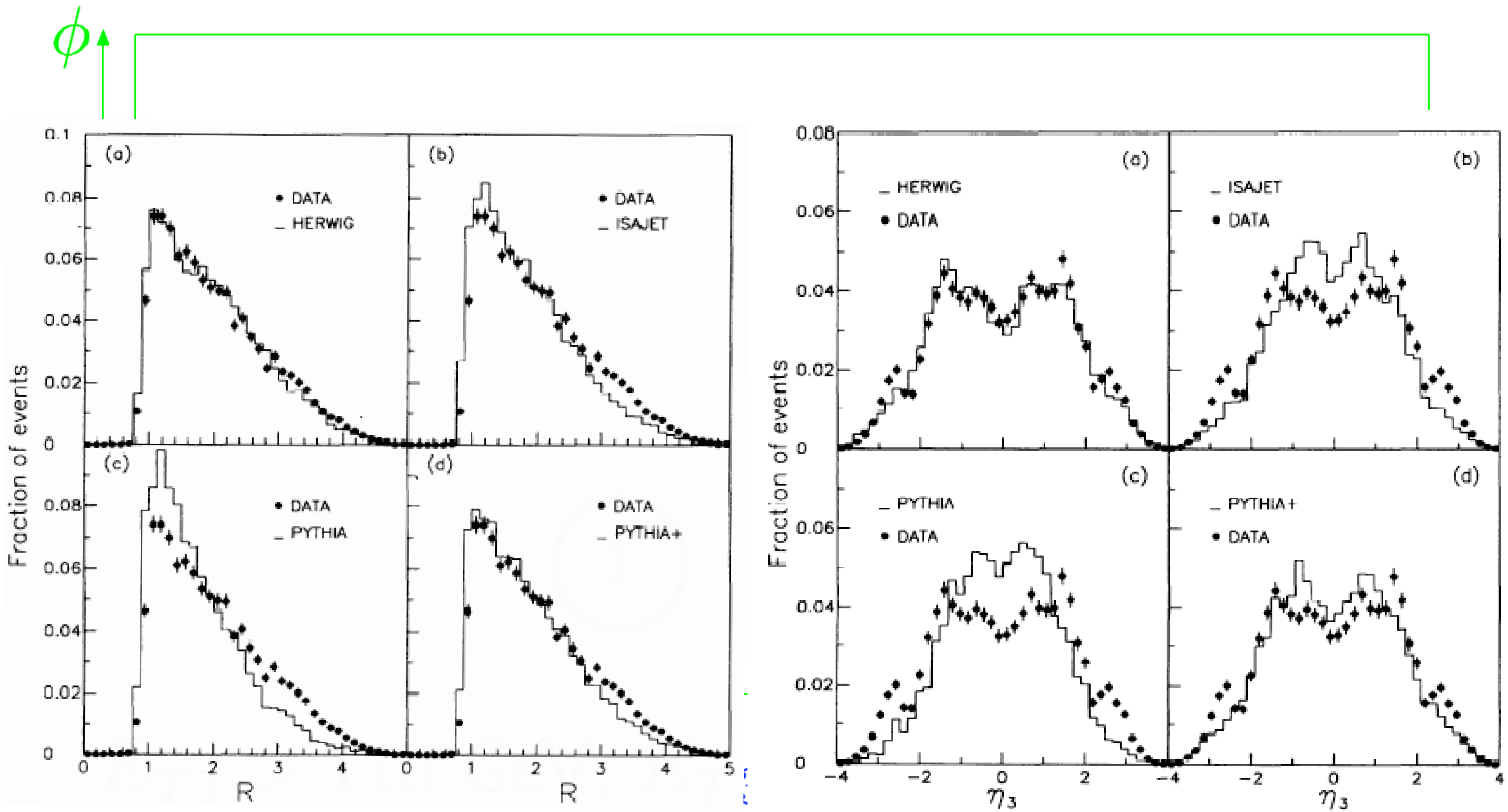
Sets up initial conditions for parton showers.

Colour coherence important here too.



Emission from each parton confined to cone stretching to its colour partner

Essential to fit Tevatron data…



Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial

The Colour Dipole Model

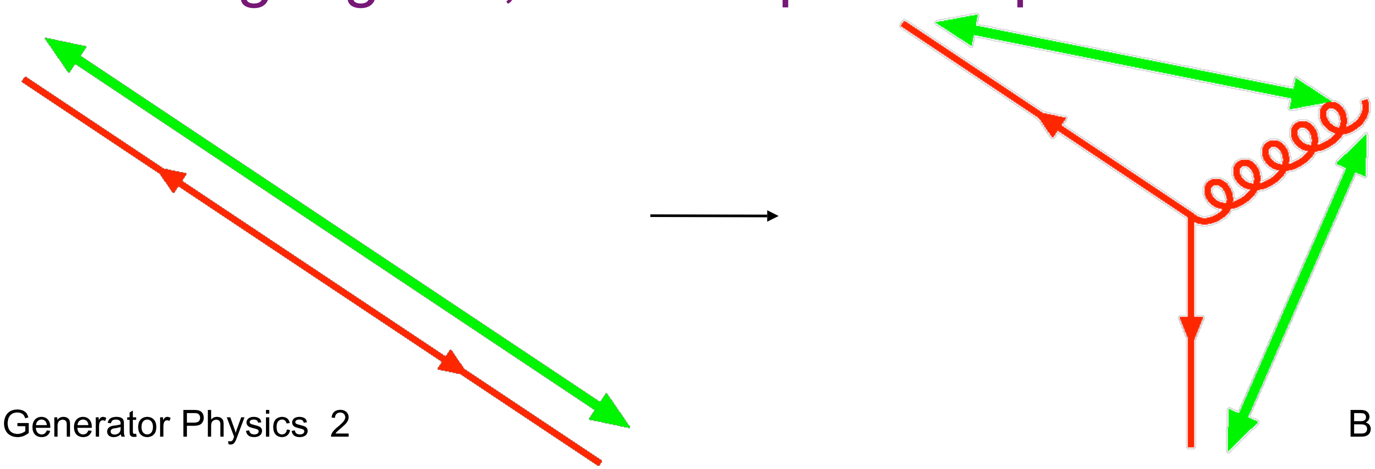
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \text{rapidity} = \log\tan\theta/2$$

After emitting a gluon, colour dipole is split:

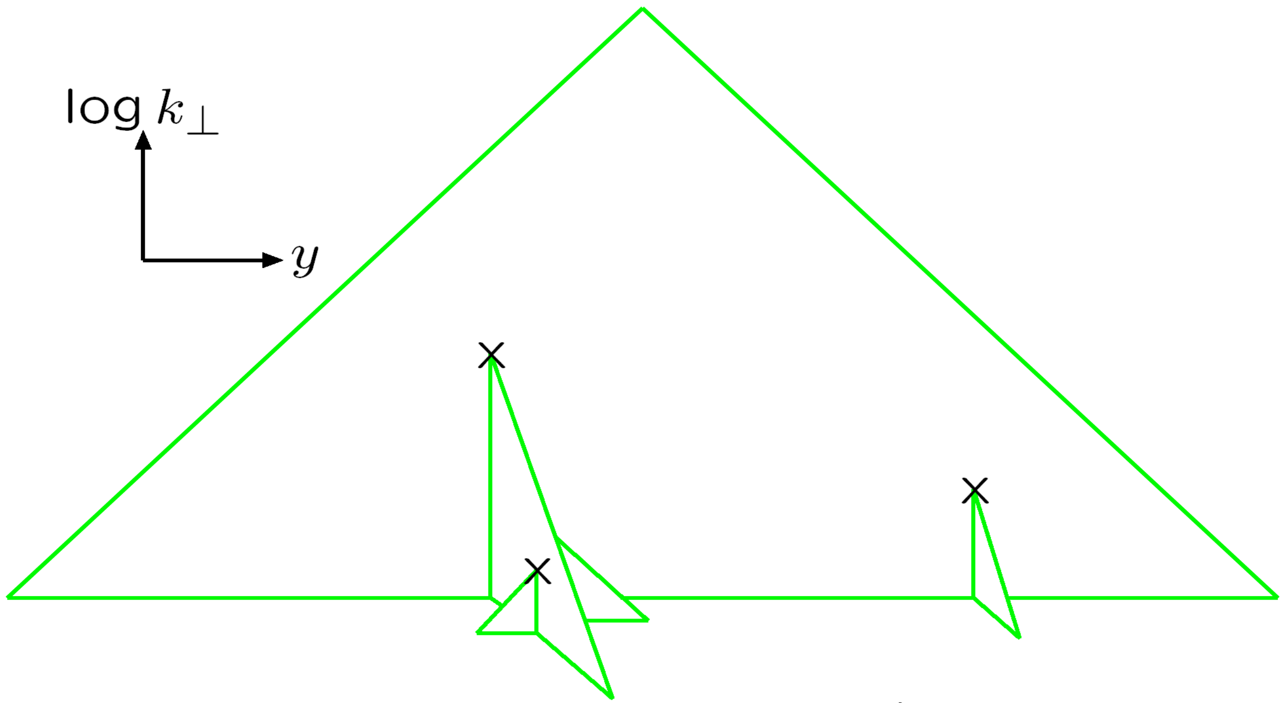


Subsequent dipoles continue to cascade

c.f. parton shower: one parton → two

CDM: one dipole → two = two partons → three

Represented in ‘origami diagram’:



Similar to angular-ordered parton shower for e^+e^- annihilation

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Summary

- Accelerated colour charges radiate gluons.

  Gluons are also charged → cascade.

- Probabilistic language derived from factorization theorems of full gauge theory.

  Colour coherence → angular ordering.

- Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models…