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Electroweak Interactions in the SM and Beyond

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A short course on the EW Theory

We start from the basic principles and formalism (a fast recall).
Then we go to present status and challenges

Content

- Formalism of gauge theories
- The SU(2)xU(1) symmetric lagrangian
- The symmetry breaking sector
- Beyond tree level
- Precision tests
- Problems of the SM
- Beyond the SM

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General formalism of non abelian gauge theories

$\Phi_a(x)$: multiplet of fields ($a=1,2,\ldots,n$)

Internal symmetry: $\Phi_a(x) \longrightarrow \Phi'_a(x) = U_{ab}\,\Phi_b(x)$

internal: x unchanged

$\Phi' = U\Phi$

A=1,2,…,N

$U = \exp[i\sum_A t^A \varepsilon^A] \sim 1 + i\sum_A t^A \varepsilon^A + o(\varepsilon^2)$

Infinitesimal transformation

t^A: generators
ε^A: parameters

Generators may: commute — abelian
not commute — non abelian

$[t^A, t^B] = iC_{ABC}\,t^C$

C_{ABC}: structure constants
define the group
depend on normalisation

$\mathrm{Tr}\; t^A t^B = 1/2\; \delta_{AB}$ $\longleftarrow$
in fund. repres.

$\mathrm{Tr}[t^A, t^B]\, t^C = \frac{i}{2} C_{ABC}$ $\longrightarrow$ compl. antisymmetric

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$U = \exp[i\sum_A t^A \varepsilon^A]$

Global symm.: ε^A constant
Local or gauge symm.: $\varepsilon^A = \varepsilon^A(x)$

Consider a lagrangian density invariant under a global symmetry:

$$L[\Phi, \partial_\mu \Phi] = L[\Phi', \partial_\mu \Phi'] = L[U\Phi, \partial_\mu U\Phi]$$

In general it is not invariant under gauge symmetry:

$$\partial_\mu(U\Phi) = U(\partial_\mu \Phi) + (\partial_\mu U)\Phi \neq U(\partial_\mu \Phi)$$

But $L[\Phi, D_\mu \Phi]$ is gauge invariant if $(D_\mu \Phi)' = U(D_\mu \Phi)$

D_μ is the covariant derivative, a linear operator that generalizes ∂_μ $\longrightarrow$ $D_\mu = \partial_\mu + ig\sum_A t^A V_\mu^A(x) = \partial_\mu + igV_\mu(x)$

gauge fields

Def.: $V_\mu = \sum_{A=1}^{N} t^A V_\mu^A$

Solution: $V'_\mu = UV_\mu U^{-1} - \frac{1}{ig}(\partial_\mu U)U^{-1}$

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This is how the gauge fields must transform

$$D_\mu = \partial_\mu + ig\sum_A t^A V^A_\mu(x) = \partial_\mu + igV_\mu(x)$$

$$V'_\mu = UV_\mu U^{-1} - \frac{1}{ig}(\partial_\mu U)U^{-1}$$

Here is the proof that $(D_\mu\Phi)' = U(D_\mu\Phi)$

$$(D_\mu\Phi)' = (\partial_\mu + igV_\mu')\Phi' =$$

$$= [\partial_\mu + igUV_\mu U^{-1} - (\partial_\mu U)U^{-1}]U\Phi =$$

$$= U\partial_\mu\Phi + \cancel{(\partial_\mu U)}\Phi + igUV_\mu\Phi - \cancel{(\partial_\mu U)}\Phi =$$

$$= U(\partial_\mu + igV_\mu)\Phi = U(D_\mu\Phi)$$

Electric charge

Note: The abelian case (QED) $U = \exp[iQ\varepsilon(x)]$

$$V_\mu = \sum_{A=1}^{N} t^A V^A_\mu \rightarrow QV_\mu \rightarrow QV'_\mu = QV_\mu - \frac{1}{ie}\cdot iQ\partial_\mu\varepsilon(x)\cancel{e^{iQ\varepsilon}}\cdot\cancel{e^{-iQ\varepsilon}}$$

g = e

finally:

$$\boxed{V'_\mu = V_\mu - \frac{1}{e}\cdot\partial_\mu\varepsilon(x)}$$

Ordinary gauge invariance for the photon

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Kinetic term for V^A_μ

$[D_\mu, D_\nu]\Phi \equiv igF_{\mu\nu}\Phi$

From $(D_\mu\Phi)' = U(D_\mu)\Phi$
one gets $(F_{\mu\nu}\Phi)' = U\, F_{\mu\nu}\Phi$
or $F_{\mu\nu}'\Phi' = U\, F_{\mu\nu}U^{-1}U\Phi$

$$\boxed{F_{\mu\nu}' = U\, F_{\mu\nu}U^{-1}}$$

Adjoint representation

Thus:

$$\mathrm{Tr}\, F_{\mu\nu}'\, F^{\mu\nu\prime} = \mathrm{Tr}\, U\, F_{\mu\nu}U^{-1}\, U\, F^{\mu\nu}U^{-1} = \mathrm{Tr}\, U^{-1}\, U\, F_{\mu\nu}F^{\mu\nu} = \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$$

Note: $F_{\mu\nu} = \sum_A F^A_{\mu\nu} t^A$ and

$$\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} = \sum_{A,B} F^A_{\mu\nu}F^{B\mu\nu}\underbrace{\mathrm{Tr}\, t^A\, t^B}_{1/2\, \delta^{AB}} = 1/2\sum_A F^A_{\mu\nu}F^{A\mu\nu}$$

Thus a gauge invariant lagrangian is given by:

$$L_{YM} = -1/2\, \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + L[\Phi, D_\mu\Phi]$$

Yang, Mills

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$[D_\mu, D_\nu]\Phi = igF_{\mu\nu}\Phi$ $\Longrightarrow$

$$[\partial_\mu + igV_\mu, \partial_\nu + igV_\nu]\Phi =$$
$$ig\{\partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]\}\Phi$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]$$

or, from $F_{\mu\nu} = \sum_A F^A_{\mu\nu} t^A$ and $[t^A, t^B] = iC_{ABC} t^C$

$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - g C_{ABC} V^B_\mu V^C_\nu$$

Note the abelian limit

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

The Electro-Weak Theory

At first sight unification of electromagnetism and of weak interactions looks difficult:

- QED is a vector theory, charged weak currents are V-A, neutral currents are a mixture of V and A
  $\longrightarrow$ violation of C and P
- γ is massless, $W^{\pm}$, Z are very massive

In the SM the first problem is solved by making particles of different chiralities to transform differently:

the SM is a “chiral” theory

The second problem leads to the concept of spontaneously broken gauge symmetry and the Higgs mechanism.

Chirality

ψ : Dirac field

Def.:

$$\psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi \quad \Longrightarrow \quad \begin{cases} \overline{\psi_L} = \psi_L{}^{\dagger}\gamma_0 = \bar{\psi}\dfrac{1+\gamma_5}{2} \\ \overline{\psi_R} = \psi_R{}^{\dagger}\gamma_0 = \bar{\psi}\dfrac{1-\gamma_5}{2} \end{cases}$$

$$\gamma^5 = \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \gamma_5{}^+ = \gamma_5, \quad \gamma_5{}^2 = 1, \quad \{\gamma_\mu, \gamma_5\} = 0$$

In the Bjorken-Drell basis: $\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}, \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(all entries are 2x2 matrices)

$P_\pm = 1/2(1 \pm \gamma_5)$ are projectors:

$P_+P_+ = P_+$; $P_-P_- = P_-$; $P_+P_- = P_-P_+ = 0$;

$P_+ + P_- = 1$

$P_\pm$ project over definite "chirality". For a massless fermion chirality = helicity

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$$\bar{\psi}\Gamma\psi = \bar{\psi}\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\Gamma\left(\frac{1+\gamma_5}{2}+\frac{1-\gamma_5}{2}\right)\psi$$

Two classes of Dirac matrices:

$\Gamma_C = 1, \gamma_5, \sigma_{\mu\nu}$: commute with γ_5

$$\Longrightarrow \quad \bar{\psi}\Gamma_C\psi = \overline{\psi_L}\Gamma_C\psi_R + \overline{\psi_R}\Gamma_C\psi_L$$

e.g. a mass term

$$\bar{\psi}M\psi = \overline{\psi_L}M\psi_R + \overline{\psi_R}M\psi_L$$

chirality flip

$\Gamma_A = \gamma_\mu, \gamma_\mu\gamma_5$: anticommute with γ_5

$$\Longrightarrow \quad \bar{\psi}\Gamma_A\psi = \overline{\psi_L}\Gamma_A\psi_L + \overline{\psi_R}\Gamma_A\psi_R$$

e.g. cov. derivative term

chirality no-flip

$$\bar{\psi}i\widehat{D}\psi = \overline{\psi_L}i\widehat{D}\psi_L + \overline{\psi_R}i\widehat{D}\psi_R \qquad (\widehat{D} \equiv \gamma_\mu D^\mu)$$

Note:

$$\bar{\psi} M \psi = \overline{\psi_L} M \psi_R + \overline{\psi_R} M \psi_L$$

A mass term can be symmetric only if Ψ_L and Ψ_R have the same transformation properties.

$$\bar{\psi} i \widehat{D} \psi = \overline{\psi_L} i \widehat{D} \psi_L + \overline{\psi_R} i \widehat{D} \psi_R$$

A covariant derivative term can be symmetric also if Ψ_L and Ψ_R have different transformation properties.

In the SM the symmetry group is SU(2)XU(1), but all Ψ_L are SU(2) doublets and all Ψ_R are SU(2) singlets.

$$\begin{bmatrix} u \\ d \end{bmatrix}_L, \quad u_R, \quad d_R \qquad\qquad \begin{bmatrix} \nu \\ e \end{bmatrix}_L, \quad \nu_R\,(?), \quad e_R$$

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The Standard Electro-Weak Theory

$L = L_{symm} + L_{Higgs}$

Glashow, Weinberg, Salam

L_{symm} (introduced by Glashow in '61 for leptons) is a gauge theory for massless fermions based on $SU(2) \times U_Y(1)$

$$L_{symm} = -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \overline{\psi_L} i\hat{D}\psi_L + \overline{\psi_R} i\hat{D}\psi_R$$

* There is a $\Psi_{L,R}$ term for each quark or lepton multiplet

* $D_\mu \Psi_{L,R} = [\partial_\mu + ig\sum_A t^A_{L,R} W^A_\mu + ig' \cdot \frac{1}{2} Y_{L,R} B_\mu]\Psi_{L,R}$

* $F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu - g\varepsilon_{ABC} W^B_\mu W^C_\nu$

* $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$[t^A, t^B] = i\varepsilon_{ABC} t^C$ ← Levi-Civita SU(2)

$\mathrm{Tr}\, t^A t^B = 1/2\, \delta^{AB}$ fixes norm of g,g'

Embedding of the electric charge in $SU(2) \times U(1)$

$$Q = t^3_L + \frac{Y_L}{2} = t^3_R + \frac{Y_R}{2}$$

All Ψ_L are weak isospin doublets
All Ψ_R are weak isospin singlets

$$Q = t^3 + Y/2$$

	t^3_L	t^3_R	Y_L	Y_R	Q
u_L	$+1/2$		$1/3$		$2/3$
d_L	$-1/2$		$1/3$		$-1/3$
u_R		0		$4/3$	$2/3$
d_R		0		$-2/3$	$-1/3$
ν_L	$+1/2$		-1		0
e_L	$-1/2$		-1		-1
e_R		0		-2	-1

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Gauge couplings to fermions

$$D_\mu = [\partial_\mu + ig\sum_A t^A_{L,R} W^A_\mu + ig' \cdot \frac{1}{2} Y_{L,R} B_\mu]$$

Charged Currents

$$g(t^1 W^1 + t^2 W^2) = g[\frac{t^1 + it^2}{\sqrt{2}} \cdot \frac{W^1 - iW^2}{\sqrt{2}} + hc] = g\left(\frac{t^+ W^-}{\sqrt{2}} + \frac{t^- W^+}{\sqrt{2}}\right)$$

$$t^\pm = t^1 \pm it^2 \qquad W^\pm = \frac{W^1 \pm iW^2}{\sqrt{2}}$$

Putting together L and R:

$$g\bar{\psi}\gamma^\mu\left[\frac{t_L^+}{\sqrt{2}} \cdot \frac{1-\gamma_5}{2} + \frac{t_R^+}{\sqrt{2}} \cdot \frac{1+\gamma_5}{2}\right]\psi W_\mu^- + \mathrm{h.c.}$$

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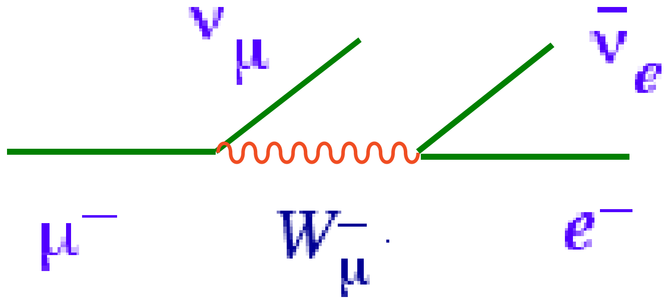
As $t^+_R = 0$ for quarks and leptons, CC are pure V-A

$$\mu^- \Rightarrow \nu_\mu + e^- + \bar{\nu}_e$$



$t^+_R=0$: $$g\bar{\psi}\gamma^\mu\left[\frac{t_L^+}{\sqrt{2}}\cdot\frac{1-\gamma_5}{2}\right]\psi W^-_\mu$$

$$(\nu_\mu,\mu^-)t^+\begin{bmatrix}\nu_\mu\\ \mu^-\end{bmatrix} = (\nu_\mu,\mu^-)\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\nu_\mu\\ \mu^-\end{bmatrix} = \nu_\mu\mu^-$$

$$g^2\,\overline{\nu_\mu}\gamma^\lambda\frac{(1-\gamma_5)}{2\sqrt{2}}\mu\cdot\frac{1}{q^2-m_W^2}.\bar{e}\gamma_\lambda\frac{(1-\gamma_5)}{2\sqrt{2}}\nu_e$$

negligible (q^2) — we anticipate the W mass

$$\frac{g^2}{8m_W^2}\overline{\nu_\mu}\gamma^\lambda(1-\gamma_5)\mu.\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e = \frac{G_F}{\sqrt{2}}\overline{\nu_\mu}\gamma^\lambda(1-\gamma_5)\mu.\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e$$

Relation with old Fermi theory (tree level)

$G_F=1.16639(1).10^{-5}$ GeV^{-2}

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}}$$

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Neutral Currents

$$g t^3 W^3 + g' \frac{Y}{2} B$$

Relation with γ and Z:

$$\begin{cases} W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{cases}$$

Def. of $\sin\theta_W$

Photon couplings: pure vector, ~Q

A_μ multiplies: $g \sin\theta_W \cdot t^3 + g' \cos\theta_W \cdot \frac{Y}{2}$

Since $(t^3+Y/2)_{L,R}=Q$ for $g\sin\theta_W = g'\cos\theta_W = e$ or $g'/g = tg\theta_W$
we obtain:

$$e\bar{\psi}\gamma_\mu[(t^3_L + \frac{Y_L}{2}) \cdot \frac{1-\gamma_5}{2} + (t^3_R + \frac{Y_R}{2}) \cdot \frac{1+\gamma_5}{2}]\psi A^\mu = e\bar{\psi}\gamma_\mu Q \psi A^\mu$$

Neutral Currents

$$g t^3 W^3 + g' \frac{Y}{2} B$$

Relation with γ and Z:

$$\begin{cases} W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{cases}$$

Def. of $\sin\theta_W$

Z couplings are now fixed:

$$g\cos\theta_W \cdot t^3 - g'\sin\theta_W \cdot \underbrace{\frac{Y}{2}}_{Q-t^3} = (g\cos\theta_W + g'\sin\theta_W)t^3 - g'\sin\theta_W \cdot Q$$

$$g' = g \cdot \frac{\sin\theta_W}{\cos\theta_W}$$

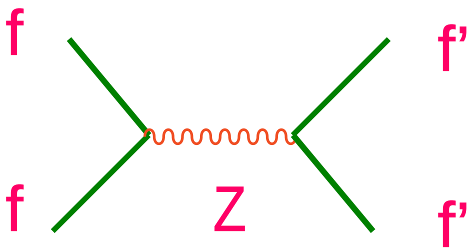
$$= \frac{g}{\cos\theta_W}(t^3 - Q\sin^2\theta_W)$$

Finally:

$$\frac{g}{\cos\theta_W}\bar{\psi}\gamma_\mu[t^3_L \cdot \frac{1-\gamma_5}{2} + t^3_R \cdot \frac{1+\gamma_5}{2} - Q\sin^2\theta_W]\psi Z^\mu$$

$$\frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu[t^3_L(1-\gamma_5) + t^3_R(1+\gamma_5) - 2Q\sin^2\theta_W]\psi Z^\mu = \frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu[\ldots]\psi Z^\mu$$

As for CC we can derive the effective 4-fermion interaction at low energies



$$\frac{g^2}{4\cos^2\theta_W}\bar{\psi}\gamma_\mu[\ldots]\psi \cdot \frac{1}{q^2 - m_Z^2} \cdot \bar{\psi}'\gamma^\mu[\ldots]\psi'$$

At $q^2 << m_Z^2$, recalling that $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$

$$L_{eff} = \sqrt{2}G_F \cdot \underbrace{\frac{m_W^2}{m_Z^2\cos^2\theta_W}}_{\rho_0} \cdot \bar{\psi}\gamma_\mu[\ldots]\psi \cdot \bar{\psi}'\gamma^\mu[\ldots]\psi'$$

We shall see that $\rho_0=1$ to a very good approximation.
Thus the intensities of NC and CC processes are comparable

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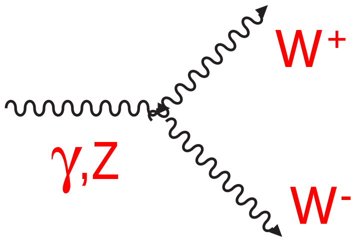
3- and 4-gauge couplings

$$L_{symm} = -\frac{1}{4}\sum_{A=1}^{3} F^{A}_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \dots$$

$$F^{A}_{\mu\nu} = \partial_\mu W^{A}_{\nu} - \partial_\nu W^{A}_{\mu} - g\varepsilon_{ABC}W^{B}_{\mu}W^{C}_{\nu} \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

3-gauge coupling: $-\frac{1}{4}\sum_{A=1}^{3} F^{A}_{\mu\nu}F^{A\mu\nu} \text{--->} 2\cdot 2\cdot\frac{1}{4}g\varepsilon_{ABC}\partial_\mu W^{A}_{\nu}W^{\mu B}W^{\nu C}$



must be ε_{123}

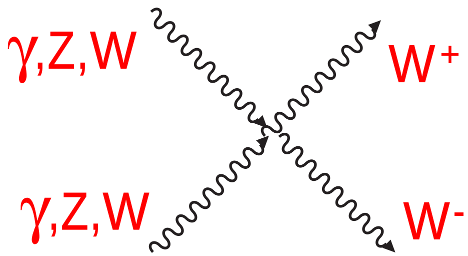
1,2 -> $W^{+,-}$

3 -> γ,Z

Only W_3 not B!

$W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$

$g_{\gamma WW} = g\sin\theta_W = e$
(obvious)
$g_{ZWW} = g\cos\theta_W = e\,\mathrm{cotg}\,\theta_W$
(larger by factor ~1.83)



4-gauge coupling: $\frac{1}{4}g^2 W^{B}_{\mu}W^{C}_{\nu}(W^{\mu B}W^{\nu C} - W^{\mu C}W^{\nu B})$

3-gauge coupling: The SM prediction is very special

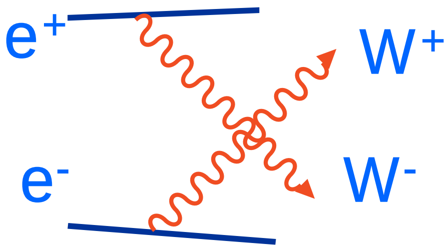
In general, assuming em gauge invariance and CP there are 6 parameters (5 for P and C conservation) for $(\gamma,Z)WW$

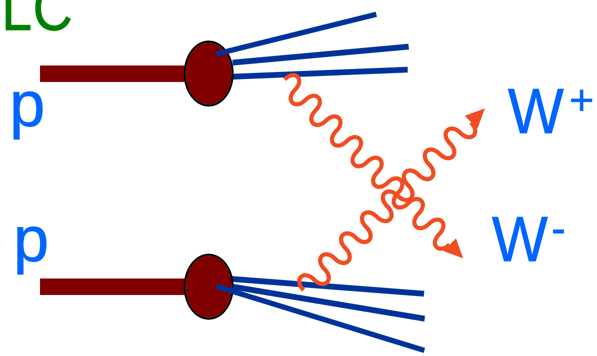
	SM
k_γ, k_Z	1
λ_γ, λ_Z	0
g_Z,(f_Z)	0

W magnetic moment: $e/2m_W(1+k_\gamma+\lambda_\gamma)$
W electric quad. mom: $-e/m_W^2\,(k_\gamma-\lambda_\gamma)$

Data are obtained from cross-section and distributions for $e^+e^- \to W^+W^-$ at LEP

The 4-gauge coupling is for LHC, NLC





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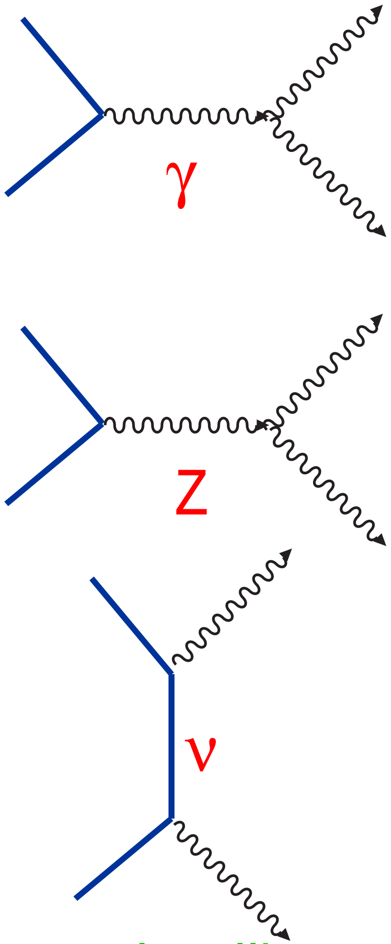
$e^+e^- \to W^+W^-$



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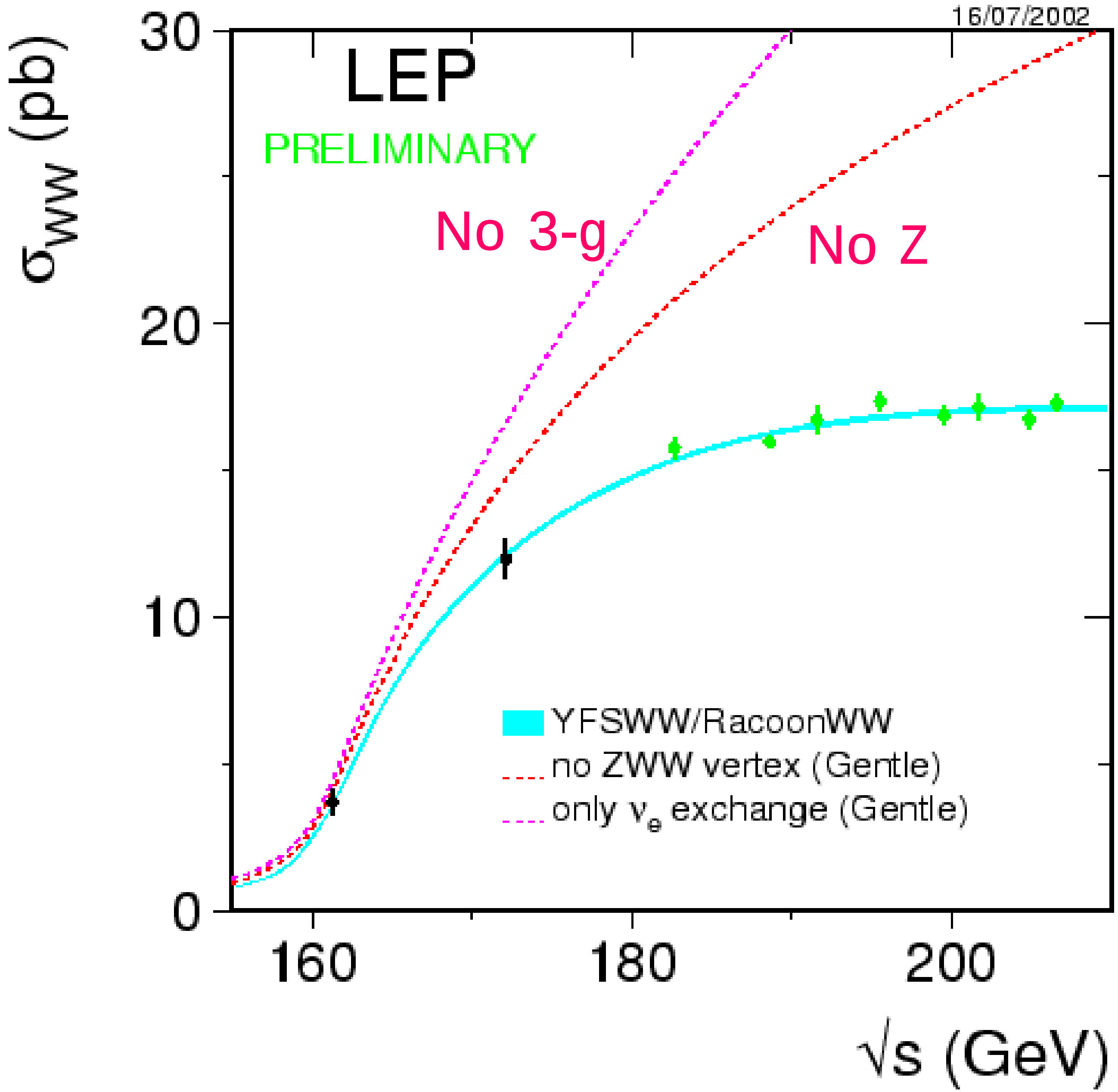

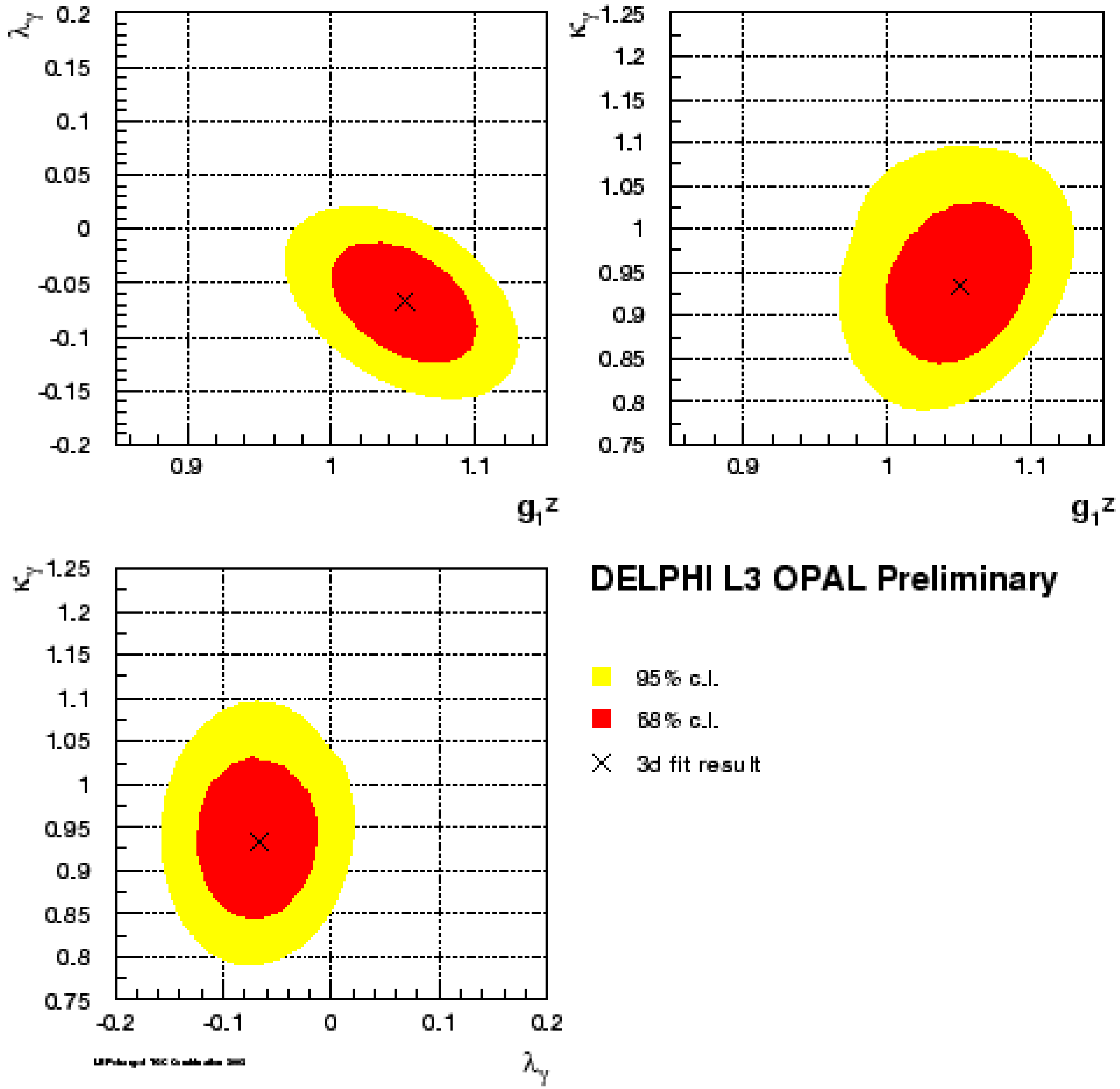

DELPHI L3 OPAL Preliminary
95% c.l.
68% c.l.
3d fit result
λ_γ
κ_γ
g_1^Z
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.9
1
1.1
-0.2
-0.1
0
0.1
0.2

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Spontaneous Symmetry Breaking

Borrowed from the theory of phase transitions:

Ferromagnet (Landau-Ginzburg, classical)

At zero magnetic field B

$$F = F(M, T) = F_0(T) + \frac{1}{2}\mu^2(T)\vec{M}^2 + \frac{1}{4}\lambda(T)(\vec{M}^2)^2 + \ldots$$

Free energy

Magnetisation

Temperature

M small
(analogue of renorm.ty)
$\lambda(T) \geq 0$: stability

F is rotation invariant.

Minimum condition: $\frac{\partial F}{\partial M} = 0 \rightarrow [\mu^2(T) + \lambda(T)\vec{M}^2]\vec{M} = 0$

Two cases:

A $\mu^2(T) \geq 0$
Solution: $M_0 = 0$

B $\mu^2(T) < 0$
Solution: $M_0^2 = -\mu^2/\lambda$

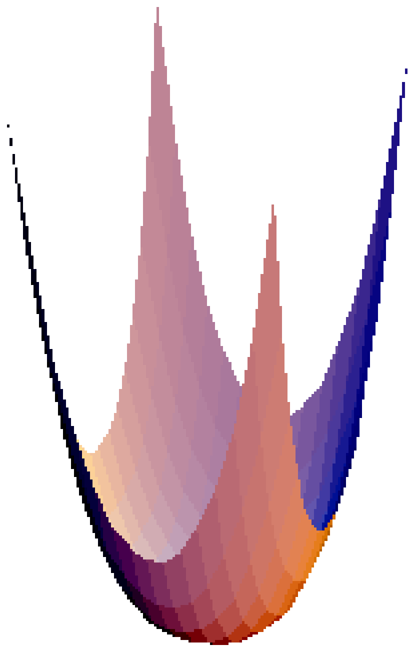
Critical temperature T_C: $\mu^2(T_C) = 0$

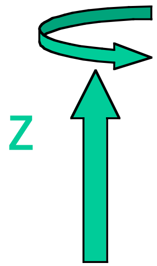
A $\mu^2(T) \geq 0$

Solution: $M_0 = 0$

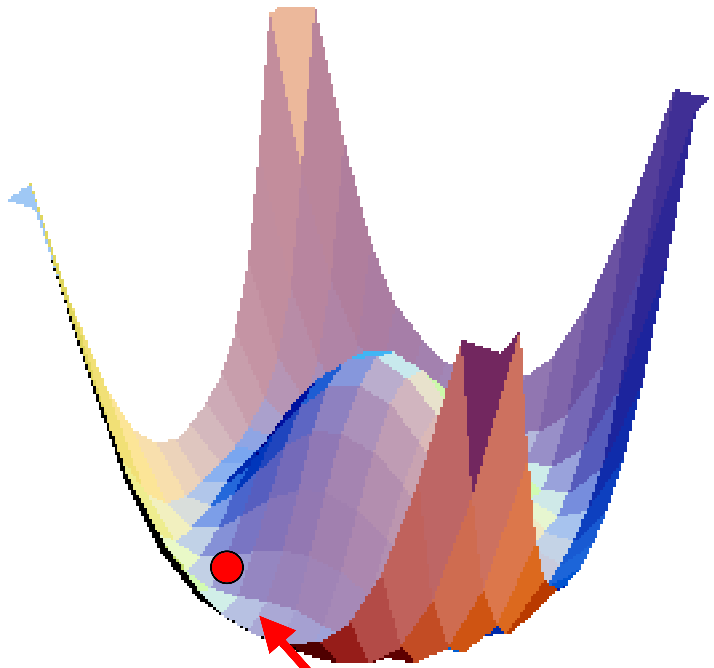
B $\mu^2(T) < 0$

Solution: $M_0^2 = -\mu^2/\lambda$





Here the actual symm. is rotation around z



Unique minimum: no SSB

A line of minima: SSB

The symmetry is broken when the system chooses one particular minimum point

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Goldstone Theorem: When SSB of a continous symmetry occurs there is a zero mass mode in the spectrum with the quantum numbers of the broken generator.

$$\Phi_i(x) \longrightarrow \Phi'_i(x) = U_{ij}\,\Phi_j(x)$$

$$\delta\phi_a \sim i\sum \varepsilon^A t^A_{ij}\phi_j \sim i\varepsilon t_{ij}\phi_j$$

$$U = \exp[i\sum_A t^A\varepsilon^A] \sim 1 + i\sum_A t^A\varepsilon^A + o(\varepsilon^2)$$

t^A: generators
ε^A: parameters

Hamiltonian density $\longrightarrow$ $H = \left|\partial_\mu\phi\right|^2 + V(\phi)$

ϕ^0: minimum of H (note constant: no gradients)

- minimum $\longrightarrow$ $\left.\frac{\partial V}{\partial\phi_i}\right|_{\phi=\phi^0} = 0$
- symmetry $\longrightarrow$ $\delta V = \frac{\partial V}{\partial\phi_i}\cdot\delta\phi_i = \frac{\partial V}{\partial\phi_i}t_{ij}\phi_j = 0$
- another derivative at the minimum $\longrightarrow$ $\left.\frac{\partial^2 V}{\partial\phi_k\delta\phi_i}\right|_{\phi=\phi^0} t_{ij}\phi_j^0 + \cancel{\left.\frac{\partial V}{\partial\phi_i}\right|_{\phi=\phi^0}} t_{ik} = 0$

$$\left.\frac{\partial^2 V}{\partial \phi_k \delta \phi_i}\right|_{\phi=\phi^0} t_{ij}\phi_j^0 = M_{ki}^2 t_{ij}\phi_j^0 = M^2\overrightarrow{(t\phi_0)} = 0$$

This is an eigenvalue equation for the (mass)^2 matrix M^2:

Either $\overrightarrow{(t\phi_0)} = 0$ for all t^A ⇒ All generators leave ϕ^0 ("the vacuum") inv. symmetry

Or for some t^A $\overrightarrow{(t\phi_0)} \neq 0$ ⇒ Non vanishing eigenvector of M^2 with zero eigenvalue Goldstone boson

For each broken generator t^A, there is a GB with the quantum numbers of t^A

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SSB: quantum versus classical

- For finite ‡ d.o.f. quantum effects remove degeneracy

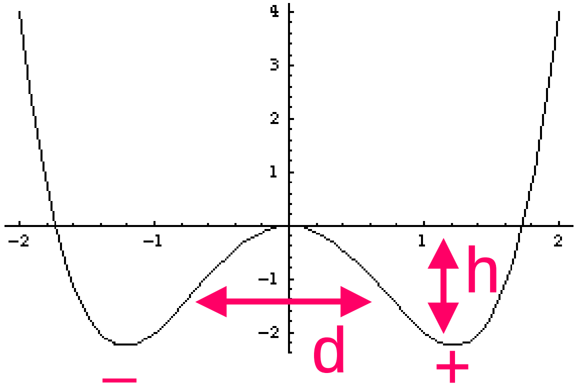
e.g. Schroedinger eqn.: $V(x) = -\mu^2 x^2 + \lambda x^4$

<+|V|+> = <-|V|-> = a
<+|V|-> = <-|V|+> = b

$$V = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \rightarrow$$

Eigenvectors:
~ |+>±|->
Eigenvalues:
= a±b

b~ exp[-dh] (tunnel)



Vacuum is unique!

While, for ∞ d.o.f. and ∞ volume

$<v|H|v'> = \delta_{vv'}$

and vacuum is degenerate

- Also, classical potential corrected by quantum effects

$$V_{eff} \sim -\mu^2\Phi^2 + \lambda\Phi^4 + \gamma\Phi^4(\log\Phi^2/\mu^2 + c) + \ldots$$

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Classical
tree level

Quantum corr's
loop expansion

SSB in gauge theories: Higgs mechanism

In general SSB $\longrightarrow$ Goldstone bosons with quantum numbers of broken generators t^A

$$M^2_{ki} = \left.\frac{\partial^2 V}{\partial \phi_k \delta \phi_i}\right|_{\phi=\phi^0}$$

$\boxed{M^2 t^A \Phi^0 = 0}$

$t^A \Phi^0 \neq 0$

In gauge theory with Higgs mechanism

Symmetry broken by vacuum expectation values (vev) of Higgs field (scalar fields otherwise Lorentz also broken)

$\longrightarrow$ No physical Goldstone bosons. Become 3rd helicity state of gauge bosons with t^A quantum numb's that take mass

The Higgs potential has an orbit of minima, and the Higgs fields, like magnetisation, take a particular direction

Symmetry restauration possible at high T (early Universe)

Simplest abelian U(1) model (Higgs)

"wrong" sign

$$L = -\frac{1}{4}F_{\mu\nu}^2 + \left|(\partial_\mu - ieA_\mu)\phi\right|^2 + \frac{1}{2}\mu^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$

Invariant under (U = exp[iQeε(x)]):

$$\begin{cases} A_\mu \Rightarrow A_\mu' = A_\mu + \partial_\mu \varepsilon(x) \\ \phi \Rightarrow \phi' = e^{ie\varepsilon(x)}\phi \end{cases}$$

($Q\phi=\phi$)

If $\phi^0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{\lambda}}$ (real $\neq$ 0) (ϕ^0 = constant=$\langle 0|\phi|0\rangle$)

one must shift (small oscill.s about field=0):

$$\phi(x) \Rightarrow \frac{\rho(x)+v}{\sqrt{2}}\exp[ie\chi(x)/\sqrt{2}] \qquad A_\mu \Rightarrow A_\mu + \frac{1}{v}\partial_\mu\chi(x)$$

($\langle 0|\rho|0\rangle = \langle 0|\chi|0\rangle = 0$)

$$L = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\rho^2A_\mu^2 + e^2\rho v A_\mu^2 + L_v(\rho)$$

mass term

No $\chi(x)$, A_μ massive
(same number of d.o.f.!)

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$$L = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}\mu^2|\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$

$$\phi^0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\phi(x) \Rightarrow \frac{\rho(x)+v}{\sqrt{2}}\exp[ie\chi(x)/\sqrt{2}]$$

$$L = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\rho^2A_\mu^2 + e^2\rho vA_\mu^2 + L_v(\rho)$$

$$L_v(\rho) = \frac{1}{2}\mu^2 \cdot \frac{(\rho(x)+v)^2}{2} - \frac{1}{4}\lambda \cdot \frac{(\rho(x)+v)^4}{4}$$

Expanding:

$$L_v(\rho) = \frac{1}{2}\rho^2\left(\frac{1}{2}\mu^2 - \frac{3}{4}\lambda v^2\right) + \ldots = \frac{1}{2}\rho^2\left(\frac{1}{2}\mu^2 - \frac{3}{2}\mu^2\right) + \ldots = -\frac{1}{2}\rho^2\mu^2 + \ldots$$

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The ρ mass has the right sign!

The Higgs mechanism was discovered in condensed matter physics. e.g.: Superconductor in Landau-Ginzburg approx'n

Free energy

Wrong sign

$$F = F_0 + \frac{1}{2}\vec{B}^2 + \frac{1}{4m}\left|(\vec{\nabla} - 2ie\vec{A})\phi\right|^2 - \alpha|\phi|^2 + \beta|\phi|^4$$

$|\phi|^2$: Cooper pair density (e-e-: charge -2e and mass 2m)

"Wrong" sign of α leads to $\phi \neq 0$ at minimum

- No propagation of massless phonons ($\omega = k\, v$)
- Mass term for A -> exponential decrease of B
  Inside the superconductor
  (Meissner effect)

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$L = L_{symm} + L_{Higgs}$

In general $\phi = \phi^i$ (several multiplets)

$$L_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) - [\bar{\psi}_L \Gamma \psi_R \phi + \text{h.c.}]$$

$$V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

No more than quartic for renormalisation

Only weak-isospin doublet Higgs ϕ contribute to fermion masses (ψ_L doublets, ψ_R singlets)

All non trivial repres.s break SU(2)xU(1) and give masses to $W^\pm$ and Z

Minimal model: only one Higgs ϕ doublet

Fermion masses: $[\bar{\psi}_L \Gamma \psi_R \phi + \mathrm{h.c.}]$

doublet singlet doublet

With one Higgs doublet: $g_f \bar{\Psi}_{fL} \Psi_{fR} \phi \longrightarrow m_f = g_f v$

Ugly: each mass one new coupling

Large mass ratios (m_t/m_e, m_t/m_u...) imply large coupling ratios

Fermion masses demand a more fundamental theory (at M_{Pl}?)

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Gauge Boson Masses

$$L_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) + \ldots.$$

$$D^\mu\phi = \left[\partial_\mu + ig\sum_A t^A W^A_\mu + ig'\frac{Y}{2}B^\mu\right]\phi$$

Recall:
$W_3 = c_W Z + s_W A$
$B = -s_W Z + c_W A$
$tg\theta_W = s_w/c_w = g'/g$

Zero photon mass -> Q unbroken
$Qv = (t^3 + Y/2)v = 0$: only neutral components of ϕ have vev$\neq 0$ → e.g for a doublet:

$$\phi = \begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix} = \begin{bmatrix}0\\ v\end{bmatrix} \equiv v$$

$|t^+ v|^2 = v^2$
$|t^3 v|^2 = v^2/4$

- $$m_W^2 W_\mu{}^\dagger W^\mu = g^2\left|\frac{t^+}{\sqrt{2}}v\right|^2 W_\mu{}^\dagger W^\mu$$
- $$\frac{1}{2}m_Z^2 Z_\mu Z^\mu = \left|\left(g c_W t^3 - g' s_W \frac{Y}{2}\right)v\right|^2 Z_\mu Z^\mu =$$

$Qv=0$ → $$= (g c_W + g' s_W)^2 |t^3 v|^2 Z_\mu Z^\mu = \left(\frac{g}{c_W}\right)^2 |t^3 v|^2 Z_\mu Z^\mu$$

Thus, for one doublet ϕ: $$\boxed{m_W^2 = \frac{1}{2}g^2 v^2 = m_Z^2\cos^2\theta_W}$$

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For doublet ϕ:

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \qquad \text{(Tree level)}$$

In general:

$$\rho_0 = \frac{\sum_\phi \frac{1}{2} \langle t^+ t^- + t^- t^+ \rangle v_\phi^2}{\sum_\phi 2 \langle t^3 t^3 \rangle v_\phi^2} = \frac{\sum_\phi \langle t(t+1) - t^3 t^3 \rangle v_\phi^2}{\sum_\phi 2 \langle t^3 t^3 \rangle v_\phi^2} \quad \begin{matrix} < 1 \\ \geq 1 \end{matrix}$$

In general, at tree level, $\rho_0 = 1 + \Delta\rho_0$. In the SM with radiative corrections: $\rho_{SM} = (1 + \Delta\rho_{SM})\, \rho_0$

$$\Delta\rho_{SM} = \frac{3 G_F m_t^2}{8\pi^2 \sqrt{2}} + \ldots.. \quad \leq 1\%$$

Exp. puts a strong bound on $\Delta\rho_0$:

$(\rho_0)_{Exp} = 1.0004 \pm 0.0006$
$(m_H \sim 115$ GeV) PDG'03

Note: $v = 2^{-3/4} G_F^{-1/2} \sim 174$ GeV

$$m_W^2 = \frac{1}{2} g^2 v^2 \quad \text{and} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

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Higgs couplings

H: physical Higgs field

$$\phi(x) = \begin{bmatrix} \phi^+(x) \\ \phi^0(x) \end{bmatrix} = \begin{bmatrix} 0 \\ v + \frac{H(x)}{\sqrt{2}} \end{bmatrix}$$

Note: normalisation

Charged $\partial_\mu \phi^\dagger \partial^\mu \phi$

Neutral $\frac{1}{2}\partial_\mu H \partial^\mu H$

$$D^\mu \phi = \left[\partial_\mu + ig\sum_A t^A W^A_\mu + ig'\frac{Y}{2}B^\mu\right]\phi$$

Recall:

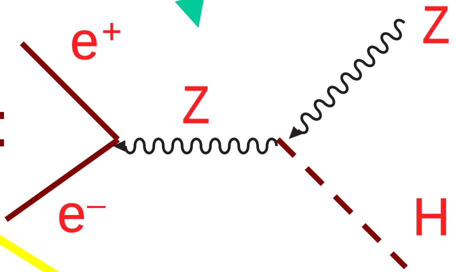
$$m_W^2 = \frac{1}{2}g^2 v^2 = m_Z^2 \cos^2\theta_W$$

$$L_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) + \ldots\ldots\ldots = \frac{1}{2}\partial_\mu H \partial^\mu H + L(H, W, Z)$$

$$L(H,W,Z) = g^2\frac{v}{\sqrt{2}}W_\mu{}^\dagger W^\mu H + \frac{g^2}{4}WWHH + g^2\frac{v}{2\sqrt{2}c_W^2}ZZH + \frac{g^2}{8c_W^2}ZZHH$$

$$g^2\frac{v}{\sqrt{2}}W_\mu{}^\dagger W^\mu H = g m_W W_\mu{}^\dagger W^\mu H$$

LEP:



Fermions (after diag.)

$$\frac{m_f}{v}\bar{\psi}_{fL}\psi_{fR}\frac{H}{\sqrt{2}} \sim 2^{1/4}G_F^{1/2}m_f\bar{\psi}_{fL}\psi_{fR}H$$

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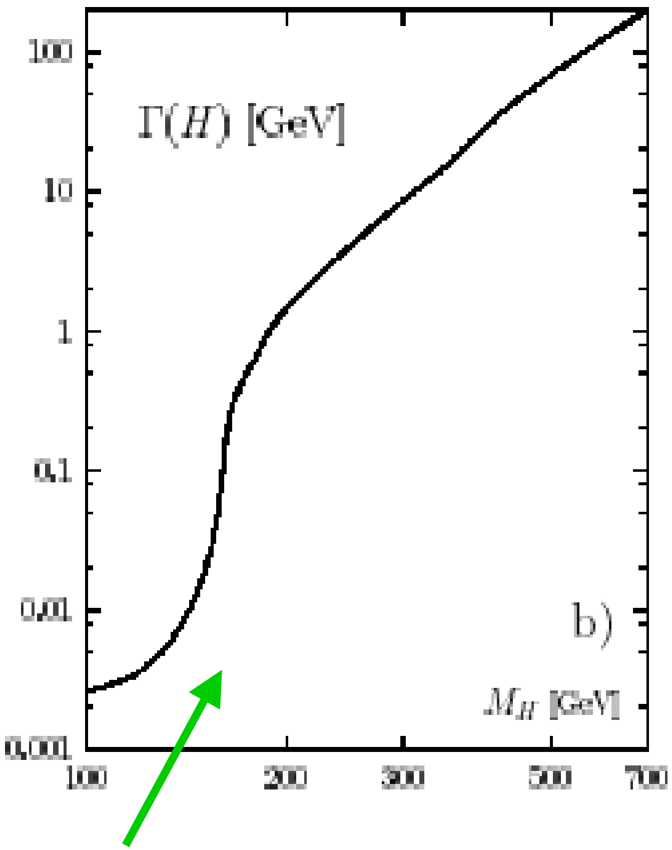
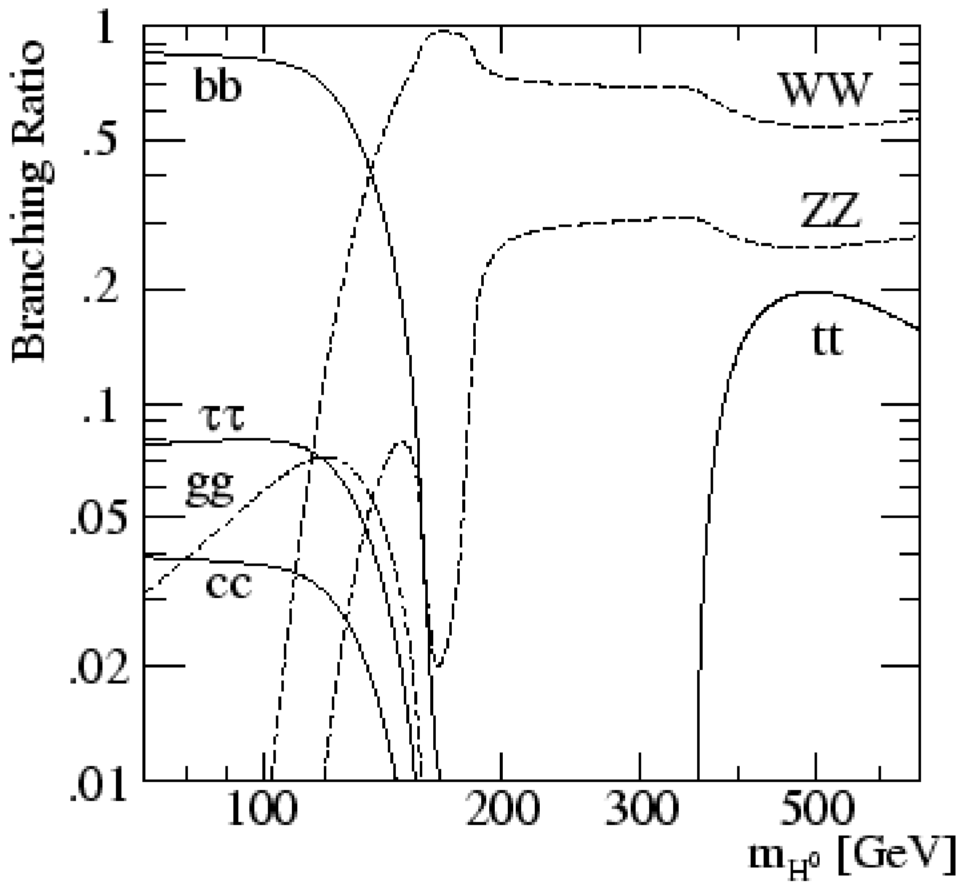
Higgs width and branching ratios





Γ_H: ~few MeV near the LEP limit,
~few GeV for intermediate mass, ~1/2$(m_H)^3$
(Γ_H, m_H in TeV) for heavy mass.

Note

- In spite of $m_D \sim m_\tau$, B(H->$\tau\tau$) ~ 3B(H->cc)

  Due to QCD running masses m_c-> $m_c(m_H)$ ~ 0.6 GeV

- In spite of $m_t > m_W$, B(H->WW)~3-4 B(H->tt) for heavy H

  Due to behaviour of W polarization sums

$(k+k')^2 = m_H^2$

$$\sum_{A,B} e_\mu^{A*} e_\nu^{A} e^{B\mu *} e^{B\nu} = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right)\left(-g^{\mu\nu} + \frac{k'^\mu k'^\nu}{m_W^2}\right) = \frac{1}{4}\left(\frac{m_H}{m_W}\right)^4 - \left(\frac{m_H}{m_W}\right)^2 + 3$$

and Γ(H->tt)~β_t^3 (P-wave), Γ (H->WW)~β_W

$\beta_i^2 = 1 - 4 m_i^2 / m_H^2$

$$\Gamma_t = N_C \frac{g^2}{32\pi}\left(\frac{m_t}{m_H}\right)^2 \beta_t^3 m_H$$

$$\Gamma_W = \frac{g^2}{64\pi}\left(\frac{m_H}{m_W}\right)^2 \beta_W m_H \left[1 - \frac{4 m_W^2}{m_H^2} + 12\left(\frac{m_W}{m_H}\right)^4\right]$$

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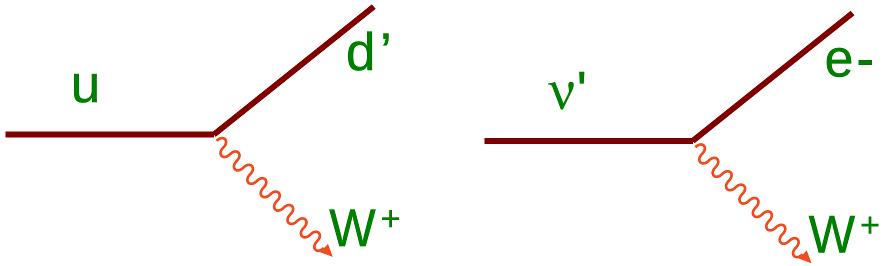
Quarks and leptons exist in different flavours

within one family and across families

$$\begin{bmatrix} u & u & u & \nu_e \\ d & d & d & e \end{bmatrix} \quad \begin{bmatrix} c & c & c & \nu_\mu \\ s & s & s & \mu \end{bmatrix} \quad \begin{bmatrix} t & t & t & \nu_\tau \\ b & b & b & \tau \end{bmatrix}$$

At tree level only charged-current weak int’s change flavour



QCD

QED

Neutral curr.s
GIM needed

Higgs
only 1 Higgs
per charge sector

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Fermion masses

$$L_{Higgs} = \ldots - [\bar{\psi}_L \Gamma \psi_R \phi + \text{h.c.}]$$

Yukawa matrix

Only Higgs doublets ϕ can contribute

Masses arise when ϕ is replaced by its vev v

If more doublets

$$M_\psi = \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L \qquad M = \Gamma v \; (= \Sigma_i \Gamma^i v^i)$$

By separate rotations of the L and R fields one can make M_ψ real and diagonal: $\quad U^+_{L,R} U_{L,R} = U_{L,R} U^+_{L,R} = 1$

$\psi_L^{diag} = U_L \psi_L$

$\psi_R^{diag} = U_R \psi_R$

$$\Longrightarrow \quad \boxed{M_{diag} = U^+_L M U_R = U^+_R M^+ U_L}$$

M commutes with Q $\Longrightarrow$ Separate rotations for up, down, ch. leptons, ν's

e.g $U^u_{L,R}$, $U^d_{L,R}$ etc

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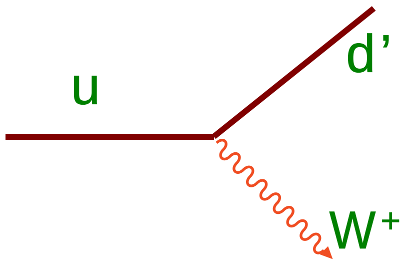
CKM Matrix



$\Longrightarrow$ Left-doublets $\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$

W-eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

mass eigenstates

V_{CKM} unitary (change of basis): $V^+V = VV^+ = 1$

Neutral current diagonal in both bases:

$$(\bar{d}', \bar{s}', \bar{b}') \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (\bar{d}, \bar{s}, \bar{b}) \underbrace{V^+V}_{1} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

or

$$\bar{d}'d' + \bar{s}'s' + \bar{b}'b' = \bar{d}d + \bar{s}s + \bar{b}b$$

An equal number of up and down needed

Glashow-Iliopoulos-Maiani '70

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The neutral current couplings are:

$$\frac{g}{\cos\theta_W}\bar{\psi}\gamma_\mu[t_L^3\cdot\frac{1-\gamma_5}{2}+t_R^3\cdot\frac{1+\gamma_5}{2}-Q\sin^2\theta_W]\psi Z^\mu$$

zero for q&l

For GIM to work all states with equal Q must have the same t^3_L and t^3_R

was not true in old Cabibbo theory:
$(u,d_C)_L$ doublet , s_{CL} singlet

$d_C=\cos\theta_C d+\sin\theta_C s$
$s_C=-\sin\theta_C d+\cos\theta_C s$

In the t^3 part there is $\bar{d}_C d_C$ but not $\bar{s}_C s_C$ and the FC terms $\cos\theta_C\sin\theta_C(\bar{d}s+\bar{s}d)$ are present

The charged current couplings are:

$$\frac{g}{2\sqrt{2}}\bar{u}\gamma^\mu(1-\gamma_5)d\cdot W_\mu \Longrightarrow V_{CKM}=U_L^{u\dagger}U_L^d$$

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Note: kinetic terms diagonal in both bases $\overline{u_L}i\gamma^\mu\partial_\mu u_L+\ldots$

More Higgs doublets?

Beware of FCNC, e. g.

To avoid FCNC (and CP viol) in the Higgs sector you need to have at most 1 Higgs for u-type quarks,
1 Higgs for d-type quarks, 1 Higgs for e-type leptons,
(1 Higgs for ν-type leptons)

In fact diagonalisation of masses $M = \Gamma^1 v^1 + \Gamma^2 v^2 + \ldots$ guarantees diagonalisation of couplings $\Gamma^1 \phi^1 + \Gamma^2 \phi^2 + \ldots$ only for a single term (then masses and couplings are proportional)

For example, in SUSY models there are H^u and H^d that give mass to $t^3 = +1/2$ and $t^3 = -1/2$ states, respectively.

Counting Parameters in V_{CKM}

Assume there are N down quarks: D' = V D, V ~ NxN unitary matrix

V ~ NxN unitary matrix → N^2 complex numbers - N^2 unitary conditions → N^2 real parameters

Freedom of phase def.:
2N quarks -> 2N - 1 relative phases
(currents $\bar{\Psi}\Psi$ insensitive to overall phase)

TOTAL:
$N^2-(2N-1)=(N-1)^2$
physical parameters

cfr: a NxN orthogonal matrix has N(N-1)/2 parameters

$OO^T=O^TO=1 \rightarrow N^2-N(N+1)/2=N(N-1)/2$

N	$(N-1)^2$	N(N-1)/2	angles	phases
2	1	1	1 (θ_C)	0
3	4	3	3	1
4	9	6	6	3

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A phase in V_{CKM} $\Longrightarrow$ CP Violation

$$\bar{U}_L\gamma_\mu V_{CKM} D_L W^\mu + \bar{D}_L\gamma_\mu V^+_{CKM} U_L W^{+\mu}$$ h.c.

Parity: $P\psi_L P^{-1} = P\psi_R$

Charge conj.: $C\psi_L C^{-1} = C\bar{\psi}_R{}^T$

Time Rev.: $T\psi_L T^{-1} = TK\psi_L$

$\bar{\psi}$: creates f , ann. $\bar{f}$

ψ : ann. f, creates $\bar{f}$

Complex conj. of c-numbers: T antiunitary

$Tc\psi T^{-1}=c^*T\psi T^{-1}$ $[x,p]=i\hbar$

$$(CP)\bar{U}_L\gamma_\mu V_{CKM} D_L W^\mu (CP)^{-1} = \bar{D}_L\gamma_\mu V^T_{CKM} U_L W^{+\mu}$$

If V is real then $V^T=V^+$ and CP invariance holds, otherwise is violated. Note CPT always holds:

$$(CPT)\bar{U}_L\gamma_\mu V_{CKM} D_L W^\mu (CPT)^{-1} = \bar{D}_L\gamma_\mu V^+_{CKM} U_L W^{+\mu}$$

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Any Lorentz inv, hermitian, local L is CPT inv.

A simple example

Three charged scalar fields A, B, C for the decay A -> B+C

$L = \lambda AB^{+}C^{+} + \text{h.c.} = \lambda AB^{+}C^{+} + \lambda^{*}A^{+}BC$

All products are normal-ordered

$(CP)L\ (CP)^{-1} = \lambda A^{+}BC + \lambda^{*}AB^{+}C^{+}$ (Under CP $A \leftrightarrow A^{+}$ etc)

$(TCP)L\ (TCP)^{-1} = \lambda^{*}A^{+}BC + \lambda AB^{+}C^{+}$

TCP is always true while CP invariance holds for λ real

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

Maiani

$s_{12} = \sin\theta_C$

$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \ldots & \ldots & c_{13}\,s_{23} \\ \ldots & \ldots & c_{13}\,c_{23} \end{bmatrix}$$

PDG'02

$s_{12} \sim 0.2196 \pm 0.0026$

$s_{23} \sim (41.2 \pm 2.0)10^{-3}$

$s_{13} \sim (3.6 \pm 0.7)10^{-3}$

Wolfenstein parametrisation:

$s_{12} = \lambda$

$s_{23} = A\lambda^2$

$s_{13}\, e^{-i\delta} = A\lambda^3(\rho - i\eta)$

$$V \sim \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + o(\lambda^4)$$

$A = 0.85 \pm 0.05$

$(\rho^2 + \eta^2)^{1/2} = 0.40 \pm 0.08$

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More precisely

$s_{12} = \lambda$
$s_{23} = A\lambda^2$
$s_{13}\, e^{-i\delta} = A\lambda^3(\rho - i\eta)$

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4, \qquad V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2),$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4, \qquad V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\varrho + i\eta)],$$

$$V_{us} = \lambda + \mathcal{O}(\lambda^7), \qquad V_{ub} = A\lambda^3(\varrho - i\eta), \qquad V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^8),$$

$$V_{ts} = -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\varrho + i\eta)], \qquad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta})$$

$\bar{\rho} = \rho(1 - \lambda^2/2)$
$\bar{\eta} = \eta(1 - \lambda^2/2)$

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Unitarity Triangles

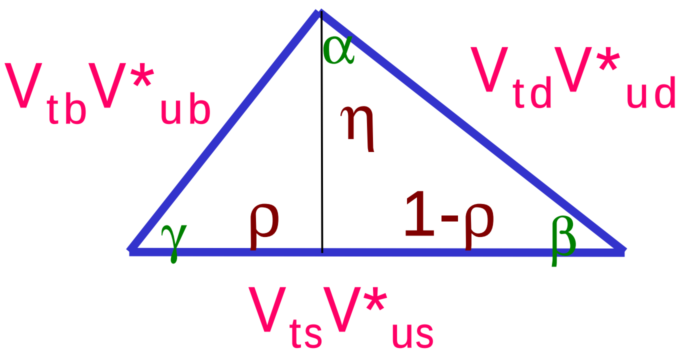
$VV^{+} = 1 \rightarrow V_{hk}V^{*}_{hl} = \delta_{kl}$

For example: $V_{ta}V^{*}_{ua} = 0$

a → d s b

$$A\lambda^3(1-\rho-i\eta) - A\lambda^3 + A\lambda^3(\rho+i\eta) = 0$$

Can be drawn as a triangle



(other 5 triangles are either equivalent $[V_{ab}V^{*}_{ad}]$ or too flat)
All have same area ~J

In SM all CP violation is proportional to J

$$2 \bullet \text{Area} = J = \eta A^2\lambda^6 \sim \eta(0.85)^2(0.224)^6 \sim \eta\ 9 \cdot 10^{-5}$$

$J \sim s_{12}\, s_{13}\, s_{23} \sin\delta$ Jarlskog

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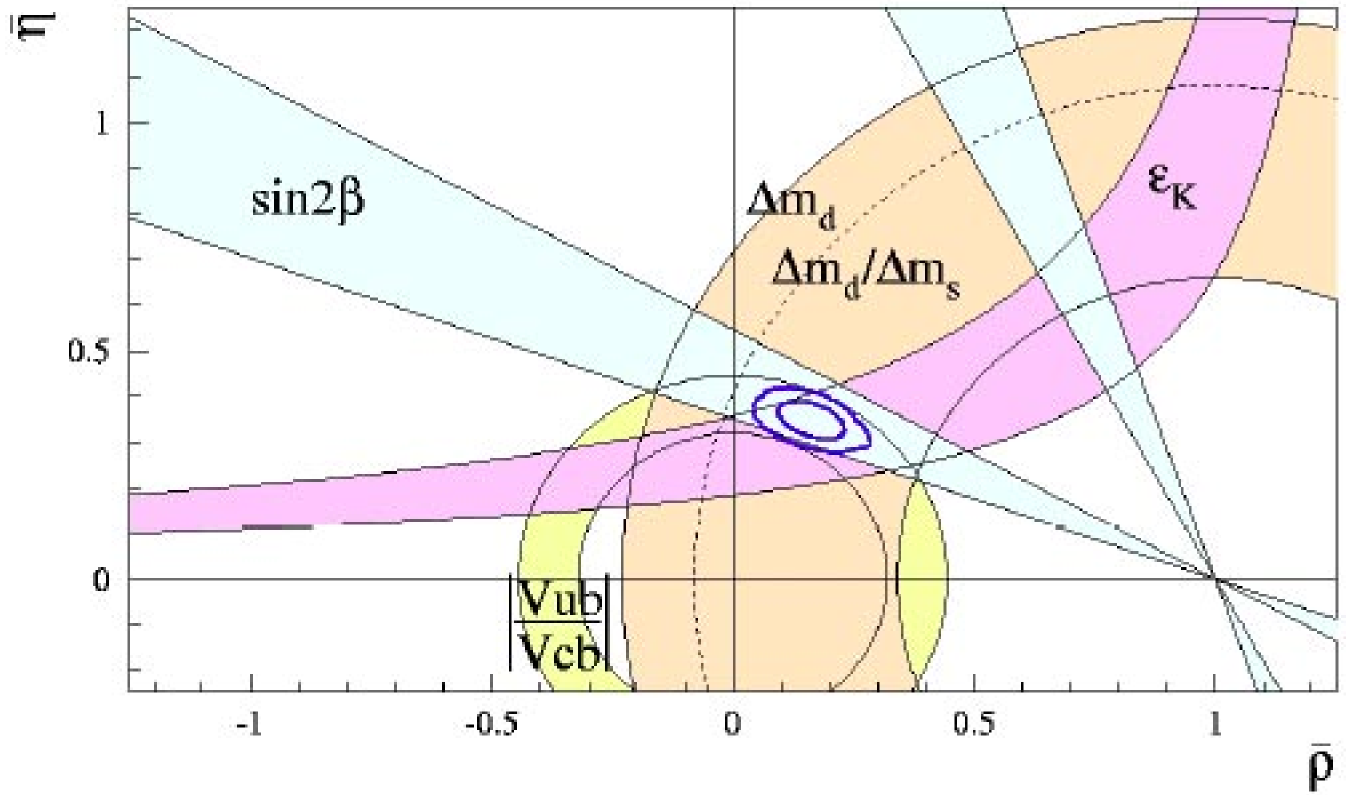
Note: $V_{td} = |V_{td}|e^{-i\beta}$, $V_{ub} = |V_{ub}|e^{-i\gamma}$

Lubicz, Durham '03, hep-ph/0307195



$\bar{\rho} = \rho(1 - \lambda^2/2) = 0.178 \pm 0.046$

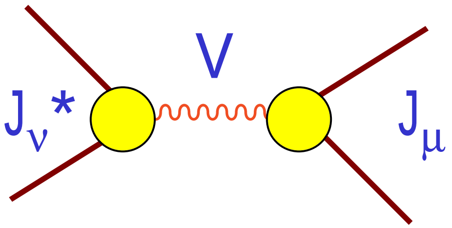
$\bar{\eta} = \eta(1 - \lambda^2/2) = 0.341 \pm 0.028$

Gauge theories broken by the Higgs mechanism are renormalisable ‘t Hooft, Veltman

Masses are given to W, Z and fermions while gauge Ward identities and current conservation remain valid.

Essential for renormalisation!

e.g. massive V propagator (V=W,Z)



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}}{q^2 - m_W^2}$$

Bad UV behaviour

But current conservation $q_\mu J^\mu = 0$ dumps it

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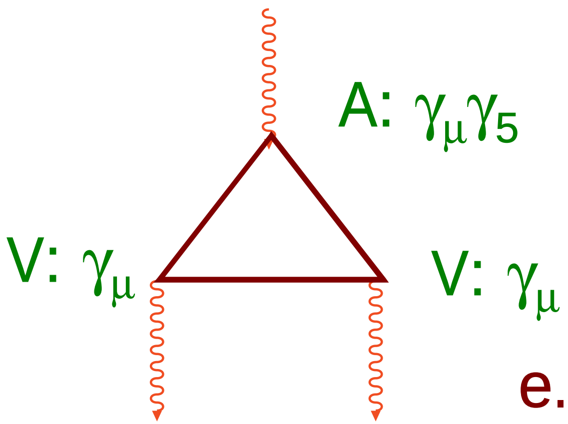
Current conservation crucial for renormalisation

But beware of chiral anomalies   Adler,Bell, Jackiw

A remarkable cancellation occurs   Bouchiat, Iliopoulos, Meyer



We need $\mathrm{Tr}[\underset{A}{t^3}\,\underset{V}{(t^3-2Qs^2_W)}\,\underset{V}{(t^3-2Qs^2_W)}]=0$

In fact it is true! For each family

e.g. $\mathrm{Tr}[t^3Q^2] = \underbrace{\frac{1}{2}\cdot 3\cdot\frac{4}{9}}_{u} - \underbrace{\frac{1}{2}\cdot 3\cdot\frac{1}{9}}_{d} - \underbrace{\frac{1}{2}\cdot 1}_{e} + \underbrace{\frac{1}{2}\cdot 0}_{\nu} = 0$

colour

Similarly for $\mathrm{Tr}[t^3t^3Q] = \mathrm{Tr}[t^3t^3t^3] = 0$

Great!! But why??   Grand unification?   SU(5): 5 -> $[ddde^+\bar{\nu}]$

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Anomaly

In QFT when a symmetry of the classical theory is broken by quantisation, regularisation and renormalisation

Examples

- Scale A. -> Breaking of scale inv. due to reg./ren. that introduces a mass scale (cut-off, subtraction point or....)
  massless QED, QCD

- Axial A. -> Breaking of chiral symmetry $\psi' = \exp(i\gamma_5\theta)\psi$ due to a clash of reg./ren. with gauge inv.

$$\boxed{\partial_\mu j_5^\mu = \frac{\alpha}{2\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}}$$

$$\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Important for $\pi^0 \to \gamma\gamma$, polarized DIS,....

Beyond tree level: radiative corrections

From the tree level relations $\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}$ and $g^2 s_W^2 = e^2 = 4\pi\alpha$

$$\Rightarrow \quad s_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F} \cdot \frac{1}{m_W^2}$$

$$s_W^2 \equiv sin^2\theta_W$$

Combining with $\frac{m_W^2}{m_Z^2\cos^2\theta_W} = \rho_0 = 1 \quad \Rightarrow \quad s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$

one obtains: $\left(1 - \frac{m_W^2}{m_Z^2}\right) \cdot m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F}$

Large pure QED effect

With radiative corr's: $\left(1 - \frac{m_W^2}{m_Z^2}\right) \cdot m_W^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F} \cdot \frac{1}{1 - \Delta r_W}$

$$\frac{m_W^2}{m_Z^2\cos^2\theta_W} = (1 + \Delta\rho_m)$$

Depends on def. of $\sin^2\theta_W$ beyond tree level

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$\sin^2\theta_W$ is usually defined from the Z->$\mu\mu$ vertex:

$$\frac{g}{2\cos\theta_W}\bar{\psi}\gamma_\mu(g_V^f - g_A^f\gamma_5)\psi Z^\mu$$

$$\begin{cases} g_A^f = \pm\frac{1}{2} \\ g_V^f/g_A^f = 1-4|Q^f|\sin^2\theta_W \end{cases} \Longrightarrow \quad g_A^{\mu 2} \equiv \frac{1}{4}(1+\Delta\rho) \qquad g_V^\mu/g_A^\mu \equiv 1-4\sin^2\theta_{eff}$$

$\sin^2\theta_{eff}$ differs from $s^2{}_0$ defined as:

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F}\cdot\frac{1}{m_Z^2}$$

$$\left\{ \text{Recall:} \quad \left(1-\frac{m_W^2}{m_Z^2}\right)\cdot m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F} \right\}$$

by a rad. corr.:

$\sin^2\theta_{eff} = (1+\Delta k')\, s^2{}_0$

Δr_W, $\Delta\rho$, $\Delta k'$ at one loop all contain terms of order:
$G_F m^2{}_W[1,\ m^2{}_t/m^2{}_W,\ \log(m^2{}_H/m^2{}_W)]$

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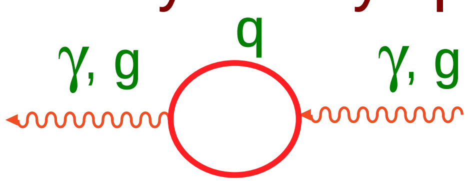
m_t, m_H do not decouple!

In the standard EW theory heavy loops do not decouple

Decoupling: for M -> ∞ we can drop diagrams with internal M lines

For example: running of α, α_s not affected by heavy quarks



Conditions for decoupling: Applequist, Carazzone

- The theory with no M should still be renorm.
- Couplings should not blow up with M -> ∞

In QED, QCD one can decouple m_t

In EW sector one cannot decouple m_t, m_H:

* breaking of gauge inv. (t-b doublet, $G_F(m^2_t-m^2_b)$)

* couplings of longitudinal W, Z grow with masses (Higgs mechanism)

* ...

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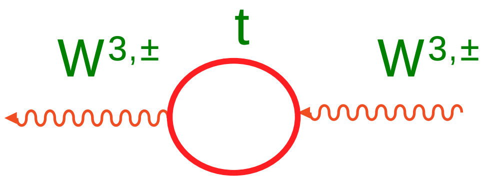
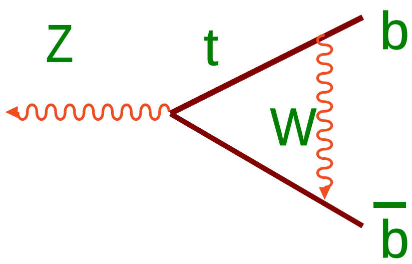
One-loop diagrams leading to $G_F m^2_t$ terms:

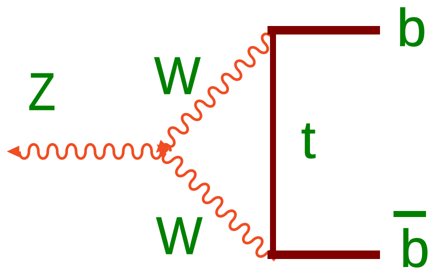


Δr_W, $\Delta\rho$, $\Delta k'$

Enough sensitivity
to correctly estimate
m_t from rad. corr.s

Note: self-energies universal. All heavy particles enter.





ε_b

At one-loop $G_F m^2_H$ terms are absent. While $m_t >> m_b$
directly breaks SU(2), Higgs couplings are invariant
in lowest order. At two-loops $(G_F m^2_H)^2$ terms are present

Veltman, Van der Bij

This is unfortunate: small sensitivity of rad. corr.
to m_H -> $G_F m^2_W \log(m^2_H/m^2_W)$

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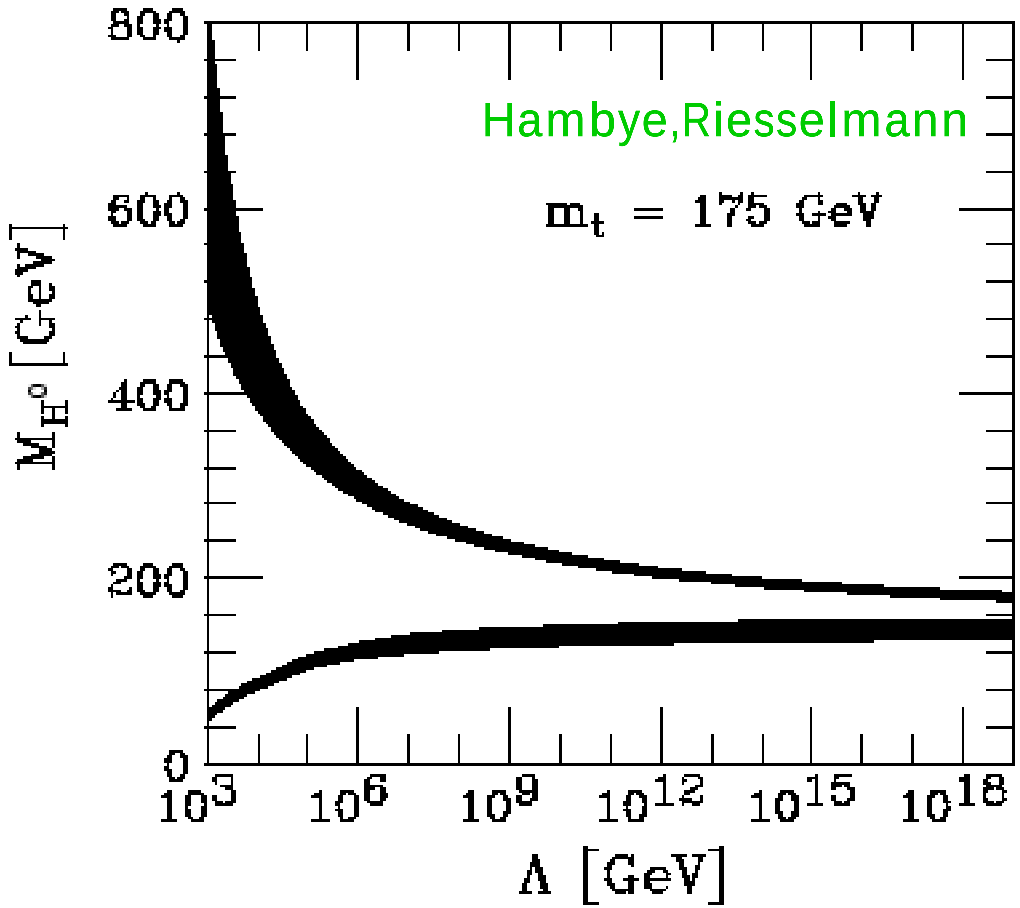
Theoretical bounds on the SM Higgs mass

Λ: scale of new physics beyond the SM

Upper limit: No Landau pole up to Λ

Lower limit: Vacuum (meta)stability



If the SM would be valid up to M_{GUT}, M_{Pl} then m_H would be limited in a small range

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Higgs potential

"Wrong" sign

Classic: $V[\phi] = -\mu^2\phi^2 + \lambda\phi^4$ $\mu^2>0,\ \lambda>0$

$$\phi \Rightarrow v + \frac{H}{\sqrt{2}} \quad \Longrightarrow \quad v^2 = \frac{\mu^2}{2\lambda} = \frac{m_H^2}{4\lambda}$$

Quantum loops: $\lambda\phi^4 \Rightarrow \lambda\phi^4\left(1 + \gamma \ln\frac{\phi^2}{\Lambda^2} + \dots\right) \xrightarrow{\text{RG}} \lambda(\Lambda)\phi'^4(\Lambda)$

(Ren. group improved pert. th)

$$\phi' = [\exp\int\gamma(t)dt]\phi$$

Running coupling

$t=\ln\Lambda/v$ h_t=top Yukawa

$$\frac{d\lambda(t)}{dt} = \beta_\lambda(t) = const[\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + small]$$

Initial conditions (at Λ=v) $\lambda_0 = \frac{m_H^2}{4v^2}$ and $h_{0t} = \frac{m_t}{v}$

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Running coupling $t = \ln\Lambda/v$ h_t=top Yukawa

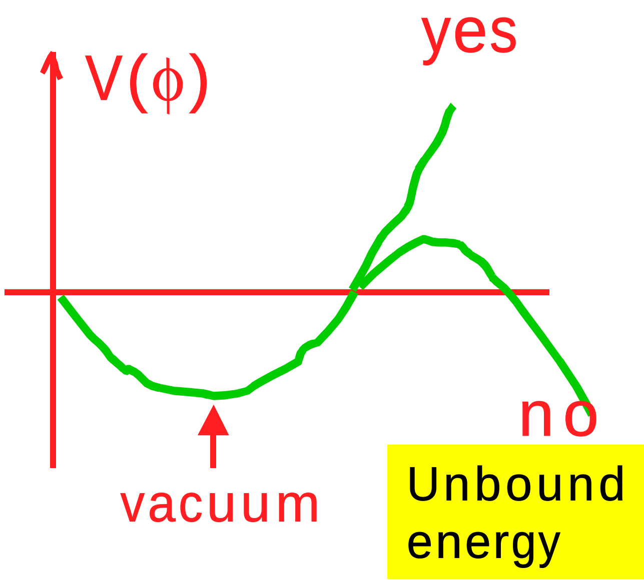
$$\frac{d\lambda(t)}{dt} = \beta_\lambda(t) = const[\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + small]$$

Initial conditions (at Λ=v) $\lambda_0 = \frac{m_H^2}{4v^2}$ and $h_{0t} = \frac{m_t}{v}$

Too small m_H? h_t wins, $\lambda(t)$ decreases.
But $\lambda(t)$ must be >0 below Λ for the vacuum to be stable

$\Rightarrow$ $m_H \geq$ ~135 GeV if $\Lambda \sim M_{GUT}$

(or at least metastable with lifetime $\tau > \tau_{Universe}$)



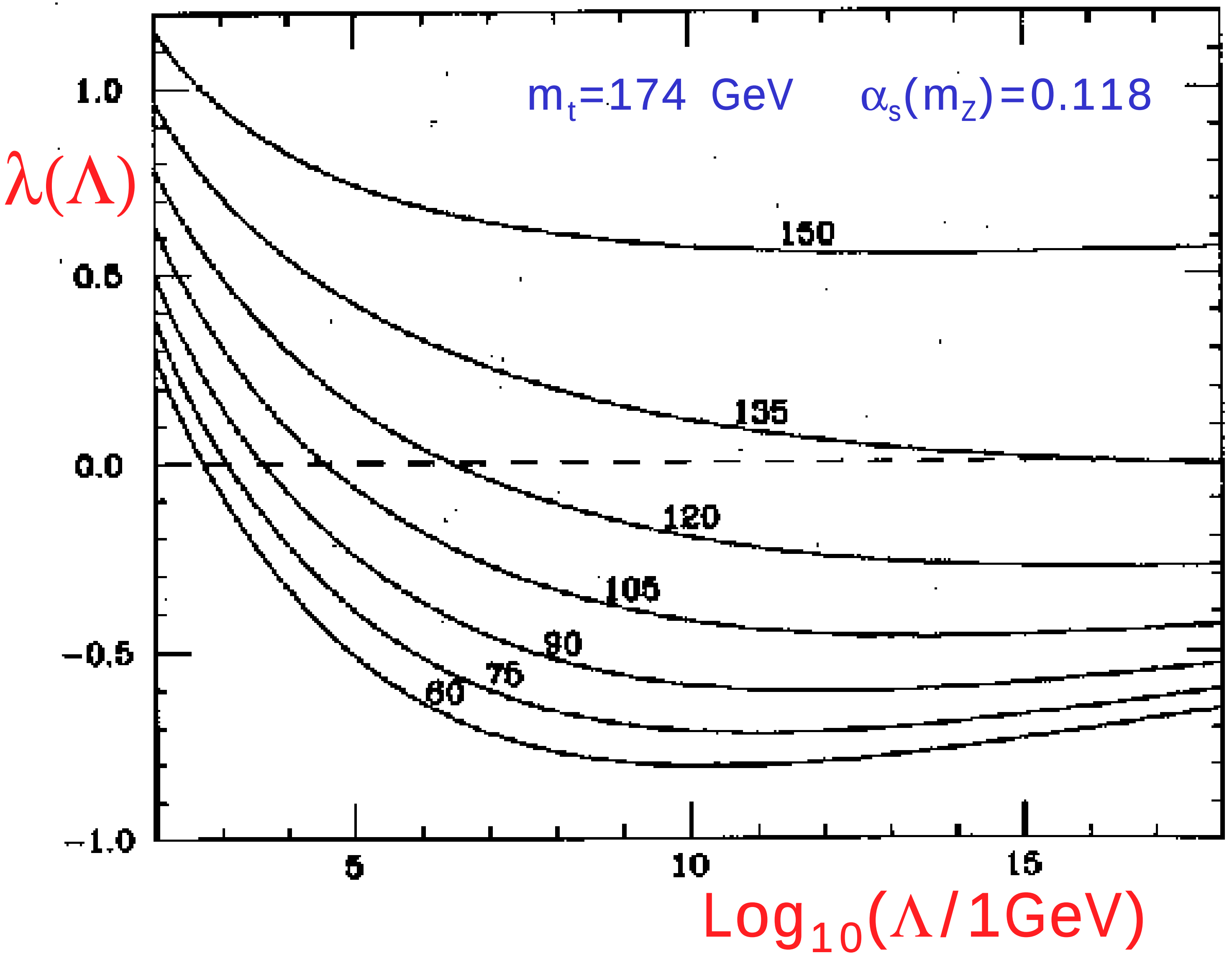
Cabibbo et al, Sher, Altarelli, Isidori

stability $$m_H(\mathrm{GeV}) > 133 + 2.0\,[m_t(\mathrm{GeV}) - (175 \pm 2)] - 1.6\left[\frac{\alpha_s(m_Z) - 0.118}{0.002}\right]$$

metastability $$m_H(\mathrm{GeV}) > 117 + 2.9\,[m_t(\mathrm{GeV}) - (175 \pm 2)] - 2.5\left[\frac{\alpha_s(m_Z) - 0.118}{0.002}\right]$$

Isidori, Ridolfi, Strumia

Altarelli, Isidori



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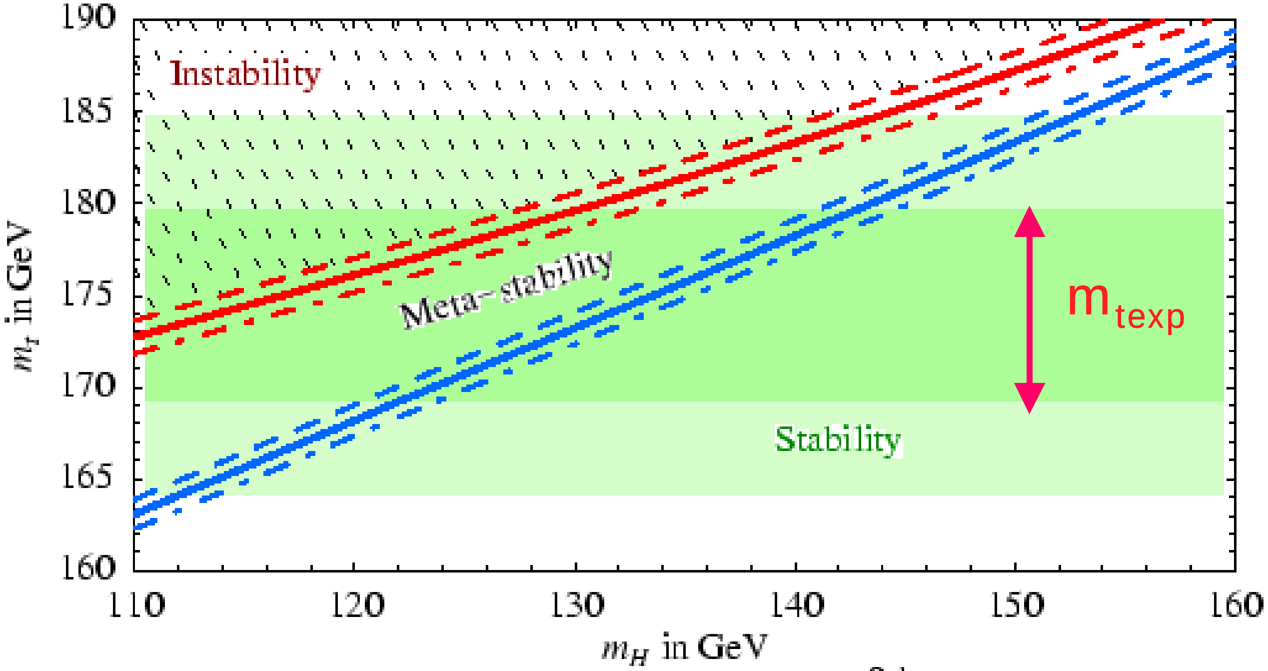
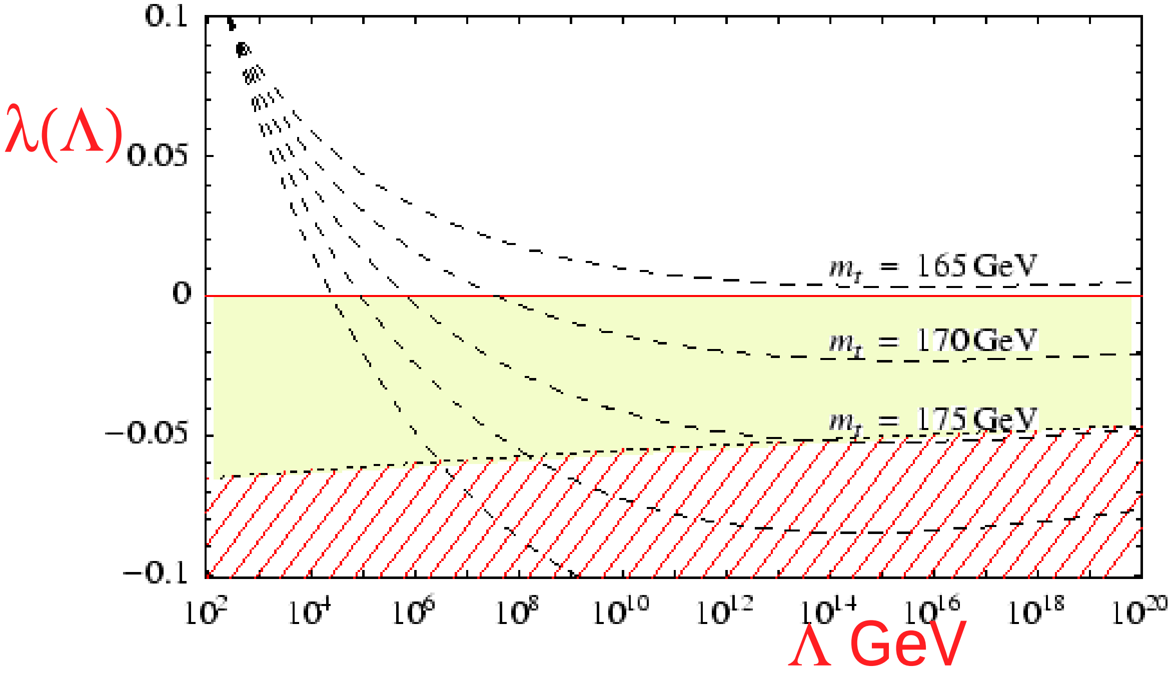


Isidori, Ridolfi, Strumia

Here m_H=115 GeV
$\alpha_s(m_Z)$=0.118



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Running coupling $t = \ln \Lambda / v$ h_t=top Yukawa

$$\frac{d\lambda(t)}{dt} = \beta_\lambda(t) = const[\lambda^2 + 3\lambda h_t^2 - 9h_t^4 + small]$$

b

Initial conditions (at L=v) $\lambda_0 = \frac{m_H^2}{4v^2}$ and $h_{0t} = \frac{m_t}{v}$

Too large m_H? λ^2 wins, $\lambda(t)$ increases.

$$\lambda(t) \sim \frac{\lambda_0}{1 - b\lambda_0 t}$$

Landau pole

The upper limit on m_H is obtained by requiring that no Landau pole occurs below Λ

$m_H \leq$ ~180 GeV if $\Lambda \sim M_{GUT}$
~ 600-800 GeV if $\Lambda \sim o(TeV)$

Rather than a bound says where non pert effects are important

Caution: near the pole pert. theory inadequate.
Simulations on the lattice appear to confirm the bound
Kuti et al, Hasenfratz et al, Heller et al

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Precision tests of the SM

Input parameters:
α, G_F, m_Z, m_{flight}, $\alpha_s(m_Z)$, m_t, m_H

in practice replaced by $\alpha(m_Z)$

Some are well known
α, G_F, m_Z
Some are less precise
$\alpha(m_Z)$, $\alpha_s(m_Z)$, m_t
m_H is unknown

Computed rad corr:
- complete 1-loop diagrams
- ren group improvements (large logs)
- Dyson resumm's of some large terms
- selected dominant 2-loop corr's.

  eg $G_F m_t^2 \alpha_s$, $G_F^2 m_t^4$, $G_F^2 m_H^2$....

Precision data: Γ_Z, R_h, σ_h, R_b, A^l_{FB}, A^τ_{pol}, A_{LR}, A^b_{FB}, m_W, Q_{APV}....

Output: check consistency of SM, constrain m_H...

Status of the SM Higgs fit

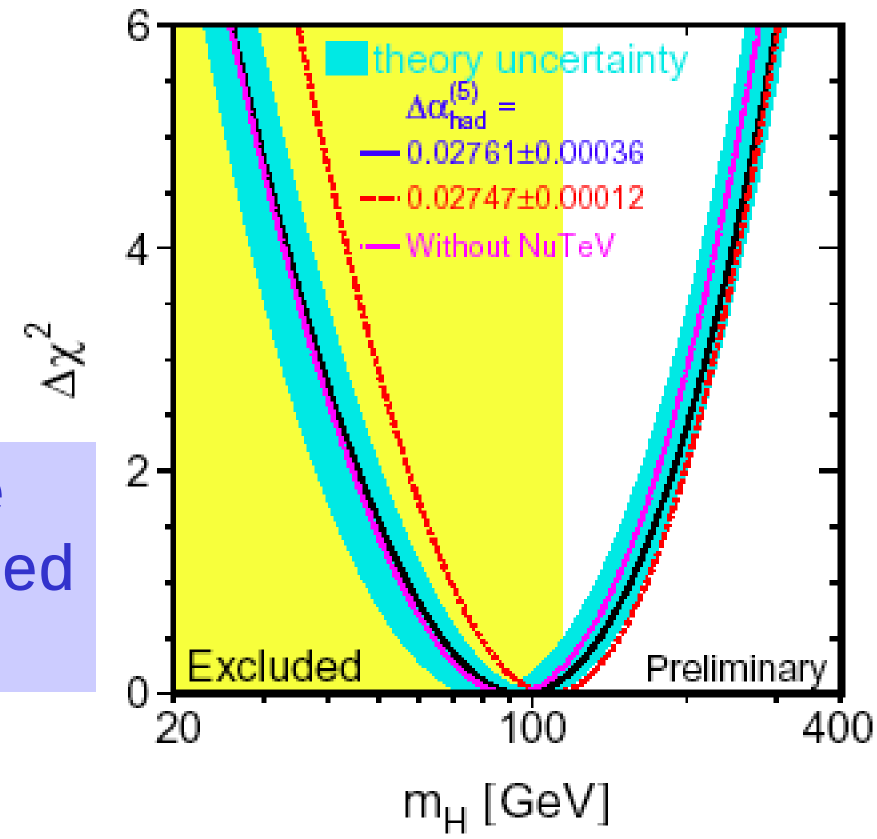
Summer'03

Sensitive to log m_H

Rad Corr.s ->

$\log_{10} m_H(\mathrm{GeV}) = 1.96 \pm 0.21$

This is a great triumph for the SM: right in the narrow allowed window $\log_{10} m_H \sim 2 - 3$



Direct search: m_H> 114 GeV

	All Z pole	All data	All but NuTeV
m_t (GeV)	$171.5^{+11.9}_{-9.4}$	$174.3^{+4.5}_{-4.4}$	$175.3^{+4.4}_{-4.3}$
m_H (GeV)	89^{+122}_{-45}	96^{+60}_{-38}	91^{+55}_{-36}
$\alpha_s(M_Z^2)$	0.1187 ± 0.0027	0.1186 ± 0.0027	0.1185 ± 0.0027
χ^2/dof (P)	14.7/10(14.3%)	25.4/15(4.5%)	16.7/14(27.5%)

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$\log_{10} m_H$ ~2 is a very important result

Drop H from SM -> renorm. lost -> divergences -> cut-off Λ

$$\log m_H \to \log\Lambda + \text{const}$$

Any alternative mechanism amounts to change the prediction of finite terms.

The most sensitive quantities to $\log m_H$ are $\varepsilon_1 \sim \Delta\rho$ and ε_3:

$\log_{10} m_H$ ~2 means that $f_{1,3}$ are compatible with the SM prediction

$$\varepsilon_1 = -\underbrace{\frac{3 G_F m_W^2}{4\pi^2\sqrt{2}}\,\mathrm{tg}^2\theta_W}_{-1.2\ 10^{-3}}\left[\log\frac{m_H}{m_Z} + f_1\right]$$

$$\varepsilon_3 = \underbrace{\frac{G_F m_W^2}{12\pi^2\sqrt{2}}}_{0.45\ 10^{-3}}\left[\log\frac{m_H}{m_Z} + f_3\right]$$

New physics can change the bound on m_H (different $f_{1,2}$)

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The EW theory: $\mathcal{L} = \mathcal{L}_{\text{symm}} + \mathcal{L}_{\text{Higgs}}$

$$\mathcal{L}_{symm} = -\frac{1}{4}[\partial_\mu W^A_\nu - \partial_\nu W^A_\mu - ig\varepsilon_{ABC} W^A_\mu W^B_\nu]^2 + -\frac{1}{4}[\partial_\mu B_\nu - \partial_\nu B_\mu]^2 + +\bar{\psi}\gamma^\mu[i\partial_\mu + gW^A_\mu t^A + g'B_\mu \frac{Y}{2}]\psi$$

$$\mathcal{L}_{Higgs} = |[\partial_\mu - igW^A_\mu t^A - ig'B_\mu \frac{Y}{2}]\phi|^2 + + V[\phi^\dagger\phi] + \bar{\psi}\Gamma\psi\phi + h.c$$

with

$$V[\phi^\dagger\phi] = \mu^2(\phi^\dagger\phi)^2 + \lambda(\phi^\dagger\phi)^4$$

$$\psi = \psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

A chiral theory:

$$(t, Y)_R \neq (t, Y)_L$$

$$Q = t^3 + Y/2$$

$\mathcal{L}_{\text{symm}}$: well tested (LEP, SLC, Tevatron…), $\mathcal{L}_{\text{Higgs}}$: ~ untested

Rad. corr's -> $m_H \leq 193$ GeV

LEP: 2.1σ

but no Higgs seen: $m_H > 114.4$ GeV; ($m_H = 115$ GeV ?)

Only hint $m_W = m_Z \cos\theta_W$ $\longrightarrow$ doublet Higgs

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Overall the EW precision tests support the SM and a light Higgs.

The χ^2 is reasonable but not perfect:

χ^2/ndof=25.5/15 (4.4%)

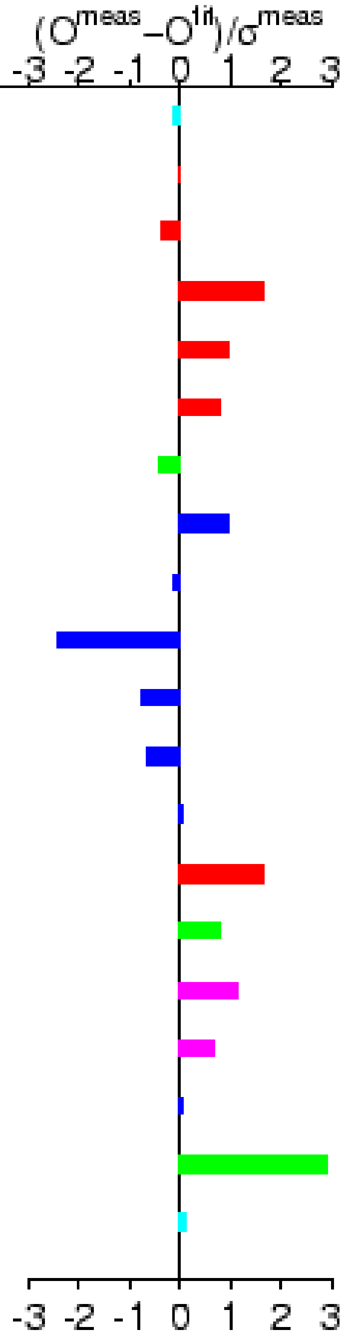
Note: includes NuTeV and APV [not $(g-2)_\mu$]

Without NuTeV:
(th. error questionable)

χ^2/ndof=16.7/14 (27.3%)

Winter 2003

	Measurement	Pull
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-0.16
m_Z [GeV]	91.1875 ± 0.0021	0.02
Γ_Z [GeV]	2.4952 ± 0.0023	-0.36
σ^0_{had} [nb]	41.540 ± 0.037	1.67
R_l	20.767 ± 0.025	1.01
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.79
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.42
R_b	0.21644 ± 0.00065	0.99
R_c	0.1718 ± 0.0031	-0.15
$A^{0,b}_{fb}$	0.0995 ± 0.0017	-2.43
$A^{0,c}_{fb}$	0.0713 ± 0.0036	-0.78
A_b	0.922 ± 0.020	-0.64
A_c	0.670 ± 0.026	0.07
A_l(SLD)	0.1513 ± 0.0021	1.67
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.82
m_W [GeV]	80.426 ± 0.034	1.17
Γ_W [GeV]	2.139 ± 0.069	0.67
m_t [GeV]	174.3 ± 5.1	0.05
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	2.94
$Q_W(Cs)$	-72.83 ± 0.49	0.12



NuTeV →

APV →

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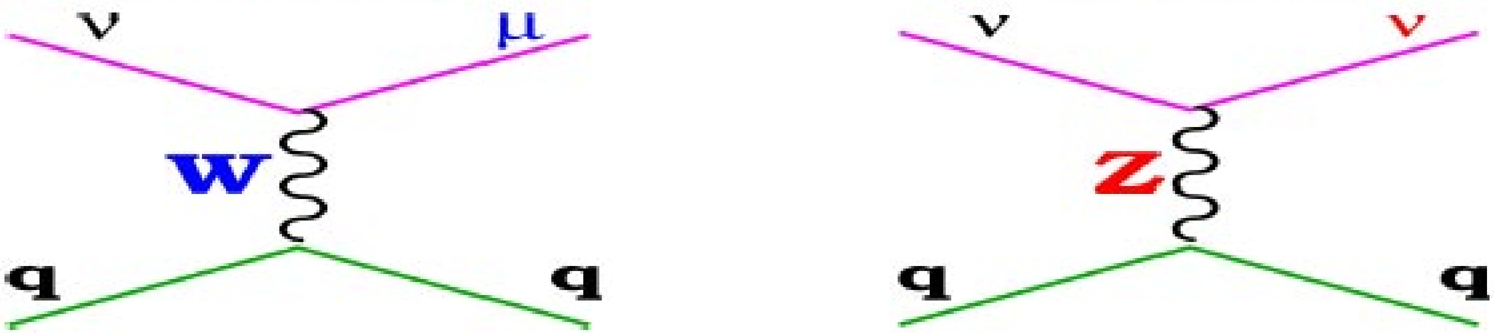
NuTeV Neutrino-Nucleon Scattering

Muon-(anti-)neutrino quark scattering:

charged current (CC) neutral current (NC)



Paschos-Wolfenstein relation (iso-scalar target):

$$R_- = \frac{\sigma_{NC}(\nu)-\sigma_{NC}(\bar{\nu})}{\sigma_{CC}(\nu)-\sigma_{CC}(\bar{\nu})} = 4g^2_{L\nu}\sum_{q_V}\left[g^2_{Lq}-g^2_{Rq}\right] = \rho_\nu\rho_{ud}\left[\frac{1}{2}-\sin^2\theta_W^{(on-shell)}\right]$$

+ electroweak radiative corrections

Insensitive to sea quarks

Charm effects only through d_V quarks (CKM suppressed)

Need neutrino and anti-neutrino beam!

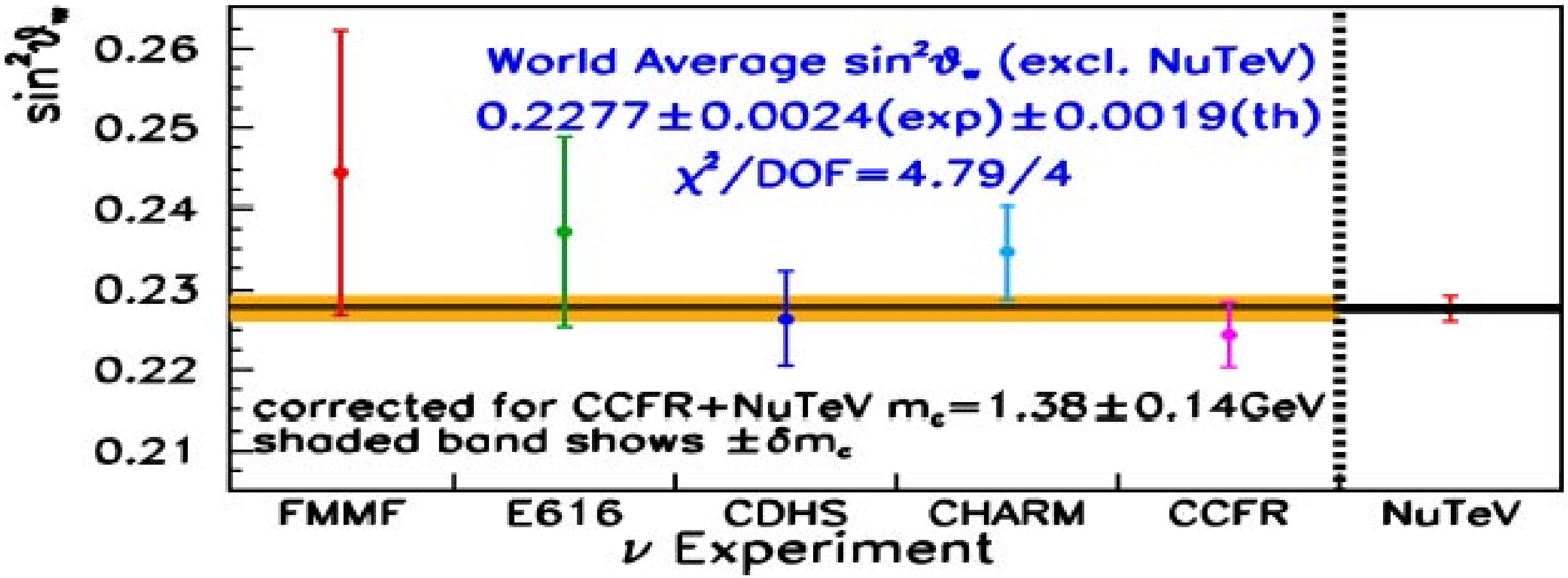
G. Altarelli [copied from Grunewald, Amsterdam ‘02 talk]

NuTeV's Result

$$\sin^2\theta_W^{(on-shell)} = 1 - \frac{M_W^2}{M_Z^2} = 0.2277 \pm 0.0013(stat.) \pm 0.0009(syst.)$$

$$- 0.00022\frac{M_{top}^2 - (175\,GeV)^2}{(50\,GeV)^2} + 0.00032\ln\frac{M_{Higgs}}{150\,GeV} \qquad [\rho = \rho_{SM}]$$

Factor two more precise than previous νN world average



Global SM analysis predicts: 0.2227(4) Difference of 3.0 σ!

G. Altarelli [copied from Grunewald, Amsterdam '02 talk]

My opinion: the NuTeV anomaly could simply arise from a large underestimation of the theoretical error

- The QCD LO parton analysis is too crude to match the required accuracy

- A small asymmetry in the momentum carried by s-$\bar{s}$ could have a large effect

They claim to have measured this asymmetry from dimuons. But a LO analysis of s-$\bar{s}$ makes no sense and cannot be directly transplanted here
(α_s*valence corrections are large and process dependent)

- A tiny violation of isospin symmetry in parton distrib's can also be important.

S. Davidson, S. Forte, P. Gambino, N. Rius, A. Strumia

G. Altarelli

Atomic Parity Violation (APV)

- Q_W is an idealised pseudo-observable corresponding to the naïve value for a N neutron-Z proton nucleus

- The theoretical ”best fit” value from ZFITTER is

$$(Q_W)_{th} = -72.880 \pm 0.003$$

- The “experimental” value contains a variety of QED and nuclear effects that keep changing all the time:

Since the 2002 LEP EWWG fit (showing a 1.52σ deviation) a new evaluation of the QED corrections led to

$$(Q_W)_{exp} = -72.83 \pm 0.49$$

Kuchiev, Flambaum ’02
Milstein et al ‘02

So in this very moment (winter ‘03) APV is OK!

Gambino, LP’03

Such a strange asymmetry...(I)

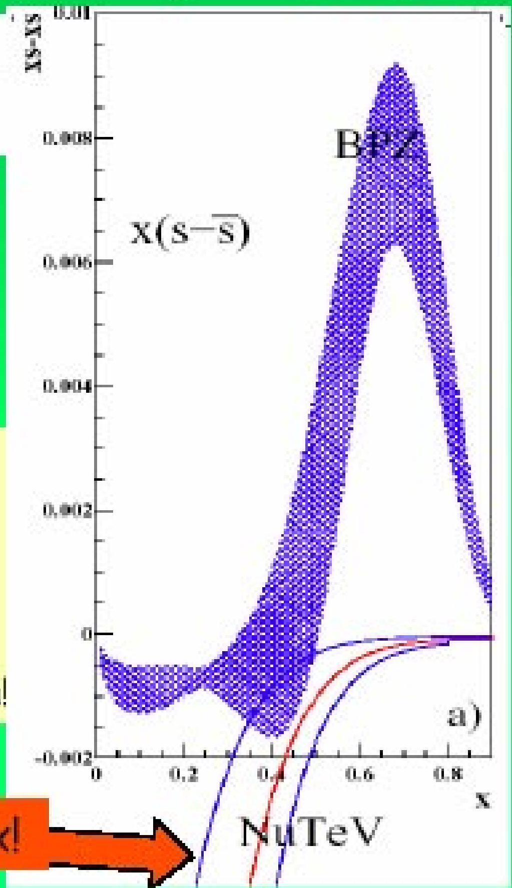
$$R_{PW} = \frac{1}{2} - s_W^2 + \frac{\tilde{g}^2}{Q^-}\left[u^- - d^- + c^- - s^-\right]\{1 + O(\alpha_s)\}$$

Strange quark asymmetry
Non-perturbatively induced by p <-> KΛ
A positive s^- reduces the anomaly

Only ν-induced processes are sensitive to $s^-(x)$

Inclusive ν-DIS
Barone et al (BPZ99) found s^-=0.002
Recently updated (Portheault et al) couldn't access dimuon data!

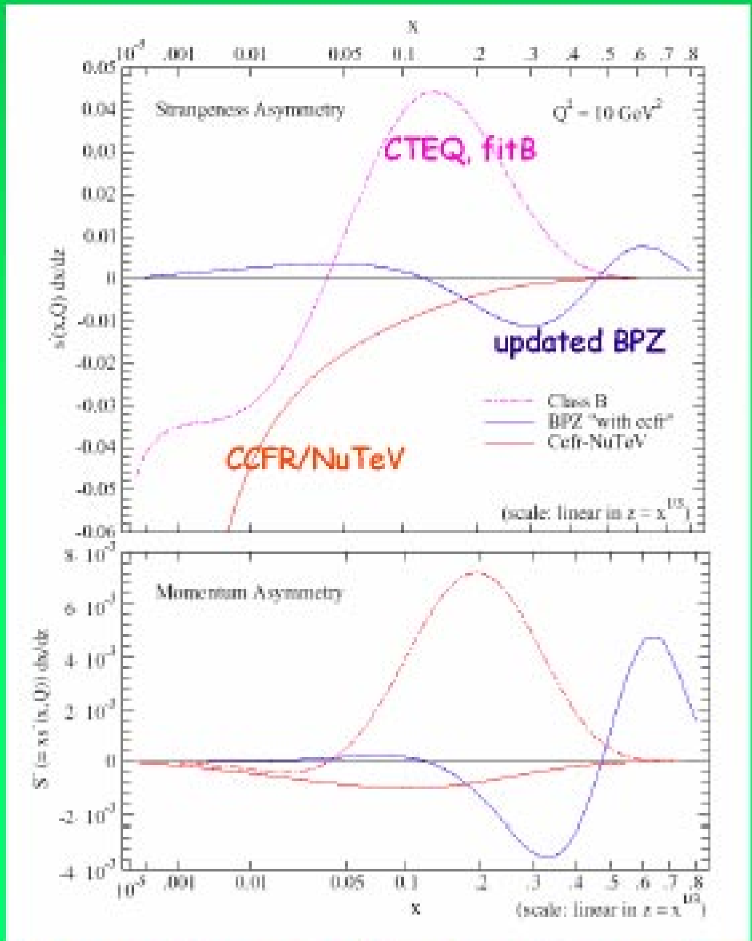
Dimuons (charm production)
NuTeV has found (low x) s^-=-0.0027±0.0013



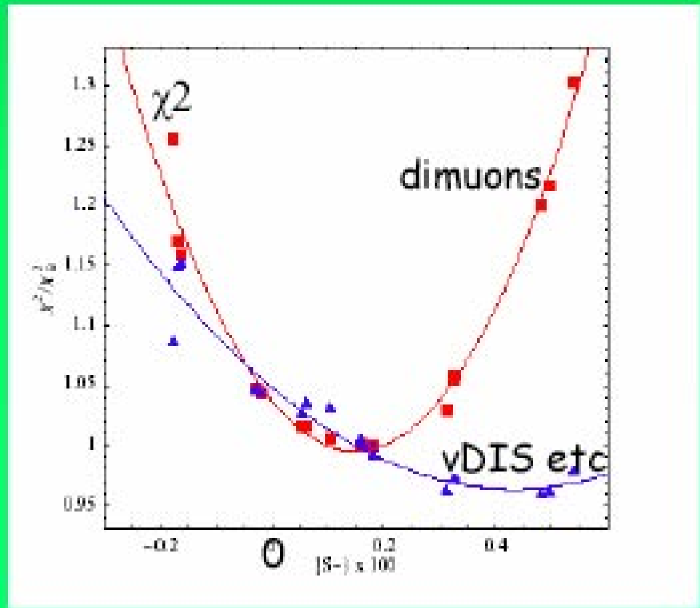
negative s^- at small x!

BUT NuTeV fit to s^-
a) relies on inconsistent parameterization (strangess not conserved)
b) does not fit s^- in the context of global fit

The new CTEQ fit



- includes all available data
- explores full range of parametrns with $S_N=0$
- fits s,sbar together with other pdfs



Most reasonable range 0.001< s- <0.003

Kretzer, Olness, Pumplin, Stump, Tung et al.

A strange end?

- Negative s^- strongly disfavoured, acceptable fits have $0.001 < s^- < 0.0031$, depending on low-x behavior

  Possible new info from W+charmed jet, lattice

fit	$[S^-] \times 100$	χ^2_{dimuon}	$\chi^2_{\text{inclusive}}$	δR^-
B^+	0.540	**1.30**	0.98	-0.0065
A	0.312	1.02	0.97	-0.0037
B	0.160	*1.00*	*1.00*	-0.0019
C	0.103	1.01	1.03	-0.0012
B^-	-0.177	**1.26**	**1.09**	0.0023

Kretzer et al

- Impact on R_{PW} in NuTeV setup estimated wrt to CTEQ s=sbar fit: $0.0012 < \delta s^2_w < 0.0037$ very likely to carry on to NuTeV analysis

NuTeV error ±0.0016

- NuTeV : a few minor issues open. In my opinion, large sea uncertainties and shift from s^- reduce discrepancy below 2σ

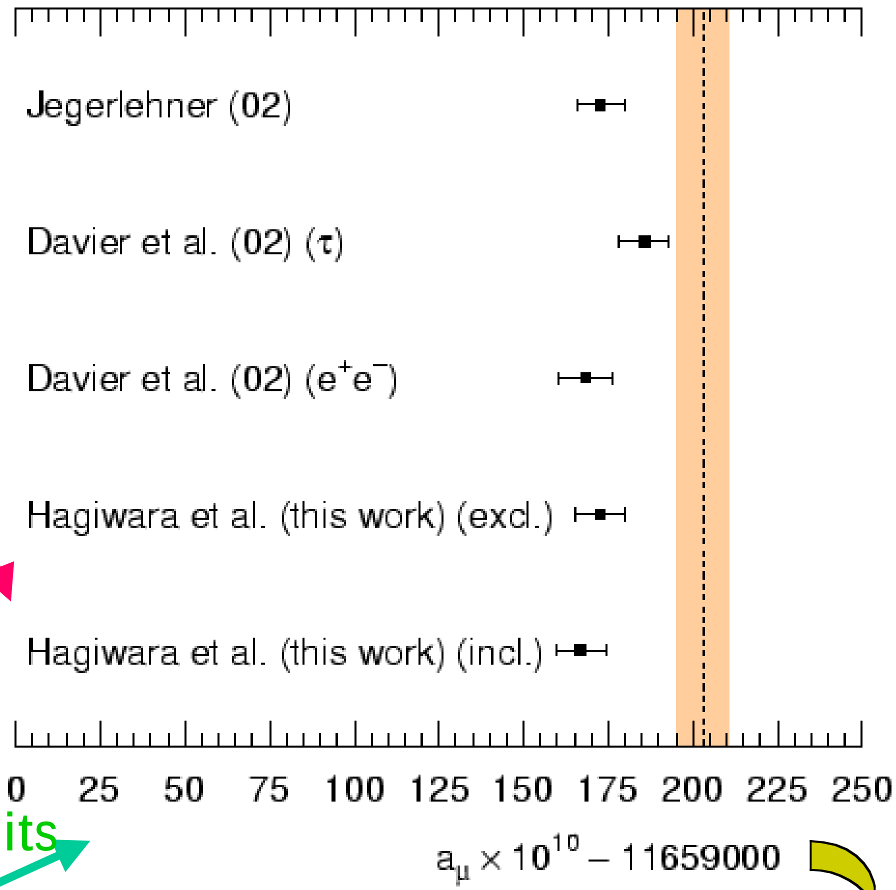
Given present understanding of hadron structure, R_{PW} is no good place for high precision physics

$(g\text{-}2)_\mu$ ~3σ discrepancy shown by the BNL'02 data

(Numbers in units 10^{-10})

LO hadr.	688.8 ± 6.2	HMNT, 'excl.'
	$683.1 \pm 5.9 \pm 2.0_{rad}$	HMNT, 'incl.'
full a_μ	11659172.6 ± 7.7	'excl.'
	11659166.9 ± 7.4	'incl.'
BNL E821	11659203 ± 8	new world av.
		(0.7 ppm!)
EXP–TH	30.4 ± 11.1	$\sim 2.7\sigma$, 'excl.'
	36.1 ± 10.9	$\sim 3.3\sigma$, 'incl.'

Th. and Exp. accuracy comparable!



EW ~ 15.2±0.4

LO hadr ~ 683.1±6.2

NLO hadr ~ -10±0.6

Light-by-Light ~ 8±4

(was ~ -8.5±2.5)

These units

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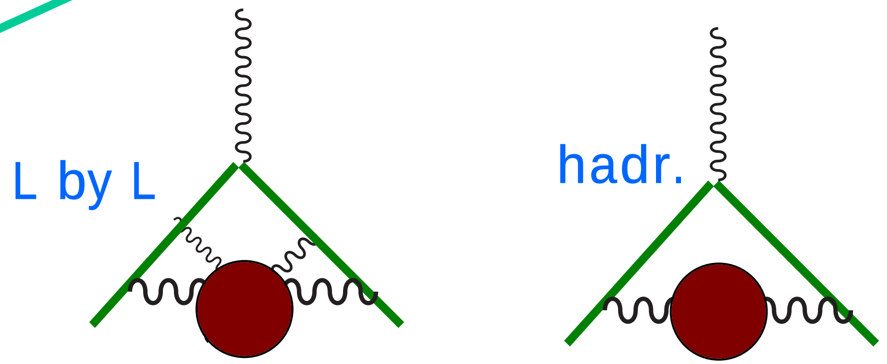


The spectral function from e^+e^-

Tau data below 1.8GeV

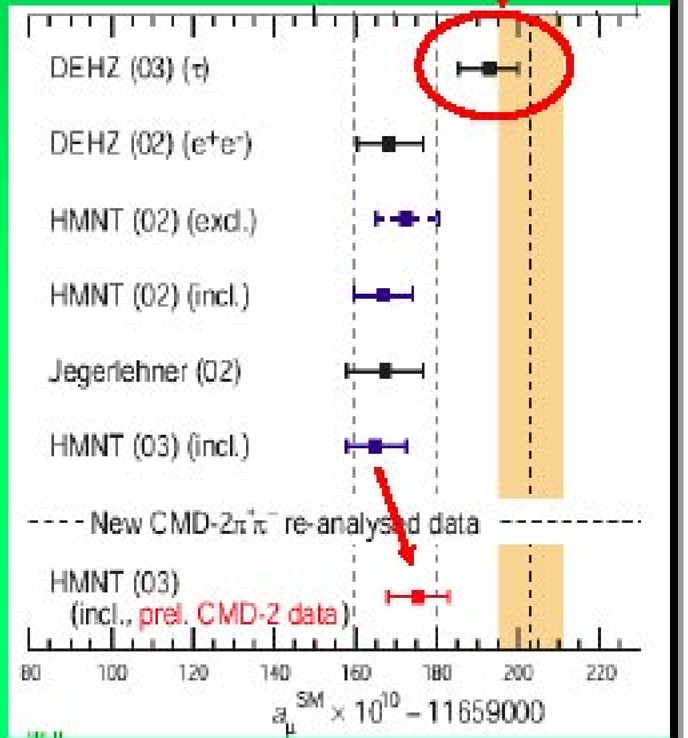
Final CMD-2 $\pi\,\pi$ data (2002) 0.6% syst error!
CMD-2 have recently reanalyzed their data

Hagiwara et al (HMNT) NEW result:
$a_\mu^{had,LO}=691.7\pm5.8_{exp}\pm2.0_{r.c.}$

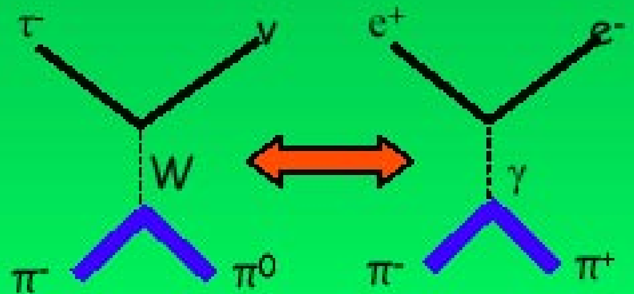
This translates in a ~2-2.5σ discrepancy depending on which e^+e^- analysis

Using τ data below 1.8 GeV Davier at al (DEHZ)
$a_\mu^{had,LO}=709.0\pm5.1_{exp}\pm1.2_{r.c}\pm2.8_{SU(2)}$

Good agreement between Aleph, CLEO, Opal τ data

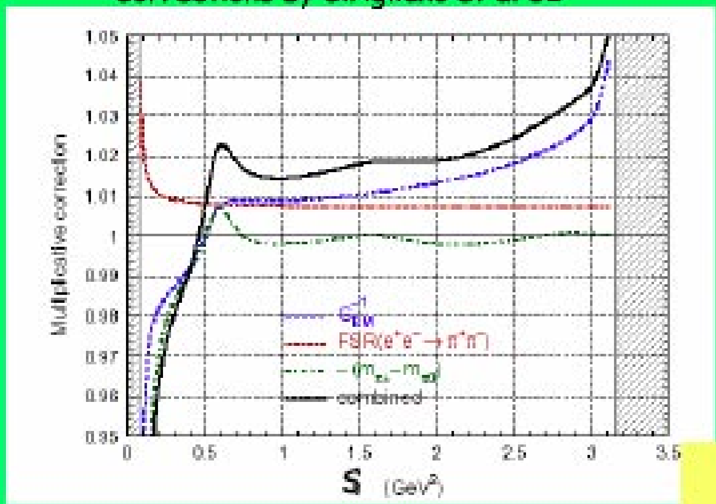


The spectral function from τ decays



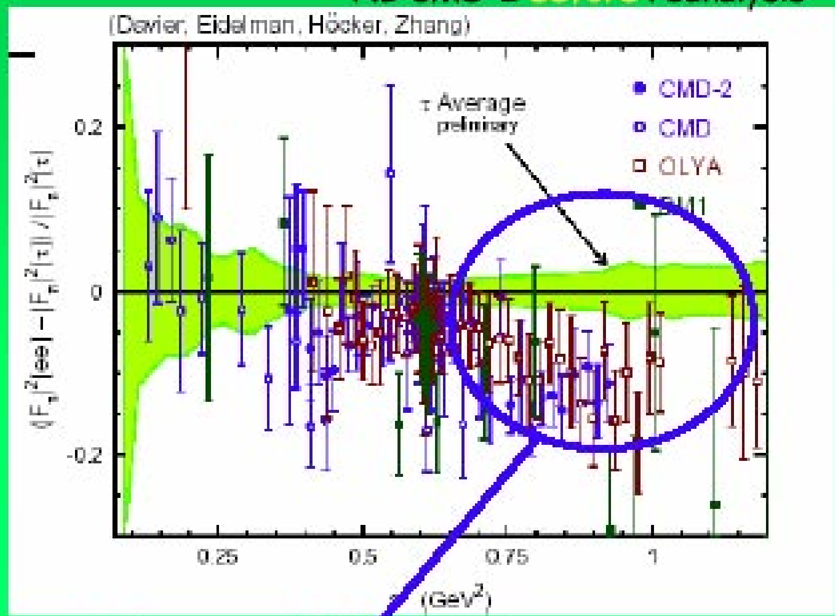
CVC + isospin symmetry

Corrections by Cirigliano et al 02



Corrections applied to τ data

NB CMD-2 before reanalysis



Relative difference between π form f. from τ and e^+e^-

>5% difference! cannot be isospin breaking. Needs further study. Data?

After new CMD-2 for $\Delta_{\pi\pi}$=(11-13±7) 10^{-10}(was 21)

Impact on $\alpha(M_Z)$

$\alpha(M_Z)$ appropriate parameter for EW

Spectral function enters its calculation similarly, but higher energy data have more weight. Results are converging

$\delta\alpha(M_Z)$ is no more the main bottleneck for precision EW

Further improvements expected
Conservative estimate
(upper bound of uncertainty)

$\Delta\alpha_{had}=0.02768\pm0.00036$

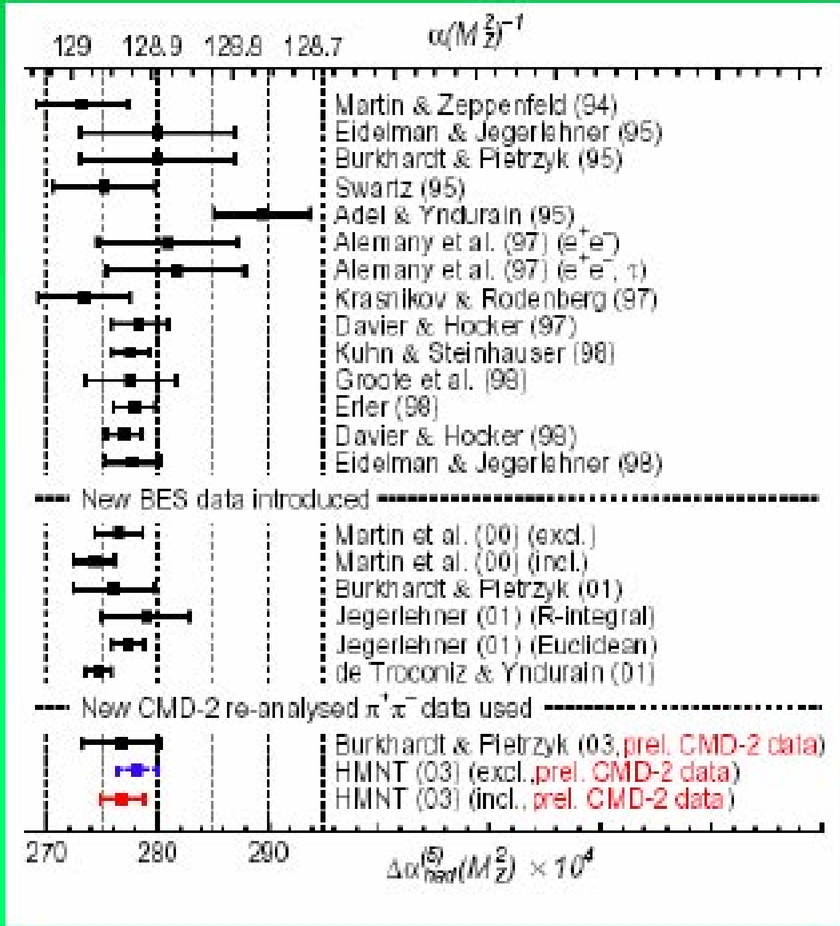
Burkhardt & Pietrzyk 2003

With more efficient use of exp data

$\Delta\alpha_{had}=0.02769\pm0.00018$

Hagiwara et al 2003

Use of τ data + ~0.002



Question Marks on EW Precision Tests

- The measured values of $\sin^2\theta_{eff}$ from leptonic (A_{LR}) and from hadronic (A^b_{FB}) asymmetries are ~3σ away

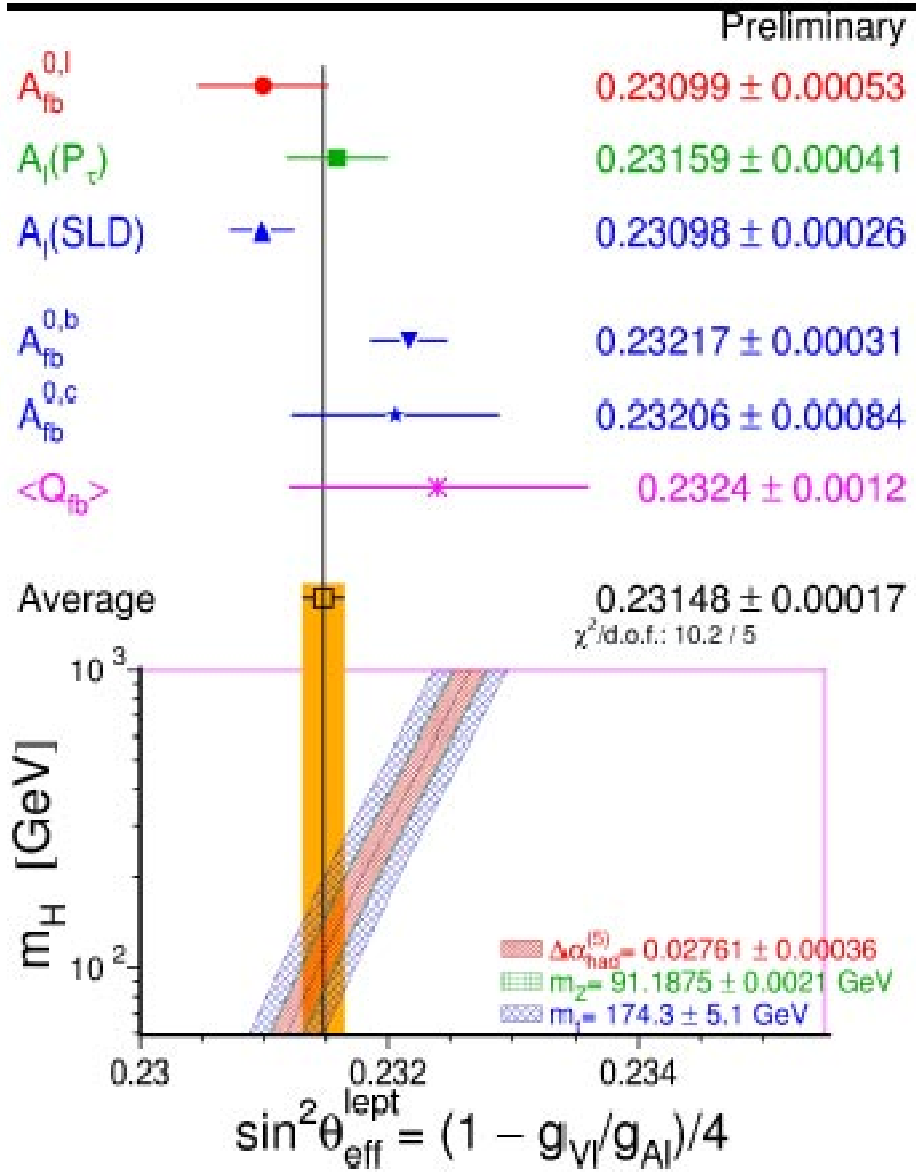
- The measured value of m_W is somewhat high

- The central value of m_H (m_H=83+50-33 GeV) from the fit is below the direct lower limit ($m_H \geq$114.4 GeV at 95%)
[more so if $\sin^2\theta_{eff}$ is close to that from leptonic (A_{LR}) asymm. m_H < ~110 GeV]

Chanowitz;
GA, F. Caravaglios, G. Giudice, P. Gambino, G. Ridolfi

Hints of new physics effects??

Comparison of all Z-Pole Asymmetries



Effective electroweak mixing angle:

$\sin^2\Theta_{eff} = 0.23148\ (17)$

$\chi^2/\text{ndof} = 10.2/5\ \ [7.0\%]$

A-posteriori observation:

0.23113 (21) leptons

0.23217 (29) hadrons

But is really:

A_l(SLD) vs. $A_{fb}b$(LEP)

Both:

$2.9\ \sigma$ difference

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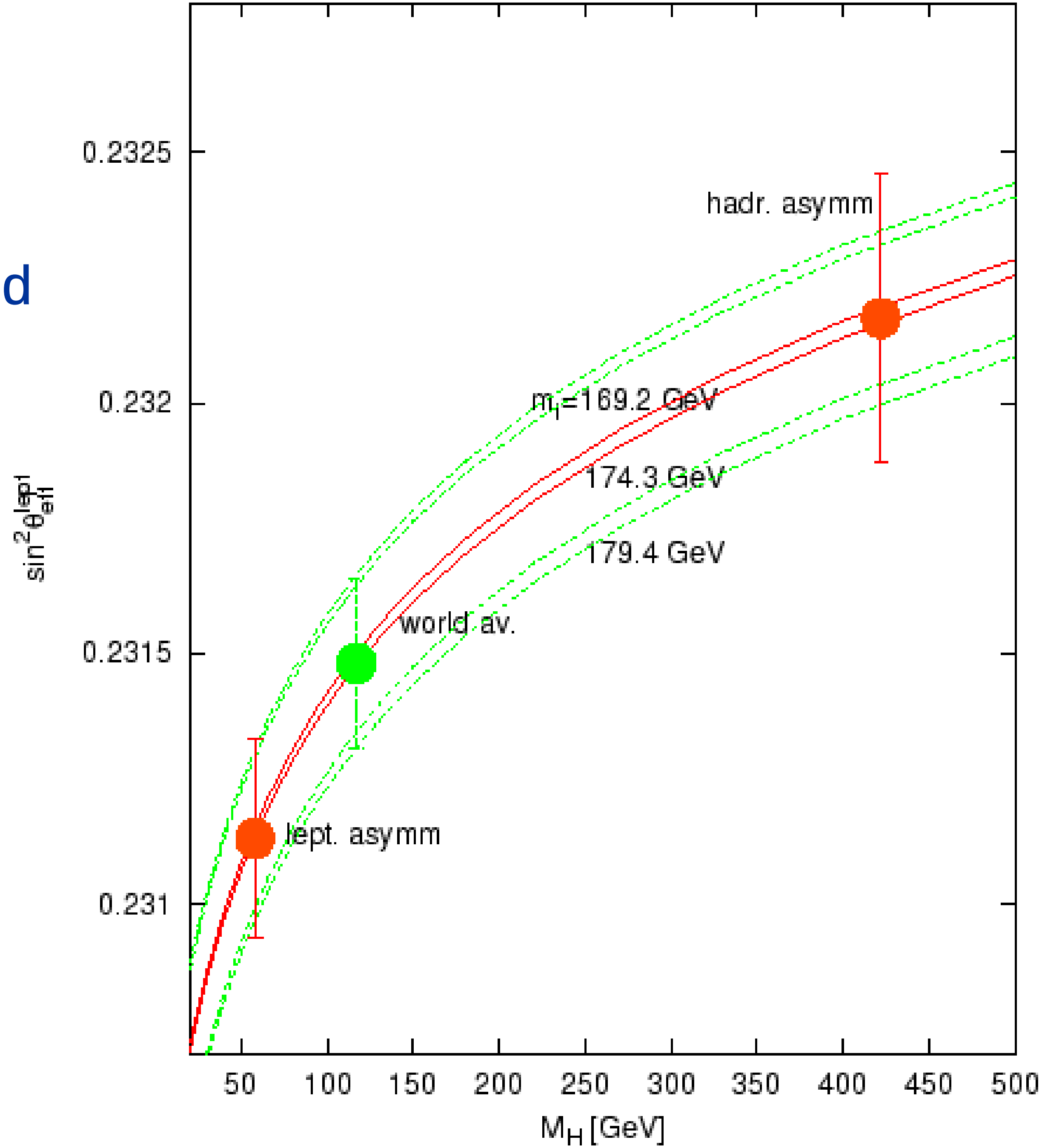
[copied from Grunewald, Amsterdam ‘02 talk]

Plot $\sin^2\theta_{eff}$ vs m_H

Exp. values are plotted at the m_H point that better fits given m_{texp}



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Question Marks on EW Precision Tests

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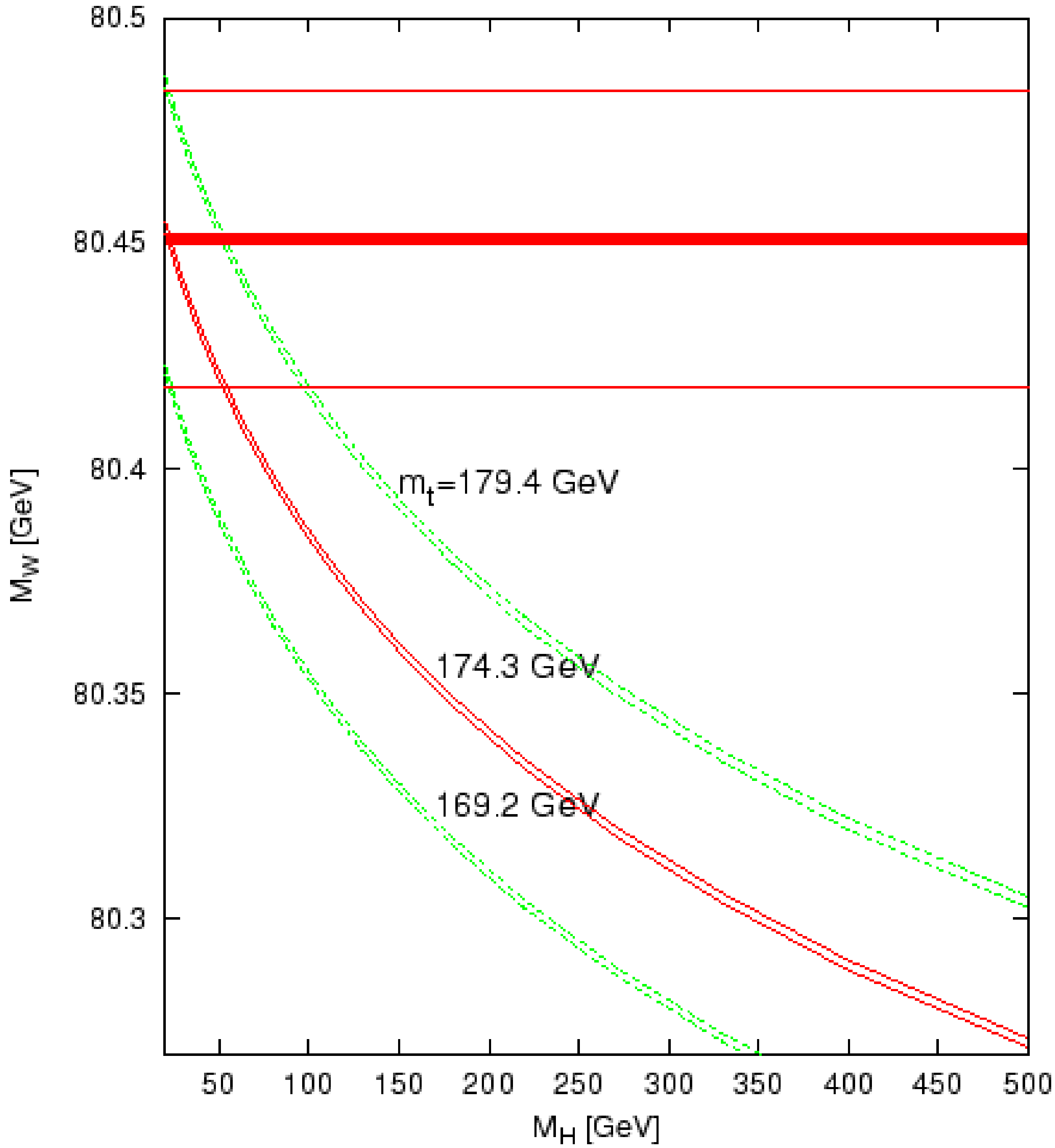
Chanowitz;
GA, F. Caravaglios, G. Giudice, P. Gambino, G. Ridolfi

Hints of new physics effects??

Plot m_W vs m_H

m_W points to a light Higgs

Like $[\sin^2\theta_{eff}]_l$



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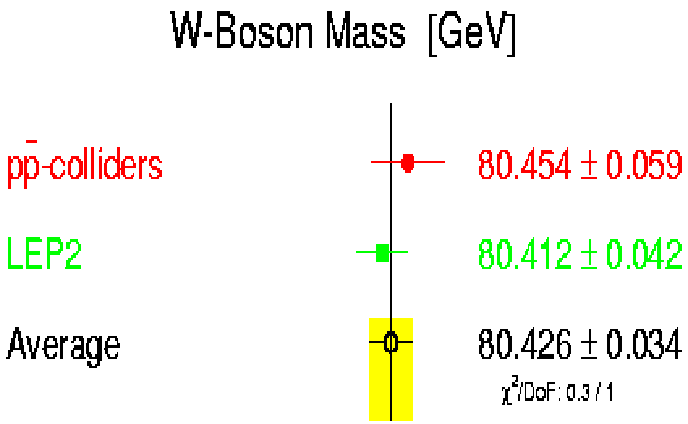
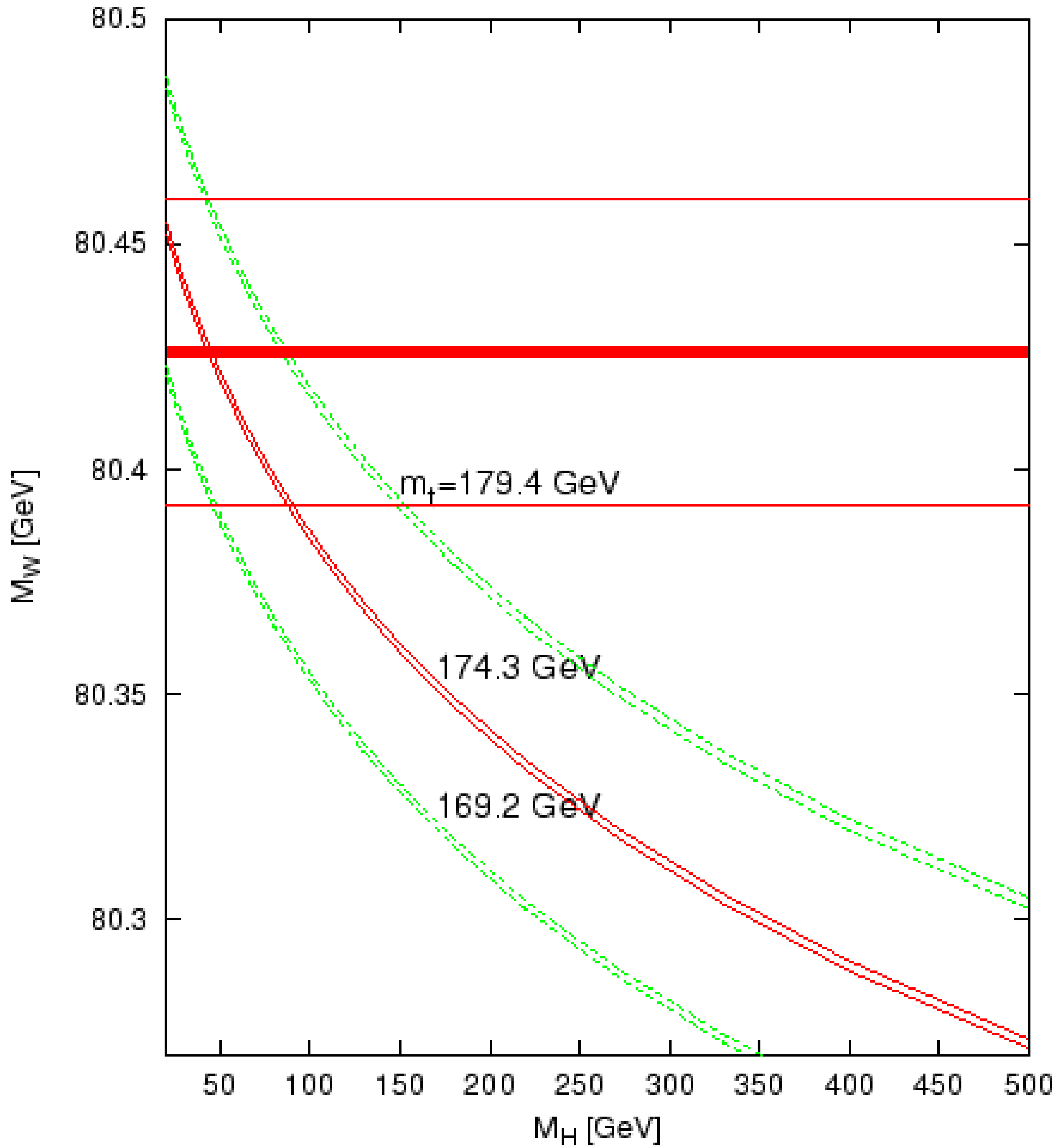
NEW!

New developments (winter '03)

m_W went down (ALEPH: -79 MeV). Still the central value points to m_H~50 GeV





Now: 80.426±0.034

Was: 80.449±0.034

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Question Marks on EW Precision Tests

- The measured values of $\sin^2\theta_{eff}$ from leptonic (A_{LR}) and from hadronic (A^b_{FB}) asymmetries are ~3σ away

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[more so if $\sin^2\theta_{eff}$ is close to that from leptonic (A_{LR}) asymm. m_H < ~110 GeV]

Chanowitz;
GA, F. Caravaglios, G. Giudice, P. Gambino, G. Ridolfi

Hints of new physics effects??

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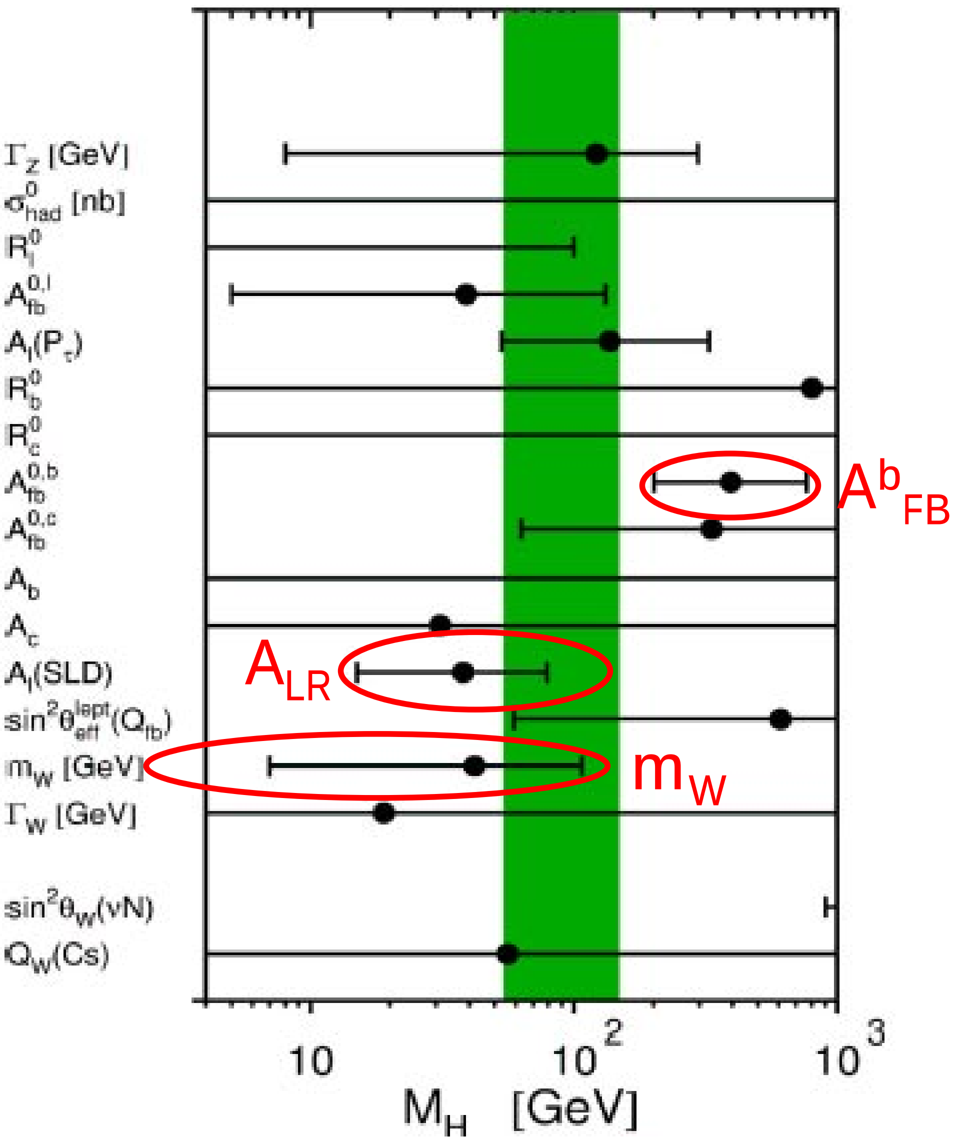
Sensitivities to m_H

The central value of m_H would be even lower if not for A^b_{FB}

One problem helpes the other:
A^b_{FB} vs A_{LR} cures the problem of A_{LR}, m_W clashing with m_H>114.4 GeV



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Some indicative fits

Note: here 2001 data

Most important observables:

m_t, m_W, Γ_l, R_b, $\alpha_s(m_Z)$, α_{QED}, $\sin^2\theta_{eff}$

Taking $\sin^2\theta_{eff}$ from leptonic or hadronic asymmetries as separate inputs, $[\sin^2\theta_{eff}]_l$ and $[\sin^2\theta_{eff}]_h$, with $\alpha^{-1}_{QED}=128.936\pm0.049$ (BP'01) we obtain:

$\chi^2/\mathrm{ndof}=18.4/4$, CL=0.001; $m_{H\mathrm{central}}=100$ GeV,
$m_H\leq$ 212 GeV at 95%

Taking $\sin^2\theta_{eff}$ from only hadronic asymm. $[\sin^2\theta_{eff}]_h$

$\chi^2/\mathrm{ndof}=15.3/3$, CL=0.0016;

Taking $\sin^2\theta_{eff}$ from only leptonic asymm. $[\sin^2\theta_{eff}]_l$

$\chi^2/\mathrm{ndof}=2.5/3$, CL=0.33; $m_{H\mathrm{central}}=42$ GeV,
$m_H\leq$ 109 GeV at 95%

Much better χ^2 but clash with direct limit!

- It is not simple to explain the difference $[\sin^2\theta]_l$ vs $[\sin^2\theta]_h$ in terms of new physics.

A modification of the Z->bb vertex (but R_b and A_b(SLD) look ~normal)?

- Probably it arises from an experimental problem

- Then it is very unfortunate because $[\sin^2\theta]_l$ vs $[\sin^2\theta]_h$ makes the interpretation of precision tests ambigous

Choose $[\sin^2\theta]_h$: bad χ^2 (clashes with m_W, ...)
Choose $[\sin^2\theta]_l$: good χ^2, but m_H clashes with direct limit

- In the last case, SUSY effects from light s-leptons, charginos and neutralinos, with moderately large $\tan\beta$ can solve the m_H problem and lead to a better fit of the data

GA, F. Caravaglios, G. Giudice, P. Gambino, G. Ridolfi

A^b_{FB} vs $[\sin^2\theta]_{lept}$: New physics in Zbb vertex?

Unlikely!! (but not impossible->)

$$A^b_{FB} = \frac{3}{4} A_e A_b \qquad\qquad A_f = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}$$

For b: $g_L = g_V - g_A = -1 + \frac{2}{3}s^2 = -0.846$

$$g_R = g_V + g_A = \frac{2}{3}s^2 = 0.154$$

$$g_L^2 \approx 0.72 >> g_R^2 \approx 0.02$$

$$(A_b)_{SM} \approx 0.936$$

From A^b_{FB}=0.0995±0.0017, using $[\sin^2\theta]_{lept}$ =0.23113±0.00020 or A_e=0.1501±0.0016, one obtains A_b=0.884±0.018

$(A_b)_{SM} - A_b$ = 0.052 ± 0.018 -> 2.9 σ

A large δg_R needed (by about 30%!)

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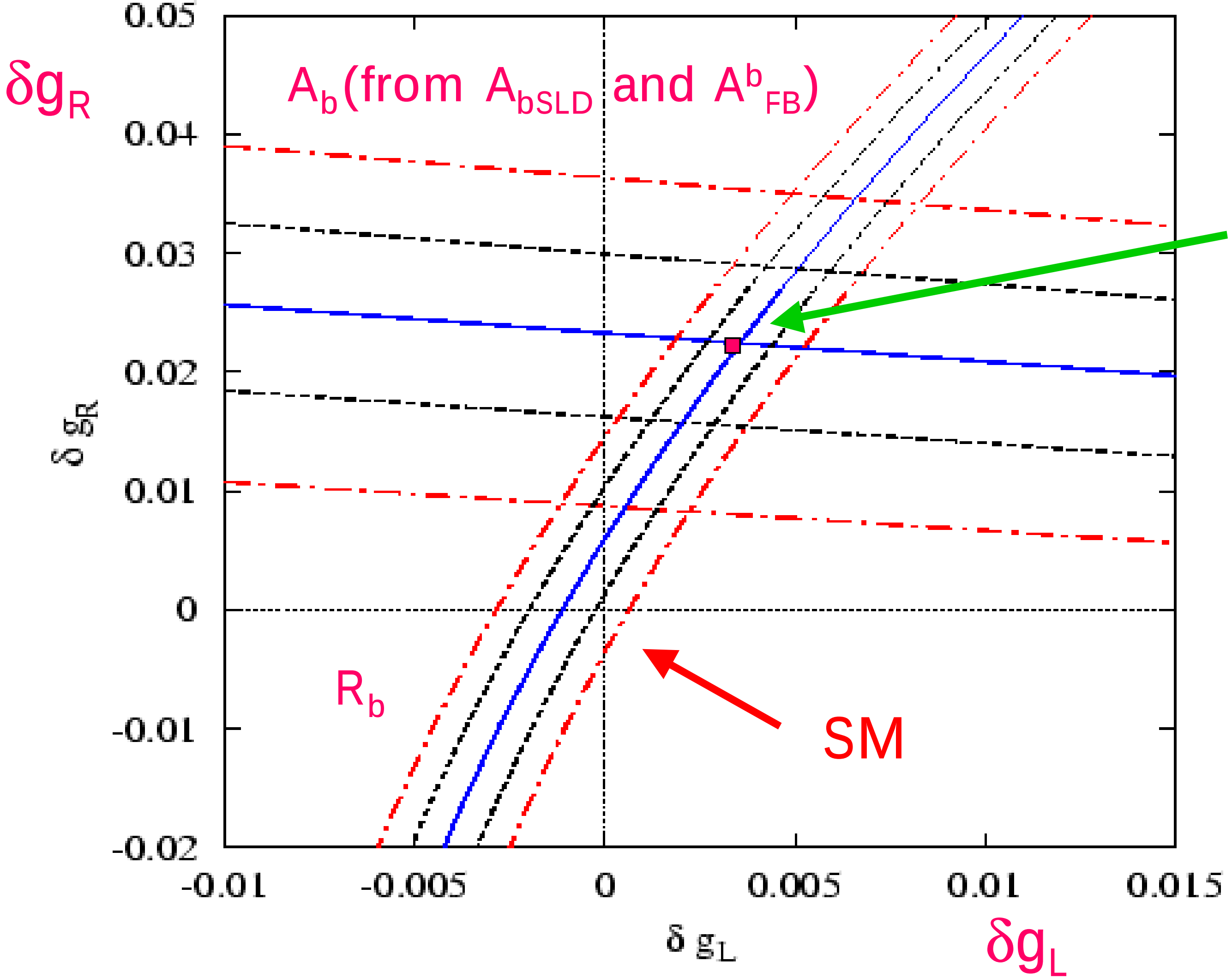
But note: $(A_b)_{SLD}$ = 0.922±0.020, R_b=0.21644±0.00065 (R_{bSM}~0.2157)



Choudhury, Tait, Wagner

A possible model involves mixing of the b quark with a vectorlike doublet (ω,χ) with charges (-1/3, -4/3)

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A modification of the Z->bb vertex (but R_b and A_b(SLD) look ~normal)?

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GA, F. Caravaglios, G. Giudice, P. Gambino, G. Ridolfi

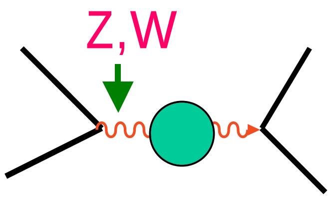
EW DATA and New Physics

For an analysis of the data beyond the SM we use the ε formalism GA, R.Barbieri, F.Caravaglios, S. Jadach

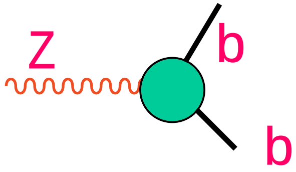
One introduces ε_1, ε_2, ε_3, ε_b such that:

- Focus on pure weak rad. correct's, i.e. vanish in limit of tree level SM + pure QED and/or QCD correct's
[a good first approximation to the data]

- Are sensitive to vacuum pol. ε_1, ε_2, ε_3 →



and Z->bb vertex corr.s
(but also include non oblique terms) ε_b →



- Can be measured from the data with no reference to m_t and m_H (as opposed to S, T, U)

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One starts from a set of defining observables:

$$O_i = m_W/m_Z, \quad \Gamma_\mu, \quad A^\mu_{FB}, \quad R_b$$

$$\underbrace{\varepsilon_1 \qquad \varepsilon_3}$$

$$\varepsilon_2 \qquad\qquad\qquad \varepsilon_b$$

$$O_i[\varepsilon_k] = O_i^{\text{"Born"}}[1 + A_{ik}\,\varepsilon_k + \ldots]$$

$O_i^{\text{"Born"}}$ includes pure QED and/or QCD corr's.

A_{ik} is independent of m_t and m_H

Assuming lepton universality: Γ_μ, A^μ_{FB} --> Γ_l, A^l_{FB}

To test lepton-hadron universality one can add Γ_Z, σ_h, R_l to Γ_l etc.

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ε_1,ε_2 and ε_3 are related to Δr_W, $\Delta\rho$ and $\Delta k'$

Large $G_F m^2_t$ terms in Δr_W, $\Delta\rho$ and $\Delta k'$ $\Rightarrow$ $\Delta r_W \sim \frac{c_W^2 - s_W^2}{s_W^2}\Delta k' \sim -\frac{c_W^2}{s_W^2}\Delta\rho$

$$\varepsilon_1 \equiv \Delta\rho$$

$$\varepsilon_2 \equiv c_W^2\Delta\rho + \frac{s_W^2}{c_W^2 - s_W^2}\Delta r_W - 2s_W^2\Delta k'$$

$$\varepsilon_3 \equiv c_W^2\Delta\rho + (c_W^2 - s_W^2)\Delta k'$$

$$\Delta\rho \sim \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}$$

- Large $G_F m^2_t$ terms are only in ε_1
- Main m_H sensitivity in ε_3
- m_W sensitivity through Δr_W in ε_2

In addition ε_b arises from the Z->bb vertex

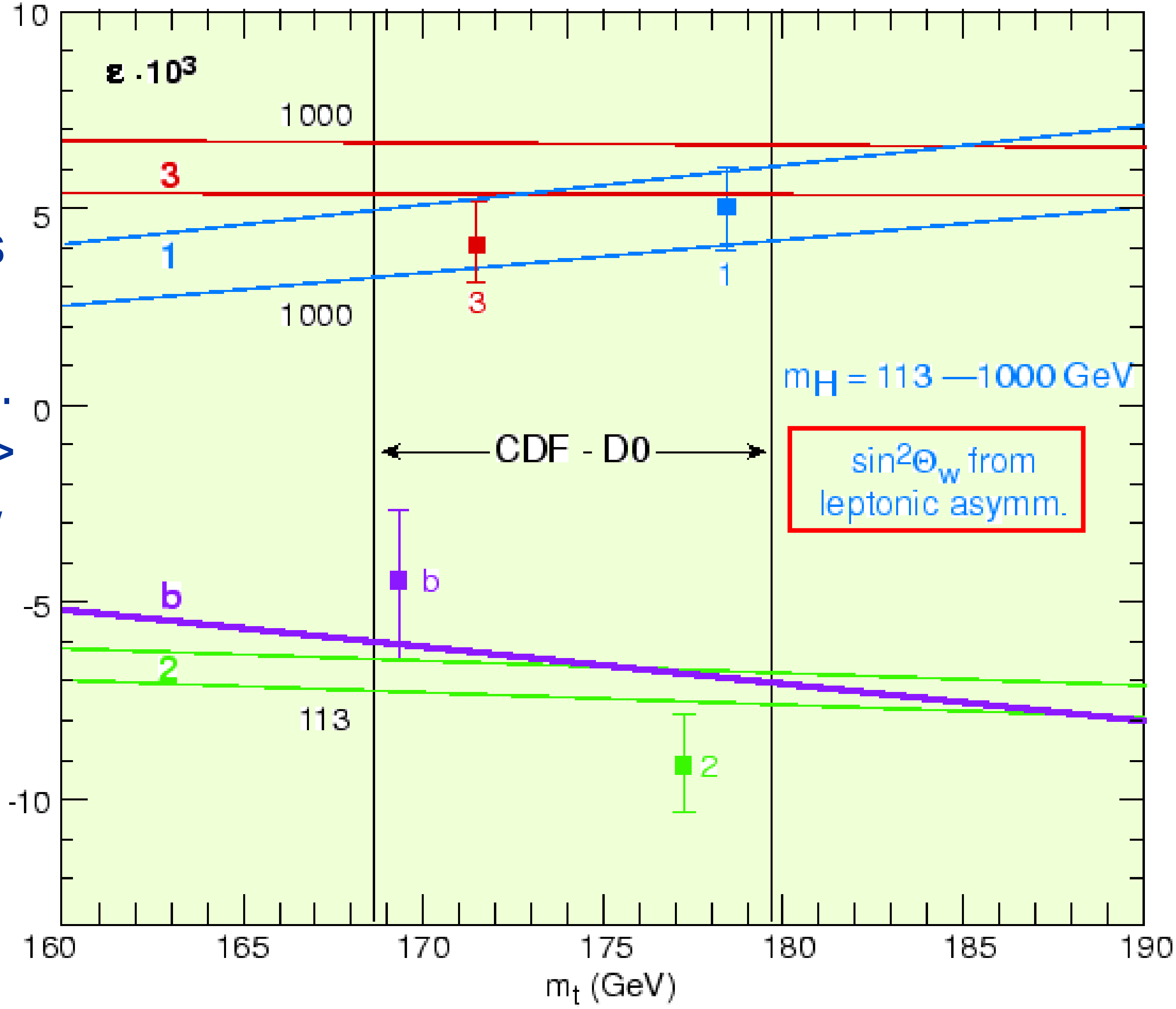
$$\varepsilon_b \sim -\frac{G_F m_t^2}{4\pi^2\sqrt{2}}$$

Relation with S, T, U: the shifts from new physics are proportional $\Delta S \sim \Delta\varepsilon_3$, $\Delta T \sim \Delta\varepsilon_1$, $\Delta U \sim \Delta\varepsilon_2$

ε vs m_{top}

GA, R. Barbieri, F.Caravaglios

With $\sin^2\theta_{eff}$ from leptons
ε_2 low ->
-> m_W large.
ε_3 also low->
-> m_H below direct limit.
$\varepsilon_{1,b}$ OK



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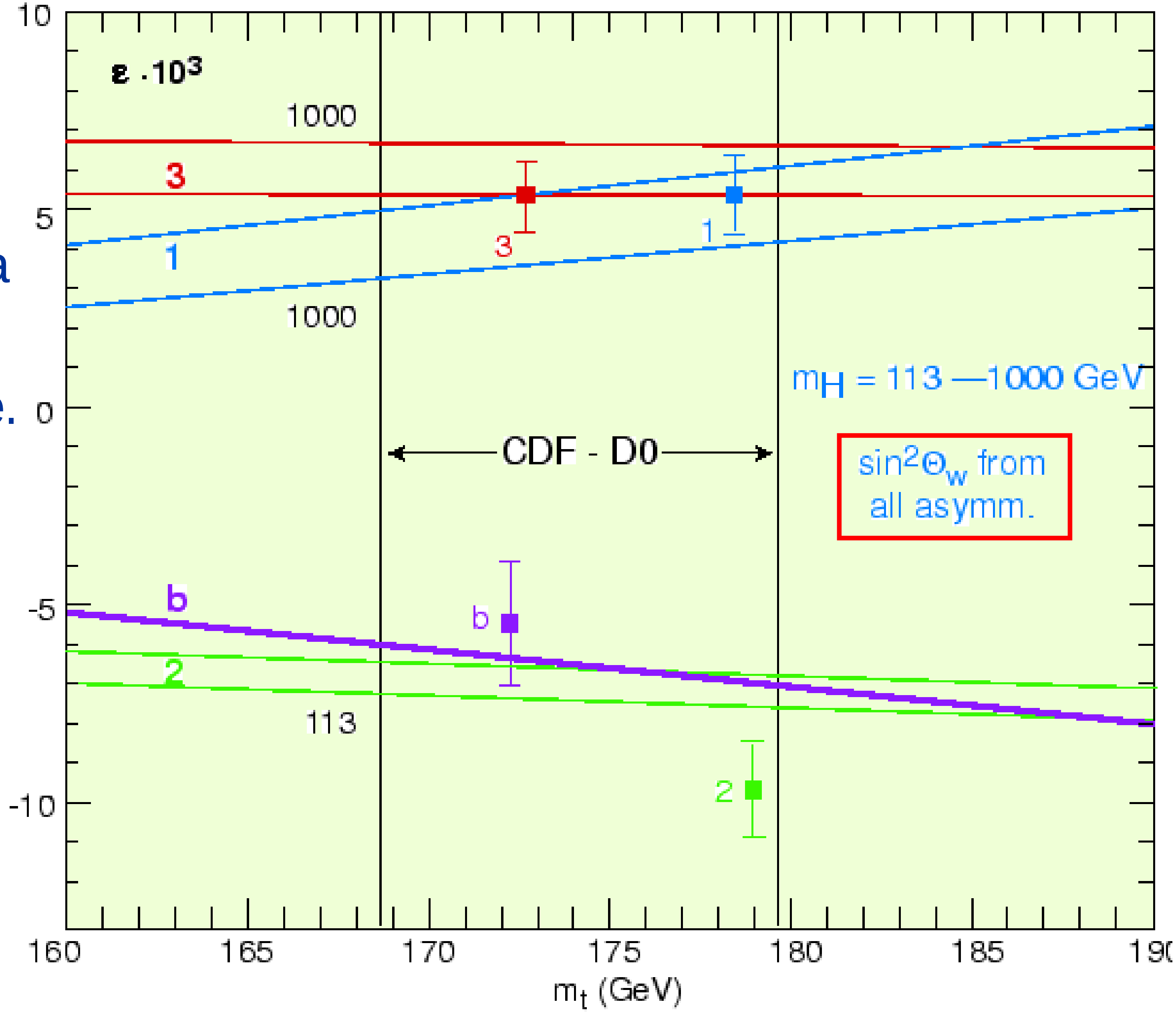
GA, R. Barbieri, F.Caravaglios

ε vs m_{top}

With $\sin^2\theta_{eff}$ from all data
ε_2 low ->
-> m_W large.
$\varepsilon_{1,3,b}$ OK



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The EWWG gives (summer ‘03):

$\varepsilon_1 = 5.4 \pm 1.0 \; 10^{-3}$

$\varepsilon_2 = -\,9.7 \pm 1.2 \; 10^{-3}$

$\varepsilon_3 = 5.25 \pm 0.95 \; 10^{-3}$

$\varepsilon_b = -\,4.7 \pm 1.6 \; 10^{-3}$

Non-degenerate
much larger shift of ε_1

For comparison:
a mass degenerate fermion multiplet gives

$$\Delta\varepsilon_3 = N_C \frac{G_F m_W^2}{8\pi^2\sqrt{2}} \cdot \frac{4}{3}[T_{3L} - T_{3R}]^2$$

For each member of the multiplet

One chiral quark doublet (either L or R):

$$\Delta\varepsilon_3 = +\,1.4 \; 10^{-3}$$

(Note that ε_3 if anything is low!)

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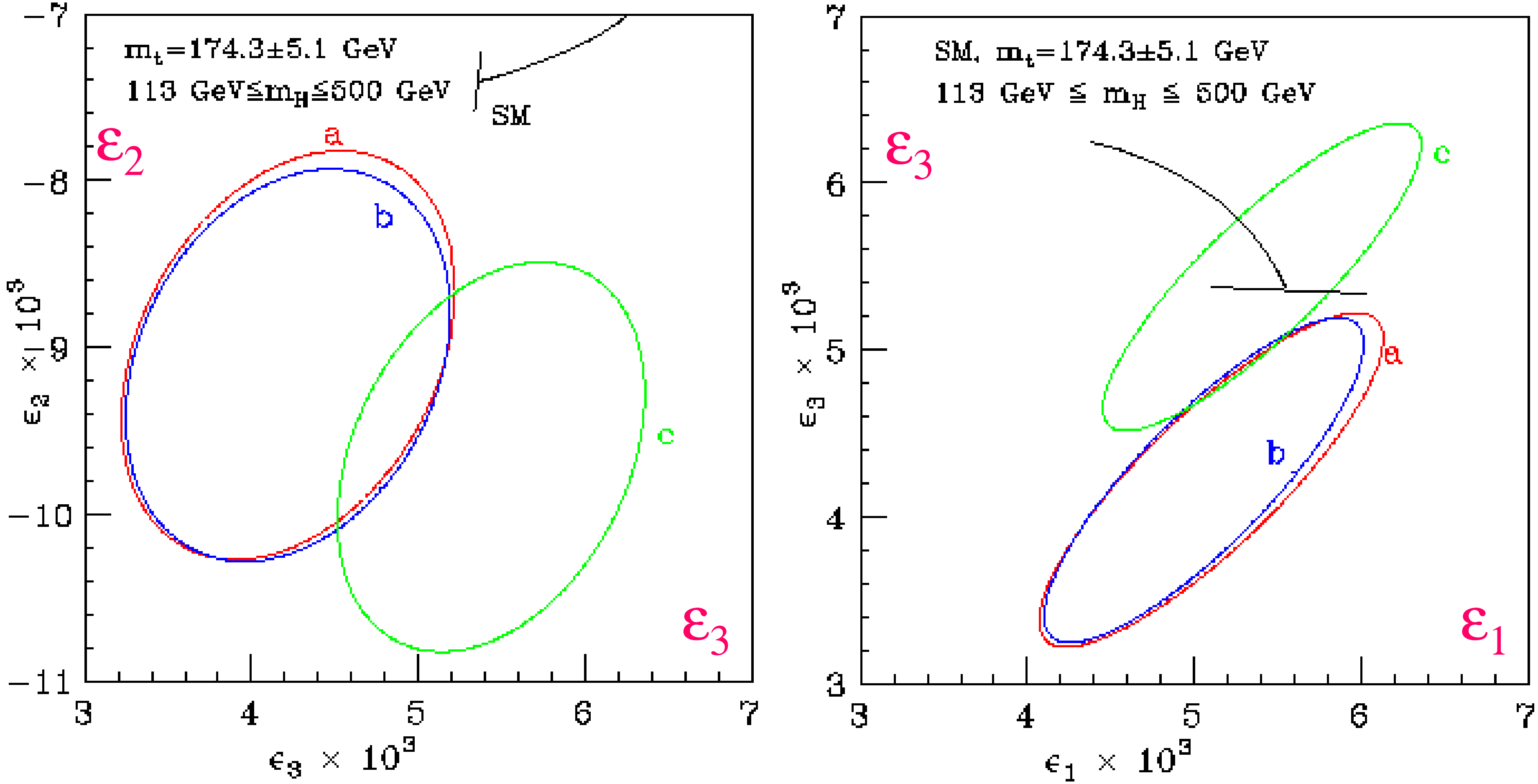
a: m_W, Γ_l, R_b, $[\sin^2\theta]_l$

b: m_W, Γ_l, R_b, Γ_Z, σ_h, R_l, $[\sin^2\theta]_l$

c: m_W, Γ_l, R_b, Γ_Z, σ_h, R_l, $[\sin^2\theta]_l+[\sin^2\theta]_h$

Note:
1σ ellipses (39% cl)



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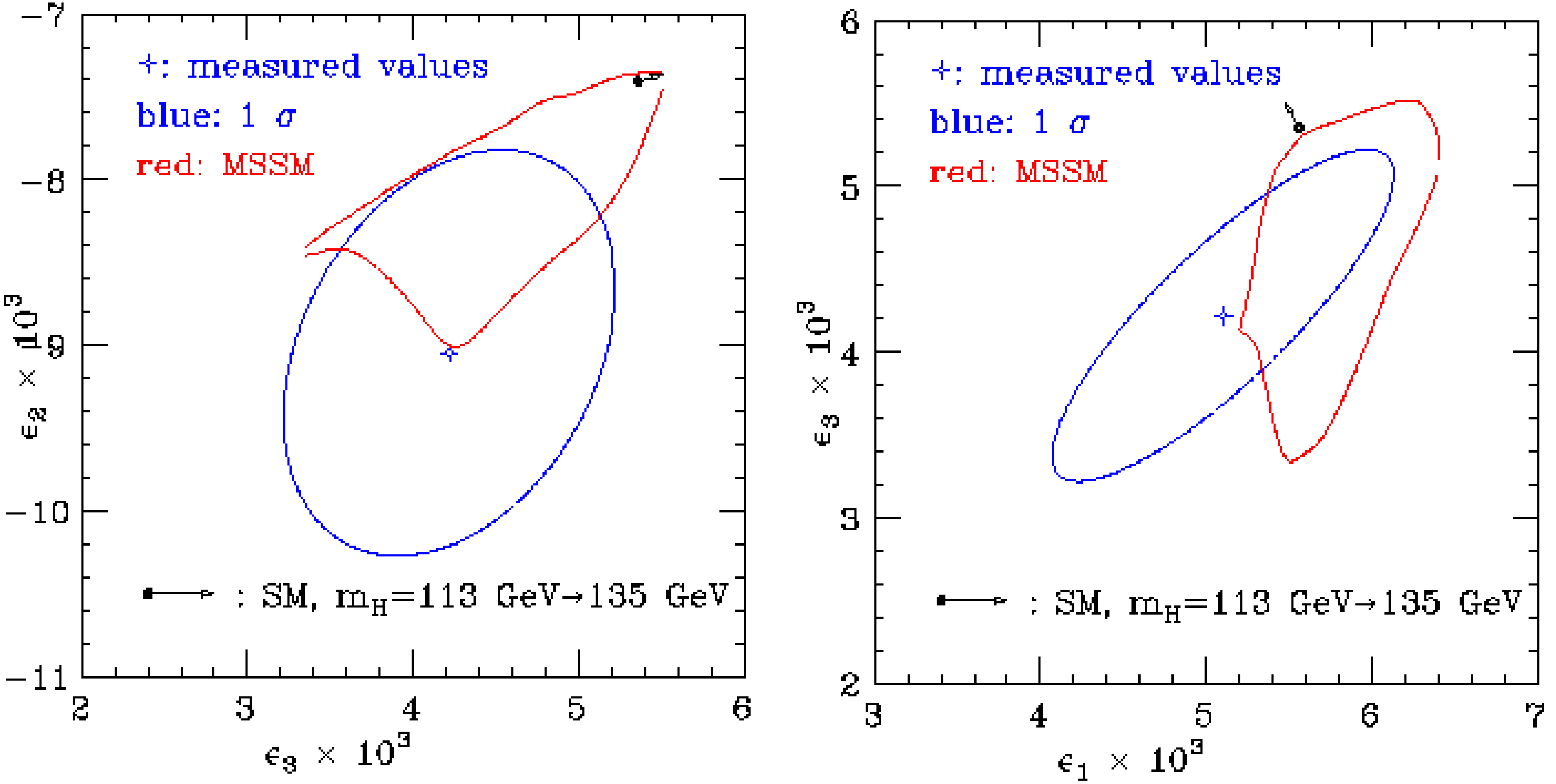
ε_1 is OK, ε_2 is low (m_W),
ε_3 depends on $\sin^2\theta$: low for $[\sin^2\theta]_l$ (m_H)

MSSM: $m_{\tilde{e}\text{-}L}$ = 96-300 GeV, m_{χ^-} = 105-300 GeV,
μ = (-1)-(+1) TeV, tgβ = 10, m_h = 113 GeV,
$m_A = m_{\tilde{e}\text{-}R} = m_{\tilde{q}}$ =1 TeV



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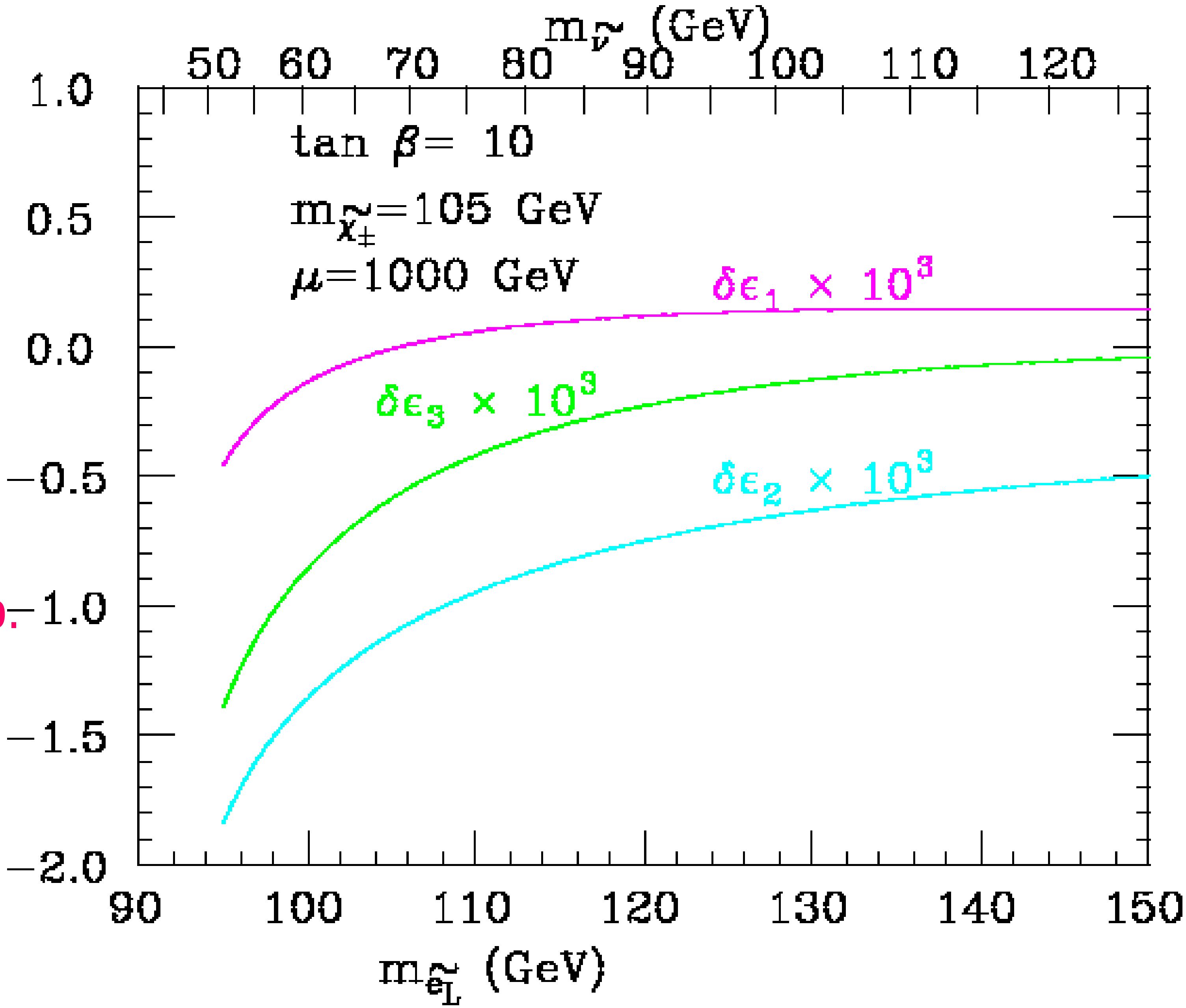
s-leptons
<u>and</u> s-ν's
plus
gauginos
must be
as light as
possible
given the
present exp.
bounds!



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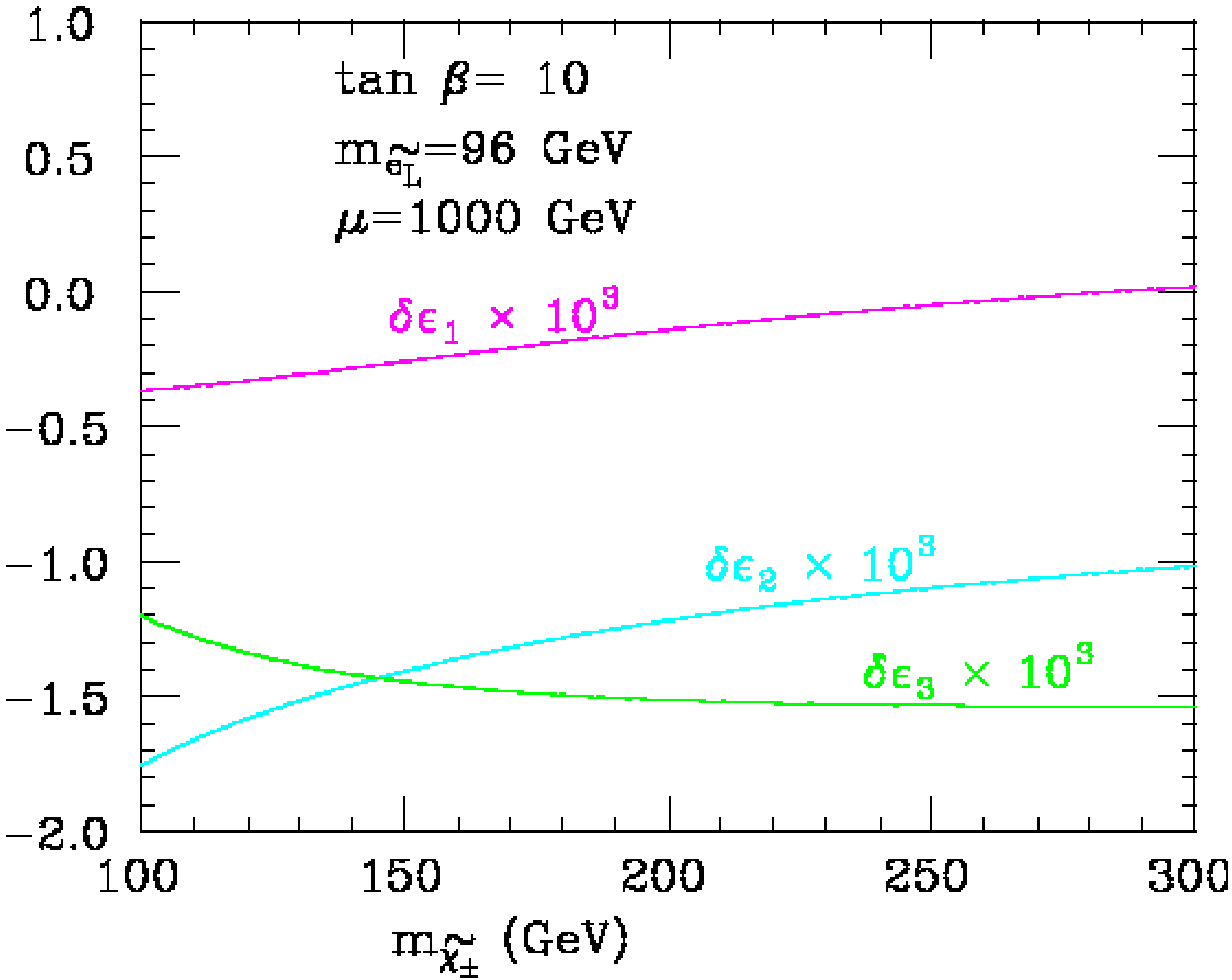
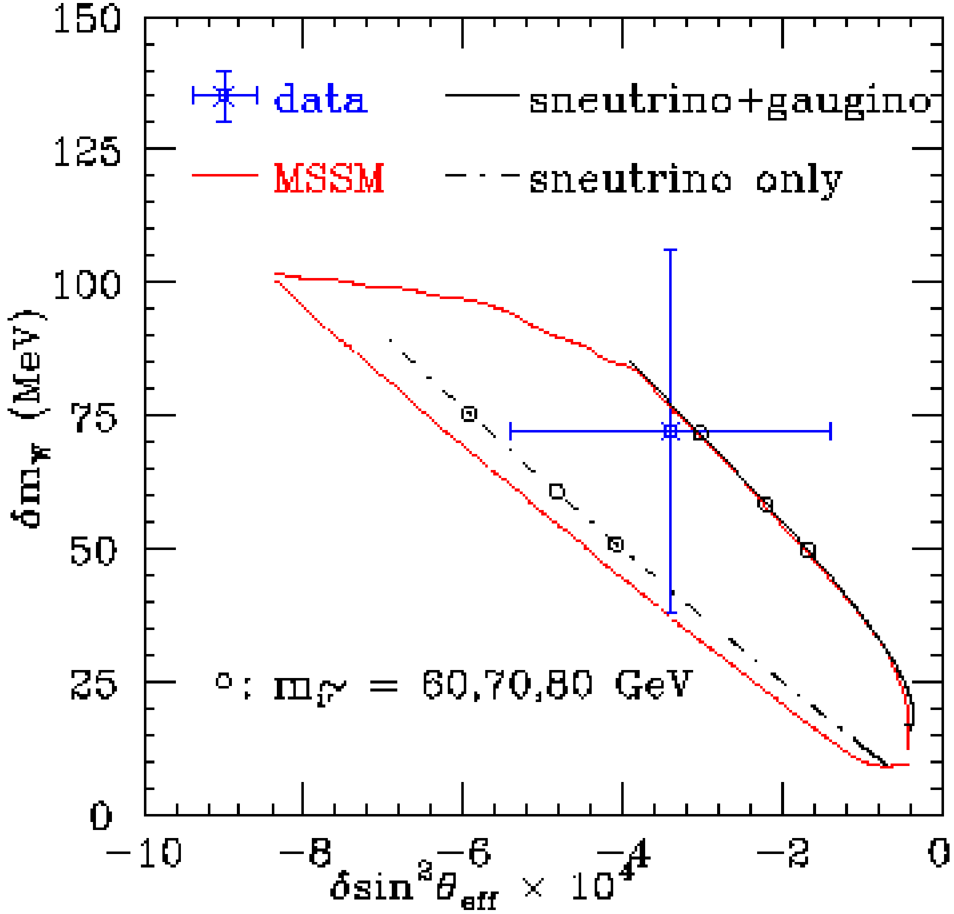
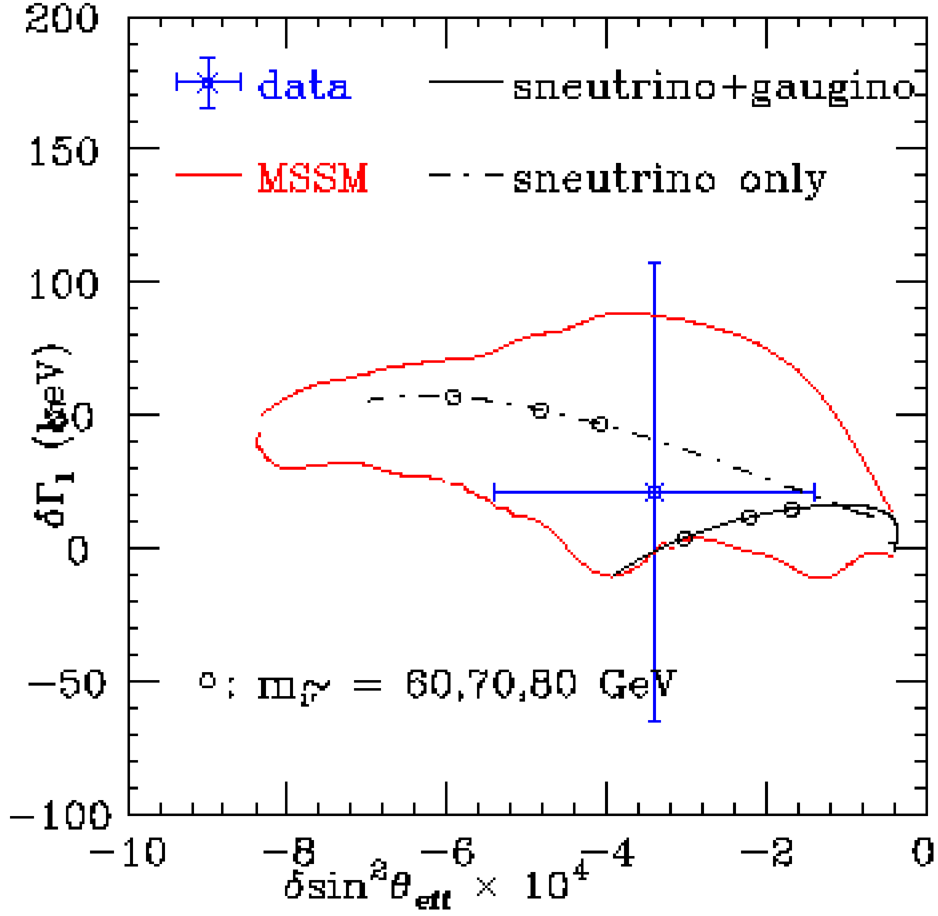
In general in MSSM: $m^2_{\tilde{e}-} = m^2_{\tilde{\nu}} + m^2_W|\cos 2\beta|$

Light charginos also help by making ε_2 corr's larger than those of ε_3





data
sneutrino+gaugino
MSSM
sneutrino only
○: $m_{\tilde{f}}$ = 60,70,80 GeV
δm_W (MeV)
$\delta \sin^2\theta_{eff} \times 10^4$
150
125
100
75
50
25
0
−10
−8
−6
−4
−2
0



data
sneutrino+gaugino
MSSM
sneutrino only
○: $m_{\tilde{f}}$ = 60,70,80 GeV
$\delta \Gamma_l$ (keV)
$\delta \sin^2\theta_{eff} \times 10^4$
200
150
100
50
0
−50
−100
−10
−8
−6
−4
−2
0

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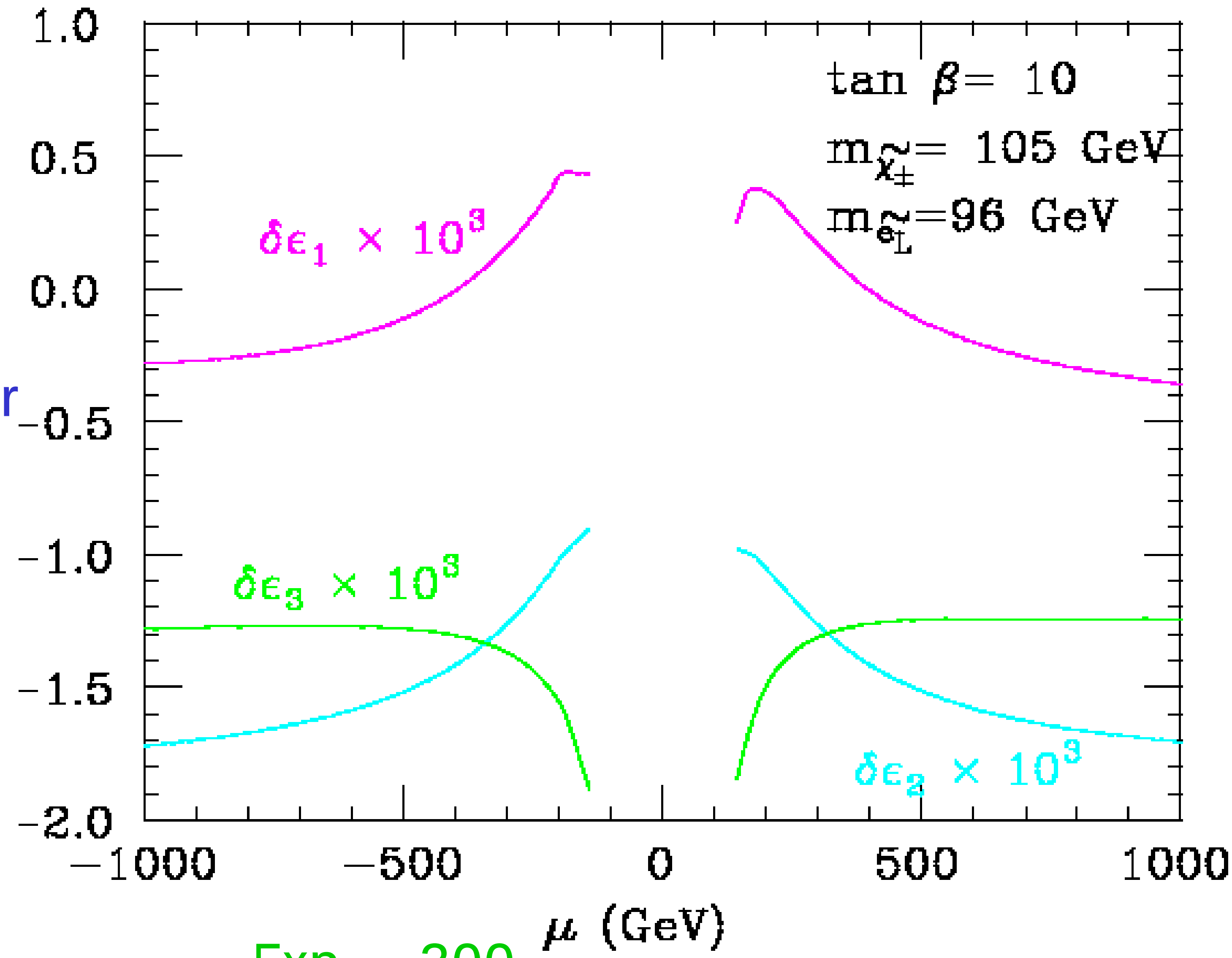
The sign of μ is irrelevant here.
But crucial for $(g-2)_\mu$



This model can also fit $(g-2)_\mu$

Approx. at large tgβ:

Exp. ~300

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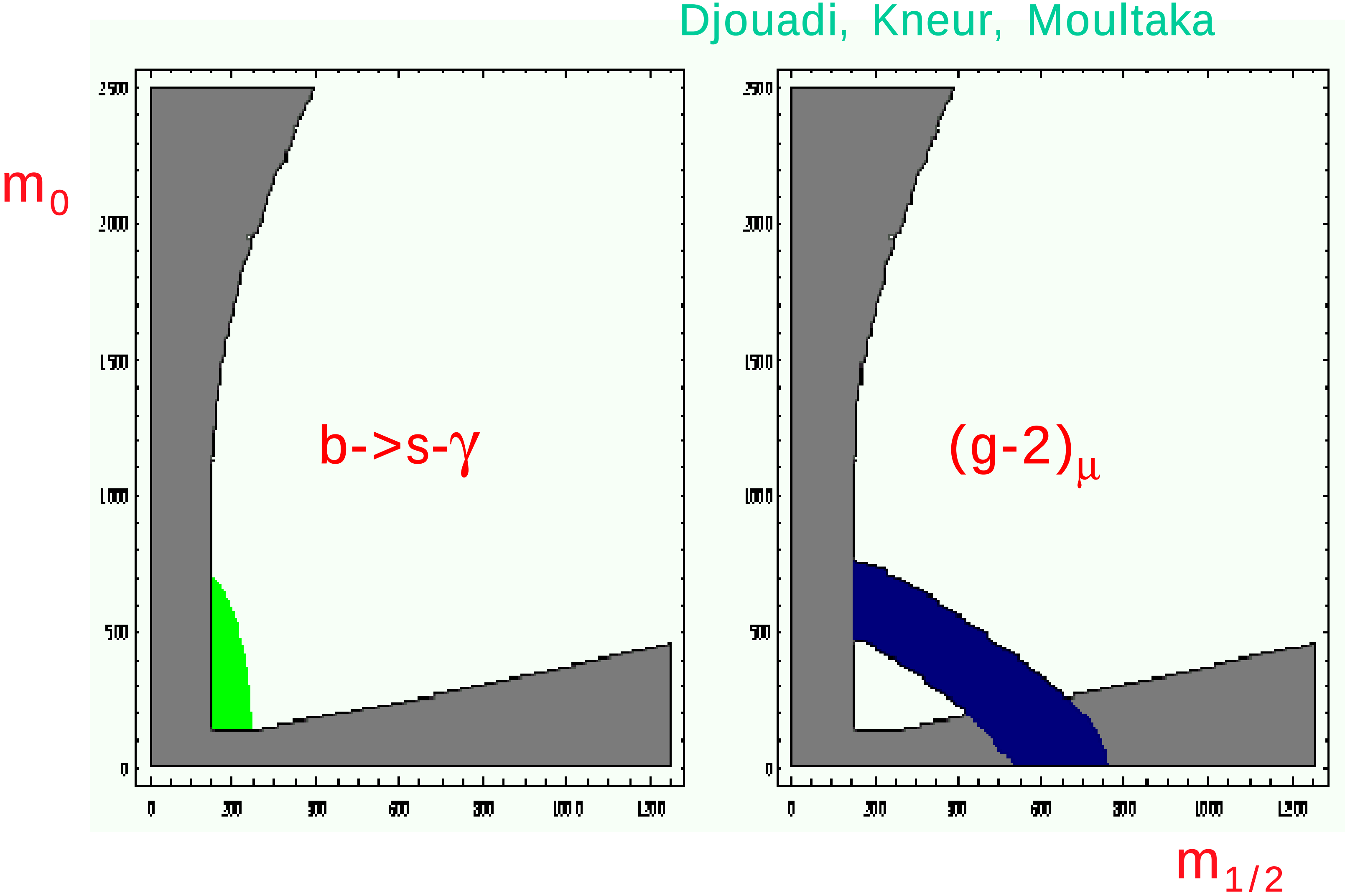
$$a_\mu \sim 130\ 10^{-11}(100\ \mathrm{GeV}/\tilde{m})^2\ \mathrm{tg}\beta$$

tanβ=40, A=0, sign(μ)>0

Djouadi, Kneur, Moultaka



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The Standard Model works very well

So, why not find the Higgs and declare particle physics solved?

First, you have to find it! $\Rightarrow$ LHC

Because of both:

Conceptual problems

- Quantum gravity
- The hierarchy problem
- •••••

and experimental clues:

- Coupling unification
- Neutrino masses
- Baryogenesis
- Dark matter
- Vacuum energy
- •••••

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Conceptual problems of the SM

Most clearly:

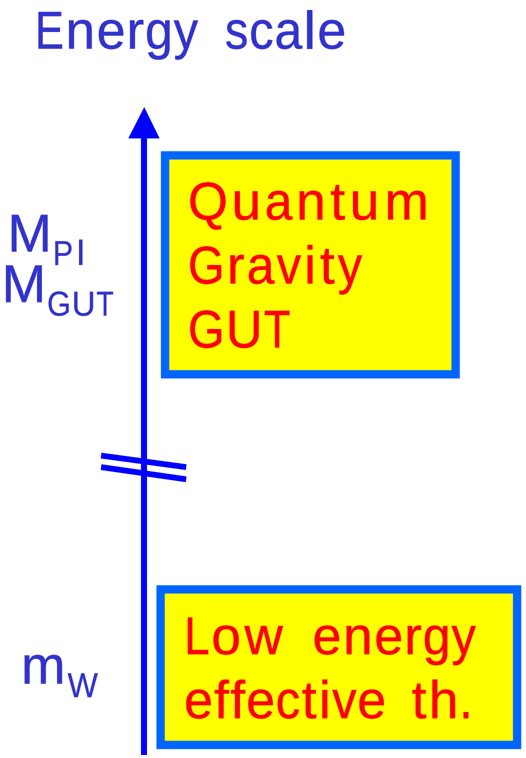
- No quantum gravity ($M_{Pl} \sim 10^{19}$ GeV)
- But a direct extrapolation of the SM leads directly to GUT's ($M_{GUT} \sim 10^{16}$ GeV)

M_{GUT} close to M_{Pl}

- suggests unification with gravity as in superstring theories
- poses the problem of the relation m_W vs M_{GUT}- M_{Pl}

Can the SM be valid up to M_{GUT}- M_{Pl}?? ← The hierarchy problem

Not only it looks very unlikely, but the new physics must be near the weak scale!



The hierarchy problem

Assume:

- A TOE at $\Lambda \sim M_{GUT} \sim M_{Pl}$
- A low en. th at o(TeV)
- A "desert" in between

The low en. th must be renormalisable as a necessary condition for insensitivity to physics at Λ.

[the cutoff can be seen as a parametrisation of our ignorance of physics at Λ]

But, as Λ is so large, in addition the dep. of ren. masses and couplings on Λ must be reasonable:
e.g. a mass of order m_W cannot be linear in Λ if $\Lambda \sim M_{GUT}$, M_{Pl}.

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With new physics at Λ the low en. th is only an effective theory. After integration of the heavy d.o.f.:

$\mathcal{L}_i$: operator of dim i

$$\mathcal{L} = \underbrace{o(\Lambda^2)\mathcal{L}_2 + o(\Lambda)\mathcal{L}_3 + o(1)\mathcal{L}_4}_{\text{Renorm.ble part}} + + \underbrace{o(1/\Lambda)\mathcal{L}_5 + o(1/\Lambda^2)\mathcal{L}_6 + \ldots}_{\text{Non renorm.ble part}}$$

In absence of special symmetries or selection rules, by dimensions $c_i\mathcal{L}_i \sim o(\Lambda^{4-i})\mathcal{L}_i$

$\mathcal{L}_2$: Boson masses ϕ^2. In the SM the mass in the Higgs potential is unprotected: $c_2 \sim o(\Lambda^2)$

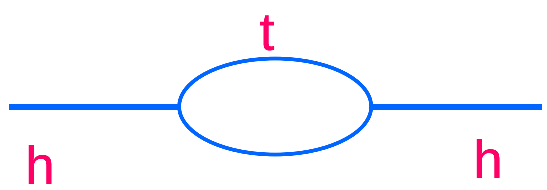
$\mathcal{L}_3$: Fermion masses $\bar{\psi}\psi$. Protected by chiral symmetry and SU(2)xU(1): $\Lambda \to m\log\Lambda$

$\mathcal{L}_4$: Renorm.ble interactions, e.g. $\bar{\psi}\gamma^\mu\psi A_\mu$

$\mathcal{L}_{i>4}$: Non renorm.ble: suppressed by $1/\Lambda^{i-4}$ e.g. $1/\Lambda^2\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma^\mu\psi$

Indeed in SM m_h, m_W... are linear in Λ!

e.g. the top loop (the most pressing): $m_h^2 = m^2_{bare} + \delta m_h^2$



$$\delta m^2_{h|top} = \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 \Lambda^2 \sim (0.3\Lambda)^2$$

The hierarchy problem demands new physics near the weak scale

$\Lambda \sim o(1\text{TeV})$

Λ: scale of new physics beyond the SM

- $\Lambda >> m_Z$: the SM is so good at LEP
- $\Lambda \sim$ few times $G_F^{-1/2} \sim o(1\text{TeV})$ for a natural explanation of m_h or m_W

Barbieri, Strumia

The LEP Paradox: m_h light, new physics must be so close but its effects are not directly visible

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Examples:

SUSY

- Supersymmetry: boson-fermion symm.

  exact (unrealistic): cancellation of $\delta\mu^2$

  approximate (possible): $\Lambda \sim m_{SUSY}$-m_{ord}

The most widely accepted

- The Higgs is a $\bar{\psi}\psi$ condensate. No fund. scalars. But needs new very strong binding force: $\Lambda_{new} \sim 10^3 \Lambda_{QCD}$ (technicolor).

Strongly disfavoured by LEP

- Large extra spacetime dimensions that bring M_{Pl} down to o(1TeV)

Elegant and exciting. Does it work?

- Models where extra symmetries allow m_h only at 2 loops and non pert. regime starts at $\Lambda \sim 10$ TeV

"Little Higgs" models. Now extremely popular around Boston.

Does it work?

SUSY at the Fermi scale

- Many theorists consider SUSY as established at M_{Pl} (superstring theory).

- Why not try to use it also at low energy to fix some important SM problems.

- Possible viable models exists:

  MSSM softly broken with gravity mediation
  or with gauge messengers
  or with anomaly mediation
  •••

- Maximally rewarding for theorists

  Degrees of freedom identified
  Hamiltonian specified
  Theory formulated, finite and computable up to M_{Pl}

Unique!

Fully compatible with, actually supported by GUT's

SUSY fits with GUT's

From $\alpha_{QED}(m_Z)$, $\sin^2\theta_W$ measured at LEP predict $\alpha_s(m_Z)$ for unification (assuming desert)

EXP: $\alpha_s(m_Z)=0.119\pm0.003$
Present world average

- Coupling unification: Precise matching of gauge couplings at M_{GUT} fails in SM and is well compatible in SUSY

Non SUSY GUT's
$\alpha_s(m_Z)=0.073\pm0.002$

SUSY GUT's
$\alpha_s(m_Z)=0.130\pm0.010$

Langacker, Polonski

Dominant error: thresholds near M_{GUT}

- Proton decay: Far too fast without SUSY
- $M_{GUT} \sim 10^{15}$GeV non SUSY ->10^{16}GeV SUSY
- Dominant decay: Higgsino exchange

While GUT's and SUSY very well match, (best phenomenological hint for SUSY!) in technicolor , large extra dimensions, little higgs etc., there is no ground for GUT's

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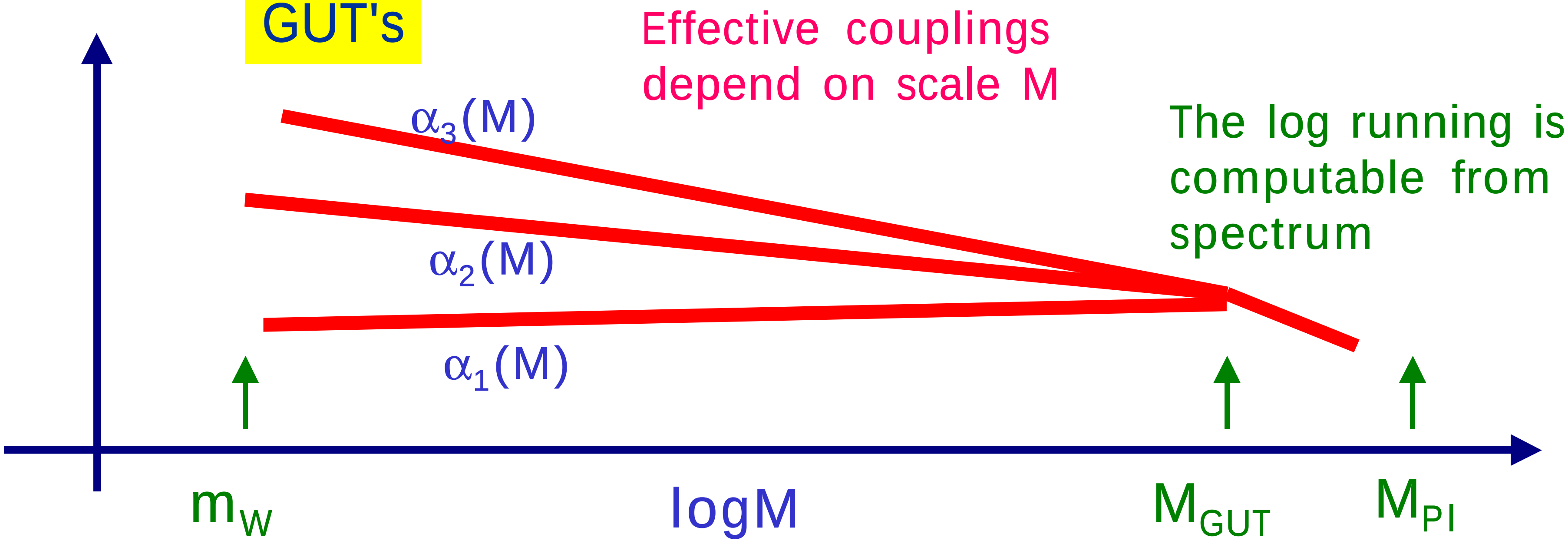
GUT's



The large scale structure of particle physics:

- $SU(3) \otimes SU(2) \otimes U(1)$ unify at M_{GUT}
- at M_{Pl}: quantum gravity

$$G_{Newton} = \frac{\hbar c}{M_{Pl}^2}$$

$r \sim 10^{-33}$ cm

Superstring theory:
a 10-dimensional non-local, unified theory of all interact's

The really fundamental level

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By now GUT's are part of our culture in particle physics

- Unity of forces: $G \supset SU(3) \otimes SU(2) \otimes U(1)$

  unification of couplings
- Unity of quarks and leptons

  different "directions" in G
- B and L non conservation

  ->p-decay, baryogenesis, ν masses
- Family Q-numbers

  e.g. in SO(10) a whole family in 16
- Charge quantisation: $Q_d = -1/3 \rightarrow -1/N_{colour}$

• • • • •

Most of us believe that Grand Unification must be a feature of the final theory!

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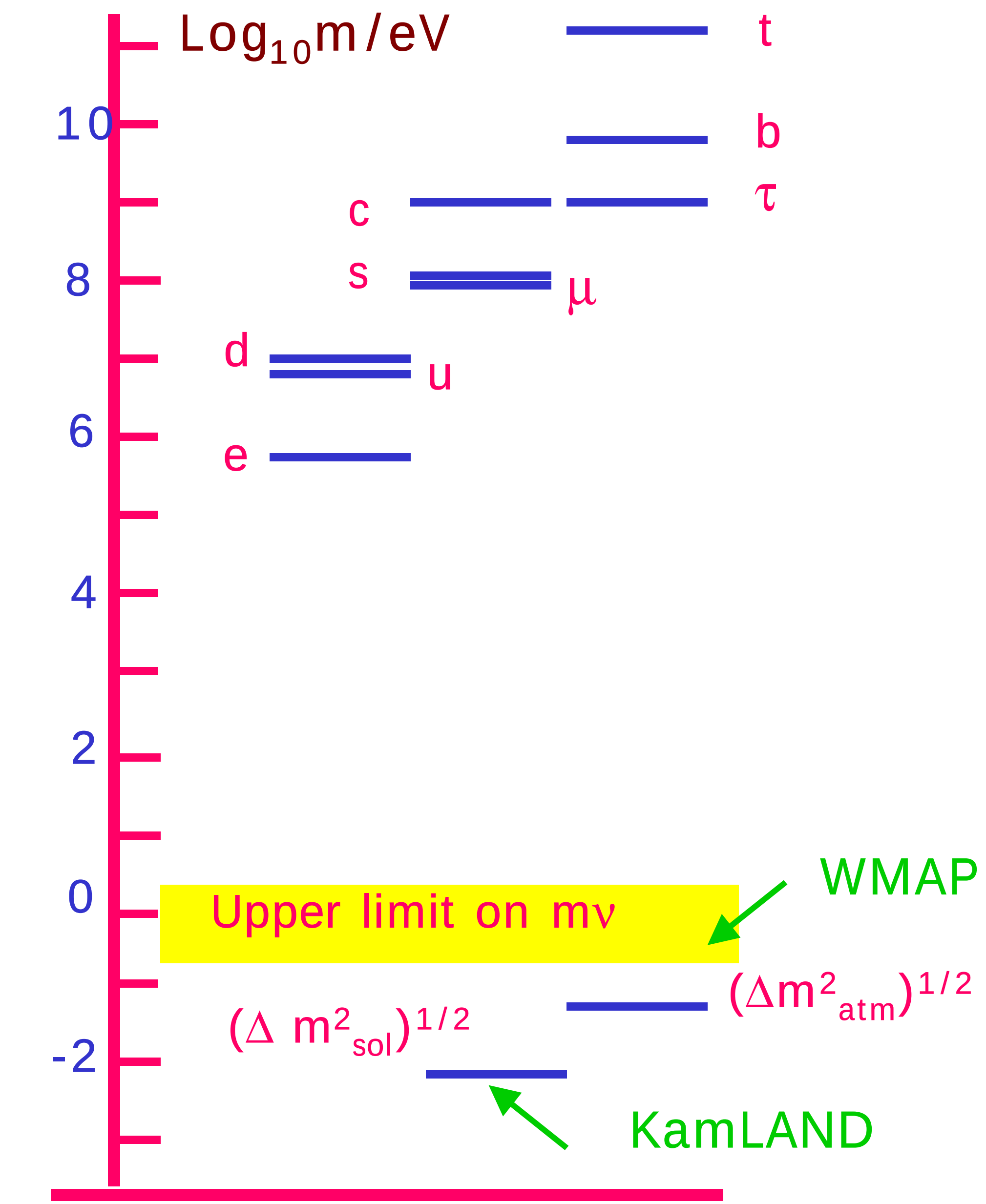
Neutrino masses point to M_{GUT}, well fit into the SUSY picture and in GUT's and have added considerable support to this idea.

Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$



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Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m \leq m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

Baryogenesis

$n_B/n_\gamma \sim 10^{-10}$, $n_B << \bar{n}_B$

Conditions for baryogenesis: (Sacharov '67)

- B non conservation (obvious) —
- C, CP non conserv'n (B-B odd under C, CP)
- No thermal equilib'm ($n = \exp[\mu - E/kT]$; $\mu_B = \mu_{\bar{B}}$, $m_B = m_{\bar{B}}$ by CPT

If several phases of BG exist at different scales the asymm. created by one out-of-equilib'm phase could be erased in later equilib'm phases: BG at lowest scale best

Possible epochs and mechanisms for BG:

- At the weak scale in the SM Excluded
- At the weak scale in the MSSM Disfavoured
- Near the GUT scale via Leptogenesis

Very attractive

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Possible epochs for baryogenesis

- BG at the weak scale: $T_{EW} \sim 0.1\text{-}\ 10$ TeV

  Rubakov, Shaposhnikov; Cohen, Kaplan, Nelson; Quiros....

In SM:

- B non cons. by instantons ('t Hooft)
  (non pert.; negligible at T=0 but large at $T=T_{EW}$
  B-L conserved!
- CP violation by CKM phase. Enough??
  By general consensus far too small.
- Out of equilibrium during the EW phase trans.
  Needs strong 1st order phase trans. (bubbles)
  Only possible for $m_H <\sim 80$ GeV
  Now excluded by LEP

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Is BG at the weak scale possible in MSSM?

- Additional sources of CP violation

  Sofar no signal at beauty factories

- Constraint on m_H modified by presence of extra scalars with strong couplings to Higgs sector (e.g. s-top)

- Requires:
  m_h<80-100 GeV; $m_{s\text{-}topl}<m_t$; tg$\beta\sim$1.2-5 preferred

  Espinosa, Quiros, Zwirner; Giudice; Myint; Carena, Quiros, Wagner; Laine; Cline, Kainulainen; Farrar, Losada…..

Disfavoured by LEP

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Baryogenesis A most attractive possibility:

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al …..

Only survives if Δ(B-L)$\neq$0

(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R (M~10^{12} GeV)

L non conserv. in ν_R out-of-equilibrium decay:

B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived

$m_i \leq 10^{-1}$ eV

Close to WMAP

Buchmuller, Di Bari, Plumacher

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Dark Matter

Most of the Universe is not made up of atoms: $\Omega_{tot}\sim1$, $\Omega_b\sim0.04$, $\Omega_m\sim0.3$

Most is non baryonic dark matter and dark energy

Cold

Non relativistic at freeze out

Good clustering at small distances (galaxies, ...)

SUSY:

Neutralino: Good candidate

Axions not excluded

Hot

Relativistic at freeze out $\rightarrow$ Could be ν's

But: $\Omega_\nu<0.015$ (WMAP)

Relevant for large scale mass distrib'ns

Conclusion:

Most Dark Matter is Cold (Neutralinos, Axions...)
Significant Hot Dark matter is disfavoured
Neutrinos are not much cosmo-relevant.

But: Lack of SUSY signals at LEP + lower limit on m_H

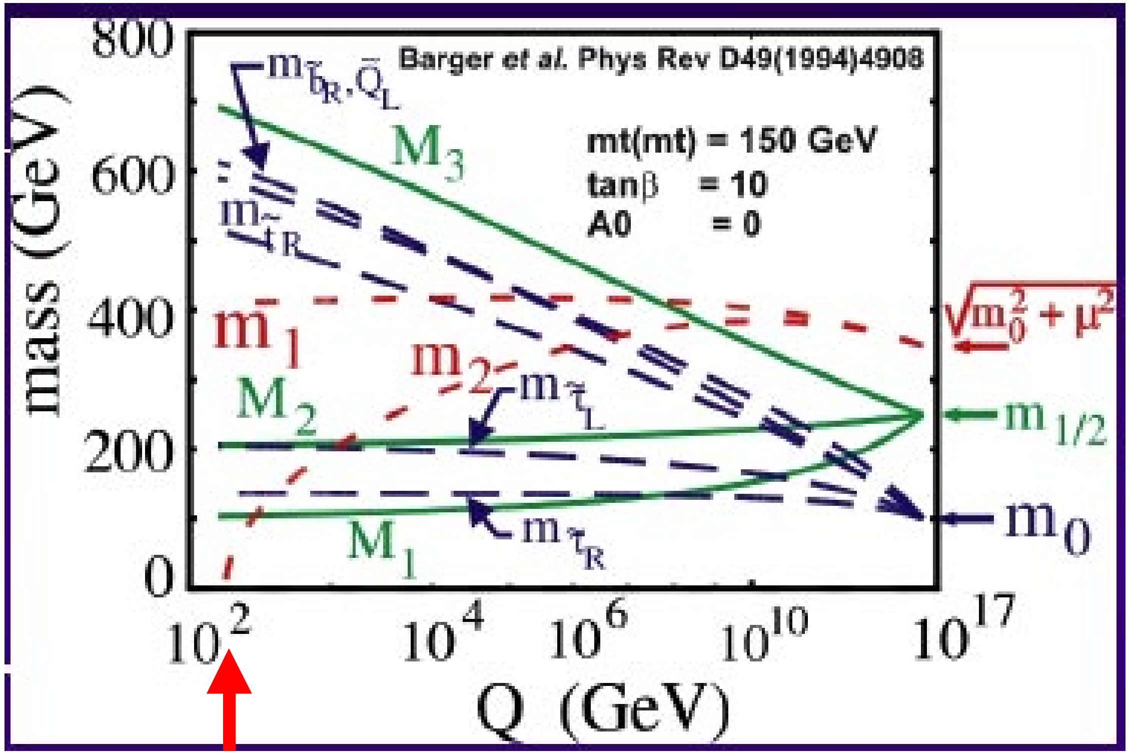
→ problems for minimal SUSY

- In MSSM: $$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3\alpha_w m_t^4}{4\pi m_W^2 \sin^2\beta} \ln \frac{\tilde{m}_t^4}{m_t^4} < \sim 130 \text{ GeV}$$

So m_H > 114 GeV considerably reduces available parameter space.

- In SUSY EW symm. breaking is induced by H_u running

Exact location implies constraints →



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m_Z can be expressed in terms of SUSY parameters

For example, assuming universal masses at M_{GUT} for scalars and for gauginos

$$m_Z^2 \approx c_{1/2} m_{1/2}^2 + c_0 m_0^2 + c_t A_t^2 + c_\mu \mu^2$$

$c_a = c_a(m_t, \alpha_i, \ldots)$

Clearly if $m_{1/2}$, m_0,... >> m_Z: Fine tuning!

LEP results (e.g. $m_{\chi+}$ >~100 GeV) exclude gaugino universality if no FT by > ~20 times is allowed

Without gaugino univ. the constraint only remains on m_{gluino} and is not incompatible

$$m_Z^2 \approx 0.7 m_{gluino}^2 + \ldots$$

[Exp. : m_{gluino} >~200GeV]

Barbieri, Giudice; de Carlos, Casas; Barbieri, Strumia; Kane, King; Kane, Lykken, Nelson, Wang......

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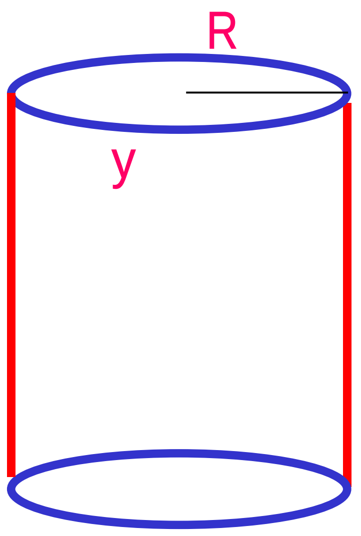
Large Extra Dimensions

Solve the hierachy problem by bringing gravity down from M_{Pl} to o(1TeV)

Arkani-Hamed, Dimopoulos, Dvali+Antoniadis; Randall,Sundrun….

Inspired by string theory, one assumes:

- Large compactified extra dimensions
- SM fields are on a brane
- Gravity propagates in the whole bulk



y: extra dimension
R: compact'n radius

y=0 "our" brane

$G_N \sim 1/M^2_{Pl}$: Newton const.

M_{Pl} large as G_N weak

The idea is that gravity appears weak as a lot of lines of force escape in extra dimensions

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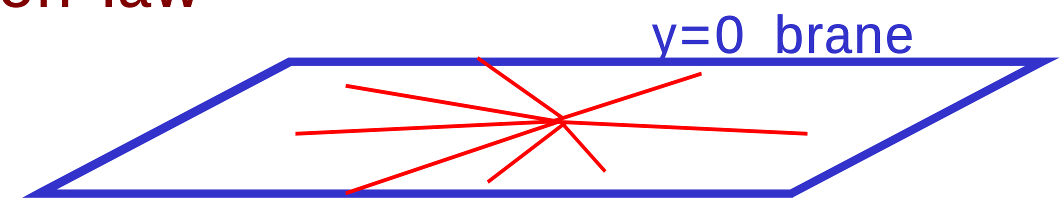
r >> R: ordinary Newton law

$$F \sim \frac{G_N}{r^2} \sim \frac{1}{M_{Pl}^2 r^2}$$

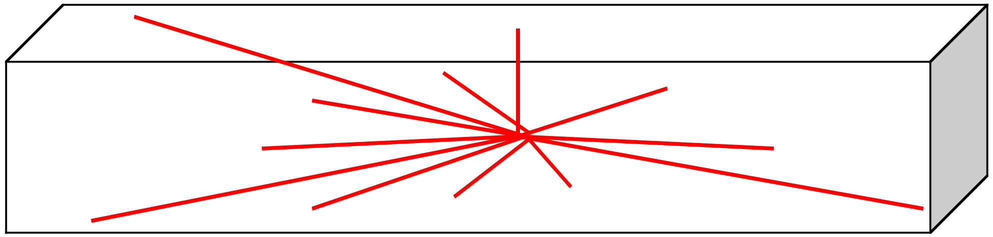


r << R: lines in all dimensions

Gauss in d dim:

$r^{d-2}\,\rho \sim m$

$$F \sim \frac{1}{m^2 (mr)^{d-4} \cdot r^2}$$



By matching at r=R

$$\left(\frac{M_{Pl}}{m}\right)^2 = (Rm)^{d-4}$$

For m = 1 TeV, (d-4 = n)

n = 1 R= 10^{15} cm (excluded)
n = 2 R= 1mm (close to limits)
n = 4 R= 10^{-9} cm
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Limits on deviations from Newton law

$$V(r) = -G\frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

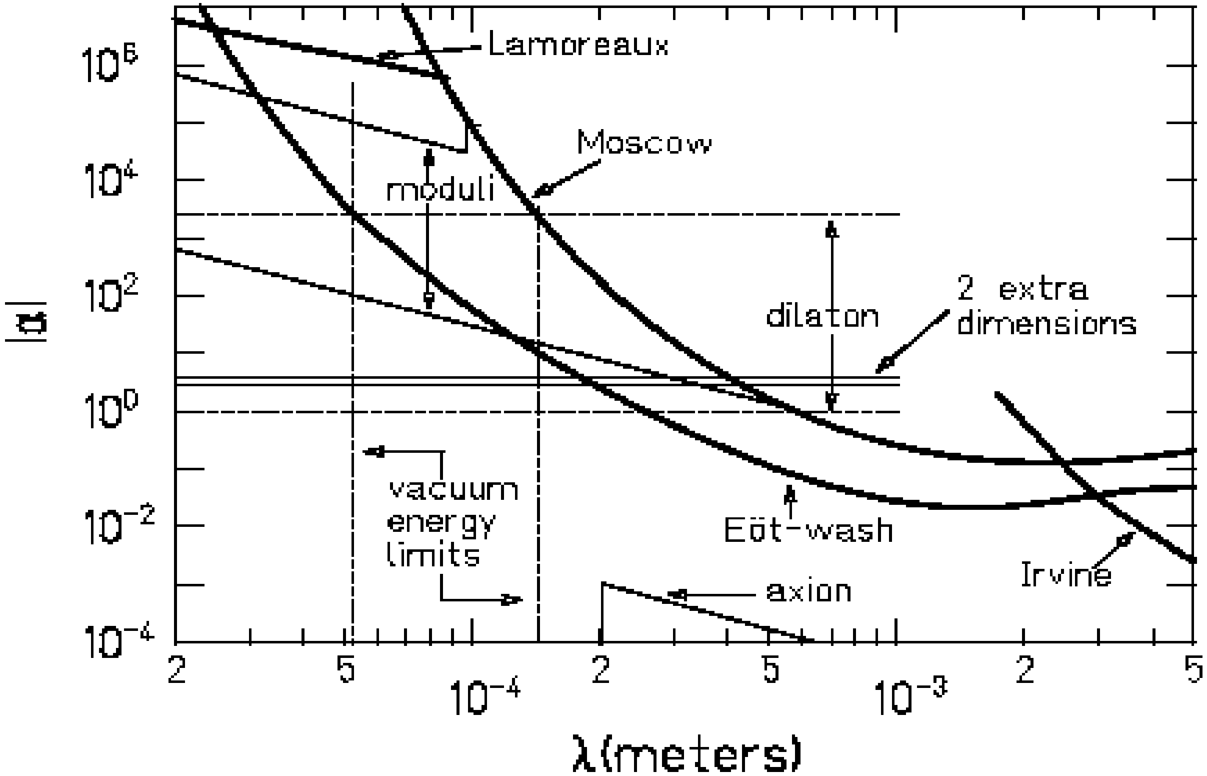

FIG. 4. 95% confidence upper limits on $1/r^2$-law violating interactions of the form given by Eq. (2). The region excluded by previous work [2,3,20] lies above the heavy lines labeled Irvine, Moscow and Lamoreaux, respectively. The data in Fig. 3 imply the constraint shown by the heavy line labeled Eöt-wash. Constraints from previous experiments and the theoretical predictions are adapted from Ref. [8], except for the dilaton prediction which is from Ref. [14].

Hoyle et al,

PRL 86,1418,2001

Generic feature:

compact dim. $\longrightarrow$ Kaluza-Klein (KK) modes

$p=n/R \longrightarrow m^2=n^2/R^2$ (quantization in a box)

Many possibilities:

- SM fields on a brane

The brane can itself have a thickness r:

$1/r >\sim 1\text{TeV} \longrightarrow r <\sim 10^{-17}$ cm

$\longrightarrow$ KK recurrences of SM fields: W_n, Z_n etc

cfr: • Gravity on bulk

$1/R >\sim 10^{-3}$ eV $\longrightarrow$ $R <\sim 0.1$ mm

- Factorized metric:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + h_{ij}(y) dy^i dy^j$$

- Warped metric: Randall-Sundrum

$$ds^2 = e^{-2kR|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - R^2 \varphi^2$$

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- Large Extra Dimensions is a very exciting scenario.

- However, by itself it is difficult to see how it can solve the main problems (hierarchy, the LEP Paradox)

  * Why (Rm) not 0(1)? $$\left(\frac{M_{Pl}}{m}\right)^2 = (Rm)^{d-4}$$

  * $\Lambda \sim 1/R$ must be small (m_H light)

  * But precision tests put very strong lower limits on Λ (several TeV)

  In fact in typical models of this class there is no mechanism to sufficiently quench the corrections

- No simple baseline model has yet emerged

- But could be part of the truth

The scale of the cosmological constant is a big mystery.

$\Omega_\Lambda \sim 0.65$ → $\rho_\Lambda \sim (2\ 10^{-3}\ \mathrm{eV})^4 \sim (0.1\mathrm{mm})^{-4}$ → Similar to m_ν!?

In Quantum Field Theory: $\rho_\Lambda \sim (\Lambda_{cutoff})^4$

If $\Lambda_{cutoff} \sim M_{Pl}$ → $\rho_\Lambda \sim 10^{123}\ \rho_{obs}$

Exact SUSY would solve the problem: $\rho_\Lambda = 0$

But SUSY is broken: $\rho_\Lambda \sim (\Lambda_{SUSY})^4 \geq 10^{59}\ \rho_{obs}$

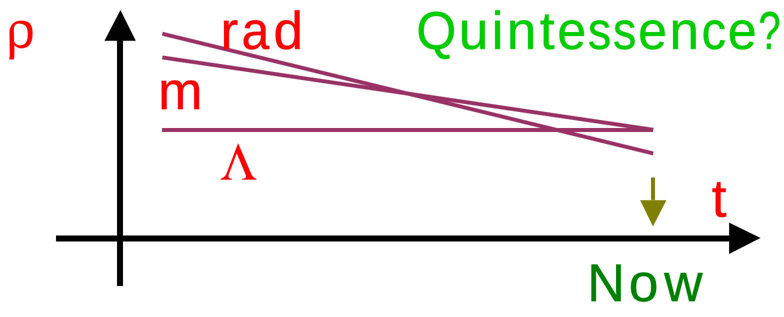
It is interesting that the correct order is $(\rho_\Lambda)^{1/4} \sim (\Lambda_{EW})^2 / M_{Pl}$

So far no solution:

- A modification of gravity at 0.1mm?(large extra dim.)
- Leak of vac. energy to other universes (wormholes)?

•••

Other problem:
Why now?



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Little Higgs Models

Georgi (moose),
Arkani-Hamed & C.

$$G \supset [SU(2) \otimes U(1)]^2 \supset SU(2) \otimes U(1)$$

global   gauged   SM

H is (pseudo)-Goldstone boson of G: takes mass only at 2-loops (needs breaking of 2 subgroups or 2 couplings)

cut off Λ ~10 TeV

Λ^2 divergences canceled by:

$\delta m^2_{H|top}$ new coloured fermion χ
$\delta m^2_{H|gauge}$ W', Z', γ'
$\delta m^2_{H|Higgs}$ new scalars

$\left.\right\}$ ~1 TeV

2 Higgs doublets ~0.2 TeV

E-W Precision Tests? Problems
GUT's? But signatures at LHC clear

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e.g.: enlarge $SU(2)_{weak}$ $\longrightarrow$ global SU(3)

quark doublet $\longrightarrow$ triplet $\begin{bmatrix} t_L \\ b_L \\ \chi_L \end{bmatrix}$

SU(3) broken spont.ly

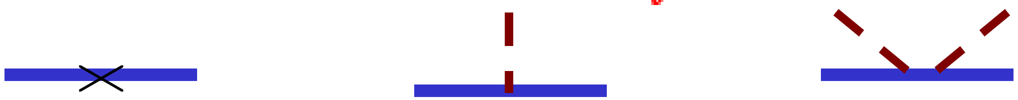
$$\varphi = \exp i \frac{\begin{bmatrix} - & h \\ h^\dagger & - \end{bmatrix}}{f} \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$$

Yukawa coupling:

expl. SU(3) breaking

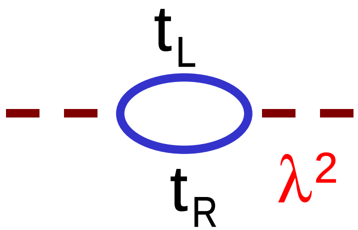
$$\lambda \begin{bmatrix} t^\dagger{}_L & b^\dagger{}_L & \chi^\dagger{}_L \end{bmatrix} \exp i \frac{\begin{bmatrix} - & h \\ h^\dagger & - \end{bmatrix}}{f} \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix} t_R + M \chi^\dagger{}_L \chi_R$$

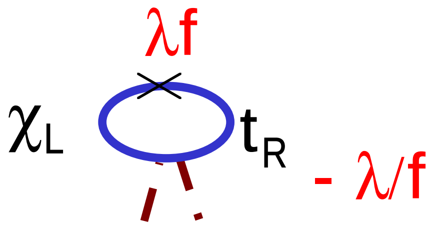
$$\lambda f \chi^\dagger{}_L t_R + i\lambda \begin{bmatrix} t^\dagger{}_L & b^\dagger{}_L \end{bmatrix} h t_R - \frac{\lambda}{2f} \chi^\dagger{}_L t_R h^\dagger h + \ldots.$$



top loop:

coeff. Λ^2





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Little Higgs: Big Problems with Precision Tests

Hewett, Petriello, Rizzo/ Csaki, Hubisz, Kribs, Meade, Terning

Even with vectorlike new fermions large corrections arise mainly from W_i', Z' exchange.
[lack of custodial SU(2) symmetry]

A combination of LEP and Tevatron limits gives:

$$f > 4 \text{ TeV at } 95\% \ (\Lambda = 4\pi f)$$

Fine tuning > 100 needed to get $m_h \sim 200$ GeV

Presumably can be fixed by complicating the model

Summarizing

- SUSY remains the Standard Way beyond the SM
- What is unique of SUSY is that it works up to GUT's .

GUT's are part of our culture!
Coupling unification, neutrino masses, dark matter, ...
give important support to SUSY

- It is true that the train of SUSY is already a bit late

(this is why there is a revival of alternative model building)

- No complete, realistic alternative so far developed

(not an argument! But…)

- Extra dim.s is an attractive, exciting possibility.
- Little Higgs models look as just a postponement

(both interesting to keep in mind)