Introduction into
Standard Model and Precision Physics
– Lecture I –

Stefan Dittmaier

MPI München

General overview

1 Electroweak phenomenology before the GSW model

Some phenomenological facts:

- discovery of the weak interaction via radioactive β-decay of nuclei:
  $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}, \qquad \mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$ (not possible for free protons)
- terminology "weak": interaction at low energy has very short range
  $\hookrightarrow$ long life time of weakly decaying particles:

strong int.:	$\rho \to 2\pi,$	$\tau \sim 10^{-22}$s
elmg. int.:	$\pi \to 2\gamma,$	$\tau \sim 10^{-16}$s
weak int.:	$\pi^- \to \mu^- + \bar{\nu}_\mu$	$\tau \sim 10^{-8}$s
	$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu,$	$\tau \sim 10^{-6}$s

- lepton-number conservation: $\mu^- \not\to \mathrm{e}^- + \gamma$ (BR $\lesssim 10^{-11}$)
  $\Rightarrow$ $L_\mathrm{e}, L_\mu, L_\tau$ individually conserved:
  $L_\mathrm{e} = +1$ for $\mathrm{e}^-, \nu_\mathrm{e}, \qquad L_\mathrm{e} = -1$ for $\mathrm{e}^+, \bar{\nu}_\mathrm{e},$ etc.

  (For massive ν's with different masses, only $L_\mathrm{e}+L_\mu+L_\tau$ is conserved.)
- parity violation (Wu et al. 1957):
  e.g.: $K^+ \to \underbrace{2\pi, 3\pi}$ final states of different parity
  $^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  $\hookrightarrow$ polarization inversion does not yield inversion of spectra

The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for "current–current interaction" of four fermions:

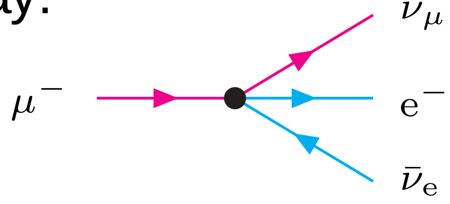
$$\mathcal{L}_{\rm Fermi}(x) = -2\sqrt{2}G_\mu J_\rho^\dagger(x) J^\rho(x), \qquad G_\mu = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

with $\quad J_\rho(x) = J_\rho^{\rm lep}(x) + J_\rho^{\rm had}(x) =$ charged weak current

- Leptonic part $J_\rho^{\rm lep}$ of J_ρ:

$$J_\rho^{\rm lep} = \overline{\psi_{\nu_e}}\gamma_\rho\omega_-\psi_e + \overline{\psi_{\nu_\mu}}\gamma_\rho\omega_-\psi_\mu \qquad \omega_\pm = \tfrac{1}{2}(1 \pm \gamma_5) = \text{chirality projectors}$$

  - ◇ only left-handed fermions $(\omega_-\psi)$, right-handed anti-fermions $(\overline{\psi}\omega_+)$ feel (charged-current) weak interactions $\quad\Rightarrow\quad$ maximal P-violation
  - ◇ doublet structure: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$, $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$, later completed by $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$
  - ◇ $(J^{{\rm lep},\rho})^\dagger J_\rho^{\rm lep}$ induces muon decay:



- Hadronic part $J_\rho^{\rm had}$ of J_ρ:

  Relevant quarks for energies $\lesssim 1\,{\rm GeV}$: $\quad {\rm u,d,s,c}$

  $\hookrightarrow$ meson ($q\bar{q}$) and baryon (qqq) spectra

  Question: doublet structure $\begin{pmatrix} {\rm u} \\ {\rm d} \end{pmatrix}, \begin{pmatrix} {\rm c} \\ {\rm s} \end{pmatrix}$?

  Problem: e.g. annihilation of ${\rm u\bar{s}}$ pair would not be allowed,
  but is observed: $\quad \underbrace{K^+}_{{\rm u\bar{s}}\text{ pair in quark model}} \to \mu^+\nu_\mu$

  Solution (Cabibbo 1963):

  u-c-mixing and d-s-mixing in weak interaction

  $\hookrightarrow$ doublets $\begin{pmatrix} {\rm u} \\ {\rm d'} \end{pmatrix}, \begin{pmatrix} {\rm c} \\ {\rm s'} \end{pmatrix}$ with $\begin{pmatrix} {\rm d'} \\ {\rm s'} \end{pmatrix} = U_{\rm C} \begin{pmatrix} {\rm d} \\ {\rm s} \end{pmatrix}$,

  orthogonal Cabbibo matrix $\quad U_{\rm C} = \begin{pmatrix} \cos\theta_{\rm C} & \sin\theta_{\rm C} \\ -\sin\theta_{\rm C} & \cos\theta_{\rm C} \end{pmatrix}$,

  empirical result: $\quad \theta_{\rm C} \approx 13^\circ$

$$J_\rho^{\rm had} = \overline{\psi_{\rm u}}\gamma_\rho\omega_-\psi_{\rm d'} + \overline{\psi_{\rm c}}\gamma_\rho\omega_-\psi_{\rm s'}$$

Remarks on the Fermi model:

- universal coupling G_μ for all transitions
  ($U_{\mathrm{C}}^\dagger U_{\mathrm{C}} = \mathbf{1}$ is part of universality)
- no (pseudo-)scalar or tensor couplings, such as $(\overline{\psi}\psi)(\overline{\psi}\psi)$, $(\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi)$, etc., necessary to describe low-energy experiments ($E \lesssim 1\,\mathrm{GeV}$)
- Problems:
  - ◇ cross sections for $\nu_\mu \mathrm{e} \to \nu_\mathrm{e} \mu$, etc., grow for energy $E \to \infty$ as E^2

    $\hookrightarrow$ unitarity violation !
  - ◇ no consistent evaluation of higher perturbative orders possible (no cancellation of UV divergences)

    $\hookrightarrow$ non-renormalizability !

“Intermediate-vector-boson (IVB) model”

Idea: “resolution” of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\mathcal{L}_{\mathrm{IVB}} = \mathcal{L}_{0,\mathrm{ferm}} + \mathcal{L}_{0,\mathrm{W}} + \mathcal{L}_{\mathrm{int}},$$

$$\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f,$$ (summation over f assumed)

$$\mathcal{L}_{0,\mathrm{W}} = -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_\mathrm{W}^2 W_\mu^+ W^{-,\mu},$$

with $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp \mathrm{i}W_\mu^2)$, W_μ^i real

$\mathrm{W}^\pm$ are vector bosons with electric charge $\pm e$ and mass M_W.

Propagator:

$$G_{\mu\nu}^{\mathrm{WW}}(k) = \frac{-\mathrm{i}}{k^2 - M_\mathrm{W}^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_\mathrm{W}^2}\right),$$ k = momentum

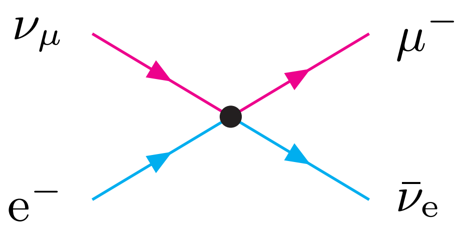
Interaction Lagrangian:

$$\mathcal{L}_{\mathrm{int}} = \frac{g_\mathrm{W}}{\sqrt{2}}\left(J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-\right),$$

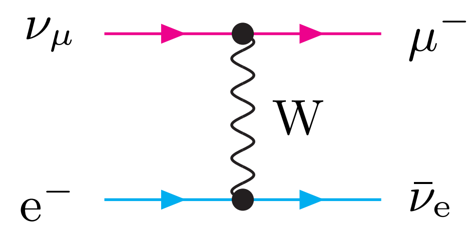
J^ρ = charged weak current as in Fermi model

Four-fermion interaction in process $\nu_\mu \mathrm{e}^- \to \mu^- \nu_\mathrm{e}$

Fermi model:



IVB model:



$$-\mathrm{i}2\sqrt{2}G_\mu\, g_{\rho\sigma} \qquad\qquad \tfrac{\mathrm{i}}{2}g_\mathrm{W}^2 \frac{1}{k^2 - M_\mathrm{W}^2}\left(g_{\rho\sigma} - \frac{k_\rho k_\sigma}{M_\mathrm{W}^2}\right)$$

$$\times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right]\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right] \qquad \times \left[\bar{u}_{\mu^-}\gamma^\rho\omega_- u_{\nu_\mu}\right]\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma\omega_- u_{\mathrm{e}^-}\right]$$

$\Rightarrow$ identification for $|k| \ll M_\mathrm{W}$: $\quad 2\sqrt{2}G_\mu = \dfrac{g_\mathrm{W}^2}{2M_\mathrm{W}^2}$

Consequences for the high-energy behaviour:

- k^ρ terms: $\quad \bar{u}_{\nu_\mathrm{e}} \not{k}\omega_- u_{\mathrm{e}^-} = \bar{u}_{\nu_\mathrm{e}}(\not{p}_\mathrm{e} - \not{p}_{\nu_\mathrm{e}})\omega_- u_{\mathrm{e}^-} = m_\mathrm{e}\bar{u}_{\nu_\mathrm{e}}\omega_- u_{\mathrm{e}^-}$
  $\hookrightarrow$ no extra factors of scattering energy E
- propagator $1/(k^2 - M_\mathrm{W}^2) \;\sim\; 1/E^2$ for $|k| \sim E \gg M_\mathrm{W}$
  $\hookrightarrow$ damping of amplitude in high-energy limit by factor $1/E^2$

$\Rightarrow$ cross section $\underset{E\to\infty}{\sim}$ const$/E^2$, $\qquad \Rightarrow$ No unitarity violation !

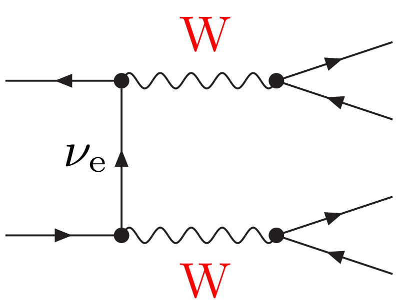
Comments on the IVB model:

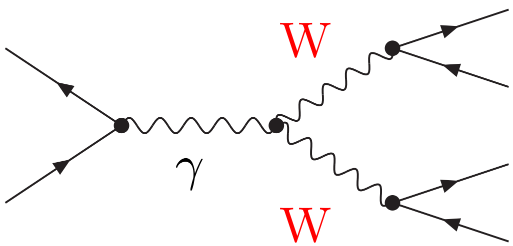
- Formal similiarity with QED interaction: $J^\rho W_\rho^+$ + h.c. $\longleftrightarrow$ $j^\rho_{\mathrm{elmg.}} A_\rho$
- Intermediate vector bosons can be produced, e.g.

$$\underbrace{\mathrm{u\bar{d}}}_{\text{in pp collision}} \longrightarrow \underbrace{\mathrm{W}^+ \to f\bar{f}'}_{\mathrm{W}^\pm \text{ unstable}}$$

(discovery 1983 at CERN)

- Problems:
  - ◇ unitarity violations in cross sections with longitudinal W bosons, e.g.





  - ◇ non-renormalizability
    (no consistent treatment of higher perturbative orders)

$\hookrightarrow$ Solution by spontaneously broken gauge theories !

2 The principle of local gauge invariance

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f$ has global phase symmetry:

$$\psi_f \to \psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}\psi_f, \quad \overline{\psi_f} \to \overline{\psi'_f} = \overline{\psi_f}\exp\{+\mathrm{i}Q_f e\theta\}$$

with space-time-independent group parameter θ

"Gauging the symmetry": demand local symmetry, $\theta \to \theta(x)$

To maintain local symmetry, extend theory by "minimal substitution":

$\partial^\mu \to D^\mu = \partial^\mu + \mathrm{i}Q_f e A^\mu(x)$ = "covariant derivative",

$A^\mu(x)$ = spin-1 gauge field (photon).

Transformation property of photon $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \to (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - Q_f e A\!\!\!/ - m_f)\psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting fields with "internal symmetry":

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

  $\Phi \to \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-\mathrm{i}gT^a\theta^a\}$ = unitary,

  T^a = group generators, $\quad [T^a, T^b] = \mathrm{i}C^{abc}T^c$, $\quad \mathrm{Tr}\left\{T^aT^b\right\} = \frac{1}{2}\delta^{ab}$

- $\mathcal{L}_\Phi$ is invariant under G: $\quad \mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu \Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

(m = common boson mass, λ = coupling strength)

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) \to \mathcal{L}_\Phi(\Phi, D_\mu \Phi) \quad \text{with} \quad D_\mu = \partial_\mu + \mathrm{i}gT^a A_\mu^a(x),$$

g = gauge coupling, T^a = generator of G in Φ representation, $A_\mu^a(x)$ = gauge fields

Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu\Phi)$ local invariant if $\quad D_\mu\Phi \;\to\; (D_\mu\Phi)' = D'_\mu\Phi' = U(\theta)(D_\mu\Phi)$

  $\Rightarrow\; T^a A'^a_\mu \;=\; UT^a A^a_\mu U^\dagger - \frac{\mathrm{i}}{g}U(\partial_\mu U^\dagger), \qquad A^a_\mu A^{a,\mu}$ = not gauge invariant

  infinitesimal form: $\quad \delta A^a_\mu = gC^{abc}\delta\theta^b A^c_\mu + \partial_\mu\delta\theta^a$

- covariant definition of field strength: $\quad [D_\mu, D_\nu] = \mathrm{i}gT^a F^a_{\mu\nu}$

  $\Rightarrow\; T^a F^a_{\mu\nu} \;\to\; T^a F'^a_{\mu\nu} = UT^a F^a_{\mu\nu}U^\dagger, \qquad F^a_{\mu\nu}F^{a,\mu\nu}$ = gauge invariant

  explicit form: $\quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gC^{abc}A^b_\mu A^c_\nu$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu\Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2$, A^4 in F^2 part

  $\hookrightarrow$ cubic and quartic gauge-boson self-interactions

- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction $\to$ unification of interactions

- mass term $M^2(A^a_\mu A^{a,\mu})$ for gauge bosons forbidden by gauge invariance

  $\hookrightarrow$ gauge bosons of unbroken Yang–Mills theory are massless

Quantum chromodynamics — gauge theory of strong interactions

- **Gauge group:** SU(3)$_c$, dim. $= 8$

  structure constants f^{abc}, gauge coupling g_s, $\alpha_\mathrm{s} = \dfrac{g_\mathrm{s}^2}{4\pi}$

- **Gauge bosons:** 8 massless gluons g with fields $A^a_\mu(x)$, $a = 1, \dots, 8$

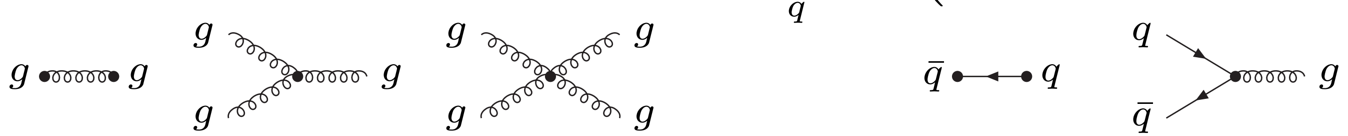
- **Matter fermions:** quarks q (spin-$\frac{1}{2}$) with flavours $q = \mathrm{d, u, s, c, b, t}$
  in fundamental representation:

$$\psi_q(x) \;\equiv\; q(x) \;=\; \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix} \;=\; \text{colour triplet}$$

$$T^a = \frac{\lambda^a}{2}, \quad \text{Gell-Mann matrices } \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

- **Lagrangian:**

$$\begin{aligned} \mathcal{L}_\mathrm{QCD} &= -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \overline{\psi}_q (\mathrm{i}\not{D} - m_q)\psi_q \\ &= -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_\mathrm{s} f^{abc} A^b_\mu A^c_\nu\right)^2 + \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_\mathrm{s}\frac{\lambda^a}{2}\not{A}^a - m_q\right)\psi_q \end{aligned}$$



3 The Standard Model of electroweak interaction (Glashow–Salam–Weinberg model)

— matter, Yang–Mills, and Higgs sector

3.1 The gauge group for electroweak interaction

Why unification of weak and elmg. interaction ?

- similiarity: spin-1 fields couple to matter currents formed by spin-$\frac{1}{2}$ fields
- elmg. coupling of charged $\mathrm{W}^{\pm}$ bosons

$\gamma, \mathrm{W}^+, \mathrm{W}^-$ as gauge bosons of group SU(2) ? – No!

Reason: charge operator Q cannot be SU(2) generator, since Tr $\{Q\} \neq 0$

for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_\mathrm{e} \\ \mathrm{e}^- \end{pmatrix}$, etc.

Possible way out: additional heavy fermions like E^+ as partner to e^- ?
$\hookrightarrow$ no experimental confirmation !

Minimal solution: SU(2)$_{\mathrm{I}}$ $\times$ U(1)$_{\mathrm{Y}}$

- SU(2)$_{\mathrm{I}}$ $\rightarrow$ weak isospin group with gauge bosons $\mathrm{W}^+, \mathrm{W}^-, \mathrm{W}^0$
- U(1)$_{\mathrm{Y}}$ $\rightarrow$ weak hypercharge with gauge boson B

W^0 and B carry identical quantum numbers

$\hookrightarrow$ two neutral gauge bosons γ, Z as mixed states

Experiment: 1973 discovery of neutral weak currents at CERN

$\hookrightarrow$ indirect confirmation of Z exchange

1983 discovery of $\mathrm{W}^\pm$ and Z bosons at CERN

3.2 Fermion sector and minimal substitution

Multiplet structure:

Distinguish between left-/right-handed parts of fermions: $\psi^{\mathrm{L}} = \omega_- \psi$, $\psi^{\mathrm{R}} = \omega_+ \psi$

- ψ^{L} couple to $\mathrm{W}^\pm$ $\quad\rightarrow$ group ψ^{L} into SU(2)$_{\mathrm{I}}$ doublets, weak isospin $T_{\mathrm{I}}^a = \frac{\sigma^a}{2}$
- ψ^{R} do not couple to $\mathrm{W}^\pm$ $\quad\rightarrow$ ψ^{R} are SU(2)$_{\mathrm{I}}$ singlets, weak isospin $T_{\mathrm{I}}^a = 0$
- $\psi^{\mathrm{L/R}}$ couple to γ in the same way

  $\hookrightarrow$ adjust coupling to U(1)$_{\mathrm{Y}}$ (i.e. fix weak hypercharges $Y^{\mathrm{L/R}}$ for $\psi^{\mathrm{L/R}}$)
  such that elmg. coupling results: $\mathcal{L}_{\mathrm{int,QED}} = -Q_f e \overline{\psi_f} \not{A} \psi_f$

Fermion content of the SM:
(ignoring possible right-handed neutrinos)

					T_{I}^3	Q
leptons:	$\Psi_L^{\mathrm{L}} =$	$\begin{pmatrix} \nu_{\mathrm{e}}^{\mathrm{L}} \\ \mathrm{e}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \nu_\mu^{\mathrm{L}} \\ \mu^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \nu_\tau^{\mathrm{L}} \\ \tau^{\mathrm{L}} \end{pmatrix}$,	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1
	$\psi_l^{\mathrm{R}} =$	e^{R},	μ^{R},	τ^{R},	0	-1
quarks: (Each quark exists in 3 colours!)	$\Psi_Q^{\mathrm{L}} =$	$\begin{pmatrix} \mathrm{u}^{\mathrm{L}} \\ \mathrm{d}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \mathrm{c}^{\mathrm{L}} \\ \mathrm{s}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \mathrm{t}^{\mathrm{L}} \\ \mathrm{b}^{\mathrm{L}} \end{pmatrix}$,	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$
	$\psi_u^{\mathrm{R}} =$	u^{R},	c^{R},	t^{R},	0	$+\frac{2}{3}$
	$\psi_d^{\mathrm{R}} =$	d^{R},	s^{R},	b^{R},	0	$-\frac{1}{3}$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \mathrm{i}\overline{\psi_f}\partial\!\!\!/\psi_f = \mathrm{i}\overline{\Psi^{\mathrm{L}}_L}\partial\!\!\!/\Psi^{\mathrm{L}}_L + \mathrm{i}\overline{\Psi^{\mathrm{L}}_Q}\partial\!\!\!/\Psi^{\mathrm{L}}_Q + \mathrm{i}\overline{\psi^{\mathrm{R}}_l}\partial\!\!\!/\psi^{\mathrm{R}}_l + \mathrm{i}\overline{\psi^{\mathrm{R}}_u}\partial\!\!\!/\psi^{\mathrm{R}}_u + \mathrm{i}\overline{\psi^{\mathrm{R}}_d}\partial\!\!\!/\psi^{\mathrm{R}}_d$$

Minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu - \mathrm{i}g_2 T^a_{\mathrm{I}} W^a_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y B_\mu = D^{\mathrm{L}}_\mu \omega_- + D^{\mathrm{R}}_\mu \omega_+,$$

$$D^{\mathrm{L}}_\mu = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}} \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} - \frac{\mathrm{i}}{2} \begin{pmatrix} g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu & 0 \\ 0 & -g_2 W^3_\mu - g_1 Y^{\mathrm{L}} B_\mu \end{pmatrix},$$

$$D^{\mathrm{R}}_\mu = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}} B_\mu$$

Photon identification:

"Weinberg rotation": $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & s_{\mathrm{W}} \\ -s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix},$ $\quad c_{\mathrm{W}} = \cos\theta_{\mathrm{W}},\ s_{\mathrm{W}} = \sin\theta_{\mathrm{W}},$

θ_{W} = weak mixing angle

$$D^{\mathrm{L}}_\mu\Big|_{A_\mu} = -\frac{\mathrm{i}}{2} A_\mu \begin{pmatrix} -g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} & 0 \\ 0 & g_2 s_{\mathrm{W}} - g_1 c_{\mathrm{W}} Y^{\mathrm{L}} \end{pmatrix} \stackrel{!}{=} \mathrm{i}eA_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1 \quad \to \quad g_2 = \dfrac{e}{s_{\mathrm{W}}}$
- normalize $Y^{\mathrm{L/R}}$ such that $\quad g_1 = \dfrac{e}{c_{\mathrm{W}}}$

$\hookrightarrow$ Y fixed by "Gell-Mann–Nishijima relation": $\quad Q = T^3_{\mathrm{I}} + \dfrac{Y}{2}$

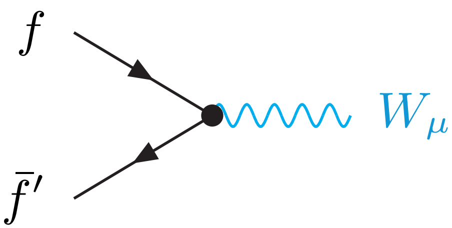
Fermion–gauge-boson interaction:

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{W}}}\overline{\Psi^{\mathrm{L}}_F}\begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix}\Psi^{\mathrm{L}}_F + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\overline{\Psi^{\mathrm{L}}_F}\sigma^3\not{Z}\Psi^{\mathrm{L}}_F$$

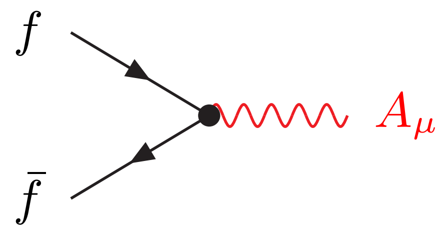
$$- e\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f\overline{\psi_f}\not{Z}\psi_f - eQ_f\overline{\psi_f}\not{A}\psi_f$$

(f=all fermions, F= all doublets)

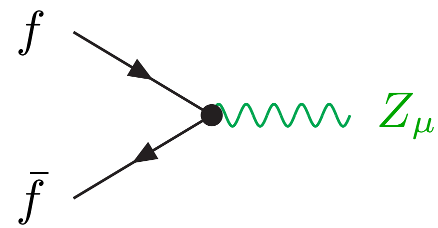
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2}s_{\mathrm{W}}}\gamma_\mu\omega_-$$



$$-\mathrm{i}Q_f e\gamma_\mu$$



$$\mathrm{i}e\gamma_\mu(g_f^+\omega_+ + g_f^-\omega_-) = \mathrm{i}e\gamma_\mu(v_f - a_f\gamma_5)$$

with $g_f^+ = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f, \quad g_f^- = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{c_{\mathrm{W}}s_{\mathrm{W}}},$

$$v_f = -\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}Q_f + \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}, \quad a_f = \frac{T^3_{\mathrm{I},f}}{2c_{\mathrm{W}}s_{\mathrm{W}}}$$

3.3 Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

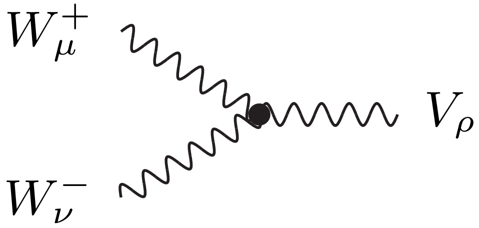
$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu})$$
$$- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$+$ (trilinear interaction terms involving AW^+W^-, ZW^+W^-)

$+$ (quadrilinear interaction terms involving AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$)

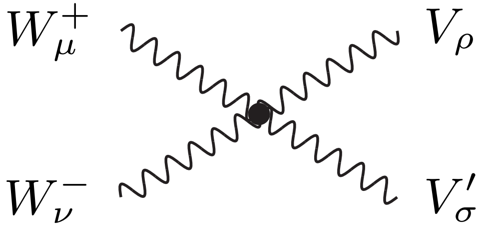
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$\mathrm{i}e C_{WWV} \Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu \Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{W}}{s_\mathrm{W}}$



$$\mathrm{i}e^2 C_{WWVV'} \Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho} \Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{W}}{s_\mathrm{W}},$

$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

3.4 Higgs sector and spontaneous symmetry breaking

Idea: spontaneous breakdown of $\mathrm{SU(2)_I \times U(1)_Y}$ symmetry $\rightarrow$ $\mathrm{U(1)_{elmg}}$ symmetry

$\hookrightarrow$ masses for $\mathrm{W}^\pm$ and Z bosons, but γ remains massless

Note: choice of scalar extension of massless model involves freedom

GSW model:

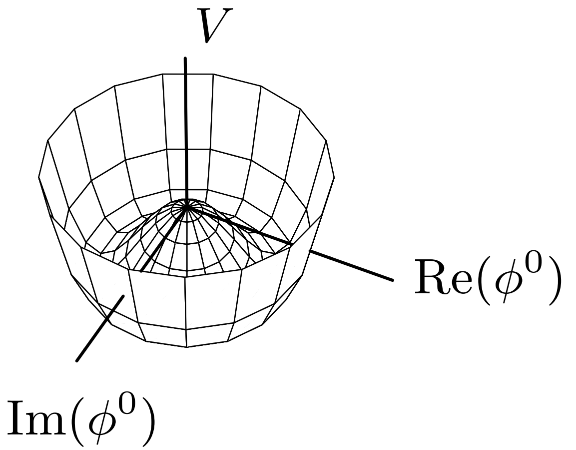
Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$$= \mathrm{SU(2)_I \times U(1)_Y} \text{ symmetric}$$

$$V(\Phi) = \text{minimal for} \quad |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $\quad Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\Big(v + H(x) + \mathrm{i}\chi(x)\Big) \end{pmatrix}$$

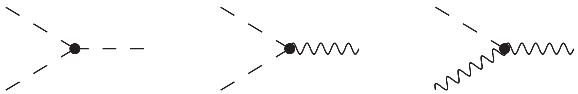
Gauge-invariant Lagrangian of Higgs sector: $(\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with } D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W_\mu^a + \mathrm{i}\frac{g_1}{2}B_\mu$$

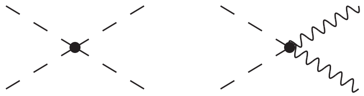
$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{W}}(W_\mu^+\partial^\mu\phi^- - W_\mu^-\partial^\mu\phi^+) + \frac{e^2v^2}{4s_\mathrm{W}^2}W_\mu^+W^{-,\mu}$$

$$+\frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{W}s_\mathrm{W}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c_\mathrm{W}^2s_\mathrm{W}^2}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (trilinear SSS, SSV, SVV interactions)



$+$ (quadrilinear $SSSS$, $SSVV$ interactions)



Implications:

- gauge-boson masses: $M_\mathrm{W} = \frac{ev}{2s_\mathrm{W}}, \quad M_\mathrm{Z} = \frac{ev}{2c_\mathrm{W}s_\mathrm{W}} = \frac{M_\mathrm{W}}{c_\mathrm{W}}, \quad M_\gamma = 0$
- physical Higgs boson H: $M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ: unphysical degrees of freedom

3.5 ρ-parameter and custodial SU(2) symmetry

Observation: Higgs potential of SM invariant under larger symmetry

$$V(\Phi) = f(\Phi^\dagger\Phi), \quad \Phi^\dagger\Phi = \mathrm{Re}\{\phi^+\}^2 + \mathrm{Im}\{\phi^+\}^2 + \mathrm{Re}\{\phi^0\}^2 + \mathrm{Im}\{\phi^0\}^2$$

$=$ invariant under O(4) = 4-dim. rotations

Relation between O(4) $\simeq$ SU(2)$\times$SU(2) and SU(2)$_\mathrm{I}\times$U(1)$_\mathrm{Y}$ symmetry

$\hookrightarrow$ matrix notation:

$$\Pi \equiv (\,\tilde{\Phi}, \Phi\,) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \qquad \to \ \tfrac{1}{2}\,\mathrm{Tr}\left\{\Pi^\dagger\Pi\right\} = \Phi^\dagger\Phi$$

SU(2)$_\mathrm{I}\times$U(1)$_\mathrm{Y}$ transformation: $U_\mathrm{I} \ = \ \exp\{\mathrm{i}g_2\theta^a T^a_\mathrm{I}\}, \quad U_\mathrm{Y} \ = \ \exp\{-\mathrm{i}g_1\theta^Y T_\mathrm{Y}\}$

$$\Pi \ \to \ \Pi' = U_\mathrm{I}\,\Pi\,U_\mathrm{Y}^\dagger, \qquad T^a_\mathrm{I} = \sigma^a/2, \quad T_\mathrm{Y} = \sigma^3/2$$

covariant derivative:

$$D_\mu\Pi \ = \ \partial_\mu\Pi - \mathrm{i}g_2\mathcal{W}_\mu\Pi - \mathrm{i}g_1\Pi B_\mu T_\mathrm{Y}, \qquad \mathcal{W}_\mu \equiv W^a_\mu T^a_\mathrm{I}$$

transformation of gauge fields:

$$\mathcal{W}_\mu \ \to \ \mathcal{W}'_\mu = U_\mathrm{I}\left(\mathcal{W}_\mu + \frac{\mathrm{i}}{g_2}\partial_\mu\right)U_\mathrm{I}^\dagger \qquad B_\mu \ \to \ B'_\mu = B_\mu + \partial_\mu\theta^Y$$

O(4) symmetry: $\Phi^\dagger\Phi$ invariant under SU(2)$_\mathrm{I}\times$SU(2)$_{\mathrm{I}'}$ transformation

$$\Pi \ \to \ \Pi' = U_\mathrm{I}\,\Pi\,U_{\mathrm{I}'}^\dagger, \qquad U_{\mathrm{I}'} \ = \ \exp\{-\mathrm{i}g_1\theta^b T^b_{\mathrm{I}'}\}, \quad T^b_{\mathrm{I}'} \ = \ \sigma^b/2$$

Situation after spontaneous symmetry breaking:

ground state $\Pi_0 = (\tilde{\Phi}_0, \Phi_0) \propto \mathbf{1}$ still "diagonal" SU(2) symmetric:

$\Pi_0 \;\to\; \Pi_0' = U\,\Pi_0\,U^\dagger = \Pi_0,$ i.e. $[T^a, \Pi_0] = 0$ for SU(2) generators T^a

$\hookrightarrow$ under global transformation U

- W_μ^a transforms as 3-vector: $W_\mu^a \to W_\mu'^a = R_U^{ab} W_\mu^b$ (R_U = rotation matrix)
- B_μ transforms as 3rd component of a fictive triplet B_μ^a with R_U

$\hookrightarrow$ mass terms for gauge bosons

$$\mathcal{L}_{\mathrm{WZ,mass}} = \frac{1}{2}\,\mathrm{Tr}\left\{(D_\mu\Pi_0)^\dagger(D^\mu\Pi_0)\right\} = \frac{1}{2}\,\mathrm{Tr}\Big\{\Pi_0^\dagger\Pi_0\,\underbrace{\left(g_2 W_\mu^a T^a + g_1 T^3 B_\mu\right)^2}_{\text{invariant under } U \;\hookrightarrow\; \text{length of 3-vector}}\Big\}$$

$$\propto g_2^2(W^1W^1 + W^2W^2) + (g_2 W^3 + g_1 B)^2 \propto c_{\mathrm{W}}^2 W^+W^- + \tfrac{1}{2}Z^2$$

$\Rightarrow$ Relation for the ρ-parameter: $\rho \equiv \dfrac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2 c_{\mathrm{W}}^2} = 1$

Role of the ρ-parameter in low-energy physics:

effective four-fermion interaction (cf. IVB model) with charged and neutral currents:

$$\mathcal{L}_{4f,\mathrm{eff}} = -2\sqrt{2}G_\mu\left(J^\dagger_{\mathrm{CC},\mu}J^\mu_{\mathrm{CC}} + \rho\, J^0_{\mathrm{NC},\mu}J^{0,\mu}_{\mathrm{NC}}\right),$$

ρ = ratio of NC to CC interaction

Literature

- Böhm/Denner/Joos:
  "Gauge Theories of the Strong and Electroweak Interaction"
- Cheng/Li:
  "Gauge Theory of Elementary Particle Physics"
- Ellis/Stirling/Webber:
  "QCD and Collider Physics"
- Peskin/Schroeder:
  "An Introduction to Quantum Field Theory"
- Weinberg:
  "The Quantum Theory of Fields, Vol. 2: Modern Applications"