PARMA - SEMINARIO DI FISICA TEORICA 2005

CONFINEMENT & MAGNETIC MONOPOLES

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PLAN OF THE LECTURES

- INTRODUCTION
- CONFINEMENT & EXPERIMENT
- THE DECONFINING TRANSITION
- ORDER - DISORDER DUALITY
- HOW TO DEFINE & DETECT CONFINEMENT AND DECONFINEMENT
- DUAL EXCITATIONS IN QCD ; MONOPOLES VERSUS VORTICES
- THE CONFINING VACUUM AS A DUAL SUPERCONDUCTOR
- A DISORDER PARAMETER FOR DUAL S.C.
- TEST IN OTHER MODELS
- DECONFINEMENT IN FULL QCD THE CASE OF $N_f = 2$
- G_2 GAUGE GROUP.
- SUMMARY & CONCLUSIONS.

INTRODUCTION

- HISTORY

EXISTENCE OF QUARKS FIRST ~~HYPOTIZED~~ HYPOTESIZED BY GELL-MANN (~ 60's)

- ISOSPIN CURRENT COUPLED TO WEAK INTERACTIONS (C.V.C)
- $Q = T_3 + \frac{Y}{2}$ $\quad Y = N + S$ (Gell Mann - Nishijima)

  NOT AN ISOSPIN GENERATOR
- TO UNIFY WEAK & E.M ⇒ A RANK 2 SYMMETRY GROUP ∋ SU(2) ~~G_2~~, SU(3)
- PHENOMENOLOGY → EIGHTFOLD WAY $1, 8, \overline{10}, 10$.
- WHAT ABOUT $3; \bar{3}$?

  ⇓
- QUARKS ARE THE FUNDAMENTAL CONSTITUENTS OF MATTER (GELL-MANN 63)

  THEY HAVE CHARGES $\pm\frac{2}{3}$ (u) $\pm\frac{1}{3}$ (d, s)
- **40 YEARS OF EXTENSIVE SEARCH: NO QUARK FOUND**
- PAULI PRINCIPLE ($\Delta^{++\uparrow}_{3/2}$) ⇒ COLOR

  [Nambu, Gell Mann]
- 1973 COLOR AS A GAUGE SYMMETRY (QCD)

  [GELL-MANN, FRITZSH, LEUTWYLER]
- ASYMPTOTIC FREEDOM, PARTONS & Q.C.D.
- QUARKS EXIST AT SHORT DISTANCES, DO NOT APPEAR AT LARGE DISTANCES. [PR2 [Z2

- EXPERIMENT (P.D.G).

- $R = \frac{n_q}{n_p} < 10^{-27}$

$\begin{cases} n_q & \text{ABUNDANCE OF } q\ (\bar{q}) \\ n_p & \text{〃 OF NUCLEONS} \end{cases}$

MILLIKAN LIKE EXPERIMENTS
~1 kg OF MATTER

- $\sigma(p+p \to q(\bar{q}) + X) < 10^{-40}\,cm^2$

[OKUN]

- EXPECTATION FOR R **S.C.M.** (NO CONFINEMENT)

10^{-9} sec AFTER BIG-BANG $T \simeq 10$ GeV

$m_q \simeq T$ $q\ \bar{q}$ BURN $\to$ HADRONS

$q + \bar{q} \longrightarrow$ hadrons

$q + q \longrightarrow \bar{q} +$ hadrons

$\sigma_0 = \lim_{v \to 0} v\sigma$

BURNING RATE $= n_q \sigma_0$

EXPANSION RATE $= G_N^{1/2} T^2$

DECOUPLING OF RELIC QUARKS

$$n_q \sigma_0 = G_N^{1/2} T^2$$

$$\frac{n_q}{T^3} = \frac{G_N^{1/2}}{T\sigma_0} = \frac{10^{-19}}{m_p T \sigma_0}$$

$n_\gamma \simeq T^3$

$$\frac{n_q}{n_p} = \frac{n_q}{n_\gamma}\frac{n_\gamma}{n_p} = 10^9 \frac{n_q}{T^3} = 10^9 \cdot 10^{-21} = 10^{-12}$$

$\boxed{R_{expected} \simeq 10^{-12}}$

$\sigma_0 = m_\pi^{-2}$ $T = 10$ GeV

• EXPECTATION FOR $\sigma(p+p \to q(\bar{q})+X)$:
[QCD & NO CONFINEMENT]

$$\sigma_{expected}(p+p \to q(\bar{q})+X) \simeq \sigma_{Tot} = 10^{-25}\,cm^2$$

• $\dfrac{R_{observed}}{R_{expected}} < 10^{-15}$

• $\dfrac{\sigma_{observed}}{\sigma_{expected}} < 10^{-15}$

$< 10^{-15}$ IS A SMALL NUMBER. THE ONLY NATURAL EXPLANATION IS THAT IT IS $= 0$ (SYMMETRY) [superconductivity $\rho < 10^{-10}\rho_0$; $m_\gamma < 10^{-19}\,eV$ ..] ['t Hooft 78]

QUARKS ARE CONFINED : THEY ARE FORBIDDEN TO APPEAR IN ASYMPTOTIC STATES

NO EXPERIMENTAL EVIDENCE FOR GLUONS

$\Downarrow$

CONFINEMENT : ONLY COLORLESS PARTICLES CAN EXIST AS FREE PARTICLES

A SYMMETRY OF THE VACUUM MUST EXIST WHICH PRODUCES CONFINEMENT.

- QCD (chiral)

$$\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\{G_{\mu\nu} G_{\mu\nu}\} + \sum_f \bar{\psi}_f \, i \not{D} \, \psi_f$$

$$A_\mu = \sum_a A^a_\mu T^a \qquad (a = 1 \ldots 8)$$

$$[T^a, T^b] = i f^{abc} T^c \qquad \mathrm{Tr}\{T^a T^b\} = \frac{1}{2}\delta^{ab}$$

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$D_\mu = \partial_\mu - ig A_\mu$$

1 parameter g.

QCD MOST PROBABLY EXISTS AS A FIELD THEORY:

$$Z[J] = \int [\mathcal{D}\varphi] \exp\left[-S[\varphi] - \int J\varphi \, d^4x\right]$$

FEYNMAN INTEGRAL: DISCRETIZE A FINITE VOLUME OF SPACE TIME (E.G. TO A LATTICE), SO AS TO HAVE AN ORDINARY INTEGRAL, TAKE THE LIMIT OF ZERO SPACING, AND FINALLY THE LIMIT OF INFINITE VOLUME.

WILSON'S FORMULATION $dA_\mu \longrightarrow dU_\mu$

AN INTEGRAL ON A COMPACT SET. $\beta = \frac{2N}{g_0^2}$

$$a \underset{\beta \to \infty}{\doteq} \frac{1}{\Lambda} \exp[\beta / 2b_0] \qquad \text{(REN. GROUP)}$$

$$b_0 = -\frac{1}{(4\pi)^2}\left(\frac{11 N_c}{3} - \frac{2}{3} N_f\right) \qquad \text{(ASYMPT. FREEDOM)}$$

AS $\beta \to \infty$ THE LATTICE SPACING BECOMES EXPONENTIALLY SMALL IN PHYSICAL UNITS: THE U.V. LIMIT EXISTS, DUE TO ASYMPTOTIC FREEDOM

- THE THEORY DEVELOPS A MASS GAP (DIMENSIONAL TRANSMUTATION), SO THAT THE "THERMODYNAMICAL" LIMIT $V \to \infty$ EXISTS.

- A LATTICE WITH $a \ll \Lambda$ (Λ THE PHYSICAL SCALE) AND $\Lambda \ll L a$ (L the lattice size) GIVES A GOOD APPROXIMANT TO $Z[J]$, AND THE CORRECT GROUND STATE (VACUUM)

-

EXPLAINING CONFINEMENT: A CHALLENGE FOR FIELD THEORY

- THE DECONFINING TRANSITION

- HAGEDORN'S LIMITING TEMPERATURE T_H IN HADRONIC PHYSICS CAN BE AN INDICATION OF A DECONFINING TRANSITION, TO A PLASMA OF QUARKS & GLUONS. [Cabibbo_Parisi 75]

- TRANSITION NOT YET OBSERVED EXPERIMENTALLY [CERN SPS; RHIC]

- OBSERVED IN VIRTUAL REALITY [LATTICE]

- BOTH IN NATURE AND ON THE LATTICE THE PROBLEM IS TO DEFINE & DETECT THE TRANSITION. THIS WILL BE ONE OF THE MAIN SUBJECTS OF MY LECTURES.

- FINITE TEMPERATURE FIELD THEORY

$$Z = \mathrm{Tr}\left\{ e^{-H/kT} \right\}_{t\ \text{fixed}}$$

$$Z = \int [d\varphi]\, e^{-\int d^3x \int_0^{1/T} \mathcal{L}[\varphi(\vec{x},\tau)]\, d\tau}$$

P.B.C. FOR BOSONS ; A.B.C FOR FERMIONS

~~RAKE~~

- THE PARTITION FUNCTION IS THE EUCLIDEAN FEYNMAN INTEGRAL WITH THE TIME DIRECTION COMPACTIFIED $(0, 1/T)$

- ON THE LATTICE THE SIMULATION IS DONE ON $L_t \times L_s^3$ $\quad L_s \gg L_t$ AND THE APPROPRIATE B.C

$$T = \frac{1}{a\, L_t}$$

$a = a(\beta, m)$

PURE GAUGE $\qquad a \underset{\beta\to\infty}{=} \frac{1}{\Lambda} \exp\left[\beta/2b_0\right] \qquad b_0 < 0$

OR

$$T = \frac{\Lambda}{L_t} \exp\left(\frac{\beta}{2|b_0|}\right) \qquad \beta = \frac{2N}{g^2}$$

AS A CONSEQUENCE OF ASYMPTOTIC FREEDOM

T large — WEAK COUPLING — ORDER

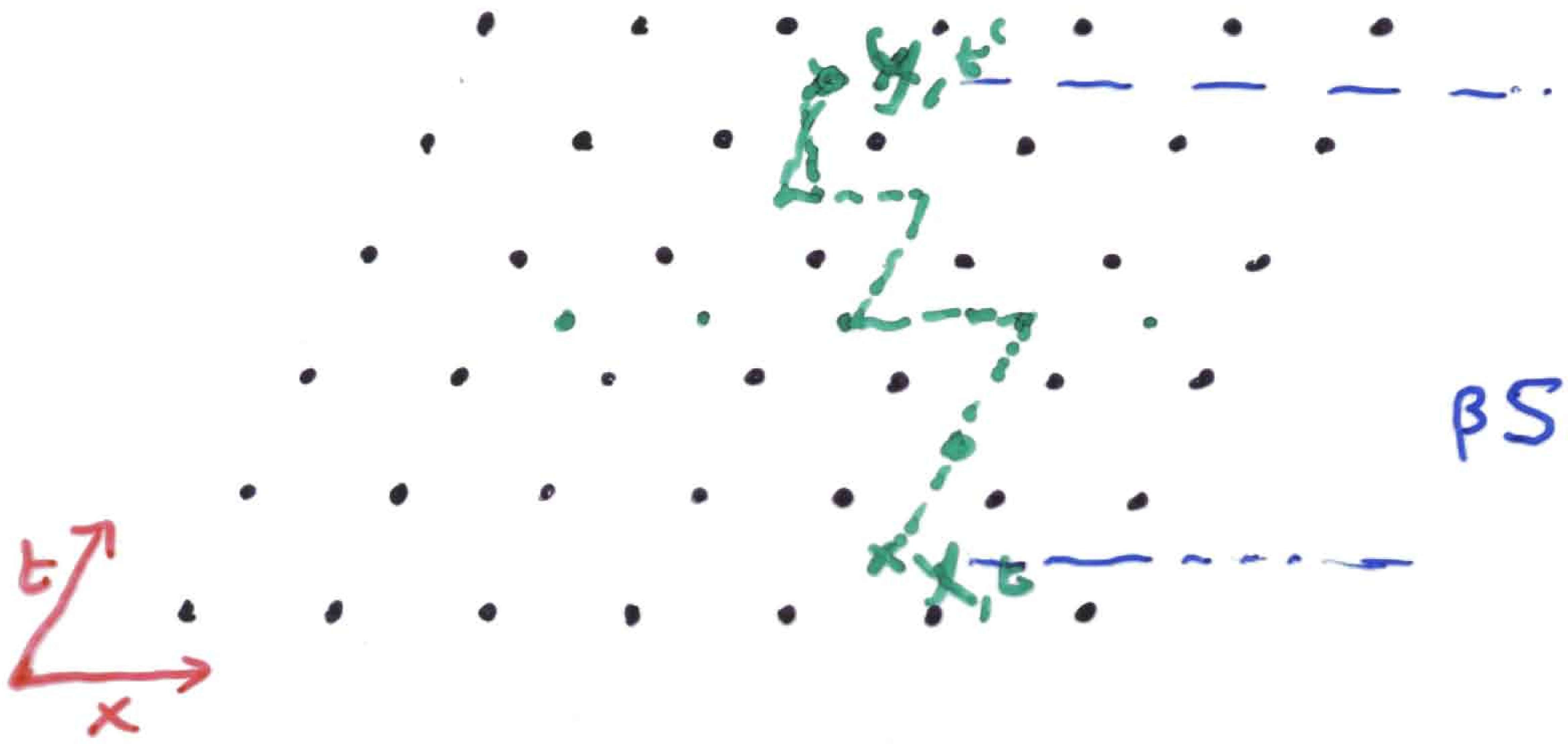
T small — STRONG COUPLING — DISORDER

CONTRARY TO WHAT HAPPENS IN ORDINARY SPIN SYSTEMS HIGH T IS ORDERED, LOW T IS DISORDERED

DUALITY (ORDER - DISORDER)

[KRAMERS & WANNIER 43, KADANOFF-CEVA 73]

SYSTEM WITH TOPOLOGICALLY NON TRIVIAL EXCITATIONS
2d ISING MODEL (1+1) d



$\sigma(n) = \pm 1$

$$\beta S = \beta \sum_{\substack{i,j \\ n.n.}} \sigma_i \sigma_j$$

$$\left(\beta = \frac{J}{T}\right)$$

$$Z[\sigma, \beta] = \sum_{\{\sigma\}} e^{-S(\sigma)\cdot\beta}$$

LOW T $\langle\sigma\rangle \neq 0$ (ORDER)

HIGH T $\langle\sigma\rangle = 0$ (DISORDER)

EXACTLY SOLVABLE
[ONSAGER 44]

$\frac{T_c}{J} = .8814$

DUAL LATTICE PLAQUETTE → SITE
LINK → ⊥ LINK

$$\langle \mu(\vec{x},t)\, \mu(\vec{y},t) \rangle = \frac{Z(S')}{Z(S)}$$

- $S \to S'$ BY CHANGING SIGN TO THE LINKS CROSSED BY GREEN LINES
- INDEPENDENT OF THE PATH (PROOF)
- A SPECIAL CHOICE: SEND THE LINES AT $x \to \infty$ CONST t [CARMONA ET AL

DEF $\mu(\bar{x}, \bar{t})$: CHANGE SIGN TO THE LINKS
$t \to t+1$ $x \geq \bar{x}$

$\mu^2 = 1$ $\mu = \pm 1$

$$\beta S = \frac{\beta}{2} \sum_{\mu, n} [\Delta_\mu \sigma(n)]^2 + c$$

- EQS MOTION $\Delta^2 \sigma = 0$
- CONSERVED CURRENT $J_\mu = \frac{1}{2} \varepsilon_{\mu\nu} \Delta_\nu \sigma$
- CHARGE $Q = \sum_x J_0(\vec{x}, t) = \sum \frac{\varepsilon_{01}}{2} \Delta_1 \sigma = \frac{\sigma(+\infty) - \sigma(-\infty)}{2}$
  (TOPOLOGICAL) $Q = 0, +1, -1$
- TOPOLOGICAL EXCITATIONS KINKS (ANTIKINKS)
  (TYPICAL OF 1+1 d)

THEOREM

$$Z[\sigma, \beta] = Z[\mu, \beta^*]$$

$$\sinh 2\beta = \frac{1}{\sinh 2\beta^*} \qquad \beta \sim \frac{1}{\beta^*}$$

- THE SYSTEM ADMITS NON LOCAL EXCITATIONS WITH NON TRIVIAL TOPOLOGY (KINKS - ANTIKINKS)

TWO COMPLEMENTARY DESCRIPTIONS

DIRECT	DUAL
LOC. FIELD VARIABLES σ.	TOPOLOGICAL EXCITATIONS
$\langle\sigma\rangle$ ORDER PARAMETERS	LOCAL $\langle\mu\rangle$ DISORDER PARAMETERS
CONVENIENT IN THE ORDERED PHASE ($T<T_c$)	σ NON LOCAL
TOPOLOGICAL EXCITATIONS μ NON LOCAL (KINKS)	CONVENIENT IN THE DISORDERED PHASE
	STRONG COUPLING IS MAPPED WEAK COUPLING ($\beta \sim 1/\beta^*$)

$$\beta S = \frac{\beta}{2} \sum_{\mu, n} [\Delta_\mu \sigma(n)]^2 + c$$

- EQS MOTION $\Delta^2 \sigma = 0$
- CONSERVED CURRENT $J_\mu = \frac{1}{2} \varepsilon_{\mu\nu} \Delta_\nu \sigma$
- CHARGE $Q = \sum_x J_0(\vec{x}, t) = \sum \frac{\varepsilon_{01}}{2} \Delta_1 \sigma = \frac{\sigma(+\infty) - \sigma(-\infty)}{2}$
  (TOPOLOGICAL) $Q = 0, +1, -1$
- μ TOPOLOGICAL EXCITATIONS KINKS (ANTIKINKS)
  (TYPICAL OF 1+1 d)

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DUALITY:

A GENERAL PROPERTY OF SYSTEMS WHICH HAVE EXCITATIONS WITH NON TRIVIAL TOPOLOGY

	FIELD	DUAL	
3d X-Y	θ	VORTICES	[TYPICAL OF (2+1)d]
		[DI CECIO, DI GIACOMO, PAFFUTI, TRIGIANTE 96]	
3d Heisenberg	$\vec{n}(x)$ $\vec{n}^2=1$	SOLITONS OF 3D O(3) σ MODEL	
		[DI GIACOMO, MARTELLI PAFFUTI, 99]	
N=2 SUSY QCD	A_μ	MONOPOLES	TYPICAL OF (3+1)d
		[SEIBERG - WITTEN]	
4D COMPACT U(1)	A_μ	MONOPOLES	TYPICAL OF (3+1) d
		[FROLICH-MARCHETTI, 86 ..... A. DI GIACOMO G. PAFFUTI, 96	
STRING THEORY	[MALDACENA]		

QCD "MONOPOLES"?

PR3c

WORKING HYPOTHESIS

CONFINEMENT IS AN ABSOLUTE PROPERTY BASED ON SYMMETRY

THE DECONFINING TRANSITION IS AN ORDER-DISORDER TRANSITION, I.E. A CHANGE OF SYMMETRY

- TO BE CHECKED BY LATTICE SIMULATIONS

- GUESS THE DUAL SYMMETRY & DUAL EXCITATIONS, VERIFY ON LATTICE

DECONFINING TRANSITION ON THE LATTICE

- SAME PROBLEM AS IN EXPERIMENTS: HOW TO DEFINE AND DETECT CONFINED & DECONFINED

- PURE GAUGE (NO QUARKS) : POLYAKOV CRITERION. LOOK FOR THE STATIC $Q\bar{Q}$ POTENTIAL: IF IT GROWS LINEARILY WITH DISTANCE R AT LARGE R

$$V(R) \underset{R\to\infty}{=} \sigma R$$

$\sigma \equiv$ string tension
CONFINEMENT

THERE IS CONFINEMENT

IF

$$V(R) \underset{R\to\infty}{\sim} \text{const} + \frac{1}{R}$$

DECONFINEMENT.

THE SYSTEM IS DECONFINED.

- POLYAKOV LINE

$$L(x) = T \int_0^{1/T} \exp(ig A(\vec{x}, t)\, dt)$$

PARALLEL TRANSPORT ALONG THE TIME AXIS ACROSS THE LATTICE

$\mathrm{Tr}\, L(\vec{x})$ GAUGE INVARIANT (P.B.C)

$$G(\vec{x}-\vec{y}) = \langle L(\vec{x})\, L^{+}(\vec{y}) \rangle \underset{x\to\infty}{\simeq} |\langle L \rangle|^2 + c\,\exp\left[-\frac{|\vec{x}-\vec{y}|\sigma}{T}\right]$$

$- T \ln G(\vec{z}) = V(\vec{z})$ (STATIC POTENTIAL)

$|\langle L\rangle| = 0 \qquad V(z) \underset{z\to\infty}{=} \sigma z$ (CONFINEMENT)

$|\langle L\rangle| \neq 0 \qquad V(z) \underset{z\to\infty}{\sim} \text{const}$ (DECONFINEMENT)

$\langle L\rangle = \exp(-F_Q/T)$

A TRANSITION OBSERVED IN LATTICE SIMULATIONS

T_c, $T<T_c$ $\langle L\rangle = 0$ $T>T_c$ $\langle L\rangle \neq 0$

- SU(2) $\quad T_c/\sqrt{\sigma} = .50 \quad$ 2nd ORDER (3d ising)

- SU(3) $\quad T_c/\sqrt{\sigma} = .630(5) \quad$ ~~weakest~~ weak first order.

$\downarrow$

$T_c \simeq 270$ MeV

$\langle L\rangle$ THE ORDER PARAMETER

Z_N ~~ZCAD~~ THE RELEVANT SYMMETRY $\longrightarrow$

- FREE ENERGY AT $T \sim T_c$

$$\mathcal{F} = \mathcal{F}(\langle L\rangle)$$

scaling $\quad \mathcal{F}\cancel{/L_s^{d}} = \mathcal{F}\left(\frac{\xi}{L_s}, \cancel{\frac{a}{\xi}}\right)$

$\tau = 1 - \frac{T}{T_c} \qquad \xi \sim \tau^{-\nu} \qquad \frac{\xi}{L_s} \rightleftarrows \tau L_s^{1/\nu}$

$$\mathcal{F} = \Phi(\tau L_s^{1/\nu}) \qquad C_V = -\frac{1}{V}\frac{\partial^2}{\partial T^2}\ln Z$$

$$C_V - C_0 = L_s^{\alpha/\nu} f_c(\tau L_s^{1/\nu})$$

$$\chi_{\langle L\rangle} \equiv \int d^3x \langle L(x) L^+(0)\rangle = L_s^{\gamma/\nu} f_{\langle L\rangle}(\tau L_s^{1/\nu})$$

α, γ, ν DEPEND ON THE UNIVERSALITY CLASS

	α	γ	ν
3d ISING	.11	1.43	.63
Weak 1st ORDER	1	1	1/3

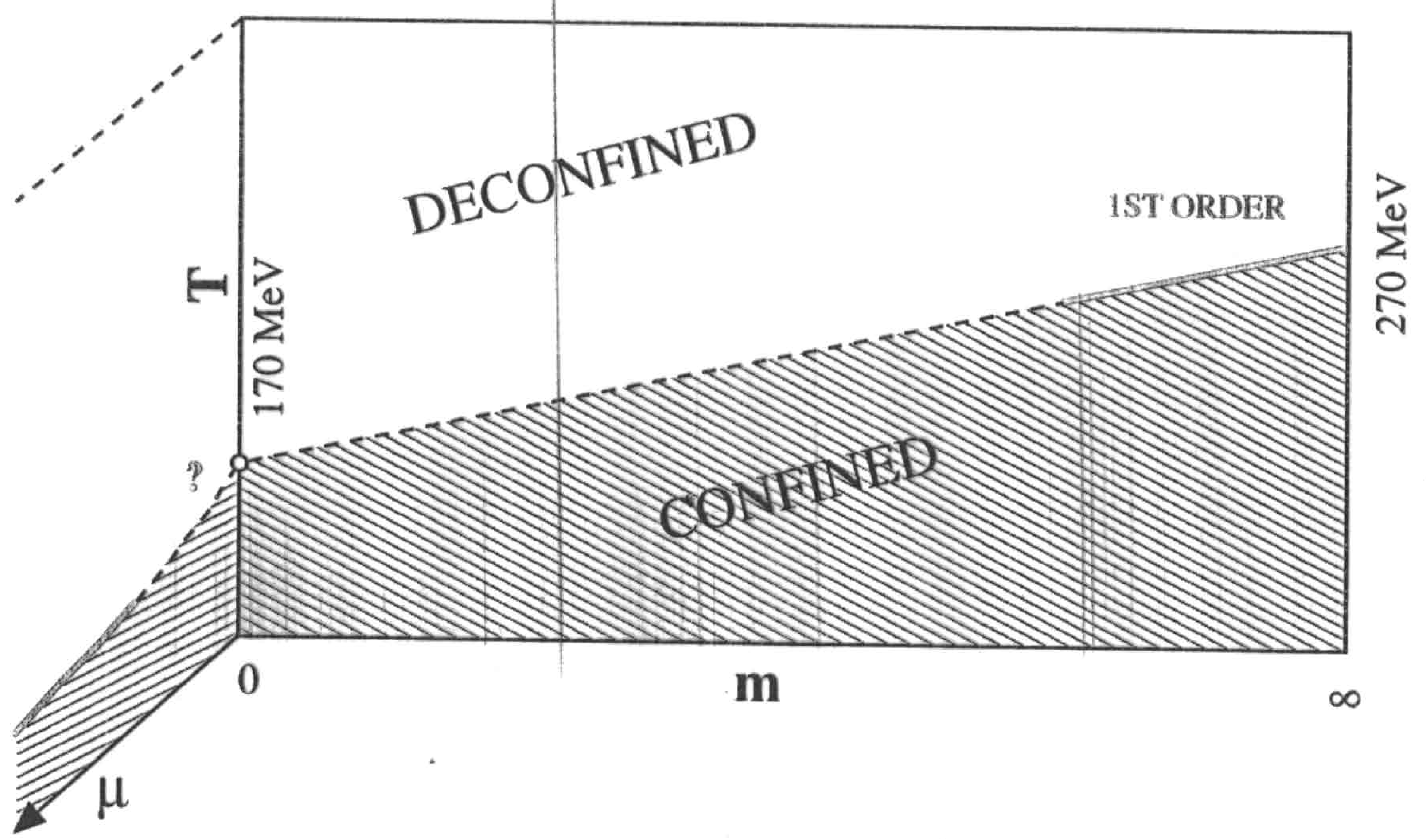
PR 11 $\quad$ Z 11

- THE CONTINUUM LIMIT IS REACHED AS $\beta \to \infty$ IF $L_t \to$ LARGE SINCE

$T_c = \frac{1}{a(\beta_c) \cdot L_t}$, AND T_c IS A PHYSICAL QUANTITY THE TRANSITION OCCURRS AT LARGER AND LARGER β, AND $a(\beta_c) \to 0$

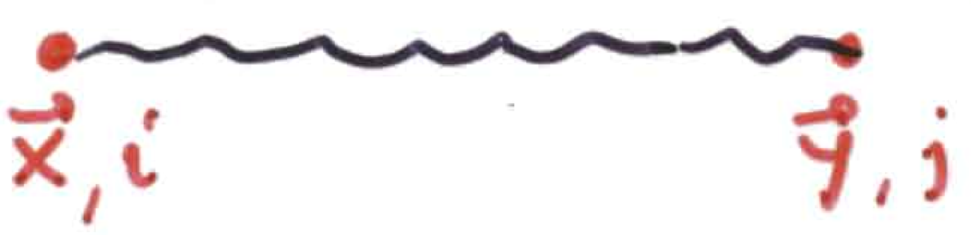
IN THIS LIMIT $\langle L \rangle \to 0$ ALSO FOR $T > T_c$ SINCE THE SELF ENERGY OF A SINGLE QUARK DIVERGES

A RENORMALIZATION OF $\langle L \rangle$ IS NEEDED TO DEFINE AN ORDER PARAMETER.


DECONFINED
CONFINED
1ST ORDER
T
170 MeV
270 MeV
?
0
m
∞
μ

THE FREDENHAGEN-MARCU CRITERION FOR CONFINEMENT (PRL 56, 223, 1986)

(IN PRESENCE OF QUARKS)



$$\Psi^{0}_{ij}(\vec{x},\vec{y}) = \bar{\Psi}_i(\vec{x})\, S(\vec{x},\vec{y})\, \psi_j(\vec{y})$$

$j, i \equiv$ FLAVOR

$$S(\vec{x},\vec{y}) = P\left[\exp ig\int_{\vec{x}}^{\vec{y}} A_\mu dx^\mu\right]$$

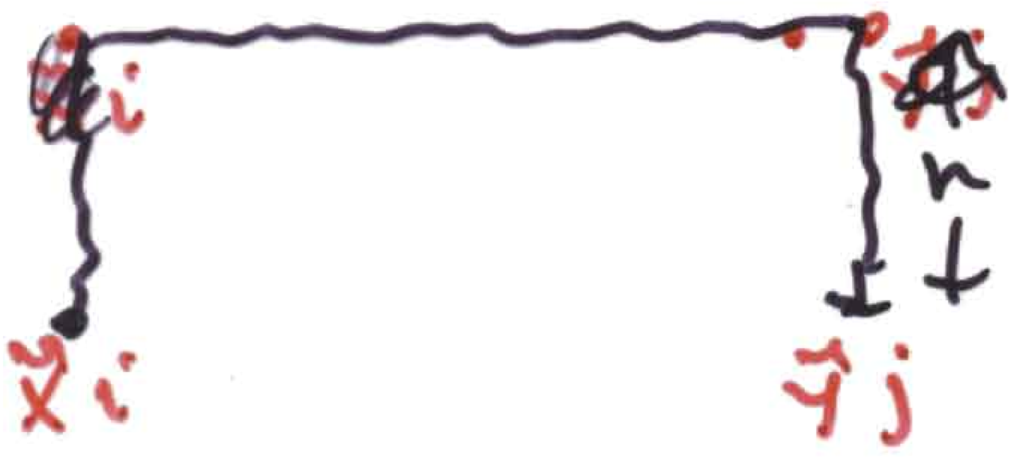
ψ IN THE FUNDAMENTAL REPRESENTATION

$$|s^{0}_{ij}\rangle = \Psi^{0}_{ij}(\vec{x},\vec{y})\,|0\rangle$$

DEF $\quad \Psi^{(n)}_{ij}(\vec{x},\vec{y}) = \bar{\Psi}_i(\vec{x})\, T^n S(\vec{x},\vec{y})\, T^{-n}\, \psi_j(\vec{y})$

$T \equiv$ TRANSFER MATRIX

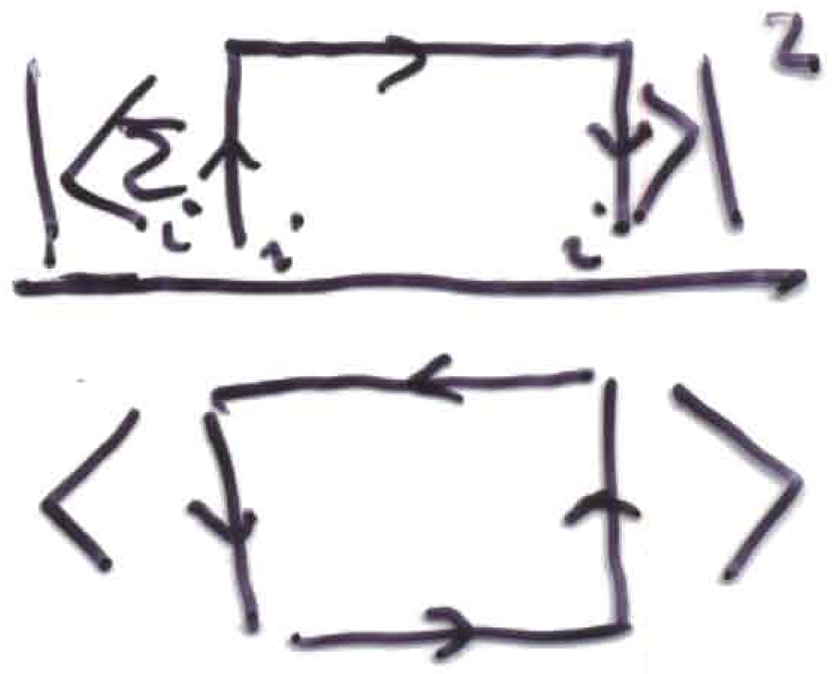
$$|s^{n}_{ij}(\vec{x},\vec{y})\rangle = \Psi^{(n)}_{ij}(\vec{x},\vec{y})\,|0\rangle$$



$|\vec{x}-\vec{y}| \to \infty \qquad n \sim \frac{1}{2}|\vec{x}-\vec{y}|$

PERIMETER LAW $\Rightarrow E^{n}(\vec{x},\vec{y})$ FINITE

$$\rho \equiv \lim_{\substack{|\vec{x}-\vec{y}|\to\infty \\ n=\frac{1}{2}|\vec{x}-\vec{y}|}} \frac{\left|\sum_i \langle 0 | s^{n}_{ii}(\vec{x},\vec{y})\rangle\right|^2}{\sum_i \langle s^{n}_{ii}(\vec{x},\vec{y}) | s^{n}_{ii}(\vec{x},\vec{y})\rangle} =$$



IF AT $|\vec{x}-\vec{y}| \to \infty$ ALL STATES ARE HADRONS

$\rho =$ finite $\neq 0$

IF ISOLATED QUARKS EXIST $\dfrac{\langle 0 | s^{n}\rangle}{\| s^{n} \|} \to 0$ (ORTHOGONAL TO HADRON STATES)

AND $\rho = 0$

DOES NOT APPLY AT FINITE T

- IN THE PRESENCE OF QUARKS
  (i) Z_3 IS NOT A SYMMETRY $\langle L \rangle$ NOT AN ORD. P.

  (ii) THE STRING BREAKS EVEN IN THE CONFINING PHASE WITH PRODUCTION OF PIONS.

- HOW TO DEFINE CONFINED & DECONFINED?

$N_F = 2$ (SEE FIG)

A LINE EXISTS ACROSS WHICH $\langle L \rangle$, $\langle E \rangle$, $\langle \bar{\psi}\psi \rangle$ EXPERIENCE A RAPID CHANGE, OR THEIR SUSCEPTIBILITIES HAVE A PEAK: CONVENTIONALLY THE PHASE BELOW IT IS CALLED CONFINED, THE ONE ABOVE IT DECONFINED

- FERMIONS IN THE ADJOINT REPRESENTATION
  2 DIFFERENT LINES : CHIRAL TRANSITION & DECONFINEMENT TRANSITION?
  [KARSCH LAT 2005 POSTER]

ORDER PARAMETER NEEDED:
I.E. UNDERSTANDING OF DUAL EXCITATIONS

PRI

GENERAL STRATEGY

- THE DENSITY OF FREE-ENERGY $\mathcal{F}$ AROUND THE PHASE TRANSITION IS THE EFFECTIVE LAGRANGEAN OF THE ORDER PARAMETERS.
- IN THE SPIRIT OF $4-\varepsilon$ EXPANSION IT IS A RENORMALIZABLE LAGRANGEAN (IRRELEVANT TERMS WITH $D>4$ ARE NEGLIGIBLE).
- THE FORM OF $\mathcal{F}$ IS DICTATED BY THE SYMMETRY UP TO A FEW PARAMETERS

EXAMPLE: SUPERCONDUCTIVITY [LANDAU-GINZBORG]

$$\mathcal{F} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \phi)^+ (D_\mu \phi) - \mu^2 (\phi^+\phi) - \frac{\lambda}{2} (\varphi^+\varphi)^2$$

φ COOPER PAIRS

SYMMETRY GAUGE INVARIANCE

PARAMETERS $\mu^2(T)$ $\lambda(T)$

$\mu^2(T) < 0$ SUPERCONDUCTOR

$\mu^2(T) > 0$ NORMAL

- IN PRINCIPLE NO TRANSITION EXISTS FOR A FINITE LATTICE: THE PARTITION FUNCTION IS ANALYTIC IN T FOR A FINITE # OF DEGREES OF FREEDOM. ONY IN THE THERMODYNAMICAL LIMIT $\boxed{V \to \infty}$ SINGULARITIES CAN SHOW UP [Lee Yang 52]

SUPERCONDUCTIVITY [Landau-Ginzburg 50 S. Weinberg 86]

φ CHARGED FIELD OF COOPER PAIRS

$$\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \phi)^* (D_\mu \phi) - V(\phi)$$

$$V(\varphi) = \mu^2 \varphi^* \varphi + \frac{\lambda}{2} (\varphi^* \varphi)^2$$

$\mu^2(T)$, $\lambda(T)$ PARAMETERS. FOR THE REST

$\mathcal{L}_{eff}$ FIXED BY SYMMETRY AND SCALE INVARIANCE (IRRELEVANT TERMS NEGLECTED)

$T < T_c \qquad \mu^2(T) < 0$

$T > T_c \qquad \mu^2(T) > 0$

$\varphi^* \varphi = -\frac{\mu^2}{\lambda}$

$\varphi^* \varphi = 0$

$\varphi = \frac{\rho}{\sqrt{2}} e^{i\theta} \qquad \rho > 0$

GAUGE TRANSF $\quad \rho \to \rho$

$\theta \to \theta + e\Lambda$

$A_\mu \to A_\mu + \partial_\mu \Lambda$

$$D_\mu \varphi = e^{i\theta} [\partial_\mu \rho + i \rho \overbrace{(\partial_\mu \theta - e A_\mu)}^{\equiv \tilde{A}_\mu}]$$

$\bar{\rho}^2 = -\frac{\mu^2}{\lambda}$

$\tilde{A}_\mu$ GAUGE INVARIANT

$$F_{\mu\nu} = \tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$$

EXAMPLE.

SPECIFIC HEAT C_V

RATE OF VARIATION OF ENERGY DENSITY $\mathcal{E}$ WITH RESPECT TO T

$T \sim T_c$ C_V ROUNDED BY FINITE SIZE EFFECTS

AS $L_s \to \infty$ $(L_s \gg \xi)$ A JUMP IN $\mathcal{E}$ APPEARS IF THE TRANSITION IS FIRST ORDER (LATENT HEAT) AND A PEAK $\propto L_s^3$ IN C_V.

IF TRANSITION IS SECOND ORDER THE INCREASE OF THE PEAK IS MILDER

C_V IS THE SUSCEPTIBILITY OF THE ENERGY DENSITY

$$C_V \sim \frac{1}{V} \int \langle E(x) E(0) - \langle E \rangle^2 \rangle$$

- THE VOLUME DEPENDENCE OF SUSCEPTIBILITIES IS DICTATED BY RENORMALIZATION GROUP. CRITICAL INDEXES $\rightleftarrows$ ANOMALOUS DIMENSIONS. EFFECTIVE THEORY $\rightleftarrows$ UNIVERSALITY CLASS.

- C_V HAS AN ADDITIVE RENORM. AND NEES A SUBTRACTION

$$C_V - C_0 = L_s^{\alpha/\nu} f_c(\tau L_s^{1/\nu})$$

SCALING OF C_V INDEPENDENT ON THE CHOICE OF ~~[illegible]~~ THE ORDER PARAMETER

- THE SUSCEPTIBILITY χ OF THE ORDER PARAMETER SCALES AS

$$\chi = L_s^{\gamma/\nu} f_\chi(\tau L_s^{1/\nu})$$

ν MUST AGREE WITH C_V

THE DUAL EXCITATIONS OF QCD

GENERAL IDEA: FOR $T < T_c$ (CONFINED PHASE) QCD IS BETTER DESCRIBED IN TERMS OF DUAL EXCITATIONS μ WITH NON TRIVIAL TOPOLOGY.

μ ARE NON LOCAL IN THE DIRECT FORMULATION (A_μ, ψ), BUT POSSIBLY LOCAL (AND WEAKLY COUPLED) IN THE DUAL FORMULATION.

['tHOOFT 78; SEIBERG WITTEN 93...]

2 PROPOSALS FOR μ

- MONOPOLES [S. Mandelstam 75, G. 'tHOOFT 75 G. 'tHOOFT 81]
- VORTICES [G. 'tHOOFT 78]

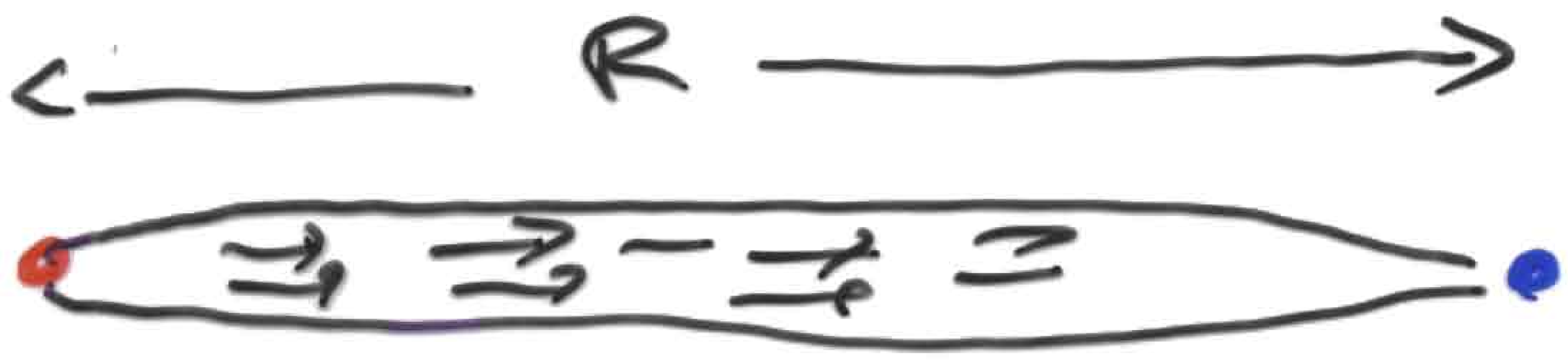
$\Downarrow$

- DEFINITION
- TOPOLOGY
- SYMMETRY

- MONOPOLES

PHYSICAL IDEA: MONOPOLES CONDENSE IN THE CONFINED PHASE. VACUUM BECOMES A DUAL SUPERCONDUCTOR.

FLUX TUBES BETWEEN $\bar{Q}Q$ BY DUAL MEISSNER EFFECT



$E = \sigma R$ $\quad\quad$ σ = ENERGY/UNIT LENGTH.

IN THE DECONFINED PHASE MAGNETIC CHARGE IS SUPERSELECTED.

$$\mathcal{L}_{eff} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}_{\mu\nu} - \frac{\bar{\rho}^2 e^2}{2}\tilde{A}_\mu \tilde{A}_\mu + \frac{1}{2}\partial_\mu\rho\,\partial_\mu\rho - V(\rho)$$

$$\partial_\mu \tilde{F}_{\mu\nu} = \bar{\rho}^2 e^2 \tilde{A}_\nu$$ (London current)

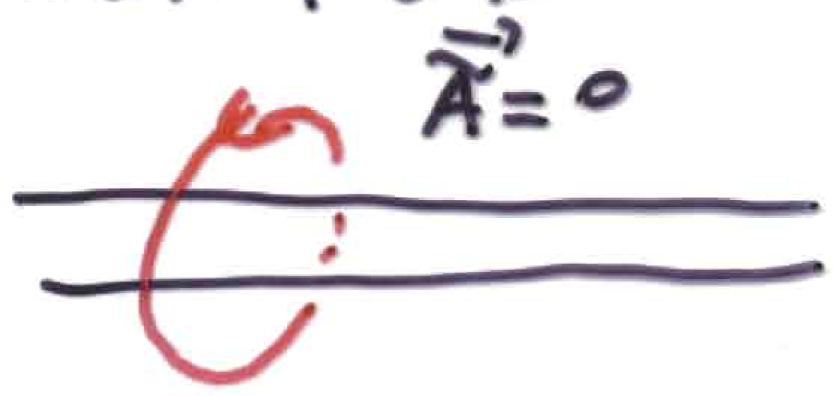
$A^0 = 0$ STATIC SOLUTION $\tilde{A}_0 = 0$ $\vec{E} = 0$

$\partial_i \tilde{F}_{ij} = \bar{\rho}^2 e^2 \tilde{A}_j$ $\vec{E} = 0$ $\vec{j} \neq 0$ $\Rightarrow$ $\vec{E} = \rho\vec{j}$, $\rho = 0$

NO RESISTIVITY

$\vec{\nabla}\wedge\vec{H} = \bar{\rho}^2 e^2 \vec{A}$ $(\nabla^2 - e^2\bar{\rho}^2)\vec{H} = 0$ (MEISSNER EFFECT)

FLUXTUBE



$$0 = \oint \vec{A}\,d\vec{x} = \oint \vec{\nabla}\theta\,dx - e\oint \vec{A}\,dx$$

$e\,\Phi(H) = 2\pi n$ DIRAC QUANTIZATION

$4\pi m e = 2\pi n$ $me = \frac{n}{2}$

ORDER PARAMETER $\bar{\rho} = \langle \phi \rangle_{\text{unitary gauge}}$

ϕ IS A CHARGED FIELD:

VACUUM IS A SUPERPOSITION OF STATES WITH DIFFERENT CHARGES (CONDENSATION OF COOPER PAIRS) (BOGOLUBOV 58, VALATIN 58 NUOVO CIMENTO

- CONSTRUCT A GAUGE INVARIANT ORDER PARAMETER: A GAUGE INVARIANT CHARGED FIELD $\tilde{\varphi}$ [Dirac Can. Journ. Phys 33, 650, 1955

$$\varphi(x) \to \tilde{\varphi}(x) = \varphi(x)\, e^{ie\int d^4y\, A_\mu(y)\, h_\mu(x-y)}$$

$$\partial_\mu h_\mu(x-y) = \delta^4(x-y)$$

GAUGE TRANSF $\quad A_\mu \to A_\mu + \partial_\mu \Lambda \qquad \Lambda(x) \xrightarrow[|x|\to\infty]{} 0$

$$\varphi(x) \to \varphi(x)\, e^{ie\Lambda(x)}$$

$$\int d^4y\, A_\mu(y)\, h_\mu(x-y) \to \int d^4x\, A_\mu(x) h_\mu(x-y) + \int d^4x\, \Lambda(y)\partial_\mu h_\mu(x-y)$$

$$= -e\Lambda(x)$$

$$\tilde\varphi(x) \to \tilde\varphi(x)\, e^{ie(\Lambda(x)-\Lambda(x))} = \tilde\varphi(x)$$

UNDER A GLOBAL TRANSFORMATION

$A_\mu \to A_\mu \qquad \varphi \to \varphi e^{ie\Lambda}$

$$\tilde\varphi(x) \to \tilde\varphi(x)\, e^{ie\Lambda}$$

$\tilde\varphi$ IS A CHARGED GAUGE INVARIANT FIELD

$\tilde\varphi^\dagger\tilde\varphi = \varphi^*\varphi$

— $h_\mu(x-y)$

1d SUPPORT $\qquad h_\mu(x-y) = \delta_{\mu 0}\, \theta(x_0-y_0)\, \delta^3(\vec{x}-y)$

$$\tilde\varphi = \varphi\, e^{ie\int_{\infty}^{x_0} A^0(\vec{x},t)\, dt}$$

(PARALLEL TRANSPORT

2d SUPPORT $\qquad h_\mu(x-y) = \left(0,\ 0,\ \delta(x_0-y_0)\,\delta(x_3-y_3)\,\frac{\vec{x}_\perp}{2\pi x^2}\right)$

3d SUPPORT $\qquad h_\mu(x-y) = \left(0,\ \frac{\delta(x_0-y_0)}{4\pi}\frac{\vec{r}}{r^3}\right)$ $\quad$ DIRAC CHOICE

4d SUPPORT $\qquad h_\mu(x-y) = \frac{x_\mu}{(x^2)^2}\frac{1}{2\pi^2}$

1d & 2d HAVE INFRARED PROBLEMS

$\langle\tilde\varphi(x)\,\varphi(0)\rangle \xrightarrow[x\to\infty]{} C \neq 0$ EVEN IF $\langle\varphi\rangle = 0$

NO CLUSTER PROPERTY

CLUSTER PROPERTY <u>OK</u> FOR $d=3$, $d=4$

4d VIOLATES REFLECTION POSITIVITY:

IN THE QUANTIZED VERSION OF THE THEORY IF $\tilde{\varphi}|0\rangle$ HAVE TO BE STATES WITH POSITIVE METRIC AND OBEY SUPERSELECTION OF CHARGE IN THE NORMAL PHASE 3-d SUPPORT IS NEEDED.

$$\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - (\partial_\mu \tilde{\varphi})^* (\partial_\mu \tilde{\varphi}) - V(\tilde{\varphi})$$

$$\dot{\tilde{\varphi}} = \tilde{\varphi}(x, A) \qquad \partial_\mu \tilde{\varphi} = (\partial_\mu - ieA_\mu)\phi\, e^{ie(A_\mu)}$$

$\langle \tilde{\varphi} \rangle$ IS A GOOD ORDER PARAMETER FOR SUPERCONDUCTIVITY

$\langle \tilde{\varphi} \rangle \neq 0$ HIGGS BREAKING

$\langle \tilde{\varphi} \rangle = 0$ NORMAL

ANY OPERATOR CHARGED & GAUGE INVARIANT CAN BE AN ORDER PARAMETER FOR SUPERCONDUCTIVITY.

MONOPOLES [Coleman ERICE 75, DIRAC (PROC ROY SOC A133, 60 (1931)]

- MAXWELLS EQ'S

$$\partial_\mu F_{\mu\nu} = j_\nu \qquad \partial_\mu F^*_{\mu\nu} = 0 \quad \text{(BIANCHI IDENTITIES)}$$

$$\vec{\nabla}\vec{E} = \rho \qquad \vec{\nabla}\vec{B} = 0$$

$$\vec{\nabla}\wedge\vec{H} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla}\wedge\vec{E} + \frac{\partial \vec{H}}{\partial t} = 0$$

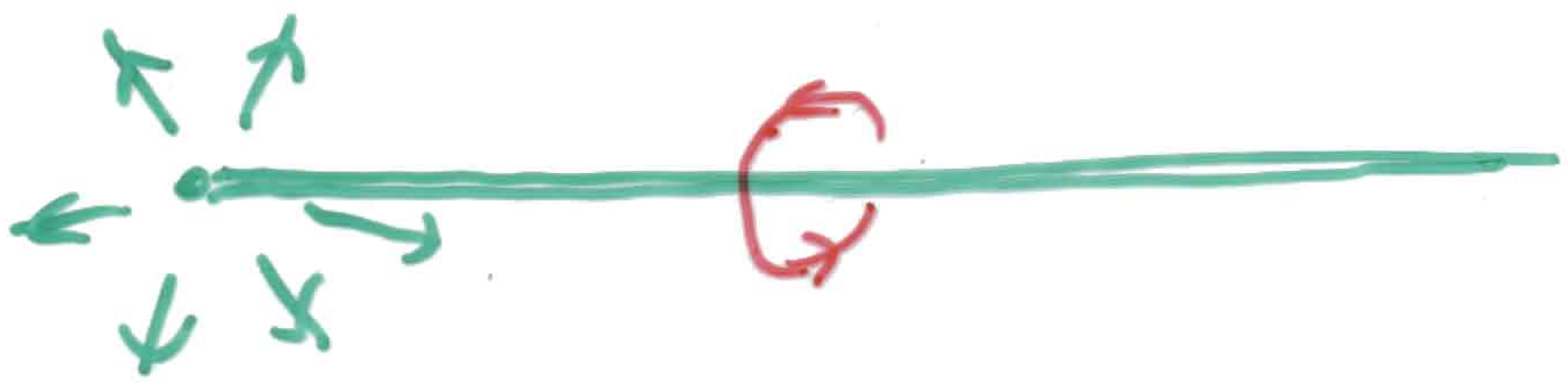
$$\Downarrow$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{(EXACT FORM)}$$

INTRODUCE MAGNETIC CHARGES

$$\partial_\mu F^*_{\mu\nu} = j^M_\nu \neq 0 .$$

DIRAC MONOPOLE:



$$\vec{H} = m\frac{\vec{r}}{r^3} + \text{STRING}$$

STRING INVISIBLE IF $ie\oint \vec{A}\,d\vec{x} = 2\pi n$

$$\left(e^{ie\oint \vec{A}\,d\vec{x}} = 1 \right). \qquad 4\pi m e = 2\pi n \qquad me = \frac{n}{2}$$

$$F_{\mu\nu} \longrightarrow F^R_{\mu\nu} + F^S_{\mu\nu} \qquad \partial_\mu F^*_{\mu\nu} = 0$$

$$\partial_\mu F^{R*}_{\mu\nu} = j^M_\nu = -\partial_\mu F^{S*}_{\mu\nu} \qquad F^S_{\mu\nu} \text{ INVISIBLE}$$

TO INCLUDE MONOPOLES A FORMULATION IN TERMS OF PARALLEL TRANSPORT IS CONVENIENT

[MANDELSTAM 59; CABIBBO FERRARI 62]
N.C.

WILSON'S FORMULATION O.K.

COMPACT U(1) GAUGE THEORY

$$S = \beta \sum_{n\mu\nu} [U_{\mu\nu}(n) - 1] \qquad n \in \text{CUBIC LATTICE}$$

$$U_{\mu\nu}(n) = U_\mu(n)\, U_\nu(n+\hat\mu)\, U^+_\mu(n+\hat\nu)\, U^+_\nu(n)$$

$$U_\mu(n) = e^{i\theta_\mu(n)} \qquad \text{ABELIAN } \theta_\mu = ea A_\mu$$

$$U_{\mu\nu}(n) = e^{i\theta_{\mu\nu}(n)} \qquad \theta_{\mu\nu} = \Delta_\mu \theta_\nu - \Delta_\nu \theta_\mu .$$

$$-\pi \leq \theta_\mu(n) \leq \pi \qquad -4\pi \leq \theta_{\mu\nu}(n) \leq 4\pi$$

$$\theta_{\mu\nu} = \theta^R_{\mu\nu} + \theta^S_{\mu\nu} \qquad -\pi \leq \theta^R_{\mu\nu} \leq \pi \qquad \theta^S_{\mu\nu} = 2\pi n_{\mu\nu}$$

$n_{\mu\nu}$ integer

$\theta^S_{\mu\nu}$ INVISIBLE $\boxed{e^{i\theta^S_{\mu\nu}} = 1}$

$$\theta^*_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\theta_{\rho\sigma}$$

$$\Delta_\mu \theta^*_{\mu\nu} = 0$$

$$\Delta_\mu \theta^{R*}_{\mu\nu} = -\Delta_\mu \theta^{S*}_{\mu\nu} = -2\pi \Delta_\mu n^*_{\mu\nu} = j^M_\nu$$

$$\Delta_\nu j^M_\nu = 0$$

[DE GRAND, TOUSSAINT 80]

COMPACT U(1) HAS A CONFINING PHASE FOR $\beta < \beta_c$, DECONFINED FOR $\beta \geqslant \beta_c$

$\beta_c = 1.01$