Introduction into Standard Model and Precision Physics

– Lecture IV –

Stefan Dittmaier

MPI München

General overview

9 Unstable particles in quantum field theory

9.1 Introduction

Almost all interesting elementary particles are unstable:

- known: leptons μ, τ and massive gauge bosons Z, $\mathrm{W}^{\pm}$, etc.
- Higgs bosons: H_{SM}, $\{\mathrm{h}, \mathrm{H}, \mathrm{A}, \mathrm{H}^{\pm}\}_{\mathrm{MSSM}}$
- postulated new particles, e.g. in SUSY: $\tilde{l}$, $\tilde{q}$, $\tilde{g}$, $\tilde{\chi}$ (maybe apart from LSP)

Lifetimes τ too short for detection (e.g. $\tau_{\mathrm{W,Z}} \sim 10^{-25}\mathrm{s} \;\to\; \Delta l = c\tau \sim 10^{-16}\mathrm{m}$)

$\hookrightarrow$ only decay products detected,
unstable particles appear as resonances in certain observables

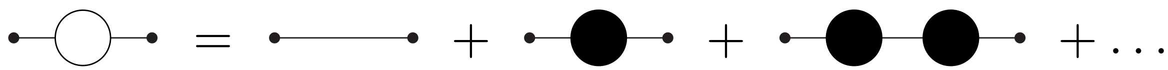
Examples: $\mathrm{e}^+\mathrm{e}^- \to \mathrm{Z} \to f\bar{f}$, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{t\bar{t}} \to 6f$,
$\mathrm{pp} \to \mathrm{W/Z} \to 2l$, $\mathrm{pp} \to \mathrm{H}+2q \to \mathrm{ZZ}+2q \to 4l+2\mathrm{jets}$, etc.

$\Rightarrow$ Consistent treatment of unstable particles needed
in perturbative evaluation of quantum field theories

9.2 Mass and width of unstable particles

Dyson series and propagator poles

Propagator near resonance: (scalar example)



$$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2-m^2} + \ldots = \frac{\mathrm{i}}{p^2-m^2+\Sigma(p^2)}$$

$\Sigma(p^2)$ = renormalized self-energy, m = ren. mass

Stable particle: $\mathrm{Im}\{\Sigma(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma(m^2) = 0$

Unstable particle: $\mathrm{Im}\{\Sigma(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole shifted into complex p^2 plane,
definition of mass and width non-trivial

Commonly used mass/width definitions:

- "on-shell mass/width" $M_{\rm OS}/\Gamma_{\rm OS}$: $\quad M_{\rm OS}^2 - m^2 + {\rm Re}\{\Sigma(M_{\rm OS}^2)\} \stackrel{!}{=} 0$

$$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2\to M_{\rm OS}^2}{\widetilde{\qquad}} \frac{1}{(p^2 - M_{\rm OS}^2)(1+{\rm Re}\{\Sigma'(M_{\rm OS}^2)\}) + {\rm i}\,{\rm Im}\{\Sigma(M_{\rm OS}^2)\}}$$

comparison with form of Breit–Wigner resonance $\quad \dfrac{R_{\rm OS}}{p^2 - m^2 + {\rm i}m\Gamma}$

yields: $\quad M_{\rm OS}\Gamma_{\rm OS} \equiv {\rm Im}\{\Sigma(M_{\rm OS}^2)\} \,/\, (1+{\rm Re}\{\Sigma'(M_{\rm OS}^2)\}), \qquad \Sigma'(p^2) \equiv \frac{\partial\Sigma(p^2)}{\partial p^2}$

- "pole mass/width" M/Γ: $\quad \mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

complex pole position: $\quad \mu^2 \equiv M^2 - {\rm i}M\Gamma$

$$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2\to \mu^2}{\widetilde{\qquad}} \frac{1}{(p^2-\mu^2)[1+\Sigma'(\mu^2)]} = \frac{R}{p^2 - M^2 + {\rm i}M\Gamma}$$

Note: $\quad \mu =$ gauge independent for any particle (pole location is property of S-matrix)

$M_{\rm OS} =$ gauge dependent at 2-loop order $\quad$ Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Relation between “on-shell” and “pole” definitions:

Subtraction of defining equations yields:

$$M_{\rm OS}^2 + {\rm Re}\{\Sigma(M_{\rm OS}^2)\} \;=\; M^2 - {\rm i}M\Gamma + \Sigma(M^2 - {\rm i}M\Gamma)$$

Equation can be uniquely solved via recursion in powers of coupling α:

ansatz:
$$M_{\rm OS}^2 = M^2 + c_1\alpha^1 + c_2\alpha^2 + \dots$$
$$M_{\rm OS}\Gamma_{\rm OS} = M\Gamma + d_2\alpha^2 + d_3\alpha^3 + \dots, \qquad c_i, d_i = \text{real}$$

counting in α: $\quad M_{\rm OS}, M = \mathcal{O}(\alpha^0), \quad \Gamma_{\rm OS}, \Gamma, \Sigma(p^2) = \mathcal{O}(\alpha^1)$

Result:

$$M_{\rm OS}^2 \;=\; M^2 + {\rm Im}\{\Sigma(M^2)\}\,{\rm Im}\{\Sigma'(M^2)\} \;+\; \mathcal{O}(\alpha^3)$$

$$M_{\rm OS}\Gamma_{\rm OS} \;=\; M\Gamma + {\rm Im}\{\Sigma(M^2)\}\,{\rm Im}\{\Sigma'(M^2)\}^2 + \tfrac{1}{2}\,{\rm Im}\{\Sigma(M^2)\}^2\,{\rm Im}\{\Sigma''(M^2)\} \;+\; \mathcal{O}(\alpha^4)$$

i.e. $\{M_{\rm OS}, \Gamma_{\rm OS}\} \;=\; \{M, \Gamma\} \;+$ gauge-dependent 2-loop corrections

Important examples: W and Z bosons

In good approximation: $\mathrm{W} \to f\bar{f}'$, $\mathrm{Z} \to f\bar{f}$ with masses fermions f, f'

so that: $$\mathrm{Im}\{\Sigma_{\mathrm{T}}^{\mathrm{V}}(p^2)\} = p^2 \times \frac{\Gamma_{\mathrm{V}}}{M_{\mathrm{V}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

$$\hookrightarrow \; M_{\mathrm{OS}}^2 = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3) \qquad M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} = M\Gamma + \frac{\Gamma^3}{M} + \mathcal{O}(\alpha^4)$$

In terms of measured numbers:

W boson: $M_{\mathrm{W}} \approx 80\,\mathrm{GeV}$, $\Gamma_{\mathrm{W}} \approx 2.1\,\mathrm{GeV}$

$\hookrightarrow \; M_{\mathrm{W,OS}} - M_{\mathrm{W,pole}} \approx 28\,\mathrm{MeV}$

Z boson: $M_{\mathrm{Z}} \approx 91\,\mathrm{GeV}$, $\Gamma_{\mathrm{Z}} \approx 2.5\,\mathrm{GeV}$

$\hookrightarrow \; M_{\mathrm{Z,OS}} - M_{\mathrm{Z,pole}} \approx 34\,\mathrm{MeV}$

Exp. accuracy: $\Delta M_{\mathrm{W,exp}} = 29\,\mathrm{MeV}$, $\Delta M_{\mathrm{Z,exp}} = 2.1\,\mathrm{MeV}$

$\hookrightarrow$ Difference in definitions phenomenologically important !

A closer look into resonance shapes:

- "on-shell mass/width" $M_{\rm OS}/\Gamma_{\rm OS}$: $\qquad M_{\rm OS}^2 - m^2 + {\rm Re}\{\Sigma(M_{\rm OS}^2)\} \stackrel{!}{=} 0$

$$
\begin{aligned}
G^{\phi\phi}(p) &= \frac{1}{p^2 - M_{\rm OS}^2 + \Sigma(p^2) - {\rm Re}\{\Sigma(M_{\rm OS}^2)\}} \\
&\underbrace{}_{p^2\to M_{\rm OS}^2} \frac{1}{(p^2 - M_{\rm OS}^2)[1 + {\rm Re}\{\Sigma'(M_{\rm OS}^2)\}] + {\rm i}\,{\rm Im}\{\Sigma(p^2)\} \;+\; \mathcal{O}[(p^2 - M_{\rm OS}^2)^2]} \\
&= \frac{R_{\rm OS}}{p^2 - M_{\rm OS}^2 + {\rm i} M_{\rm OS}\Gamma_{\rm OS}(p^2) \;+\; \mathcal{O}[(p^2 - M_{\rm OS}^2)^2]}
\end{aligned}
$$

with the "running on-shell width" $\quad \Gamma_{\rm OS}(p^2) = \dfrac{{\rm Im}\{\Sigma(p^2)\}}{M_{\rm OS}[1 + {\rm Re}\{\Sigma'(M_{\rm OS}^2)\}]}$

- "pole mass/width" M/Γ: $\qquad \mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

$$
\begin{aligned}
G^{\phi\phi}(p) &= \frac{1}{p^2 - \mu^2 + \Sigma(p^2) - \Sigma(\mu^2)} \\
&\underbrace{}_{p^2\to \mu^2} \frac{1}{(p^2 - \mu^2)[1 + \Sigma'(\mu^2)] \;+\; \mathcal{O}[(p^2 - \mu^2)^2]} = \frac{R}{(p^2 - \mu^2) \;+\; \mathcal{O}[(p^2 - \mu^2)^2]}
\end{aligned}
$$

Example of W and Z bosons continued:

Approximation of massless decay fermions:

$$\Gamma_{\mathrm{V,OS}}(p^2) \;=\; \Gamma_{\mathrm{V,OS}} \times \frac{p^2}{M^2_{\mathrm{V,OS}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

Fit of W/Z resonance shapes to experimental data:

- ansatz $\left|\dfrac{R'}{p^2 - m'^2 + \mathrm{i}\gamma' p^2/m'}\right|^2$ yields: $m' = M_{\mathrm{V,OS}}, \quad \gamma' = \Gamma_{\mathrm{V,OS}}$
- ansatz $\left|\dfrac{R}{p^2 - m^2 + \mathrm{i}\gamma m}\right|^2$ yields: $m = M_{\mathrm{V,pole}}, \quad \gamma = \Gamma_{\mathrm{V,pole}}$

Note: the two forms are equivalent:

$$R = \frac{R'}{1 + \mathrm{i}\gamma'/m'}, \quad m^2 = \frac{m'^2}{1 + \gamma'^2/m'^2}, \quad m\gamma = \frac{m'\gamma'}{1 + \gamma'^2/m'^2}$$

$\hookrightarrow$ consistent with relation between “on-shell” and “pole” definitions !

9.3 Complex mass and decay widths

Free propagator with finite width:

$$G(x-y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}p(x-y)} \frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}M\Gamma}, \qquad \tilde{E}_p = \sqrt{\mathbf{p}^2 + M^2 - \mathrm{i}M\Gamma}$$

$$= \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{p}(\mathbf{x}-\mathbf{y})} \int \frac{\mathrm{d}p_0}{2\pi}\, \mathrm{e}^{-\mathrm{i}p_0(x_0-y_0)} \frac{\mathrm{i}}{2\tilde{E}_p} \left(\frac{1}{p_0 - \tilde{E}_p} - \frac{1}{p_0 + \tilde{E}_p} \right)$$

Contour integration in p_0 plane yields

$$G(x-y) = \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{p}(\mathbf{x}-\mathbf{y})} \frac{1}{2\tilde{E}_p} \left[\theta(x_0-y_0)\mathrm{e}^{-\mathrm{i}(x_0-y_0)\tilde{E}_p} + \theta(y_0-x_0)\mathrm{e}^{\mathrm{i}(x_0-y_0)\tilde{E}_p} \right]$$

For $\Gamma \ll M$: $\quad \tilde{E}_p \approx E_p - \mathrm{i}\Gamma M/(2E_p), \quad E_p = \sqrt{\mathbf{p}^2 + M^2}$

$$G(x-y) = \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\mathbf{p}(\mathbf{x}-\mathbf{y})} \frac{1}{2E_p} \Big[\theta(x_0-y_0)\mathrm{e}^{-\mathrm{i}(x_0-y_0)E_p}\, \mathrm{e}^{-(x_0-y_0)\Gamma m/(2E_p)} + \theta(y_0-x_0)\mathrm{e}^{\mathrm{i}(x_0-y_0)E_p}\, \mathrm{e}^{-(y_0-x_0)\Gamma m/(2E_p)} \Big]$$

Exponential decay in particle and antiparticle propagation $x_0 - y_0 \gtrless 0$:

$|G|^2 \propto \mathrm{e}^{\mp(x_0-y_0)\Gamma_p}$ with $\Gamma_p = \Gamma M/E_p = \Gamma/\gamma =$ width of particle with momentum p

Conventional definition of decay widths via amplitudes:

Partial decay widths for $\phi \to f$:

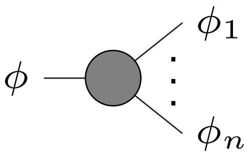
$$\Gamma_{\phi\to f,\mathrm{conv}} = \underbrace{\frac{1}{2m}}_{\text{flux factor}} \int \mathrm{d}\Phi_{\phi\to f}\, \left|\mathcal{M}_{\phi\to f}\right|^2$$

Comments:

- Lorentz-invariant phase space for final state $|f\rangle = |\phi_1(k_1),\dots,\phi_n(k_n)\rangle$:

$$\int \mathrm{d}\Phi_{\phi\to f} = \left[\prod_{l=1}^{n} \int \frac{\mathrm{d}^4 k_l}{(2\pi)^4}\,(2\pi)\delta(k_l^2 - m_l^2)\theta(k_l^0)\right](2\pi)^4\delta\left(p - \textstyle\sum_{m=1}^{n} k_m\right)$$

- Transition matrix element $\mathcal{M}_{\phi\to f}$ calculated from diagrams



  Note: $\mathcal{M}_{\phi\to f}$ involves external unstable particle ϕ

  $\hookrightarrow$ problems expected in higher orders !

- Mass definition of ϕ relevant

  $\hookrightarrow$ usual choice at 1–2 loops: $m = M_{\mathrm{OS}}$

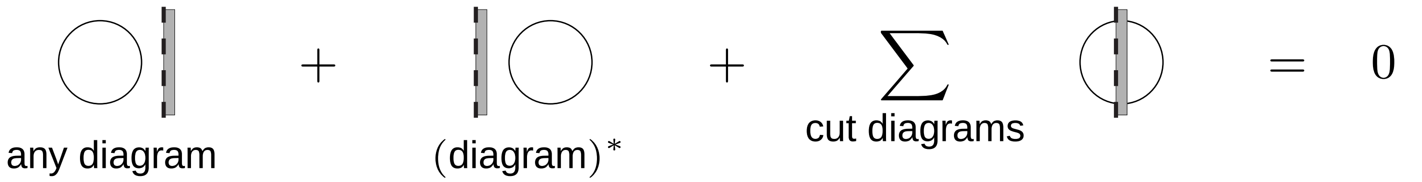
Total decay width:

$$\Gamma_{\mathrm{conv}} = \sum_f \Gamma_{\phi\to f,\mathrm{conv}}$$

$\hookrightarrow$ Relation between Γ_{conv} and “on-shell” / pole definitions ? Answer by unitarity...

9.4 Unstable particles and unitarity in (perturbative) QFT

Causality implies Cutkowsky cut rules for diagrams:



cut propagators: $2\pi\,\delta(p^2 - m^2)\,\theta(p_0)$

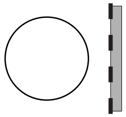
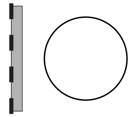
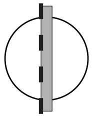
$=$ phase space for free propagation

$\hookrightarrow$ sum over connected diagrams: $(S - \mathbf{1}) + (S - \mathbf{1})^\dagger + (S - \mathbf{1})(S - \mathbf{1})^\dagger = 0$

$\Rightarrow$ unitarity of S-matrix: $SS^\dagger = \mathbf{1}$

9.4 Unstable particles and unitarity in (perturbative) QFT

Causality implies Cutkowsky cut rules for diagrams:

 $+$  $+$ $\sum$  $= \quad 0$

any diagram (diagram)* cut diagrams

cut propagators: $2\pi\,\delta(p^2 - m^2)\,\theta(p_0)$

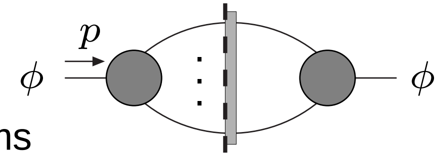
$=$ phase space for free propagation

$\hookrightarrow$ sum over connected diagrams: $(S - \mathbf{1}) + (S - \mathbf{1})^\dagger + (S - \mathbf{1})(S - \mathbf{1})^\dagger = 0$

$\Rightarrow$ unitarity of S-matrix: $SS^\dagger = \mathbf{1}$

Application to self-energy $\quad \rightarrow$ relation to conventional decay width $\Gamma_{\rm conv}$

$$\mathrm{Im}\{\Sigma^{\phi\phi}(p^2)\}\Big|_{p^2=m^2} = \frac{1}{2} \sum_{\text{cut diagrams}}$$



$$= \frac{1}{2} \sum_{\text{decay channels } f} \underbrace{\int \mathrm{d}\Phi_{\phi\to f}\, |\mathcal{M}_{\phi\to f}|^2}_{= 2m\,\Gamma_{\phi\to f,\rm conv}} = m\,\Gamma_{\rm conv}$$

$\Rightarrow \quad \Gamma_{\phi\to f,\rm conv} = \Gamma_{\phi\to f,\rm OS}$ for mass definition $m = M_{\rm OS}$

Note: "derivation" quite sloppy (ignores problems in defining $\mathcal{M}_{\phi\to f}$ in higher orders !)

Subtleties with S-matrix elements and unitarity

S-matrix elements:

Definition for $|i\rangle \to |f\rangle$: $$\langle f|S|i\rangle = \lim_{\substack{t_0\to-\infty \\ t_1\to+\infty}} \langle f,t_1|S|i,t_0\rangle$$

But: for unstable states $|f\rangle$: $$\lim_{t_1\to+\infty} \langle f,t_1| = 0$$

$\hookrightarrow$ S-matrix elements for external unstable particles do not exist,
application of LSZ reduction not justified !

Unitarity:

Cut equations not consistent for external or internal unstable particles !

But: important result of Veltman '63

Toy field theory with stable and unstable scalar fields
$\hookrightarrow$ theory is unitary, causal, and renormalizable on space of stable external states

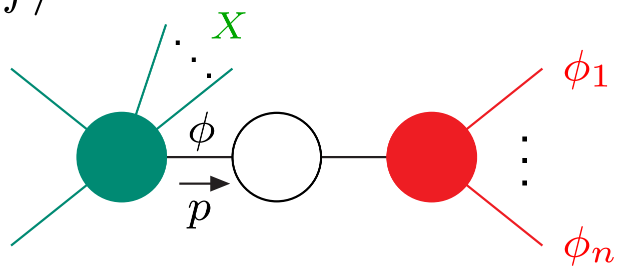
Comments on Veltman's result:

- cut equations: no cuts of internal propagators for unstable particles
- statement rests on consideration of "complete" (resummed) propagators
  $\hookrightarrow$ does not provide a practical method for standard perturbation theory

9.5 Resonances – factorization into production and decay subprocesses

Transition rate near a resonance: $|i\rangle \;\to\; |X,\phi(p)\rangle \;\to\; |X,f\rangle$

$$\int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to Xf}|^2 \;\underset{p^2\to m^2}{\sim}\; \int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to X\phi\to Xf}|^2$$



Phase-space factorization:

$$\int \mathrm{d}\Phi_{i\to Xf} \;=\; \int \frac{\mathrm{d}p^2}{2\pi} \int \mathrm{d}\Phi_{i\to X\phi(p)} \int \mathrm{d}\Phi_{\phi(p)\to f}$$

Decomposition of resonance diagrams:

$$\mathcal{M}_{i\to X\phi\to Xf} \;=\; \sum_\lambda \mathcal{M}^{(\lambda)}_{i\to X\phi}\,\frac{1}{p^2-m^2+\mathrm{i}m\Gamma}\,\mathcal{M}^{(\lambda)}_{\phi\to f}, \qquad \lambda = \text{polarization index of } \phi$$

$\hookrightarrow$ total rate proportional to (hat on $\hat{\Phi}$, $\hat{\mathcal{M}}$ means $p^2{=}m^2$ used)

$$\int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to Xf}|^2 \;\underset{p^2\to m^2}{\sim}\; \sum_{\lambda,\lambda'} \int \mathrm{d}\hat{\Phi}_{i\to X\phi(p)}\,\hat{\mathcal{M}}^{(\lambda)}_{i\to X\phi}\left(\hat{\mathcal{M}}^{(\lambda')}_{i\to X\phi}\right)^*$$

$$\times \int \frac{\mathrm{d}p^2}{2\pi}\,\frac{1}{|p^2-m^2+\mathrm{i}m\Gamma|^2}\,\underbrace{\int \mathrm{d}\hat{\Phi}_{\phi(p)\to f}\,\hat{\mathcal{M}}^{(\lambda)}_{\phi\to f}\left(\hat{\mathcal{M}}^{(\lambda')}_{\phi\to f}\right)^*}_{=D_{\lambda\lambda'}\ \text{(“decay correlation”)}}$$

Manipulations for total rate:

- Rotational invariance in ϕ rest frame implies for full integral $\int \mathrm{d}\Phi_{\phi(p)\to f}$
  $$D_{\lambda\lambda'} = \delta_{\lambda\lambda'}\, \bar{D}, \qquad \text{with spin average} \quad \bar{D} = 2m\Gamma_{\phi\to f,\mathrm{conv}}$$
- "Narrow-width approximation" (NWA) for resonance factor:
  $$\frac{1}{|p^2-m^2+\mathrm{i}m\Gamma|^2} = \frac{1}{(p^2-m^2)^2+m^2\Gamma^2} \underset{\Gamma\to 0}{\widetilde{\quad}} \frac{\pi}{m\Gamma}\,\delta(p^2-m^2)$$

Resulting NWA for total rate and total cross section:

$$\int \mathrm{d}\Phi_{i\to Xf}\,|\mathcal{M}_{i\to Xf}|^2 \underset{p^2\to m^2}{\longrightarrow} \sum_\lambda \int \mathrm{d}\hat{\Phi}_{i\to X\phi(p)}\,|\hat{\mathcal{M}}^{(\lambda)}_{i\to X\phi}|^2 \underbrace{\frac{\Gamma_{\phi\to f,\mathrm{conv}}}{\Gamma}}_{=\mathrm{BR}_{\phi\to f}} \text{ ("branching ratio")}$$

$$\Rightarrow \sigma^{\mathrm{NWA}}_{i\to Xf} = \sigma_{i\to X\phi}\,\mathrm{BR}_{\phi\to f} \qquad \text{with} \quad \sum_f \mathrm{BR}_{\phi\to f} = 1 \quad \text{if } \Gamma = \Gamma_{\mathrm{conv}}$$

Note: NWA insufficient to describe

- invariant-mass distributions of decay products (needed for resonance shape)
- angular distributions of decay products (needed for spin determination)
- "off-shell effects" resulting from regions with $|p^2-m^2| \gg m\Gamma$
  (in particular because of neglect of non-resonant diagrams)

9.6 The issue of gauge invariance

Gauge invariance implies...

- Slavnov–Taylor or Ward identites
  = algebraic relations of or between Greens functions
  $\hookrightarrow$ guarantee cancellation of unitarity-violating terms,
  crucial for proof of unitarity of S-matrix
- compensation of gauge-fixing artefacts
  = gauge-parameter independence of S-matrix
  although Greens function (e.g. self-energies) are gauge dependent

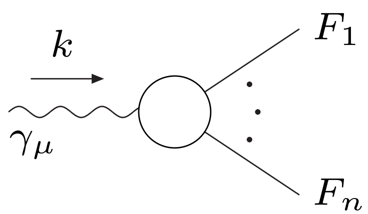
Both statements hold order by order in standard perturbation theory !

But: Resonances require Dyson summation of resonant propagators
$\hookrightarrow$ perturbative orders mixed
$\hookrightarrow$ gauge invariance jeopardized !

Note: Gauge-invariance-violating terms are formally of higher order,
but can be dramatically enhanced

Important Ward identities for processes with EW gauge bosons:

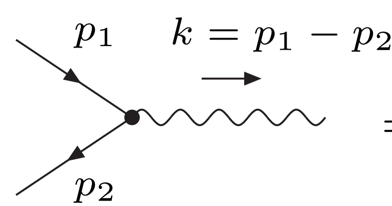
Elmg. U(1) gauge invariance implies

k^μ 

$= 0$ for any on-shell fields F_l

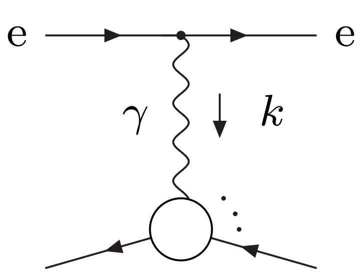
$\hookrightarrow$ Identity becomes crucial for collinear light fermions:

for fermion momenta $p_1 \sim c\, p_2$:



$$= \bar{u}_2(p_2)\gamma^\mu u_1(p_1) \;\propto\; k^\mu$$

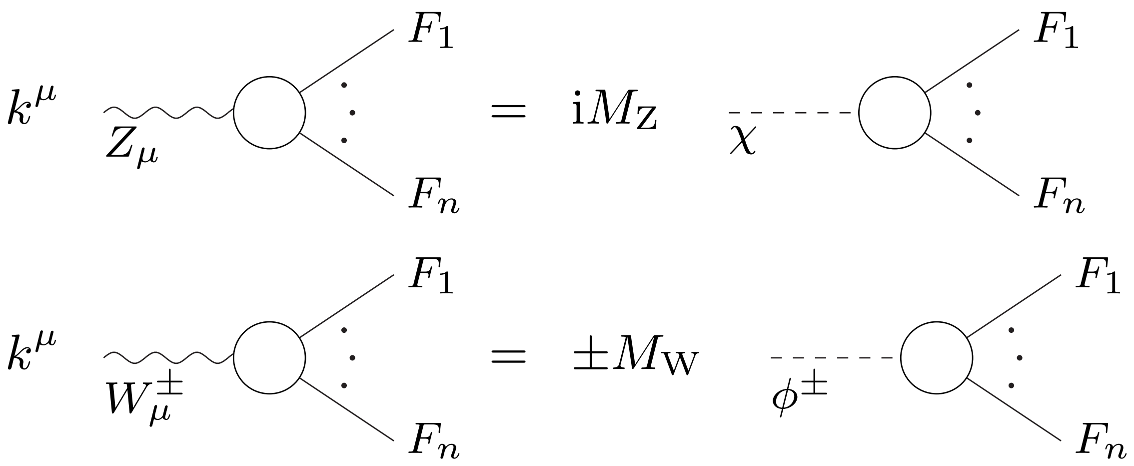
A typical situation: quasi-real space-like photons



$$\sim \frac{1}{k^2}\, k^\mu\, T^\gamma_\mu \quad \text{for} \quad k^2 \to \mathcal{O}(m_\mathrm{e}^2) \ll E^2$$

Identity $k^\mu\, T^\gamma_\mu = 0$ needed to cancel $1/k^2$,
otherwise gauge-invariance-breaking terms enhanced by E^2/m_e^2 ($\sim 10^{10}$ for LEP2)

Electroweak SU(2) gauge invariance implies

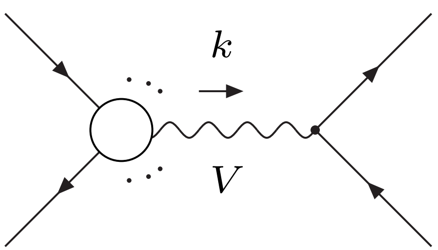


F_l = on-shell fields

$\chi, \phi^\pm$ = would-be Goldstone fields

A typical situation: high-energetic quasi-real longitudinal vector bosons

$\hookrightarrow$ fermion current attached to $\mathrm{V}(k)$ again $\propto k^\mu$



$$\sim \frac{1}{k^2 - M_\mathrm{V}^2} \, k^\mu \, T_\mu^V \qquad \text{for } k^0 \gg M_\mathrm{V}$$

Identity $k^\mu T_\mu^V = c_V M_\mathrm{V} T^S$ needed to cancel factor k^0,
otherwise gauge-invariance/unitarity-breaking terms enhanced by k^0/M_V

10 Lowest-order descriptions of resonance processes

10.1 Motivation

The final aim: a method to describe resonance processes in lowest order that

- is mathematically consistent, but simple to apply
- valid in resonant and non-resonant regions of phase space
- supports arbitrary differential distributions
- respects gauge invariance (at least controls breaking effects)
- respects unitarity (at least controls breaking effects)
- can be generalized to higher orders

$\hookrightarrow$ Aim is highly demanding,
different solutions proposed (with different strengths and weaknesses)

Discussed in the following:

naive "solutions" (propagator modifications, fudge factors, etc.),
"complex-mass scheme", "fermion-loop scheme", pole expansions

Not discussed:

proposals of effective field theories Beenakker et al. '00,'03; Beneke et al. '03,'04; Hoang,Reisser '04

Counting of orders in resonance processes:

- self-energy = loop effect: $\Sigma(p^2) \; = \; \mathcal{O}(\alpha)$
- width = higher-order effect: $m\Gamma \; = \; m^2\,\mathcal{O}(\alpha)$

$\hookrightarrow$ Propagator in resonance region and in the continuum:

$$\frac{m\Gamma}{p^2-m^2+\Sigma(p^2)} \sim \frac{m\Gamma}{p^2-m^2+\mathrm{i}m\Gamma} = \begin{cases} \mathcal{O}(1) & \text{for } |p^2-m^2| \sim m\Gamma \\ \mathcal{O}(\alpha) & \text{for } |p^2-m^2| \gg m\Gamma \end{cases}$$

Implications: [resonant part counted as $\mathcal{O}(1)$]

- higher-order corrections to resonant parts are of $\mathcal{O}(\alpha)$

  $\hookrightarrow$ (virtual+real) corrections to scattering matrix elements and to total width Γ in resonant denominator

- off-shell effects are generically of $\mathcal{O}(\Gamma/m) \; = \; \mathcal{O}(\alpha)$

  or in presence of phase-space cuts:

$$\int_{(m-\Delta)^2}^{(m+\Delta)^2} \mathrm{d}p^2\,|\mathcal{M}_{\mathrm{res}}|^2 \;\propto\; \int_{(m-\Delta)^2}^{(m+\Delta)^2} \mathrm{d}p^2\,\frac{1}{(p^2-m^2)^2+m^2\Gamma^2} \;\sim\; \frac{\pi}{m\Gamma}$$

$$\int_{(m-\Delta)^2}^{(m+\Delta)^2} \mathrm{d}p^2\,|\mathcal{M}_{\text{non-res}}|^2 \;\propto\; \mathcal{O}(\Delta) \qquad \Rightarrow\; \sigma_{\text{non-res}}/\sigma_{\text{res}} \;\sim\; \frac{\Delta\Gamma}{m^2} \quad \text{for } \Delta \gg \Gamma$$

10.2 Naive approaches

Naive propagator substitutions in full tree-level amplitudes:

$$\frac{1}{k^2 - m^2} \to \frac{1}{k^2 - m^2 + im\Gamma(k^2)}$$ for resonant or all propagators

- constant width $\Gamma(k^2) = \text{const.}$ $\to$ U(1) respected (if all propagators dressed), SU(2) “mildly” violated
- step width $\Gamma(k^2) \propto \theta(k^2)$ $\to$ U(1) and SU(2) violated
- running width $\Gamma(k^2) \propto \theta(k^2) \times k^2$ $\to$ U(1) and SU(2) violated
  $\hookrightarrow$ results can be totally wrong !

Fudge factor approaches:

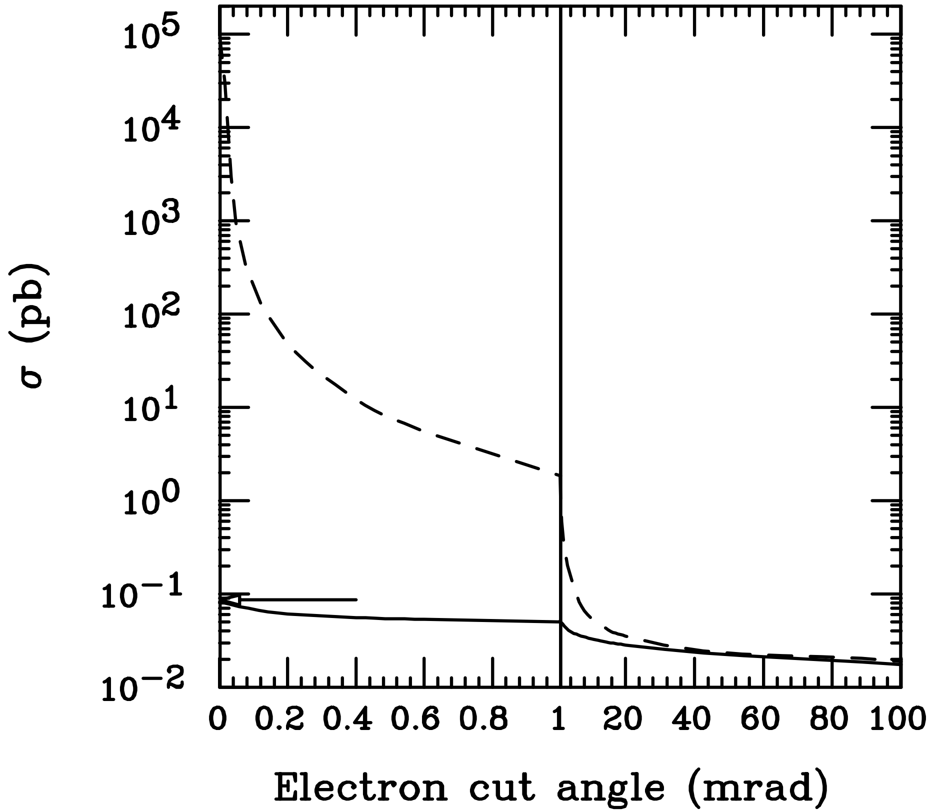
Multiply full amplitudes without widths with

factors $\dfrac{p^2 - m^2}{p^2 - m^2 + im\Gamma}$ for each potentially resonant propagator

$\hookrightarrow$ procedure preserves gauge invariance,
but introduces spurious factors of $\mathcal{O}(\Gamma/m)$

Note: none of these schemes preserves unitarity

An example: $e^-e^+ \to e^-\bar{\nu}_e u\bar{d}$ result of Kurihara, Perret-Gallix, Shimizu ’95



$\sqrt{s} = 180\,\mathrm{GeV}$

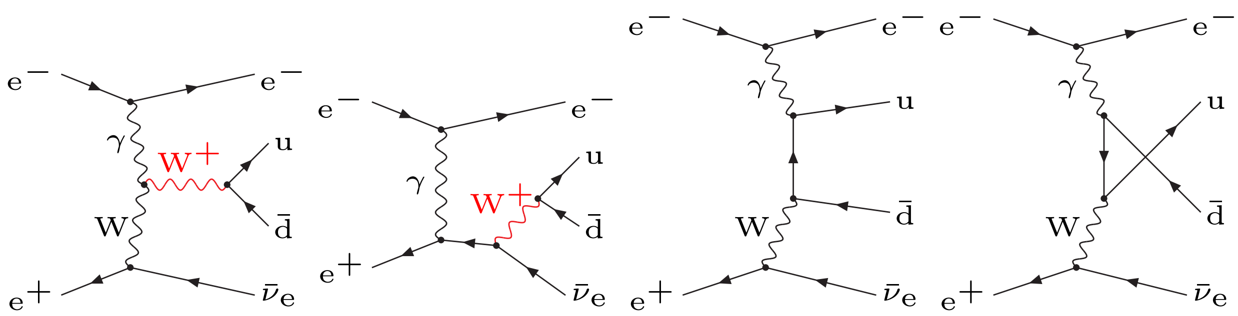
solid: gauge-invariant
(fudge factor) scheme

dashed: constant width
only in resonant propagator

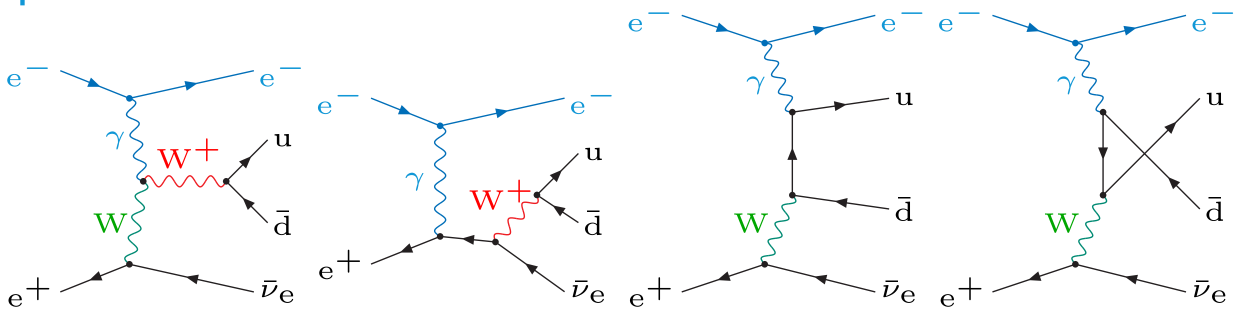
$\hookrightarrow$ crude U(1) gauge-invariance violation

Dominant diagrams:

nearly real photon !



Example continued:



Partial amplitude from above "photon diagrams":

$$\mathcal{M}_\gamma = Q_\mathrm{e} e\, \bar{u}_\mathrm{e}(k_\mathrm{e})\gamma^\mu u_\mathrm{e}(p_\mathrm{e})\, \frac{1}{k_\gamma^2}\, T_\mu^\gamma$$

Elmg. Ward identity:

$$0 \stackrel{!}{=} k_\gamma^\mu T_\mu^\gamma \propto (p_+^2 - p_-^2) Q_\mathrm{W} P_\mathrm{W}(p_+^2) P_\mathrm{W}(p_-^2) + Q_\mathrm{e} P_\mathrm{W}(p_+^2) - (Q_\mathrm{d} - Q_\mathrm{u}) P_\mathrm{W}(p_-^2)$$

With $\quad Q_\mathrm{W} = Q_\mathrm{e} = Q_\mathrm{d} - Q_\mathrm{u} \quad$ and $\quad P_\mathrm{W}(p^2) = [p^2 - M_\mathrm{W}^2 + \mathrm{i} M_\mathrm{W} \Gamma_\mathrm{W}(p^2)]^{-1}$

one obtains: $\quad \Gamma_\mathrm{W}(p_+^2) \stackrel{!}{=} \Gamma_\mathrm{W}(p_-^2)$

$\hookrightarrow$ Elmg. gauge invariance demands
common width on s- and t-channel propagators in "naive fixed width scheme"

10.3 Complex-mass scheme at tree level

Denner, Dittmaier, Roth, Wackeroth '99

Basic idea: mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ consistent use of complex masses everywhere !

Application to gauge-boson resonances:

- replace $M_\mathrm{W}^2 \to \mu_\mathrm{W}^2 = M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}, \qquad M_\mathrm{Z}^2 \to \mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i}M_\mathrm{Z}\Gamma_\mathrm{Z}$

  and define (complex) weak mixing angle via $c_\mathrm{W}^2 = 1 - s_\mathrm{W}^2 = \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$

  $\hookrightarrow$ preserves all algebraic relations among parameters and amplitudes

- virtue: gauge-invariant result !

  (Slavnov–Taylor identities and gauge-parameter independence)

  $\hookrightarrow$ unitarity cancellations respected !

- drawbacks:
  - $\diamond$ spurios terms of $\mathcal{O}(\frac{\Gamma}{m}) = \mathcal{O}(\alpha)$ (from off-shell propagators and complex mixing angle)

    $\hookrightarrow$ but these terms are beyond tree-level accuracy !
  - $\diamond$ cut equations not valid anymore (reformulation not yet worked out)

    $\hookrightarrow$ unitarity not yet understood, but possible unitarity violation is of $\mathcal{O}(\frac{\Gamma}{m})$

Examples: results from RACOONWW (Denner et al. '99-'01) and LUSIFER (Dittmaier,Roth '02)

- $\sigma[\,\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$

$\sqrt{s}$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	703.5(3)	237.4(1)	13.99(2)	0.624(3)
running width	703.4(3)	238.9(1)	34.39(3)	498.8(1)
complex mass	703.1(3)	237.3(1)	13.98(2)	0.624(3)

- $\sigma[\,\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu + \gamma$ (separation cuts for "visible" γ: $E_\gamma, \theta_{\gamma f} >$ cut)

$\sqrt{s} =$	189 GeV	500 GeV	2 TeV	10 TeV
constant width	224.0(4)	83.4(3)	6.98(5)	0.457(6)
running width	224.6(4)	84.2(3)	19.2(1)	368(6)
complex mass	223.9(4)	83.3(3)	6.98(5)	0.460(6)

- $\sigma[\,\mathrm{fb}]$ for $\mathrm{e^+e^-} \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mu^-\bar{\nu}_\mu\mathrm{u\bar{d}}$ (phase-space cuts applied)

$\sqrt{s}$	500 GeV	800 GeV	2 TeV	10 TeV
constant width	1.633(1)	4.105(4)	11.74(2)	26.38(6)
running width	1.640(1)	4.132(4)	12.88(1)	12965(12)
complex mass	1.633(1)	4.104(3)	11.73(1)	26.39(6)

10.4 Fermion-loop scheme

Argyres et al. ’95; Beenakker et al. ’96; Passarino ’99; Accomando et al. ’99

Procedure: Dyson summation of *all* closed fermion-loop graphs

Benefits of the scheme:

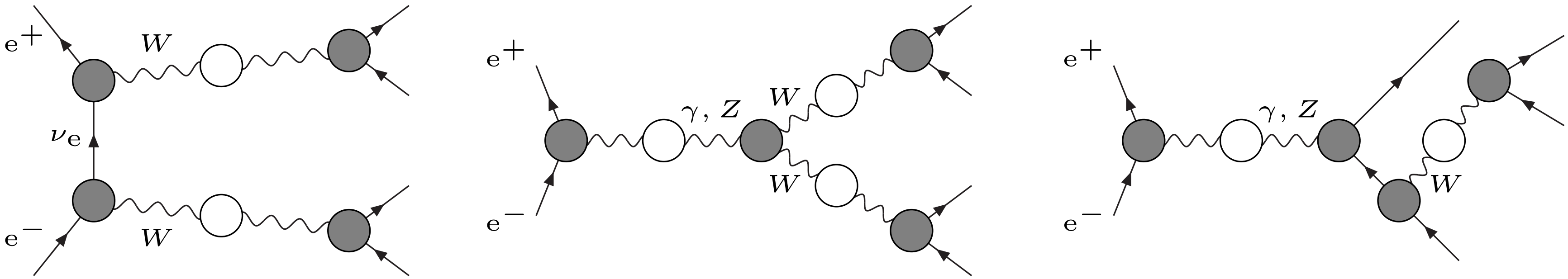
- introduction of widths via resummed self-energies
  for particles that decay into fermions only, e.g. W and Z bosons
- Ward identites (WI) maintained,
  because full set of diagrams of the form $\sum_f N_f^{\mathrm{colour}}$ is considered
- gauge-parameter independence,
  because gauge parameters do not enter loops, and WI are valid for “trees”
- natural inclusion of running-coupling effects possible
- no spurious terms included (selection of diagrams!)
- scheme has natural generalization to remaining “bosonic loops”
  $\hookrightarrow$ background-field quantization Denner, Dittmaier ’96

Drawbacks / limitations:

- width in one-loop self-energy is tree-level quantity
  $\hookrightarrow$ scheme does not include fermion-loop corrections to width
- no applicability to unstable particles that decay into bosons (top, Higgs)

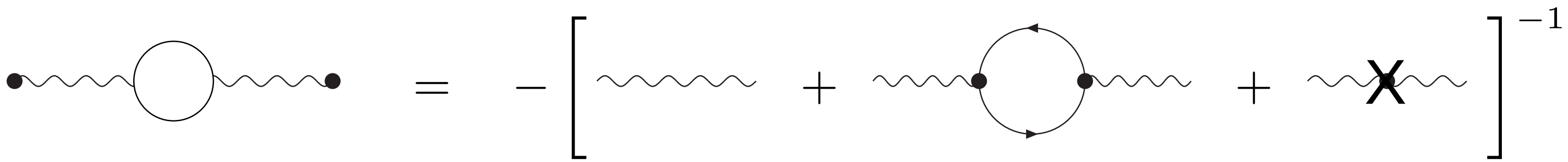
Example: $e^-e^+ \to 4f$

Structural diagrams:

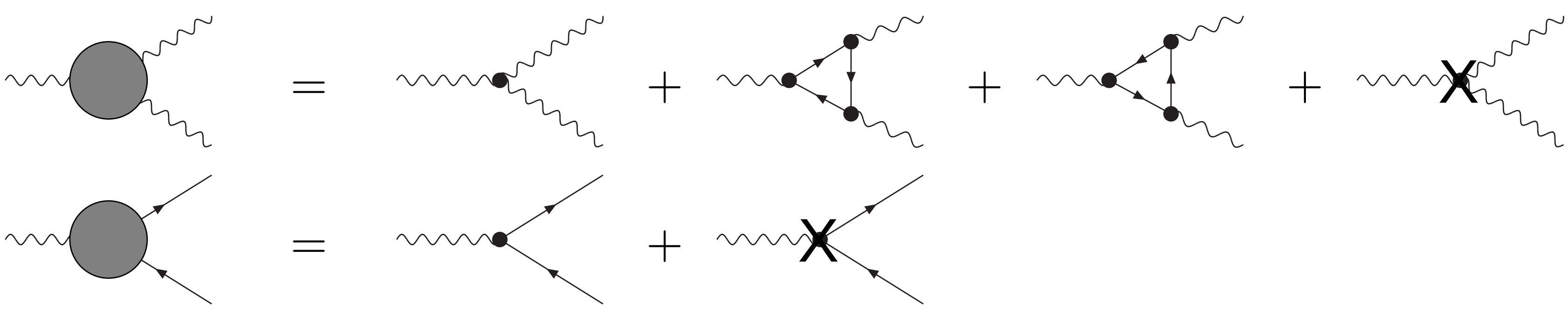


Building blocks:

- resummed propagators:



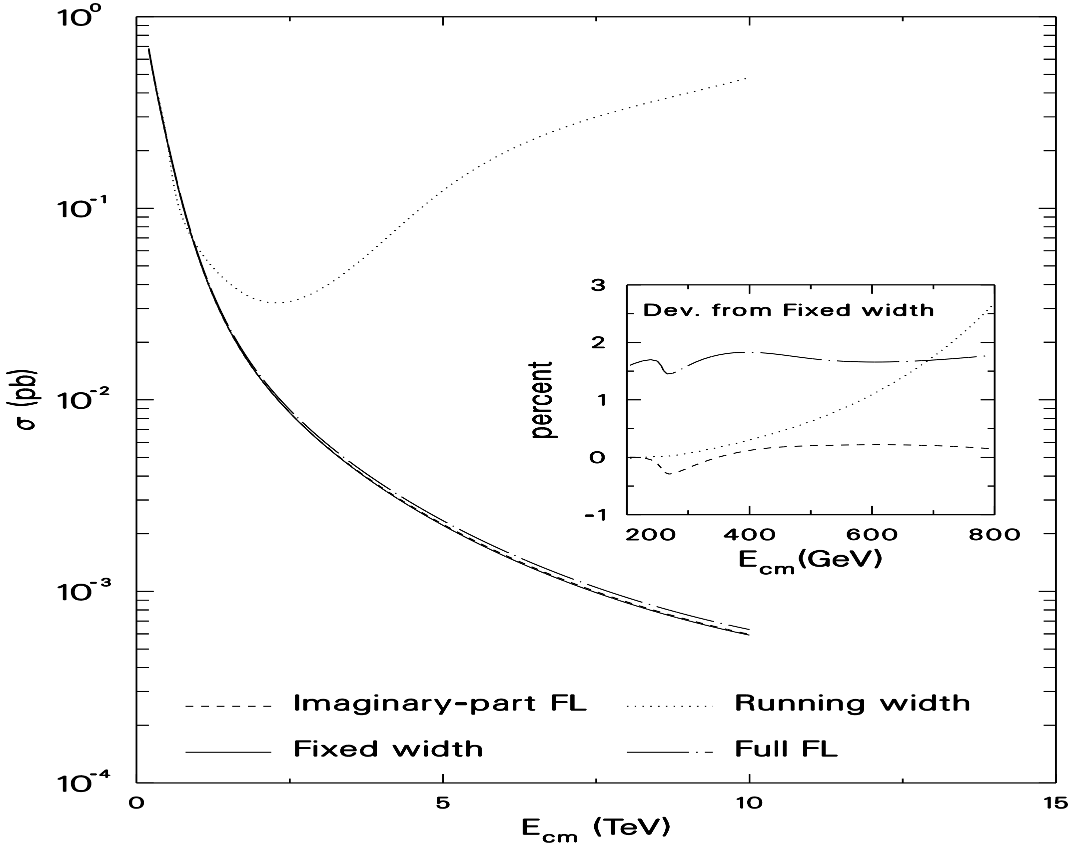
- corrected vertices:



Specific example: $\sigma[\mathrm{fb}]$ for $\mathrm{e}^-\mathrm{e}^+ \to \mu^-\bar{\nu}_\mu \mathrm{u}\bar{\mathrm{d}}$ at high energies

WTO (Passarino '96)

$\sqrt{s}$	200 GeV	500 GeV	1 TeV	2 TeV	5 TeV	10 TeV
running width	672.96(3)	225.45(3)	62.17(1)	33.06(1)	123.759(8)	481.18(5)
constant width	673.08(4)	224.05(3)	56.90(1)	13.19(1)	2.212(6)	0.591(4)
imaginary-part FLS	673.1(1)	224.5(7)	56.8(1)	13.18(4)	2.24(3)	0.597(6)
full FLS	683.7(1)	227.9(2)	58.0(1)	13.57(4)	2.34(3)	0.632(6)



A (not exhaustive) selection of literature

- Unstable particles in quantum field theory
  - ◇ mass and width of unstable particles:
    A. Sirlin, Phys. Rev. Lett. 67 (1991) 2127; Phys. Lett. B 267 (1991) 240;
    R. G. Stuart, Phys. Lett. B 262 (1991) 113; Phys. Rev. Lett. 70 (1993) 3193;
    M. Passera and A. Sirlin, Phys. Rev. Lett. 77 (1996) 4146 [hep-ph/9607253];
    P. Gambino and P. A. Grassi, Phys. Rev. D 62 (2000) 076002 [hep-ph/9907254];
    P. A. Grassi, B. A. Kniehl and A. Sirlin, Phys. Rev. D 65 (2002) 085001 [hep-ph/0109228].
  - ◇ unitarity and causality:
    M. J. G. Veltman, Physica 29 (1963) 186.
- Schemes for treating unstable particles in lowest-order amplitudes (see also references therein)
  - ◇ complex-mass scheme:
    A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B 560 (1999) 33 [hep-ph/9904472].
  - ◇ fermion-loop scheme:
    E. N. Argyres *et al.*, Phys. Lett. B 358(1995) 339 [hep-ph/9507216];
    W. Beenakker *et al.*, Nucl. Phys. B 500 (1997) 255 [hep-ph/9612260];
    G. Passarino, Nucl. Phys. B 574 (2000) 451 [hep-ph/9911482];
    E. Accomando, A. Ballestrero and E. Maina, Phys. Lett. B 479 (2000) 209 [hep-ph/9911489].
  - ◇ more discussions of gauge-invariance violation:
    Y. Kurihara, D. Perret-Gallix and Y. Shimizu, Phys. Lett. B 349 (1995) 367 [hep-ph/9412215];
    S. Dittmaier and M. Roth, Nucl. Phys. B 642 (2002) 307 [hep-ph/0206070].