Introduction into
Standard Model and Precision Physics
– Lecture V –

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General overview

Lecture I – Standard Model (part 1)

Lecture II – Standard Model (part 2)

Lecture III – Quantum Corrections

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13 $e^+e^- \to WW \to 4f$: double-pole approximation vs. complex-mass scheme

11 The pole scheme for radiative corrections to resonance processes

11.1 General strategy for a single resonance

Stuart '91; H.Veltman '92; Aeppli, v.Oldenborgh, Wyler '94

The idea: expansion about resonance pole

$$\mathcal{M} = \frac{R(p^2)}{p^2-m^2} + N(p^2) = \frac{R(m^2)}{p^2-m^2} + \frac{R(p^2)-R(m^2)}{p^2-m^2} + N(p^2)$$

$$\hookrightarrow \underbrace{\frac{R(m^2)}{p^2-m^2+\mathrm{i}m\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2)-R(m^2)}{p^2-m^2} + N(p^2)}_{\text{non-resonant}}$$

Benefits / drawbacks / subtleties:

- procedure is gauge invariant, because residue $R(m^2)$ is gauge invariant
- scheme is applicable to higher orders
- $R(p^2)$ in general not analytic at $p^2 = m^2$

  $\hookrightarrow$ "non-factorizable corrections" (i.e. not of the form const.×Breit–Wigner)
- $R(m^2)$ is "ambiguous", because it depends on other phase-space variables

  $\hookrightarrow$ $R(m^2)$ depends on choice of phase-space parametrization
- reliability questionable in presence of small scales,

  e.g. γ radiation with $E_\gamma \sim \Gamma$, vicinity of thresholds: $E - E_{\text{threshold}} \sim \Gamma$

The pole expansion including higher orders:

Starting point: complete matrix element $\mathcal{M} = \underbrace{\frac{W(p^2)}{p^2 - m^2 + \Sigma(p^2)}}_{=\mathcal{M}'} + N(p^2)$

Isolation of pole structure:

recall:
$$\begin{aligned} p^2 - m^2 + \Sigma(p^2) &= p^2 - M^2 + \Sigma(p^2) - \Sigma(M^2) \\ &= (p^2 - M^2)[1 + \Sigma'(M^2)] \; + \; \mathcal{O}((p^2 - M^2)^2) \end{aligned}$$

$$\begin{aligned} \Rightarrow \mathcal{M}' &= \frac{W(M^2)}{p^2 - M^2}\frac{1}{1 + \Sigma'(M^2)} + \left[\frac{W(p^2)}{p^2 - m^2 + \Sigma(p^2)} - \frac{W(M^2)}{p^2 - M^2}\frac{1}{1 + \Sigma'(M^2)}\right] \\ &\equiv \frac{w}{p^2 - M^2} \; + \; n(p^2) \end{aligned}$$

Comments:

- complex pole mass M as well as w and $\Sigma(M^2)$ are gauge invariant
- evaluation of $W(M^2)$ for complex $p^2 = M^2$ not straightforward !

  But: w and $n(p^2)$ can be perturbatively obtained from quantities with real momenta

  Aeppli et al. '94

Perturbative evaluation of w and $n(p^2)$:

Alternative expansion of resonant diagrams about real mass m^2:

$$\mathcal{M}' = \frac{W(p^2)}{p^2-m^2}\sum_{n=0}^{\infty}\left(\frac{-\Sigma(p^2)}{p^2-m^2}\right)^n = \bar{N}(p^2) + \frac{W_{-1}}{p^2-m^2} + \sum_{n=2}^{\infty}\frac{W_{-n}}{(p^2-m^2)^n}$$

$\hookrightarrow$ perturbative expansion for coefficients:

$$W_{-1} = W(m^2) + \frac{\mathrm{d}}{\mathrm{d}p^2}\Big[-W(p^2)\Sigma(p^2)\Big]_{p^2=m^2} + \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}(p^2)^2}\Big[W(p^2)\Sigma^2(p^2)\Big]_{p^2=m^2} + \dots$$

$$\bar{N}(p^2) = \frac{W(p^2)-W(m^2)}{p^2-m^2} - \frac{W(p^2)\Sigma(p^2) - W(m^2)\Sigma(m^2) - (p^2-m^2)\frac{\mathrm{d}}{\mathrm{d}p^2}\Big[W(p^2)\Sigma(p^2)\Big]_{p^2=m^2}}{(p^2-m^2)^2} + \dots$$

One can show to all orders: (see next slides)

$$w = W_{-1}, \qquad n(p^2) = \bar{N}(p^2)$$

$\hookrightarrow$ residue and non-resonant remainder can be obtained
from perturbative calculation with real $p^2 = m^2$

Proof that $w = W^{-1}$:

$$W_{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\frac{\mathrm{d}^n}{\mathrm{d}s^n} W(s) \Big(-\Sigma(s) \Big)^n \right]_{s=m^2}$$

Expand $[\ldots]$ with $s = M^2 + (m^2 - M^2)$ about $s = M^2$:

$$W_{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k=0}^{\infty} \frac{1}{k!} \left[\frac{\mathrm{d}^{n+k}}{\mathrm{d}s^{n+k}} W(s) \Big(-\Sigma(s) \Big)^n \right]_{s=M^2} \underbrace{(m^2 - M^2)}_{=\Sigma(M^2)}{}^k$$

$$= \sum_{n,k=0}^{\infty} \frac{1}{(n+k)!} \binom{n+k}{n} \left[\frac{\mathrm{d}^{n+k}}{\mathrm{d}s^{n+k}} W(s) \Big(-\Sigma(s) \Big)^n \Big(\Sigma(M^2) \Big)^k \right]_{s=M^2}$$

$$= \sum_{r=0}^{\infty} \frac{1}{r!} \frac{\mathrm{d}^r}{\mathrm{d}s^r} \left[W(s) \quad \underbrace{\Big(\Sigma(M^2) - \Sigma(s) \Big)^r}_{=[-\Sigma'(M^2)]^r (s-M^2)^r + \ldots} \quad \right]_{s=M^2}, \qquad r = n + k$$

Only the terms $\propto [-\Sigma'(M^2)]^r$ survive after setting $(s - M^2)$:

$$\Rightarrow \quad W_{-1} \; = \; \sum_{r=0}^{\infty} W(M^2)[-\Sigma'(M^2)]^r \; = \; \frac{W(M^2)}{1 + \Sigma'(M^2)} \; = \; w$$

Proof that $n(p^2) = \bar{N}(p^2)$:

Formal manipulations with Taylor series:

$$\bar{N}(s) = \frac{W(s) - W(m^2)}{s - m^2} - \frac{W(s)\Sigma(s) - W(m^2)\Sigma(m^2) - (s - m^2)\frac{\mathrm{d}}{\mathrm{d}s}\Big[W(s)\Sigma(s)\Big]_{s=m^2}}{(s - m^2)^2} + \dots$$

$$= \sum_{n=0}^{\infty} (s - m^2)^{-n-1} \left(W(s)\Big(-\Sigma(s)\Big)^n \underbrace{- \sum_{k=0}^{n} \frac{1}{k!} \left[\frac{\mathrm{d}^k}{\mathrm{d}s^k} W(s)\Big(-\Sigma(s)\Big)^n \right]_{s=m^2} (s - m^2)^k \right)}_{\text{subtraction of first } (n+1) \text{ Taylor terms}}$$

$$= \sum_{n=0}^{\infty} \sum_{k=n+1}^{\infty} \frac{1}{k!} \left[\frac{\mathrm{d}^k}{\mathrm{d}s^k} W(s)\Big(-\Sigma(s)\Big)^n \right]_{s=m^2} (s - m^2)^{k-n-1}, \qquad k = n + \ell$$

$$= \sum_{\ell=1}^{\infty} (s - m^2)^{\ell-1} \sum_{n=0}^{\infty} \frac{1}{(n+\ell)!} \left[\frac{\mathrm{d}^{n+\ell}}{\mathrm{d}s^{n+\ell}} W(s)\Big(-\Sigma(s)\Big)^n \right]_{s=m^2}, \qquad n = r - \ell$$

$$= \sum_{\ell=1}^{\infty} (s - m^2)^{\ell-1} \left\{ \sum_{r=0}^{\infty} - \sum_{r=0}^{\ell-1} \right\} \frac{1}{r!} \left[\frac{\mathrm{d}^r}{\mathrm{d}s^r} \frac{W(s)}{[-\Sigma(s)]^{\ell}} \Big(-\Sigma(s)\Big)^r \right]_{s=m^2}$$

$$= -\frac{W(M^2)}{s - M^2} \frac{1}{1 + \Sigma'(M^2)} + \frac{W(s)}{s - m^2 + \Sigma(s)} \qquad \text{(see next page)}$$

$$= n(s)$$

Proof that $n(p^2) = \bar{N}(p^2)$: (continued)

First term in curly brackets:

$$\sum_{\ell=1}^{\infty}(s-m^2)^{\ell-1} \underbrace{\sum_{r=0}^{\infty} \frac{1}{r!}\left[\frac{\mathrm{d}^r}{\mathrm{d}s^r}\frac{W(s)}{[-\Sigma(s)]^\ell}\Big(-\Sigma(s)\Big)^r\right]_{s=m^2}}_{\text{known from proof that } w = W_{-1}}$$

$$= \sum_{\ell=1}^{\infty}(s-m^2)^{\ell-1}\frac{W(M^2)}{[-\Sigma(M^2)]^\ell}\frac{1}{1+\Sigma'(M^2)}$$

$$= \frac{1}{-\Sigma(M^2)}\frac{W(M^2)}{1+\Sigma'(M^2)}\sum_{\ell'=0}^{\infty}\frac{(s-m^2)^{\ell'}}{[-\Sigma(M^2)]^{\ell'}} = -\frac{W(M^2)}{s-M^2}\frac{1}{1+\Sigma'(M^2)}$$

Second term in curly brackets:

$$-\sum_{\ell=1}^{\infty}\sum_{r=0}^{\ell-1}(s-m^2)^{\ell-1}\frac{1}{r!}\left[\frac{\mathrm{d}^r}{\mathrm{d}s^r}\frac{W(s)}{[-\Sigma(s)]^\ell}\Big(-\Sigma(s)\Big)^r\right]_{s=m^2}, \qquad \ell = \ell' + r$$

$$= -\sum_{\ell'=1}^{\infty}\sum_{r=0}^{\infty}(s-m^2)^{\ell'+r-1}\frac{1}{r!}\left[\frac{\mathrm{d}^r}{\mathrm{d}s^r}\frac{W(s)}{[-\Sigma(s)]^{\ell'}}\right]_{s=m^2}$$

$$= -\sum_{\ell'=1}^{\infty}(s-m^2)^{\ell'-1}\frac{W(s)}{[-\Sigma(s)]^{\ell'}} = \frac{W(s)}{s-m^2+\Sigma(s)}$$

Perturbative ordering in pole scheme:

First step: calculate M^2 from $M^2 = m^2 - \Sigma^{(1)+\dots+(n+1)}(M^2)$

$\hookrightarrow$ yields Γ in M^2 up to n-loop order

Expansion of matrix element: ($A^{(n)} \equiv n$-loop contribution to A)

$$\mathcal{M} = \frac{W(p^2)}{p^2 - m^2 + \Sigma(p^2)} + N(p^2)$$

$$= \frac{W(M^2)}{p^2 - M^2} \frac{1}{1 + \Sigma'(M^2)} + n(p^2) + N(p^2)$$

$$= \frac{W^{(0)}(m^2)}{p^2 - M^2}$$

$\Big\}$ LO:
leading order
in pole approximation

$$+ \frac{W^{(1)}(m^2)}{p^2 - M^2} - \frac{W^{(0)}(m^2)\Sigma^{(1)\prime}(m^2)}{p^2 - M^2} - \frac{W^{(0)\prime}(m^2)\Sigma^{(1)}(m^2)}{p^2 - M^2}$$

$$+ \underbrace{\frac{W^{(0)}(p^2) - W^{(0)}(m^2)}{p^2 - m^2}}_{= n^{(0)}(p^2)} + N^{(0)}(p^2)$$

$+$ non-factorizable corrections

$\Bigg\}$ NLO:
correction to residue
and
leading-order off-shellness
and
non-fact. corrections

$+$ higher orders

Modified (improved!) version of the pole expansion:

Inclusion of lowest order without pole expansion:

$$
\begin{aligned}
\mathcal{M} = {} & \mathcal{M}^{(0)} && \Big\} \begin{array}{l}\text{LO:}\\ \text{complete leading order}\end{array}\\
& \left.\begin{array}{l} + \dfrac{W^{(1)}(m^2)}{p^2-M^2} - \dfrac{W^{(0)}(m^2)\Sigma^{(1)\prime}(m^2)}{p^2-M^2} \\ + \text{non-factorizable corrections}\end{array}\right\} && \begin{array}{l}\text{NLO:}\\ \text{correction to residue}\\ \text{and}\\ \text{non-fact. corrections}\end{array}\\
& \qquad + \text{higher orders}
\end{aligned}
$$

Comments:

- inclusion of $\mathcal{M}^{(0)}$ is usually easier than its expansion
- wave-function correction $\Sigma^{(1)\prime}(m^2) = 0$ in on-shell renormalization scheme
- naive estimate of relative theoretical uncertainty (TU) in NLO:

$$
\mathrm{TU} \quad \sim \quad \begin{cases} \frac{\alpha}{\pi} \times \frac{\Gamma}{m} \times \mathrm{const.} & \text{in resonance region } |p^2 - m^2| \lesssim m\Gamma \\ \frac{\alpha}{\pi} \times \mathrm{const.} & \text{off resonance } |p^2 - m^2| \gg m\Gamma \end{cases}
$$

Factorizable corrections:

$$\mathcal{M}^{(1)}_{\text{fact.}} = \frac{W^{(1)}(m^2) - W^{(0)}(m^2)\Sigma^{(1)\prime}(m^2)}{p^2 - M^2}$$

$$= \sum_\lambda \frac{\mathcal{M}^{(1)}_{\text{production}}(\lambda)\mathcal{M}^{(0)}_{\text{decay}}(\lambda) + \mathcal{M}^{(0)}_{\text{production}}(\lambda)\mathcal{M}^{(1)}_{\text{decay}}(\lambda)}{p^2 - M^2}$$

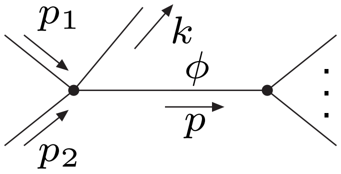
Spin correlations: identical definitions of polarized states $|\phi(\lambda)\rangle$ needed in $\mathcal{M}^{(n)}_{\text{production}}(\lambda)$ and $\mathcal{M}^{(n)}_{\text{decay}}(\lambda)$

Subtlety in kinematics:

gauge invariance of $\mathcal{M}^{(n)}_{\text{production/decay}}$ requires $p^2 = m^2$

$\hookrightarrow$ "on-shell projection" of momenta needed !

Example:



off-shell phase space: $(p_1 + p_2 - k)^2 = p^2 \neq m^2$

$\hookrightarrow$ define $\hat{k}$ (e.g. from angle of k) such that $(p_1 + p_2 - \hat{k})^2 = m^2$

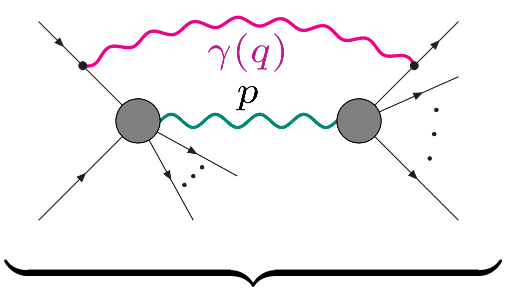
Non-factorizable corrections:

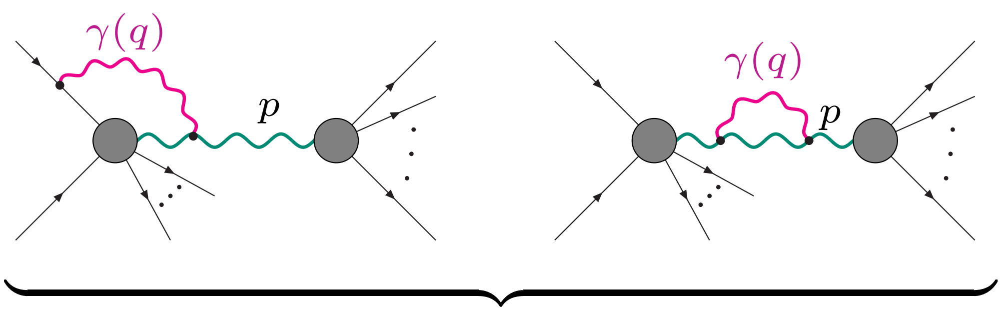
Melnikov, Yakovlev ’96; Beenakker, Berends, Chapovsky ’97;
Denner, Dittmaier, Roth ’97,’98

Origin:

on-shell limit ($p^2 \to m^2$) and IR regularization (e.g. $m_\gamma \to 0$) do not commute

in diagrams with exchange of γ/g between external and/or resonant lines:





“manifestly non-factorizable”

- diagram has no explicit propagator factor $(p^2 - m^2)^{-1}$
- resonant IR-divergent contribution in loop integral from region $q \to 0$

“not manifestly non-factorizable” diagrams

- diagram has explicit propagator factor $(p^2 - m^2)^{-1}$ and contributes also to factorizable corrections $W^{(1)}(m^2)$
- non-factorizable part:
  $$W^{(1)}_{\text{non-fact.}}(p^2) \equiv \left[W^{(1)}(p^2) - W^{(1)}(m^2)\right]_{p^2 \to m^2}$$
  $\hookrightarrow$ receives only contributions from $q \to 0$

Evaluation of NLO non-factorizable corrections:

Only leading behaviour of loop integrands for soft-photon momentum $q \to 0$ relevant

$\hookrightarrow$ “Extended soft-photon (or gluon) approximation”:

- neglect q in numerator of diagrams $\quad \to$ scalar loop integrals only
- q only kept in propagators that become singular for $q \to 0$
- resonance propagators are dressed with complex mass: $\quad [(p+q)^2 - M^2]^{-1}$
- take limits $p^2, M^2 \to m^2$ in final result whenever possible

Result factorizes from Born amplitude: $\quad \mathcal{M}^{\mathrm{virt}}_{\text{non-fact.}} = \delta^{\mathrm{virt}}_{\text{non-fact.}} \, \mathcal{M}^{(0)}$

Features of $\delta^{\mathrm{virt}}_{\text{non-fact.}}$:

- gauge independent by definition
- contains contributions like $\quad \alpha \ln\left(\dfrac{p^2 - M^2}{m_\gamma M}\right)$

  from non-commutativity of on-shell and soft-photon limits
- free of collinear singularities from external particles
- various cancellations after addition of corresponding real-photon contributions:
  - ◇ no resonant contribution from photon exchange between initial and final states
  - ◇ non-local cancellation of whole effect after integration over p^2

11.2 Real corrections to resonance processes

Calculation of real NLO corrections:

NLO: 1-particle bremsstrahlung in LO (tree-level diagrams)

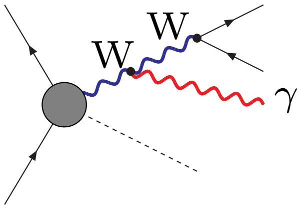
$\hookrightarrow$ LO prescriptions for resonances applicable

But: real $|\mathcal{M}_{i\to f+\gamma/\mathrm{g}}|^2$ is related to $2\,\mathrm{Re}\{\mathcal{M}^{(0)*}_{i\to f}\mathcal{M}^{(1)}_{i\to f}\}$ in soft and collinear limits,

$\hookrightarrow$ matching between resonance descriptions in virtual and real corrections !

Pole expansions for real corrections:

Split diagrams with radiating resonances (2 resonant propagators) as follows:



$$\frac{1}{[(p+k)^2-M^2](p^2-M^2)} = \frac{1}{2pk}\left[\frac{1}{p^2-M^2} - \frac{1}{(p+k)^2-M^2}\right]$$

$E_\gamma \gg \Gamma_\mathrm{W}$ (hard photon): photon can be assigned to production or decay, resonances are well separated in phase space

$\hookrightarrow$ pole-scheme decomposition contains two leading on-shell contributions

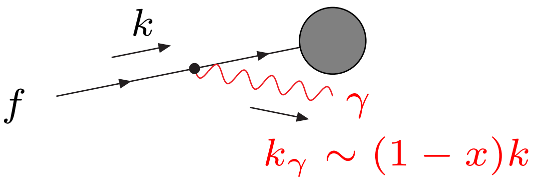
$E_\gamma = \mathcal{O}(\Gamma_\mathrm{W})$ ("semi-soft photon"): two resonances overlap in phase space

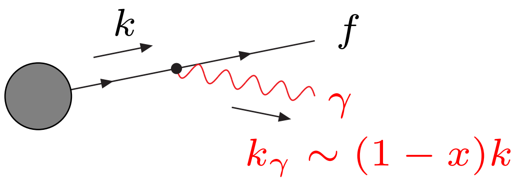
$\hookrightarrow$ definition of leading-pole approximation potentially problematic

(definition depends on specific observable; keep p^2 or $(p-k)^2$ fixed ?)

Enhancement of real-photon emission due to collinear singularities

Collinear photon emission off light particles:





$\hookrightarrow$ leads to mass-singular universal corrections

which can be described via “structure functions” in leading-log approximation:

$$\Gamma_{ff}(x,M^2) \;=\; \delta(1-x) + \frac{Q_f^2\alpha}{2\pi}\,\ln\!\left(\frac{M^2}{m_f^2}\right)\left(\frac{1+x^2}{1-x}\right)_+ \;+\;\ldots$$

$\hookrightarrow$ e.g. $\sigma^{\mathrm{ISR}}_{\mathrm{e^+e^-}\to X}(p_+,p_-) \;\approx\; \int_0^1 \mathrm{d}x_1\,\Gamma_{\mathrm{ee}}(x_1,M^2)\int_0^1 \mathrm{d}x_2\,\Gamma_{\mathrm{ee}}(x_2,M^2)\;\sigma^{\mathrm{Born}}_{\mathrm{e^+e^-}\to X}(x_1p_+,x_2p_-)$

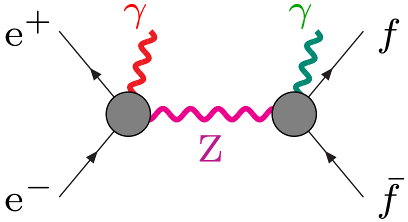
Comments:

- $M =$ QED factorization scale $=$ typical scale of process (set by full calculation)
- structure fucntions Γ_{ff}, etc., known up to $\mathcal{O}(\alpha^5)$ $\oplus$ IR exponentiation
- unitarity / KLN theorem demands $\int_0^1 \mathrm{d}x\,\Gamma_{ff}(x,M^2) = 1$

  $\hookrightarrow$ mass singularities cancel for FSR if $f + n\gamma$ is treated inclusively for collinear γs
- ISR / FSR can lead to large effects, e.g. distortion of resonances

Distortion of resonance shapes by real radiation:

Initial state fixed:



Typical situations: $e^+e^- \to Z \to f\bar{f}$,

$\mu^+\mu^- \to Z, H, ? \to f\bar{f}$

$\hookrightarrow$ scan over s-channel resonance in $\sigma(s)$ by changing CM energy $\sqrt{s}$

Initial-state radiation (ISR):

Z can become resonant for $s = (p_+ + p_-)^2 \; > \; (p_+ + p_- - k_\gamma)^2 \sim M_Z^2$

$\hookrightarrow$ radiative tail for $s > M_Z^2$ due to "radiative return"

Final-state radiation (FSR):

$s = k_Z^2 \sim M_Z^2$ for FSR

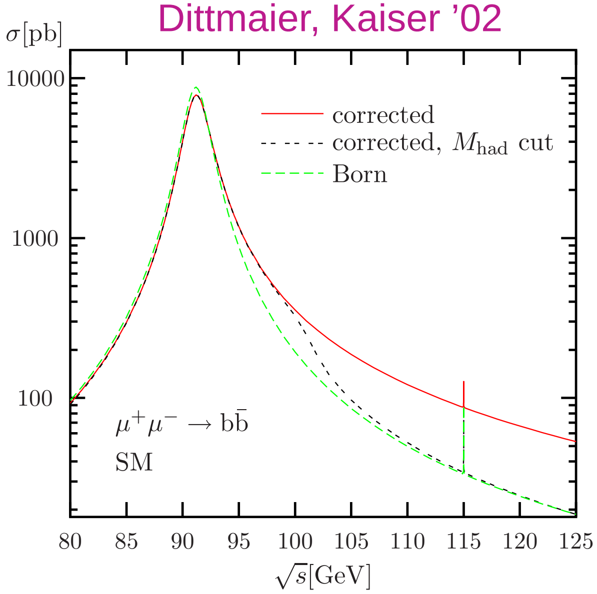
$\hookrightarrow$ only rescaling of resonance

An example:

cross section for $\mu^-\mu^+ \to b\bar{b}$ in lowest order and including photonic and QCD corrections, with and without invariant-mass cut

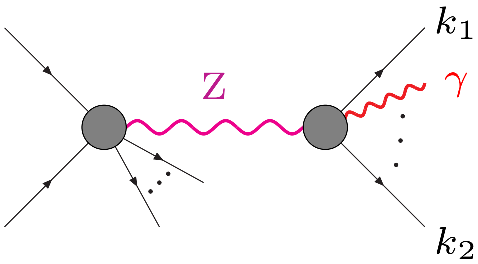
$\sqrt{s} - M(b\bar{b}) < 10\,\mathrm{GeV}$

Dittmaier, Kaiser '02



Distortion of resonance shapes by real radiation: (continued)

Resonance reconstructed from decay products:



Typical situations: $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW}/\mathrm{ZZ} \to 4f$,
$\mathrm{pp} \to \mathrm{Z} \to f\bar{f} + X$

$\hookrightarrow$ resonance in invariant-mass distribution $\frac{\mathrm{d}\sigma}{\mathrm{d}M}$ of reconstructed invariant mass M

Final-state radiation (FSR):

resonance for $M^2 = (k_1 + k_2)^2 \; < \; (k_1 + k_2 + k_\gamma)^2 \sim M_\mathrm{Z}^2$

$\hookrightarrow$ radiative tail for $M < M_\mathrm{Z}$

An example:

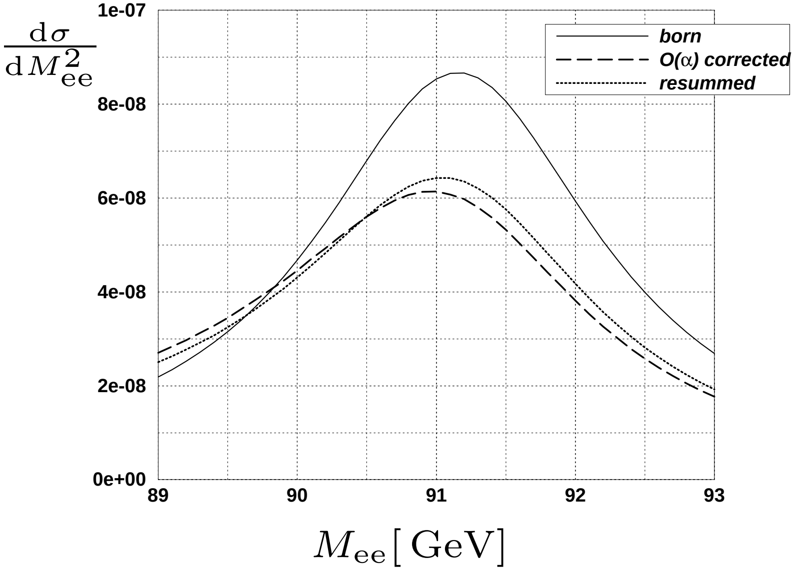
Z in $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ} \to 4l$

reconstructed via $M_\mathrm{ee} = (p_{l_1} + p_{l_2})^2$

lowest order, $\mathcal{O}(\alpha)$ FSR,

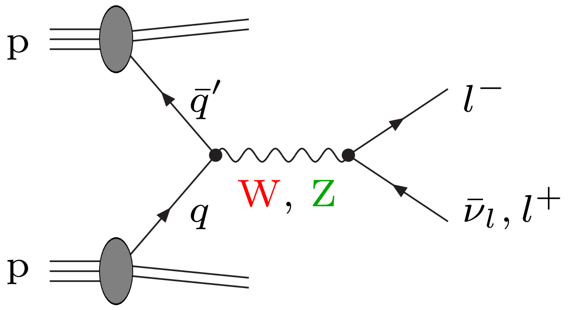
and higher-order FSR beyond $\mathcal{O}(\alpha)$

Beenakker, Berends, Chapovsky ’98



12 Single-W production at hadron colliders

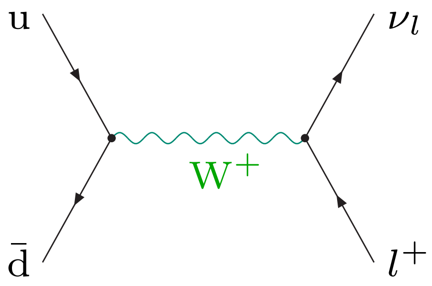
Drell–Yan-like W and Z production:

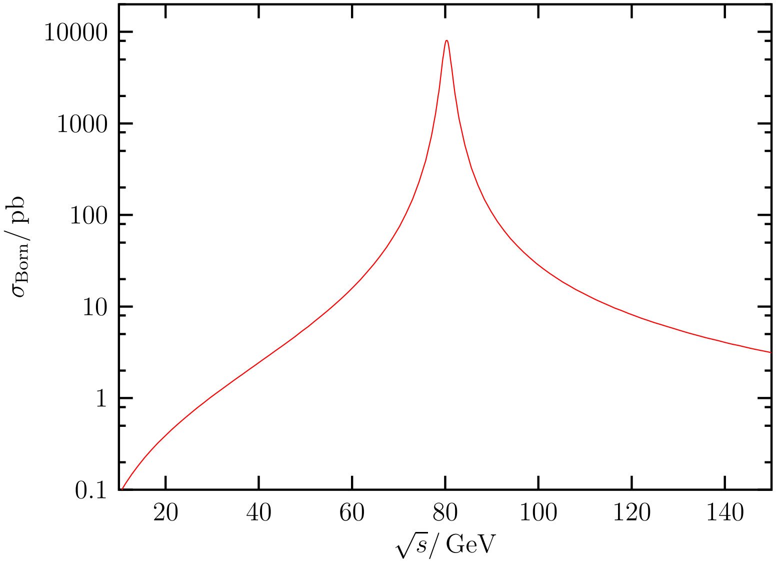


Physics goals:

- M_Z → detector calibration by comparing with LEP1 result
- $\sin^2\theta_{\text{eff}}^{\text{lept}}$ → comparison with results of LEP1 and SLC
- M_W → improvement to $\Delta M_W \sim 15\,\text{MeV}$
- decay widths Γ_Z and Γ_W from M_{ll} or $M_{T,l\nu_l}$ resonance tails
- search for Z' and W' at high M_{ll} or $M_{T,l\nu_l}$
- information on PDFs

Partonic cross section and W-boson resonance:





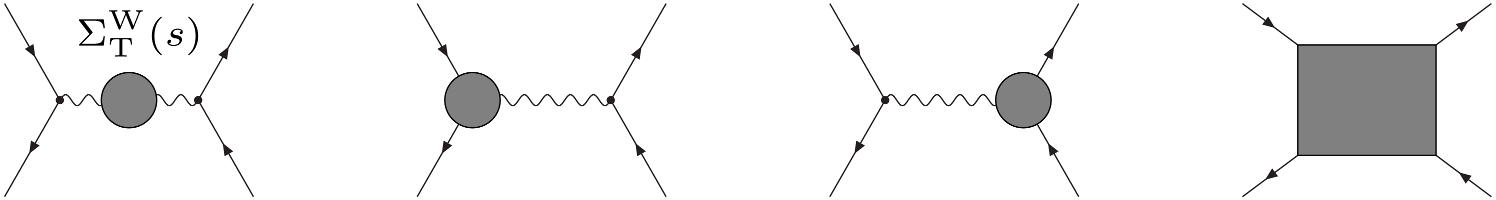
Born amplitude:

$$\mathcal{M}_0 = \frac{e^2}{2s_\mathrm{W}^2}\left[\bar{v}_\mathrm{d}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)u_\mathrm{u}\right]\frac{1}{s-M_\mathrm{W}^2+\mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}}\left[\bar{u}_{\nu_l}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)v_l\right]$$

Electroweak corrections:

Dittmaier, Krämer ’02; Baur, Wackeroth ’04
Arbuzov et al. ’05; Carloni Calame et al. ’06

- virtual corrections:



W self-energy — Wud and $\mathrm{W}l\nu_l$ vertex corrections — box diagrams

inclusion in factorized form: $|\mathcal{M}_0+\mathcal{M}_1|^2 = (1+2\,\mathrm{Re}\{\delta^\mathrm{virt}\})|\mathcal{M}_0|^2+\ldots$

with $\delta^\mathrm{virt} = \delta_\mathrm{self}(\hat{s})+\delta_{Wdu}(\hat{s})+\delta_{W\nu_l l}(\hat{s})+\delta_\mathrm{box}(\hat{s},\hat{t})$

$\hookrightarrow$ δ^virt gauge independent in limit $\Gamma_\mathrm{W}\to 0$,

non-analytic terms in δ^virt described via $\ln(\hat{s}-M_\mathrm{W}^2) \to \ln(\hat{s}-M_\mathrm{W}^2+\mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})$

- real photon corrections:

full amplitude calculation for $\mathrm{u}\bar{\mathrm{d}}\to\nu_l l^+\gamma$ with complex W mass

$\hookrightarrow$ gauge invariant with correct IR (soft and collinear) limits

Electroweak corrections in Pole Approximation (PA):

Baur, Keller Wackeroth ’99
Dittmaier, Krämer ’02

↪ decomposition into factorizable and non-factorizable contributions:

$$
\begin{aligned}
\delta^{\mathrm{virt}}_{\mathrm{PA}} &= \delta^{\mathrm{virt}}_{\mathrm{fact}} + \delta^{\mathrm{virt}}_{\mathrm{nonfact}}(\hat{s},\hat{t}) \\
\delta^{\mathrm{virt}}_{\mathrm{fact}} &= \delta_{W\,du}(M_{\mathrm{W}}^2)|_{\Gamma_{\mathrm{W}}=0} + \delta_{W\,\nu_l l}(M_{\mathrm{W}}^2)|_{\Gamma_{\mathrm{W}}=0} \\
\delta^{\mathrm{virt}}_{\mathrm{nonfact}}(\hat{s},\hat{t}) &= \delta^{\mathrm{virt}}|_{\hat{s}\to M_{\mathrm{W}}^2,\Gamma_{\mathrm{W}}\to 0} - \delta^{\mathrm{virt}}_{\mathrm{fact}} \\
&= -\frac{\alpha}{2\pi}\Bigg\{-2 + Q_d\,\mathrm{Li}_2\left(1+\frac{M_{\mathrm{W}}^2}{\hat{t}_{\mathrm{res}}}\right) - Q_u\,\mathrm{Li}_2\left(1+\frac{M_{\mathrm{W}}^2}{\hat{u}_{\mathrm{res}}}\right) \\
&\qquad + 2\ln\left(\frac{M_{\mathrm{W}}^2 - \mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}} - \hat{s}}{m_\gamma M_{\mathrm{W}}}\right)\left[1 + Q_d\ln\left(-\frac{M_{\mathrm{W}}^2}{\hat{t}_{\mathrm{res}}}\right) - Q_u\ln\left(-\frac{M_{\mathrm{W}}^2}{\hat{u}_{\mathrm{res}}}\right)\right]\Bigg\}
\end{aligned}
$$

PA versus full $\mathcal{O}(\alpha)$ correction:

$\sqrt{\hat{s}}/\mathrm{GeV}$	40	80	120	200	500	1000	2000
$\hat{\sigma}_0/\mathrm{pb}$	2.646	7991.4	8.906	1.388	0.165	0.0396	0.00979
$\delta/\%$	0.7	2.42	−12.9	−3.3	12	19	23
$\delta_{\mathrm{PA}}/\%$	0.0	2.40	−12.3	−0.7	18	31	43

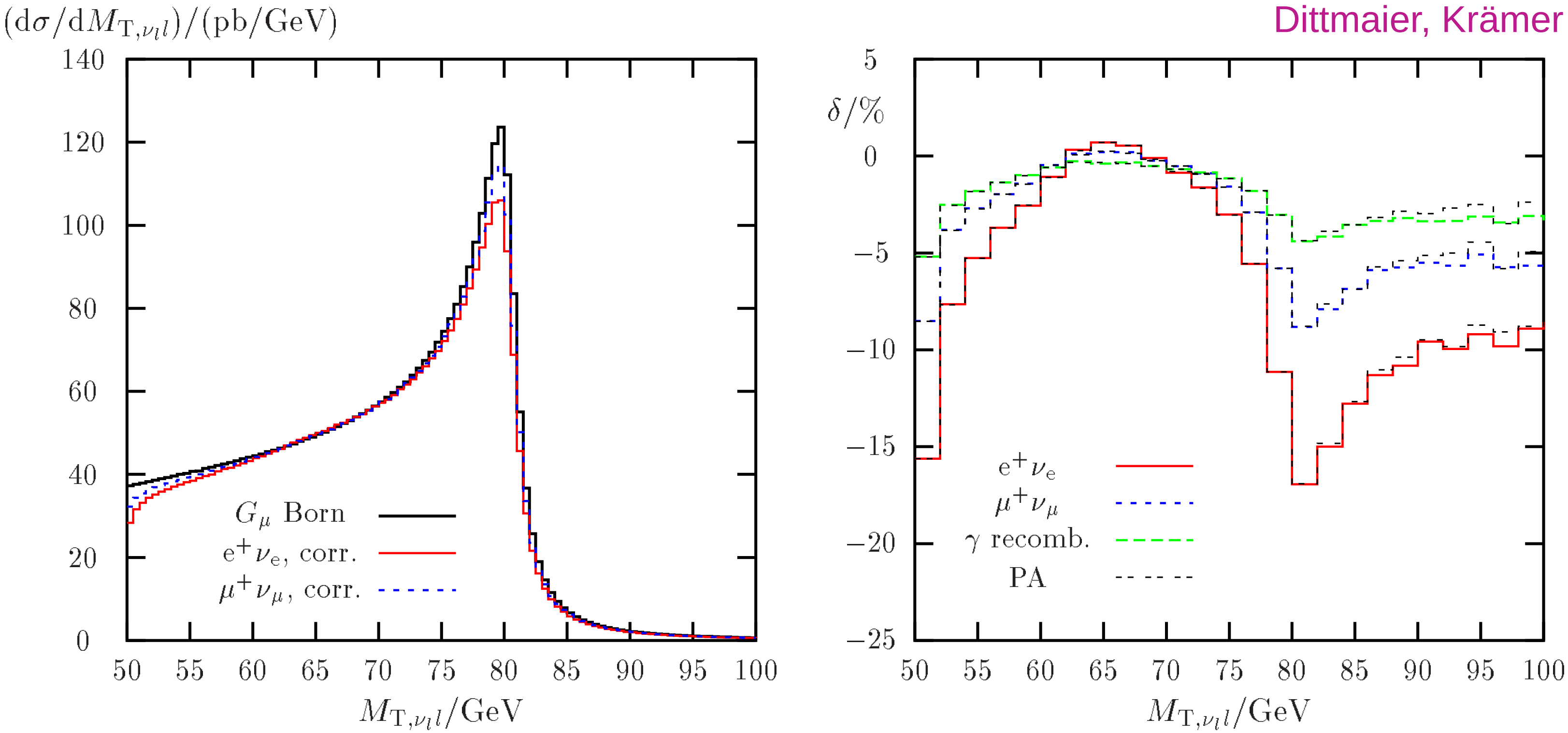
error estimate: $|\delta^{\mathrm{virt}}_{\mathrm{A}} - \delta^{\mathrm{virt}}| \sim \frac{\alpha}{\pi}\max\left\{\frac{\Gamma_{\mathrm{W}}}{M_{\mathrm{W}}}, \ln\left(\frac{\hat{s}}{M_{\mathrm{W}}^2}\right), \ln^2\left(\frac{\hat{s}}{M_{\mathrm{W}}^2}\right)\right\} \times \mathrm{const.}$

Hadronic pp cross section and Jacobian peak:

Note: ν_l not detectable → e.g. study “transverse W mass”:

$$M^2_{\mathrm{T},\nu_l l} = (E_{\mathrm{T,miss}} + E_{\mathrm{T},l})^2 - (\mathbf{p}_{\mathrm{T,miss}} + \mathbf{p}_{\mathrm{T},l})^2$$

Dittmaier, Krämer ’02



- pole approximation (PA) for W resonance
  sufficient near Jacobian peak, but not for large $M_{\mathrm{T},\nu_l l}$
- EW corrections sensitively depend on treatment of photon radiation
  $\hookrightarrow$ issue of inclusiveness / KLN violation causes large effects

13 $e^+e^-\to WW\to 4f$: double-pole approximation vs. complex-mass scheme

13.1 Double-pole approximation (DPA)

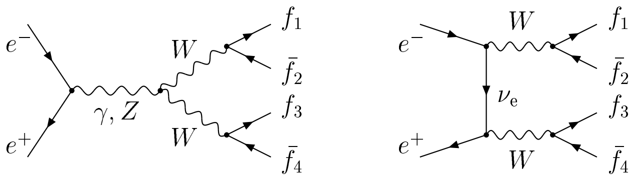
Structure of Monte Carlo generators with EW corrections used at LEP2:

RacoonWW (Denner, Dittmaier,Roth,Wackeroth) and

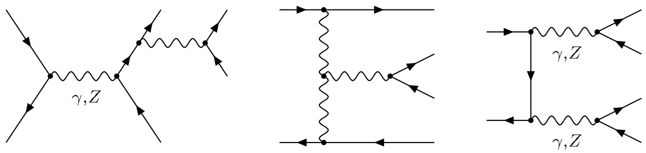
KoralW ⊕ *YFSWW* (Jadach,Płaczek,Skrzypek,Ward) include

- full lowest-order matrix elements for $\mathrm{e}^+\mathrm{e}^- \to 4f(+\gamma)$

signal diagrams

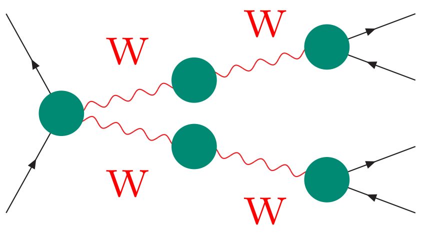


background diagrams



etc.

- non-universal electroweak corrections DPA



leading term in expansion about W resonances

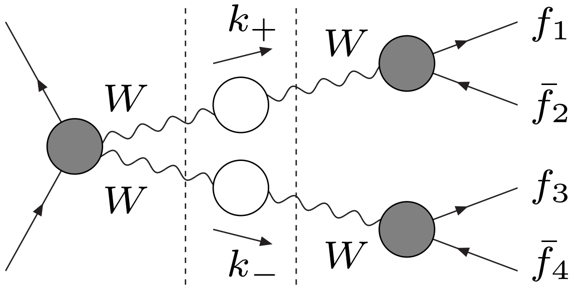
$\hookrightarrow$ contributions:

  - corrections to $\mathrm{ee} \to \mathrm{WW}$ Böhm et al. '88; Fleischer, Jegerlehner, Zralek '89
  - corrections to $\mathrm{W} \to f\bar{f}'$ Bardin, S. Riemann, T. Riemann '86 Jegerlehner '86; Denner, Sack '90
  - non-factorizable photonic corrections Melnikov, Yakovlev '96 Beenakker, Berends, Chapovsky '97 Denner, Dittmaier, Roth '97

- improvements by leading higher-order corrections

Virtual corrections in DPA:

- Factorizable corrections:



on-shell production on-shell decays

$$\mathcal{M}^{e^+e^-\to WW\to 4f}_{virt,fact,DPA} = \frac{R(M_W^2, M_W^2)}{(k_+^2 - M_W^2 + iM_W\Gamma_W)(k_-^2 - M_W^2 + iM_W\Gamma_W)}$$

with the gauge-independent residue

$$R(M_W^2, M_W^2) = \sum_{\text{W-pols}} \Big(\delta\mathcal{M}^{e^+e^-\to W^+W^-} \mathcal{M}^{W^+\to f_1\bar{f}_2}_{Born} \mathcal{M}^{W^-\to f_3\bar{f}_4}_{Born} + \mathcal{M}^{e^+e^-\to W^+W^-}_{Born} \delta\mathcal{M}^{W^+\to f_1\bar{f}_2} \mathcal{M}^{W^-\to f_3\bar{f}_4}_{Born} + \mathcal{M}^{e^+e^-\to W^+W^-}_{Born} \mathcal{M}^{W^+\to f_1\bar{f}_2}_{Born} \delta\mathcal{M}^{W^-\to f_3\bar{f}_4} \Big)$$

containing the corrections to on-shell production and decay

- Non-factorizable corrections:

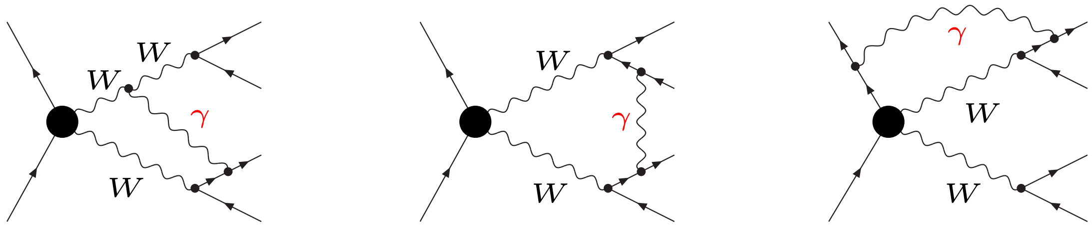
$$\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,nonfact,DPA}} = \delta\mathcal{M}^{\mathrm{e^+e^-\to 4}f}\Big|_{\text{doubly-resonant part}} - \mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}}$$

$$= \mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{Born,DPA}}\;\delta_{\mathrm{virt,nonfact,DPA}}$$

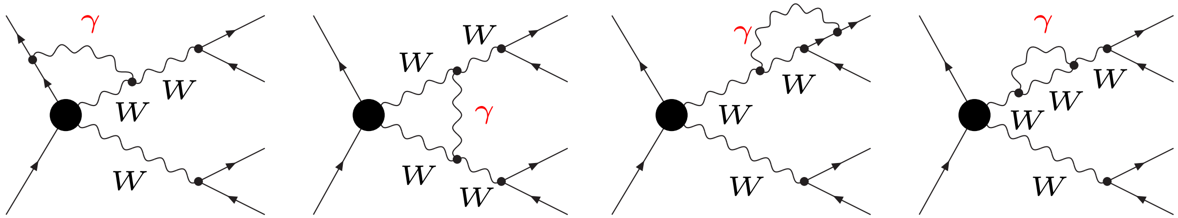
Features of $\delta_{\mathrm{virt,nonfact,DPA}}$ analogous to single-resonance case:

- ◇ gauge independent, no mass singularities
- ◇ compensates IR singularities of W bosons in $\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}}$
- ◇ no factorization of Breit–Wigner-type resonances (complicated dependence on off-shellness $k_\pm^2$ of W bosons)

Manifestly non-factorizable diagrams:



Diagrams contributing to factorizable and non-factorizable RCs:



Combination of contributions: (as implemented in RacoonWW)

$$\begin{aligned}\int \mathrm{d}\sigma = \frac{1}{2s}\Bigg\{ \int \mathrm{d}\Phi_{4f} \Bigg[& |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to 4f}_{\mathrm{Born}}|^2 \\ & + 2\,\mathrm{Re}\Big((\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to \mathrm{WW}\to 4f}_{\mathrm{Born},\mathrm{DPA}})^* \, \delta\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to \mathrm{WW}\to 4f}_{\mathrm{virt,fact},\mathrm{DPA}} \\ & \qquad + |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to \mathrm{WW}\to 4f}_{\mathrm{Born},\mathrm{DPA}}|^2 \, \delta_{\mathrm{virt,nonfact},\mathrm{DPA}} \Big) \Bigg] \\ & + \int \mathrm{d}\Phi_{4f\gamma} \; |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to 4f\gamma}_{\mathrm{Born}}|^2 \Bigg\}\end{aligned}$$

Note: virtual corrections in DPA $\oplus$ real from full amplitudes

Combination of contributions: (as implemented in RacoonWW)

$$
\begin{aligned}
\int \mathrm{d}\sigma \;=\; \frac{1}{2s}\Bigg\{ \int \mathrm{d}\Phi_{4f} \Bigg[\; & |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to 4f}_{\mathrm{Born}}|^2 \\
& \left.\begin{aligned}
& + 2\,\mathrm{Re}\bigg((\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to\mathrm{WW}\to 4f}_{\mathrm{Born,DPA}})^*\, \delta\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to\mathrm{WW}\to 4f}_{\mathrm{virt,fact,DPA}} \\
& \qquad + |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to\mathrm{WW}\to 4f}_{\mathrm{Born,DPA}}|^2\, \delta_{\mathrm{virt,nonfact,DPA}} \bigg) \\
& + |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to\mathrm{WW}\to 4f}_{\mathrm{Born,DPA}}|^2\, \delta^{4f}_{\mathrm{sub,1}}
\end{aligned}\right\}\ \text{non-singular} \\
\text{explicit mass singularities} \to \qquad & + |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to 4f}_{\mathrm{Born}}|^2 \otimes \delta^{4f}_{\mathrm{sub,2}} \;\Bigg] \\
+ \int \mathrm{d}\Phi_{4f\gamma} \Bigg[\; & \underbrace{|\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to 4f\gamma}_{\mathrm{Born}}|^2 - |\mathcal{M}^{\mathrm{e}^+\mathrm{e}^-\to 4f}_{\mathrm{Born}}|^2\, \delta^{4f\gamma}_{\mathrm{sub}}}_{\text{non-singular}} \;\Bigg] \Bigg\}
\end{aligned}
$$

Note: virtual corrections in DPA $\oplus$ real from full amplitudes

↪ redistribution of singular contributions to avoid mismatch in cancellations

From LEP2 to the ILC:

Experimental vs. theoretical uncertainties for some observables:

Observable	$\Delta_{\rm exp}(\rm LEP2)$	$\Delta_{\rm exp}(\rm ILC)$	$\Delta_{\rm th}(\rm DPA/IBA)$
$\sigma_{\rm WW}$	$\sim 1\%$	$\lesssim 0.5\%$	2% for $\sqrt{s} < 170\,\mathrm{GeV}$ (IBA range) 0.7% for $170\,\mathrm{GeV} < \sqrt{s} < 180\,\mathrm{GeV}$ 0.5% for $180\,\mathrm{GeV} < \sqrt{s} < 500\,\mathrm{GeV}$
$M_{\rm W}(\rm threshold)$	$\sim 200\,\mathrm{MeV}$	$\sim 7\,\mathrm{MeV}$	? but $> 50\,\mathrm{MeV}$
$M_{\rm W}(\rm reconstr.)$	$\sim 30\,\mathrm{MeV}$	$\sim 10\,\mathrm{MeV}$	$5{-}10\,\mathrm{MeV}$
TGC	some $\%$	$\sim 0.1\%$	$\lesssim 1\%$ at LEP2 ? at $\sqrt{s} \gg 200\,\mathrm{GeV}$

Exceptional case: threshold region and below ($\sqrt{s} < 170\,\mathrm{GeV}$)

error estimate of DPA not reliable

$\to$ description at LEP2 via IBA = “Improved Born Approximation”
(off-shell Born calculation dressed with universal corrections such as ISR)

$\Rightarrow$ DPA/IBA approach sufficiently accurate at LEP2

but precision beyond DPA needed at ILC

$\hookrightarrow$ recent treatment beyond DPA in complex-mass scheme

13.1 The complex-mass scheme at one loop and application to $\mathrm{e}^+\mathrm{e}^- \longrightarrow 4f$

The complex-mass scheme at one loop Denner, Dittmaier, Roth, Wieders ’05

mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ complex mass renormalization: $\underbrace{M^2_{\mathrm{W},0}}_{\text{bare mass}} = \mu^2_{\mathrm{W}} + \underbrace{\delta\mu^2_{\mathrm{W}}}_{\text{ren. constant}},$ etc.

$\hookrightarrow$ Feynman rules with complex masses and counterterms

Virtues and drawbacks:

- perturbative calculations as usual
- no double counting of contributions (bare Lagrangian unchanged !)
- spurios terms are of $\mathcal{O}(\alpha^2)$, but spoil unitarity
- complex gauge-boson masses also in loop integrals

Convenient choice:

complex field renormalization $\underbrace{W_0^\pm}_{\text{bare field}} = \left(1 + \tfrac{1}{2}\underbrace{\delta\mathcal{Z}_W}_{\text{ren. constant}}\right) W^\pm,$ etc.

- complex $\delta\mathcal{Z}_W$ applies to W^+ and W^- $\Rightarrow$ $(W^\pm)^\dagger \neq W^\pm$
- $\delta\mathcal{Z}_W$ drops out in S-matrix elements without external W bosons

Complex renormalization for W bosons explicitly:

On-shell renormalization conditions for renormalized (transverse) self-energy

$$\hat{\Sigma}_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2) \;=\; 0, \qquad \hat{\Sigma}_{\mathrm{T}}^{\prime W}(\mu_{\mathrm{W}}^2) \;=\; 0$$

$\hookrightarrow$ μ_{W}^2 is location of propagator pole, and residue = 1

Solution of renormalization conditions:

$$\delta\mu_{\mathrm{W}}^2 \;=\; \Sigma_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2), \qquad \delta\mathcal{Z}_W \;=\; -\Sigma_{\mathrm{T}}^{\prime W}(\mu_{\mathrm{W}}^2)$$

Note: evaluation of $\Sigma_{\mathrm{T}}^{W}(p^2)$ at complex p^2 can be avoided

$$\Sigma_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2) \;=\; \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2) \;+\; (\mu_{\mathrm{W}}^2 - M_{\mathrm{W}}^2)\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2) \;+\; \underbrace{\mathcal{O}(\alpha^3)}_{\text{beyond one loop and finite}}$$

$\Rightarrow$ Renormalized W self-energy:

$$\hat{\Sigma}_{\mathrm{T}}^{W}(p^2) \;=\; \Sigma_{\mathrm{T}}^{W}(p^2) - \delta M_{\mathrm{W}}^2 + (p^2 - M_{\mathrm{W}}^2)\delta Z_W$$

with $\quad \delta M_{\mathrm{W}}^2 \;=\; \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2), \qquad \delta Z_W \;=\; -\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2)$

Differences to the usual on-shell scheme:

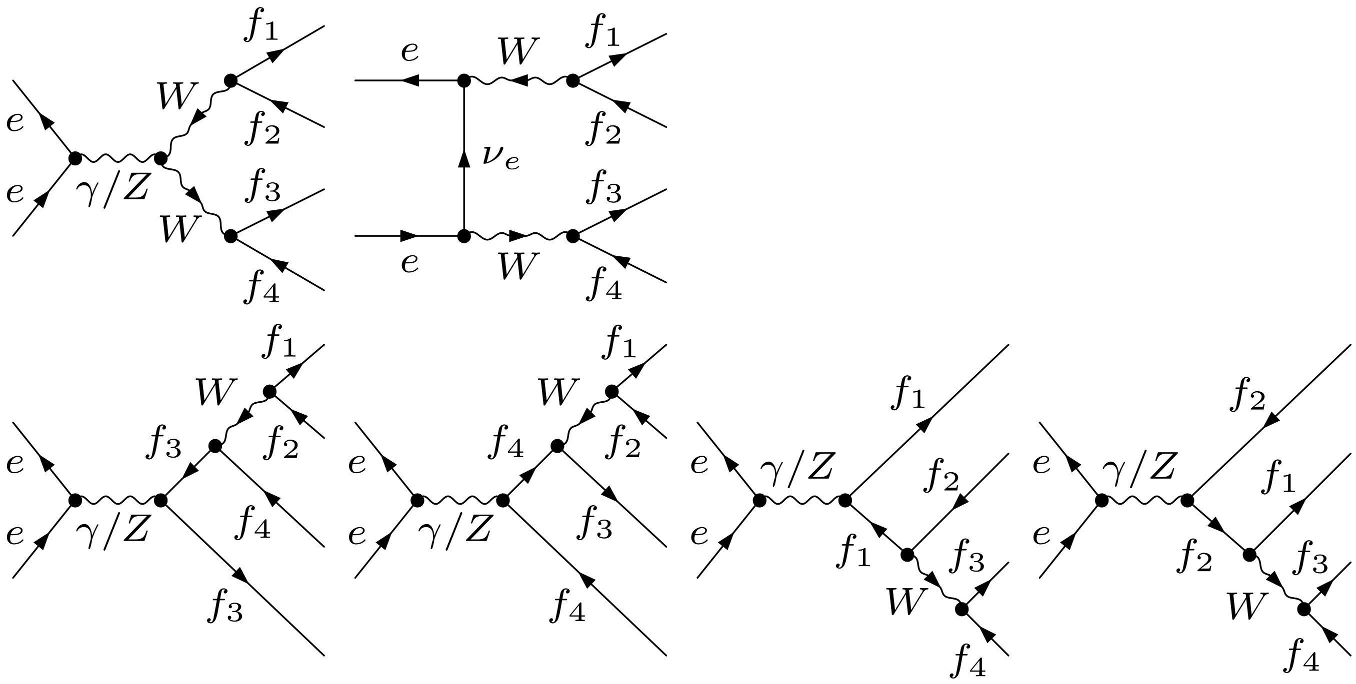
- no real parts taken from Σ_{T}^{W}
- Σ_{T}^{W} evaluated with complex masses and couplings

Full $\mathcal{O}(\alpha)$ corrections to (charged-current) $\mathrm{e}^+\mathrm{e}^- \to 4f$

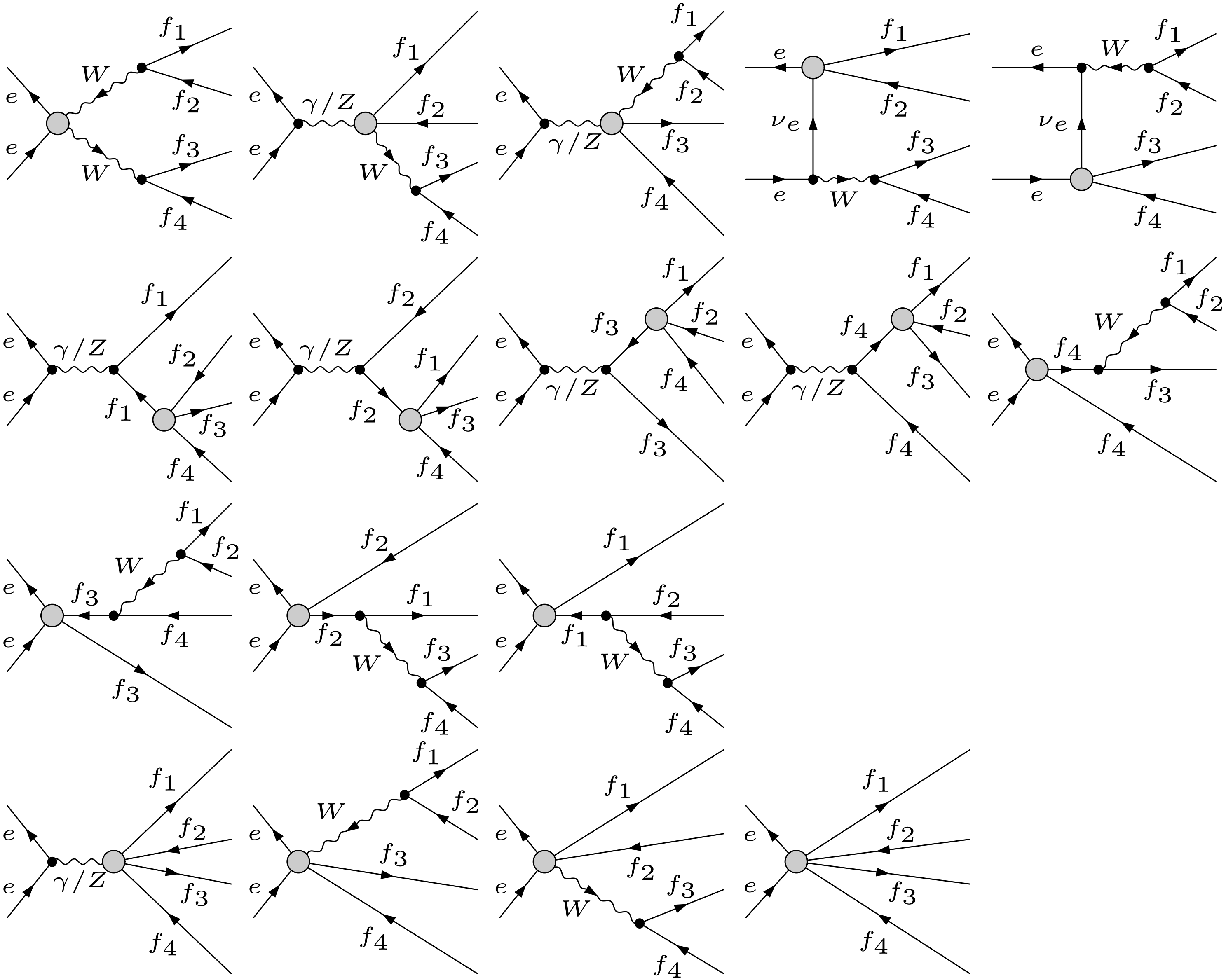
Features of the calculation:

- \# 1-loop diagrams $\sim$ 1200, loops up to 6-point integrals
- W resonances treated in the complex-mass scheme
- all loop integrals with complex W/Z masses
- new tensor reduction methods for stability in exceptional phase-space points
- real-photonic corrections taken from RACOONWW

11 lowest-order diagrams: (“CC11 class”)

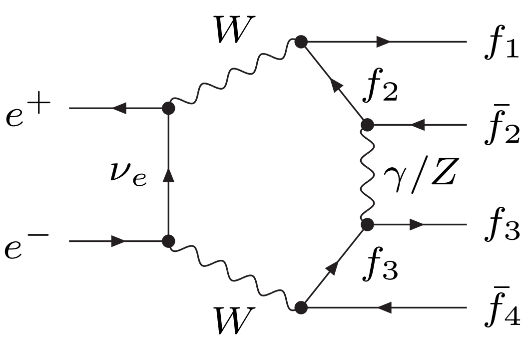


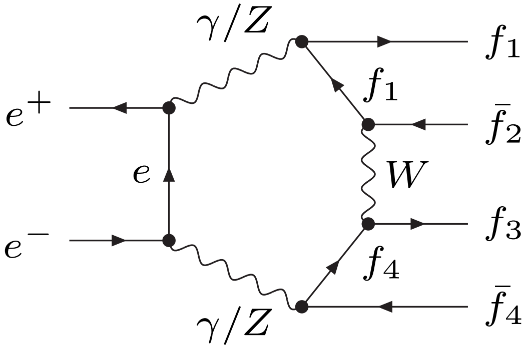
Generic diagrams for loop insertions (4-, 5-, 6-point functions)

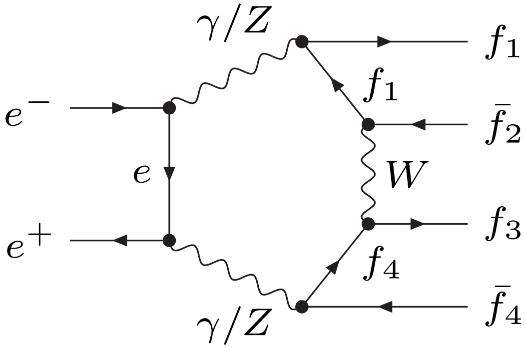


$\mathcal{O}(1200)$ one-loop diagrams per channel:

- 40 hexagons





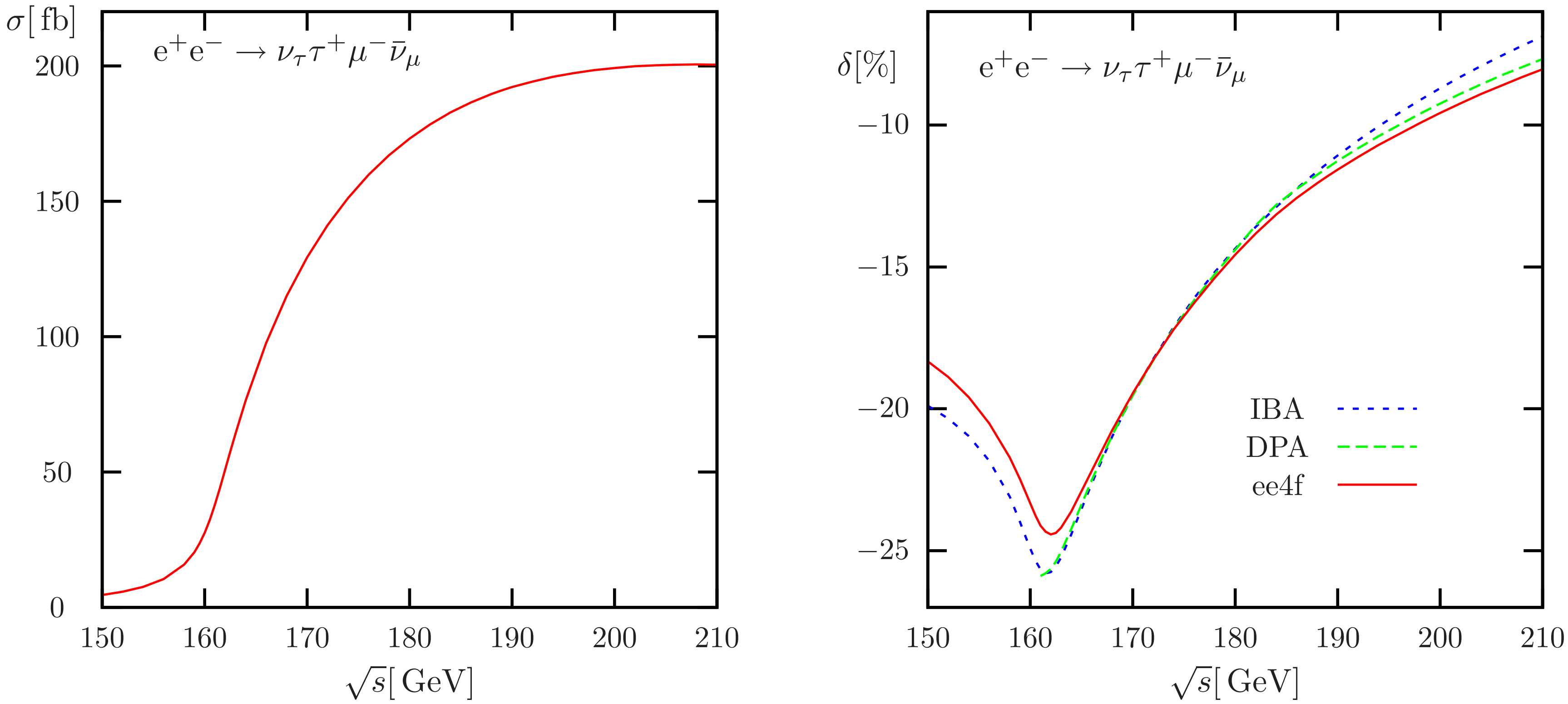


+ graphs with reversed fermion-number flow in final state

- 112 pentagons
- 227 boxes ('t Hooft–Feynman gauge)
- many vertex corrections and self-energy diagrams

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section – LEP2 energies

Denner, Dittmaier, Roth, Wieders ’05



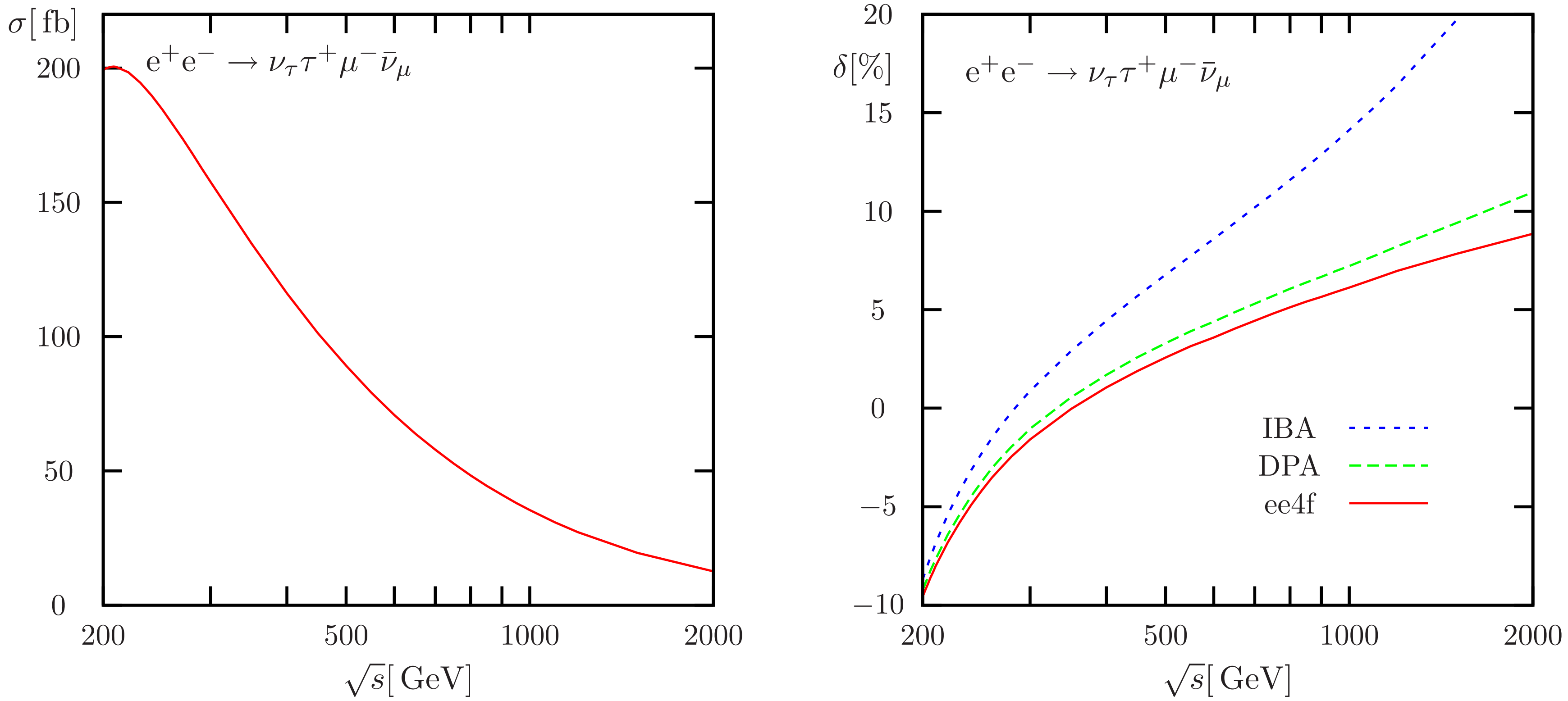
- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \quad 0.5\%$ for $170\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 210\,\mathrm{GeV}$
- $|\mathrm{ee4f} - \mathrm{IBA}| \quad \sim \quad 2\%$ for $\sqrt{s} \lesssim 170\,\mathrm{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and IBA

Remaining theoretical uncertainty from higher-order EW effects $\sim$ a few 0.1%

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section – ILC energies

Denner, Dittmaier, Roth, Wieders ’05



- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \ 0.7\%$ for $200\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 500\,\mathrm{GeV}$

  $\hookrightarrow$ agreement with error estimate of DPA

- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \ 1{-}2\%$ for $500\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 1{-}2\,\mathrm{TeV}$

A (not exhaustive) selection of literature

- Radiative corrections to resonance processes (see also references therein)
  - ◇ expansion about resonance poles (“pole scheme”):
    R. G. Stuart, Phys. Lett. B 262 (1991) 113;
    A. Aeppli, G. J. van Oldenborgh and D. Wyler, Nucl. Phys. B 428 (1994) 126 [hep-ph/9312212];
    H. G. J. Veltman, Z. Phys. C 62 (1994) 35.
  - ◇ electroweak corrections to Drell–Yan-like W production:
    U. Baur, S. Keller and D. Wackeroth, Phys. Rev. D 59 (1999) 013002 [hep-ph/9807417];
    S. Dittmaier and M. Krämer, Phys. Rev. D 65 (2002) 073007 [hep-ph/0109062];
    U. Baur and D. Wackeroth, Phys. Rev. D 70 (2004) 073015 [hep-ph/0405191].
  - ◇ $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$ in DPA:
    W. Beenakker, F. A. Berends and A. P. Chapovsky, Nucl. Phys. B 548 (1999) 3 [hep-ph/9811481];
    S. Jadach *et al.*, Phys. Rev. D 61 (2000) 113010 [hep-ph/9907436];
    A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B 587 (2000) 67 [hep-ph/0006307].
  - ◇ $\mathrm{e}^+\mathrm{e}^- \to 4f$ and complex-mass scheme at one loop:
    A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Phys. Lett. B 612 (2005) 223 [hep-ph/0502063] and Nucl. Phys. B 724 (2005) 247 [hep-ph/0505042].