Parma 11-12 Sept. 2003

Lectures on Supersymmetric Gauge Theories III:

Recent developments

(Dijkgraaf-Vafa hep-th/0208048, Cachazo-Douglas-Seiberg-Witten hep-th/0211170)

K. Konishi

- **Chiral Rings of Operators**
- **Generalized Konishi anomaly and Determination of W_{eff}**
- **Confinement Index**
- $\mathcal{N}=2$ **vs** $\mathcal{N}=1$ **approaches**
- **Phases and Multiplication Maps**

Veneziano-Yankielovicz Eff. Action

- $\mathcal{N}=1$ **susy** $SU(N)$ **Yang-Mills (** $W_\alpha = -i\lambda + \frac{i}{2}(\sigma^\mu\,\bar\sigma^\nu)_\alpha^\beta\,F_{\mu\nu}\,\theta_\beta + \ldots$ **)**

$$\mathcal{L}^{bare} = \int d^2\theta\,\frac{1}{g_0^2}WW = \int d^2\theta\,\frac{1}{g_0^2}\,S$$

- $$S \equiv W^\alpha W_\alpha = -\lambda\lambda + \ldots - \frac{1}{2}F_{\mu\nu}^2 - i\lambda\sigma^\nu\mathcal{D}_\nu\bar\lambda +$$

$$\mathcal{L}^{VY} = kin.term - \int d^2\theta\,S\,[\log\frac{S^N}{\Lambda^{3N}} - N] + h.c. \qquad (\&)$$

- 1**-loop renormalization**

$$[\frac{1}{g_0^2} + b_0\,\log\frac{M}{S^{1/3}}]\,S = \frac{1}{g(S)^2}\,S = b_0\,S\log\frac{S^{1/3}}{\Lambda}, \qquad b_0 = 3N$$

- $\mathcal{L}^{VY} \to \langle S\rangle = \Lambda^3\,\exp 2\pi ik/N,$ **with** $k = 1,2,\ldots N$ **(**$Z_{2N}\subset U_A(1)$ **broken to** Z_2**)**

- **Under** $\lambda \to e^{i\alpha}\lambda$

$$\Delta\mathcal{L}^{VY} = 2N\,\alpha\,F_{\mu\nu}\tilde F^{\mu\nu}$$

- $\int e^{i\,S}$ **invariant under** Z_{2N} **while (&) not invariant under** Z_{2N} **!?!**

  $\to$ **Chirally symmetric vacuum (**Kovner, Shifman**) ? No.**

Chiral Rings in Theory with adjoint field Φ

- $\mathcal{N}=1$ **susy** $U(N)$ **gauge theory**

$$\mathcal{L} = \frac{1}{8\pi}\mathrm{Im}\,\tau_{cl}\left[\int d^4\theta\,\Phi^\dagger e^V \Phi + \int d^2\theta\,\frac{1}{2}WW\right] + \int d^2\theta\,\mathcal{W}(\Phi) + h.c.$$

$$\Delta\mathcal{L} = \int d^2\theta\;\mu\,\mathrm{Tr}\,\Phi^2 + h.c., \qquad \tau_{cl} \equiv \frac{\theta_0}{\pi} + \frac{8\pi i}{g_0^2}$$

- **(**$\mathcal{N}=1$**) multiplets** $\Phi = \phi + \sqrt{2}\,\theta\,\psi + \ldots;\; W_\alpha = -i\lambda + \frac{i}{2}\,(\sigma^\mu\,\bar\sigma^\nu)_\alpha^\beta\,F_{\mu\nu}\,\theta_\beta + \ldots$

$$\mathcal{W}(\Phi) = \sum_{k=0}^{n}\frac{g_k}{k+1}\mathrm{Tr}\,\Phi^{k+1}. \tag{1}$$

- **Gauge inv. chiral composites, modulo** $\{\bar{Q},\ldots\} \rightarrow$

$$\{\;\mathrm{Tr}\,\Phi^k, \qquad \mathrm{Tr}\,W_\alpha\,\Phi^k, \qquad \mathrm{Tr}\,W^\alpha W_\alpha\,\Phi^k\} \qquad (\%)$$

- **Perturbatively (for** $k > N$**), e.g.,**

$$\mathrm{Tr}\,\Phi^k = \mathcal{P}(\{u_i\}), \qquad u_i = \mathrm{Tr}\,\Phi^j, \qquad j \le N,$$

$$\frac{\partial}{\partial\Phi}\mathcal{W}(\Phi) = \bar{D}^2(\ldots) = 0, \qquad S^N = 0, \qquad (\$)$$

Problem

- **Classically** $\{a_i\}$ **= eigenvalues of** Φ**,**

$$\mathcal{W}'(z) = g_n \prod_i^n (z - a_i) \quad \rightarrow \quad U(N) \Rightarrow \prod_i U(N_i)$$

- **Low-energy eff. degrees of freedom are**

$$S_i = \frac{1}{16\pi^2} \mathrm{Tr}\, W_i^\alpha W_{\alpha i}, \qquad w_{\alpha i} = \frac{1}{4\pi} \mathrm{Tr}\, W_{\alpha i} :$$

$$\mathcal{L}_{eff} = \int d^2\theta\, \mathcal{W}_{eff}(S_i, w_{\alpha i}, g_k) + \dots$$

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$$\int \mathcal{D}\Phi\, e^{i\,S} = e^{i \int \mathcal{L}_{eff}} = \exp i \int d^6 z\, \mathcal{W}_{eff}(S_i, w_{\alpha i}, g_k)$$

- **Problem: Compute** $\mathcal{W}_{eff}(S_i, w_{\alpha i}, g_k)$

- **Idea**

$$\frac{\partial}{\partial g_k} \mathcal{W}_{eff}(S_i, w_{\alpha i}, g_k) = \langle \mathrm{Tr}\, \frac{\Phi^{k+1}}{k+1} \rangle, \tag{2}$$

etc. Determine all chiral condensates as fns of $S_i, w_{\alpha i}, g_k$

Symmetries

Fields	Δ	Q_Φ	Q_R	Q_θ
Φ	1	1	$\frac{2}{3}$	0
W_α	$\frac{3}{2}$	0	1	1
g_l	$2-l$	$-(l+1)$	$\frac{2}{3}(2-l)$	2
Λ^{2N}	$2N$	$2N$	$\frac{4N}{3}$	0

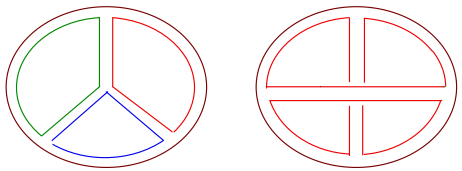
- $U_\Phi(1)$**:** $\Phi \to e^{i\alpha}\,\Phi$ **is anomalous at one loop only** $\Rightarrow$ **for** $\geq 2\,loops$

$$\mathcal{W}_{eff} = W_\alpha^2\, F(g_k\, W_\alpha^{k-1}/g_1^{(k+1)/2}), \quad \text{or}$$

$$\left[\sum_k (2-k)\, g_k \frac{\partial}{\partial g_k} + \frac{3}{2}\, W_\alpha \frac{\partial}{W_\alpha}\right] \mathcal{W}_{eff} = 3\, \mathcal{W}_{eff}$$

- **Index loops (L), vertices (k_i), genus (g)**

$$L = 2 - 2g + \frac{1}{2}\sum_i (k_i - 1)$$



- $\therefore$ **Only planar diagrams contribute to** $\mathcal{W}_{eff}$

  (Proof of the conjecture by Dijkgraaf-Vafa)

- $U(1) \subset U(N)$ **is free:** $\mathcal{W}_{eff}(S_i, w_{\alpha i}, g_k)$ **inv under**

$$W_\alpha \to W_\alpha - 4\pi\psi_\alpha$$

$\Rightarrow$ **General form of** $\mathcal{W}_{eff}$

$$U(N) \Rightarrow \prod_i U(N_i),$$

$$\mathcal{W}_{eff} = \sum N_i \frac{\partial \mathcal{F}_p(S_k, g_k)}{\partial S_i} + \frac{1}{2}\sum_{i,j} \frac{\partial^2 \mathcal{F}_p(S_k, g_k)}{\partial S_i \, \partial S_j} w_{\alpha i} w_j^\alpha.$$

$$= \int d^2\psi \; \mathcal{F}_p(\mathcal{S}_i, g_k);$$

$$\mathcal{S}_i = -\frac{1}{2}\mathrm{Tr}\,(\frac{1}{4\pi}W_{\alpha i} - \psi_\alpha)(\frac{1}{4\pi}W_i^\alpha - \psi^\alpha) = S_i + \psi_\alpha w_i^\alpha - N_i \psi_\alpha \psi^\alpha \qquad (3)$$

• **(2) ⇒**

$$\frac{\partial \mathcal{W}_{eff}}{\partial g_k} = \int d^2\psi \ \frac{\partial \mathcal{F}_p(\mathcal{S}_i, g_k)}{\partial g_k} = \langle \ \frac{\Phi^{k+1}}{k+1} \ \rangle_\Phi$$

$$= -\frac{1}{2(k+1)} \int d^2\psi \, \langle \, \mathrm{Tr} \, (\frac{1}{4\pi} W_\alpha - \psi_\alpha)^2 \, \Phi^{k+1} \rangle_\Phi$$

• **So**

$$\frac{\partial \mathcal{F}_p(\mathcal{S}_i, g_k)}{\partial g_k} = -\frac{1}{2(k+1)} \int d^2\psi \langle \mathrm{Tr}(\frac{1}{4\pi} W_\alpha - \psi_\alpha)^2 \, \Phi^{k+1} \rangle_\Phi$$

• **Problem: find the right hand side**

Generalized Konishi Anomaly

- **Konishi Anomaly**

$$\bar{D}^2 \mathrm{Tr}\{\bar{\Phi} e^V \Phi\} = \mathrm{Tr}\,\Phi \frac{\partial \mathcal{W}}{\partial \Phi} + \frac{1}{32\pi^2} \mathrm{Tr}\,(ad\, W_\alpha\, ad\, W^\alpha) \tag{4}$$

$$\bar{D}^2 \mathrm{Tr}\{\bar{\Phi} e^V \Phi\} = \mathrm{Tr}\,\Phi \frac{\partial \mathcal{W}}{\partial \Phi} + \frac{N}{16\pi^2} \mathrm{Tr}\,(W_\alpha W^\alpha) - \frac{1}{16\pi^2} \mathrm{Tr}\, W_\alpha \mathrm{Tr}\, W^\alpha$$

- **Supersymmetrized form of $U_\Phi(1)$ anomaly**

- **Taking the VEVS**

$$\langle \mathrm{Tr}\,\Phi \frac{\partial \mathcal{W}}{\partial \Phi} \rangle = -\frac{N}{16\pi^2} \langle \mathrm{Tr}\,(W_\alpha W^\alpha) \rangle.$$

But

$$L.H.S. = \langle \mathrm{Tr}\, \sum_k g_k \Phi^{k+1} \rangle = \sum_k (k+1) g_k \frac{\partial}{\partial g_k} \mathcal{W}_{eff}$$

- **(4) à la Fujikawa (Shizuya-Konishi)** $\delta\Phi = \alpha\Phi$.

- **Generalization ($J_f = \mathrm{Tr}\{\bar{\Phi} e^V f(\Phi, W_\alpha)\}$):**

$$\delta\Phi = f(\Phi, W_\alpha) \tag{5}$$

$$\bar{D}^2 J_f = \mathrm{Tr}\, f(\Phi, W_\alpha)\frac{\partial \mathcal{W}}{\partial \Phi} + \frac{1}{32\pi^2}\sum_{ij}[W_\alpha, [W^\alpha, \frac{\partial f}{\partial \Phi_{ij}}]]_{ij} \tag{6}$$

$$\langle R.H.S.\rangle = 0.$$

- **Define**

$$\mathcal{R}(z, \phi) = -\frac{1}{2}\,\mathrm{Tr}\,(\frac{1}{4\pi}W_\alpha - \psi_\alpha)^2 \frac{1}{z - \Phi}, \tag{7}$$

$$= R(z) + \psi_\alpha w^\alpha(z) - \psi^1\psi^2 T(z),$$

where generating functions are

$$T(z) = \mathrm{Tr}\,\frac{1}{z - \Phi}, \qquad w^\alpha = \frac{1}{4\pi}\mathrm{Tr}\, W_\alpha \,\frac{1}{z - \Phi},$$

$$R(z) = -\frac{1}{32\pi^2}\mathrm{Tr}\, W_\alpha W^\alpha \,\frac{1}{z - \Phi},$$

• **By choosing** $f(\Phi) = W_\alpha W^\alpha \frac{1}{z-\Phi}$ **in (5):**

$$\langle -\frac{1}{32\pi^2} \sum_{ij} [W_\alpha, [W^\alpha, \frac{\partial}{\partial \Phi_{ij}} (W_\beta W^\beta \frac{1}{z-\Phi})]]_{ij} = \langle \operatorname{Tr} [\frac{\partial \mathcal{W}}{\partial \Phi} W_\alpha W^\alpha \frac{1}{z-\Phi}] \rangle.$$

By identity

$$\sum_{ij} [\chi_1, [\chi_2, \frac{\partial}{\partial \Phi_{ij}} \frac{\chi_1 \chi_2}{z-\Phi}]]_{ij} = (\operatorname{Tr} \frac{\chi_1 \chi_2}{z-\Phi})^2$$

(valid if $\chi_1^2 = \chi_2^2 = 0,\ [\chi_i, \Phi] = 0$ **) one gets**

$$R(z,\psi)^2 = \operatorname{Tr} (\mathcal{W}'(\Phi) R(z,\psi))$$

• **Analogously, with** $f(\Phi) = \mathcal{R}$ **(r.h.s of (7) without trace)**

$$\mathcal{R}(z,\psi)^2 = \operatorname{Tr} (\mathcal{W}'(\Phi) \mathcal{R}(z,\psi))$$

which can be rewritten as

$$\mathcal{R}(z,\psi)^2 = \operatorname{Tr} (\mathcal{W}'(z) \mathcal{R}(z,\psi)) + \frac{1}{4} f(z,\psi), \tag{8}$$

or

$$\begin{aligned} R^2(z) &= \mathcal{W}'(z)R(z) + \frac{1}{4}f(z); \\ 2\,R(z)\,w^\alpha(z) &= \mathcal{W}'(z)w^\alpha(z) + \frac{1}{4}\rho^\alpha; \\ 2\,R(z)\,T(z) + w_\alpha(z)w^\alpha(z) &= \mathcal{W}'(z)T(z) + \frac{1}{4}c(z). \end{aligned}$$

with

$$\begin{aligned} f(z,\psi) = \frac{1}{8\pi^2}\mathrm{Tr}\,\frac{(\mathcal{W}'(z) - \mathcal{W}'(\Phi))(W_\alpha - 4\pi\psi_\alpha)\,(W^\alpha - 4\pi\psi^\alpha)}{z - \Phi}, \\ = f(z) + \psi_\alpha\,\rho^\alpha(z) - \psi_1\psi_2\,c(z) \end{aligned}$$

where $f(z)$ is an nth order polynomial in z

- **Solving the quadratic euation**

$$2\,\mathcal{R}(z,\psi) = \mathcal{W}'(z) - \sqrt{\mathcal{W}'(z)^2 + f(z,\psi)}$$

or

$$2\,R(z) = \mathcal{W}'(z) - \sqrt{\mathcal{W}'(z)^2 + f(z)}$$

etc.

- $R(z) = -\frac{1}{32\pi^2}\mathrm{Tr}\, W_\alpha W^\alpha \frac{1}{z-\Phi}$ **determined in terms of** f_i **where**

$$f(z) = \sum_{i=0}^{n-1} f_i\, z^i$$

- **(3) and def of** $\mathcal{R}(z,\psi) \Rightarrow$

$$\mathcal{S}_i = S_i + \psi_\alpha w_i^\alpha - N_i \psi_\alpha \psi^\alpha = \frac{1}{2\pi i}\oint_{C_i} dz\, \mathcal{R}(z,\psi)$$

$\Rightarrow$ **Relations** $\{f_i\} \leftrightarrow (S_i, w_i^\alpha)$

- **Finally**

$$\frac{\partial \mathcal{F}_p(\mathcal{S}_i, g_k)}{\partial g_k} = -\frac{1}{2(k+1)}\int d^2\psi \langle \mathrm{Tr}(\frac{1}{4\pi}W_\alpha - \psi_\alpha)^2\, \Phi^{k+1}\rangle_\Phi$$
$$= -\frac{1}{2(k+1)}\oint dz\, z^{k+1}\mathcal{R}(z,\phi)$$

- **By integrating over** g_k **and adding integration constant -** g_k **independent,** $1-loop$**, contribution (VY), we get** $\mathcal{W}_{eff}$ **in terms of** $(S_i, w_i^\alpha, \Lambda_i)$ ♡

Matrix Model (Dijkgraaf-Vafa)

- **Integral over $\hat{N} \times \hat{N}$ Hermitian matrices M**
- **Free energy (*cfr* $\mathcal{W}(\Phi)$)**

$$\exp -\frac{\hat{N}^2}{g_m^2} F_{m.m.} = \int d^{\hat{N}^2} M \;\; \exp -\frac{\hat{N}}{g_m} \operatorname{Tr} \mathcal{W}(M)$$

- $\delta M = \epsilon\, M^{n+1} \;\longrightarrow\; 0 = \int d^{\hat{N}^2} M \;\, e^{-\frac{\hat{N}}{g_m} \operatorname{Tr} \mathcal{W}(M)} \, [\operatorname{Tr} \frac{\partial}{\partial M} M^n - \frac{\hat{N}}{g_m} \operatorname{Tr} \mathcal{W}' M^n]$

$$\langle R_m(z)^2 \rangle = \langle W'(z) R_m(z) \rangle + \frac{1}{4} f_m(z), \qquad \text{where} \qquad R_m(z) = \frac{g_m}{\hat{N}} \langle \operatorname{Tr} \frac{1}{z - M} \rangle$$

- **Take now $\hat{N} \to \infty$: $\to$ factorization**

$$\langle R_m(z) \rangle^2 = \langle W' \rangle \langle R_m(z) \rangle + \frac{1}{4} f_m(z),$$

: relation identical to Eq. (6) !!!

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$$S_i = \frac{1}{2\pi i} \oint_{C_i} R_m(z)\, dz \qquad \frac{\partial F_{m.m}}{\partial g_k} = \langle \frac{\operatorname{Tr} M^{k+1}}{k+1} \rangle$$

$F_{m.m}(S_i, g_k) \Rightarrow$ **identify with** $\mathcal{F}_p(\mathcal{S}_i, g_k) \to \mathcal{W}_{eff}(S_i, w_i^{\alpha}, \Lambda_i)$

Further development

(Cachazo-Seiberg-Witten, hep-th/0301006)

$$\mathcal{L}^{U(N)} = \frac{1}{8\pi}\mathrm{Im}\,\tau_{cl}\left[\int d^4\theta\,\Phi^\dagger e^V \Phi + \int d^2\theta\,\frac{1}{2}WW\right] + \int d^2\theta\,\mathcal{W}(\Phi)$$

where $\mathcal{W}(\Phi) = \sum_{r=0}^{k} \frac{g_r}{r+1}\mathrm{Tr}\,\Phi^{r+1}$ **(superpotential)**

- $\mathcal{W}(\Phi) = 0 \Rightarrow \mathcal{N} = 2 \Rightarrow G \sim U(1)^{N-1}$ **on a generic point of QMS.**
- **Special points where some** $N-n$ **monopoles become massless (condensation and Higgs mech. for** $N-n$ **dual gauge bosons)** $\Rightarrow G \sim U(1)^n$
- **Curve factorizes (cond. on QMS)**

$$y^2 = P_N^2(x) - 4\Lambda^2 = F_{2n}\,H_{N-n}^2(x) \qquad (9)$$

- **Classically** $\{a_i\}$ = **eigenvalues of** Φ**,**

$$\mathcal{W}'(z) = g_k \prod_i^k (z - a_i) \qquad \mathrm{diag}\,\Phi = \{a_1, \ldots a_1, a_2, \ldots, \ldots, a_n, \ldots, a_n\}$$

$$U(N) \Rightarrow \prod_i^n U(N_i) \Rightarrow U(1)^n \qquad n \leq k \qquad (10)$$

- **Problem: find QM'ly the relation**

$$\text{Vacua (10)} \iff \mathcal{W}(\Phi)$$

- **Ans: the Factorization condition** (9) **with (for** $k = n$**)**

$$F_{2n}(x) = \frac{1}{g_n^2}\mathcal{W}'(x)^2 + f_n(x)$$

$f_n(x) = O(x^{n-1})$ **with** n **unknown coefficients.**

- **Generalized Konishi anomaly (electric variables) from** $\mathcal{N} = 2$ **curves (whose singularities related magnetic variables)**

- **Use of Konishi anomaly in** $\mathcal{N} = 2$, $SU(2)$ **theory with** $n_f = 1 \Leftrightarrow$ **the knowledge of the curve (**Gorsky-Vainshtein-Yung**)**

- **Detailed knowledge about the decpoupling of** Φ

Vacua of $\mathcal{N} = 2$ **theory** $\rightarrow$ **Vacua of** $\mathcal{N} = 1$ **theory**

Confinement Index

$\equiv$ **Smallest possible** $r \in Z_N^{(E)}$ **for which Wilson loop displays no area law**

- $SU(N)$ **YM:** $r = N$ **completely confining;** $r = 1$ **totally Higgs**
- $r = 1$ **in a theory with**

$$SU(N) \to SU(N-1) \times U(1)$$

- $\mathcal{N} = 1$ **Susy** $SU(N)$ **theory broken (by adjoint VEV) as**

$$SU(N) \to SU(N_1) \times SU(N_2) \times U(1)$$

Then

$$r = l.c.d\,\{N_1, N_2, r_1 - r_2\}$$

where

$$r_1 = 0, 1, 2, \ldots N_1 - 1, \qquad r_2 = 0, 1, 2, \ldots N_2 - 1$$

label the vacua in which $(n_m, n_e) = (1, r_1)$ **and** $(n_m, n_e) = (1, r_2)$ **are condensed**

Multiplication Map

$U(N) \Leftrightarrow U(tN)$ **with the same superpotential** $\mathcal{W}(\Phi)$

- **Vacua with** $\Pi_{i=1}^{n} U(N_i)$ **of** $U(N) \Leftrightarrow$ **vacua with** $\Pi_{i=1}^{n} U(tN_i)$ **of** $U(tN)$**;**
- **Confinement Index** r **gets simply multiplied by** t**;**
- *All* **confining vacua with** $r = t$ **in the** $U(tN)$ **theory, arise from the Coulomb vacua of** $U(N)$ **theory**
- **Map of the chiral condensates**

$$\langle \mathrm{Tr} \frac{1}{x - \Phi} \rangle = t \, \langle \mathrm{Tr} \frac{1}{x - \Phi_0} \rangle$$

- $USp(N) \Leftrightarrow U(N+2n)$ **map in the theory with order** $n+1$ $\mathcal{W}(\Phi)$ **(**Cachazo**)**

$$\Pi_{i=1}^{n} USp(N_i) \Leftrightarrow \Pi_{i=1}^{n} U(N_i + 2) :$$

$$\langle \mathrm{Tr} \frac{1}{z - \Phi} \rangle = \langle \mathrm{Tr} \frac{1}{z - \Phi_U} \rangle - \frac{d}{dz} \log(W'(z)^2 + f(z))$$

Authors

- Matrix model

  Dijkgraaf-Vafa, Cachazo-Intrilligator-Vafa, Argurio-Campos-Ferretti-Heise, Suzuki.H, Bena-Roiban, Tachikawa,T. Itoh, Kraus-Ryzkov-Shigemori, Abbaspur-Imaanpur-Parvizi, Berenstein, Gorsky, ...;

- Field Theory

  Cachazo-Douglas-Seiberg-Witten, Cachazo-Seiberg-Witten, Alday-Cirafici, Brandhuber-Ita-Nieder-Oz-Römelsberger, Eguchi-Sugawara, Matone, Feng, Ahn-Ookouchi, Feng, Huang-Naqvi, Ahn-Nam, Shih, Svrcek, Merlatti, Gripaios-Wheater, David-Gava-Narain, ...

Summary

- $\mathcal{W}_{eff}$ **of** $U(N)$ **theory with superpotential** $\mathcal{W}(\Phi)$ **in terms of** $(S_i, w_i^\alpha, \Lambda_i)$
- **Only planar diagrams contribute to the effective superpotential**
- **Complete set of Chiral ring relations (Ward-Takahashi Identities)**
- **e.g. for** $SU(N)$ **SYM**

$$S^N = \Lambda^{3N} + \{\bar{Q}_{\dot{\alpha}}, X^{\dot{\alpha}}\}$$

as an *operator relation* $\rightarrow$ **No chirally symmetric vacuum with** $\langle \lambda\lambda \rangle = 0.$

- **Whole results elegantly summarized by matrix model bookkeeping**
- **Addition of matter; other gauge groups (** $SO(N), USp(2N)$ **)**
- **More precise relations** $\mathcal{N}=2$ *vs* $\mathcal{N}=1$ **theories**

  (Generalized K anomaly relations from $\mathcal{N}=2$ **!)**
- **Phases, new duality (continuous and discrete maps among the vacua of the same or different theories), etc.**

Symmetry, Quantization, Phase Factor

$\sim$ Melodies of Theoretical Physics of the 20th Century

C.N. Yang

Do we need something else ?