- larger m_T? → no, otherwise $\alpha_3(m_Z)$ too large
- larger m_{SUSY}? → $m_{SUSY} \lesssim O(\mathrm{TeV})$ by naturalness
- $\tan\beta = 1$ → still possible?
- $K \neq \mathbb{1}$ → no distructive interference
- hadronic matrix element? → $\langle 0 | u u d | p \rangle \simeq a$

  $a = 0.003\,\mathrm{GeV}^3$ adopted

  cf. $a = 0.015(1)\,\mathrm{GeV}^3$

↓

minimal SU(5) is <u>dead</u>.

SO(10) in similar troubles...

non minimal GUT survive but often baroque constructions!

Are they simplest ways out?

[extra dimensions? more on this later on...]

II GAUGE COUPLING UNIFICATION BEYOND LO

$$\alpha_3(m_Z) = \frac{\alpha_3(m_Z)\big|_{LO}}{\left[1 + \delta\, \alpha_3(m_Z)\big|_{LO}\right]}$$

$$\delta = \underbrace{k}_{1,2,3} + \underbrace{\frac{1}{2\pi} \log \frac{m_{SUSY}}{m_Z}}_{2} - \underbrace{\frac{3}{5\pi} \log \frac{m_T}{M_{GUT}\big|_{LO}}}_{3}$$

1. 2-loop running
2. threshold from SUSY partners at $m_{SUSY} \approx 1\,\mathrm{TeV}$
3. thresholds from particles at M_{GUT}

$k = -1.24$ in minimal SU(5) → large correction

$$\alpha_3(m_Z)\big|_{LO} \approx 0.118 \longmapsto \overbrace{\alpha_3(m_Z) \approx 0.13 \pm 0.01}^{\text{not really a problem}}$$

$[m_{SUSY} \approx 1\,\mathrm{TeV}$
$m_T \approx M_{GUT}|_{LO}]$

uncertainty coming from spectrum at m_{SUSY} ⊕ spectrum at M_{GUT} ⊕ non-perturbative effects

$$\boxed{\text{NOTE THAT } m_T > M_{GUT}\big|_{LO} \text{ WORSENS } \alpha_3(m_Z)}$$

GUTs SUMMARY

$$G \xrightarrow{M_{GUT}} SU(3)\times SU(2)\times U(1)$$

SU(5), SO(10)

① Gauge coupling unification in SUSY version
(not the only possibility but one of the simplest)
More on that later on ...

Evidence for new physics at $M_{GUT} \approx 2\times 10^{16}$ GeV
not-so-far from M_{Pl} and $\Lambda \approx 10^{15}$ GeV

$$\frac{(HL)(HL)}{\Lambda} = m_\nu \nu\nu + \dots$$

② B/L violation: welcome for baryogenesis

p-decay e.g. in minimal SUSY SU(5)

d=6 operators $\tau(p\to e^+\pi^0) \sim 10^{35}$ yr O.K.

d=5 operators $\tau(p\to \bar\nu K^+) \sim 10^{31} \div 10^{32}$ yr

cfr. $\tau(p\to \bar\nu K^+) > 2\times 10^{33}$ yr [S.K.]

(3) Particle classification: powerful

$SO(10) \ni 16 \equiv [1 \text{SM family} + \nu^c]$

ν^c * good for ν masses (see-saw)

* baryogenesis through leptogenesis

$\cancel{B-L}$ needed as, for instance, by $M \nu^c \nu^c$

out-of-equilibrium, $\cancel{CP}$ decay of ν^c

(4) Fermion mass relations

encouraging but incomplete in minimal GUTs

(5) Hierarchy problem $\leftrightarrow$ DT splitting problem

MSSM

Why $\mu \simeq \Delta m_{SUSY}$?

$\mu H_u^D H_d^D$

GUT

Why $m_D \ll m_T$?

GUTs AND EXTRA DIMENSIONS

GUT symmetry [SU(5)] not realized as a $d=4$ gauge symmetry. SU(5) manifest only in $d>4$, e.g. $d=5$.

In $d=4$ only $SU(3)\times SU(2)\times U(1)$ is seen.

SB of SU(5) down to $SU(3)\times SU(2)\times U(1)$ is <u>not</u> through $24\sim\Sigma$

$$\langle\Sigma\rangle = \mathrm{diag}(2\sigma, 2\sigma, 2\sigma, -3\sigma, -3\sigma)$$

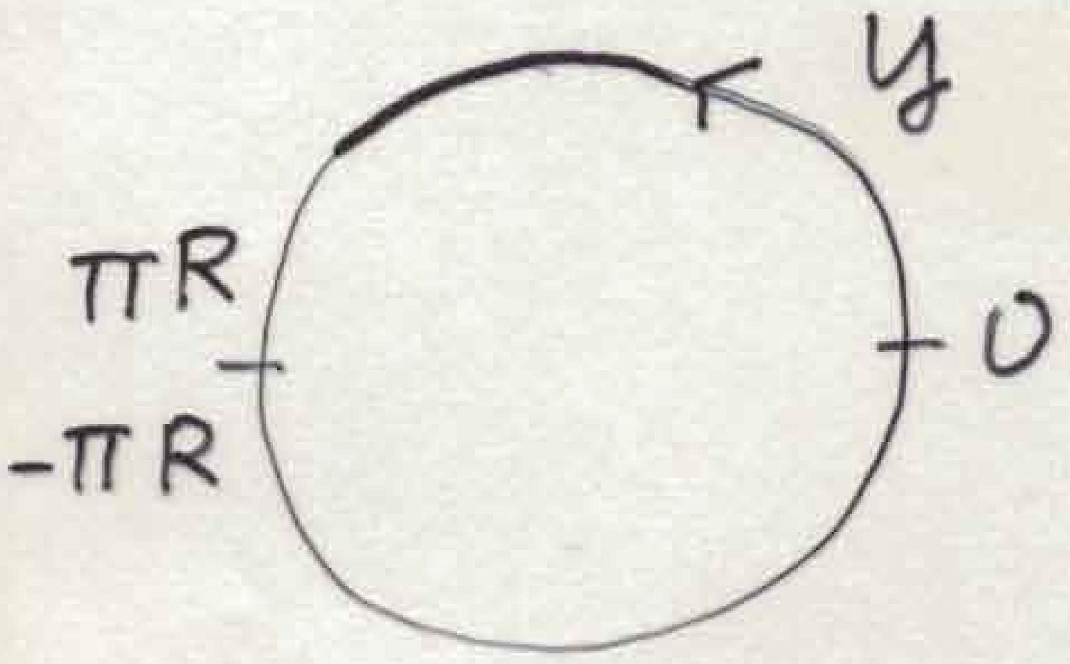
$$\sigma \approx 10^{16}\,\mathrm{GeV} \simeq M_{GUT}$$

but through compactification of the 5^{th} dimension on $\left(\frac{S^1}{Z_2}\right)$ $\quad \frac{1}{R} \approx M_{GUT}$

→ DT splitting problem : <u>solved</u>!

→ $p \to \bar{\nu} K^+$ much less constrained

COMPACTIFICATION ON A CIRCLE S^1



$$-\pi R \leq y \leq \pi R$$

Fourier expansion:

generic 'bulk' field

$$\varphi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{-\infty}^{+\infty} {}_n \, \varphi_n(x) \, e^{i n \frac{y}{R}}$$

d=4 fields
KK modes

MASS

$\varphi_n(x)$ $\quad \frac{n}{R}$ $\rightarrow$ the momentum in 5th direct.

$\varphi_0(x)$ $\quad 0$ $\rightarrow$ zero mode $\equiv$ d=4 massless particle

Problem with fermions:

$$d=5 \quad \Psi(x,y) \equiv \begin{bmatrix} \Psi_L \\ \Psi_R \end{bmatrix}$$

both in the same representation of the gauge group

$\rightarrow$ vector-like spectrum. In particular Ψ_{0L}, Ψ_{0R} both massless <u>but</u>

SM fermions are CHIRAL.

GAUGE SYMMETRY ON S^1
SCHERK - SCHWARZ MECHANISM

$G = SU(2)$

gauge fields $\quad A_M^a(x,y) \qquad (a=1,2,3)$

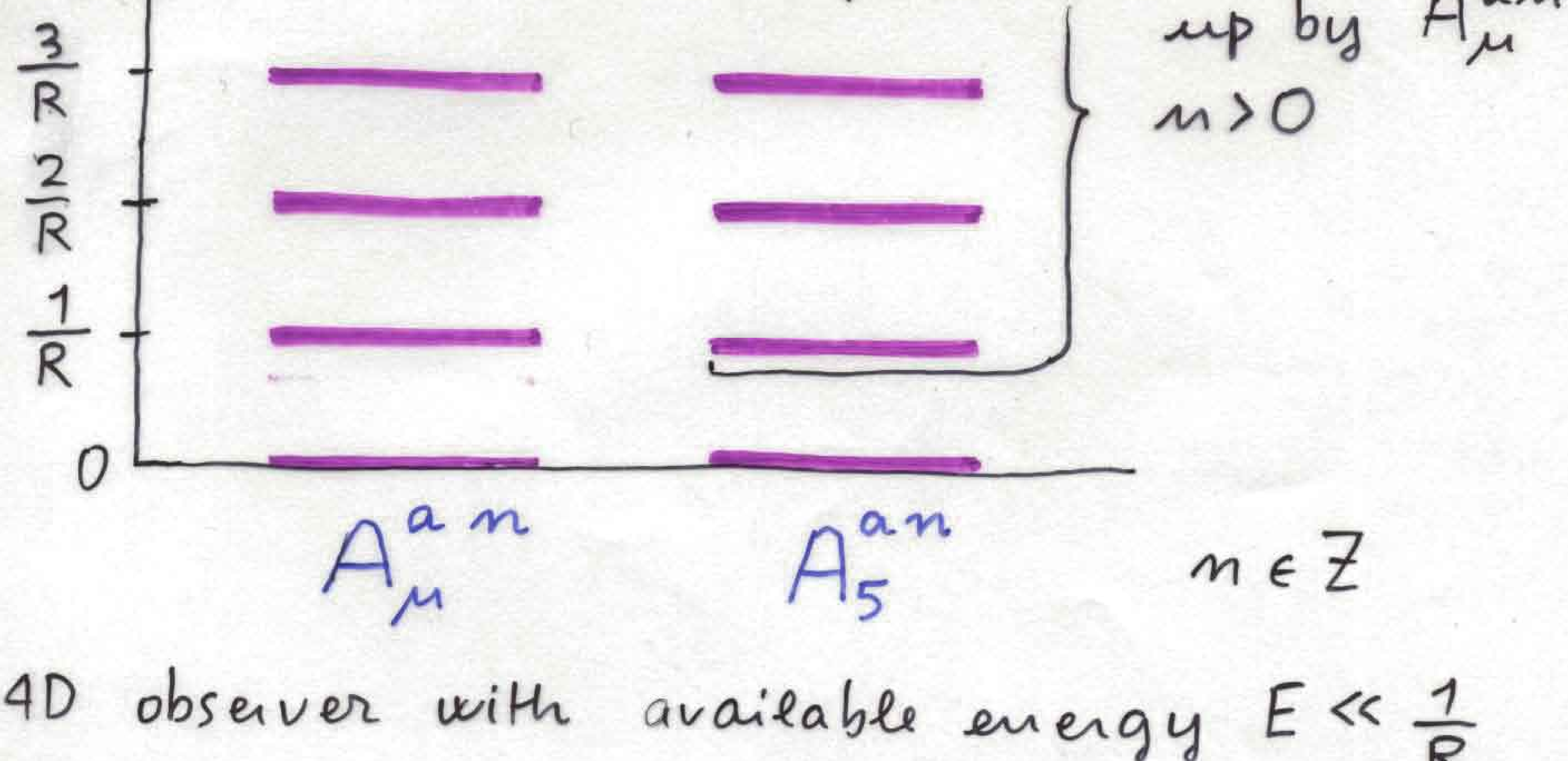
$$-\pi R \leq y \leq +\pi R$$

$$\begin{cases} A_\mu^a & \text{vector bosons in } d=4 \\ A_5^a & \text{scalars in } d=4 \end{cases}$$

Simplest boundary conditions:

$$A_M^a(x, y+2\pi R) = A_M^a(x,y)$$

Mass spectrum:



A 4D observer with available energy $E \ll \frac{1}{R}$ would detect a 4D SU(2) gauge theory with 3 massless scalar fields.

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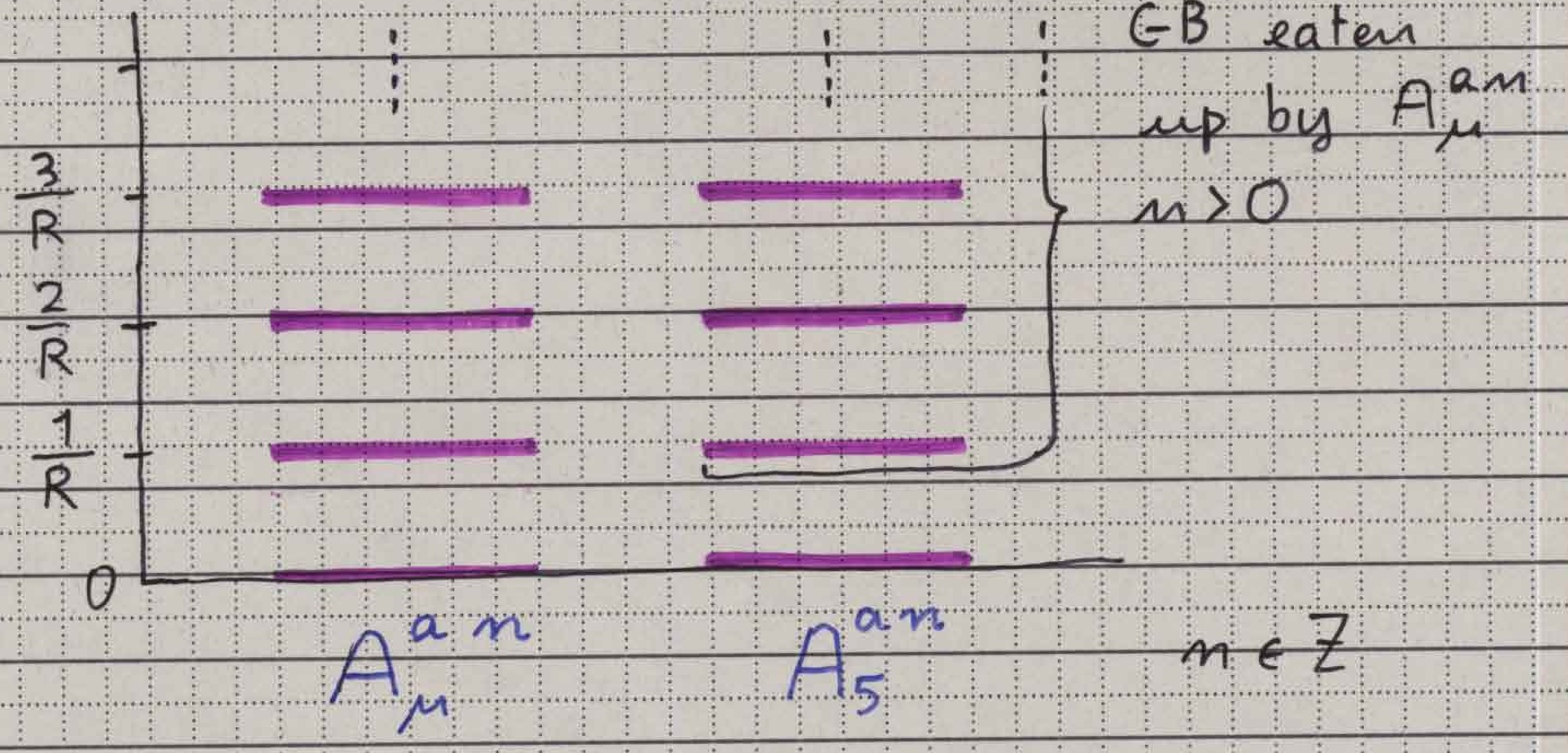

$n \in Z$

A 4D observer with available energy $E \ll \frac{1}{R}$ would detect a 4D $SU(2)$ gauge theory with 3 massless scalar fields.

A more general possibility:

$$A_M(x, y+2\pi R) = T\, A_M(x,y) \qquad A \equiv \begin{pmatrix} A^1 \\ A^2 \\ A^3 \end{pmatrix}$$

"twist"

TWISTED BOUNDARY CONDITIONS

T is an orthogonal matrix that does not upset the SU(2) algebra. For instance

$$T = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \beta \in R$$

Fields are periodic up to a global SU(2) transformation leaving the SU(2) algebra invariant.

Mass spectrum:

$$\begin{cases} m_3 = \dfrac{n_3}{R} & A_\mu^{3\,n_3} \quad A_5^{3\,n_3} \\[2ex] m_{1,2} = \dfrac{n}{R} - \dfrac{\beta}{2\pi R} & A_\mu^{1,2\,n} \quad A_5^{1,2\,n} \end{cases} \quad \text{all GB but } n_3=0$$

→ β produces a shift of the KK levels

For $\beta = 2p\pi \quad p \in Z \quad [T \equiv 1\!\!1]$

$m_{1,2} = \dfrac{n'}{R} \qquad n' = n - p \in Z$

the spectrum is unchanged.

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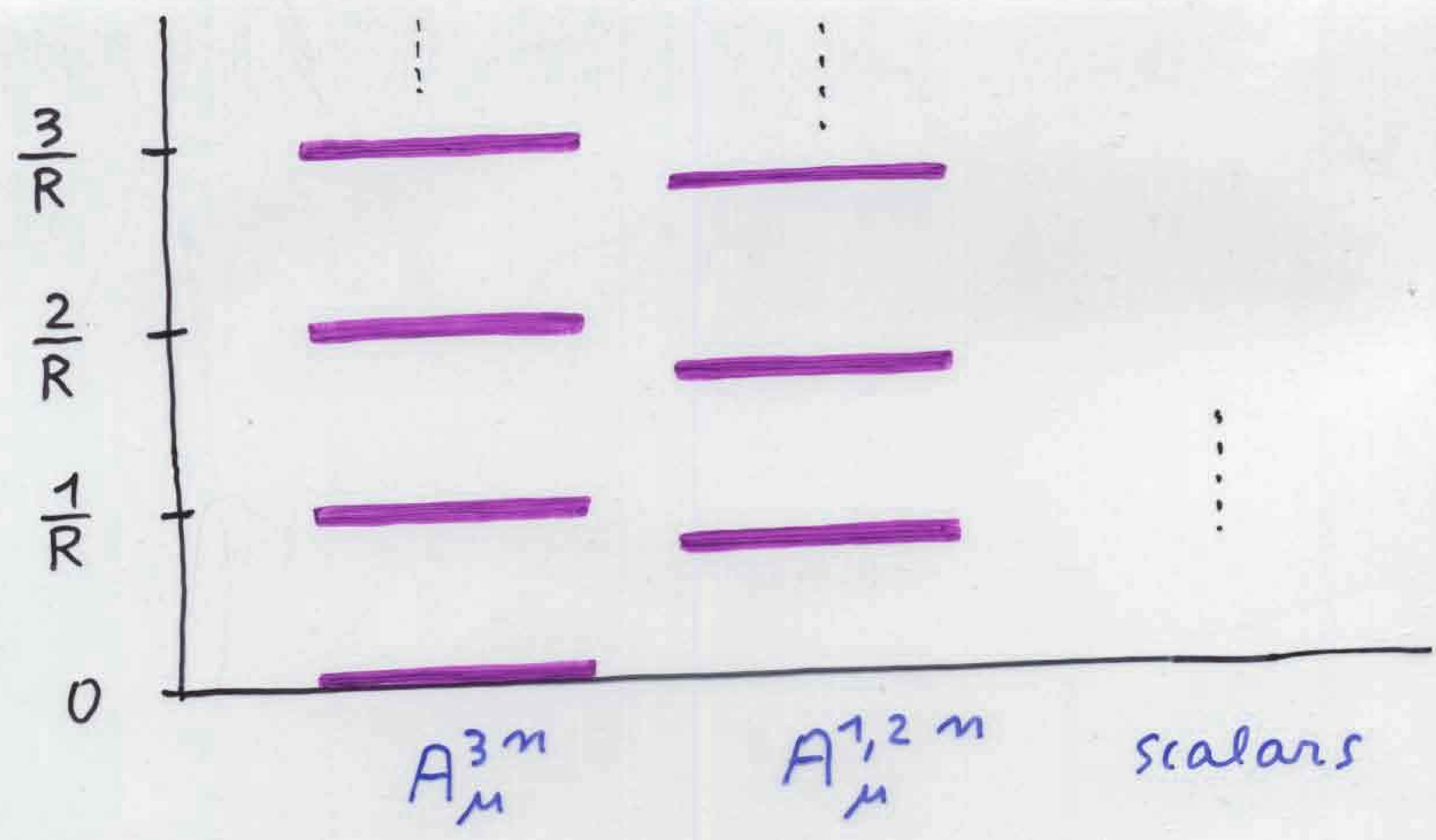


shift of KK levels $\equiv$ Scherk-Schwarz mechanism

A 4D observer with $E \ll \frac{1}{R}$ would detect a $U(1) \leftrightarrow A_\mu^{3\,0}$ gauge theory

$\underbrace{SU(2)}$ is effectively broken down to $U(1)$

strictly speaking there is no 4D $SU(2)$ gauge symmetry. $SU(2)$ is only manifest at 5D

✻

Remark:
We have assumed $\langle A_5^a \rangle = 0$ throughout.

An equivalent theory is obtained with periodic b.c. on $A_M^a(x,y)$ and $\langle A_5^3 \rangle = -\frac{\beta}{2\pi R}$.

gauge invariant order parameter:

$$W = P\left[\exp ig \int_0^{2\pi R} dy \, A_5(y)\right]$$

Most general possibility on $S^1/\mathbb{Z}_2$

$\varphi(x,y)$ set of fields $\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_N \end{pmatrix}(x,y)$

Under $\mathbb{Z}_2$ parity:

$$\varphi(x,-y) = Z\,\varphi(x,y)$$

Z is a $N \times N$ matrix such that $Z^2 = \mathbb{1}$

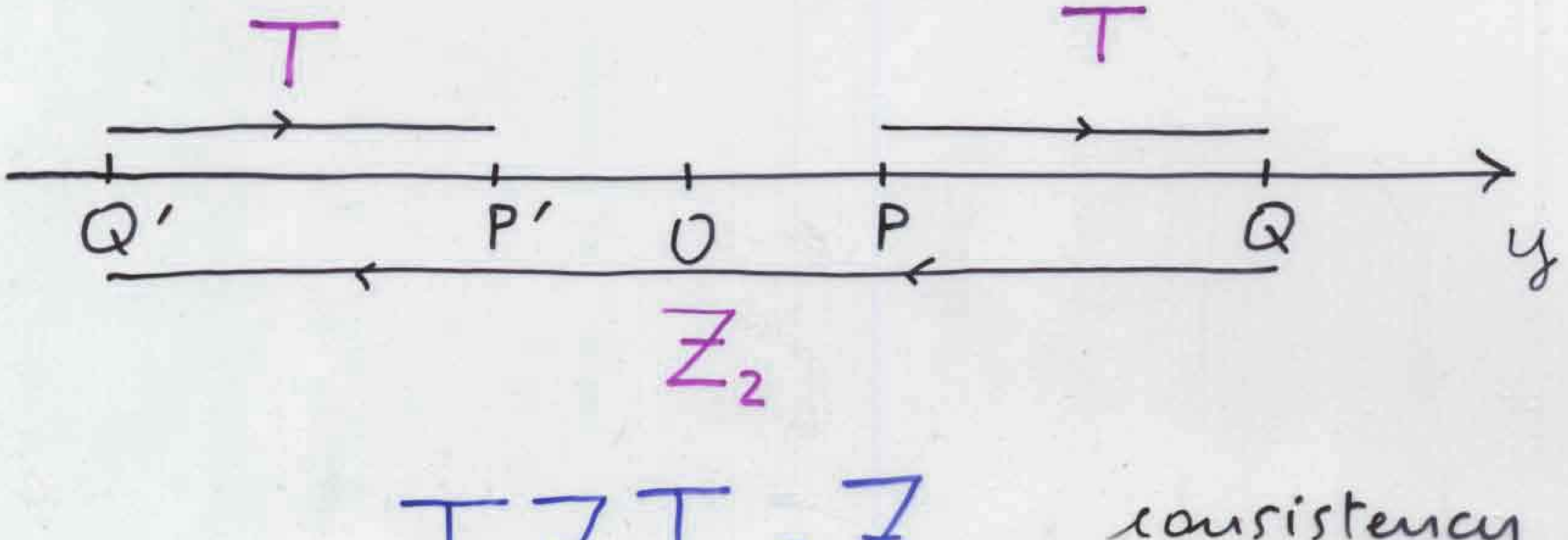
not restrictive: Z diagonal, entries $= \pm 1$

Twist:

$$\varphi(x, y+2\pi R) = T\,\varphi(x,y)$$

$T \leftrightarrow$ global symmetry of the theory

Moreover:



$$TZT = Z$$ consistency condition

5D SU(2) GAUGE THEORY ON $S^1/\mathbb{Z}_2$

Example

In the basis $\{A^1_\mu, A^2_\mu, A^3_\mu\}$ we take

$$Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[Then A^a_5 transform with $-Z$]

Consistency condition is simply $T^2 = \mathbb{1}$

If $T = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ only $\beta = 0, \pi$ allowed

$\rightarrow$ $T = \mathbb{1}$ or $T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ \hline 0 & 0 & 1 \end{bmatrix}$ antiperiodic / periodic

Spectrum:

$$\begin{cases} m_3 = \dfrac{n_3}{R} \\ m_{1,2} = \left(n_{1,2} + \dfrac{1}{2}\right)\dfrac{1}{R} \end{cases}$$

$n_i \geq 0$

half of the tower removed by Z_2 parity

$\rightarrow$ only A^{30}_μ is massless.

Mass spectrum according to (Z_2, T) assignment:

Z_2	T	mass $n \geq 0$	
$+$	$+$	$\frac{n}{R}$	zero mode only here
$+$	$-$	$\left(n+\frac{1}{2}\right)\frac{1}{R}$	
$-$	$-$	$\left(n+\frac{1}{2}\right)\frac{1}{R}$	
$-$	$+$	$(n+1)\frac{1}{R}$	

Gauge symmetry can be broken by both T and/or Z_2 non trivial assignment

Z_2 ↳ orbifold breaking

T ↳ SS breaking

SU(5) in $d=5$

$d=5$ gauge vector bosons

$$A^A_M(x,y) \qquad M = \begin{cases} \mu & d=4 \text{ vector bosons} \\ 5 & d=4 \text{ scalars} \end{cases}$$

y compactified on S^1/Z_2 with

	Z	$\mathbb{T}$
A^a_μ	+	+
$A^{\hat{a}}_\mu$	+	−

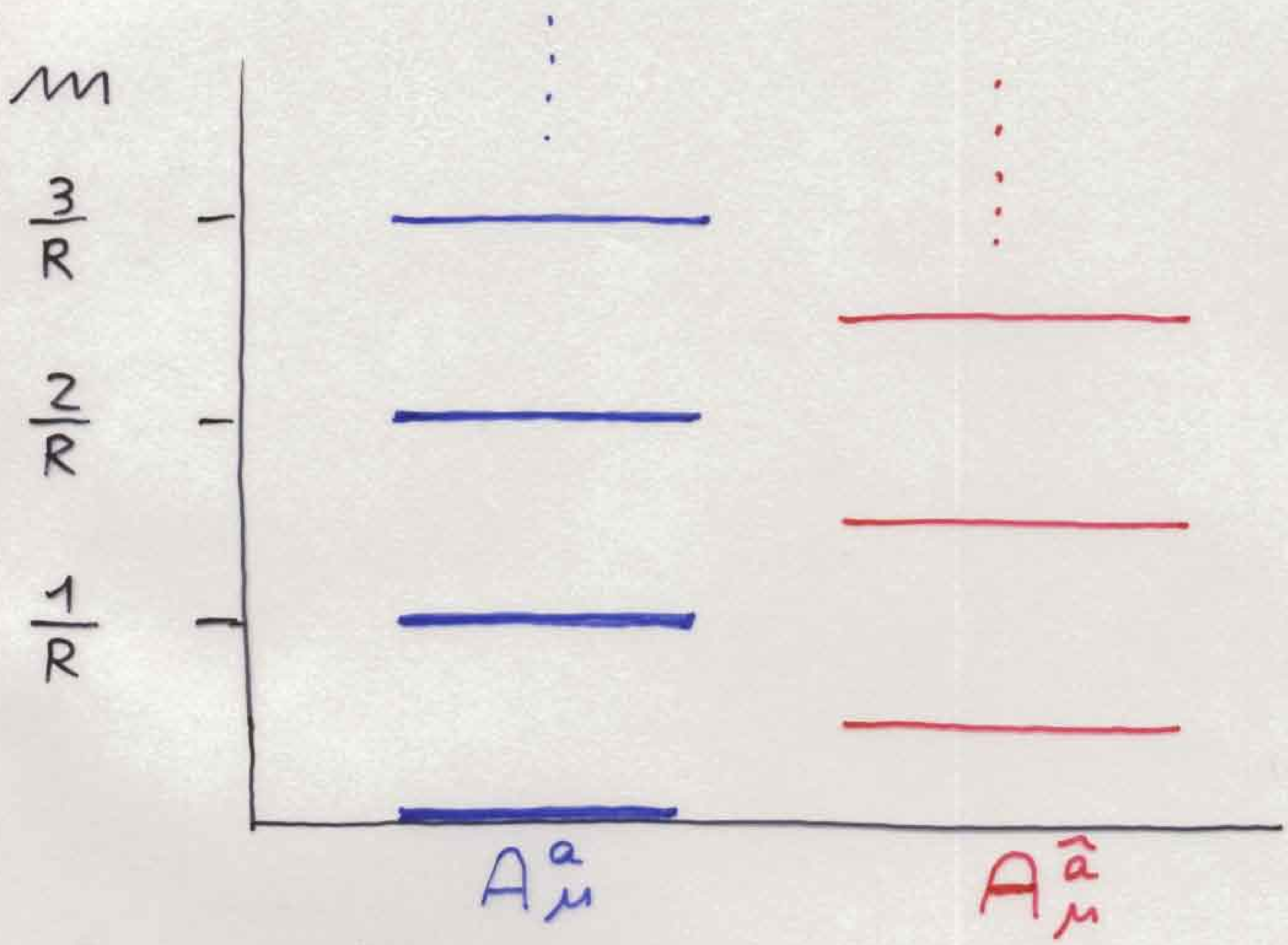
consistent with SU(5)?

$$[A^a_\mu T^a, A^b_\mu T^b] \sim A^c_\mu T^c$$
$$[A^{\hat{a}}_\mu T^{\hat{a}}, A^{\hat{b}}_\mu T^{\hat{b}}] \sim A^c_\mu T^c$$
$$[A^a_\mu T^a, A^{\hat{b}}_\mu T^{\hat{b}}] \sim A^{\hat{c}}_\mu T^{\hat{c}}$$

$\mathbb{T}$ is an automorphism of SU(5) algebra

Zero modes only in $A^a_\mu(x,y)$:

one set of massless gauge vector bosons for $SU(3) \times SU(2) \times U(1)$



SU(5) "broken" down to SU(3) × SU(2) × U(1)

breaking scale $\approx \frac{1}{R} \approx M_{GUT}$

<u>no Σ_s!</u>

symmetry restored in $R \to \infty$ limit

DT SPLITTING:

Include $H_{\mu,d} \equiv \begin{bmatrix} T \\ D \end{bmatrix}(x,y)$

From covariant derivatives:

$$D A_\mu^a D + T A_\mu^a T + D A_\mu^{\hat{a}} T + T A_\mu^{\hat{a}} D$$

Assume $Z(D) = Z(T) = +1$

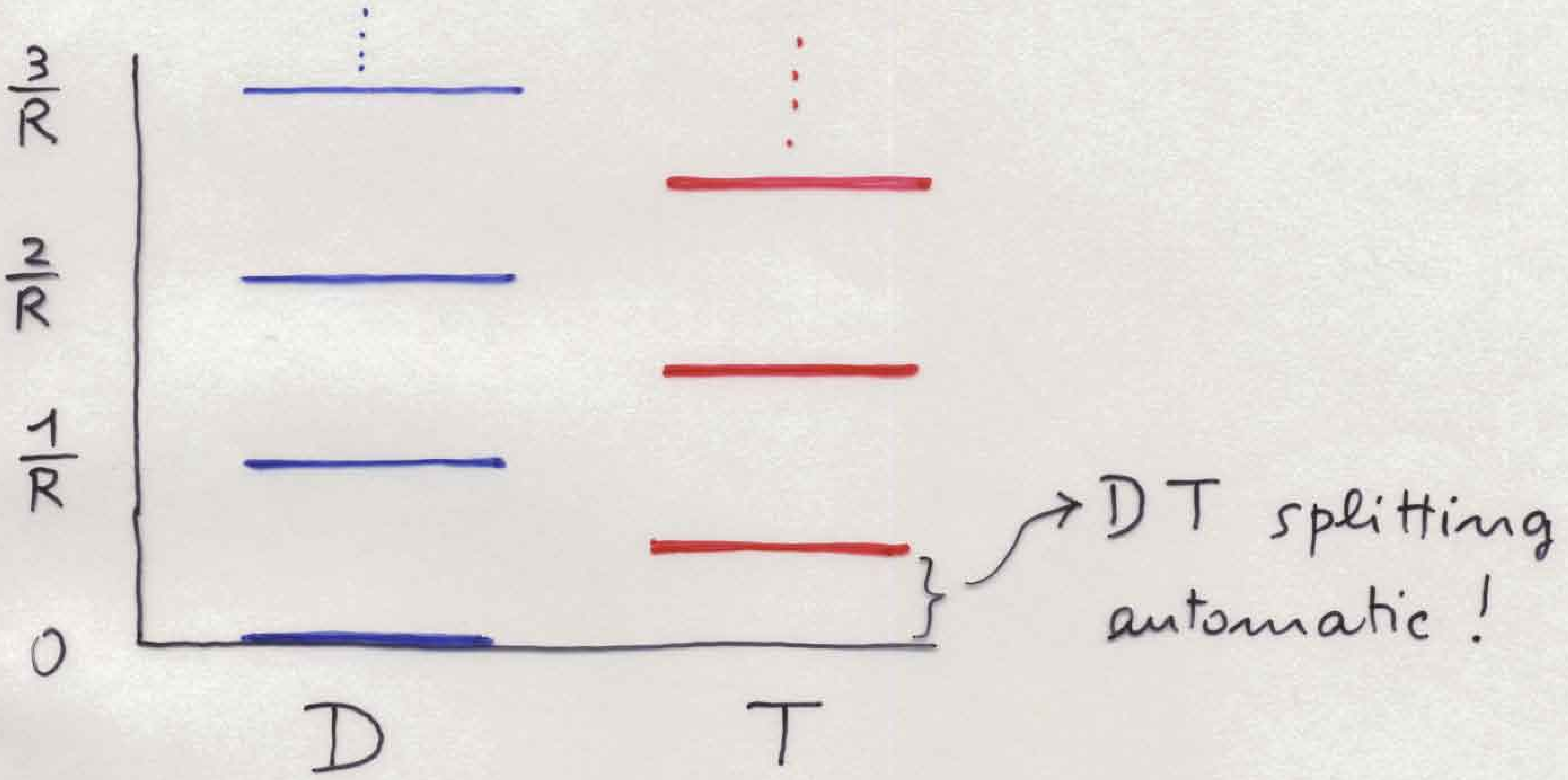
Possible $\mathbb{T}$ assignment

	Z	$\mathbb{T}$	
D	+	+	$(-)$
T	+	−	$(+)$

By choosing

	Z	$\mathbb{T}$
D	+	+
T	+	−

we have the mass spectrum



To make the model realistic we need:

1. SUPERSYMMETRY

to preserve gauge coupling unification

$[N=1 \text{ SUSY in } d=5] \equiv [N=2 \text{ SUSY in } d=4]$

$N=2$ can be reduced down to $N=1$ by orbifold breaking $\leftrightarrow$ suitable Z assignment

2. $U(1)_R$ SYMMETRY

to forbid a bulk mass term $H_u H_d$ (allowed by both $SU(5)$ and $N=2$ SUSY) that would spoil the DT splitting no $H_u H_d$ mass term $\rightarrow$ no p-decay from $d=5$ operators

dominant p-decay by gauge bosons exchange

3. GAUGE COUPLING UNIFICATION:

as good as in conventional 4D GUTs

$$\alpha_3(m_Z) = \frac{\alpha_3(m_Z)\big|_{LO}}{\left[1 + \delta\, \alpha_3(m_Z)\big|_{LO}\right]}$$

$$\delta = \delta^{(2)} + \delta^{(l)} + \delta^{(b)} + \delta^{(h)}$$

$$\left.\begin{array}{l} \delta^{(2)} \simeq -0.82 \\ \delta^{(l)} \simeq -0.50 + \dfrac{19}{28\pi} \log \dfrac{m_{SUSY}}{m_Z} \\ \delta^{(b)} \simeq \pm 1/2\pi \end{array}\right\} \alpha_3(m_Z) \simeq 0.129$$

$$\delta^{(h)} \simeq + \frac{3}{7\pi} \log\left(\frac{\Lambda}{M_c}\right)$$

$$M_c \equiv 1/R$$

can lower α_3 if $\dfrac{\Lambda}{M_c} \gg 1$

p-decay

Unfortunately quite model dependent

$$M_c = 1/R \approx 10^{15\pm 1} \text{ GeV}$$

mass of SU(5) gauge bosons mediating p-decay

$\rightarrow$ p-decay from $d=6$ operators can be important

p-lifetime and decay channels depend on how fermion fields are introduced

$$\begin{cases} \psi(x) & \text{at } y=0 \rightarrow \text{brane fields} \\ \psi(x,y) & \text{bulk fields} \end{cases}$$

For instance:

all fermions as brane fields in $y=0$ $\longleftrightarrow$ $\pi^0 e^+$ dominant almost ruled out by existing bounds

Fermions partly at $y=0$ partly in the bulk $\longleftrightarrow$ $K^+\bar{\nu}$ as "low" as $\tau_p \approx 10^{35}$ yr close to future Water Cherenkov Liquid Argon detector sensitivities