FISICA DEL SAPORE E QCD SUL RETICOLO

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1. FLAVOR PHYSICS AND ITS MOTIVATIONS

I. The Standard Model: a low energy effective theory

II. Flavor Physics in the Standard Model (and beyond)

2. LATTICE QCD AND QUARK MASSES

I. Generalities on quark masses

II. Introduction to Lattice QCD

III. Lattice calculations of quark masses

3. CKM MATRIX, UNITARITY AND CP VIOLATION

I. First row, unitarity and the Cabibbo angle

II. The Unitarity Triangle Analysis

III. Search for New Physics

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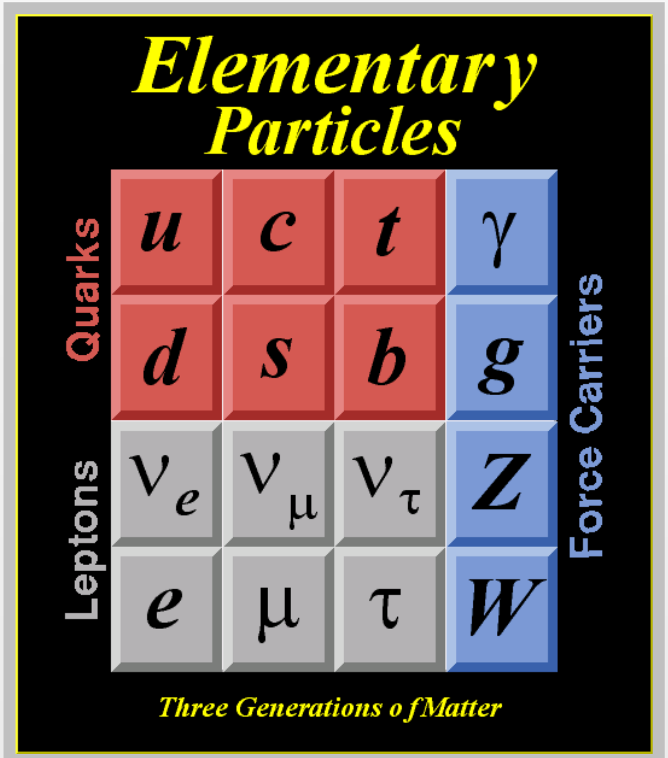
FLAVOR PHYSICS AND ITS MOTIVATIONS

I. The Standard Model: a low energy effective theory

II. Flavor Physics in the Standard Model (and beyond)

I. The Standard Model: a low energy effective theory

THE STANDARD MODEL



Flavor

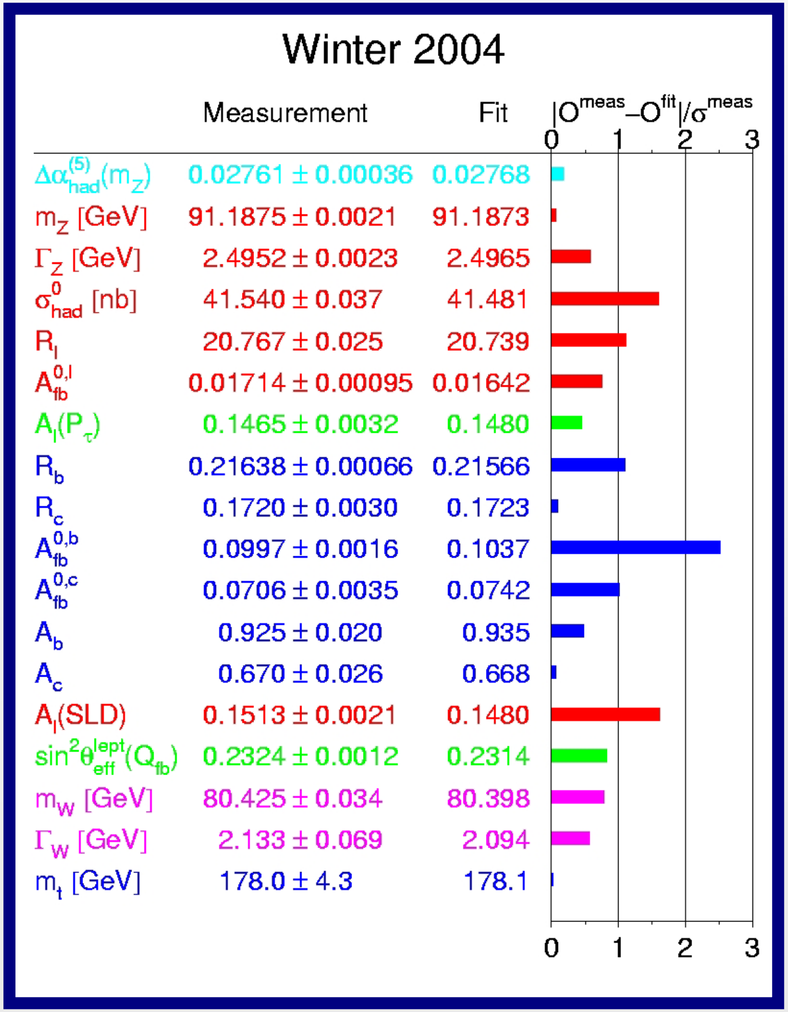
Gauge symmetry:

$$SU(3) \times SU(2)_L \times U(1)_Y$$

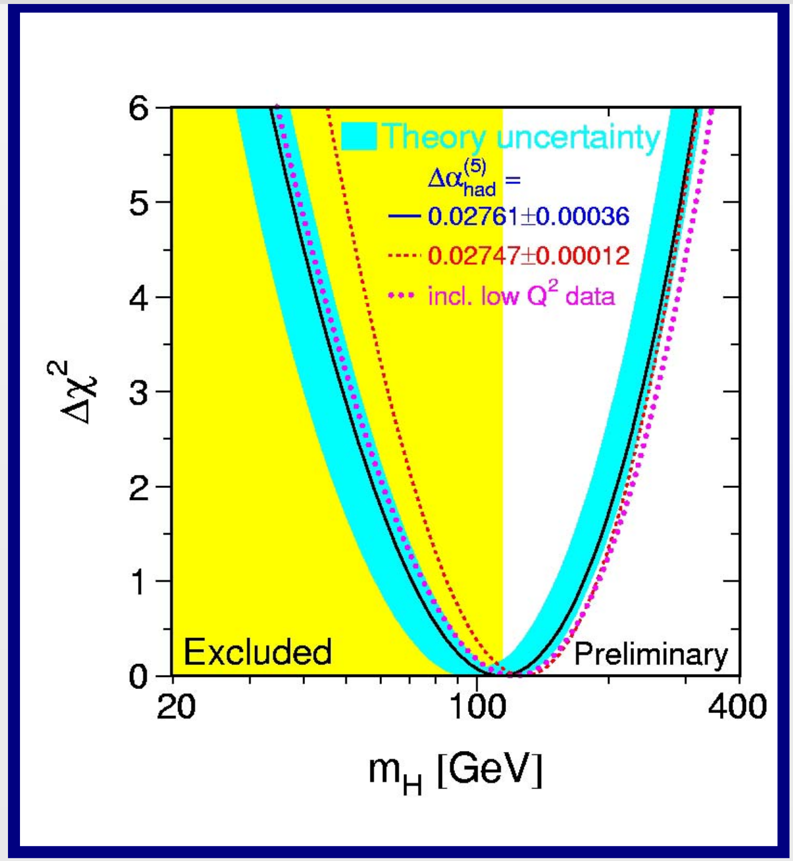
$\Downarrow$ Higgs

$$SU(3) \times U(1)_{e.m.}$$

The SM provides an extremely successful picture, up to the Fermi scale:



EW precision tests support the SM and a light Higgs



TWO OPEN QUESTIONS:

1) Which is the mechanism of gauge symmetry breaking ?

SM Higgs, more Higgs doublets, composite Higgs, ...

2) Which is the origin of flavor physics ?

Why the spectrum of quarks and leptons covers 5 orders of magnitude?

What give rise to the pattern of quark mixing encoded in the CKM matrix and the magnitude of CP violation?

Fermion masses are generated by gauge symmetry breaking

Gauge symmetry breaking and flavor physics are closely related

The Standard Model does not explain flavor

Flavor physics is an open window on physics beyond the Standard Model

THE STANDARD MODEL: A LOW ENERGY EFFECTIVE THEORY

CONCEPTUAL PROBLEMS

- o Gravity ($M_{Planck} = (\hbar c/G_N)^{1/2} \approx 10^{19}$ GeV)
- o ………………

PHENOMENOLOGICAL INDICATIONS

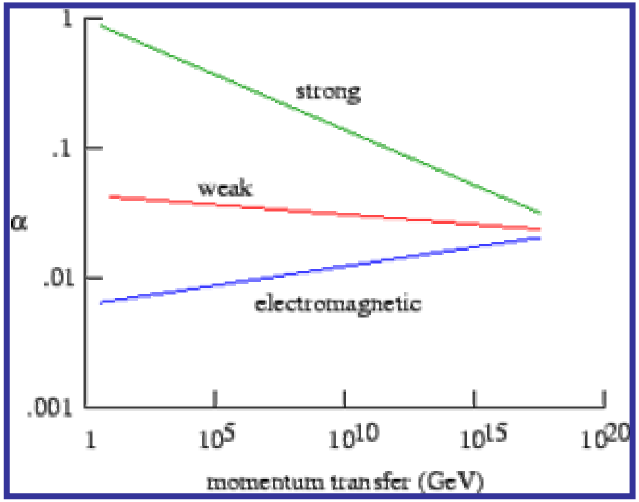
- o Unification of couplings ($M_{GUT} \approx 10^{15}$-10^{16} GeV)
- o Neutrino masses
- o Dark matter ($\Omega_M \approx 0.3$)
- o Vacuum energy ($\Omega_\Lambda \approx 0.7$)
- o Baryogenesis
- o Inflation

Unification of Couplings

The running of gauge couplings provides strong indication of unification. However:

precise unification **fails** in the SM [$\alpha_s(M_Z) \approx 0.073$].

(Well compatible in SUSY)



Grand Unification Theories (GUT) are very appealing for several reasons:

• Unity of forces • Unity of quark and leptons (different directions in G) • Family Q-numbers (in SO(10) a whole family in 16) • Charge quantization ($Q_d = -1/Nc = -1/3$) • B and L non conservation • ....

Neutrino Masses

The existence of neutrino masses and mixings is well established. But **neutrinos are massless in the SM**.

Neutrino masses are really special: $m_t/(\Delta m_{atm}) \sim 10^{12}$

→ The simple extension of the SM with the inclusion of ν_R looks very unnatural

A natural solution: ν's are Majorana particles and get masses through L violating interactions suppressed by a large scale M

$$m_\nu \sim \frac{m^2}{\mathbf{M}}$$

For $m_\nu \sim 0.05$ eV and $m \sim v \sim 200$ GeV →

$$\mathbf{M} \sim 10^{15}\,\mathrm{GeV} \sim \mathrm{M_{GUT}}$$

Energy Density of the Universe

$$\Omega_{tot} = \Omega_{mat} + \Omega_{rad} + \Omega_{vac}$$

matter radiation vacuum

$\Omega_i \equiv \rho_i / \rho_c$

$\rho_c = 3\,H^2/8\,\pi\,G_N \approx$

$\approx 5 \cdot 10^{-6}\ \mathrm{GeV\,cm^{-3}}$

$\Omega_{tot} > 1 \rightarrow\ k = +1$ **closed** universe

$\Omega_{tot} < 1 \rightarrow\ k = -1$ **open** universe

$\Omega_{tot} = 1 \rightarrow\ k = 0$ **flat** universe

$k \equiv$ curvature constant

$\mathbf{\Omega_{tot} = 1.02 \pm 0.02}$

Consistent with spatial flatness (WMAP)

Inflation: $\Omega_{tot} = 1$

Ω_{rel} negligible

$\Omega_{rad} \approx 10^{-5}$

$\mathbf{\Omega_{mat} \approx 0.3\ ,\ \ \Omega_{vac} \approx 0.7}$

← **Both problematic !**

Dark Matter

$$\Omega_{mat} = \Omega_b + \Omega_{dm}$$

Baryonic matter

Dark matter (i.e. non-luminous and non-absorbing)

$$\Omega_{mat} \approx 0.3 \ , \quad \Omega_b \approx 0.04$$

→ more than 80% of matter is dark matter !!

Cold DM ≡ non relativistic at the onset of galaxy formation

Primordial black holes, axions, WIMP, ...

(SUSY neutralino)

Hot DM ≡ relativistic at the onset of galaxy formation

Could be ν's but $\Omega_\nu < 0.015$ (WMAP)

Most of DM should be cold ← All hot DM would have not permitted galaxies to form

Vacuum Energy

$$\Omega_{vac} \approx 0.7$$

The scale of the cosmological constant is a big mystery

- In QFT the energy density of the vacuum receives an infinite contribution from the zero-point energies of the various modes of oscillation. For a bosonic scalar field:

$$H_b = \sum_{\mathbf{p}} \left(a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + \frac{1}{2} \right) \varepsilon_{\mathbf{p}} \quad \Rightarrow \quad \langle 0 | H_b | 0 \rangle = \frac{1}{2} \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}}$$

Fermionic s=1/2 fields give a negative contribution:

$$H_f = \sum_{\mathbf{p}} \left(b^\dagger_{\mathbf{p}} b_{\mathbf{p}} + c^\dagger_{\mathbf{p}} c_{\mathbf{p}} - 1 \right) \varepsilon_{\mathbf{p}} \quad \Rightarrow \quad \langle 0 | H_f | 0 \rangle = - \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}}$$

• The scale of the zero-point energy density is provided by the cutoff:

$$\rho_{vac} = \frac{1}{V}\langle 0|\, H\, |0\rangle \sim \frac{1}{V}\sum_{\varepsilon_p < \Lambda_{cut}} \varepsilon_p \quad \xrightarrow{(\varepsilon_p = cp)}$$

$$\longrightarrow \quad \boxed{\rho_{vac} \approx \frac{\Lambda_{cut}^4}{(\hbar c)^3} = \left(\frac{\Lambda_{cut}}{1\ \mathrm{GeV}}\right)^4 \cdot 10^{41}\ \mathrm{GeV\ cm^{-3}}}$$

• In elementary particle physics experiments the shift of the vacuum energy is unobservable. In cosmology its absolute value is observable through the **coupling of vacuum energy to gravity**:

$$\boxed{\Omega_{vac}^{obs} \approx 0.7} \quad \longrightarrow \quad \boxed{\rho_{vac}^{obs} \approx 3.5 \cdot 10^{-6}\ \mathrm{GeV\ cm^{-3}}}$$

$$\boxed{\text{If } \Lambda_{cut} \sim M_{Planck} \quad \longrightarrow \quad \boxed{\rho_{vac} \sim \mathbf{10^{123}}\, \rho_{vac}^{obs}}}$$

- Exact SUSY would solve the problem:

$$\langle 0|\, H_b \,|0\rangle = \frac{1}{2} \sum_p \varepsilon_p \qquad \langle 0|\, H_f \,|0\rangle = -\sum_p \varepsilon_p$$

$$\Rightarrow \quad \langle 0|\, H \,|0\rangle = \left(\frac{1}{2} n_b - n_f \right) \sum_p \varepsilon_p = 0$$

But SUSY is broken:

$$\rho_{vac} \approx \frac{\Lambda_{SUSY}^4}{(\hbar c)^3} \sim \mathbf{10^{59}}\, \rho_{vac}^{obs} \qquad (\Lambda_{SUSY} \approx 1 \text{ TeV})$$

So far, the problem of the scale of the cosmological constant has found no solution

Baryogenesis

• So far, no primordial anitimatter has been observed in the Universe. Up to distances of order 100 Mpc – 1 Gpc the Universe consists only of matter.

(1Mpc = 3.2 10^6 light years. Observable universe : $H_0^{-1} \sim 10$ Gpc)

• Furthermore, the density of baryons compared to the density of photons is extremely small

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

$$n_{\bar{b}} << n_b$$

• A very plausible assumption is that the big bang produces an equal number of quarks and antiquarks

→ WHEN AND WHY ANTIMATTER DISAPPEARED ?

THE SAKHAROV CONDITIONS: (1967)

1) Baryon number violation

2) C and CP violation

3) Departure from thermal equilibrium

In the SM:

Istanton process

Weak interactions

Electro-weak phase transition

In the SM, for $m_H \geq 80$ GeV, the e.w. phase transition is not "strong" enough: it does not provide enough termal instability necessary for baryogenesis

CP violation generated by the CKM mechanism is irrelevant for baryogenesis → Non-standard CP violation is a necessary ingredient for baryiogenesis

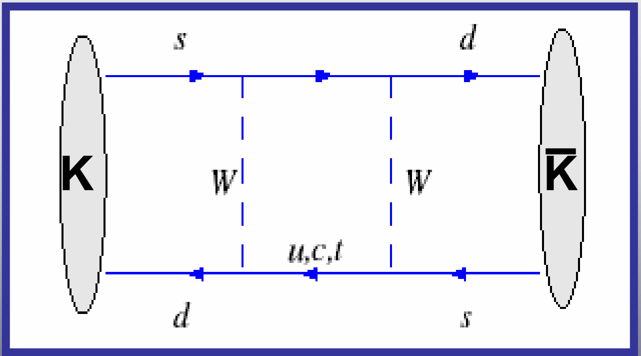
THE "FLAVOR PROBLEM"

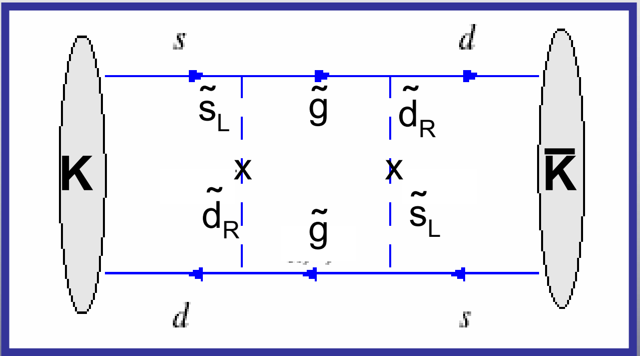
The "natural" cut-off

NEW PHYSICS MUST BE VERY "SPECIAL"

$$\delta m_H^2 = \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 \Lambda^2 \approx (0.3\,\Lambda)^2 \longrightarrow \boxed{\Lambda = O(1\ \mathrm{TeV})}$$

From higher dimensional operator in the flavor sector





$$\Lambda_{K^0\text{-}\overline{K}^0} \approx O(100\ \mathrm{TeV})$$

The flavor problem

II. Flavor Physics in the Standard Model (and beyond)

THE QUARK FIELDS

$Q = T_3 + Y/2$

$$Q_L = \left(\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \right)$$

$T_3 = 1/2$
$T_3 = -1/2$
$Y = 1/3$

$$U_R = \left(u_R \,,\, c_R \,,\, t_R \right)$$

$T_3 = 0$ $\quad$ $Y = 4/3$

$$D_R = \left(d_R \,,\, s_R \,,\, b_R \right)$$

$T_3 = 0$ $\quad$ $Y = -2/3$

$T_3 = -1/2$, $Y = 1$

Ordinary mass terms $(\bar{u}_L u_R + \text{h.c.})$ are forbidden by gauge invariance → Quark masses must be generated by spontaneous gauge symmetry breaking

Flavor and gauge symmetry breaking are closely related !!

THE HIGGS FIELD

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$ $T_3 = 1/2$, $T_3 = -1/2$, $Y = 1$

$T_3 = -1/2$, $Y = 1$

$(\bar{u}_L u_R + \text{h.c.})$

$$H^C = i\tau_2 H^* = \begin{pmatrix} \phi^0 \\ -\phi^+ \end{pmatrix}^*$$

$$H \to U H = \exp(i\,\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2)\,H$$

$$H^C \to U H^C = \exp(i\,\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2)\,H^C$$

$$\left[\, H^C \to i\,\tau_2\,(U\,H)^* = i\,\tau_2 \exp(-i\,\boldsymbol{\alpha}\cdot\boldsymbol{\tau}^*/2)\,H^* = \exp(i\,\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2)\;i\,\tau_2\,H^* = U\,H^C \,\right]$$

Spontaneous symmetry breaking

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

THE QUARK MASS TERMS

$$L^{Yukawa} = - \sum_{i,k} [\overline{Q}^i_L Y^d_{ik} D^k_R H + \overline{Q}^i_L Y^u_{ik} U^k_R H^C] + + \text{ h.c.}$$

Gauge symmetry breaking

$$L^{mass} = - \sum_{i,k} [\overline{d}^i_L m^d_{ik} d^k_R + \overline{u}^i_L m^u_{ik} u^k_R] + \text{h.c.}$$

$$m^q = Y^q v/\sqrt{2}$$

$\longleftrightarrow$ $M_W = gv/2$

Why $m^q \neq O(M_W)$ **??**

CP VIOLATION

$$L^{mass} = - \sum_{i,k} [\bar{q}_L^i m_{ik}^q q_R^k + \bar{q}_R^i m_{ik}^q q_L^k] \qquad (q=u,d)$$

DISCRETE SYMMETRIES

$P\, q_{L,R} P^{-1} = U_P\, q_{R,L}$, $P\, \bar{q}_{L,R} P^{-1} = \bar{q}_{R,L} U_P^\dagger$	$U_P = \gamma^0$
$C\, q_{L,R} C^{-1} = U_C\, \bar{q}_{R,L}^T$, $C\, \bar{q}_{L,R} C^{-1} = - q_{R,L}^T U_C^\dagger$	$U_C = i\gamma^2\gamma^0$
$T\, q_{L,R} T^{-1} = U_T\, q_{L,R}$, $T\, \bar{q}_{L,R} T^{-1} = \bar{q}_{L,R} U_T^\dagger$	$U_T = i\gamma^1\gamma^3$
T is antiunitary: $T\, c\psi T^{-1} = c^* T\, \psi T^{-1}$	

$$C P\; L^{mass} (C P)^{-1} = L^{mass} \Longleftrightarrow m_{ik} = m_{ik}^*$$

But: $T C P\; L^{mass} (T C P)^{-1} = L^{mass}$ (TCP theorem)

A necessary and sufficient condition for CP invariance is:

$$\boxed{m^{u,d} = \text{real}}$$

But there is no compelling symmetry for $m^{u,d}$ to be real. In field theory, all that may happen will happen

In the Standard Model the quark mass matrix, from which $\cancel{CP}$ originate, is determined by the Yukawa Lagrangian

CP invariant

$$\boxed{L^{\text{quarks}} = L^{\text{kinetic}} + L^{\text{gauge int.}} + L^{\text{Yukawa}}}$$

$\cancel{CP}$ and symmetry breaking are closely related !

DIAGONALIZATION OF THE MASS MATRIX

The mass matrices m^q are not Hermitean. Up to singular cases, they can be diagonalized by 2 unitary transformations:

$$\boxed{U_L^\dagger m U_R = m_D}$$

$$\begin{cases} U_L^\dagger m m^\dagger U_L = m_D m_D^\dagger \\ U_R^\dagger m^\dagger m U_R = m_D^\dagger m_D \end{cases}$$

i

$$(U_L^\dagger)_{ik} q_L^k \to q_L^i \ , \ (U_R^\dagger)_{ik} q_R^k \to q_R^i$$

($U_{L,R}$ different for u^k and d^k)

$$L^{mass} = -[m_u \bar{u}_L u_R + m_d \bar{d}_L d_R + \ldots] + h.c.$$

With respect to:

$$(U_L^\dagger)_{ik} q_L^k \to q_L^i \ , \ (U_R^\dagger)_{ik} q_R^k \to q_R^i$$

neutral currents $\bar{q}_L^i \gamma_\mu q_L^i$ and $\bar{q}_R^i \gamma_\mu q_R^i$ are invariant: quark kinetic terms, QCD couplings with gluons, QED couplings with photons, weak couplings with Z^0

No flavor changing neutral currents (FCNC) at tree level

The only effect is in the weak charged currents:

$$\bar{u}_L^i \gamma_\mu d_L^i \cdot W^\mu \to \bar{u}_L^k \gamma_\mu (U_L^{u\dagger} U_L^d)_{kj} d_L^j \cdot W^\mu$$

$$V_{CKM} = U_L^{u\dagger} U_L^d$$

$$V_{CKM} V_{CKM}^\dagger = 1$$

V_{CKM}: Counting of parameters

$V_{CKM} \sim N \times N$ unitary matrix → N^2 complex numbers – N^2 unitary conditions → N^2 real parameters

$N + 2N(N-1)/2 \quad (V_{ij} V^*_{kj} = \delta_{ik})$

$N^2 - [\, N + N(N-1)/2 \,]$

- A $N \times N$ orthogonal matrix, $OO^T=1$, has $N(N-1)/2$ real parameters

→ V_{CKM} has $N(N-1)/2$ angles and $N(N+1)/2$ phases

- Freedom of phase redefinition: $2N$ quarks $\rightarrow$ $2N-1$ relative phases ($\bar{U} V_{CKM} D$ insensitive to the overall phase)

⇒ **$N(N-1)/2$ angles, $(N-1)(N-2)/2$ phases**

V_{CKM} and CP VIOLATION

N	N(N-1)/2 angles	(N-1)(N-2)/2 phases
2	1	0
3	3	1
4	6	3

CP Violation

CP violation is natural with 3 quark generations (Kobayashi-Maskawa)

With 3 generations all CP violating phenomena are related to the same unique parameter (δ)

V_{CKM}: the PDG parameterization

(Maiani)

N = 3

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij} \quad s_{ij} = \sin\theta_{ij}$$

$$c_{ij} \geq 0 \qquad s_{ij} \geq 0 \qquad 0 \leq \delta \leq 2\pi$$

$\sin\theta_C$

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

The Wolfenstein parameterization

One small parameter: $s_{12} \approx \theta_{12} \approx 0.22$

Approximate parameterization

$$s_{12} \equiv \lambda \quad s_{23} \equiv A\,\lambda^2 \quad s_{13}\, e^{-i\delta} \equiv A\lambda^3(\rho - i\eta)$$

$\lambda \approx 0.22 \quad A \approx 0.8$

$\rho \approx 0.3 \quad \eta \approx 0.2$

$1-\lambda^2/2$	λ	$A\lambda^3(\rho-i\eta)$
$-\lambda$	$1-\lambda^2/2$	$A\lambda^2$
$A\lambda^3(1-\rho-i\eta)$	$-A\lambda^2$	1

$+\,O(\lambda^4)$

Buras et al.

$A\lambda^3(1-\bar{\rho}-i\bar{\eta})$

$$\bar{\rho} \equiv \rho\,(1-\lambda^2/2) \qquad \bar{\eta} \equiv \eta\,(1-\lambda^2/2)$$

THE UNITARITY TRIANGLES

(Bjorken-Jarlskog)

Unitarity relations:

$$V^\dagger V = 1 \longrightarrow \sum_k V^*_{ki} V_{kj} = \delta_{ij}$$

9 constraints,
6 triangular relations

Only 2 triangles have all sides with length of the same $O(\lambda^3)$:

$$V^*_{ub} V_{ud} + V^*_{cb} V_{cd} + V^*_{tb} V_{td} = 0$$

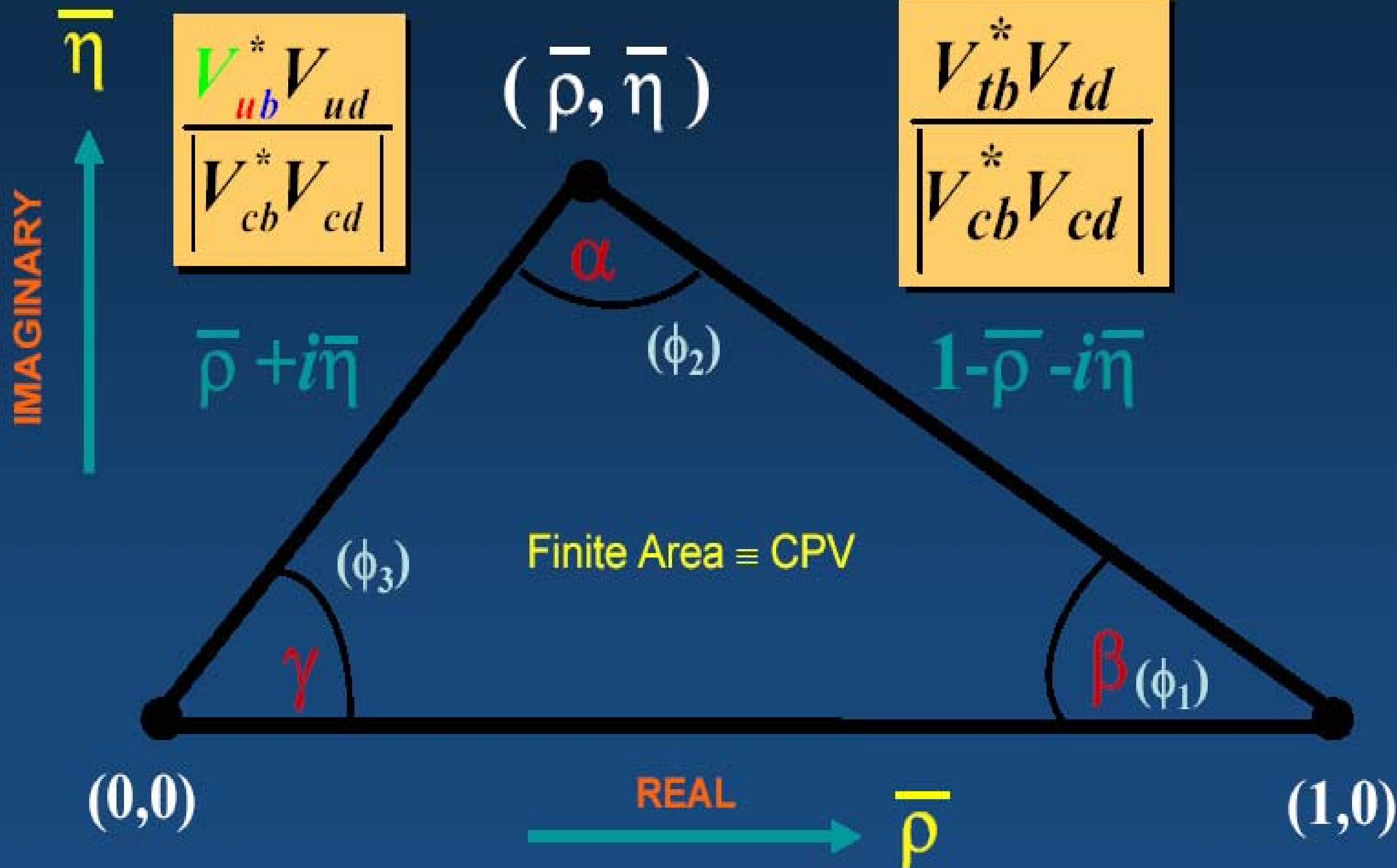
$$V_{ud} V^*_{td} + V_{us} V^*_{ts} + V_{ub} V^*_{tb} = 0$$

They are equivalent at order λ^3

Only the orientation of the triangles depends on the phase convention. The area and ~~CP~~ are proportional to:

$$J = c_{12}\, c_{13}^2\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta \approx A^2 \lambda^6 \eta \sim 10^{-5}$$

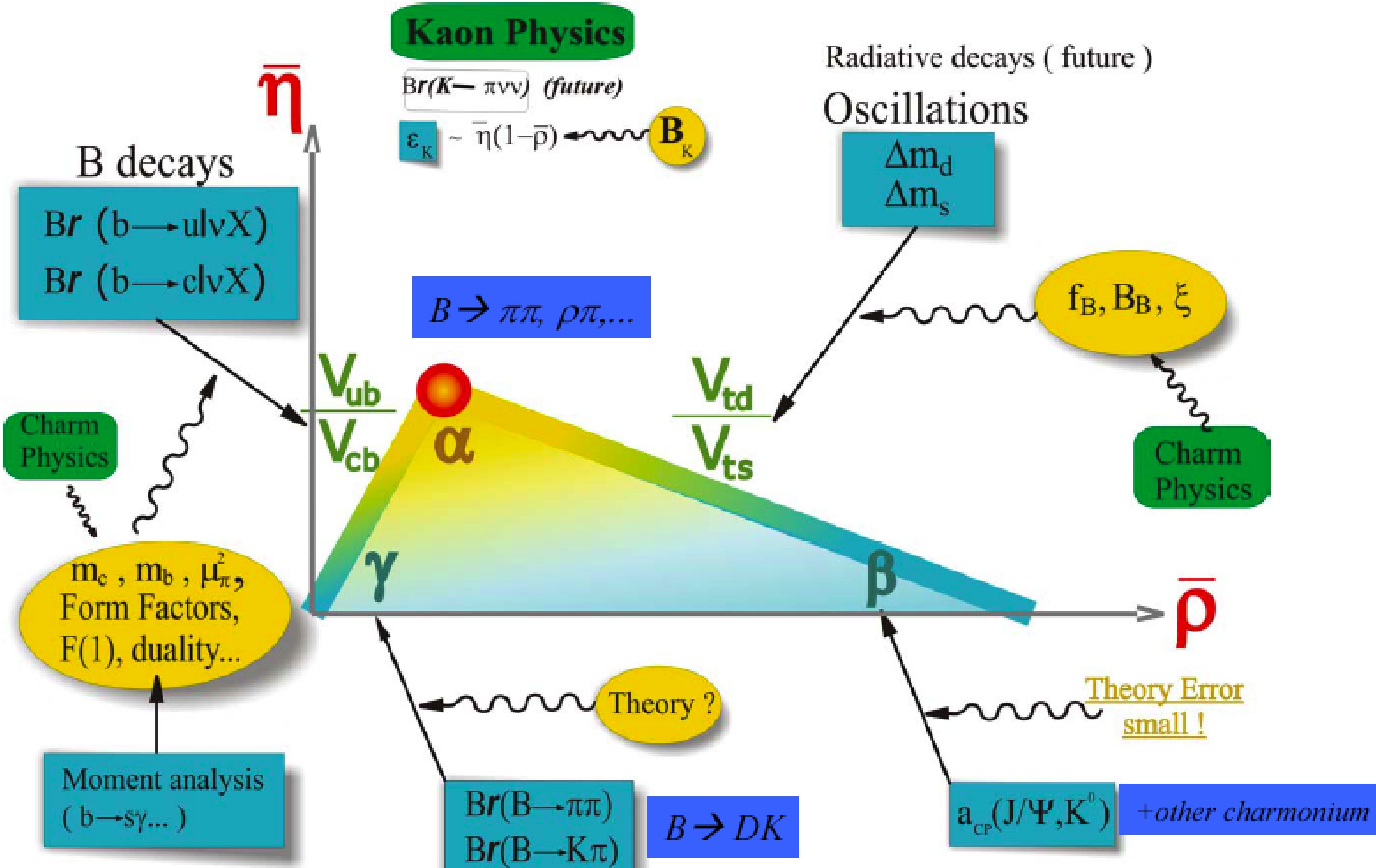
Unitarity: $V_{ub}^{*}V_{ud}+V_{cb}^{*}V_{cd}+V_{tb}^{*}V_{td}=0$



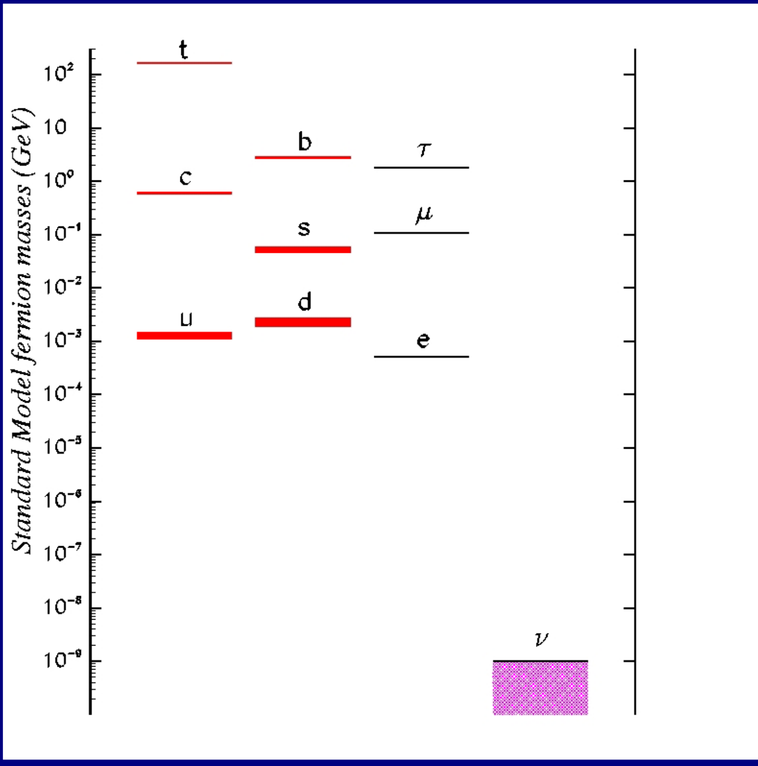
Visualization of the unitarity of the CKM matrix

Unitarity Triangle in the (ρ–η) plane

From
A. Stocchi
ICHEP 200



THE QUARK MASS SPECTRUM



Hierarchy of masses

$m_u \sim 3$ MeV
$m_c \sim 1.2$ GeV
$m_t \sim 175$ GeV

$m_d \sim 6$ MeV
$m_s \sim 100$ MeV
$m_b \sim 4.3$ GeV

Which is the origin of FLAVOR SYMMETRY BREAKING ?

THERE IS A CLEAR CORRELATION BETWEEN MASSES AND MIXINGS ANGLES

In the first 2 generations:

$$\left(\frac{m_d}{m_s}\right)^{1/2} \approx 0.24 \qquad \left(\frac{m_u}{m_c}\right)^{1/4} \approx 0.22$$

$$\Rightarrow \quad \boxed{\left(\frac{m_d}{m_s}\right)^{1/2} \approx \left(\frac{m_u}{m_c}\right)^{1/4} \approx V_{us}}$$

Can we explain this relation ?

MASS TEXTURES

Two generations:

Gatto et al.

$$\mathbf{m^d} = m_s \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix} \qquad \mathbf{m^u} = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}$$

$$\mathrm{diag}(\mathbf{m^d}) = m_s(x, 1) \quad \Longrightarrow \quad x = m_d / m_s$$

Diagonalization:

$$\begin{cases} U_L^\dagger\, m\, m^\dagger U_L = m_D\, m_D^\dagger \\ U_R^\dagger\, m^\dagger m\, U_R = m_D^\dagger\, m_D \end{cases} \qquad U_L^\dagger\, m\, U_R = m_D \qquad V_{CKM} = U_L^{u\dagger} U_L^d$$

$$V_{CKM} = U_L^{u\dagger} U_L^d = U_L^d \approx \begin{pmatrix} 1 - x/2 & \sqrt{x} \\ -\sqrt{x} & 1 - x/2 \end{pmatrix}$$

$$V_{us} = \sin\theta_C = \sqrt{x} = \sqrt{m_d/m_s} \approx 0.22$$

Which theory of flavor generates this texture?

HORIZONTAL SYMMETRIES

Example: **Horizontal U(2)** (Barbieri, Hall, ...)

$q^a \rightarrow \mathbf{U}_{\mathbf{ab}}\, q^b \quad , \quad \mathbf{U} \in \mathbf{U(2)}$ $\qquad a,b = 1,2$ (Generation indices)

$$L = \frac{1}{M_F}\, \phi_{ab}\, q^a\, q^b\, H$$

"Flavon" field

Higgs field (U(2) scalar)

Non-renorm. interaction

M_F = flavor scale

$$\phi_{ab} = S_{ab} + A_{ab}$$

Symmetric tensor

Anti-symmetric tensor

$$\mathbf{U(2)} \xrightarrow{S_{ab}} \mathbf{U(1)} \xrightarrow{A_{ab}} \{\mathbf{1}\}$$

$$\langle S_{ab}\rangle = \begin{pmatrix} 0 & 0 \\ 0 & V \end{pmatrix} \qquad \langle A_{ab}\rangle = \begin{pmatrix} 0 & -V \\ V & 0 \end{pmatrix}$$

$$L = \frac{1}{M_F}(S_{ab} + A_{ab})\, q^a q^b H \;\longrightarrow$$

Flavor symm. breaking

$$\longrightarrow \frac{V}{M_F} q^2 q^2 H + \frac{v}{M_F}(q^2 q^1 - q^1 q^2) H \equiv q^a Y_{ab} q^b H$$

Yukawa matrix

$$Y_{ab} = \begin{pmatrix} 0 & -v/M_F \\ v/M_F & V/M_F \end{pmatrix}$$

$v/M_F = \sqrt{x}$
$V/M_F = 1+x$

Is the Gatto's texture

2

LATTICE QCD AND QUARK MASSES

I. Generalities on quark masses

II. Introduction to Lattice QCD

III. Lattice Results for quark masses

I. Generalities on quark masses

QUARK MASSES from
The Review of Particle Physics
The 2004 Edition

[S. Eidelman *et al.*, (Particle Data Group)
Phys. Lett. B592, 1 (2004)]

Light Quark Masses $m_{\overline{MS}}$ (2 GeV)

$(m_u+m_d)/2$ [MeV]	4.25 (1.25)	[29 %]
m_s [MeV]	105 (25)	[24 %]

PDG 2000
120 (50) [42%]

Heavy Quark Masses $m_{\overline{MS}}$ ($m_{\overline{MS}}$)

m_c [GeV]	1.25 (10)	[8.0 %]
m_b [GeV]	4.25 (15)	[3.5 %]
m_t [GeV]	174.3 (5.1)	[2.9 %] (Pole Mass from CDF/D0)

QUARK MASSES

♦ QM ARE EXTREMELY IMPORTANT FOR BOTH PHENOMENOLOGY (cross sections, inclusive decays rates, lifetimes, …) AND THEORY (physics of flavour, GUTs, mass textures…)

♦ QM CANNOT BE "DIRECTLY" MEASURED IN THE EXPERIMENTS BECAUSE QUARKS ARE CONFINED INSIDE HADRONS

♦ QM ARE FUNDAMENTAL PARAMETERS OF THE STANDARD MODEL: THEY CANNOT BE DETERMINED BY THEORETICAL CONSIDERATIONS ONLY.

♦ QM CAN BE INTRODUCED AS SHORT-DISTANCE EFFECTIVE COUPLINGS, WHICH DEPEND ON THE RENORMALIZATION SCALE AND SCHEME. SEVERAL DEFINITIONS ARE USED, E.G. $m^{\overline{MS}}(\mu)$

♦ FOR LIGHT QUARKS, RATIOS OF QM, ARE PREDICTED BY CHIRAL PERTURBATION THEORY. E.G. $2\,m_s/(m_u+m_d) = 24.4 \pm 1.5$
LATTICE QCD ALLOWS TO DETERMINE THEIR ABSOLUTE VALUES

LIGHT QUARK MASSES AND CHIRAL PERTURBATION THEORY

- The hadronic spectrum of QCD is very simple at low energy. The only relevant degrees of freedom are 8 pseudoscalar mesons (π, K, η) separated by a mass gap from the heavier states.

- The heavier degrees of freedom can be integrated out and QCD is described by a low-energy effective theory, $L_\chi(\pi,K,\eta)$, the chiral perturbation theory (ChPT).

- The operators entering the ChPT Lagrangian are determined by the chiral symmetry $SU(3)_L \times SU(3)_R$ of QCD.

- The operators are organized in a power series, according to the number of derivatives (power of the external momenta in phyisical amplitudes) and powers of light quark masses.

- The Lagrangian at the lowest order (p^2) is:

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\,\mathrm{Tr}\left[(\partial_\mu U)^\dagger(\partial^\mu U) + 2B\,(M^\dagger U + U^\dagger M)\right]$$

$$U(\phi) = \exp(i\sqrt{2}\,\phi/F)$$

$$M = \mathrm{diag}(m_u, m_d, m_s)$$

$$\phi = \begin{bmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -\eta\sqrt{2/3} \end{bmatrix}$$

- In terms of the Goldstone boson fields:

$$\mathcal{L}_\chi^{(2)} = \frac{1}{2}\,\mathrm{Tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right] - B\,\mathrm{Tr}\left[(M\,\phi^2)\right] + O(\phi^4)$$

$$= \text{kin. terms} + B\,(m_u+m_d)\,\pi^\dagger\pi^- + B\,(m_d+m_s)\,(k^0)^2 + \ldots$$

- Thus, at leading order (LO):

$$\boxed{M_{PS}^2 = B\,(m_{q1} + m_{q2})}$$

and ratios of light quark masses can be determined from ChPT

- At the next to leading order (NLO):

Kaplan & Manohar ambiguity: $\mathbf{m} \rightarrow \alpha_1\,\mathbf{m} + \alpha_2\,(\mathbf{m}^\dagger)^{-1}\,\mathbf{det(m)}$

$\rightarrow$ ChPT + $1/N_C$ + Lowest resonances dominance

$$\boxed{\frac{m_u}{m_d} = 0.553 \pm 0.043 \qquad \frac{m_s}{(m_u + m_d)/2} = 24.4 \pm 1.5}$$

(H. Leutwyler '96)

ABSOLUTE VALUES FROM LATTICE QCD

LATTICE DETERMINATION OF QUARK MASSES

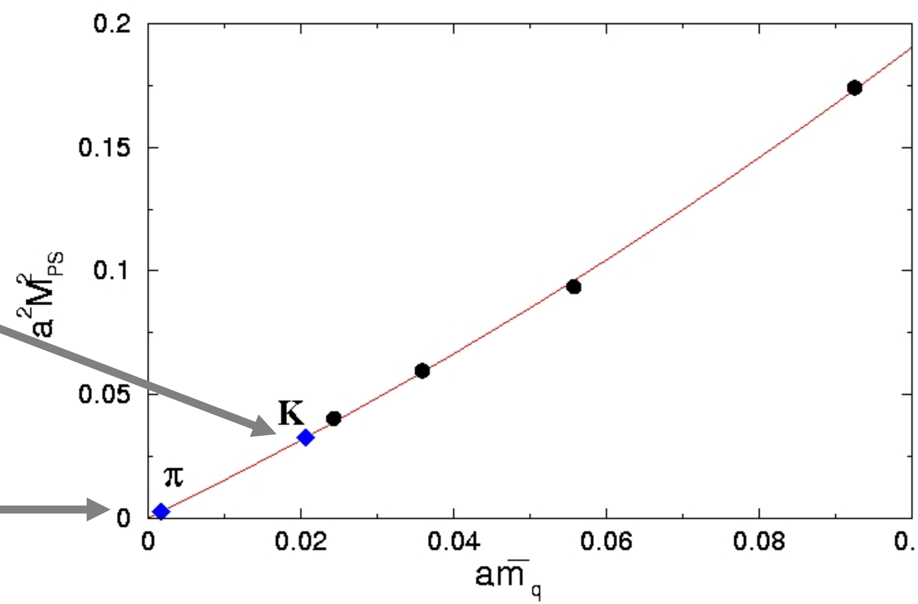
$$\hat{m}_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL
$M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY OR
NON-PERTURBATIVE METHODS

Extrapolation to
m = m_s

Extrapolation to
m = $m_{u,d}$



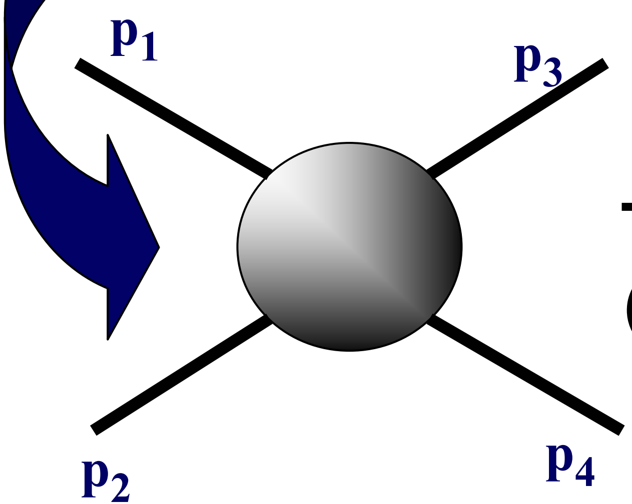
II. Introduction to Lattice QCD

The Green Functions
The basic quantities of field theory

$$L(\phi) = 1/2\,(\partial_\mu \phi(x))^2 - 1/2\, m_0^2\, \phi^2(x) - 1/4!\, \lambda_0\, \phi^4(x)$$

$$S(\phi) = \int d^4x\, L(\phi)$$

$$\mathbf{G(x_1, x_2, x_3, x_4) = \langle 0 |\, T\, [\phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)]\, | 0 \rangle_c}$$



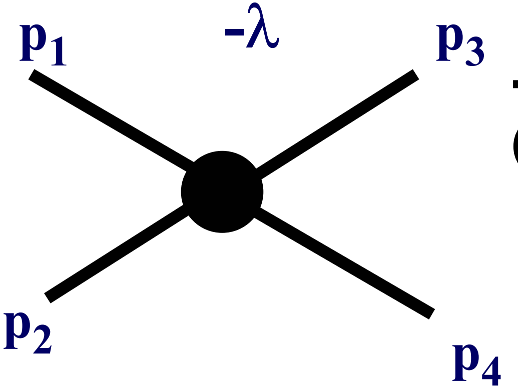
$$\frac{(Z^{1/2}{}_{\phi})^4\, M(p_1, p_2, p_3, p_4, m, \lambda)}{(p_1^2 - m^2)\,(p_2^2 - m^2)\,(p_3^2 - m^2)\,(p_4^2 - m^2)} + \ldots.$$

The S-matrix element

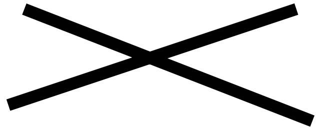
$$S(p_1+p_2 \to p_3+p_4) = \lim_{p^2_{1,2,3,4} \to m^2} M(p_1, p_2, p_3, p_4, m, \lambda)$$

$$= \lim_{p^2_{1,2,3,4} \to m^2} \frac{(p_1^2-m^2)(p_2^2-m^2)\, G(p_1, p_2, p_3, p_4, m, \lambda)\, (p_3^2-m^2)(p_4^2-m^2)}{(Z^{1/2}_\phi)^4}$$

G at lowest order in perturbation theory



$$\frac{\lambda}{(p_1^2-m_0^2)(p_2^2-m_0^2)(p_3^2-m_0^2)(p_4^2-m_0^2)}$$

$+$  $+$ 

The Functional Integral

The Green Functions can be written in terms of Functional Integrals over classical fields:

$$\mathbf{G(x_1, x_2, x_3, x_4) = \langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4) \rangle \equiv}$$

$$\boxed{Z^{-1} \int [d\phi]\, \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\, e^{-S(\phi)}}$$

where $\mathbf{Z} = \int [d\phi]\, e^{-S(\phi)}$

Wick rotation: $t \rightarrow -it_E$

In perturbation theory: $S_I(\phi) \sim O(\lambda)$

$$e^{-S(\phi)} = e^{-S_0(\phi) - S_I(\phi)} \approx e^{-S_0(\phi)} \left(1 - S_I(\phi) - S_I^2(\phi)/2 + \ldots\right)$$

The Lattice regularization

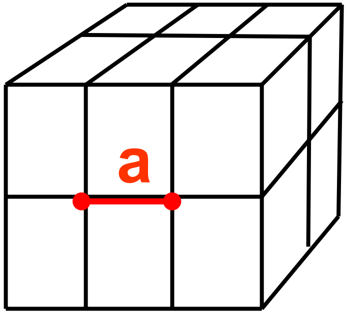
The functional integral is only a formal definition because of the infrared and ultraviolet divergences. These problems can be cured by introducing an infrared and an ultraviolet cutoff

1) The ultraviolet cutoff

The fields are defined on a (hypercubic) four dimensional lattice

$\phi(x) \rightarrow \phi(a\, n)$

$n = (n_x, n_y, n_z, n_t)$



$\partial_\mu \phi(x) \rightarrow \nabla_\mu \phi(x) = [\phi(x + a\, n_\mu) - \phi(x)]/a$

The momentum p is cutoff at the first Brioullin zone:

$|p| \leq \pi/a$

The cutoff can be in conflict with important symmetries of the theory, as for example Lorentz invariance or chiral invariance. This problem is common to all regularizations, like for example Pauli-Villars, dimensional regularization etc.

2) The infrared cutoff

The lattice is defined in a finite volume

$n_i = 1, 2, \ldots, L$

$p_i a = 2\pi k_i / L$ with $k_i = 0, 1, \ldots, L - 1$

The physical theory is obtained in the limit

$a \to 0$ Continuum limit ; $L \to \infty$ Thermodinamic limit

Non-physical quantities like Green Functions may develop divergences in this limits. S matrix elements however are finite:

$$Z_\phi(a) = 1 + \lambda \log(pa) + \ldots$$

Important sampling techniques

$$Z^{-1} \int [d\phi]\, \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\, e^{-S(\phi)}$$

The integral is like a statistical Boltzmann system with

$$\beta H = S$$

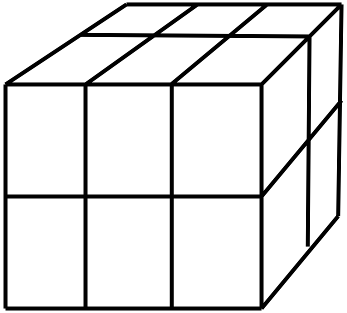
On a finite volume (L) and with a finite lattice spacing (a) this is now an integral on L^4 real variables which can be performed with important sampling techniques, for example the Metropolis technique.

The fields are extracted with weight $\boxed{e^{-S(\phi)}}$

For the 3-dimensional **Ising model**:

$$Z = \sum_{\{\sigma = \pm 1\}} e^{J_{ij}\,\sigma_i\,\sigma_j}$$

$2^N = 2^{L^3} \approx 10^{301}$ for $L = 10$!!!



For the 4-dimensional **scalar field theory** with the **important sampling technique**:

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4) \rangle =$$

$$= Z^{-1} \int [d\phi]\, \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\, e^{-S(\phi)} \approx$$

$$\approx Z^{-1} \sum_{\{\phi(x)\}_n} \phi_n(x_1)\,\phi_n(x_2)\,\phi_n(x_3)\,\phi_n(x_4) \qquad Z \approx \sum_{\{\phi(x)\}_n} 1 = N$$

→ Statistical errors

The lattice QCD action

The continuum QCD Lagrangian:

$$L = -1/4\, G^A{}_{\mu\nu} G_A{}^{\mu\nu} + \sum_{f = \text{flavor}} \overline{q}_f (i\gamma_\mu D_\mu - m_f) q_f$$

GLUONS

QUARKS (& GLUONS)

$$G^A{}_{\mu\nu} = \partial_\mu G^A{}_\nu - \partial_\nu G^A{}_\mu - g_0\, f^{ABC} G^B{}_\mu G^C{}_\nu$$

$$q_f \equiv q_f{}^a{}_\alpha(x) \qquad \gamma_\mu \equiv (\gamma_\mu)^{\alpha\beta} \qquad D_\mu \equiv \partial_\mu I + i\, g_0\, t^A{}_{ab} G^A{}_\mu$$

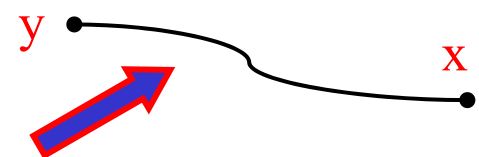
Local gauge invariance:

$[G_\mu(x) \equiv G^A{}_\mu(x)\, t^A]$

$$G_\mu(x) \to V(x)[G_\mu(x)]V^\dagger(x) + i/g_0[\partial_\mu V(x)]V^\dagger(x)$$

$$q(x) \to V(x)q(x) \qquad \bar{q}(x) \to \bar{q}(x)V^\dagger(x)$$

For non-local products of quark fields



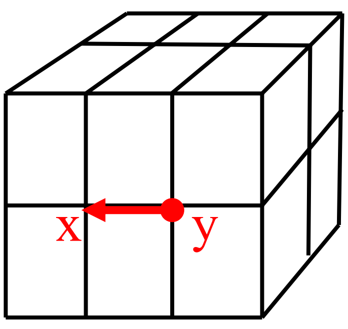
$\bar{q}(y)\, P[\exp \int_x^y i g_0 G_\mu(x) dx_\mu] q(x)$ is gauge invariant

On the lattice:

$$\bar{q}(x+a\mu)\ \boxed{\exp[i g_0 G_\mu(x+a\mu/2)]}\ q(x)$$

$$\Downarrow$$

$$\boxed{\textbf{LINK}\quad U^\dagger_\mu(x)}$$



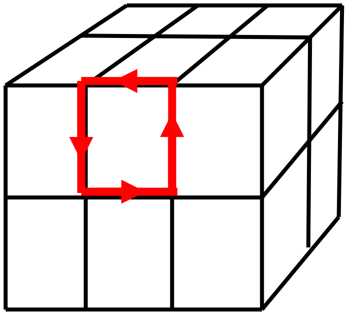
Gauge transformation:

$$U_\mu(x) \to V(x)\,[U_\mu(x)]\,V^\dagger(x+ a\,\mu\,)$$

Plaquette:

$$W_{\mu\nu}(x) = U_\mu(x)U_\nu(x + a\,\mu\,)U^\dagger{}_\mu(x + a\,\nu\,)U^\dagger{}_\nu(x)$$

$$\approx 1 + i\,a^2 g_0 G_{\mu\nu}(x) - a^4 g_0{}^2/2\, G_{\mu\nu}(x)\, G^{\mu\nu}(x) + \ldots$$



The pure gauge lattice action:

$$S_G = 1/g_0{}^2 \sum_x \sum_{\mu<\nu} \mathrm{Re\,Tr}\,[1-W_{\mu\nu}(x)]$$

$$\to a^4/4 \sum_x \sum_{\mu\nu} G_{\mu\nu}(x)\, G^{\mu\nu}(x)$$

$$\to 1/4 \int G_{\mu\nu}(x)\, G^{\mu\nu}(x) + O(a^2)$$

The fermion action(s)

$\nabla_\mu q(x) = [U_\mu(x)\, q(x + a\mu) - q(x)]/a$

∇^*_μ = backward derivative

The "Wilson" lattice Dirac operator is

$\not{D} = 1/2\, [(\nabla_\mu + \nabla^*_\mu)\gamma_\mu - a\nabla_\mu\nabla^*_\mu]$

The "Wilson term": explicit breaking of chiral symmetry

The Wilson fermion action:

$$S_F = a^4 \sum_x q(x)[\not{D} + m_0]\, q(x)$$

We may define many (an infinite number of) lattice actions which all formally converge to the same continuum QCD action: Wilson, Kogut-Susskind, Clover, Domain Wall, Overlap …

Hadron masses and simple matrix elements

$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^\dagger{}_0(\mathbf{0},0) \rangle =$$

The operator A_0 can excite 1-π, 3-π etc. states

$$= \sum_{\mathbf{x}} \sum_n \frac{\langle 0 | e^{iPx} A_0(0)\, e^{-iPx} | n \rangle \langle n | A^\dagger{}_0(0) | 0 \rangle}{2 E_n}$$

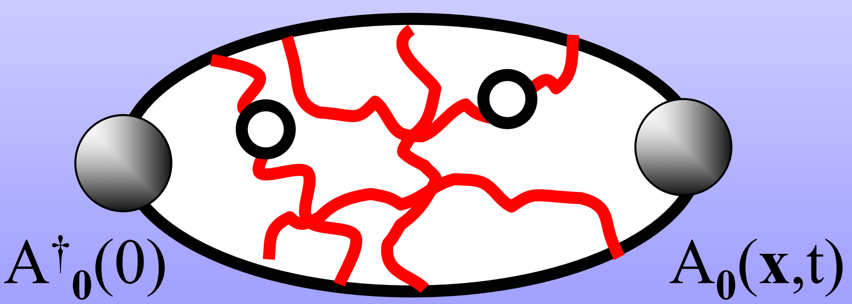
$$= \sum_n \frac{|\langle 0 | A_0 | n \rangle|^2}{2 m_n} \exp[-i\, m_n t] \;\xrightarrow{t \to -it}\; = \sum_n \frac{|\langle 0 | A_0 | n \rangle|^2}{2 m_n} \exp[-m_n t]$$

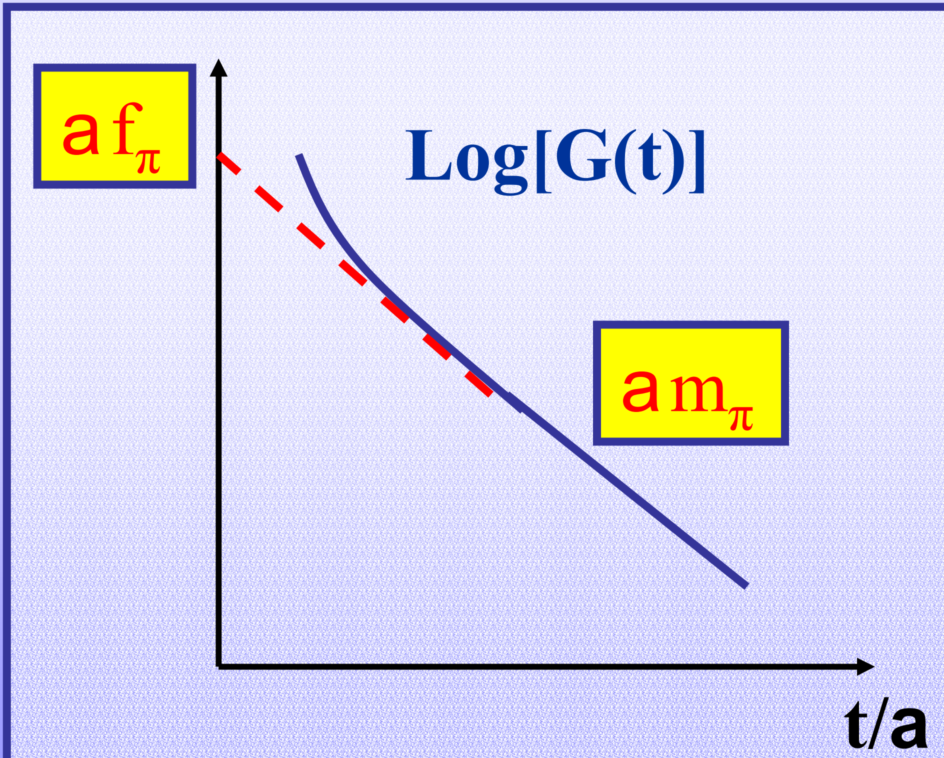
$$\xrightarrow{t \to \infty} \frac{|\langle 0 | A_0 | \pi \rangle|^2}{2 m_\pi} \exp[-m_\pi t] = \frac{f_\pi^2\, m_\pi}{2} \exp[-m_\pi t]$$

$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x},t)\, A^\dagger_0(\mathbf{0},0) \rangle \rightarrow$$

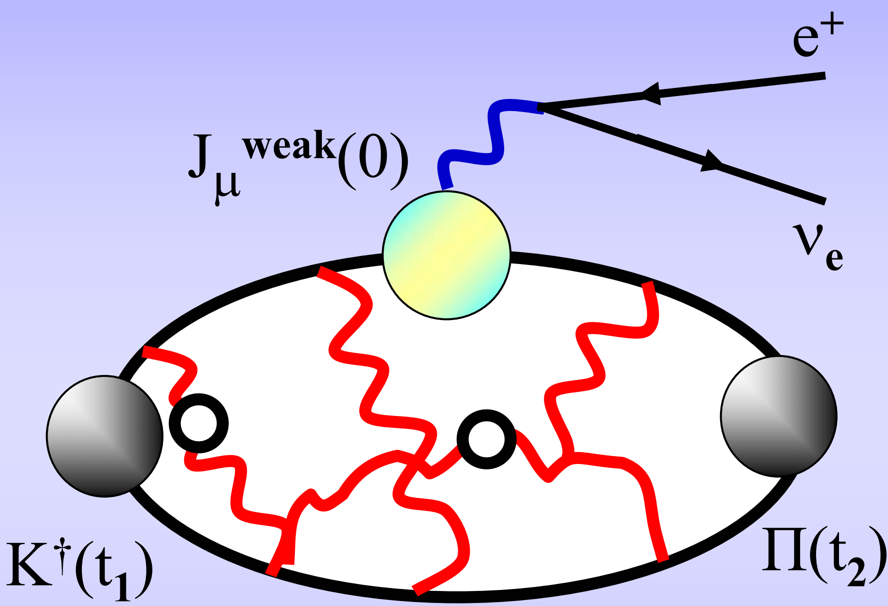
$$\rightarrow \frac{|\langle 0|A_0|\pi\rangle|^2}{2 m_\pi} \exp[-m_\pi t] = \frac{f_\pi^2 m_\pi}{2} \exp[-m_\pi t]$$

Hadron mass and $\langle 0|A|h\rangle$ matrix elements from the 2-point correlation function





3-point functions



$K^\dagger(t_1) = \sum_{\mathbf{x}} K^\dagger(\mathbf{x}, t_1) \exp[-i\mathbf{p_K x}]$
$\Pi(t_2) = \sum_{\mathbf{x}} \Pi(\mathbf{x}, t_2) \exp[+i\mathbf{p_\pi x}]$

$\langle \Pi(t_2)\, J_\mu^{\mathbf{weak}}(0)\, K^\dagger(t_1) \rangle \longrightarrow$

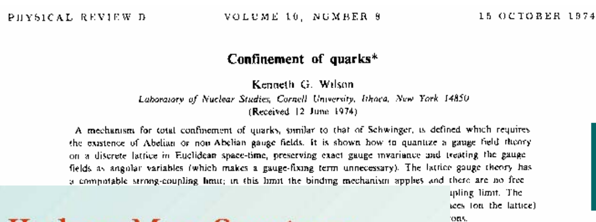
$$\frac{\langle 0|\Pi|\pi\rangle \langle K|\, K^\dagger|0\rangle \exp[-\,E_K t_1 -\, E_\pi t_2]}{(2E_K)\,(2E_\pi)} \times \langle \pi(\mathbf{p}_\pi) \mid J_\mu^{\mathbf{weak}}(0) \mid K(\mathbf{p}_K) \rangle$$

Also e.m. form factors, structure functions, etc

From S. Hashimoto
ICHEP 2004

30 years of lattice QCD

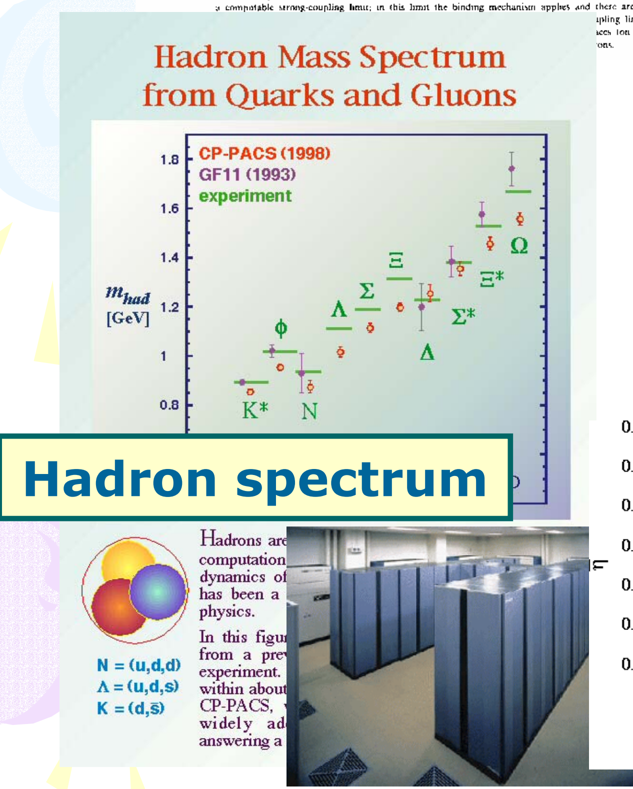
K. Wilson (1974)

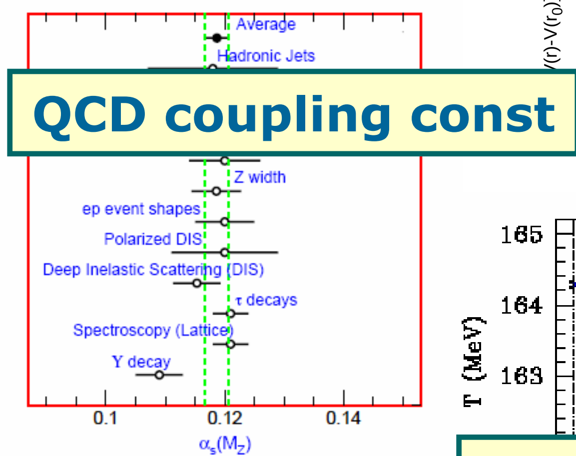

PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

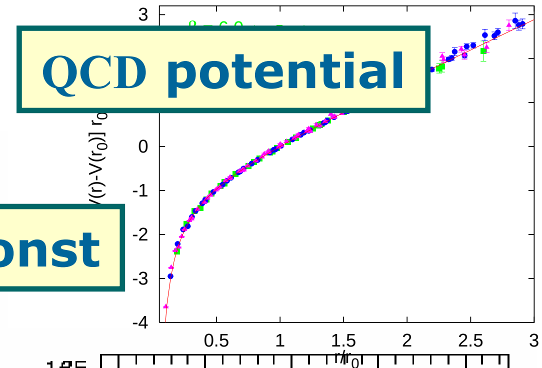
Confinement of quarks*

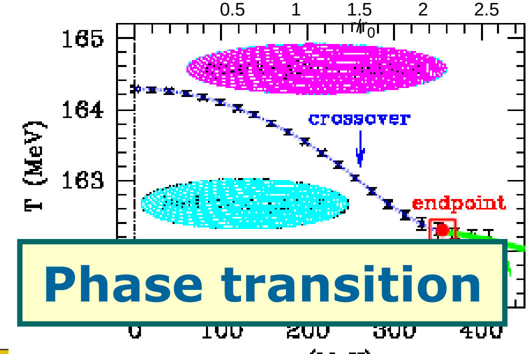
Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

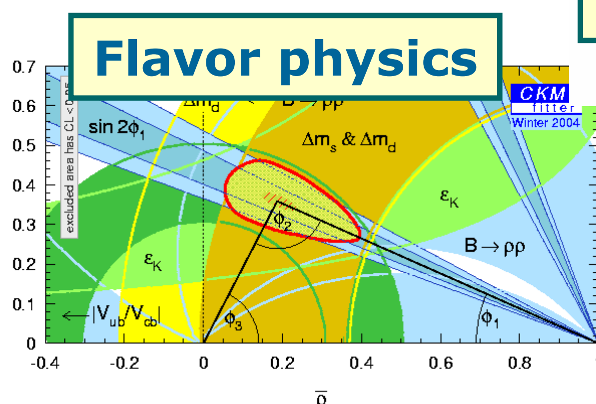
A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free

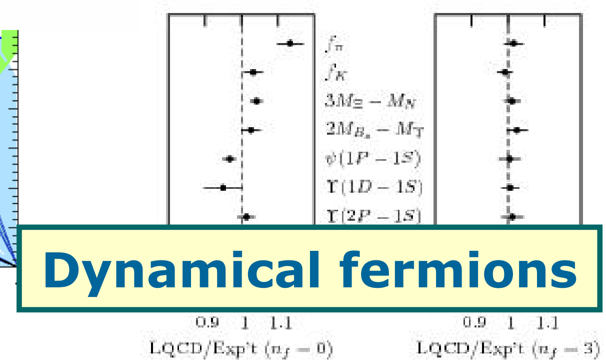












Hadronic matrix elements from Lattice QCD

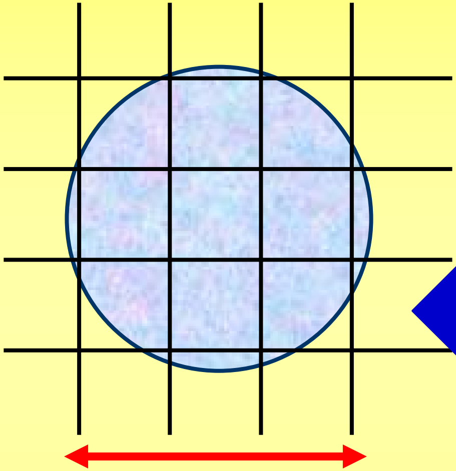
- **Leptonic decay constants:** f_π, f_K, f_D, f_{D_s}, f_B, f_{B_s}, f_ρ, ...
- **Electromagnetic form factors:** $F_\pi(Q^2)$, $G_M(Q^2)$, ...
- **Semileptonic form factors:** $K \to \pi$; $D \to K, K^*, \pi, \rho$; $B \to D, D^*, \pi, \rho$; $B \to K^* \gamma$; ...
- **B-parameters:** $\langle K^0 \,|\, Q^{\Delta S=2} \,|\, \overline{K}^0 \rangle$, $\langle B^0 \,|\, Q^{\Delta B=2} \,|\, \overline{B}^0 \rangle$
- **Weak decays:** $\langle \pi \,|\, Q^{\Delta S=1} \,|\, K \rangle$, $\langle \pi\, \pi \,|\, Q^{\Delta S=1} \,|\, K \rangle$

etc. etc. etc. ...

Lattice QCD is really a powerful approach ...

... BUT INVOLVES SYSTEMATIC ERRORS

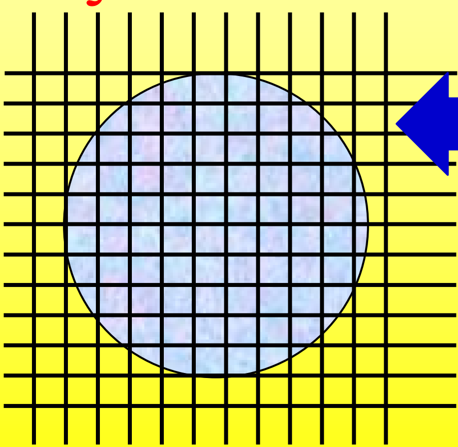
DISCRETIZATION ERRORS
(THE ULTRAVIOLET PROBLEM)



$\xi = 1/m$

$G_H(t) \sim \exp(-m_H t) \Longrightarrow \xi = 1/m$ is the correlation length (and the size of the hadron)

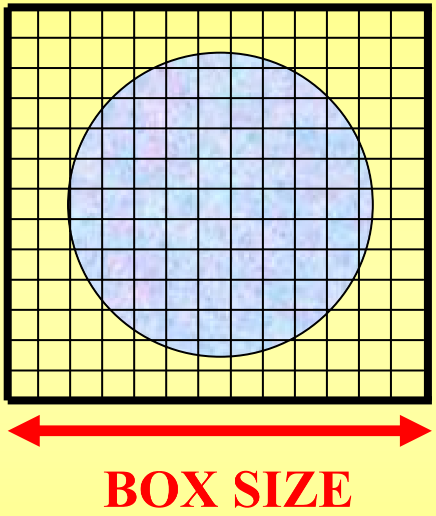
If $\xi \sim a \Longrightarrow ma \sim 1$ the size of the object is comparable to the lattice spacing



If $\xi \gg a \Longrightarrow ma \rightarrow 0$ the size of the object is much larger than the lattice spacing

$$C_{LATT} = C_{CONT}[1 + O(am, ap, a\Lambda_{QCD})]$$

FINITE VOLUME EFFECTS
(THE INFRARED PROBLEM)



$L >> \xi = 1/m$

to avoid finite size effects

For a large class of important physical amplitudes finite size effects are not really a problem:

$O(\exp[-\xi/L]) \longrightarrow L \geq 4 \div 5\ \xi$ is sufficient

But there are more problematic cases, e.g. non-leptonic decays…

QUENCHING ERROR

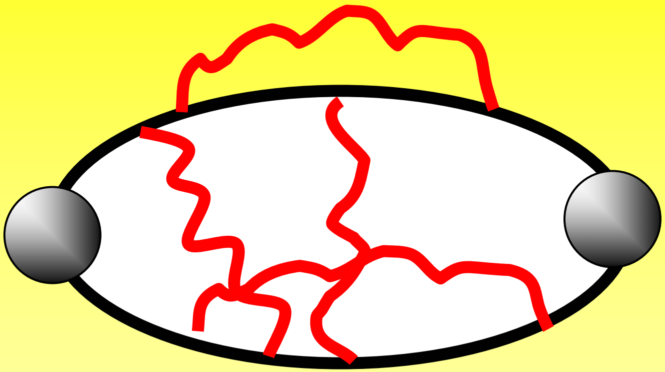
$$\int [dU][d\psi][d\bar{\psi}] \exp[-S_g - \bar{\psi} M \psi] =$$

$$\int [dU] \det M \exp[-S_g]$$

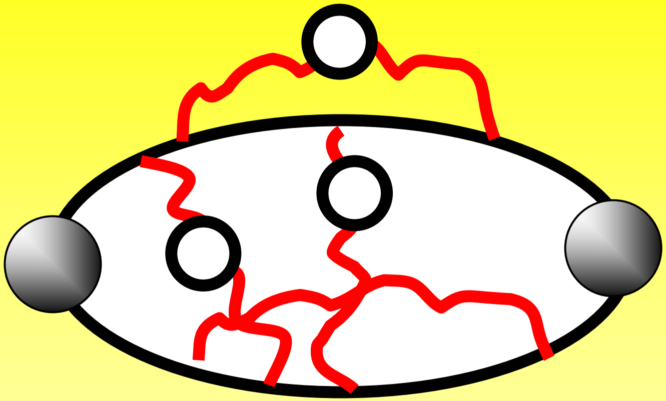
$$S^{eff} = S_g + \log M$$

$\det M = \text{cost}$

QUENCHED APPROXIMATION



QUENCHED



UNQUENCHED

ESTIMATES OF QUENCHING EFFECT:

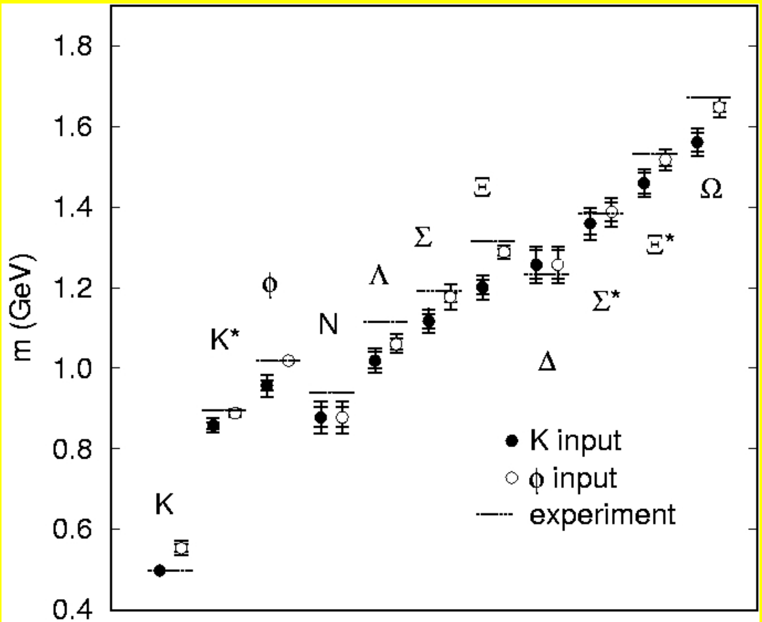
- hadron spectrum at $\leq$ 10% level
- kaon B-parameter estimated to be essentially the same
- effect on f_D and f_B at 10% level
- nucleon σ-term and polarized structure functions wrong
- problems with chiral logarithms

REAL UNQUENCHING STILL TO COME

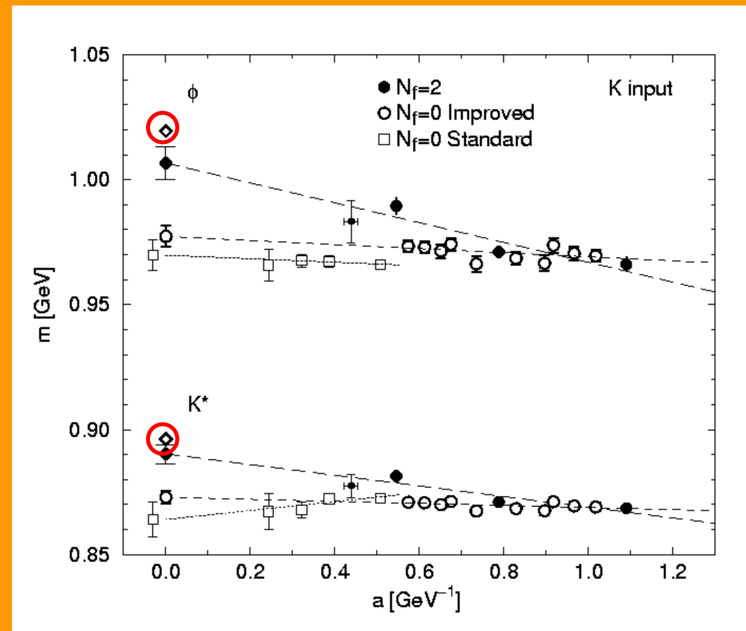
(QUARK MASSES TOO HEAVY)

LIGHT HADRON SPECTRUM FROM LATTICE QCD

CP-PACS Nf = 0



CP-PACS Nf = 2



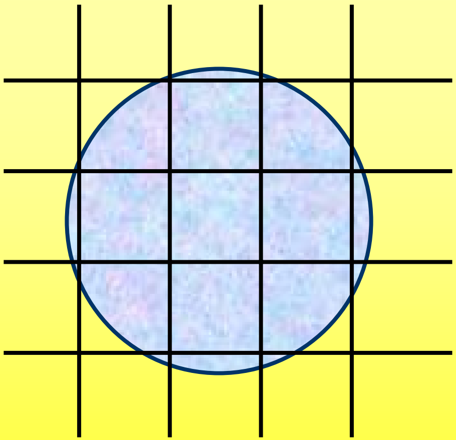
♦ IN THE QUENCHED CASE (Nf=0) THE AGREEMENT WITH THE EXPERIMENTS IS AT A 10% LEVEL (THE STATISTICAL AND SYSTEMATIC ACCURACY IS 3%).

♦ DEVIATIONS FROM EXPERIMENTS ARE CONSIDERABLY REDUCED IN FULL QCD (Nf=2).

EXTRAPOLATIONS IN QUARK MASSES

1) HEAVY QUARK MASSES

DISCRETIZATION ERRORS, THE ULTRAVIOLET PROBLEM



$\xi = 1/m$

REQUIREMENT

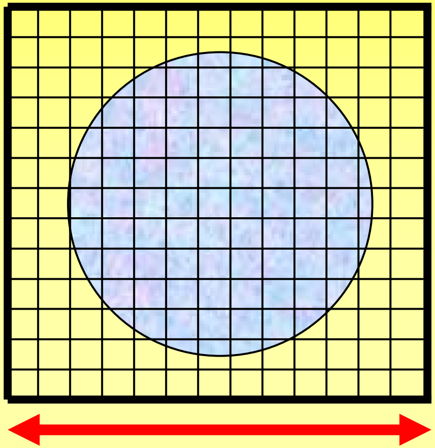
$$1/M_H >> a \Longleftrightarrow a\, M_H << 1$$

Typically $a^{-1} \sim 2 \div 5$ GeV

$m_{charm} \sim 1.3$ GeV $m_{charm}\, a \sim 0.3$

$m_{bottom} \sim 4.5$ GeV $m_{bottom}\, a \sim 1$

2) LIGHT QUARK MASSES



BOX SIZE

REQUIREMENT

$$1/M_\ell \ll L \iff L M_\ell \gg 1$$

BECAUSE OF THE LIMITATIONS IN COMPUTER RESOURCES VOLUMES CANNOT BE LARGE ENOUGH TO WORK AT THE PHYSICAL LIGHT QUARK MASSES

Typical quark mass $m_s/2 < m_q < m_s$

An extrapolation in m_{light} to the physical point is necessary. Chiral perturbation Theory (ChPT) may help in the extrapolation.

III. Lattice calculations of quark masses

LATTICE DETERMINATION OF QUARK MASSES

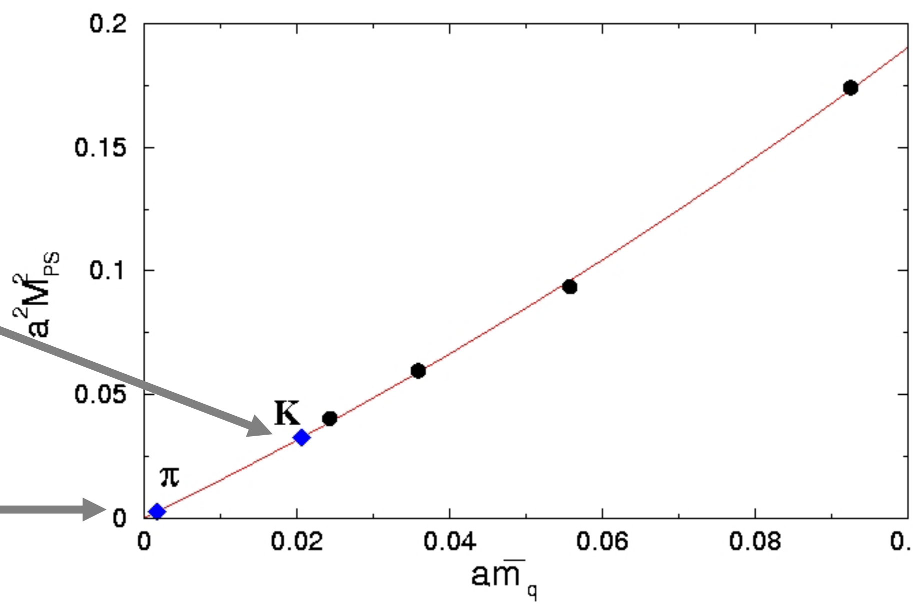
$$\hat{m}_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL
$M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY OR
NON-PERTURBATIVE METHODS

Extrapolation to
m = m_s

Extrapolation to
m = $m_{u,d}$



SYSTEMATIC ERRORS

$$m_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL
$M_H^{LATT} = M_H^{EXP}$

PERTURBATION
THEORY

O(a)

O(α^2)

CONTINUUM
EXTRAPOLATION
AND
IMPROVED ACTIONS

NON-PERTURBATIVE
RENORMALIZATION

SYSTEMATIC ERRORS

$$m_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL $M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY

$O(a)$

$O(\alpha^2)$

CONTINUUM EXTRAPOLATION AND IMPROVED ACTIONS

NON-PERTURBATIVE RENORMALIZATION

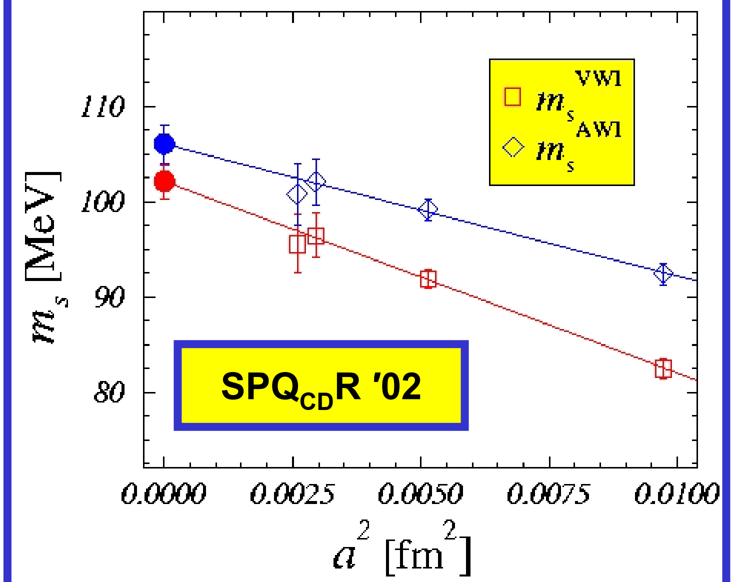
DISCRETIZATION EFFECTS

$$Q(a)_{LATT} = Q_{PHYS} + a\, Q_1 + a^2\, Q_2 + \ldots$$

$Q_1 = 0$ for improved actions

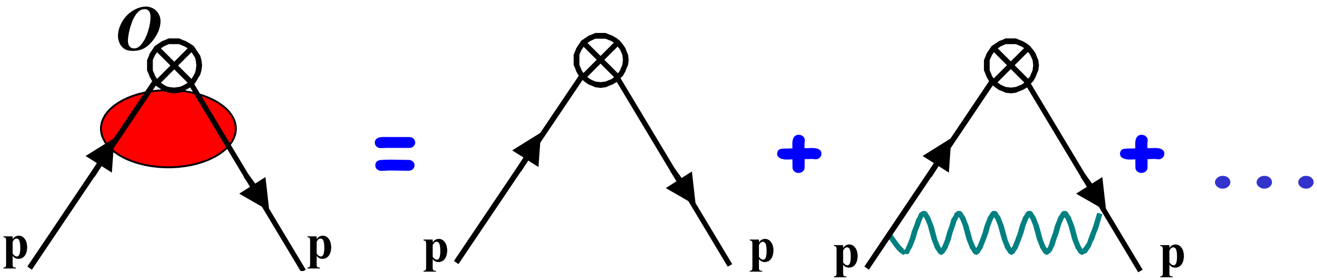
DISCRETIZATION EFFECTS CAN BE REDUCED BY:

1. USING AN IMPROVED ACTION
2. EXTRAPOLATING TO THE CONTINUUM LIMIT



NON-PERTURBATIVE RENORMALIZATION

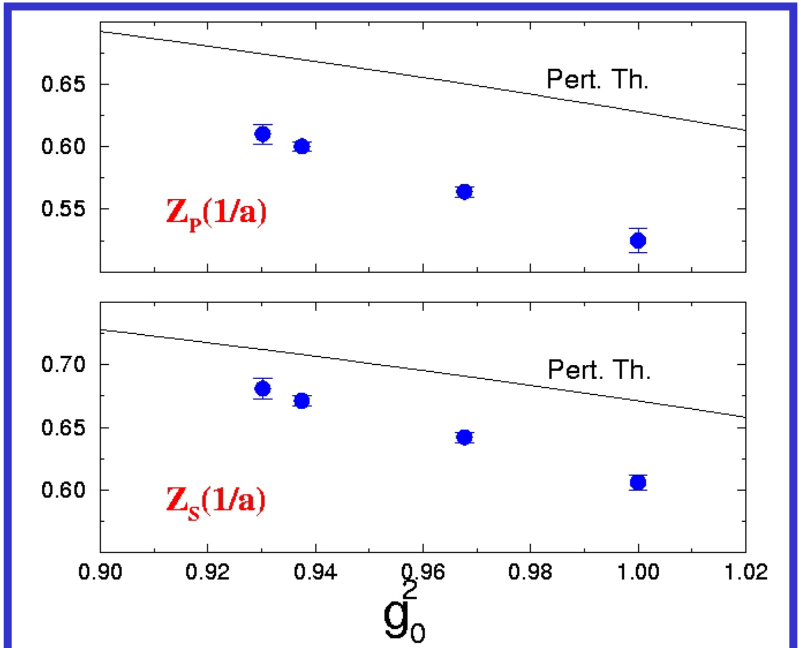
THE RI-MOM METHOD



$\Gamma_O(p^2)$

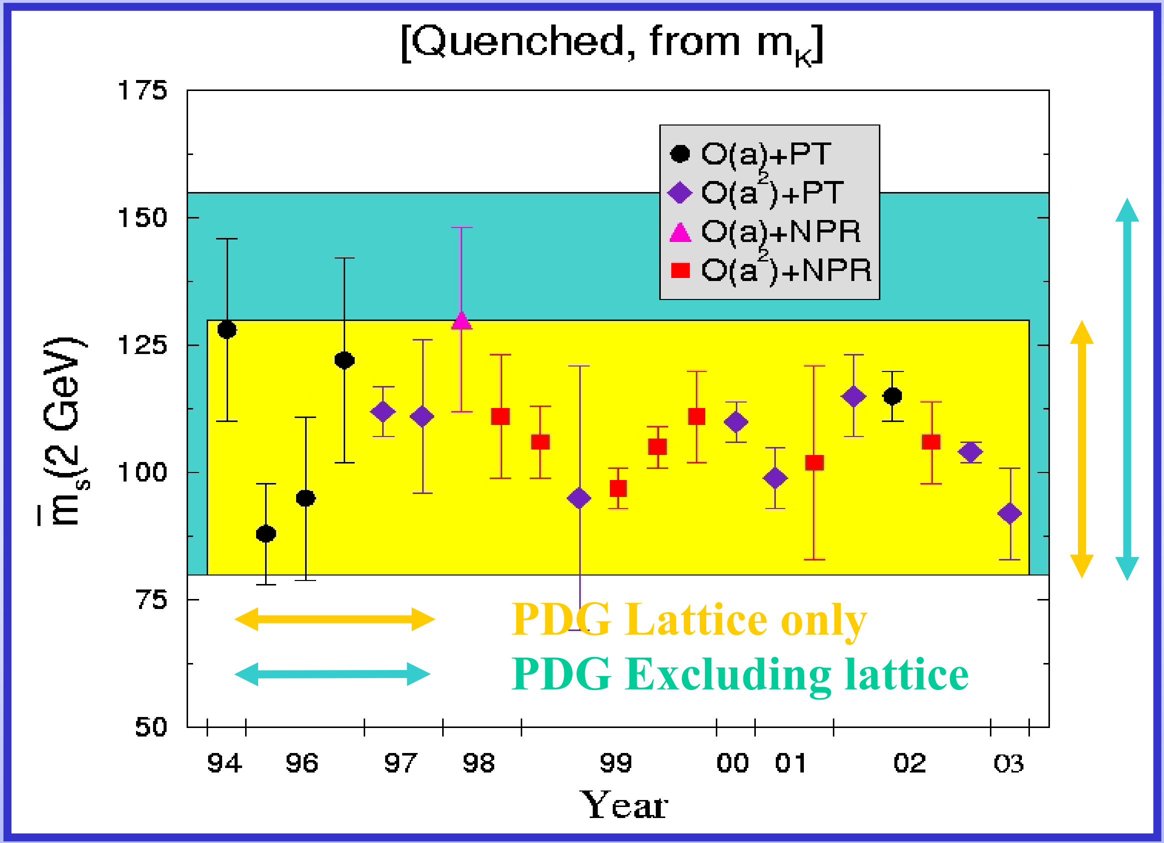
The (non-perturbative) renormalization condition:

$$Z_O(a\mu)\, \Gamma_O(p^2)|_{p^2=\mu^2} = \Gamma_{\text{Tree-Level}}$$



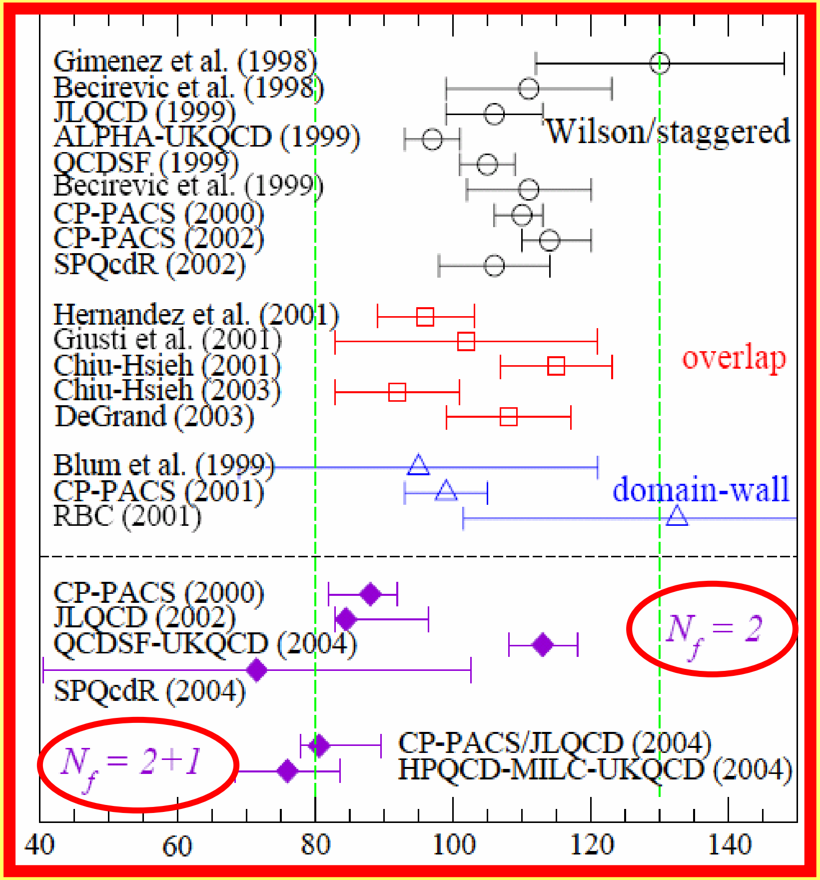
THE STRANGE QUARK MASS

History of Lattice Calculations



References: • **1994:** Allton et al. [FIRST NLO CALCULATION] • **1996:** LANL, FNAL, AGGR • **1997:** QCDSF, APETOV • **1998:** GGRT, APE • **1999:** JLQCD, RBC, ALPHẠUKQCD, QCDSF, APE • **2000:** CP-PACS • **2001:** CP-PACS, GHR • **2002:** C+H, CP-PACS, SPQ$_{\mathrm{CD}}$R, JLQCD • **2003:** C+H

THE STRANGE QUARK MASS
RECENT RESULTS



From S. Hashimoto
ICHEP 2004

N.B. some of the systematic errors are not taken into account in the error bars

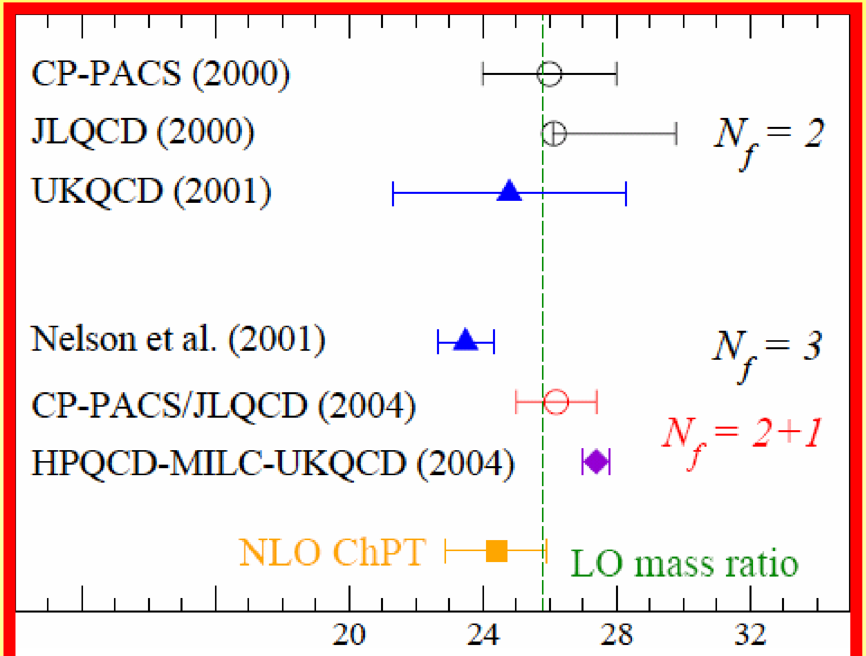
m_s seems to become lower for the see quark effects

$$\overline{m}_s\,(2\ \text{GeV}) = (105 \pm 15 \pm 20)\ \text{MeV}$$

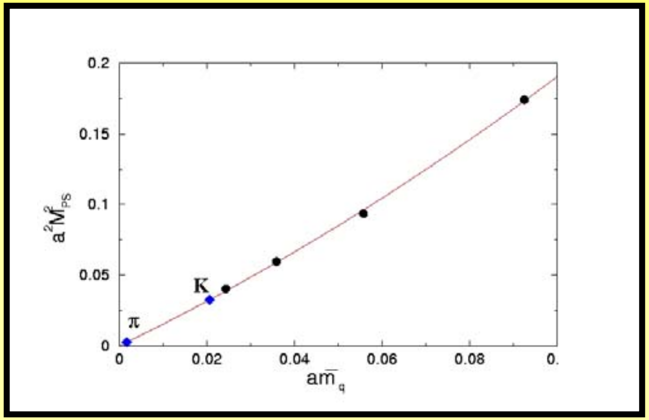
PDG LATTICE AVERAGE

THE AVERAGE UP/DOWN QUARK MASS

From S. Hashimoto ICHEP 2004



$2\, m_s/(m_u+m_d)$



It can be computed by assuming no major surprises in the chiral extrapolation

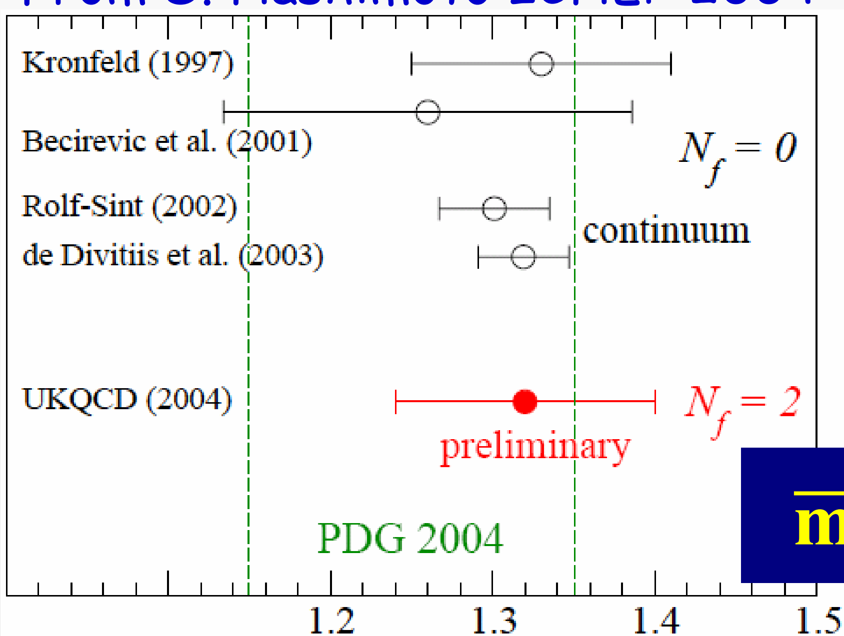
$$(\overline{m}_u + \overline{m}_d)/2 = (4.2 \pm 0.6 \pm 0.8)\ \text{MeV} \qquad (\mu = 2\ \text{GeV})$$

HEAVY QUARKS:
1) THE CHARM QUARK MASS

$m_c \approx a^{-1}$ → Potentially large O(a) effects

Require fine lattices and continuum extrapolation

From S. Hashimoto ICHEP 2004



Note: $m_c \geq \Lambda_{QCD}$ → Large $1/m$ corrections in effective theories

PDG LATTICE AVERAGE

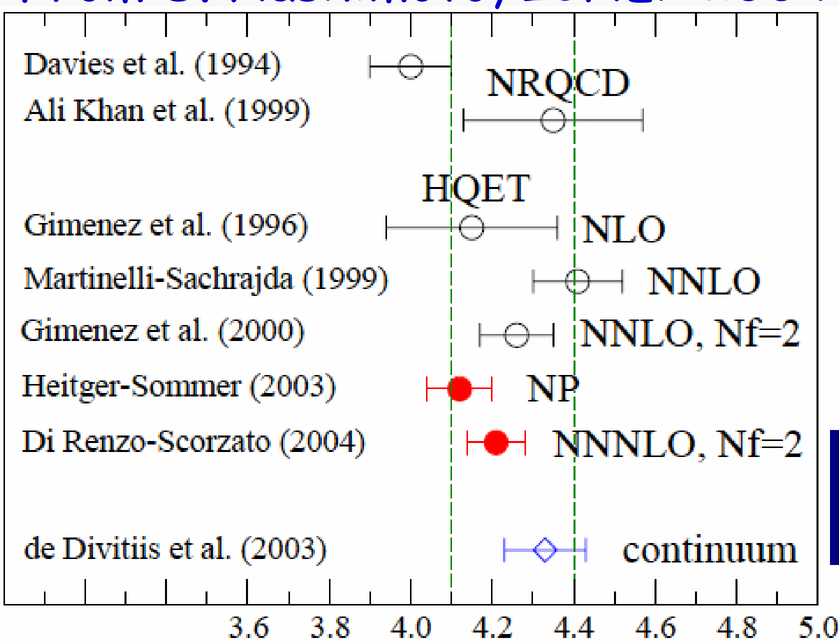
$$\overline{m}_c = (1.26 \pm 0.13 \pm 0.20)\ \mathrm{GeV}$$

2) THE BOTTOM QUARK MASS

$m_b \gg a^{-1}$ → The b quark cannot be directly simulated on the lattice

$m_b \gg \Lambda_{QCD}$ → Effective theories on the lattice: HQET, NRQCD, …

From S. Hashimoto, ICHEP 2004



N.B. In effective theories on the lattice perturbative renormalization effects are important

PDG LATTICE AVERAGE

$$\overline{m}_b = (4.26 \pm 0.15 \pm 0.15)\ \mathrm{GeV}$$

3

CKM MATRIX, UNITARITY AND CP VIOLATION

I. First row, unitarity and the Cabibbo angle

II. The Unitarity Triangle Analysis

III. Search for New Physics

I. First row, unitarity and the Cabibbo angle

III. Search for New Physics

The most stringent unitarity test:
$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

PDG 2002 quotes a 2.2 σ deviation from unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0043 \pm 0.0019$$

$|V_{ud}| = 0.9734 \pm 0.0008$ Superallowed and neutron β-decay

$|V_{us}| = 0.2196 \pm 0.0026$ $K \to \pi l \nu$

$|V_{ub}| = 0.0036 \pm 0.0010$ $b \to u$ inclusive and exclusive

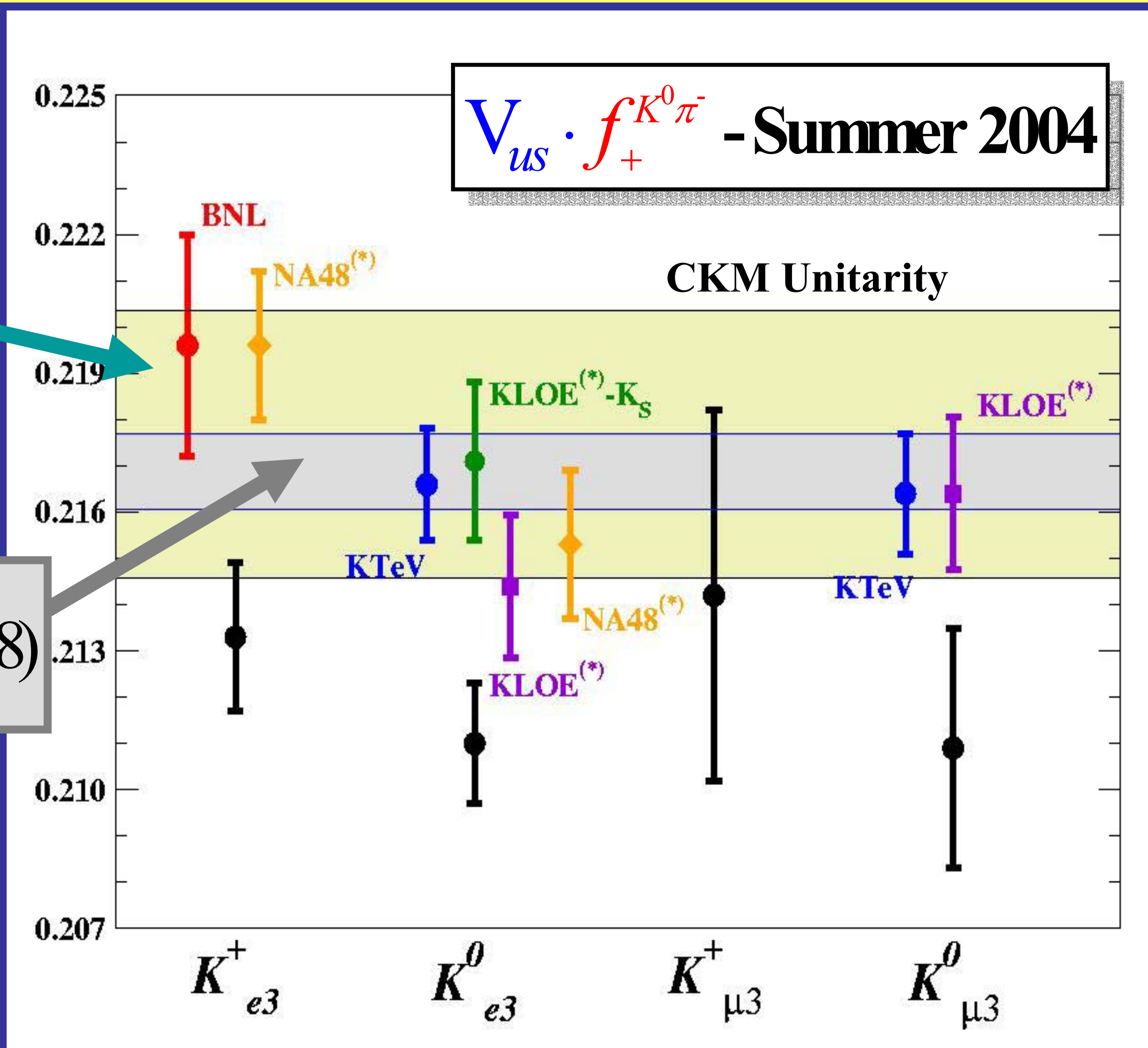
$|V_{us}|$ relies on old experimental and theoretical results of Kl3 ⟹ Examine Kl3 decays

The NEW experimental results

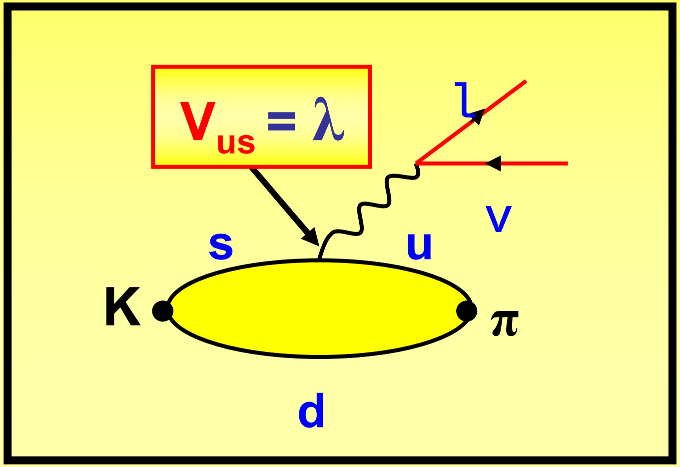
From F. Mescia, ICHEP 2004

$$V_{us}^{uni} \cdot f_+^{LR} = 0.2175(29)$$

$$\left(V_{us} \cdot f_+^{K^0\pi^-} \right)_{KTeV\text{-}E865}^{Exp\text{-}Average} = 0.2169(8)$$



Theoretical description



$$\Gamma_{K\ell 3} = C^{\ell} \frac{G_F^2 |V_{us}|^2 M_K^5}{192\,\pi^3} S_{EW} (1+\delta_K^{\ell})\, I_K^{\ell}\, f_+(0)^2$$

THE LARGEST UNCERTAINTY IS DUE TO THE FORM FACTOR AT ZERO MOMENTUM TRANSFER: $f_+(0)$

ChPT:

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation

$f_2 = -\,0.023$
Independent of L_i
(Ademollo-Gatto)

THE LARGEST UNCERTAINTY

"Standard" estimate:

Leutwyler, Roos (1984)
(QUARK MODEL)

$f_4 = -0.016 \pm 0.008$

ChPT: The complete $O(p^6)$ calculation

Post, Schilcher (2001), Bijnens, Talavera (2003)

$$\boxed{f_4 = \Delta_{loops}(\mu) - \frac{8}{F_\pi^4}\left[C_{12}(\mu) + C_{34}(\mu)\right](M_K^2 - M_\pi^2)^2}$$

$C_{12}(\mu)$ and $C_{34}(\mu)$ can be determined from the slope and the curvature of the scalar form factor. Experimental data, however, are not accurate enough.

... and models

Jamin et al., $f_4^{LOC} = -0.018 \pm 0.009$ [Coupled channel dispersive analysis]

Cirigliano et al., $f_4^{LOC} = -0.012$ [Resonance saturation]

Cirigliano et al., $f_4^{LOC} = -0.016 \pm 0.008$ [QM, Leutwyler and Roos]

μ = ??? $\Delta_{loops}(1\text{GeV}) = 0.004$ $\Delta_{loops}(M_\rho) = 0.015$ $\Delta_{loops}(M_\eta) = 0.031$

Cirigliano et al., $f_+^{K^0\pi^-}(0) = 0.981 \pm 0.010$

The Lattice QCD calculation

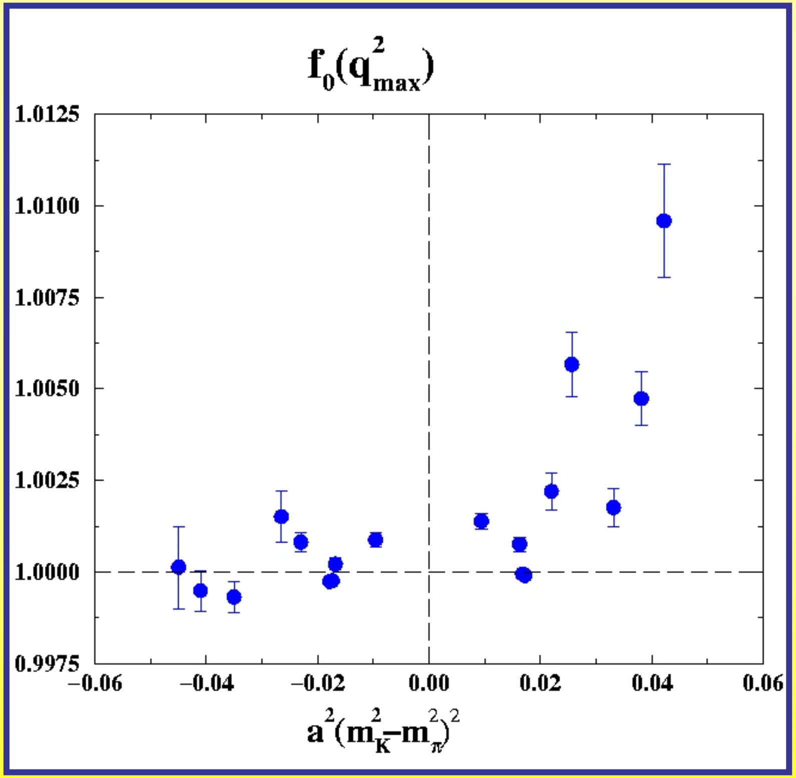
1) Evaluation of $f_0(q^2_{MAX})$

The basic ingredient is a double ratio of correlation functions:

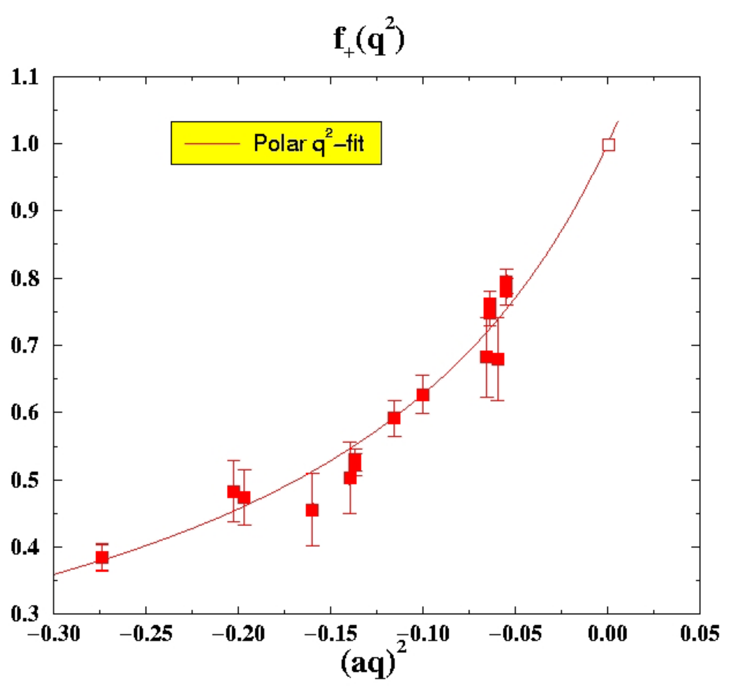
$$R = \frac{\langle \pi | \bar{s}\gamma_0 u | K \rangle \langle K | \bar{u}\gamma_0 s | \pi \rangle}{\langle \pi | \bar{u}\gamma_0 u | \pi \rangle \langle K | \bar{s}\gamma_0 s | K \rangle}$$

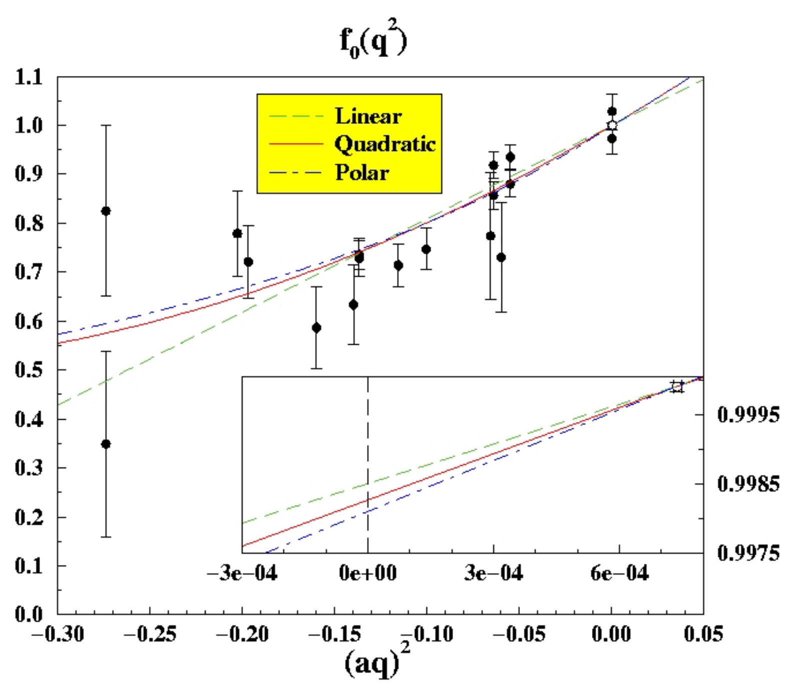
$$= \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} f_0(q^2_{max})^2$$

[FNAL for B->D*]



2) Extrapolation of $f_0(q^2_{MAX})$ to $f_0(0)$



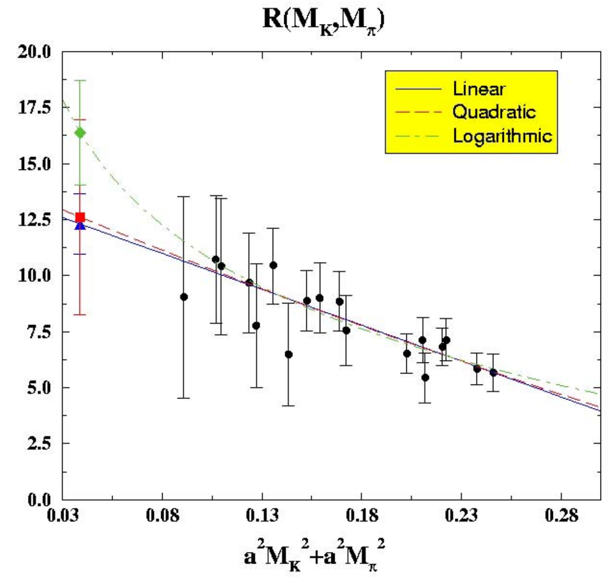


Comparison of polar fits:

LQCD: $\lambda_+ = (25 \pm 2)\,10^{-3}$ $\lambda_0 = (12 \pm 2)\,10^{-3}$

KTeV: $\lambda_+ = (24.11 \pm 0.36)\,10^{-3}$ $\lambda_0 = (13.62 \pm 0.73)\,10^{-3}$

3) Chiral extrapolation



$$R = \frac{f_+(0) - 1 - f_2^{QUEN}}{(M_K^2 - M_\pi^2)^2}$$

Computed in Quenched-ChPT

The dominant contributions to the systematic error come from the uncertainties on the q^2 and mass dependencies of the form factor

$$f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005_{stat} \pm 0.007_{syst}$$

[Quenching error is not included]

In agreement with LR!!

II. The Unitarity Triangle Analysis

UTfit Collaboration

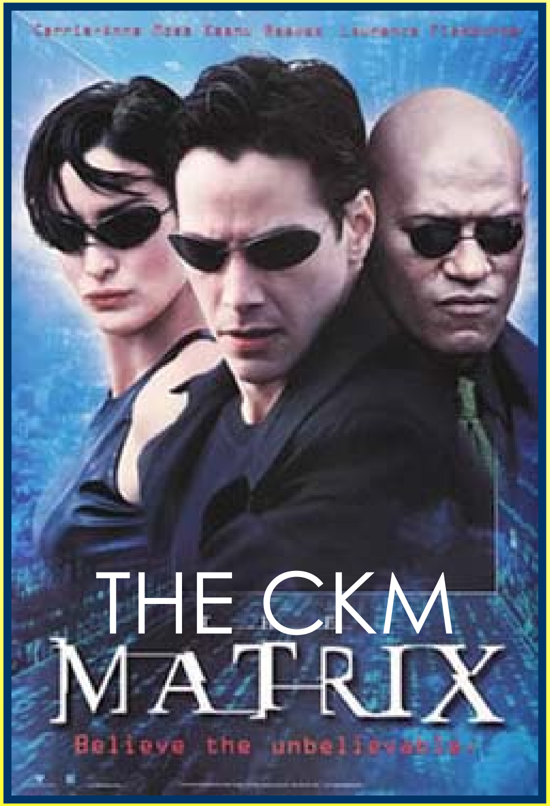
M.Bona, **M.Ciuchini**, **E.Franco**,

V.L., **G.Martinelli**, **F.Parodi**,

M.Pierini, **P.Roudeau**, **C.Schiavi**,

L.Silvestrini, **A.Stocchi**

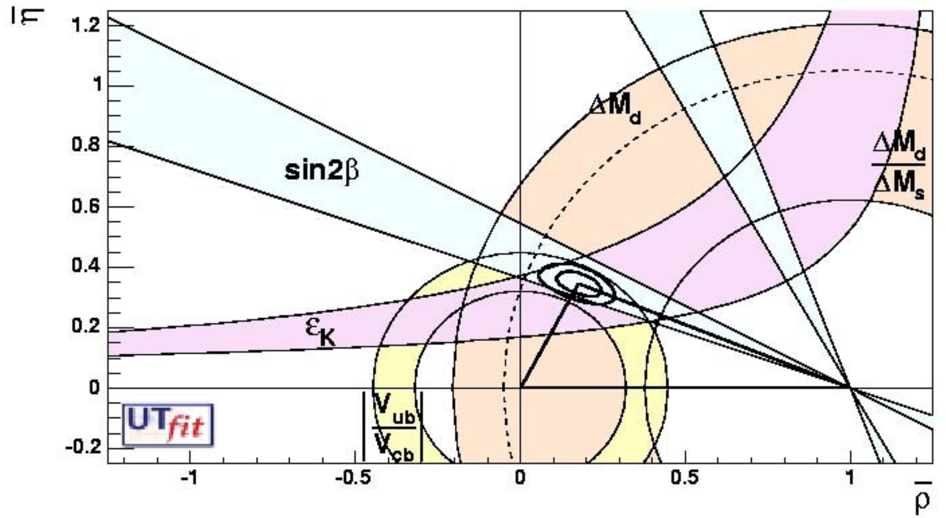
Roma, Genova, Torino, Orsay



www.utfit.org

THE UNITARITY TRIANGLE ANALYSIS

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

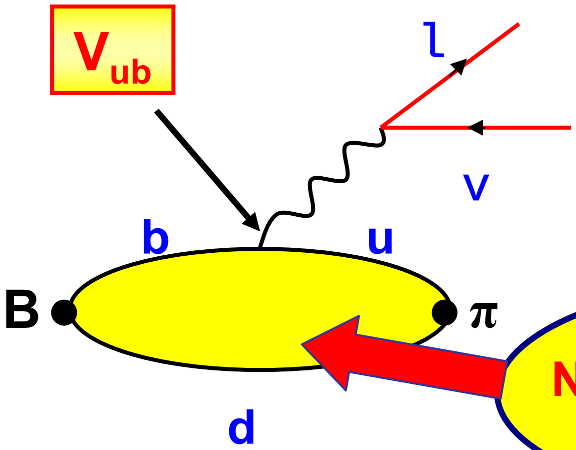


5 CONSTRAINTS
2 PARAMETERS

Hadronic Matrix Elements from LATTICE QCD

$(b\to u)/(b\to c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$f_+, F(1), \ldots$
ε_K	$\bar{\eta}\,[(1-\bar{\rho}) + P]$	B_K
Δm_d	$(1-\bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1-\bar{\rho})^2 + \bar{\eta}^2$	ξ
$A(J/\psi K_S)$	$\sin 2\beta(\bar{\rho}, \bar{\eta})$	—

V_{ub} FROM B-MESON SEMILEPTONIC DECAYS

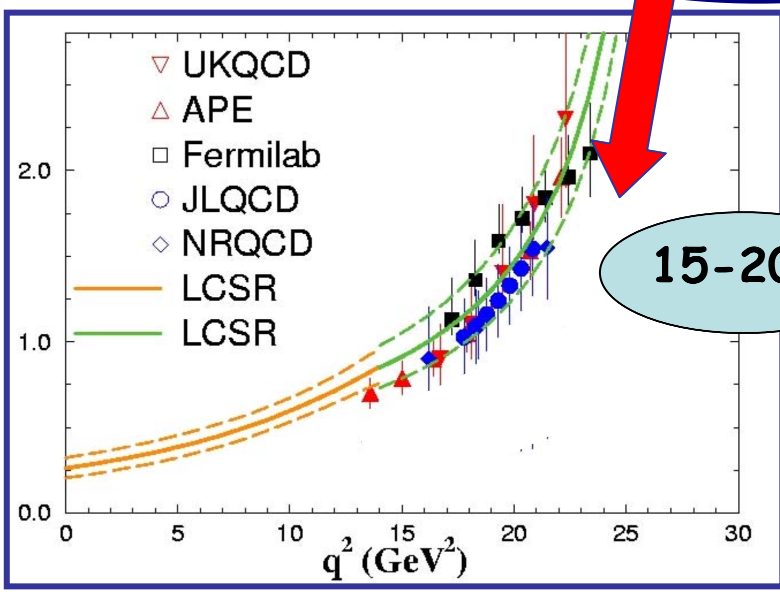


$$\Gamma(B \to \pi l \nu) = \frac{G_F^2 |V_{ub}|^2}{192\,\pi^3} \int dq^2\, \lambda(q^2)^{3/2}\, |f_+(q^2)|^2$$

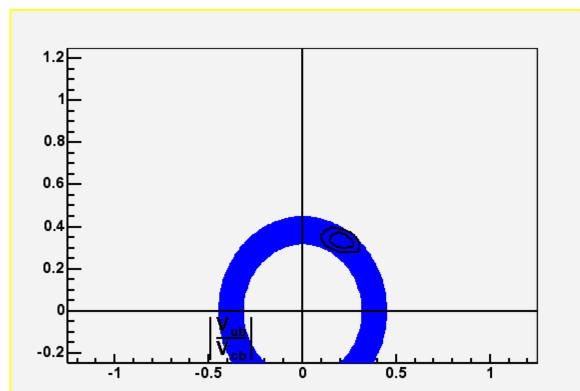
NON-PERTURBATIVE PHYSICS

<u>EXPERIMENTS:</u>

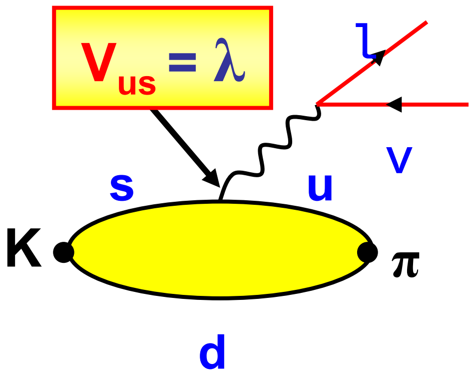
CLEO, BaBar, Belle, …

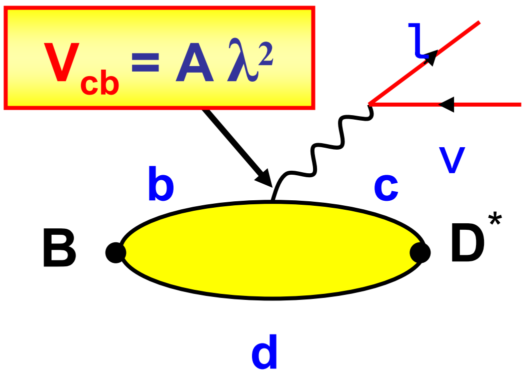


15-20%



PRECISION FLAVOUR PHYSICS ON THE LATTICE



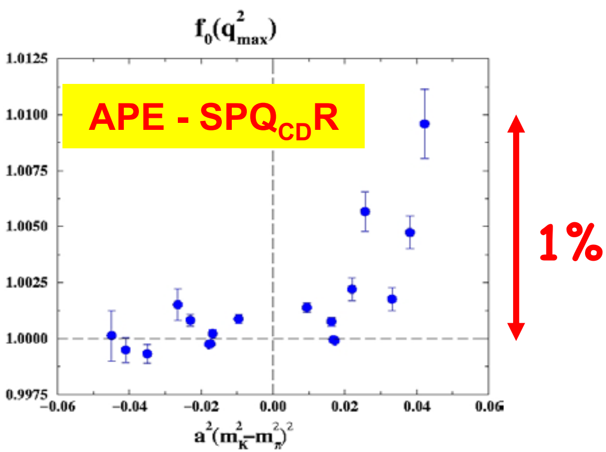


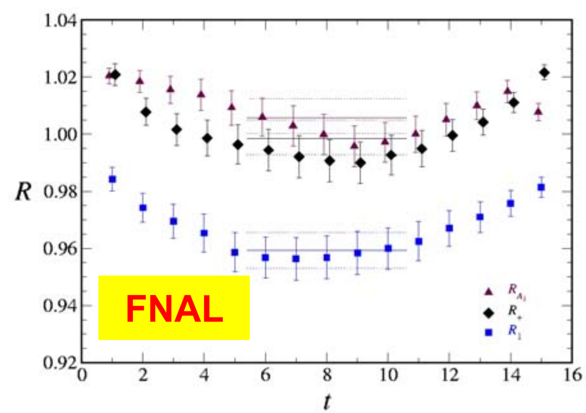
$f_+(0) = 1 - O(m_s - m_u)^2$

Ademollo-Gatto theorem

$F_{B\to D^*}(1) = \eta_A\,[1 - O(1/m_b, 1/m_c)^2]$

Luke theorem





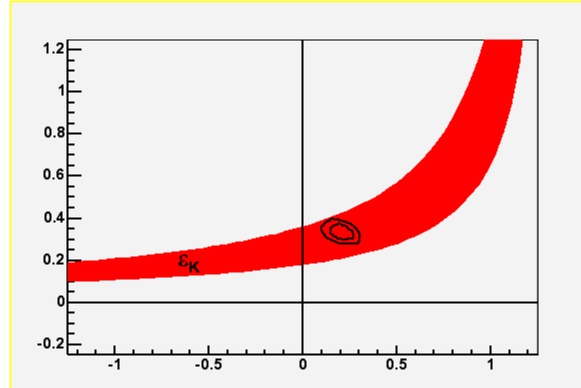
$f_+(0) = 0.960 \pm 0.005 \pm 0.007$

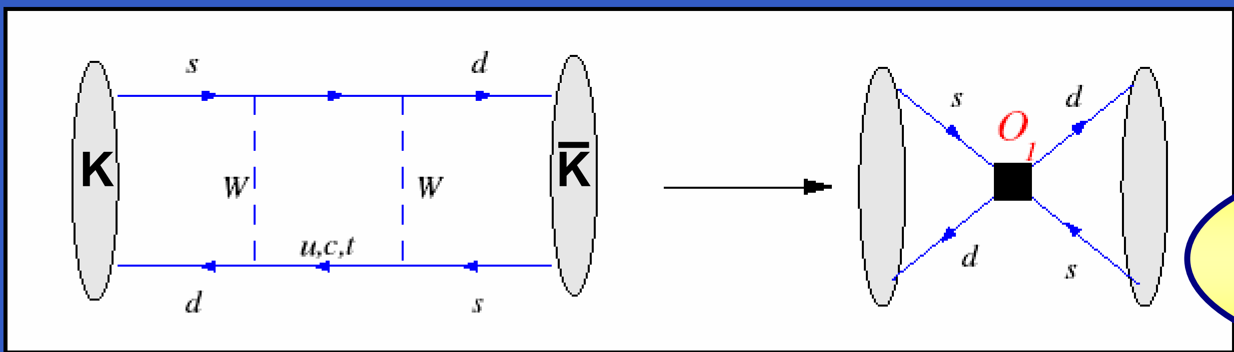
$F_{B\to D^*}(1) = 0.913\,^{+0.024}_{-0.017}\,^{+0.017}_{-0.030}$

$K - \bar{K}$ Mixing: ε_K and B_K

$$K_L \sim \underbrace{(K^0 - \bar{K}^0)}_{CP=-1} + \varepsilon_K \underbrace{(K^0 + \bar{K}^0)}_{CP=+1}$$

CP Violation





NON-PERTURBATIVE PHYSICS

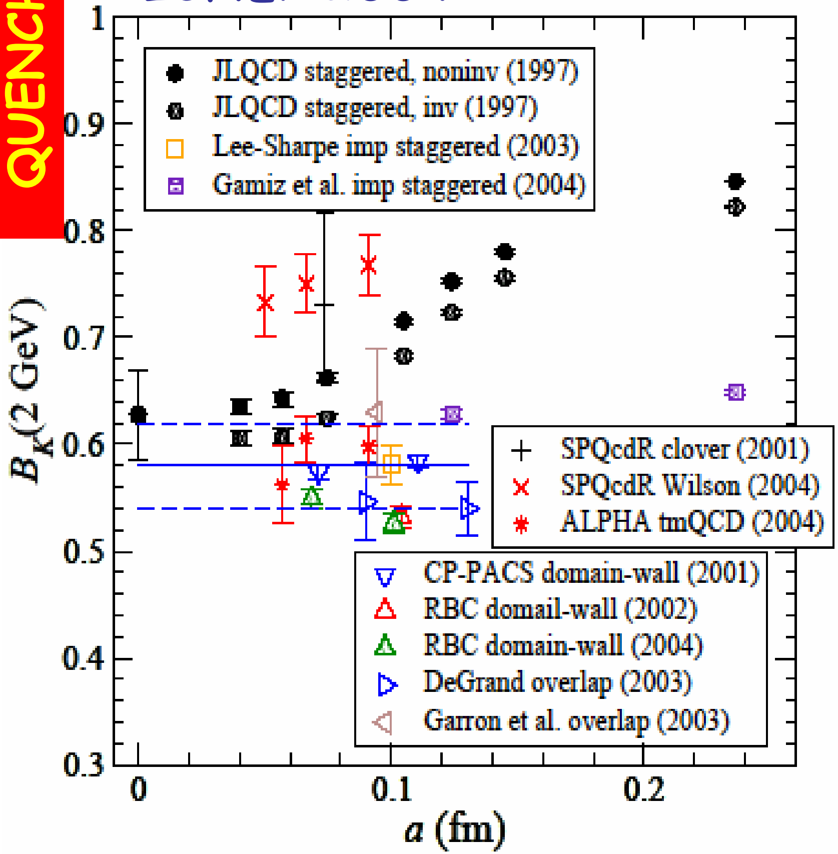
$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle$$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

Lattice Results for B_K

QUENCHED

From S. Hashimoto
ICHEP 2004



- ✓ High level of accuracy
- ✓ Discretization effects not negligible
- ✓ Estimate of quenching error from ChPT ≤ 15% (Sharpe)

QUENCHING ERROR

$\hat{B}_K = 0.86 \pm 0.06 \pm 0.14$

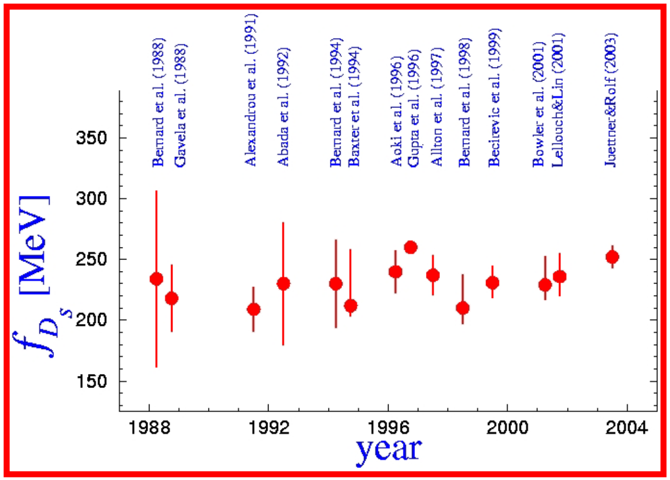
LATTICE PREDICTION (!) $\hat{B}_K = 0.90 \pm 0.20$ [Gavela et al., 1987]

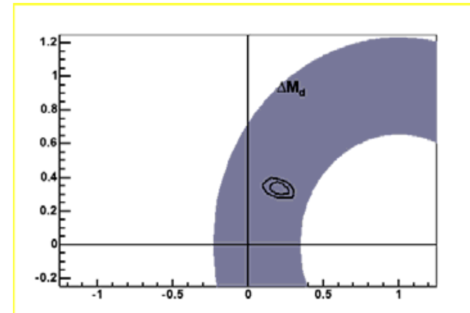
$B_{B_{d/s}} - \bar{B}_{B_{d/s}}$ Mixing: $f_{B_{d/s}}\sqrt{B_{B_{d/s}}}$

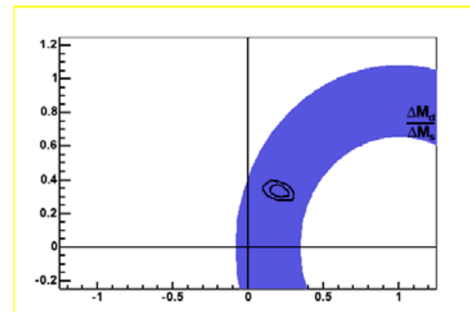
THE D_s-MESON DECAY CONSTANT

A long history of lattice calculations…

QUENCHED







$f_{Ds}^{Nf=2} / f_{Ds}^{Nf=0} = 1.08 \pm 0.05$ (CP-PACS,MILC)

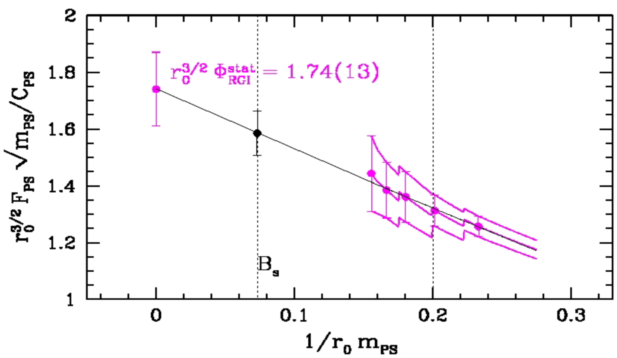
$f_{Ds} = 265 \pm 14 \pm 13$ MeV LQCD Average

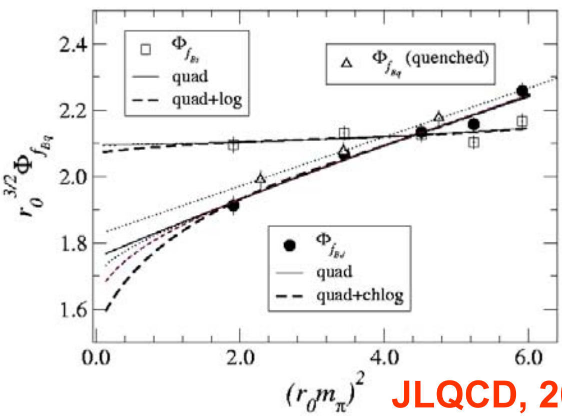
$f_{Ds} = 285 \pm 19 \pm 40$ MeV EXP. PDG 2002

New results from *CLEO-c*

From f_{Ds} to f_{Bd}

J.Heitger, EPS-HEP 2003





JLQCD, 2003

$m_Q \ll 1/a$

$m_\pi \gg 1/L$

- Extrapolations from $m_Q \sim m_c$ to m_b
- Effective theories: HQET, NRQCD, "FNAL", …
- Combine the two approaches
- Finite size approach , APE-Tov

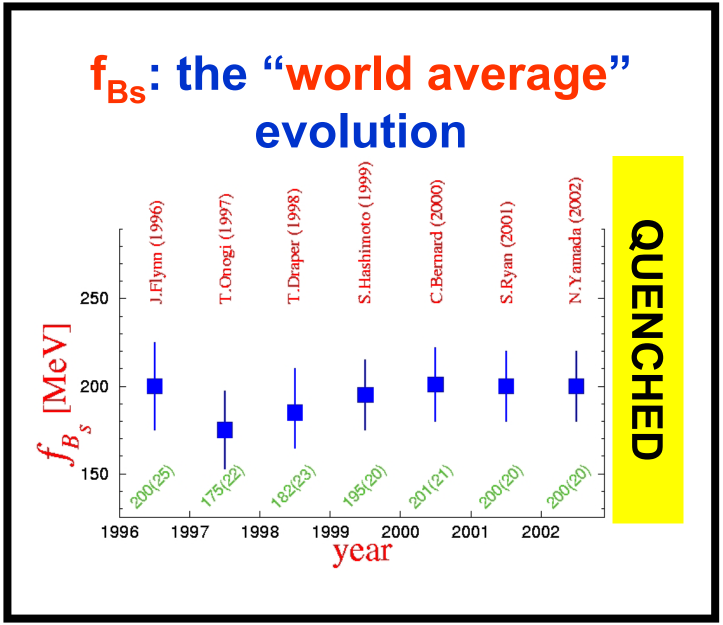
- Extrapolations from m_q to $m_{u,d}$ using ChPT as a guidance:

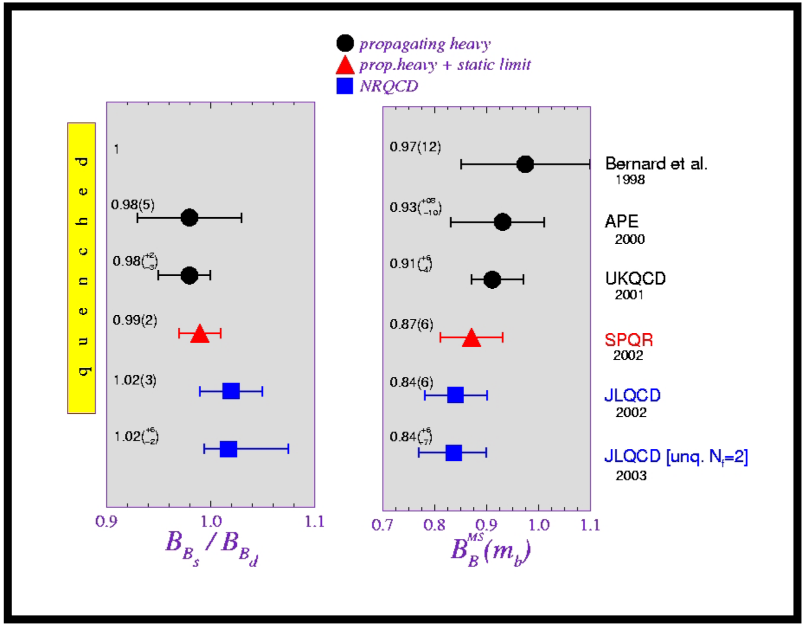
$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_{B_d}\sqrt{m_{B_d}}} = 1 + \frac{1+3\hat{g}^2}{4(4\pi f)^2}\,\chi logs + C$$

Orsay

Use $\dfrac{f_{B_s}/f_{B_d}}{f_K/f_\pi}$, Becirevic et al.

$B_{B_{d/s}} - \bar{B}_{B_{d/s}}$ Mixing: $f_{B_{d/s}}$ and $B_{B_{d/s}}$





$f_{Bs}^{Nf=2} / f_{Bs}^{Nf=0} = 1.12 \pm 0.05$ (CP-PACS,MILC)

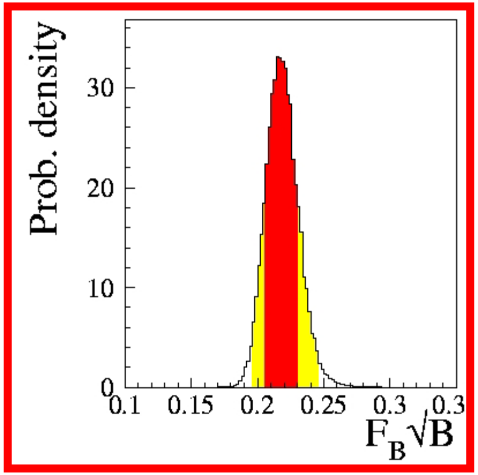
In other quantities (f_{Bs}/ f_{Bd}, B_{Bd}, B_{Bs}/ B_{Bd}) quenching effects are smaller

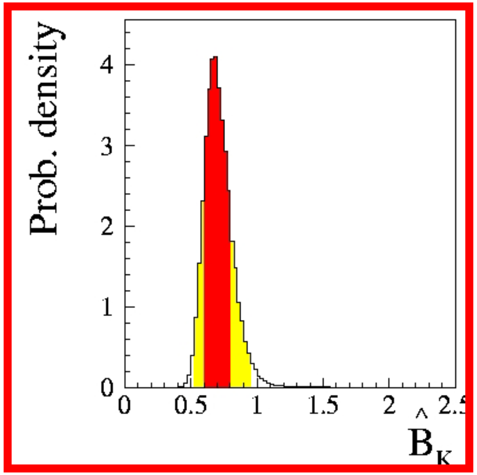
LATTICE AVERAGES

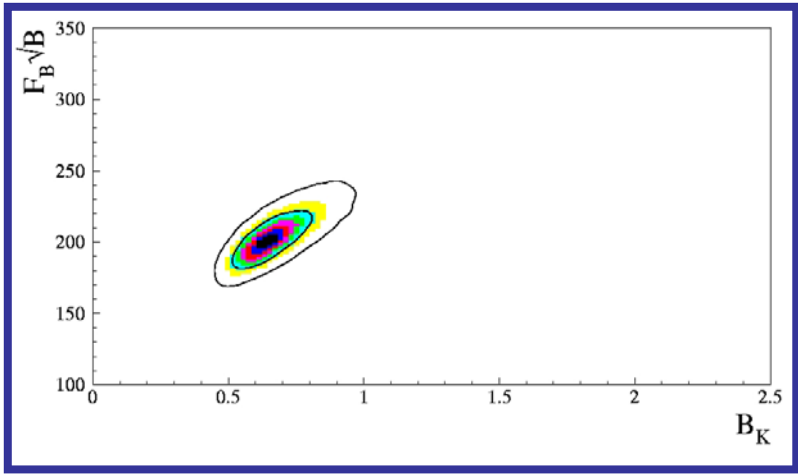
$$f_{Bs}\sqrt{B_{Bs}} = 276 \pm 38 \text{ MeV},$$

$$\xi = 1.24 \pm 0.04 \pm 0.06$$

Lattice QCD vs UT FITS

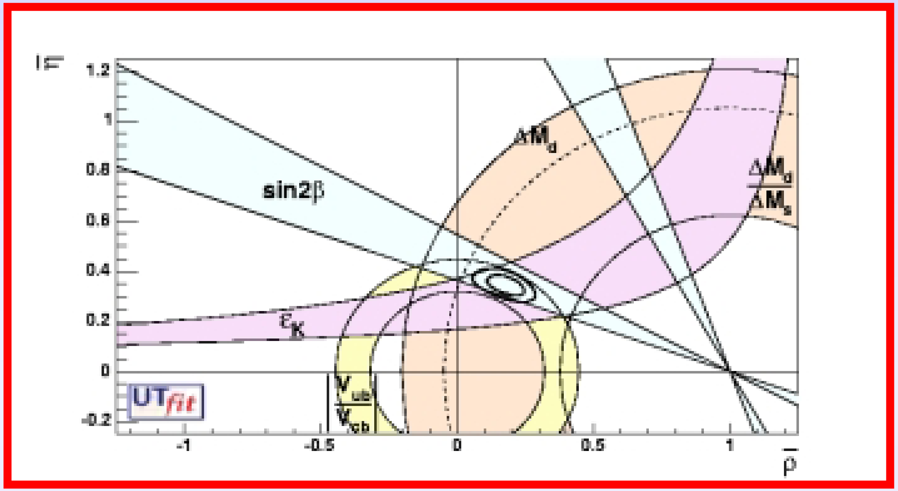






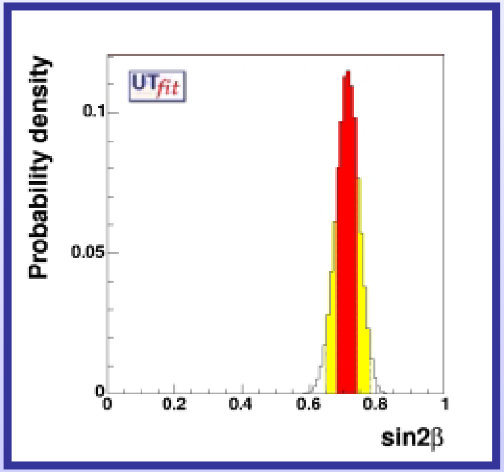
	LATTICE QCD	UT FIT
$f_B\sqrt{B_B}$	$223 \pm 33 \pm 12$ MeV	217 ± 12 MeV
B_K	$0.86 \pm 0.06 \pm 0.14$	0.71 ± 0.11

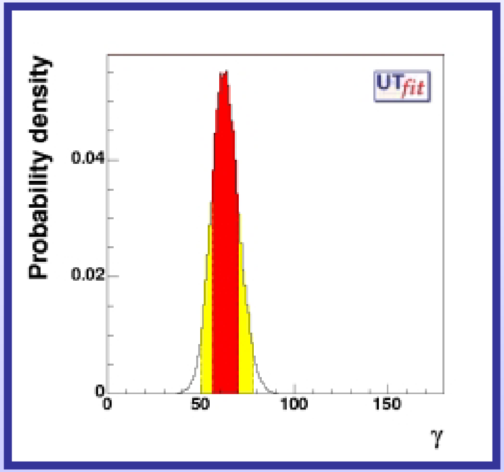
FIT RESULTS



$\bar{\rho}$ = 0.174 ± 0.048

$\bar{\eta}$ = 0.344 ± 0.027





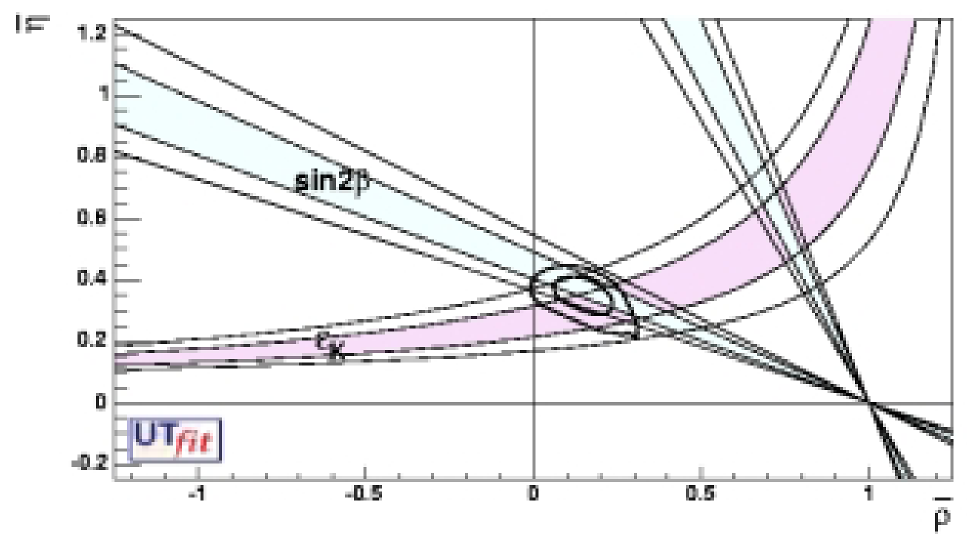
Sin2α = – 0.14 ± 0.25

Sin2β = 0.697 ± 0.036

γ = (61.9 ± 7.9)°

INDIRECT EVIDENCE OF CP VIOLATION

3 FAMILIES → - Only 1 phase - Angles from Sides

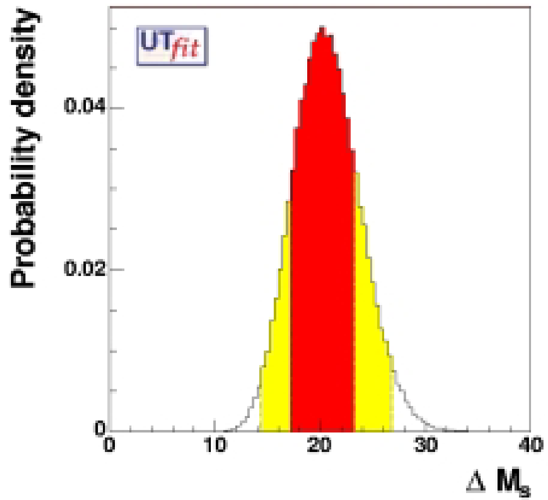
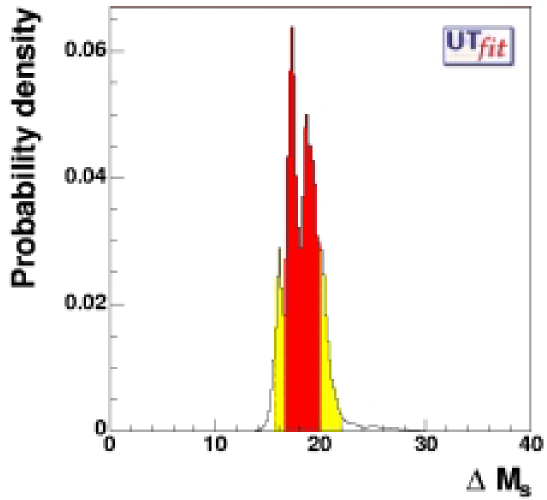


$\sin 2\beta_{\text{UT Sides}} = 0.685 \pm 0.047$

$\sin 2\beta_{J/\psi\, K_S} = 0.739 \pm 0.048$

Prediction (Ciuchini et al., 2000): **$\sin 2\beta_{\text{UTA}} = 0.698 \pm 0.066$**

Prediction for Δm_s

Δm_s NOT USED	WITH ALL CONSTRAINTS
 	
$\Delta m_s = (20.5 \pm 3.2)\ ps^{-1}$	$\Delta m_s = (18.0 \pm 1.6)\ ps^{-1}$

A measurement is expected at FERMILAB

IMPACT OF IMPROVED DETERMINATIONS

$B_K = 0.86 \pm 0.06 \pm \cancel{0.14}$

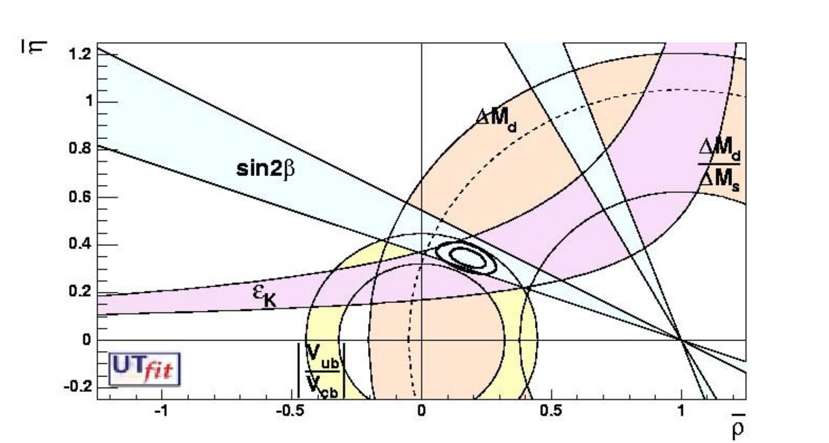
$f_{Bs}\sqrt{B_{Bs}} = 276 \pm 38$ MeV → 14

$\xi = 1.24 \pm 0.04 \pm \cancel{0.06}$

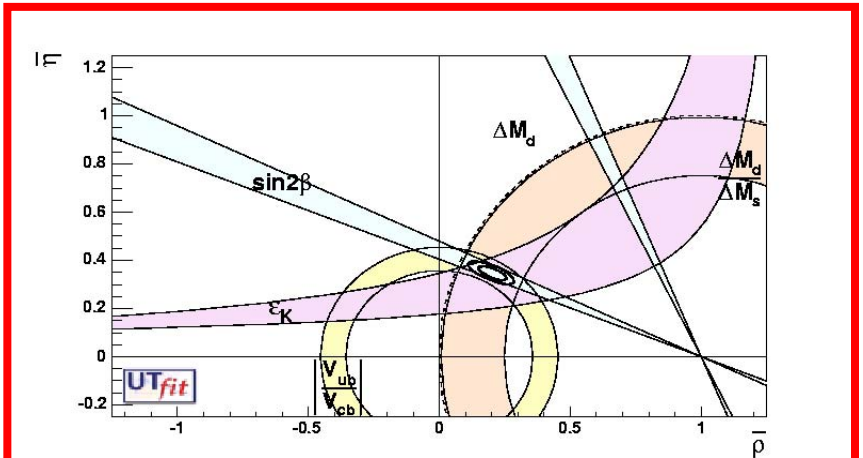
$\sin 2\beta = 0.734 \pm 0.054$ → 21

$V_{ub} = (32.4 \pm 2.4 \pm \cancel{4.6})\, 10^{-4}$ (exclusive only)

TODAY

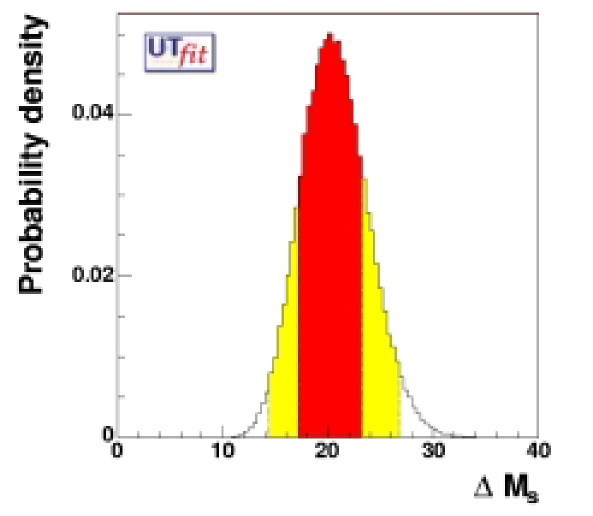


NEXT YEARS

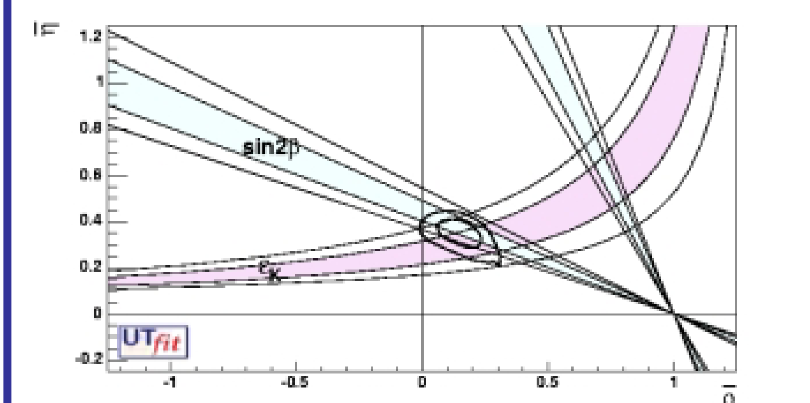


$\Delta\bar{\rho}$ = 28% → 17% (-40%) $\Delta\bar{\eta}$ = 7.8% → 5.2% (-33%)

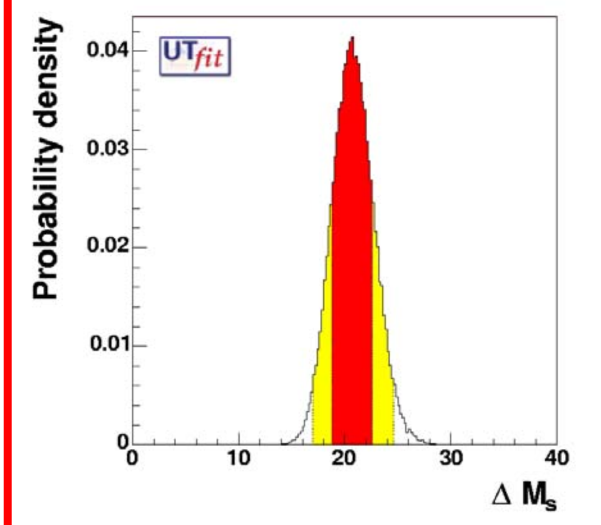
TODAY



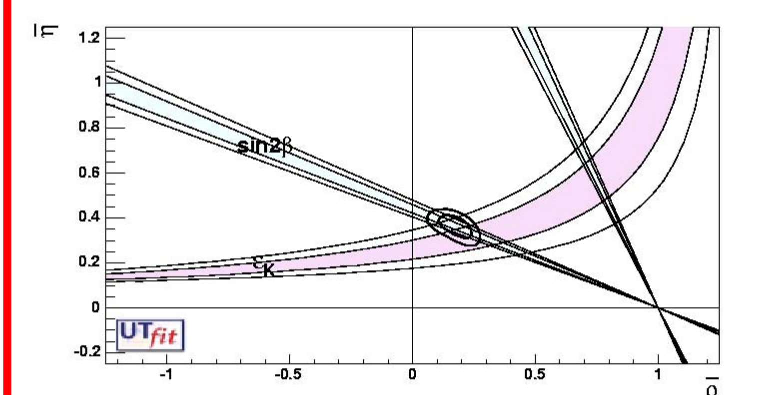
$\Delta m_s = (20.5 \pm 3.2)\ ps^{-1}$



NEXT YEARS



$\Delta m_s = (20.7 \pm 1.9)\ ps^{-1}$



III. Search for New Physics

SEARCH FOR NEW PHYSICS

2) "Given the present theoretical and experimental constraints, to which extent the UTA can still be affected by New Physics contributions?"

An interesting case:

New Physics in B_d–$\overline{B}_d$ mixing

The New Physics mixing amplitudes can be parameterized in a simple general form:

$$M_d = C_d \, e^{2i\varphi_d} (M_d)^{SM}$$

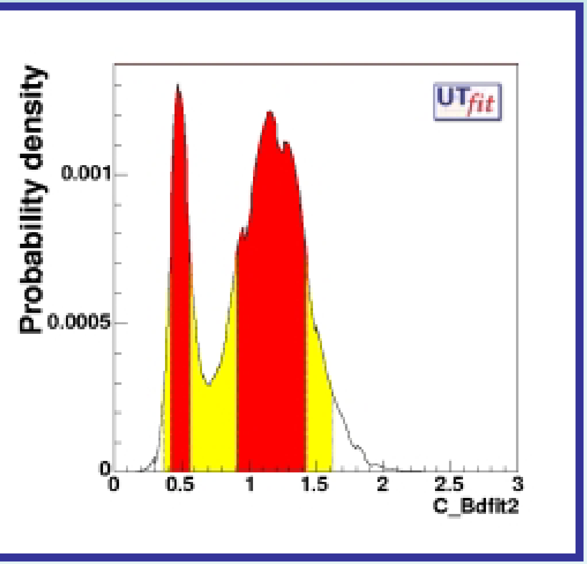
$\Rightarrow$

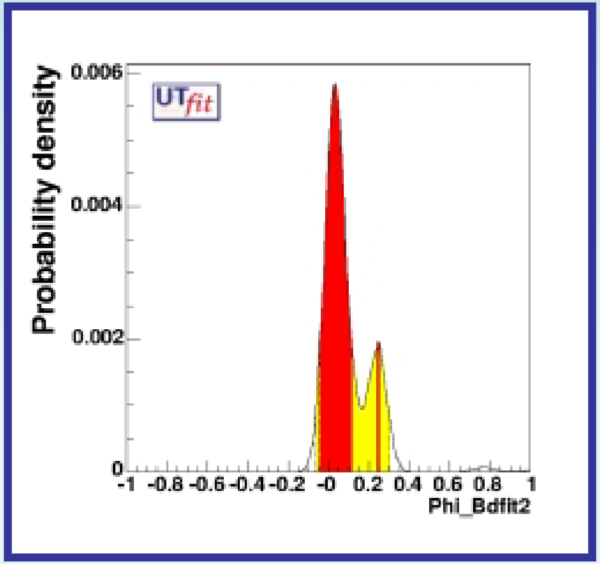
$$\Delta m_d = C_d \, (\Delta m_d)^{SM}$$

$$A(J/\psi \, K_S) \sim \sin 2(\beta + \varphi_d)$$

TWO SOLUTIONS:

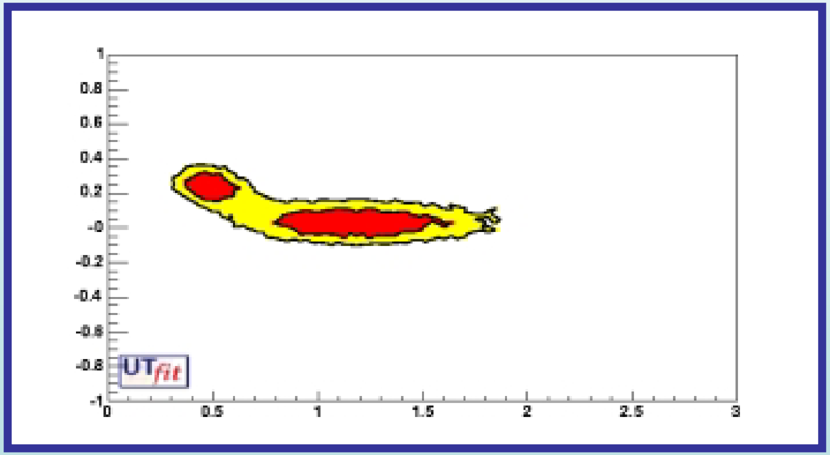
Standard Model solution:
$C_d = 1 \quad \varphi_d = 0$

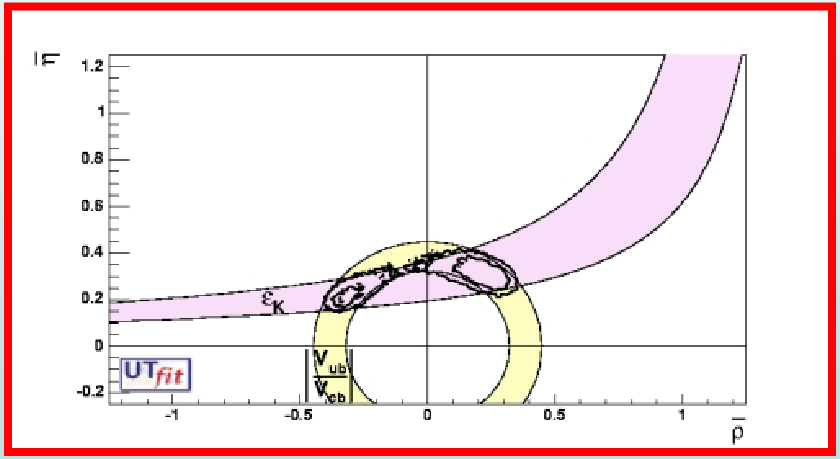




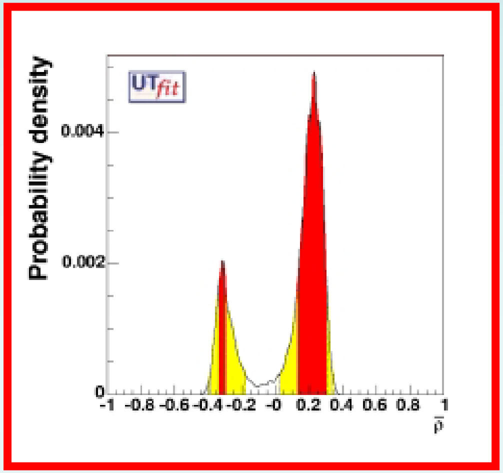
φ_d can be only determined up to a trivial twofold ambiguity:

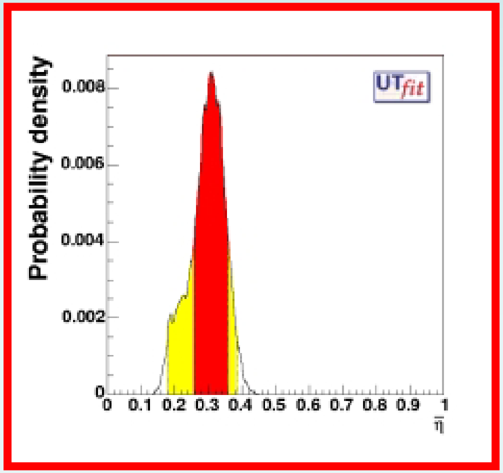
$\beta+\varphi_d \rightarrow \pi-\beta-\varphi_d$

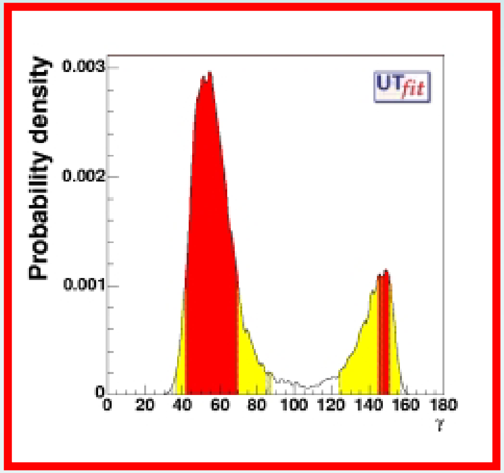




HOW CAN WE DISCRIMINATE BETWEEN THE TWO SOLUTIONS?







$\cancel{\Delta m_s}$, $\cancel{\eta\ [K_L \to \pi \nu \bar{\nu}]}$, γ [$B \to DK$], $|V_{td}|$ [$B \to \rho\gamma$], ...

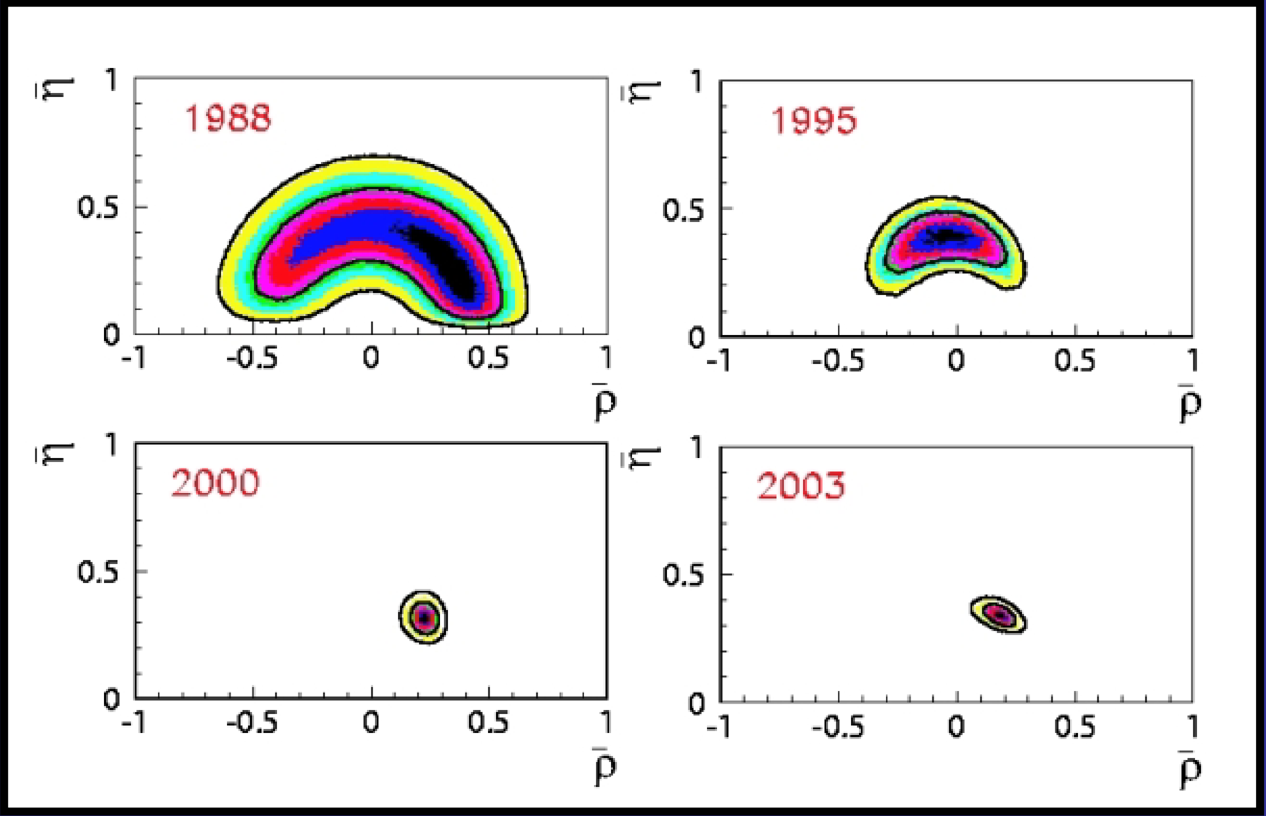
γ = 81° ± 19° ± 13° (syst) ± 11° (mod)

Belle **Independent of NP**

Belle preliminary + LQCD

Coming back to the Standard Model:

15 YEARS OF ($\bar{\rho}$-$\bar{\eta}$) DETERMINATIONS



CLOSING REMARKS

- THE STANDARD MODEL OF PARTICLE PHYSICS PROVIDES AN EXTREMELY SUCCESFULL DESCRIPTION OF FUNDAMENTAL INTERACTIONS UP TO THE FERMI SCALE (THE WHOLE ENERGY REGION EXPLORED SO FAR).

- NEVERTHELESS, THE STANDARD MODEL IS A LOW-ENERGY EFFECTIVE THEORY, AND WE ALREADY HAVE SEVERAL PHENOMENOLOGICAL INDICATIONS OF NEW PHYSICS

- FLAVOR PHYSICS IS "REPRODUCED" BUT NOT EXPLAINED IN THE STANDARD MODEL (MANY FREE PARAMETERS IN THIS SECTOR !). IT REPRESENTS A WINDOW OPEN ON NEW PHYSICS.

- IN OUR INVESTIGATION OF FLAVOR PHYSICS LATTICE QCD IS PLAYING, AND IT IS STILL EXPECTED TO PLAY, A FUNDAMENTAL ROLE.