MONOPOLES IN NON ABELIAN GAUGE THEORIES
[t'HOOFT 74, POLYAKOV '74]

SO(3) HIGGS MODEL

$$\mathcal{L} = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}(\overrightarrow{D_\mu \varphi})^+ \overrightarrow{D_\mu \varphi} - \frac{\mu^2}{2}\vec{\varphi}^2 - \frac{\lambda}{4}(\vec{\varphi}^2)^2$$

$$\vec{G}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\,\vec{A}_\mu \wedge \vec{A}_\nu$$

$$\mu^2 < 0 \quad \langle \varphi^2 \rangle = -\frac{\mu^2}{\lambda} \neq 0$$

UNITARY GAUGE $\quad \vec{\varphi} = (0, 0, |\varphi|)$

$$SO(3) \longrightarrow U(1)$$

HEDGEHOG CONFIGURATION

$$\frac{\vec{\varphi}}{|\varphi|} \equiv \frac{\vec{r}}{r}$$

A MAPPING OF S_2 ONTO $SO(3)/U(1)$

$$\pi_2(G/H) = \pi_1(H)$$

$$\pi_2(SO_3/U(1)) = \tilde{\pi}_1(U(1)) = Z_{even}$$

$\tilde{\pi}_1$ = NATURAL HOMOMORPHISM OF $\pi_1(U(1))$ INTO $\pi_1(G)$

A NON TRIVIAL TOPOLOGY $\Longrightarrow$

SOLITON (STABLE STATIC CONFIGURATION)

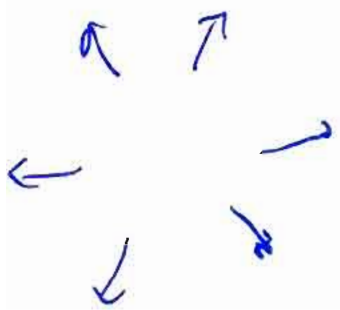
A SINGULARITY AT $\vec{r}=0$ WHEN TRANSFORMING TO UNITARY GAUGE

$- U \varphi U^+ = (\phi, 0, \phi)$

U IS SINGULAR AT ZEROS OF ϕ $\quad \phi(x) = 0$

WHERE $\hat{\varphi} = \frac{\varphi}{|\varphi|}$ IS NOT DEFINED

\- THE HEDGEHOG CONFIGURATION HAS A ZERO AT $\vec{x} = 0$



THIS IS THE ORIGIN OF THE DIRAC STRING

MONOPOLE ~~[illegible]~~ $\vec{\varphi} = 0$ OR

SINGULARITY OF U: IF $\phi = \vec{\varphi}\vec{\sigma}$ AT THE SITES WHER ϕ HAS EQUAL EIGENVALUES

ANSATZ

$$\varphi^a = \frac{r^a}{r}\varphi(r)$$

$$A^a_\mu = -\frac{1}{g}\,\varepsilon_{\mu a b}\frac{r_b}{r^2}\,W(r)$$

A REGULAR SOLUTION EXISTS WITH $\begin{cases}\varphi(r)=1 \\ W(r)=1\end{cases} r > \frac{1}{m}$

AND ENERGY $\sim m$.

t'HOOFT TENSOR

$$\hat{\varphi} \equiv \frac{\vec{\varphi}}{|\varphi|}$$

$$\boxed{F_{\mu\nu} = \hat{\varphi}\cdot\vec{G}_{\mu\nu} - \frac{1}{g}\hat{\varphi}\cdot(D_\mu\hat{\varphi}\wedge D_\nu\hat{\varphi})}$$

- GAUGE INVARIANT BY CONSTRUCTION
- BILINEAR TERMS $A_\mu A_\nu$ CANCEL BETWEEN THE TWO TERMS

$$F_{\mu\nu} = \partial_\mu(\hat{\varphi}\cdot\vec{A}_\nu) - \partial_\nu(\hat{\varphi}\cdot\vec{A}_\mu) - \frac{1}{g}\hat{\varphi}\cdot(\partial_\mu\hat{\varphi}\wedge\partial_\nu\hat{\varphi})$$

IN THE UNITARY GAUGE $\hat{\varphi} = (0,0,1)$

AND

$$F_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu \qquad \text{(ABELIAN)}$$

- $\partial_\mu F^\nu_{\mu\nu} = j^M_\nu$ $\qquad j^M_\nu = 0$ EXCEPT FOR SINGULARITIES

$$\boxed{\partial_\mu j^M_\mu = 0}$$

TOPOLOGICAL CONS. LAW.

- FOR 't HOOFT SOLUTION

$$F_{0i} = 0 \qquad H_i \equiv \frac{1}{2}\varepsilon_{ijk}F_{jk}$$

$$\vec{H} = \frac{1}{g}\frac{\hat{r}}{r^3} + \text{Dirac string}$$

GAUGE TRANSF. TO UNITARY GAUGE ≡ ABELIAN PROJECTION

MAGNETIC CHARGE $\Longleftrightarrow$ HOMOTOPY

ANY CONFIGURATION WITH NON TRIVIAL HOMOTOPY HAS MAGNETIC CHARGE, EVEN IF IT IS NOT A SOLITON

MAGNETIC CHARGED CONFIGURATION $\rightarrow$
$|\vec{\varphi}(x)| = 0$: WORLD LINE OF THE MONOPOLE

- ANY FIELD $\vec{\varphi}$ IN THE ADJOINT REPR. CAN BE USED TO DEFINE (A CONSERVED MONOPOLES) MAGNETIC CHARGE) EVEN IF ~~THERE~~ THE THEORY IS PURE GAUGE (NO HIGGS BREAKING) .

- CALL U(1) THE LITTLEGROUP OF $\vec{\varphi}$
- $SO(3)/U(1)$ HAS NON TRIVIAL HOMOTOPY
- THE CORRESPONDING 'tHOOFT TENSOR CAN BE DEFINED AND j^M_μ $\partial_\mu j^M_\mu = 0$
- THE ABELIAN PROJECTION IS THE G.T TO ~~DIAGONAL~~ $\vec{\varphi}_{\text{kel}} = (0, 0, 1)$

- THE RESIDUAL U(1) GAUGE FIELD $F_{\mu\nu}$ IS COUPLED TO j^M_μ

$$\partial_\mu F^*_{\mu\nu} = j^M_\nu \qquad \partial_\nu j^M_\nu = 0$$

THE MONOPOLE HAS THE NATURAL TOPOLOGY FOR (3+1) d.

GENERALIZE TO SU(N) 't Hooft 81;

[L. Del Debbio, A.D.G, B Lucini, G Paffuti hep-lat/02-03.023]

DEF (A GENERALIZATION OF SU(2))

$$F^a_{\mu\nu} = \mathrm{Tr}\{\phi^a G_{\mu\nu}\} - \frac{i}{g}\mathrm{Tr}\{\phi^a [D_\mu\phi^a, D_\nu\phi^a]\}$$

$A_\mu = \sum_i T^i A^i_\mu \qquad \phi^a = \sum_i T^i \varphi^a_i \qquad D_\mu\phi = \partial_\mu\phi + ig[A_\mu\phi]$

$\mathrm{Tr}\{T^i T^j\} = \delta^{ij}$

$\phi^a \in$ ADJOINT REPRESENTATION

- REQUIRE CANCELLATION OF BILINEARS IN $A_\mu A_\nu$ BETWEEN THE TWO TERMS.
- OBTAINED IFF

$$\phi^a_{(x)} = U(x)\,\phi^a_{diag}\,U^+(x)$$

U(x) AN ARBITRARY GAUGE TRANSF.

$a = 1 \ldots N-1$

$$\phi^a_{diag} = \mathrm{diag}\Big(\underbrace{\frac{N-a}{N},\ldots\frac{N-a}{N}}_{a},\underbrace{-\frac{a}{N}\ldots-\frac{a}{N}}_{N-a}\Big)$$

LITTLE GROUP $SU(a)\times SU(N-a)\times U(1)$

FOR SUCH $\phi^a(x)$

(1) $\partial_\mu F^{a*}_{\mu\nu} = 0$ BIANCHI ID. (APART FROM SINGUL.) $\partial_\mu F^{a}_{\mu\nu} = j^a_\nu \quad \partial_\nu j^a_\nu$

(2) $$F^a_{\mu\nu} = \partial_\mu \mathrm{Tr}\{\phi^a A_\nu\} - \partial_\nu \mathrm{Tr}\{\phi^a A_\mu\} - i\,\mathrm{Tr}\{\phi^a[\partial_\mu\phi^a, \partial_\nu\phi^a]\}$$

(3) IN THE UNITARY GAUGE $\phi^a = \phi^a_{diag} \quad \partial_\mu\phi^a_{diag} = 0$

AND $$F^a_{\mu\nu} = \partial_\mu \mathrm{Tr}\{\phi^a_{diag}A_\nu\} - \partial_\nu \mathrm{Tr}\{\phi^a_{diag}A_\mu\}$$

$$(A_\mu)_{diag} = \sum A^a_\mu \alpha^a \qquad \mathrm{Tr}\{\alpha^a\phi^b_{diag}\} = \delta_{ab}$$

α^a SIMPLE ROOTS

$$\alpha^a = \mathrm{diag}(0,0,0\ldots,\overset{a}{1},\overset{a+1}{-1},0,0,\ldots 0)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$ (ABELIAN PROJ)

(4) $H \equiv$ LITTLE GROUP OF ϕ^a_{diag} $SU(a)\otimes SU(N-a)\otimes U(1)$

$$\pi_2(SU(N)/H) = \pi_1(H) = Z$$

(5) $SU(N)/H$ GENERATES A SYMMETRIC SPACE $L' \in SU(N)/H$ $L_0 \in H$

$[L^0, L^0] \in L_0$ $[L_0, L'] \in L'$ $[L', L'] \in L^0$

MICHEL CONJECTURE (MICHEL 75) [WEINBERG]

(6) A MONOPOLE 'tHOOFT POLYAKOV SOLUTION EXISTS IN THE SUBSPACE SU(2) (a, a+1) OF WHICH α^a IS THE 3d COMPONENT

(7) a $\Longleftrightarrow$ ORBIT IN THE GROUP.

IF X IS ANY HERMITIAN OPERATOR $\in$ ADJOINT REPR.

$$X = U(x)\, X_{diag}\, U^+(x) \qquad (X_{diag})_{i+1,i+1} > (X_{diag})_{i,i}$$

$$X_{diag} = \sum_a c^X_a(x)\, \phi^a_{diag}$$

$$X = \sum U(x)\phi^a_{diag} U^+(x) \qquad \langle \phi^a(x)\rangle = \sum_a c^X_a(x)\, \phi^a(x)$$

$$c^X_a = (X_{diag})_{a+1,a+1} - (X_{diag})_{a,a}$$ DIFFERENCE BETWEEN EIGENVALUES.

U SINGULAR AT $c_a(x) = 0$, POSITION OF A MONOPOLE

(8) TO EACH $\mathcal{X}$ CORRESPONDS A U(X)

$$\mathcal{X} \Leftrightarrow U(x)$$

IE. AN ABELIAN PROJECTION AND A CONFIGURATION OF MONOPOLES $\mathcal{E}_a^{\mathcal{X}}(x) = 0$

HOWEVER IF $U(x)$ $U'(x)$ ARE SUCH THAT $U(x)\, U'^{+}(x)$ IS CONTINUOUS, $V(x)$ CANNOT MODIFY THE TOPOLOGY AND THE LOCATION OF SINGULARITIES.

~~A MONOPOLE~~

A CONFIGURATION WITH A MONOPOLE AT X IN SOME ABELIAN PROJECTION P, WILL HAVE A MONOPOLE AT X IN ALL THE PROJECTIONS CONNECTED TO P BY A GAUGE TRANSFORMATION WHICH IS REGULAR IN A NEIGHBOURHOOD OF X.

- MONOPOLES ON THE LATTICE: HOW TO DETECT THEM.

$$U_\mu(n) = e^{iA_\mu(n)}$$

IF (G/H) GENERATES A SYMMETRIC SPACE

A GENERIC U CAN BE UNIQUELY SPLIT AS

$$U = e^{iL_0} e^{iL'} \qquad L_0 \in H \quad L' \in G/H$$

AND $U(1)$ FACTORIZES FROM e^{iL_0}

AN ABELIAN LINK ASSOCIATED TO EACH LINK, AND MONOPOLES CAN BE DEFINED AND DETECTED, À LA DEGRAND-TOUSSAINT

- DIAGONALIZE X (ABELIAN PROJECTION)
- FACTORIZE THE $U(1)$ CORRESPONDING TO α^a:

(RAPID) SURVEY OF THE LITERATURE

(MONOPOLES)

- PIONEERING WORK [DESY GROUP 90]
  COUNTING OF MONOPOLES AT $T < T_c$ AND $T > T_c$.
  (SEE L. Del Debbio, A. DG, M. Maggiore, S. Olejnik 91]: ORDER PARAMETER
- MAX. ABELIAN GAUGE ['t HOOFT 81]
  GAUGE TRANSFORM

$$U_\mu(x) \longrightarrow V(x)\, U_\mu(x)\, V^+(x+\hat\mu) = U'_\mu(x,V)$$

SU(2) : ~~[illegible]~~ $-\text{MAX} \sum_{n,\mu} \text{TR}\{ U'_\mu(n,V)\sigma_3 U'^{+}_\mu(n,V) \cdot \sigma_3 \}$

$$D^3_\mu A^\pm_\mu \equiv \partial_\mu A^\pm_\mu - ig\,[A^3_\mu \tfrac{\sigma^3}{2}, A^\pm_\mu] = 0$$

A "RENORMALIZABLE" GAUGE.

- RESULTS U_μ 90% ORIENTED ALONG σ_3
  (ABELIAN DOMINANCE) [SUZUKI 91]
  - NEGLECT $\sigma_\pm$ COMPONENTS $\Rightarrow$ U(1) GAUGE THEORY
    COMPUTE σ, $\langle \bar\psi \psi \rangle$ ...
- - MONOPOLE DOMINANCE [STACK 92]
  - SURGICAL APPROACH: ELIMINATE MONOPOLES AND COMPUTE σ, $\langle \bar\psi\psi \rangle$ ...

ALL THAT DOES NOT HAPPEN WITH OTHER ABELIAN PROJECTIONS $\Longrightarrow$ MONOPOLES OF THE MAX ABELIAN PROJECTION ARE THE DUAL EXCITATIONS OF QCD.

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- RECONSTRUCT THE EFFECTIVE LAGRANGEAN OF MONOPOLES (M.A.G) FROM THEIR CONFIGURATIONS [ITEP - KANAZAWA]

  INCONCLUSIVE : HIGHER TERMS THAN φ^4 NEEDED.

A FEW GENERAL COMMENTS

- SURGICAL APPROACH LOGICALLY INEFFECTIVE TO CONCLUDE THAT MONOPOLES ARE THE DUAL EXCITATIONS
- WHY M.A.G.? A FUNCTIONAL INFINITY OF GAUGES, SOME OF THEM ~EQUIVALENT TO M.A.G. ARE WE OBSERVING A "KINEMATICAL" EFFECT?
- AN EFFECTIVE LAGRANGEAN MAKES SENSE IF THE SYMMETRY IS UNDERSTOOD.

$\Downarrow$

(a) LOOK FOR AN ORDER PARAMETER TO UNDERSTAND THE SYMMETRY AND TO DEFINE CONFINED & DECONFINED

(b) TRY TO IDENTIFY THE "MONOPOLES" DESCRIBING DUAL DEGREES OF FREEDOM

CONSTRUCTION OF THE ORDER PARAMETER [A.D.G 93]

BASIC PRINCIPLE

$$e^{ipa}|x\rangle = |x+a\rangle$$

p CONJUGATE MOM. TO x.

FIELD THEORY $x, p \rightarrow \varphi(x), \Pi(x)$

- SCHRODINGER REPRESENTATION $|\varphi(\vec{x},t)\rangle$

$$e^{i\int d^3y\, \Pi(\vec{y},t)\, f(\vec{y})} |\varphi(\vec{x},t)\rangle = |\varphi(\vec{x},t)+f(\vec{x})\rangle$$

- LET $\vec{b}_\perp(\vec{x}-\vec{y})$ BE THE VECTOR POTENTIAL OF A DIRAC MONOPOLE, IN THE TRANSVERSE GAUGE

$$\vec{\nabla}\vec{b}_\perp = 0 \qquad \vec{\nabla}\wedge\vec{b}_\perp = H(\vec{r}) = \frac{n}{2e}\frac{\vec{r}}{r^3} + \text{DIRAC STRING}$$

e.g. $$\vec{b}_\perp = \frac{n}{2e}\frac{\vec{r}\wedge\vec{n}_3}{r(r-\vec{r}\cdot\vec{n}_3)}$$ STRING ON 3 AXIS

SU(N) GAUGE THEORY: ABELIAN PROJECTION $U(x)$

$$\phi^a(x) = U(x)\,\phi^a_{diag}\,U^+(x)$$

$$\phi^a_{diag} = \mathrm{diag}\Big(\underbrace{\tfrac{N-a}{N}\cdots\tfrac{N-a}{N}}_{a \text{ times}}, \underbrace{-\tfrac{a}{N}\cdots -\tfrac{a}{N}}_{N-a \text{ times}}\Big)$$

DEF. $\mu(\vec{x}, t)$ [A. DG., B. Lucini, L. Montesi, G. Paffuti 00]

$$\mu(\vec{x},t) \equiv e^{i\int d^3y \, Tr\{\Phi^a(\vec{y},t)\vec{E}(\vec{y},t)\} \vec{b}_\perp(\vec{x}-\vec{y})}$$

$$\Phi^a = \sum_i \Phi_i^a T^i \qquad \vec{E} = \sum_i \vec{E}_i T^i \qquad Tr\, T^i T^j = \delta_{ij}$$

- μ IS GAUGE INVARIANT..

$$Tr\{\Phi^a(\vec{y},t)\vec{E}(\vec{y},t)\} = Tr\{U(\vec{y},t)\Phi^a_{diag}U^+(\vec{y},t)\vec{E}(\vec{y},t)\}$$
$$= Tr\{\Phi^a_{diag}\vec{E}(\vec{y},t)_{A.P}\} = \vec{E}^a_{A.P}(\vec{y},t)$$

- $\vec{E}(\vec{y},t)_{AP} = U^+(\vec{y},t)\vec{E}(\vec{y},t)U(\vec{y},t)$ IS THE ELECTRIC FIELD OPERATOR IN THE AB. PROJ. GAUGE

$$\mu(\vec{x},t) = e^{i\int d^3y \, \vec{E}^a_{AP}(\vec{y}\, t)_\perp \vec{b}_\perp(\vec{x}-\vec{y})}$$

μ RESIDUAL U(1) GAUGE INVARIANT
IN WHATEVER QUANTIZATION SCHEME
$\vec{E}_\perp$ IS THE CONJUGATE MOMENTUM TO $A_\perp$
$\Longrightarrow$

$$\mu(\vec{x},t)\,|..A^a_\perp(\vec{y})\rangle = |A^a_\perp(\vec{y}) + \vec{b}_\perp(\vec{x}-\vec{y})\rangle$$

$\mu(\vec{x},t)$ CREATES A MONOPOLE AT $\vec{x},t$

IN THE RESIDUAL U(1) OF THE GIVEN ABELIAN PROJECTION.

U(1)

$$\mu(\vec{x},t) = e^{i\int d^3y \, \vec{E}(\vec{y},t)\vec{b}_\perp(x-y)} = e^{i\int d^3y \, \vec{E}_\perp(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})}$$

- μ A NON LOCAL OPERATOR CARRYING MAGNETIC CHARGE.
- GAUGE INVARIANT , U(1) GAUGE INVARIANT

A CANDIDATE (DIS) ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY

IF

$\langle \mu \rangle \neq 0 \quad T < T_c \qquad \langle \mu \rangle = 0 \quad T > T_c$

THEN CONFINED VACUUM IS A DUAL SUPERCONDUCTOR

PROBLEMS

(i) CHECK THE DEFINITION CONSTRUCTIVELY (LATTICE) ⟹ U(1)

(ii) DEPENDENCE ON THE ABELIAN PROJECTION?

(iii) WHAT CAN BE SAID ON THE DUAL EXCITATIONS?

– CONSTRUCTIVE DEFINITION OF μ

4D U(1) COMPACT G.T. [A.D.G, G PAFFUTI 97; FRÖLICH MARCHETTI 86]

$$\beta S = \beta \sum_{n,\mu\nu} \left[\Pi_{\mu\nu}(n) - 1 \right] \qquad \beta = \frac{2}{g^2}$$

$$\Pi_{\mu\nu}(n) = U_\mu(n)\, U_\nu(n+\hat\mu)\, U_\mu^+(n+\hat\nu)\, U_\nu^+(n)$$

$$U_\mu(n) = e^{i\theta_\mu(n)} \qquad \Pi_{\mu\nu} = e^{i\theta_{\mu\nu}}$$

$$\theta_{\mu\nu} = \Delta_\mu A_\nu - \Delta_\nu A_\mu$$

$$Z = \int \prod_{n,\mu,\nu} \frac{d\theta_\mu}{2\pi}\, e^{-S\beta}$$

WEAK. FIRST ORDER TRANSITION AT $\beta = 1.01$ FROM CONFINED TO DECONFINED.

CONJUGATE MOMENTUM $\quad e\, \mathrm{Im}\, \Pi_{0i} = e \sin\theta_{0i}$

$$\mu(\vec x, n_0) = e^{-\beta \sum_{\vec n} \mathrm{Im}\, \Pi_{0i}(\vec n, n_0)\, b^i(\vec x - \vec n) \frac{m}{2}}$$

[Del Debbio et al 94]

COMPACTIFIED VERSION [ADG, PAFFUTI 97]

$$\mu(\vec x, n_0) = e^{\beta \sum_{\vec n} \left[S(\theta_0^i(\vec n, t) + b^i(\vec x - \vec n)\frac{m}{2}) - S(\theta_0^i(\vec n, t)) \right]} = e^{-\beta S'}$$

WILSON ACTION: $\cos(\theta_{i0} + b^i \frac{m}{2}) - \cos(\theta_{i0})$

$$\approx \cos\theta_{i0} - \sin\theta_{i0} \frac{m}{2} b^i(\vec x - n) - \cos\theta_{i0}$$

$$\left|\frac{m}{2} b^i\right| \ll 1.$$

$$\langle \mu(\vec x, n_0) \rangle = \frac{\int \prod d\theta\, e^{-\beta_S S} \mu(\vec x, n_0)}{\int \prod d\theta\, e^{-\beta S}} = \frac{Z[S + S']}{Z[S]}$$

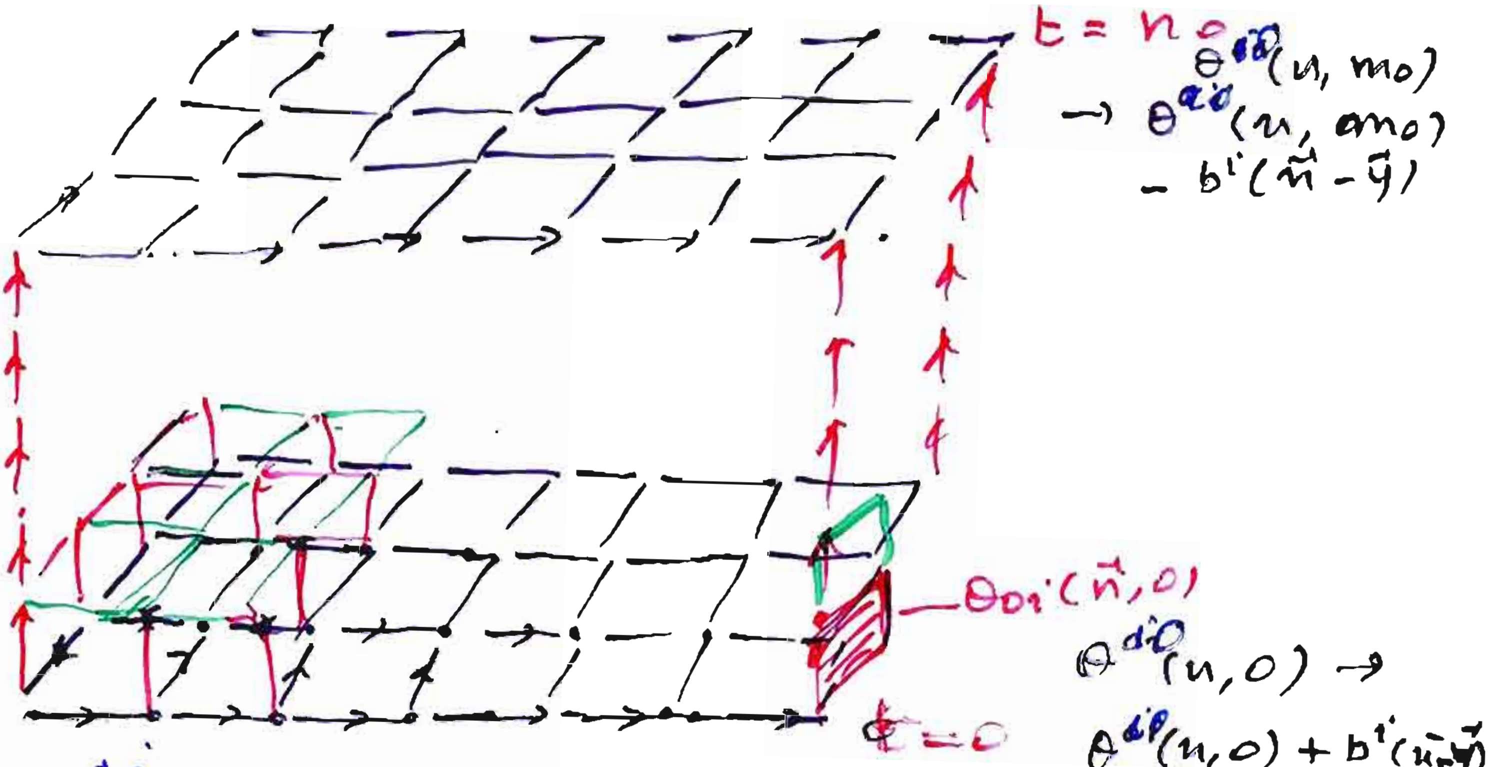
• COMPUTING $\langle\mu\rangle$

$$\langle \bar{\mu}(\vec{y}, n_0)\, \mu(\vec{y}, 0)\rangle \underset{n_0\to\infty}{\simeq} C e^{-M n_0} + \langle\mu\rangle^2$$

M = MASS OF THE LIGHTEST MONOPOLE

$\langle\mu\rangle \underset{c}{=} \langle\bar{\mu}\rangle$ DISORDER PARAMETER

$\langle\mu\rangle \neq 0$ SIGNALS DUAL SUPERCONDUCTIVITY



$\theta^{0i}(n, 0) = -\theta^i(\vec{n}, 1) + \theta^i(\vec{n}, 0) - \theta^0(\vec{n}+\hat{i}, 0) + \theta^0(\vec{n}, 0)$

$\theta^{0i} \to \theta^{0i} + b^i \quad \equiv \quad \theta^i(\vec{n}, 1) \to \theta^i(\vec{n}, 1) - b^i(\vec{n}, -y)$

CHANGE VARIABLE. $\quad \theta^{ij}(\vec{n}, 1) \to \theta^{ij}(\vec{n}, 1) + \Delta^i b_j - \Delta^j b_i$

$\theta^{i0}(\vec{n}, 1) \to \theta^{i0}(\vec{n}, 1) + b^i(\vec{n} - y)$

VORTICES ['t HOOFT NPB138(1978) 1]

- GAUGE THEORY WITH GROUP G COUPLED TO A SCALAR FIELD

SOLITON $\lim_{r\to\infty} \phi(r,\Omega) = \phi(\infty,\Omega)$

$\phi(\infty,\Omega)$ A MINIMUM OF $V(\phi)$

- A MAPPING ON G/H OF S_{d-1}
- STABILITY IF THE MAPPING IS NON TRIVIAL: $\Pi_{d-1}(G/H)$ NON TRIVIAL

- $SO(3)$ $[SU(2)]$

$SO(3)/U(1) = S_2$ $\quad d=3\ (2+1)$ $\quad \Pi_1\left(SO(3)/U(1)\right) = 0$ $\quad$ NO SOLITON, NO TOPOLOGY

$d=3+1$ $\quad \Pi_2\left(SO(3)/U(1)\right) = Z_{even}$ $\quad$ MONOPOLES

$SU(2)/Z_2$ $\quad d=2+1$ $\quad \Pi_1\left(SU(2)/Z_2\right) = Z_2$ $\quad$ VORTICES

$\Pi_2\left(SU(2)/Z_2\right) = 0$ $\quad$ NO TOPOLOGY, NO SOLITON

$$\Pi_2\left(\frac{SU(n)}{SU(n)\,SU(N-n)\,U(1)}\right) = Z_{even}$$

$$\Pi_1\left(\frac{SU(N)}{SU(n)\,SU(N-n)\,U(1)}\right) = \{0\}$$

$$\Pi_2\left(SU(N)/Z_N\right) = \{0\}$$

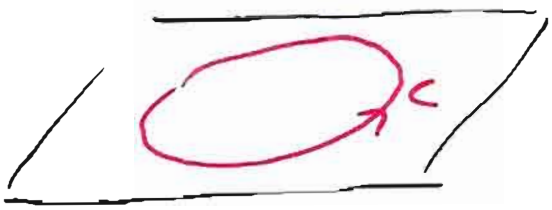
$$\Pi_1\left(SU(N)/Z_N\right) = Z_N$$

MONOPOLES ARE THE NATURAL TOPOLOGICAL EXCITATIONS IN 3+1 d; VORTICES IN 2+1.

PR 2 VORS 1

(2+1) d _ SU(N) COMPLETELY BROKEN TO

$\phi(x)$ $\quad |\phi(x)| \neq 0 = H_0 \quad \Phi(x) = U^\dagger(x) \cdot H_0$



$H_0 = U(x)\,\Phi(x)$

$U(\theta) = U(x_c)$

CURVE C : PARAMETRIZE BY $0 \leq \theta \leq 2\pi$

$U(2\pi) = U(0)\, e^{i \frac{n 2\pi}{N}} \quad -N \leq n \leq N$

element of the centre.

$n \neq 0 \implies$ VORTEX

. SHRINK C TO A POINT $\longrightarrow$ SINGULARITY

- VORTEX HAS A CHARGE $\pm n$ (mod N), WHICH IS CONSERVED TOPOLOGICALLY

- ONE CAN DEFINE THE CREATION OPERATORS OF VORTICES $\upsilon(\vec{x})$,

$\langle \upsilon \rangle = 0$ UNBROKEN PHASE

$\langle \upsilon \rangle \neq 0$ CONDENSATION OF VORTICES

$$A(C) \equiv \mathrm{Tr}\left\{ P \oint_C \exp \oint_C i g \vec{A}(x)\, d\vec{x} \right\}$$

$\vec{A}(x) = \sum_a T^a \vec{A}^a(x)$ THE VECTOR POTENTIAL

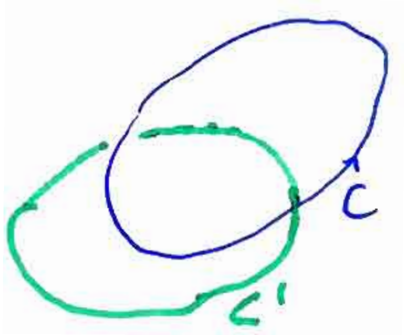
$[A(C), \upsilon(\vec{x})] = 0 \quad \vec{x}$ external to C

$A(C)\,\upsilon(\vec{x}) = \upsilon(x)\, A(C)\, e^{2\pi i n/N} \quad \vec{x}$ internal to C.

$[\upsilon(\vec{x}), \upsilon(\vec{y})] = 0$

3+1 d NO CONSERVED TOPOLOGICAL CHARGE.

$$A(c,t) = \mathrm{Tr}\left\{ \mathcal{P} \exp \oint_c i g \vec{A}(\vec{x},t)\, d\vec{x} \right\}$$



Def $B(c'\, t)$ CREATING A VORTEX

$$[A(c,t), A(c'\, t)] = 0$$

$$[B(c,t), B(c'\, t)] = 0$$

$$A(c)\, B(c') = B(c')\, A(c)\, e^{\frac{2\pi i n}{N}}$$

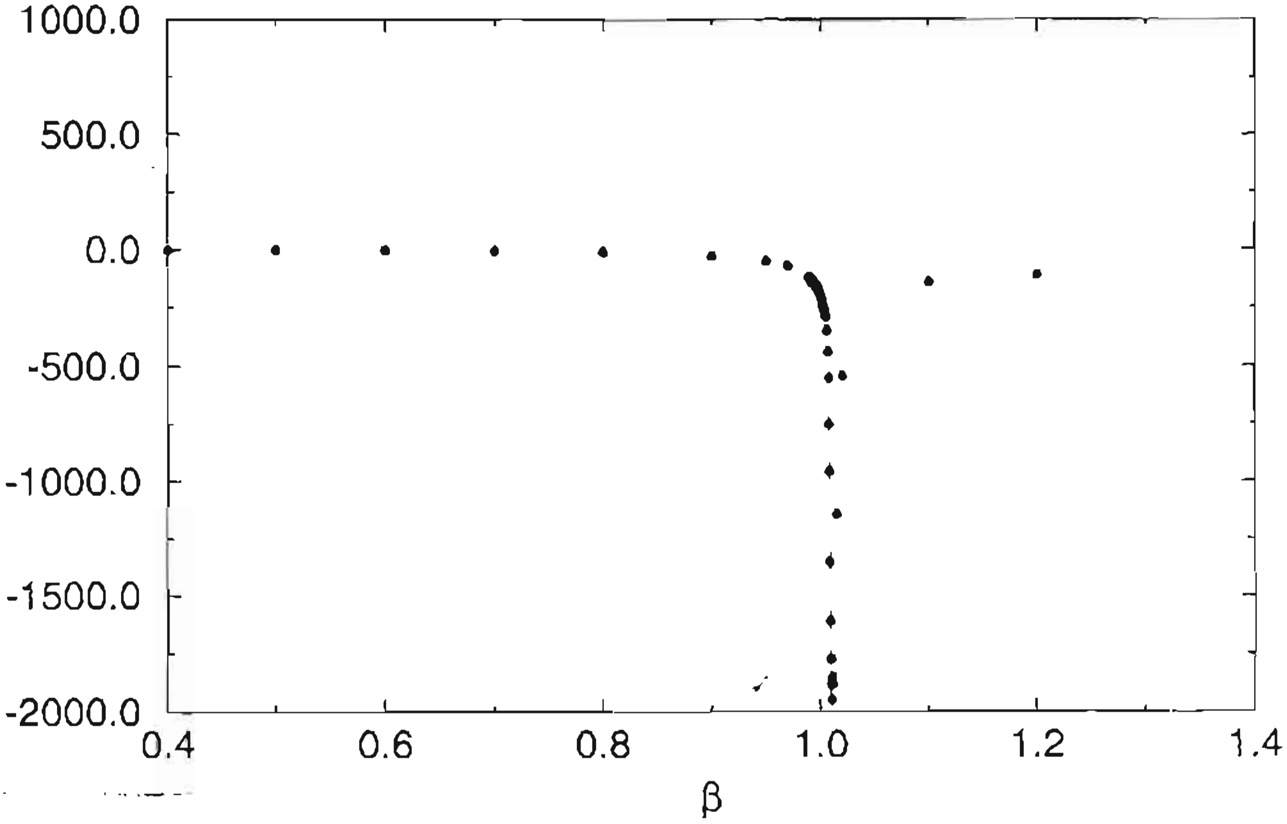
<u>THEOREM</u> IF A OBEYS THE PERIMETER LAW B OBEYS THE AREA LAW; IF A OBEYS THE AREA LAW B OBEYS THE PERIMETER LAW

$\langle L \rangle$ POLYAKOV LOOP

$\langle \tilde{L} \rangle$ 'tHOOFT LOOP

$$\langle L \rangle \langle \tilde{L} \rangle = 0 \qquad Z_N$$

<u>PROBLEM</u> WHAT ABOUT INTRODUCING QUARKS?

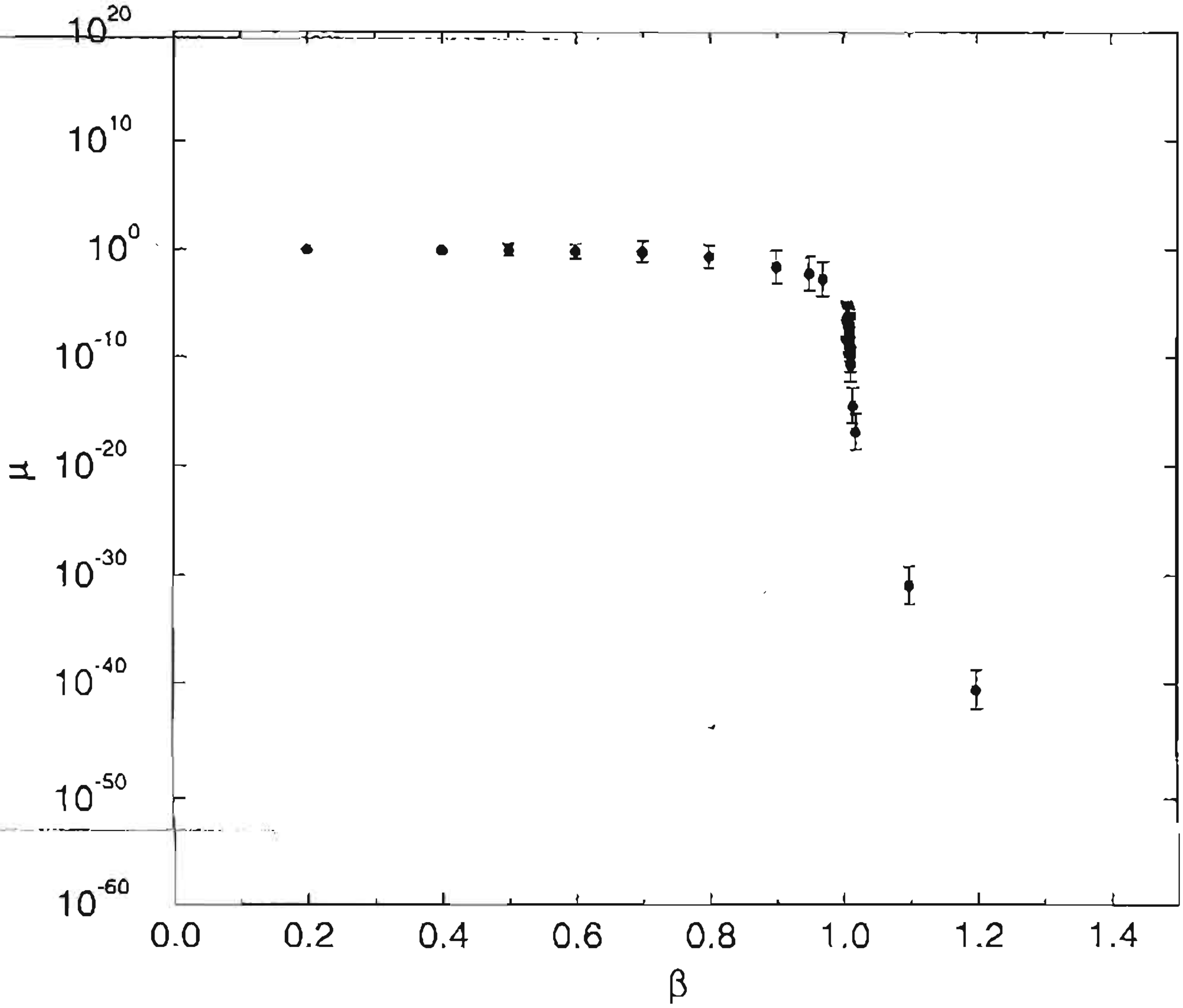


U(1) GAUGE THEORY

ρ vs β $8^3 \times 16$

A.D.G, G. PAFFUTI
PHYS REV D 56, 6816, (1997)

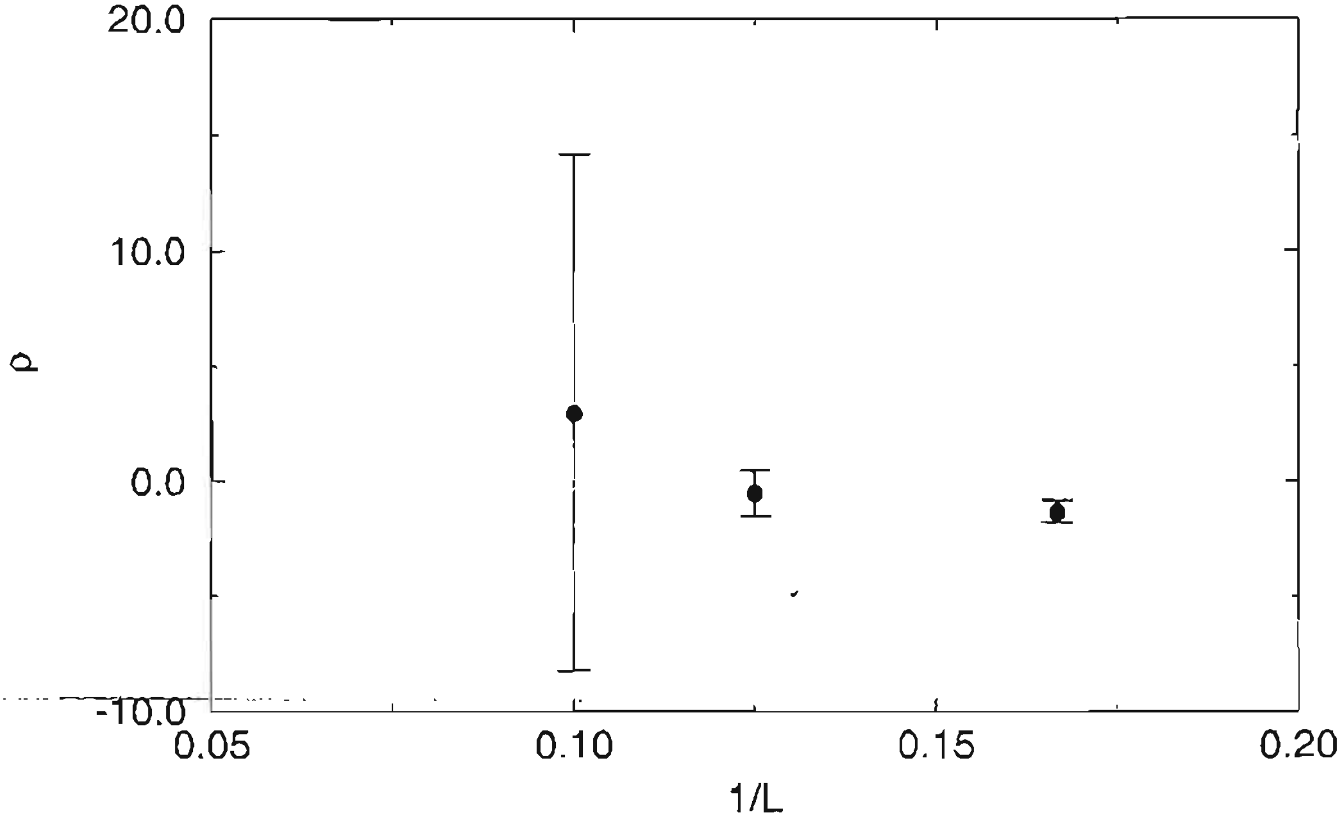
U(1) 2



$$\langle \mu \rangle = \exp\left[\int_0^\beta \rho(\beta')\, d\beta'\right]$$

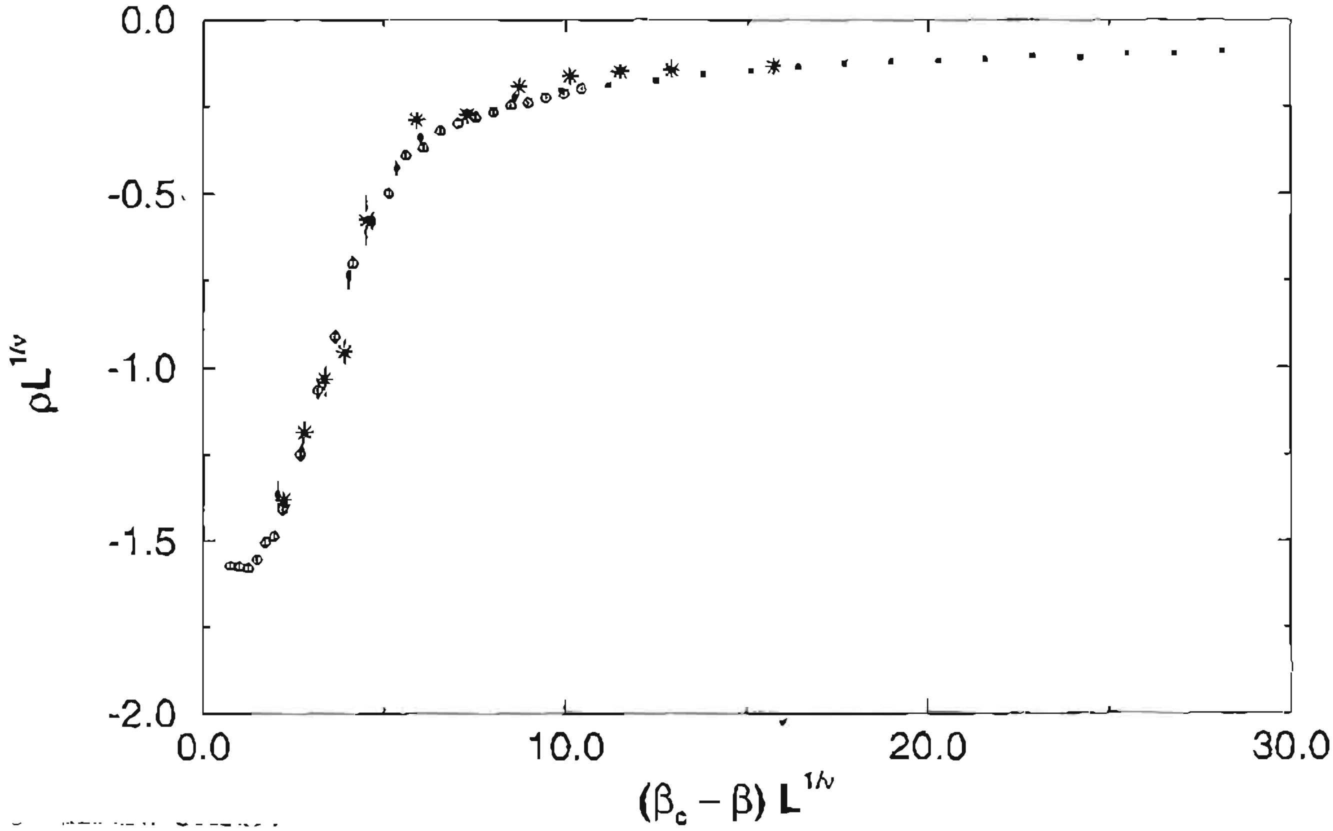
$\langle \mu \rangle$ vs β

U(1) 3



ρ vs. $\frac{1}{L}$ $\beta = 1.009$ ($\beta_c = 1.01160$)

$U(1)_4$



SCALING $\rho / L_s^{1/\nu} = f(\tau L^{1/\nu})$

$\nu = .29(2)$ $\beta_c = 1.01160(5)$

$$\langle \mu^+(\vec{0}, t) \;\; \mu(\vec{0}, 0) \rangle = \frac{Z(S+S')}{Z(S)}$$

$$S' = \sum_{\vec{n}} S\left(\theta_0^i(\vec{n}, 0) - b_i(\vec{0}-\vec{n}) \frac{m}{2} \right) - S\, \theta_0^i(\vec{n}, 0)$$

$$+ \sum_{\vec{n}} S\left(\theta_0^i(\vec{n}, t) + b_i(\vec{0}-\vec{n}) \frac{m}{2} \right) - S\, \theta_0^i(\vec{n}, t)$$

- ANY CORRELATOR $\langle \mu(x_1) \ldots \mu(x_n) \rangle$ CAN BE DEFINED BY MODIFING S AT THE APPROPRIATE TIMES

- [illegible] $\langle \mu^+(\vec{0}, t) \;\; \mu(\vec{0}, 0) \rangle \underset{t \to \infty}{\simeq} |\langle \mu \rangle|^2 + C e^{-Mt}$

(CLUSTER PROPERTY) $\Rightarrow \langle \mu \rangle$ $\Rightarrow$.

- NUMERICAL RESULTS . $\langle \mu \rangle$ fig 2.1

VERY NOISY. $\langle \mu \rangle = 0$?

DEFINE $\rho \equiv \dfrac{\partial \ln \langle \mu \rangle}{\partial \beta} =$ $\langle S \rangle_S - \langle S + \Delta S \rangle_{S+\Delta S}$

$$\frac{\partial \ln C(t)}{\partial \beta} \underset{t \to \infty}{\simeq} 2\rho$$

$$\langle \mu \rangle = \exp \int_0^\beta \rho(\beta')\, d\beta'$$

$$[\langle \mu \rangle_{\beta=0} = 1]$$

ρ. AS A FUNCTION OF $V = L^4$

THERMODYNAMICAL LIMIT $\langle \mu \rangle \neq 0 \quad \beta < \beta_c$

$\langle \mu \rangle = 0 \quad \beta > \beta_c$

$\Downarrow$

$\rho \to -\infty \quad \beta > \beta_c$ [pt theory] as $V \to \infty$

$\rho \to$ finite function of $\beta \quad \beta < \beta_c$

$\beta \sim \beta_c$ FINITE SIZE SCALING

$$\langle \mu \rangle \simeq L_s^{\eta}\, \Phi_\mu\left(\frac{a}{\xi}, \frac{L_s}{\xi}\right) \qquad \xi \sim (\beta - \beta_c)^{-\nu} \simeq \tau^{-\nu}$$

$$\tau = 1 - \frac{T}{T_c}$$

$$\frac{a}{\xi} \simeq 0 \qquad \langle \mu \rangle = L_s^{\eta}\, \Phi_\mu\left(0, \frac{L_s}{\xi}\right)$$

$$\frac{L_s}{\xi} \rightleftarrows \tau L_s^{1/\nu} \qquad \rho = \frac{\partial}{\partial \beta} \ln \tilde{\Phi}_\mu (\tau L_s^{1/\nu}) + 0$$

$$\boxed{\rho / L_s^{1/\nu} = f(\tau L_s^{1/\nu})} \Rightarrow \nu, \beta_c$$

Fig.

RESULTS

$\beta > \beta_c \qquad \rho \simeq -.505\, L_s + 4.7 \underset{L_s \to \infty}{\Rightarrow} -\infty \Rightarrow \mu = 0$

$\beta < \beta_c \qquad \rho(\beta) \to \rho_\infty(\beta)$ fig.

$\beta \simeq \beta_c \qquad$ SCALING $\Rightarrow \nu, \beta_c$

$$\beta_c = 1.01160\ (5)$$

$$\nu = .29\ (2)$$

VACUUM A DUAL SUPERCONDUCTOR FOR $\beta < \beta_c$: NORMAL $\beta > \beta_c$

A RESULT WHICH WAS ALREDY KNOWN AS A THEOREM. [FROLICH - MARCHETTI 87 (VILLAIN) CIRIGLIANO, PAFFUTI 96 WILSON]

- THE APPROACH OF F. M.

INTRODUCE A DEFECT

$$\langle \bar{\mu}(t,\vec{0})\,\mu(0,\vec{0})\rangle = \frac{Z(X)}{Z(0)} \qquad Z(0) = Z[d\theta] \qquad Z(X) = Z[d\theta + X]$$

$$d\theta = F_{\mu\nu}\, dx^\mu \wedge dx^\nu \qquad (2\ \text{form})$$

$$X = d\frac{1}{\Delta}a + \delta\frac{1}{\Delta}b$$ [HODGE DECOMPOSITION]

$$dX = d\delta\frac{1}{\Delta}b = b$$

- THE FIRST TERM IS AN EXACT FORM AND CAN BE ABSORBED BY A SHIFT IN θ

- X IS FIXED ONCE $dX = b$ IS FIXED

$$dX_{\mu\nu\alpha} = -(\partial_\alpha X_{\mu\nu} + \partial_\mu X_{\nu\alpha} + \partial_\nu X_{\alpha\mu})$$ VIOLATION OF BIANCHI IDENTITY

$$* dX = j_\mu = \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma}\, dX_{\mu\rho\sigma}$$ (MAGNETIC CURRENT)

$$\delta J_\mu = 0$$ [MAGNETIC U(1) SYMMETRY]

TO CREATE A MONOPOLE $(\vec{0},0)$ AND DESTROY IT AT $(\vec{0},t)$

$$j^0 = 2\pi q\, \delta^3(\vec{x})\,[\theta(x_0) - \theta(x_0 - t)]$$

$$\partial_\mu j^\mu = 0 \Rightarrow \vec{\nabla}\cdot\vec{j} = -2\pi q\, \delta^3(\vec{x})\,[\delta(x_0) - \delta(x_0 - t)]$$

$$\vec{j} = -\frac{2\pi q}{4\pi}\frac{\vec{x}}{x^3}[\delta(x_0) - \delta(x_0 - t)]$$ $* dX$

TWO dX DIFFERING BY A $2\pi n$ VALUED CURRENT ON A 1-d SUPPORT GIVE THE SAME $Z[X]$

PRZ-23

OUR APPROACH $\bar{X}_{\alpha\beta}$

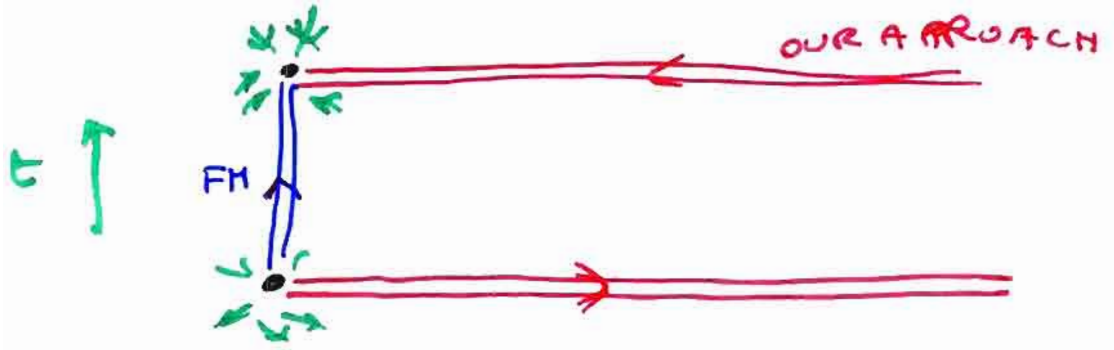
$$\bar{X}_{ij} = 0 \qquad \bar{X}_{0i} = -2\pi q\, b_i\, [\delta(x_0) - \delta(x_0 - t)]$$

$$({}^*\bar{X})_{0i} = 0 \quad ({}^*\bar{X})_{ij} = -\varepsilon_{ijk}\, 2\pi q\, b_k\, [\delta(x_0) - \delta(x_0 - t)]$$

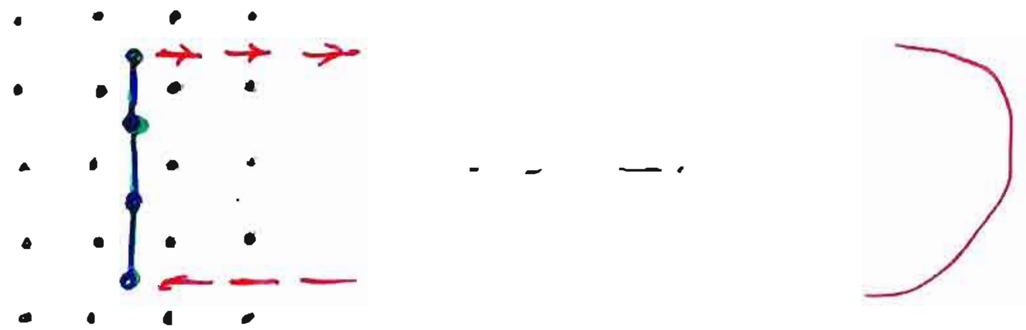
$$({}^*d\bar{X})_\mu = \delta({}^*\bar{X})_\mu = -\Delta_\beta({}^*X)_{\beta\mu}$$

$$= 2\pi q \frac{\vec{x}}{4\pi x^3} [\delta(x_0) - \delta(x_0 + t)] - 2\pi q\, \delta(x_1)\delta(x_2)\theta(x_3) [\delta(x_0) - \delta(x_0 + t)]$$

IDENTICAL TO F.M EXCEPT FOR A CLOSED STRING



SAME AS IN ISING MODEL



- THEOREMS

(1) $\langle\mu\rangle \neq 0 \quad \beta < \beta_c$ [Frohlich Marchetti 86 ; Cirigliano Paffuti 96]
$\langle\mu\rangle = 0 \quad \beta \geq \beta_c$ (SUPERSELECTED SPACE)

(2) μ IS A DIRAC LIKE (CHARGED & GAUGE INVARIANT) OPERATOR. (OBEYS CLUSTER PROPERTY)

(3) DUAL SYSTEM IS A COULOMB GAS OF MONOPOLES.

IN U(1) GAUGE THEORY μ IS A WELL DEFINED OPERATOR, GAUGE INVARIANT, DIRAC LIKE.

- μ IS NON LOCAL IN THE DIRECT DESCRIPTION
  LOCAL IN THE DUAL
- $\langle\mu\rangle$ IS NUMERICALLY FEASIBLE:
  $\rho = \frac{\partial \ln\langle\mu\rangle}{\partial\beta}$ (A SUSCEPTIBILITY) IS MORE SUITED TO NUMERICALLY DETECT DUAL SUPERCONDUCTIVITY.

- THE DISORDER PARAMETER $\langle \mu^a \rangle$ IN SU(N) GAUGE THEORY AND IN QCD. [A DG, L. Montesi, B. Lucini, G. Paffuti 00]

DEFINITION

$$\langle \mu^a(x_1)\, \mu^b(x_2) \ldots \rangle = \frac{\tilde{Z}}{Z}$$

$$Z = \mathrm{Tr}\{ e^{-\beta S} \} \qquad \tilde{Z} = \mathrm{Tr}\{ e^{-\beta(S+S')} \}$$

$S + S'$, BY ANALOGY TO U(1), IS OBTAINED BY REPLACING THE DENSITY OF ACTION Π_{0i} AT EACH TIME x_i^0 AT WHICH A MONOPOLE OR A SET OF MONOPOLES IS CREATED, WITH A MODIFIED ONE.

- $\vec{A}(\vec{y}) = \sum_i \vec{b}_\perp(\vec{x}_i - \vec{y})\, 2mq_i$ THE VECTOR POTENTIAL OF THE CLASSICAL FIELD OF THE MONOPOLES CREATED AT x_i^0. CHOOSE THE GAUGE

$$\vec{\nabla}\vec{A} = 0 \qquad \vec{\nabla}\wedge\vec{A} = \sum_i \left(2mq_i \frac{(\vec{y}-\vec{x}_i)}{4\pi|\vec{y}-\vec{x}_i|^3} + S_i \right) \quad \vec{A}_\perp$$

S_i: Dirac String

1 MONOPOLE AT $\vec{x}$ $\qquad \vec{A}_\perp(\vec{y}) = \vec{b}(\vec{x}-\vec{y})\, 2mq$.

$$\Pi_{0i} \rightarrow \Pi'_{0i}$$

$$\Pi_{i0} = U_i(\vec{n}, x^0)\, U_0(\vec{n}+\hat{i}, x_0)\, U_i^+(\vec{n}, x_0+1)\, U_0^+(\vec{n}, x^0)$$

$$\Pi'_{i0} = U_i(\vec{n}, x^0)\, U_0(\vec{n}+\hat{i}, x_0)\, U_i^{+\prime}(\vec{n}, x_0+1)\, U_0^+(\vec{n}, x_0)$$

$$U'_i(\vec{n}, x^0+1) = U_i(\vec{n}, x^0+1)\, e^{-i\vec{A}_{\perp i}(\vec{n}-\vec{x})\,\Phi^a(\vec{n}, x^0+1)}$$

$$\Phi^a(\vec{n}, x^0) = U(\vec{n}, x^0)\, \Phi^a_{diag}\, U^+(\vec{n}, x) \qquad \mathrm{Tr}\{T^b \Phi^a_{diag}\} = \delta^{ab}$$

- IN THE ABELIAN PROJECTED GAUGE

$$U'_i(\vec{n}, x^0+1) = U_i(\vec{n}, x^0+1)\, e^{-iT^a \hat{A}_{\perp i}(\vec{n}-\vec{x})}$$

Π'_{i0} DOES IN FACT CREATE A MONOPOLE OF TYPE a

- CHANGE VARIABLE IN THE FEYNMAN INTEGRAL FROM $U_i(\vec{n}, x^0+1)$ TO $U'_i(n, x^0+1)$

JACOBIAN = 1

$$\Pi'_{i0} \to \Pi_{i0}$$

Π_{ij} AT x^0+1 $\quad U_i(\vec{n}, x^0+1) \to U_i(n, x^0+1)\, e^{i\hat{A}^a_{\perp i}(\vec{n}-\vec{x})T^a}$

- THE ABELIAN PART OF Π_{ij} GETS $e^{iT_a(\Delta_i \hat{A}^a_{\perp j} - \Delta_j \hat{A}^a_{\perp i})}$, I.E. A MONOPOLE HAS BEEN PRODUCED

- AT THE SAME TIME

$$\Pi_{i0}(\vec{n}, t+1) \to U_i(\vec{n}, x^0+1)\, e^{-i\hat{A}^a_{\perp i}(\vec{n}-\vec{x})T_a}$$

$$U_0(\vec{n}+\hat{i}, x_0+1)\, U_i^+(n, x^0+2)\, U_0^+(\vec{n}, x^0+1)$$

$$= U_i(\vec{n}, x^0+1)\, U_0(\vec{n}+\hat{i}, x_0+1) \cdot \left[U_0^+(\vec{n}+\hat{i}, x^0+1)\, e^{-i\hat{A}^a_{\perp i}(\vec{n}-\vec{x})T_a}\, U_0(\vec{n}+\hat{i}, x_0+1) \right] U_i^+(n, x^0+2)\, U_0^+(\vec{n}, x^0+1)$$

$$e^{-i\hat{A}'^a_{\perp i}(\vec{n}-\vec{x})T^a} \quad \times \quad e^{-i\hat{A}^a_{\perp i}(\vec{n}-x)T^a + O(a^2)}$$

CHANGE VARIABLE $U_i \to U_i^+$

$$U_i(\vec{n}, x^0+2) \to U_i(n, x_0+2)\, U_0^+(\vec{n}+\hat{i}, x_0+1)\, e^{i\hat{A}'^a_{\perp i}(n-\vec{x})T^a}\, U_0(\vec{n}+\hat{i}, x_0+1)$$

AT x^0+2 THE ABELIAN PART OF $\Pi_{ij}(n, x^0+2)$ ACQUIRES A PHASE $e^{i(\Delta_i A_{\perp j} - \Delta_j A_{\perp i})}$ UP TO TERMS $O(a^2)$.

THE PROCEDURE CAN BE ITERATED UNTIL x^0+t IS REACHED AND THE PHASE CANCELS WITH THAT OF $\Pi_{i0}(\vec{n}, x^0+t)$.

THE ABELIAN PROJECTION IS DEFINED UP TO TERMS $O(a^2)$.

THE OPERATORS μ^a DEFINED IN THIS WAY ARE GAUGE INVARIANT.

THEY ~~OPERATE~~ ACT ON THE ABELIAN COMPONENT (AFTER ABELIAN PROJECTION) LIKE μ IN $U(1)$ GAUGE THEORY

OF COURSE THEY DEPEND A PRIORI ON THE CHOICE OF THE ABELIAN PROJECTION

- NUMERICAL RESULTS

SU(2) PURE GAUGE. [A.DG, L.Montesi, B.Lucini, G.Paffuti 00]
COLOR CONFINEMENT & DUAL.. I

MEASURE $\langle\mu\rangle$ AT $T \sim T_c$

ONE SINGLE MONOPOLE C^*-PERIODIC BOUND. CONDITIONS IN TIME. $U_\mu(\vec{n}, N_T) = U_\mu^*(\vec{n}, 0)$

* COMPLEX CONJUGATE.

$$\rho \equiv \frac{\partial \ln\langle\mu\rangle}{\partial\beta}$$

- ρ - FOR DIFFERENT ABELIAN PROJECTIONS
  - CHECK SCALING OF T_c
  - L_s DEPENDENCE
  - $T < T_c$ FINITE LIMIT $\Rightarrow \langle\mu\rangle \neq 0$
  - $T > T_c$ $\rho \sim -0.6 L_s - 12$

$T \sim T_c$ $\qquad \mu \simeq \tau^\delta \quad \tau \sim (\beta_c - \beta)$

$$\langle\mu\rangle = L_s^{-\delta/\nu} \Phi\left(\frac{\xi}{L_s}, \frac{a}{\xi}, \frac{L_t}{L_s}\right)$$

$$\xi \sim \tau^{-\nu} \qquad a/\xi \ll 1 \qquad \frac{L_t}{L_s} \ll 1$$

$$\frac{\xi}{L_s} \rightleftarrows \tau L_s^{1/\nu}$$

$$\langle\mu\rangle \simeq L_s^{-\delta/\nu} \tilde{\Phi}(\tau L_s^{1/\nu})$$

$$\rho = \frac{\partial \ln\langle\mu\rangle}{\partial\beta} = L_s^{1/\nu} \tilde{\Phi}'(\tau L_s^{1/\nu})$$

$$\boxed{\rho / L_s^{1/\nu} = f(\tau L_s^{1/\nu})}$$

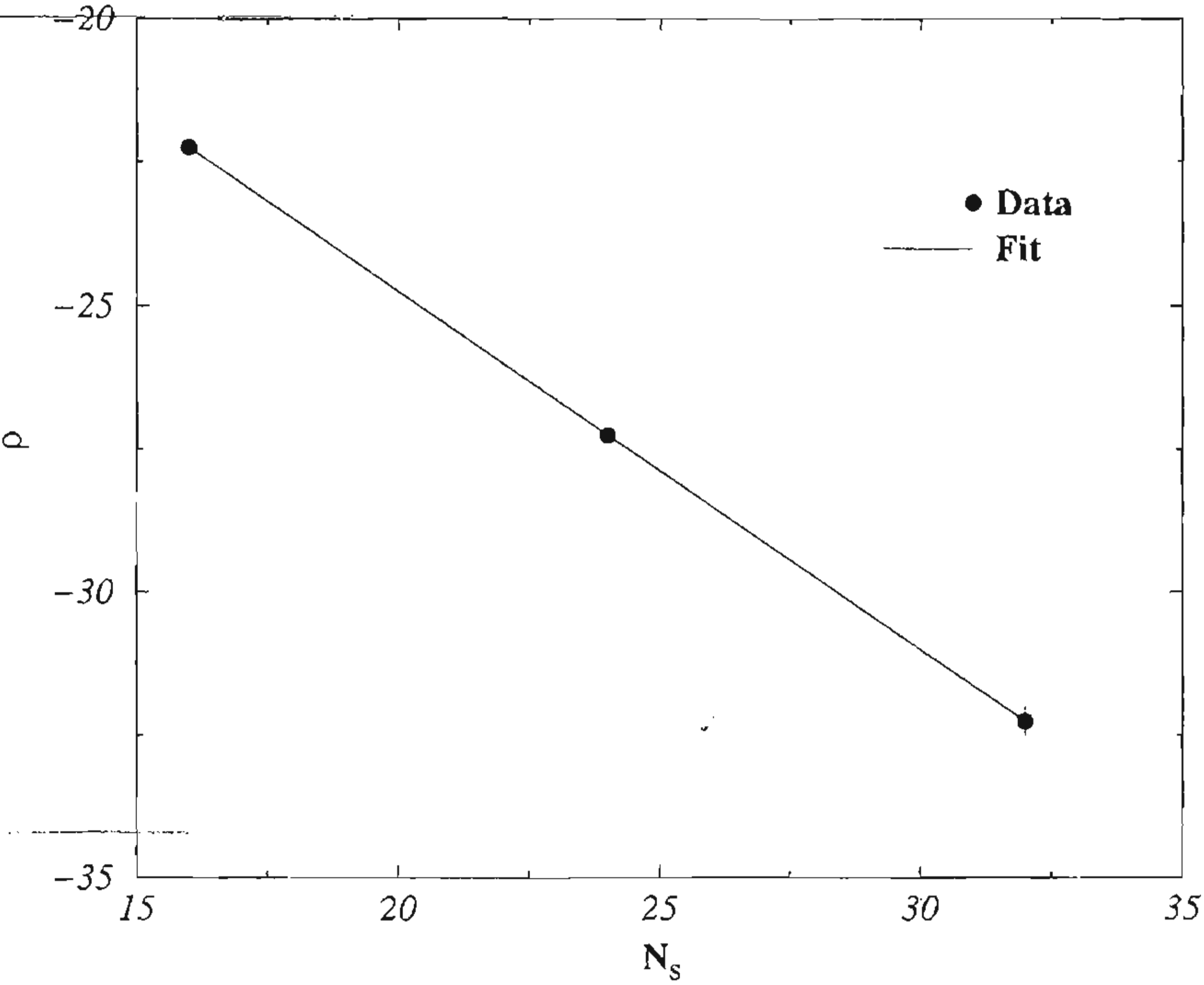
SCALING ν, β_c

$$\boxed{\rho_{PEAK} \propto L_s^{1/\nu}}$$

$\beta_c = 2.2986$ $\qquad \nu = .63(1)$ 3d ISING

CONSISTENT WITH DETERMINATION FROM $\langle L\rangle$

SU(2)5



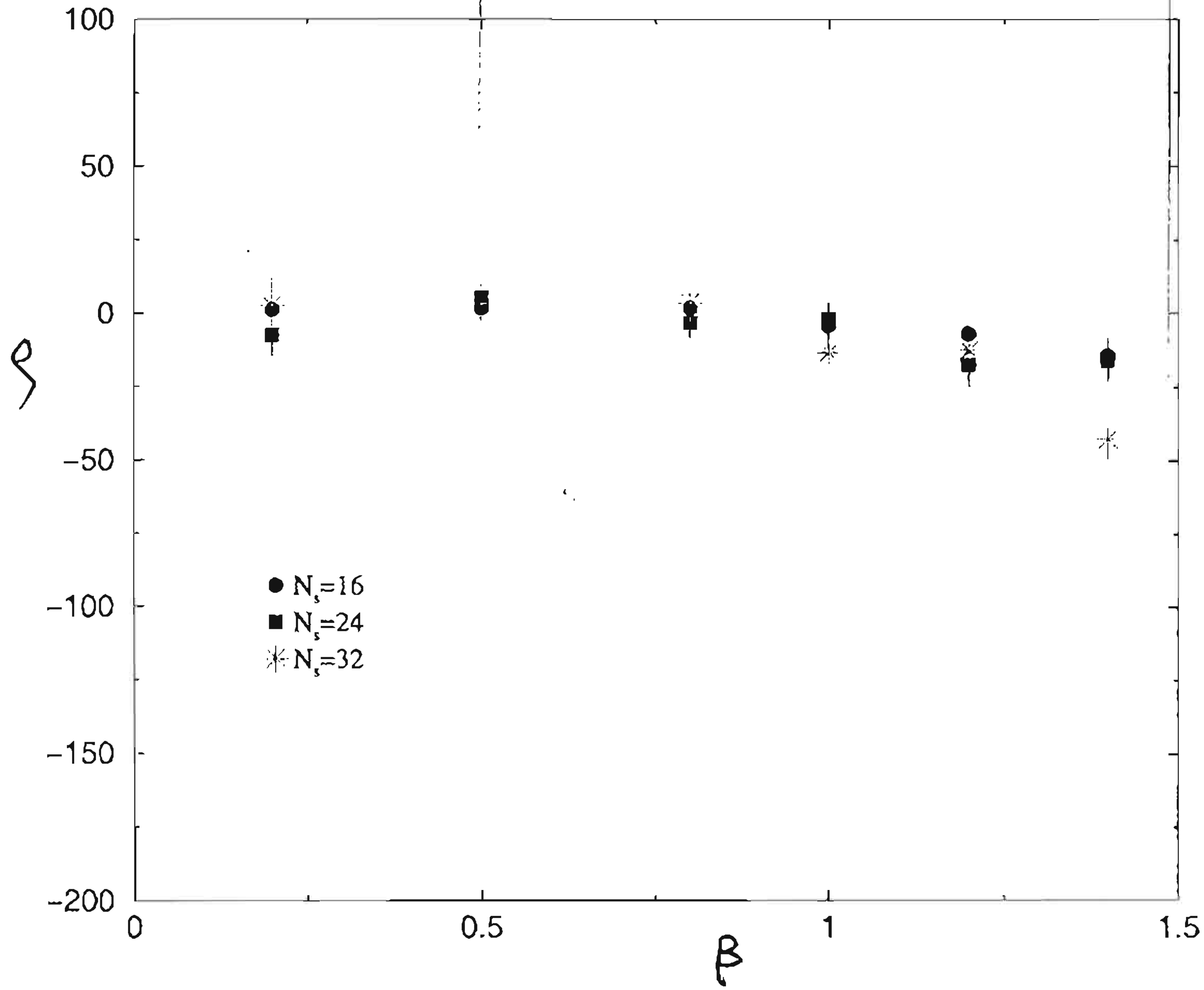
ρ vs N_S $N_t = 4$

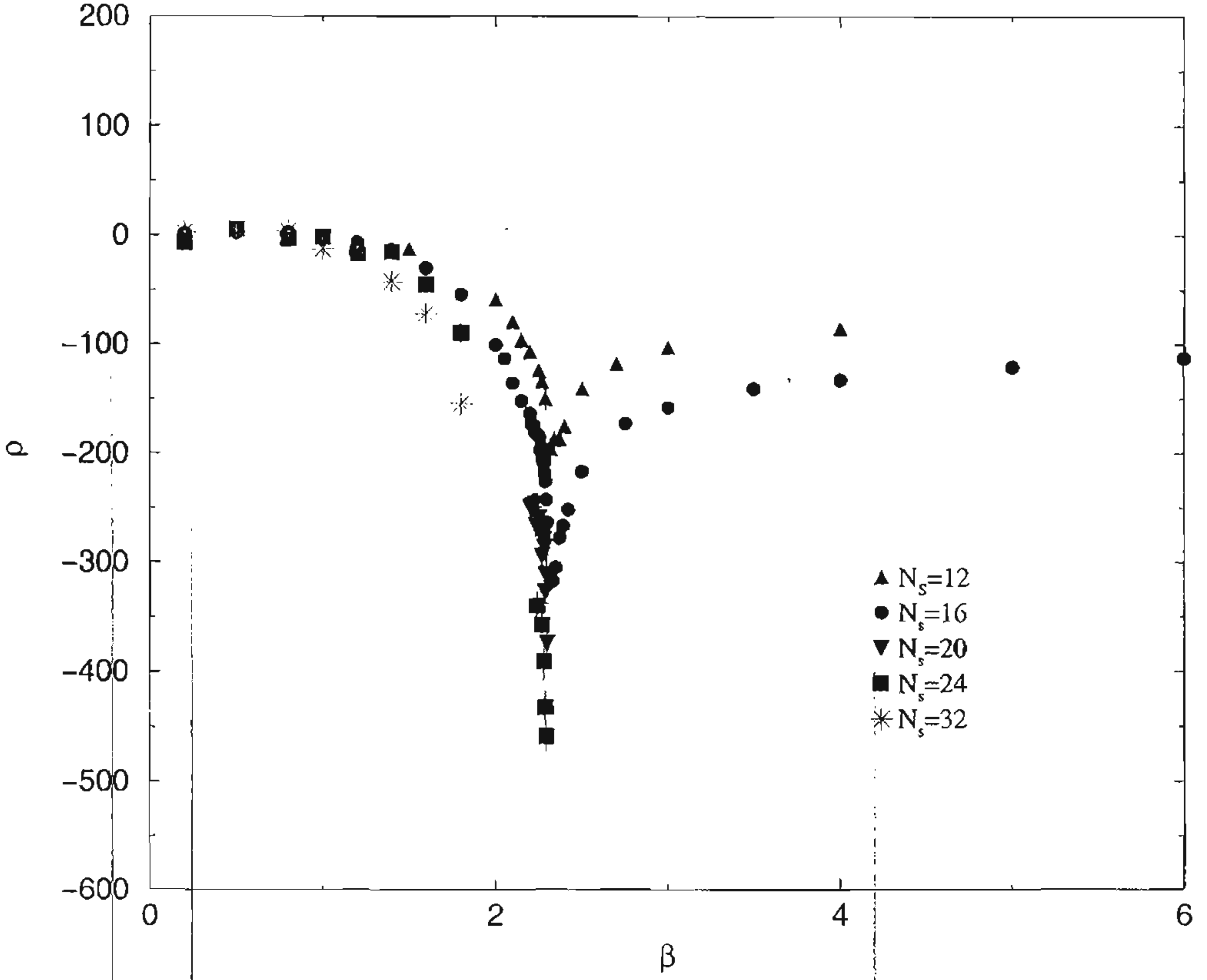
FIT $\rho = -.6\, N_S - 12.$

ρ VS β FOR $\beta < \beta_c$: $4 \times N_s^3$

NO N_s DEPENDENCE

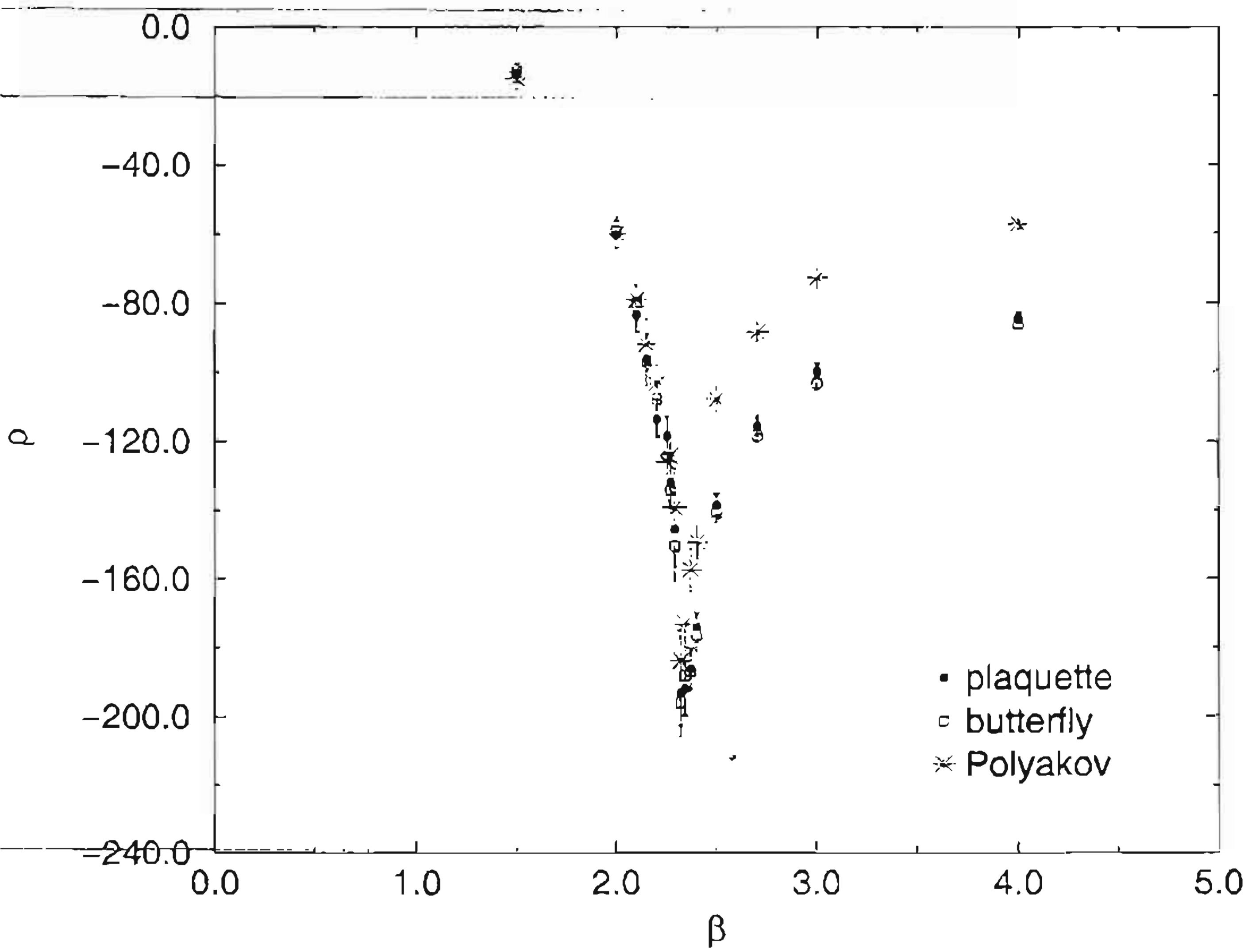
$SU(2)_4$





N_s DEPENDENCE OF ρ

SU(2) 1

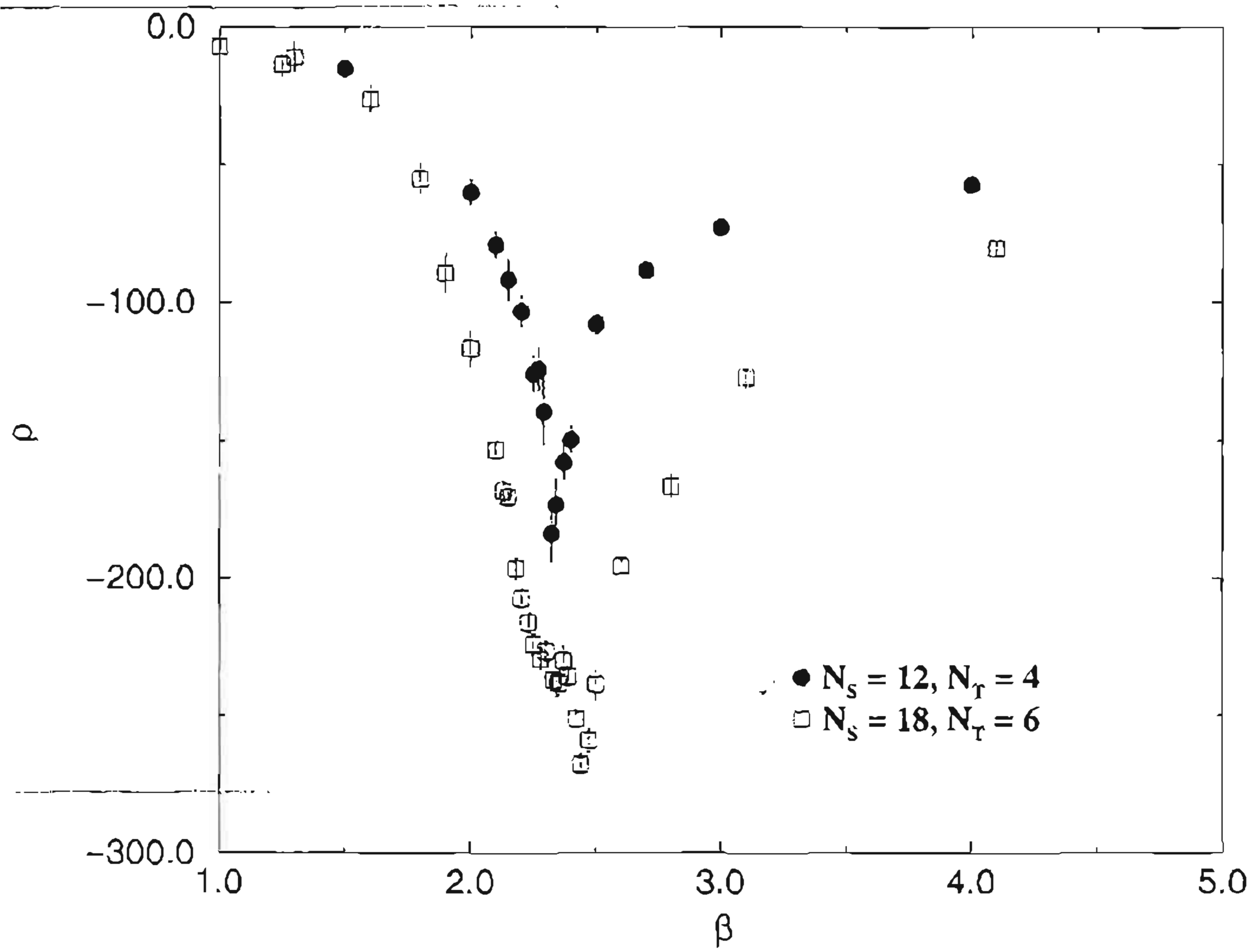


ρ VS β FOR DIFFERENT ABELIAN PROJECTIONS – LATTICE 4×12^3

A.D.G., B. LUCINI, L. MONTESI, G. PAFFUTI

PHYS. REV. D61, 034504 (2000)

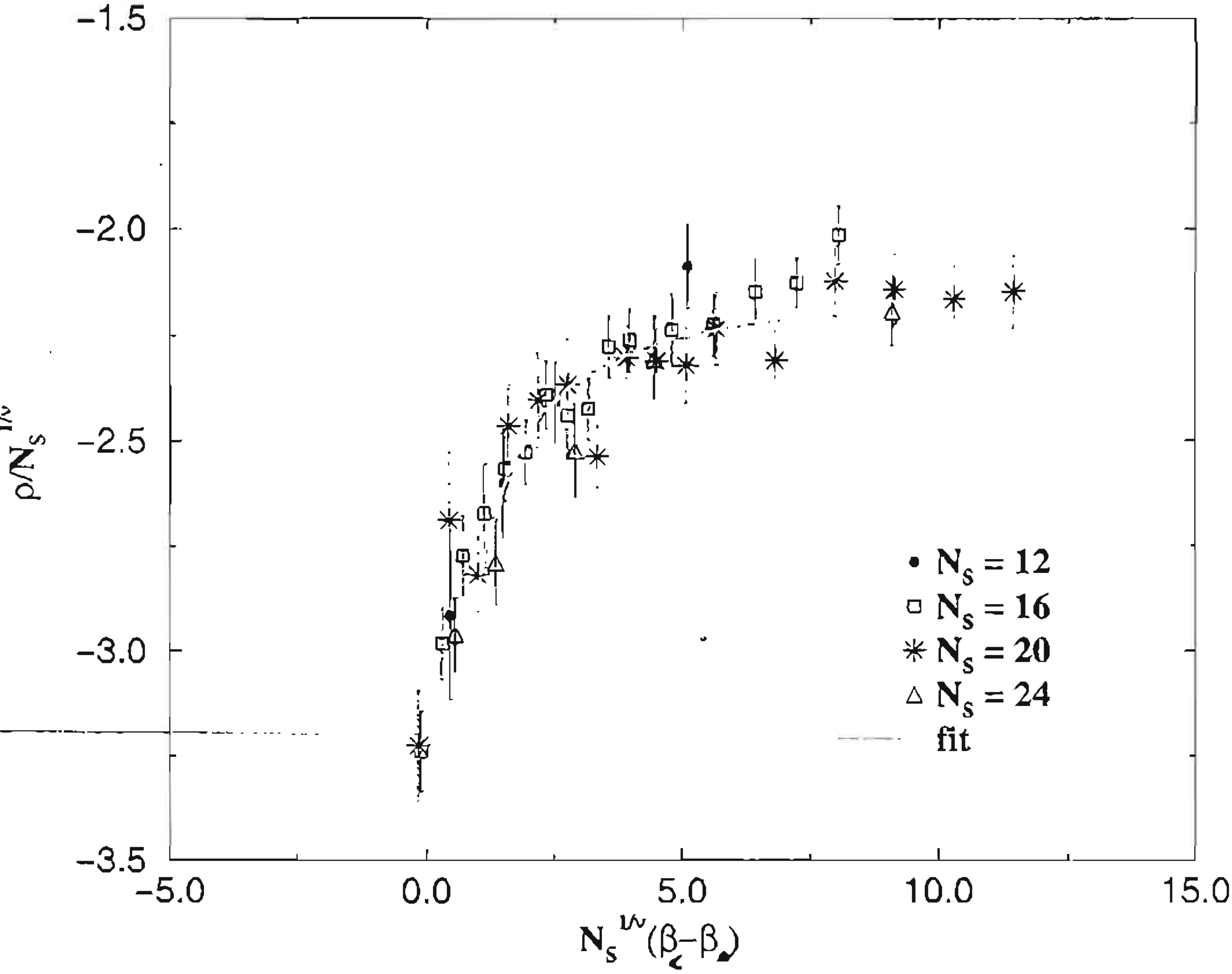
SU(2)



ρ VS β FOR DIFFERENT N_T

THE PEAK POSITION IS AT THE SAME T (SCALES WITH N_t)

SU(2) 6



SCALING $\rho/N_s^{1/\nu} = f(\tau N_s^{1/\nu})$

CURVE $\nu = .63(1)$

$\beta_c = 2.2986$