L'esperimento VIRGO

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Abstract

Queste lezioni, tenute in occasione del VI Seminario Nazionale di Fisica Teorica, erano rivolte a studenti delle scuole di dottorato. Lo scopo è stato di dare una idea generale dei principi di funzionamento di un esperimento per la rivelazione delle onde gravitazionali basato su metodi interferometrici. Si è tenuto presente che la preparazione del pubblico era eminentemente teorica e si è cercato di dare spazio a quei metodi sperimentali che sono essenziali alla comprensione dell'esperimento e non rientrano nel bagaglio culturale teorico. La scelta degli argomenti è stata condizionata dai limiti di spazio, e si è cercato di privilegiare gli aspetti generali: ne risulta una trattazione inevitabilmente incompleta e imprecisa per alcuni aspetti. Me ne scuso con i lettori, che rimando agli approfondimenti in bibliografia.

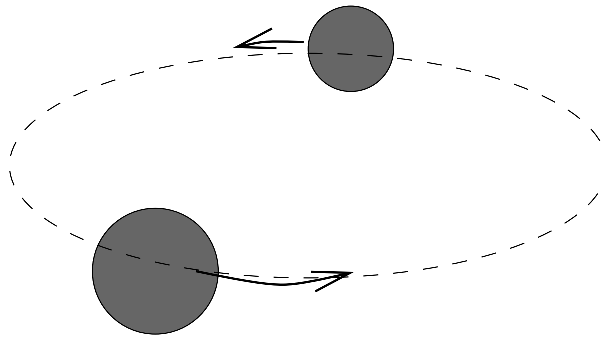
Lecture 1: the VIRGO interferometer

In this first seminar we want to give an overview of the VIRGO experiment function and concept [1].

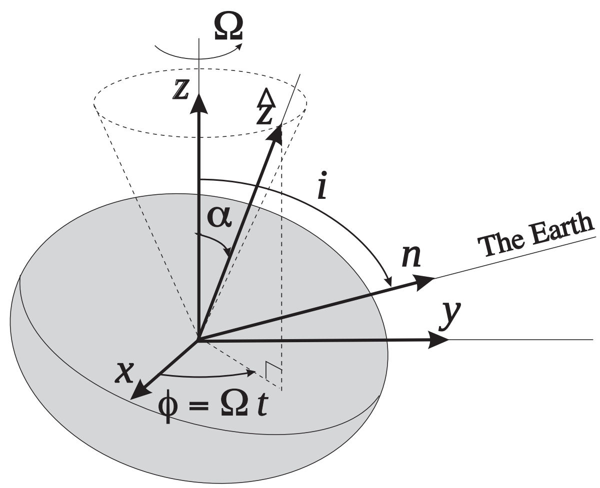
- Purpose.
- General principles.
- A few details on some of the noise sources.
- Prospected sensitivity.

Purpose: gravity wave detection

Sources [2]



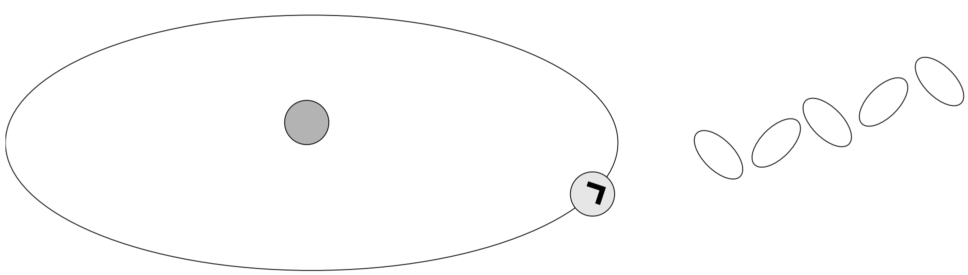
Coalescing stars



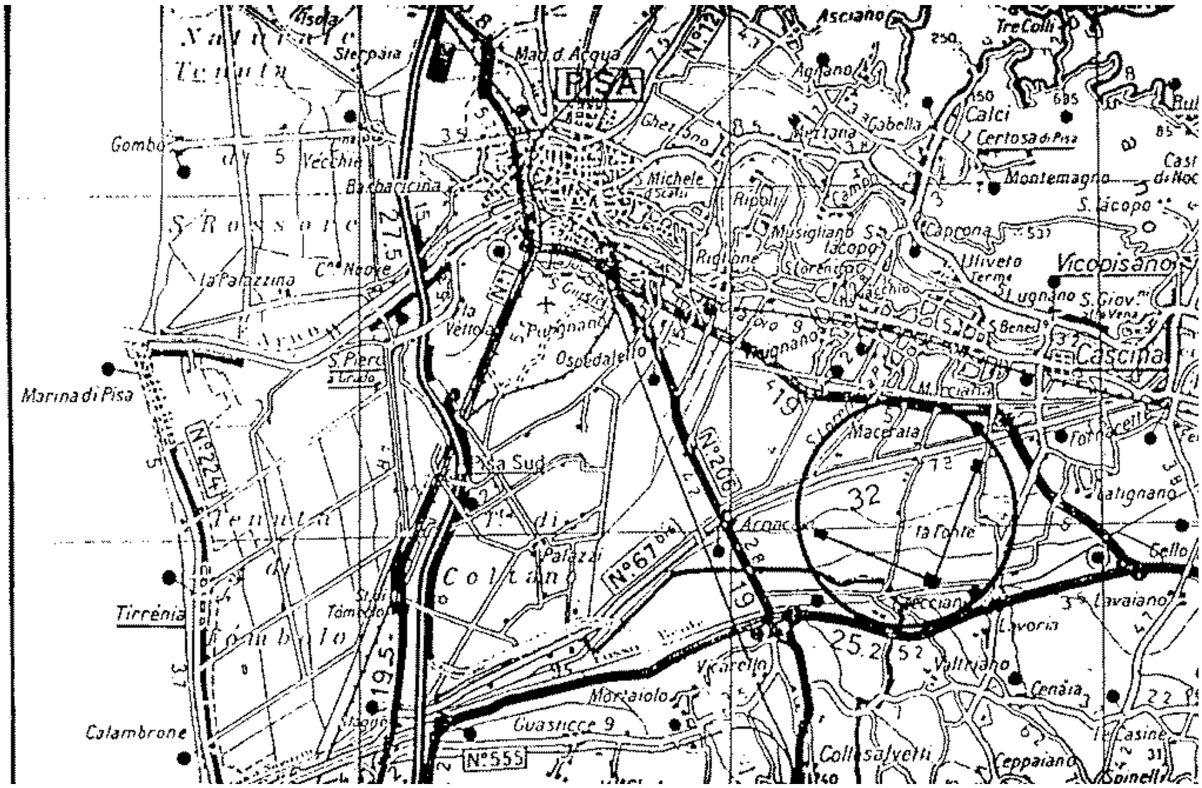
Distorted neutron star [3]

. . .

Receiver



VIRGO: a few numbers



- Two arms of 3 Km each.
- Vacuum with $p \leq 10^{-9}$ mbar.
- A laser power "stored" in the arms of the order of 1 kW.
- Mirrors positioned with an accuracy of $10^{-18} m/\sqrt{Hz}$ at 10 Hz.
- . . .

The future detection network

VIRGO is not alone: many other observatories are being built:

LIGO Laser Interferometric Gravitational Observatory is the US project [4]. Two interferometers, one in Louisiana, the other in the Washington state, arms of 4 Km. Construction already started, sensitivity similar to the one of VIRGO.

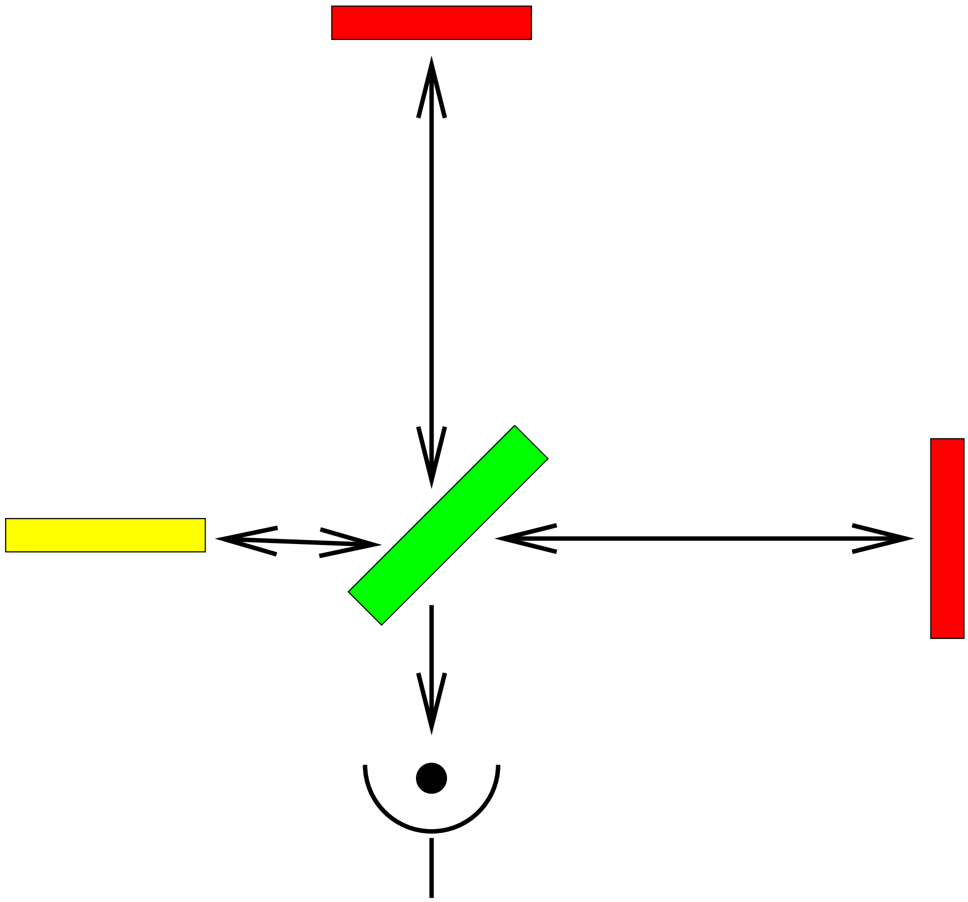
GEO600 The British - German project [5], a single interferometer near Hannover, with arms of 600 m. Less sensitive, but probably operating a couple of years before the VIRGO and LIGO

TAMA The Japanese project: a still smaller detector with arms of 300 m, which starts from good experience on a smaller instrument [6].

Having a network is really important:

- spurious *non gaussian* noise can give many false alarms. They can be rejected by requiring coincidences.
- GW observatories have little directionality. The direction of the sources can be reconstructed using at least 3 well separated detectors.

Theorist interferometer



- Purpose: compare the travel time of light along the two arms.
- Generate a light beam.
- Split the light at the *beam splitter*
- Read the interference at the output port

This device is sensitive to many effects: most of them **we** call *noise*, apart the Gravitational Waves, which **we** call *signal*.

The GW in transverse-traceless gauge

Throughout the lessons we shall assume *tiny* perturbations $h_{\mu\nu}$ of the flat metric $\eta_{\mu\nu}$ [7]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad (1)$$

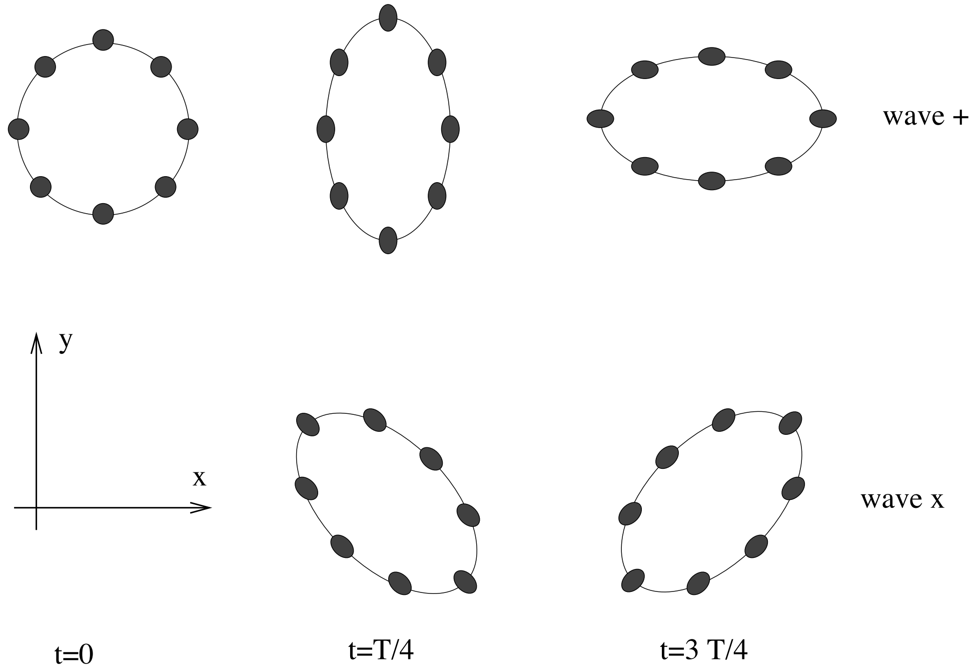
where in the weak field limit one has

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] h_{\mu\nu} = 0 \ . \qquad (2)$$

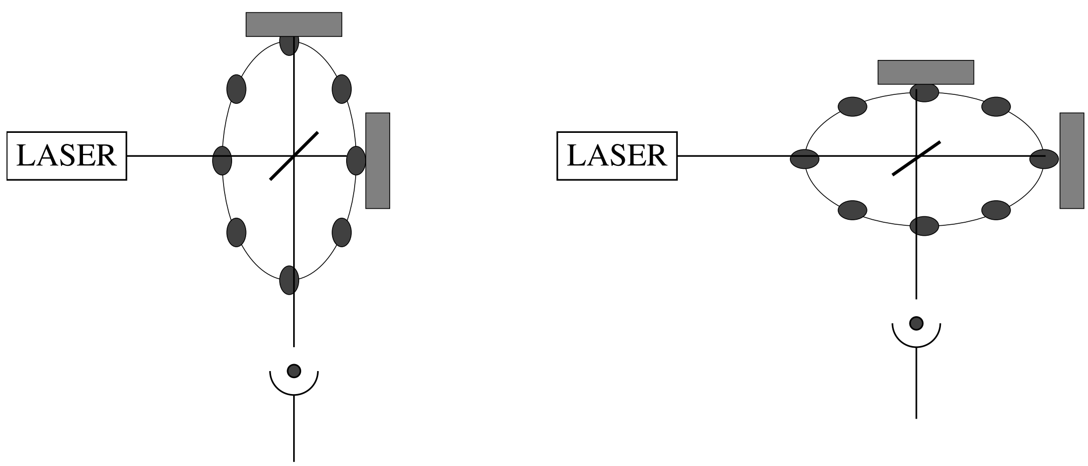
A field propagating along $\hat{z}$ will therefore be

$$h_{\mu\nu}(z,\ t) \equiv e^{i\left(2\pi f_{gw} t - kz\right)} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (3)$$

The fact that a rotation of 90^o brings the tensor on itself reminds of the spin-2 content of the field quanta.



Freely falling masses will sit on the geodesics of this "perturbated" metric.



In the TT gauge, the masses define the coordinate system! Then how can we detect any effect?

Why a Michelson-Morley is sensitive to GW?

- Light velocity is constant, and space-time events linked by the light have

$$
\begin{aligned}
ds^2 = 0 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= \left[\eta_{\mu\nu} + h_{\mu\nu}\right] dx^\mu dx^\nu
\end{aligned}
\tag{4}
$$

- Consider the length of the axis $\hat{x}$ leg of the ITF, under the influence of a *monochromatic* GW: you have

$$
c^2 dt^2 = \left[1 + h_{11}\left(2\pi f_{gw} t - \mathbf{k}\cdot\mathbf{x}\right)\right] dx^2 \tag{5}
$$

- Assume for simplicity a (complex!) $+$ polarized wave propagating along $\hat{z}$: then the wavefront leaving the BS at $t = 0$ and propagating along $\hat{x}$ comes back to the BS at (complex!) time

$$
\tau_x = \frac{2L}{c} + \frac{h}{4\pi i f_{gw}}\left[e^{i2\pi f_{gw} 2L/c} - 1\right] \; ; \tag{6}
$$

along $\hat{y}$ the light picks the effect of $h_{22} = -h_{11}$, hence one has that the two wavefronts are out of time by

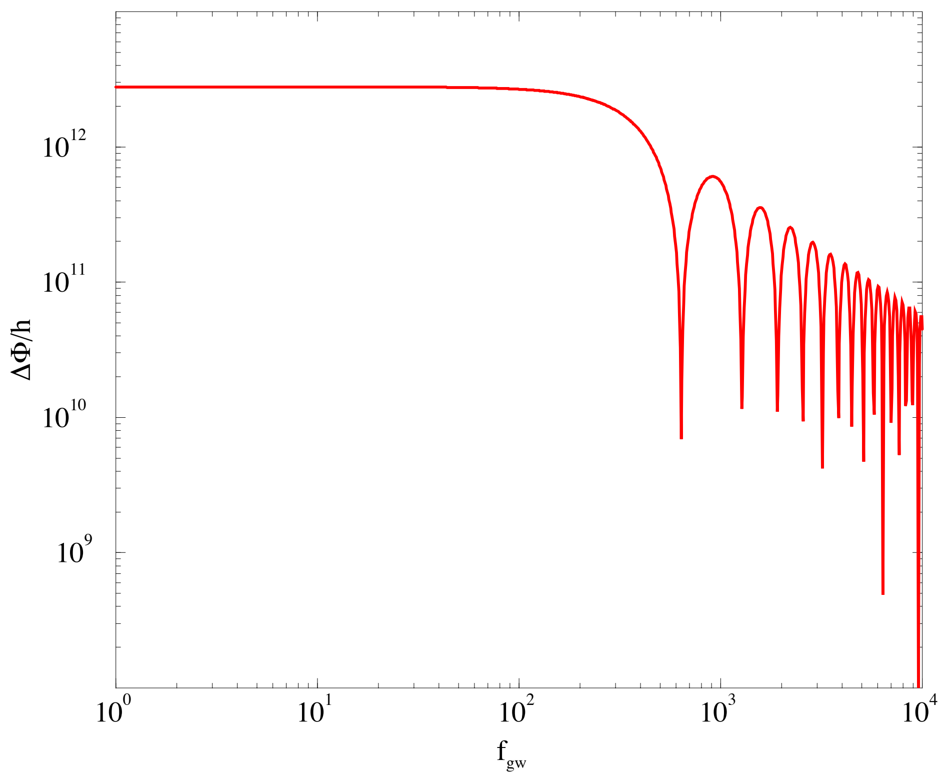
$$
\Delta\tau(t) = h(t)\tau_{rt} e^{i\pi f_{gw}\tau_{rt}} \frac{\sin\left(\pi f_{gw}\tau_{rt}\right)}{\pi f_{gw}\tau_{rt}} \tag{7}
$$

that is, they are *out of phase* by

$$
\Delta\phi(t) = \frac{2\pi c}{\lambda} h(t)\tau_{rt} e^{i\pi f_{gw}\tau_{rt}} \mathrm{sinc}(\pi f_{gw}\tau_{rt})
$$

$$= \frac{4\pi L}{\lambda} h(t) e^{i\pi f_{gw}\tau_{rt}} \mathrm{sinc}(\pi f_{gw}\tau_{rt}) \quad (8)$$

Having a fixed GW frequency f_{gw}, we have obtained the *transfer function* $h \to \Delta\phi$; the factor L/λ is the amplification which makes interferometry useful [8].



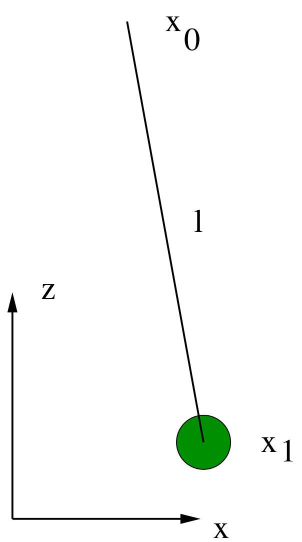
In the example, it is assumed $\lambda \sim 1\mu m$, $L \sim 100$ Km .

- The larger L, the greater the amplification, but the narrower the detection band! The first zero indeed occurs at $f = \frac{c}{4L}$. An interferometer is a GW *low-pass* filter.
- $L = 100$ Km is not a joke. What really counts is the *optical* length, which can be made much larger than the physical distance between the mirrors.

Freely falling masses??

How can we have end mirrors which behave as freely falling masses, without going in the space?

The idea is to have each mirror suspended to a *pendulum*.



Ideally, you have a motion equation

$$\ddot{x}_1 + \frac{g}{l}(x_1 - x_0) = F_{ext}/m \quad (9)$$

where F_{ext} is a force applied to the payload, that is

$$\tilde{x}_1(\omega) = \frac{\omega_0^2 \tilde{x}_0(\omega) + \tilde{F}_{ext}(\omega)/m}{\omega_0^2 - \omega^2} \quad (10)$$

we have two effects:

- Beyond the low frequency resonance, the effect of the motion of the suspension point on the *test mass* is attenuated: we shall see that this is absolutely necessary in order to isolate from the *seismic noise*.
- The effect of the force applied on the *test mass*, at high frequencies, is approximated by

$$\omega^2 m \tilde{x}_1(\omega) + F_{ext} \simeq 0 \quad (11)$$

that is it responds, along the $\hat{x}$ axis, as it were *free*.

Conceptual operation

Ideally, the VIRGO system is composed of two parts

1. The *suspension system*, which allows to keep the mirrors as close as possible to freely falling masses, isolating from all the *external* disturbances.
2. The *read-out* optical system, which measures continuosly the variation of the difference of the optical lengths along the two arms of the ITF.

Each of the systems is affected by noises:

1. Any motion $x(f)$ of the mirrors, induced either by the seismic noise or by *internal* disturbances of the suspension systems, that is *thermal* noises, reflects itself as an equivalent *displacement* noise

$$h_{disp}(f) = \frac{2}{L}x(f) \tag{12}$$

   where the factor $2 = \sqrt{4}$ comes from having 4 test masses, not only 2 as in a Michelson.
2. Any fluctuation in the photodiodes read-out, induced either by the intrinsic fluctuations of the photon counting process or by the presence of paths for the light different from the main one, are translated in a phase noise: for instance in a

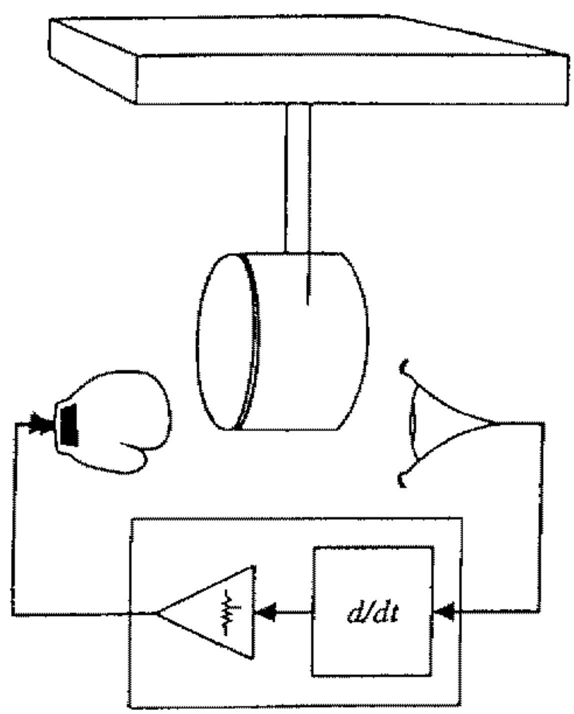
simple Michelson ITF we shall see that one has

$$h_{shot}(f) = \frac{1}{L}\sqrt{\frac{\hbar c\lambda}{2\pi P_{in}}} \tag{13}$$

where P_{in} is the laser power.

The two systems cannot be taken as independent! Indeed a *naive* ITF, as discussed so far, would not work at all. To obtain good sensitivities, it is necessary to keep the optical system at a *working point* which minimizes the effect of asymmetries in the arms and of fluctuations of the input light.

The position noise induced by external disturbances would swing the mirrors across many *fringes*: a *control* system is needed to maintain the ITF at the working point [8].



The VIRGO sensitivity: what does it mean?

We assume that our interferometer, in absence of GW, will continuosly produce a noise equivalent to some $h(t)$.

The (two-sided) noise spectrum in the continuum is defined as

$$S_h(f) = \lim_{T\to\infty} \left\langle \left| \frac{1}{\sqrt{T}} \int_{-T/2}^{T/2} h(t) e^{2\pi i f t}\, dt \right|^2 \right\rangle \quad ; \qquad (14)$$

while in the discrete is

$$S_h(f) = \lim_{N\to\infty} \left\langle \left| \sqrt{\frac{\Delta t}{N}} \sum_{n=0}^{N-1} h_n e^{2\pi i f n \Delta t} \right|^2 \right\rangle \quad . \qquad (15)$$

The sensitivity is simply given by

$$h(f) \equiv \sqrt{2 S_h(f)} \qquad (16)$$

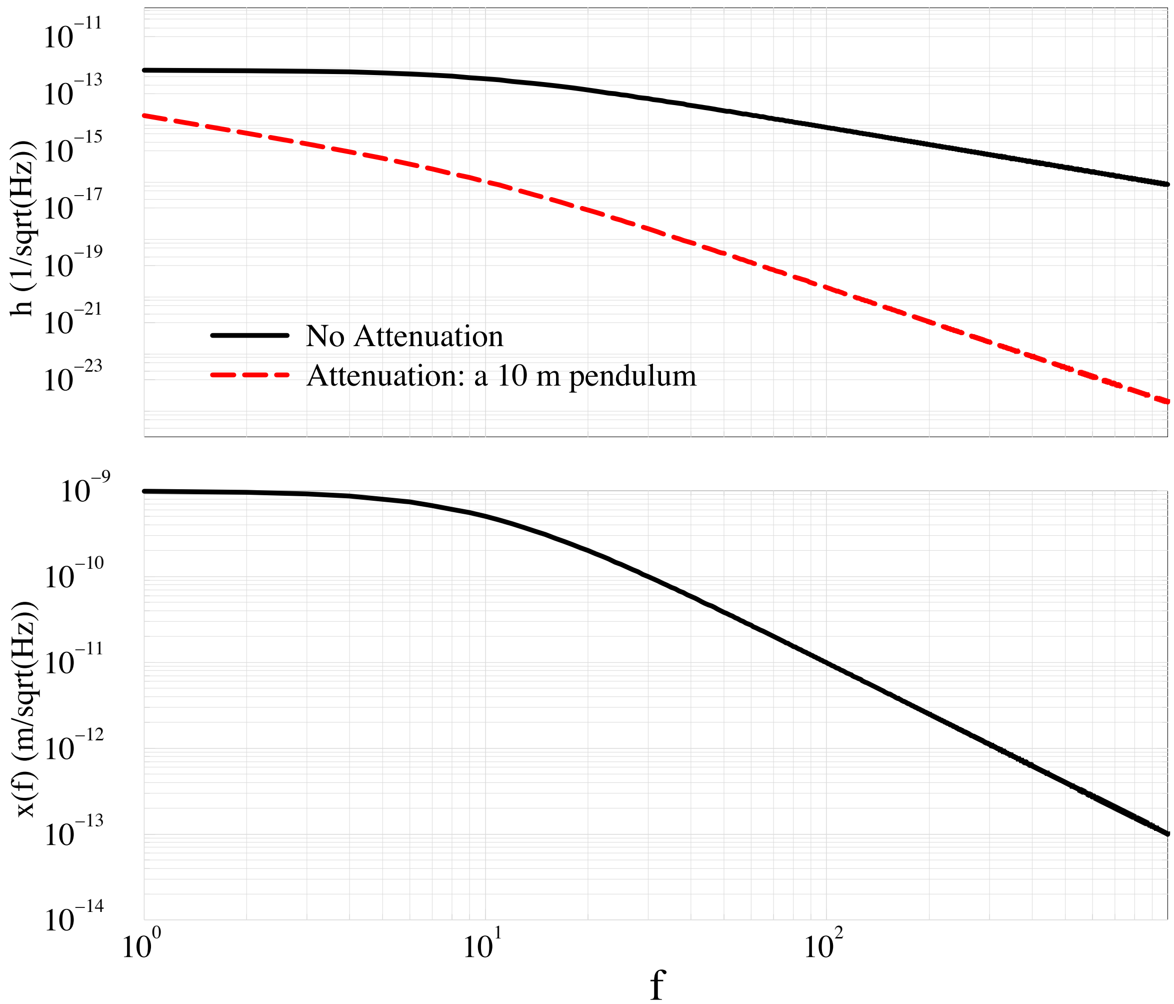
where the factor 2 means that only positive frequencies should be used. Example: if we have a sinusoid with amplitude A at frequency f, observed over a time T, one has

$$SNR \propto \frac{A\sqrt{T}}{h(f)} \qquad (17)$$

that is $h(f)$ is at fixed observation time T an "equivalent" sinusoid amplitude.

The danger from seismic noise

Is really seismic noise a problem? The displacement spectral density at a quiet site is something like [9]



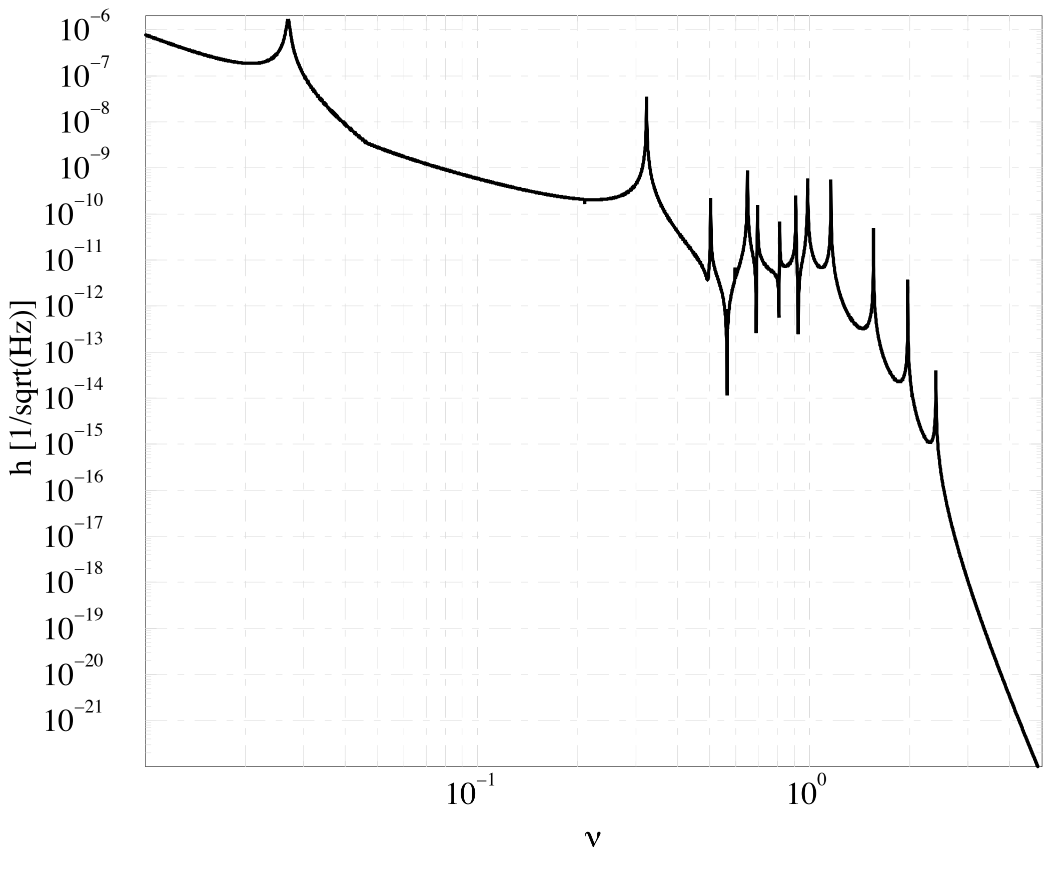
Keeping in mind the goal of a sensitivity of about 10^{-22} at 100 Hz, it is clear that seismic noise is almost 8 orders of magnitude higher!

The pendulum isolation helps: at 100 Hz a 10 m pendulum provides an attenuation factor of about 2×10^{-6}. We shall see that one can do better by building a chain of pendola.

The VIRGO **Super Attenuator**

One of the distinctive features of the VIRGO project is the ambitious isolation system [10].

Composed of multiple oscillators, it will reduce the h noise at the 10^{-22} level already at 10 Hz !



The residual seismic noise is a *wall* which prevents any GW observation below ~ 4 Hz, while above this limit is completely negligible if compared with the other noise sources.

Thermal noise effects

All the components of the ITF, from the suspension system to the mirrors, are operated at room temperature: hence they are subject to thermal random forces, which set the limit to the masses position noise [11].

The key to understanding these effects is the Fluctuation-Dissipation theorem: any *linear* system, at thermal equilibrium, which responds with an admittance Y at an external force

$$v(\omega) = Y(\omega) F_{ext}(\omega) \tag{18}$$

will also be subject to a minimal thermal force

$$F^2_{therm}(\omega) = 4\kappa_B T \mathcal{R}\left(Y^{-1}(\omega)\right) \tag{19}$$

or equivalently will undergo a random motion

$$x^2_{therm}(\omega) = \frac{4\kappa_B T}{\omega^2} \mathcal{R}\left(Y(\omega)\right) \ . \tag{20}$$

We shall see in the following that the typical form of the real part of the admittance function is a sum of Lorentzian terms, relative to the different modes of the system

$$x^2_{therm}(\omega) = \frac{4\kappa_B T}{\omega} \sum_n \frac{\phi_n \omega_n^2}{\mu_n \left[\left(\omega_n^2 - \omega^2\right)^2 + \phi_n^2 \omega_n^4\right]} \tag{21}$$

- Each mode is characterized by a resonance pulsation ω_n, an effective mass μ_n, a *loss angle* ϕ_n. In other words each mode behaves as an harmonic oscillator of defined resonant frequency, mass and quality factor (at resonance!)

$$Q_n = \frac{1}{\phi_n} \tag{22}$$

- At frequencies well above the internal resonances, the thermal noise goes as

$$x^2_{therm} \simeq \frac{4\kappa_B T}{\omega^5} \sum_n \frac{\phi_n \omega_n^2}{\mu_n} \tag{23}$$

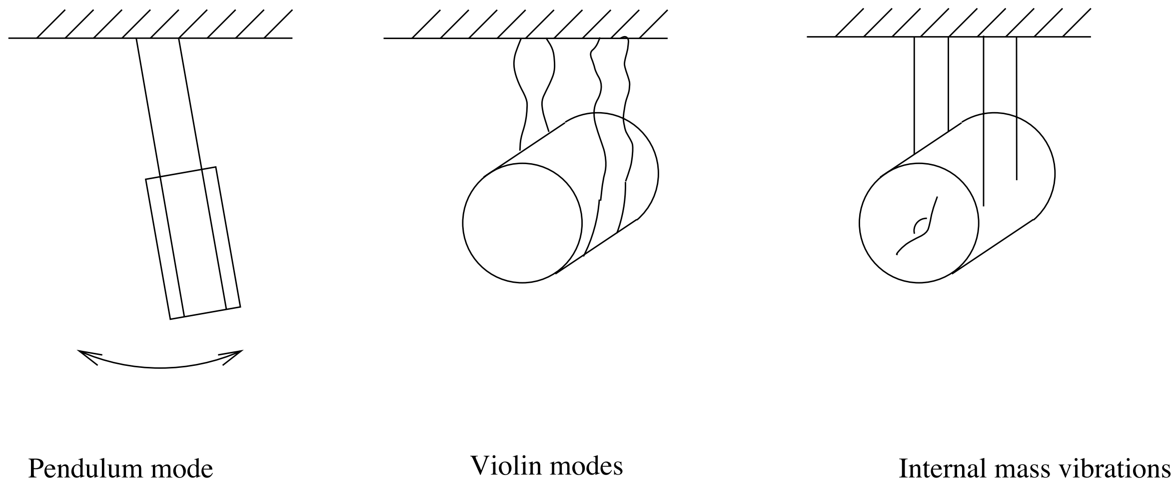
while well below the internal resonances one has

$$x^2_{therm} \simeq \frac{4\kappa_B T}{\omega} \sum_n \frac{\phi_n}{\mu_n \omega_n^2} \tag{24}$$

- In any case, the thermal noise is inversely proportional to the quality factors of the modes. To have a low noise floor one needs
  1. a system with very low dissipation (very high Q factors, very narrow resonances).
  2. A frequency band not too densely populated by resonances.

Sources of thermal noise

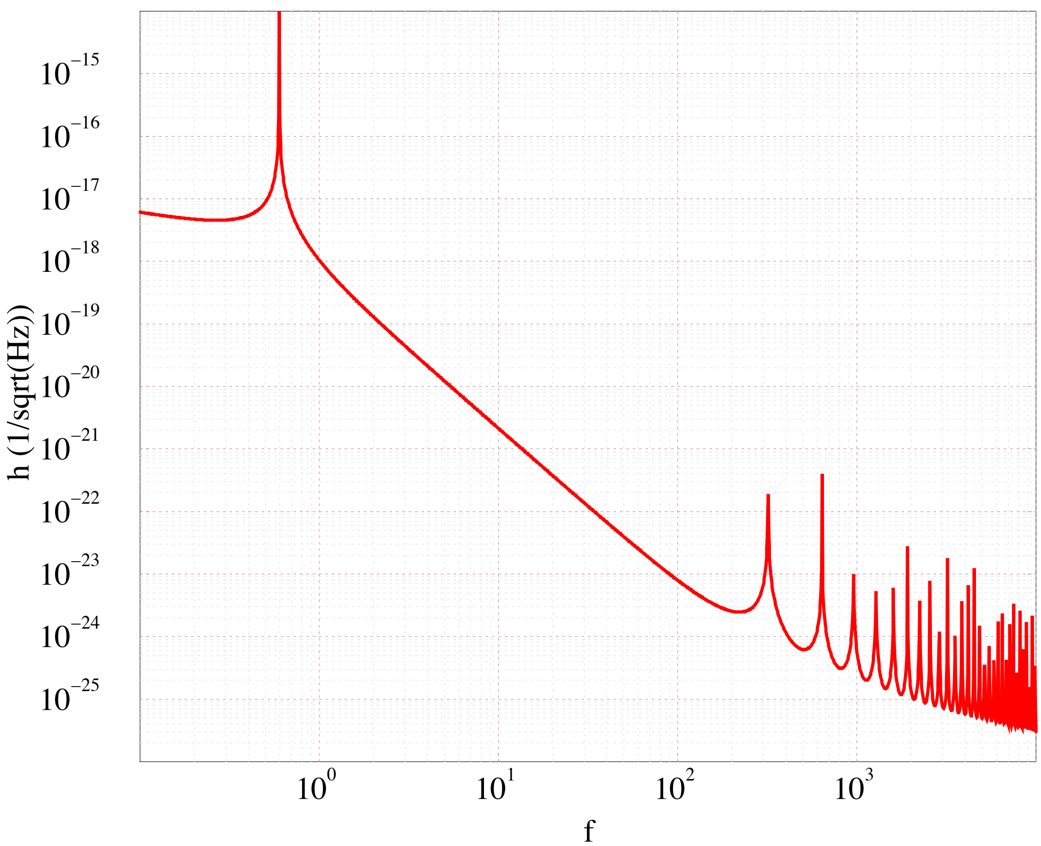
The position of the mirror surface, as seen by the laser wavefront, is affected by different motions [11]:



- Pendulum fundamental modes: affected by dissipation in the wires and in the clamps, but having a very good quality factor, because most of the recalling force comes from gravity!
- Violin modes: they too are mostly due to wire tension, and have good quality factors.
- Mirror drum modes: they are pushed at very high frequency by a careful design.

Wire oscillations

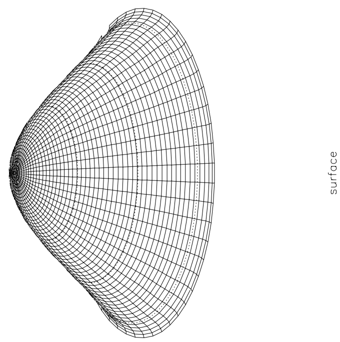
The dissipation in the wires is responsible of the noise spectral density

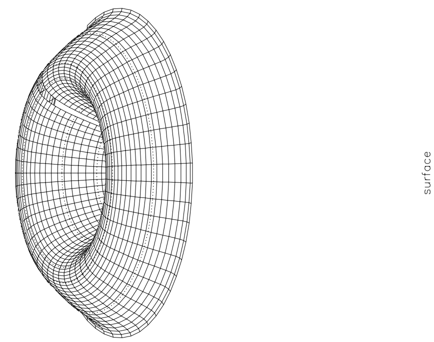


- The position of the lowest resonance is fixed by the length of the wires
- The amount of dissipation depends on the material of the wires and on their cross-section
- Thinner wires lead to higher *violin* frequencies, at the expense of working closer to the wire breaking point.

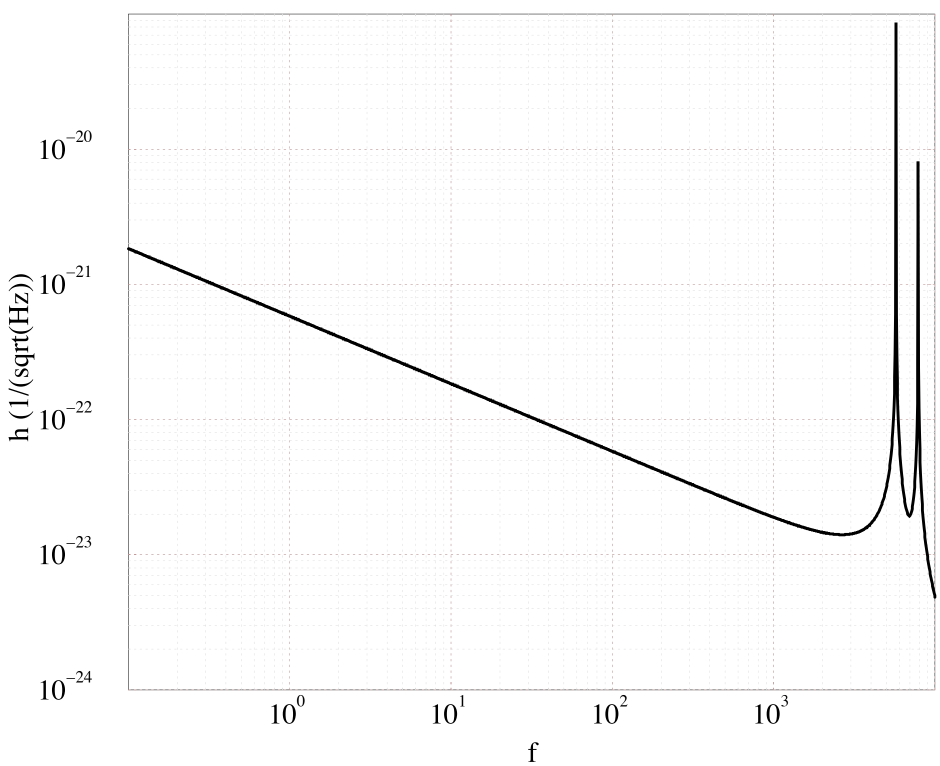
Mirror oscillations

VIRGO mirrors, on the contrary, have *drum* oscillation frequencies at relatively high frequency [11].

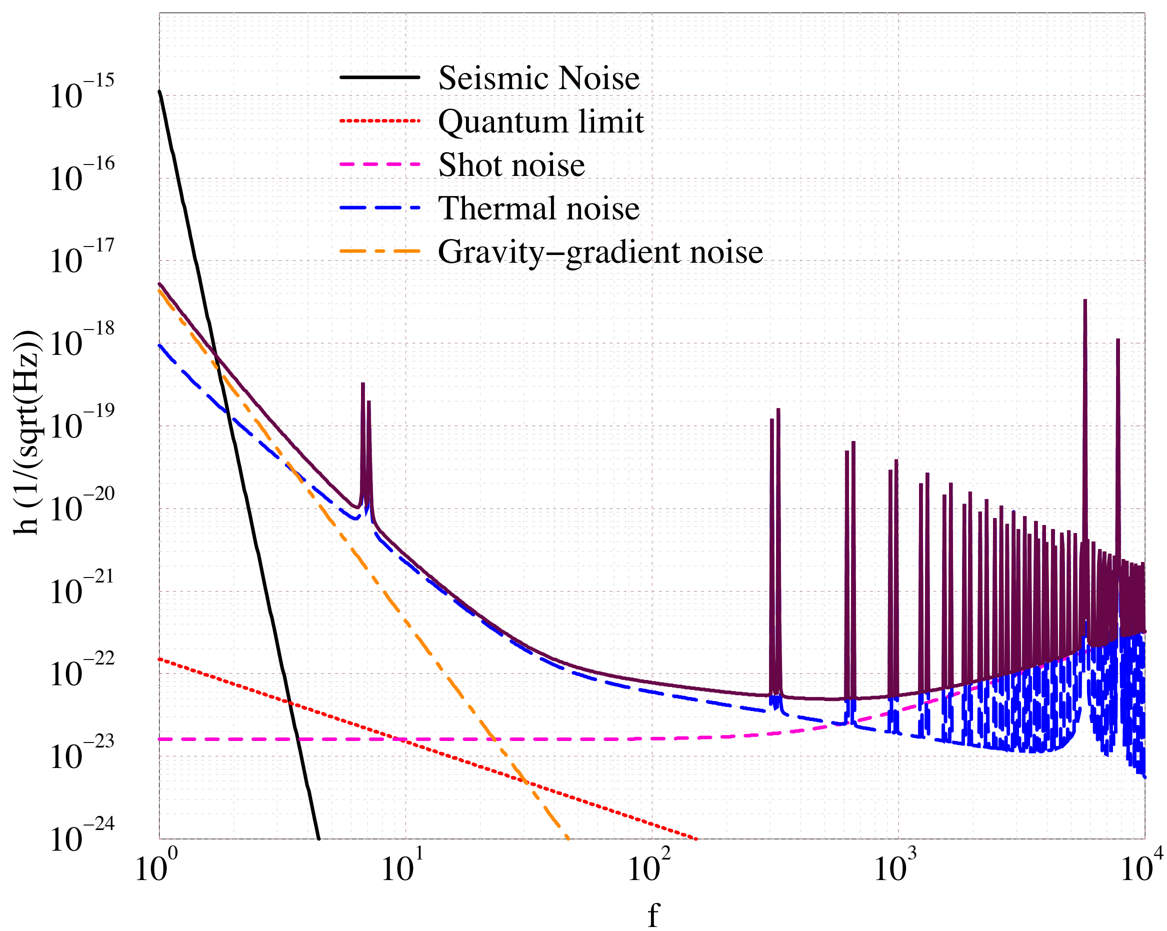




~ 5700 – 7800 Hz $\qquad \sim 15000$ Hz



The VIRGO **Sensitivity**



- Low frequency: $S_h(f) \sim \kappa_1/f^5$
- Intermediate: $S_h(f) \sim \kappa_2/f$
- High frequency: $S_h(f) \sim \kappa_3 \left(1 + (f/f_c)^2\right)$

Next topics

- How an interferometer is actually operated, as an gravito-optical transducer. We shall see that the "theorist" interferometer would not work . . .
- How the optical components are steered to their working positions.

Lecture 2: the VIRGO interferometer

Our purpose is to understand the working principle of an interferometer and the role of its parts.

- What is the noise limit? Shot noise and all that.
- How to increase GW sensitivity: arm folding.
- Fabry-Perot based interferometers.
- Rejection of noises: heterodyne detection, null instruments.
- Dark fringe operation.
- The concept of *locking*.
- Ecology! Reusing the light.
- Phase noise and (some) other beasts.

Optical read-out noise

A Michelson-Morley ITF translates a power read-out on a photodiode in a phase difference through the relations

$$\begin{aligned}\Delta\phi &= \frac{4\pi}{\lambda} L h \\ P_{out} &= P_{in}\cos^2\left[\phi_0 + (\Delta\phi)\right]\end{aligned}$$

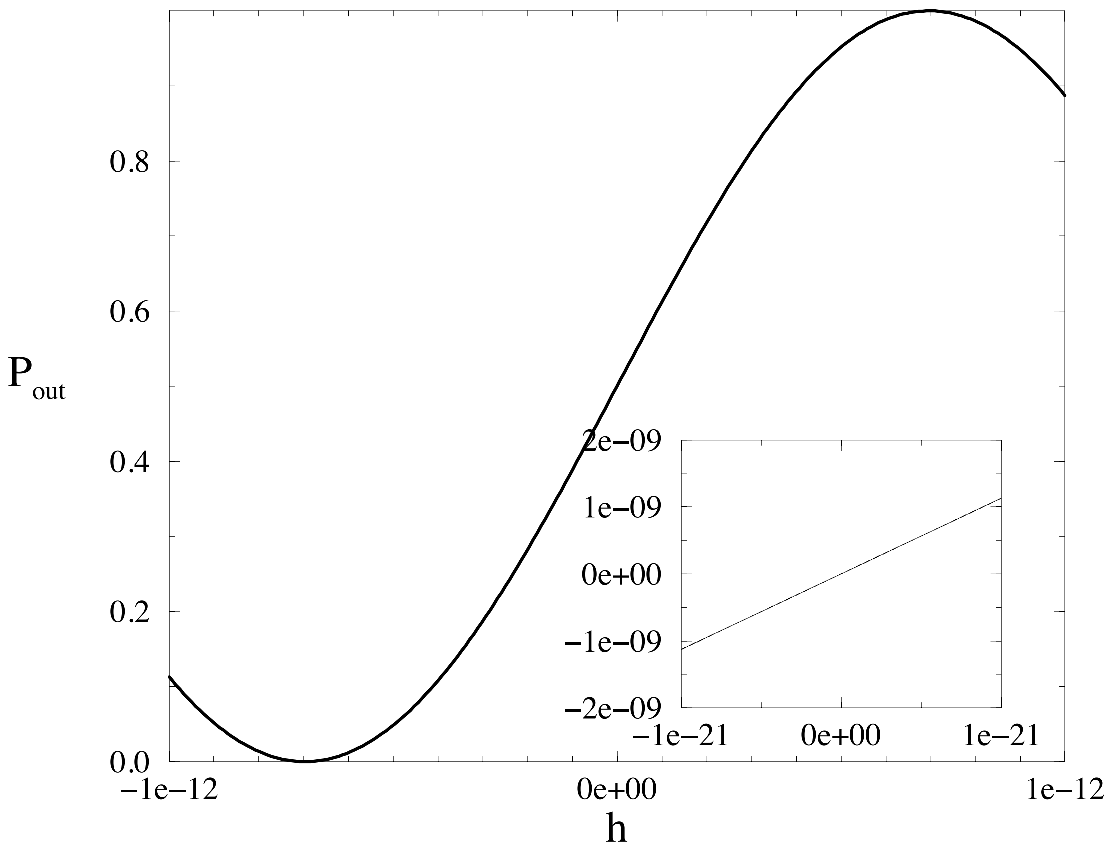
If we choose as operating point the one having maximal $\Delta P/\Delta\phi$, for instance at $\phi_0 = -\pi/4$, we have

$$P_{out} = \frac{P_{in}}{2}\left[1 + \sin 2\Delta\phi\right] \; ; \tag{25}$$

that is we have

$$\Delta P_{out} = P_{in}\Delta\phi \; ; \tag{26}$$

if we assume a reasonable optical length L, we find that to appreciate $h \sim 10^{-21}$ we need $\Delta\phi \sim 10^{-9}$.



In other words, one-billionth of a fringe is to be appreciated.

Now state-of-the art photodiodes have a quantum efficiency close to 1: hence the problem reduces to count a sufficient number of photons in a fixed time.

The fluctuation in the photon counting process is therefore the ultimate limitation to phase resolution.

Shot noise

Any counting process over a time t of *independent* events follows the Poisson distribution

$$\begin{aligned} p(N) &= \frac{1}{N!}\left[\bar{n}t\right]^{N} e^{-\bar{n}t} \\ E\left[N\right] &= \bar{n}t \\ \lim_{\bar{n}t\to\text{large}} p(N) &\sim \frac{1}{\sqrt{2\pi\bar{n}t}}\, e^{-\frac{(N-\bar{n}t)^2}{2\bar{n}t}} \end{aligned} \tag{27}$$

It means that the fractional precision of power is

$$\frac{\sigma_{\bar{N}}}{\bar{N}} = \frac{1}{\sqrt{\bar{n}t}}\ ; \tag{28}$$

recalling that the power $P_{out} = P_{in}/2$ is made by

$$\bar{n} = \frac{\lambda}{2\pi\hbar c} P_{\text{out}} \tag{29}$$

photons per unit time, we have a fractional photon number fluctuation

$$\frac{\sigma_{\bar{N}}}{\bar{N}} = \sqrt{\frac{4\pi\hbar c}{\lambda P_{in} t}} \tag{30}$$

or equivalently

$$\sigma_h = \frac{1}{L}\sqrt{\frac{\hbar c\lambda}{4\pi P_{in} t}}\ . \tag{31}$$

- Provided $P_{in}t$ is sufficiently large to allow the approximation of the Poisson process as gaussian, we interpret this result as a (one-sided) spectral density noise

$$h(f) = \frac{1}{L}\sqrt{\frac{\hbar c \lambda}{2\pi P_{in}}}\ . \tag{32}$$

- At high frequencies (small t) the approximation breaks down! We can easily estimate the cutoff frequency: it is of the order of the inverse of the time over which we count in average *one* photon

$$f_c \propto \frac{P_{out}}{2\pi\hbar\nu} \tag{33}$$

- From these equations, it appears that the noise can be reduced at will by increasing the input power.

  Apart technological limitations, there is however a *conjugate* effect: the radiation pressure.

Radiation pressure noise

Light normally reflected from the mirrors gives a net force

$$F_{rad} = \frac{P}{c} \tag{34}$$

and the force fluctuates by

$$\begin{aligned} \sigma_F &= \frac{\sigma_P}{c} \\ F(f) &= \sqrt{\frac{2\pi\hbar P_{in}}{\lambda c}} \; . \end{aligned} \tag{35}$$

Hence a suspended mirror is subject to a spectral motion *increasing* with P_{in} !

$$x_{rp}(f) = \frac{F(f)}{m\,(2\pi)^2\sqrt{\left(f_0^2 - f^2\right)^2 + f^2 f_0^2/Q^2}} \tag{36}$$

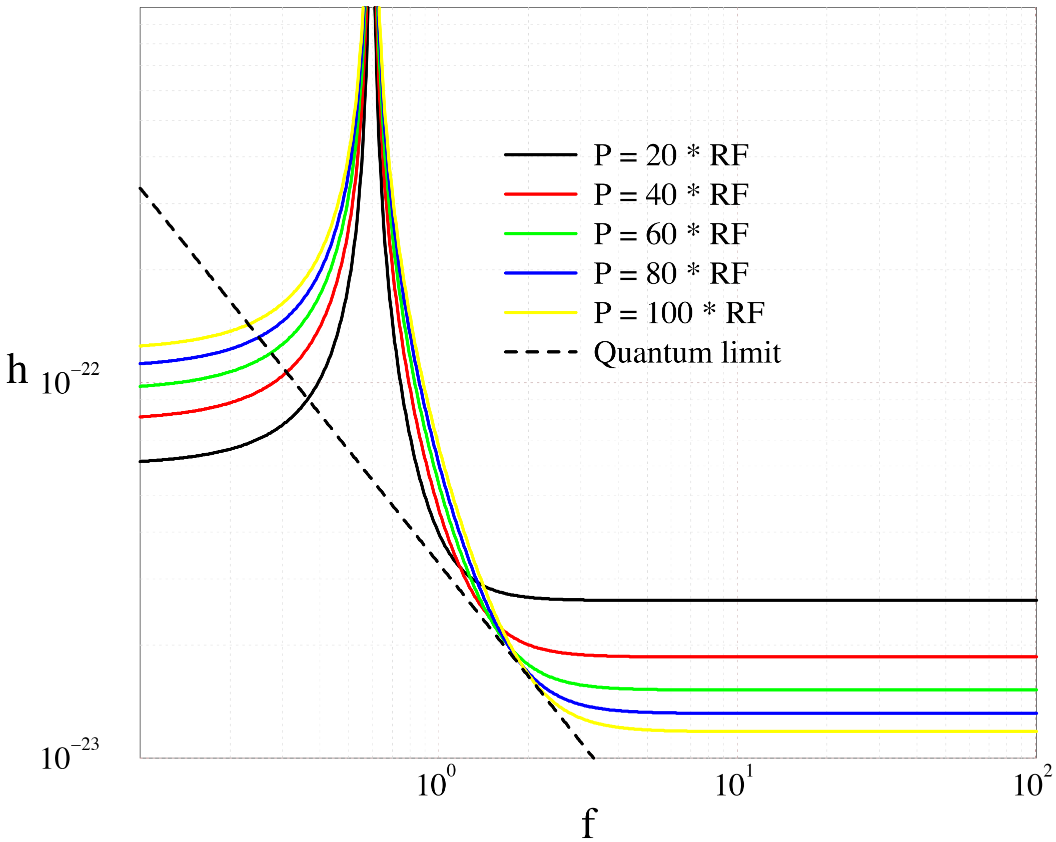
the effect is rapidly attenuated, at frequencies above f_0, by the mechanical admittance of the pendulum: in any case it translates in a *radiation pressure* noise

$$h_{rp}(f) = \frac{2}{L} x_{rp}(f) \; . \tag{37}$$

The quadrature sum of the two noises is the total *optical readout noise*

$$h_{orn} = \sqrt{h_{shot}^2(f) + h_{rp}^2(f)} \tag{38}$$

Quantum limit



The balance of the two noises is altered varying the total input power. At fixed frequency $\bar{f}$, the total noise is minimized choosing a particular power

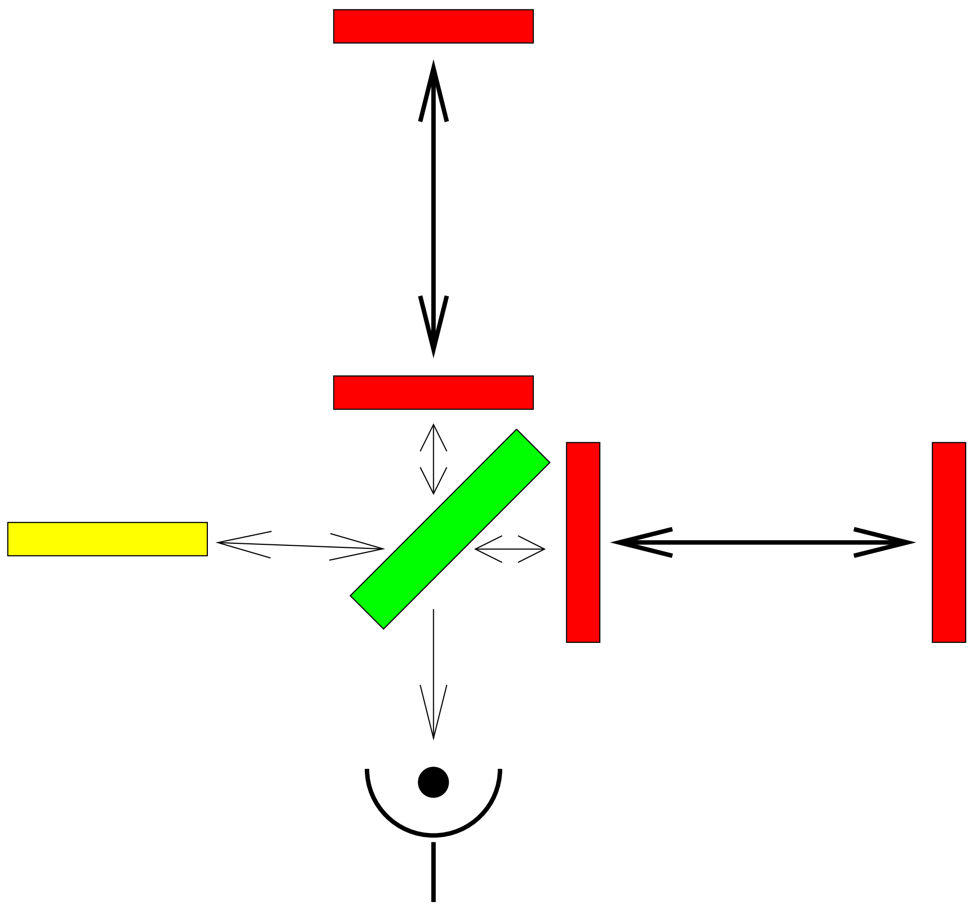
$$P(\bar{f}) = \pi c \lambda m \bar{f}^2 \tag{39}$$

which leads to the *quantum limit* noise

$$h_{QL}(\bar{f}) = \frac{1}{\pi \bar{f} L}\sqrt{\frac{\hbar}{m}}\ . \tag{40}$$

Folding the arms

We have shown up to now sensitivities obtained with optical length $L_{opt} \sim 150$ Km ! How are they possible? For instance, by having the light bounce back and forth (say) $\mathcal{N}$ times in each arm.



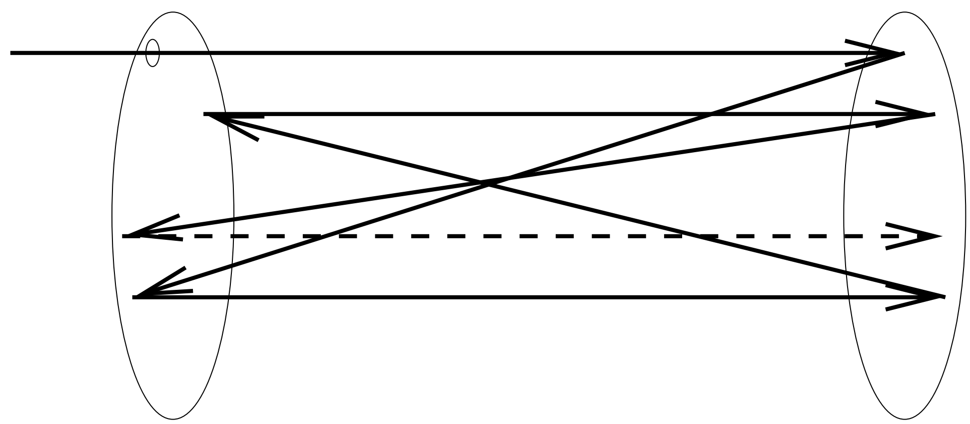
A photon will "integrate" the effect of the GW, leading to

$$\Delta\phi = h(t)\tau_s \frac{2\pi c}{\lambda} \mathrm{sinc}(\pi f_{gw}\tau_s) e^{i\pi f_{gw}\tau_s} \tag{41}$$

where $\tau_s = \mathcal{N}\,(2L/c)$ is the *storage time*.

Herriot delay lines

The conceptually simpler way to "store" the light is a delay line:



the (concave) lenses are arranged in such a way that at each encounter the light ray coordinates rotate by θ, where choosing

$$\frac{\theta}{2\pi} = \frac{\mathcal{M}}{2\mathcal{N}} \tag{42}$$

with $\mathcal{M}$ some integer, one obtains a multiplication by $\mathcal{N}$ of the optical path: in other words, a motion Δx of one mirror alters by $2\mathcal{N}\Delta x$ the optical length.

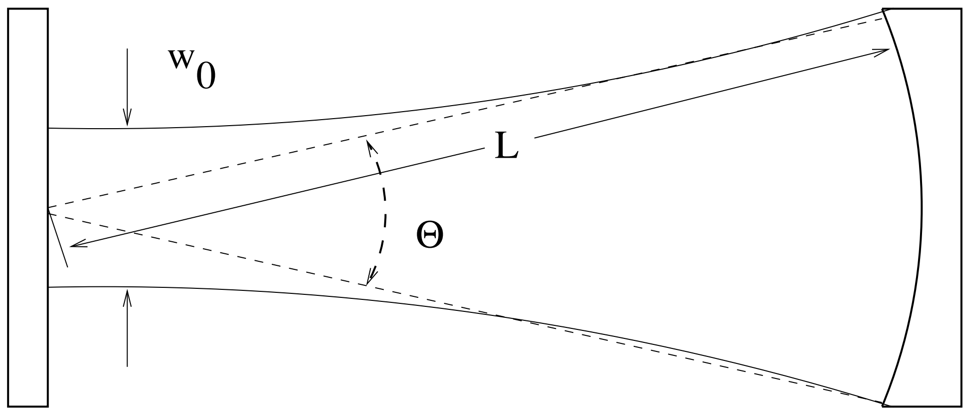
A famous application of this principle is the 1887 version of the Michelson-Morley ITF. They used 4 mirrors at each arm end, obtaining a single arm path of 22 m on an optical table of 1.5 m side.

However, a severe limitation to delay lines comes from the diffraction phenomena.

Gaussian beams and diffraction

Light beams have transverse dimensions!

The lowest order solutions of the wave equations are *gaussian* beams



$$
\begin{aligned}
u_{00}(x,\ y,\ z) &= \sqrt{\frac{2}{\pi x^2}} e^{i\phi} e^{-\left(y^2+z^2\right)} e^{-ik\left(y^2+z^2\right)/(2R)} \\
\phi &= \arctan \frac{x}{\pi w_0^2/\lambda} \\
w^2(x) &= w_0^2 \left[1 + \left(\frac{\lambda x}{\pi w_0^2}\right)^2\right] \\
\frac{1}{R(x)} &= \frac{x}{x^2 + (\pi w_0^2/\lambda^2)} \\
\Theta &\simeq \frac{\lambda}{\pi w_0} \qquad (43)
\end{aligned}
$$

If gaussian beams propagate in a Herriot delay line, one has to make sure that they do not become too large at the *far* mirror: one sets w_0 by minimizing $w(L)$ at

$$w_0 = \sqrt{\frac{\lambda L}{\pi}} \sim 3cm \; ; \tag{44}$$

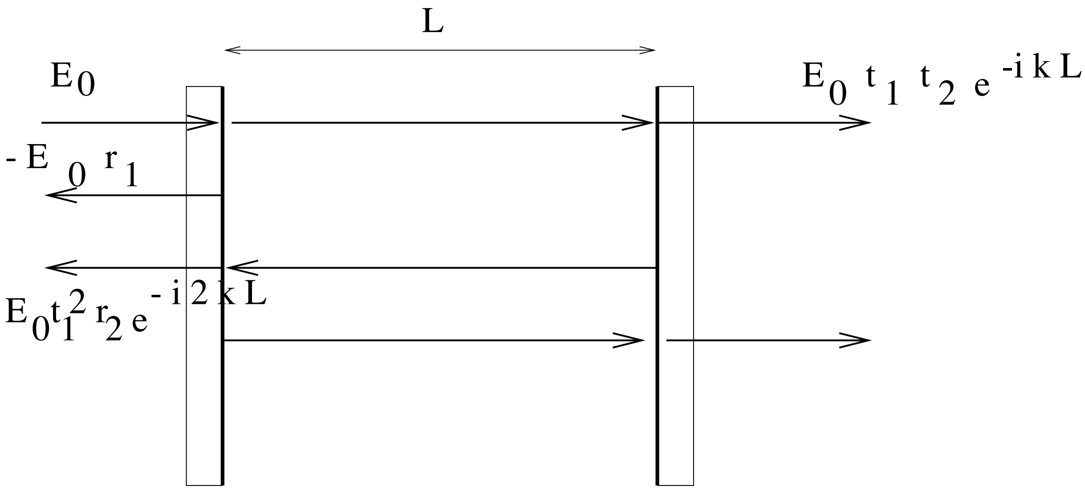
this sets stringent constraints

- To avoid edge diffraction, the hole must be a few times w_0.
- Remember the aim to $\Delta\phi \sim 10^{-9}$: little *cross-talk* is allowed between beams, which have accumulated different phases, $\Rightarrow$ widely spaced spots.
- Large mirrors are required! Typically with a diameter $\propto \mathcal{N} w_0$, that is greater than 1 m.

It has been possible to build small delay lines, however large L interferometer appear to require too large mirrors, too large vacuum enclosures, too stringent requirements over large mirror surfaces.

If you can't beat them, join them: if beams want to interfere, just allow it, by storing the light in a *Fabry-Perot* resonator [12].

Fabry-Perot principle



The full cavity behaves as a single mirror having

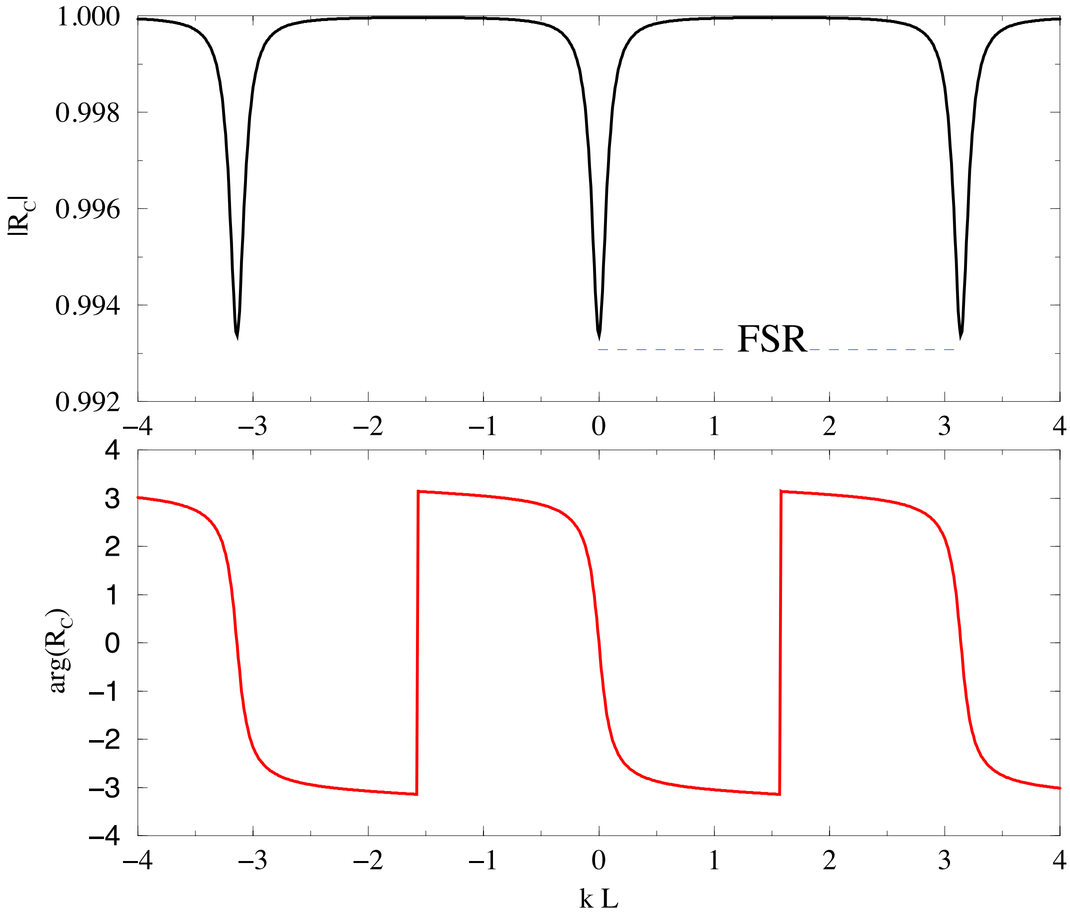
$$
\begin{aligned}
t_c &= \frac{t_1 t_2 e^{-i\kappa L}}{1 - r_1 r_2 e^{-2i\kappa L}} \\
r_c &= \frac{-r_1 + r_2 \left(r_1^2 + t_1^2\right) e^{-2i\kappa L}}{1 - r_1 r_2 e^{-2i\kappa L}} \qquad (45)
\end{aligned}
$$

which can be specialized to the case of a *perfect* end mirror $(t_2 = 0,\ r_2 = 1)$ and *lossless* close mirror to

$$
\begin{aligned}
t_c &= 0 \\
r_c &= e^{-i2\kappa L} \frac{1 - r_1 e^{+2i\kappa L}}{1 - r_1 e^{-i2\kappa L}}\ ; \qquad (46)
\end{aligned}
$$

the FP is a mirror which reflects light with a phase depending on its length [8].

Fabry-Perot as a resonator



- At fixed length, the FP is a transducer from frequency changes to phase changes.

1. The interval between resonances is called *Free Spectral Range*

$$FSR = \frac{2c}{L} \; ; \tag{47}$$

2. the sharpness of the resonance (FWHH / FSR) is marked by the finesse $\mathcal{F}$

$$\mathcal{F} = \frac{\pi\sqrt{r_1}}{1 - r_1} \tag{48}$$

- On the contrary, at fixed light frequency, the FP is a transducer from length to phase changes:
  1. One easily finds, for the change in phase of the reflected light,

$$\frac{\Delta\phi}{\Delta L} = \frac{2\,(1+r_1)}{1-r_1}\frac{2\pi}{\lambda} \simeq \frac{4}{1-r_1}\frac{2\pi}{\lambda} \tag{49}$$

     to be compared with the analogous situation in a delay line:

$$\frac{\Delta\phi}{\Delta L} = 2\mathcal{N}\frac{2\pi}{\lambda} \tag{50}$$

     that is the FP behaves as a delay line with

$$\mathcal{N}_{FP} = \frac{2\mathcal{F}}{\pi} \tag{51}$$

  2. In which sense light is *stored* ? The field (say, moving toward mirror 1) inside the cavity is

$$E_{in} = E_0\frac{t_1}{1-r_1 e^{-2i\kappa L}} \tag{52}$$

     which is maximal at resonance. If the laser is shut off, the field continues to emerge diminishing as e^{-t/τ_s} where

$$\tau_s \simeq \frac{2L}{c}\frac{\mathcal{F}}{2\pi}\ . \tag{53}$$

Response to GW

The FP response to a GW can be computed easily as the sum of the responses due to photons emerging after 1, 2, 3 . . . round trips [13]. Indeed, we know from the delay line the phase shift for n round trips in a single arm is

$$\frac{\delta\phi_n}{h} = \frac{c}{\lambda f_{gw}} e^{in\pi f_{gw} 2L/c} \sin n\pi f_{gw} 2L/c \tag{54}$$

and hence the reflected field is

$$E_r/E_0 = -r_1 + t_1^2 r_2 \sum_{n=0}^{\infty} (r_1 r_2)^n \, e^{i\delta\phi_{n+1}} \ . \tag{55}$$

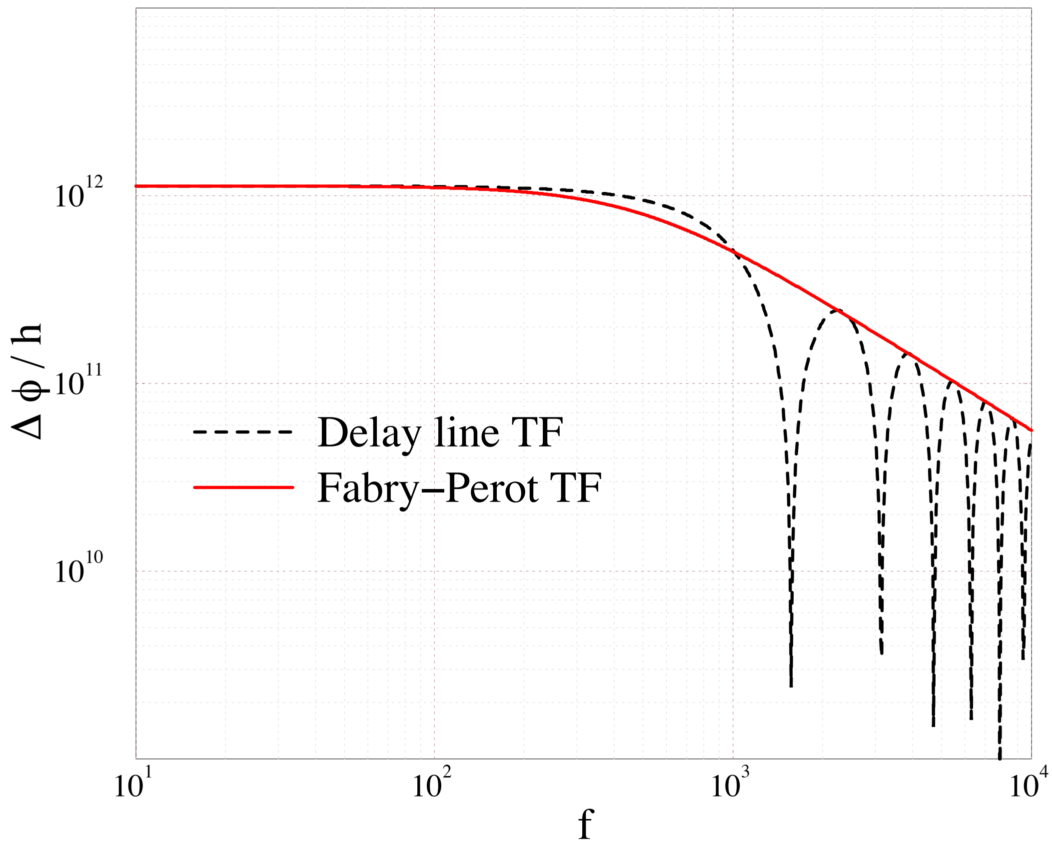
One finds the transfer function for the differential arm phase,

$$\begin{aligned} \frac{\Delta\phi}{h}(f_{gw}) &= \left[\frac{2r_2 t_1^2}{r_1 - r_2\left(r_1^2 + t_1^2\right)}\right] \frac{c}{\lambda f_{gw}} \frac{\sin\alpha}{1 - r_1 r_2 e^{i2\alpha}} e^{i\alpha} \\ \alpha &= \pi f_{gw} \frac{2L}{c} \end{aligned} \tag{56}$$

which becomes for α small, $r_2 \simeq 1, r_1^2 + t_1^2 \simeq 1$

$$\begin{aligned} \frac{\Delta\phi}{h} &\simeq \tau_s \frac{8\pi c}{\lambda} \frac{1}{\sqrt{1 + (4\pi f_{gw}\tau_s)^2}} e^{i\theta} \\ \theta &\simeq \pi f_{gw} \frac{2L}{c} + \arctan\left(4\pi f_{gw}\tau_s\right) \ . \end{aligned} \tag{57}$$

The FP transfer function does not exhibit the “blind” frequencies of the delay line: a simple pole at $1/(4\pi\tau_s)$ characterizes the response



the two systems have the same low frequency behaviour, provided that the cavity finesse $\mathcal{F}$ and the delay line number of round trips $\mathcal{N}$ are in the relation

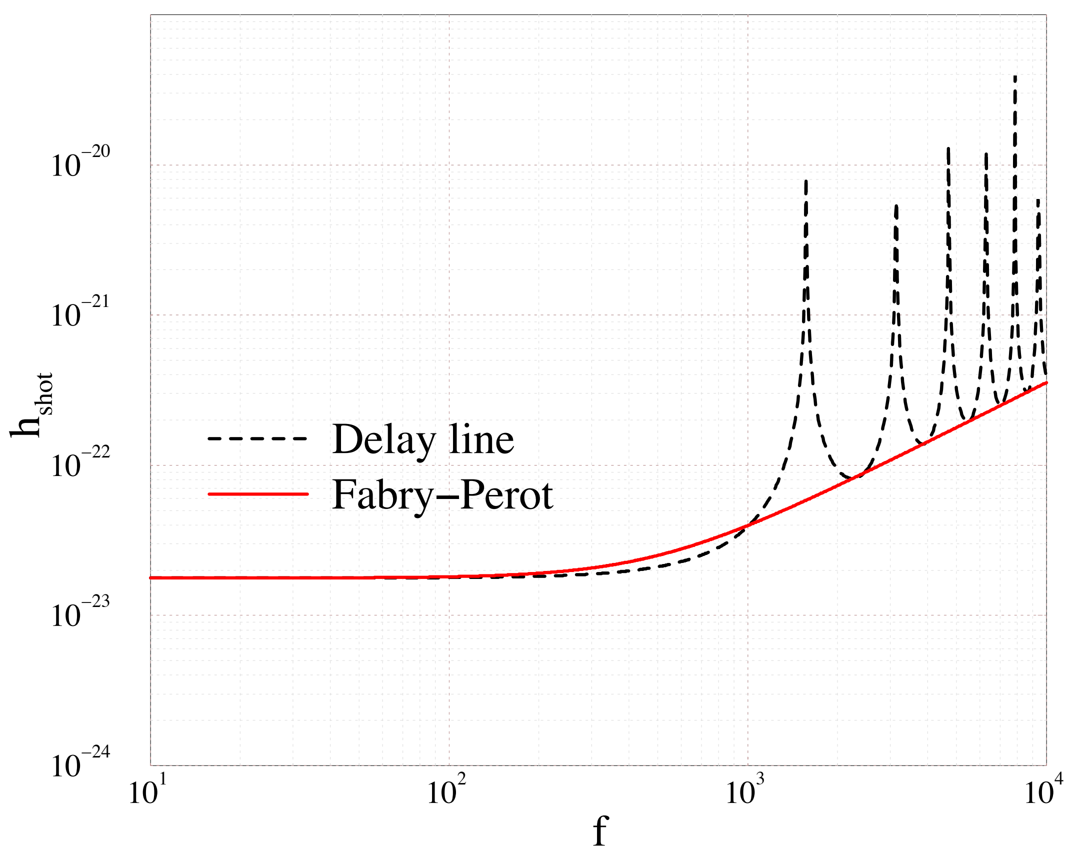
$$\mathcal{N} = \frac{2}{\pi}\mathcal{F} \tag{58}$$

Shot noise in folded ITF

The shot noise, after folding the path, is exactly identical to that of an unfolded, equal optical length, ITF

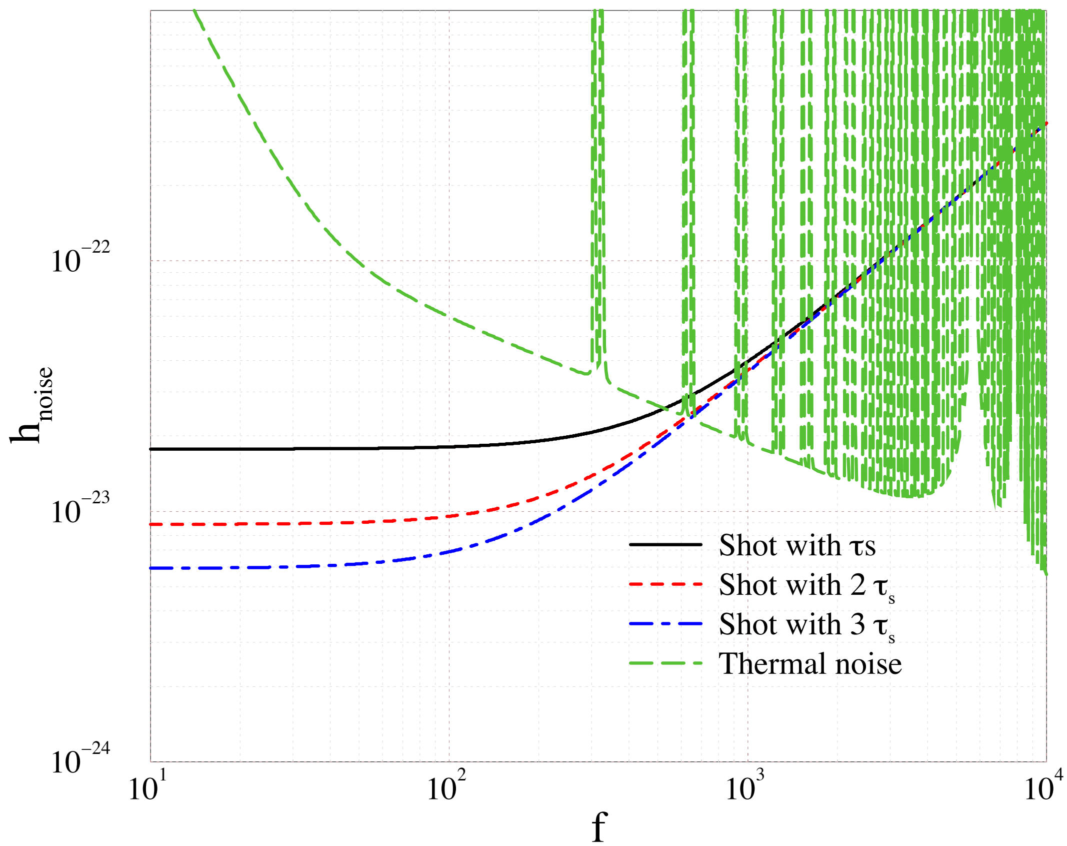
$$h_{shot}(f_{gw}) \simeq \frac{1}{\mathcal{N}L}\sqrt{\frac{\hbar c\lambda}{2\pi P_{in}}}\sqrt{1+(4\pi f_{gw}\tau_s)^2} \qquad (59)$$

where the noise increase at high frequencies results from the lower sensitivity of the instrument to higher frequencies GW.



Limits to folding

At fixed L, why not increasing $\mathcal{N}$? Because it helps only at low frequencies, where other noises dominate.



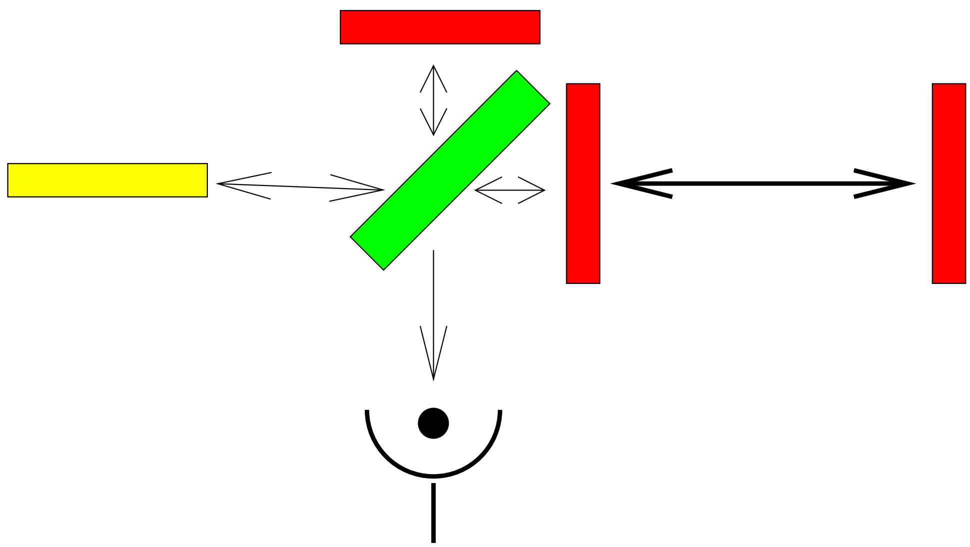
At fixed $\mathcal{N}L$, why not reducing the expensive arm length L? The fluctuations Δx in the mirror positions enter multiplied by $\mathcal{N}$! When translated in an h noise using the TF, they result in a position noise

$$h = \frac{2}{L}\Delta x \tag{60}$$

where L is the *real* length of the arm.

Saving billions!

In principle, we can recombine the light shined back by the FP with the light coming from the laser!



The output power would give a measure of the FP length, and then of the GW itself, by the usual relation

$$P_{out} = P_{in} \cos^2 \Delta\phi \ . \tag{61}$$

But the same reasoning would apply to the Michelson-Morley ITF: so why they had need of two arms?

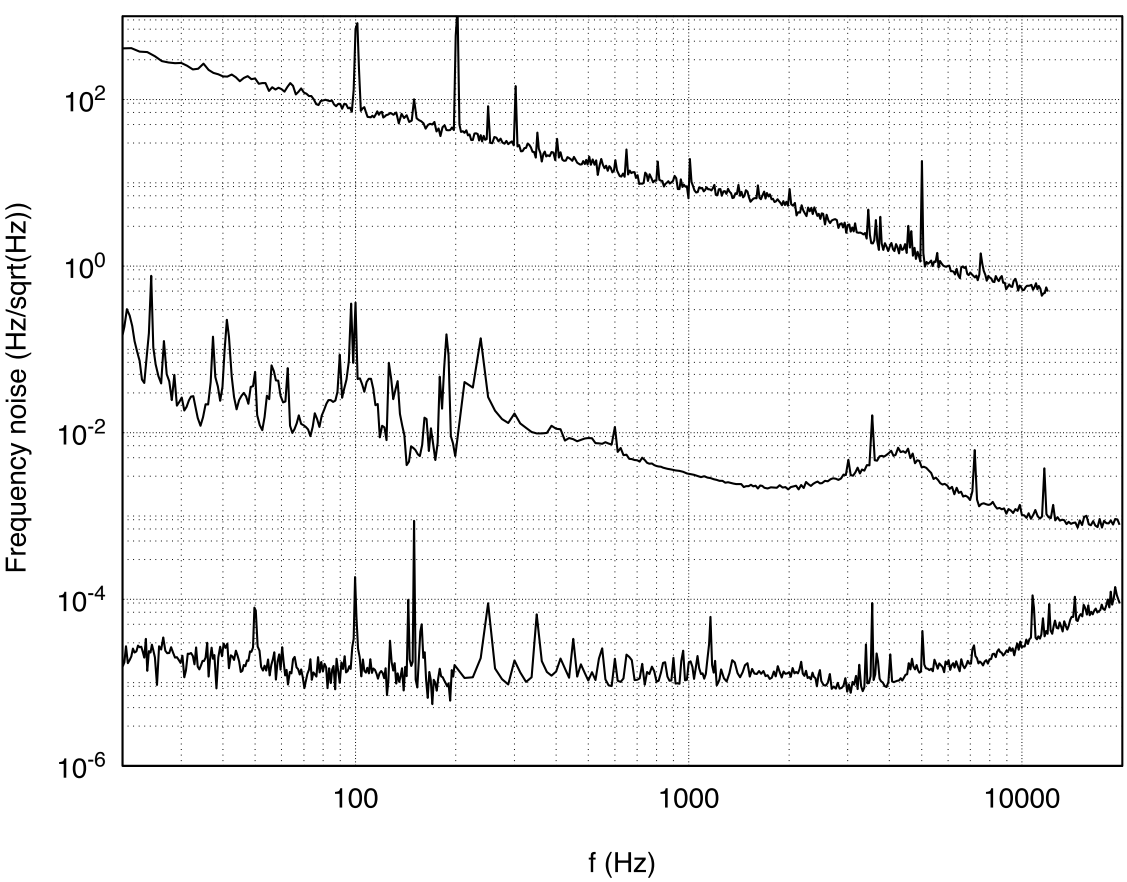
By understanding why, we shall also realize that the whole intrument discussed till now still lacks an important ingredient.

There is more than shot noise . . .

A first answer to the question "why two arms" comes noting that lasers *frequency* fluctuate. If the two arms have unequal L_{opt}, and the frequency ν of the laser fluctuates in time with spectral density $\nu(f)$ (in units $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$) one has an equivalent strain noise

$$h(f) = \frac{\nu(f)}{\nu} \frac{\Delta L_{opt}}{L_{opt}} \ . \tag{62}$$

In VIRGO, where $\lambda = 1064$ nm, $\nu = 2.82 \times 10^{14}$ Hz, if we allow just 1% asymmetry we get a requirement on frequency stability of $\nu(f) \sim 10^{-6}\mathrm{Hz}/\sqrt{\mathrm{Hz}}$, at 100 Hz [11].



Worse: power fluctuations

Reconsider our output quantity:

$$P_{out} = P_{in} \cos^2 \phi \tag{63}$$

what happens if the laser power fluctuates? We are not referring to the intrinsic error in the statistics of photon counting, which we have already considered and called *shot* noise. We refer to "true" power fluctuations.

We have

$$P_{out} = (P_{in} + \Delta P_{in}) \cos^2 (\phi_0 + \Delta\phi) \; ; \tag{64}$$

that is, working at $\phi_0 = -\pi/4$, it gives

$$\Delta P_{out} = P_{in} \Delta\phi + \frac{1}{2} \Delta P_{in} \; ; \tag{65}$$

in other words, a power fluctuation gives a spurious

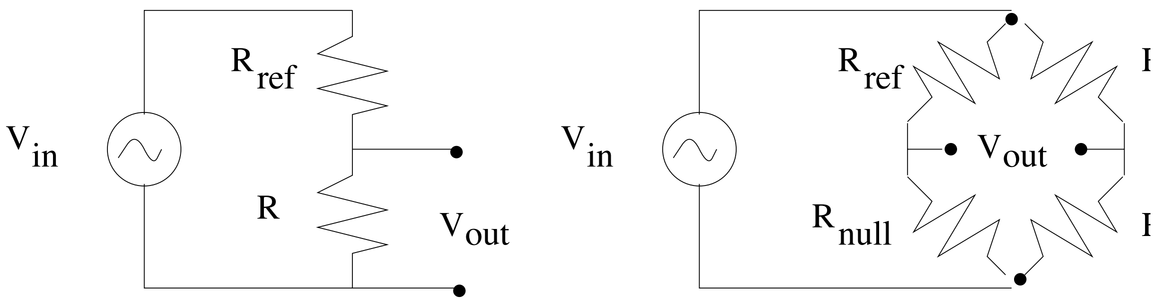
$$\Delta\phi_{pf} = \frac{1}{2} \frac{\Delta P_{in}}{P_{in}} \; . \tag{66}$$

To beat this noise we should require power stabilizations exceeding 10^{-9}, which is rather difficult.

It seems that we have chosen in any case a bad working point: the reason is that our instrument does not realize a *null* measurement, as required by good laboratory practice [8].

The Wheatstone bridge

Analogy: measuring an unknown R.



- Naive method

$$R = \frac{V_{out}}{V_{in} - V_{out}} R_{ref} \tag{67}$$

suppose $R \simeq R_{ref}$: then

$$\frac{\Delta R}{R_{ref}} \simeq 2 \frac{\Delta V_{in}}{V_{in}} \tag{68}$$

- Bridge method:

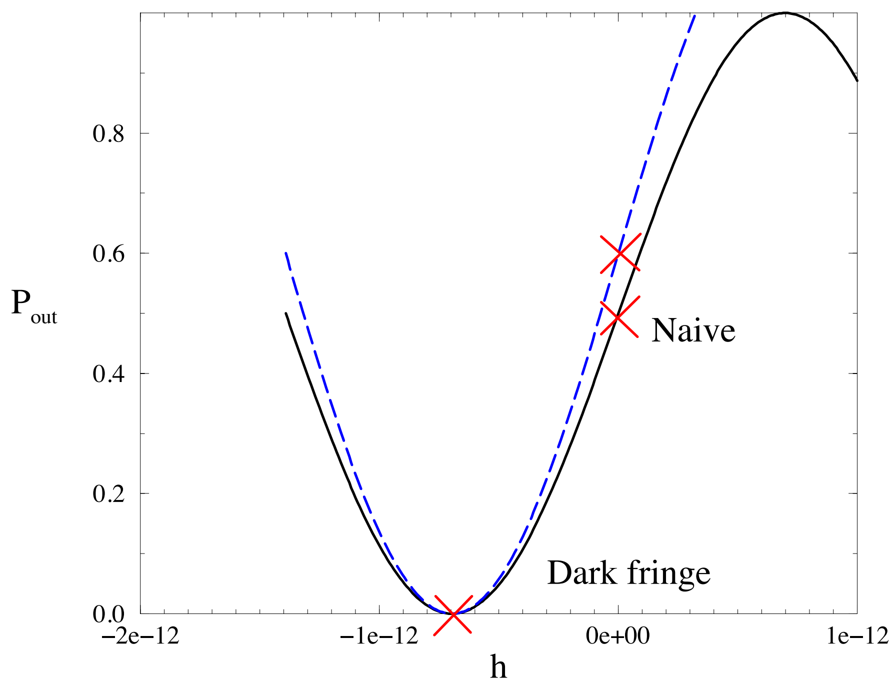
tune R_{null} so as to have $V_{out} \simeq 0$: then in a similar condition

$$\frac{\Delta R}{R_{ref}} \simeq 4 \frac{V_{out}}{V_{in}} \frac{\Delta V_{in}}{V_{in}} \tag{69}$$

The trick is in balancing the bridge in such a way that the answer (in terms of a calibrated reference) is obtained by a null measurement.

Dark fringe operation

We shall start for simplicity a non-resonant ITF, for instance with simple delay lines in the arms



$$P_{out} = \frac{P_{in}}{2}\left[1 + \cos 2\left(\phi_0 + \phi_{sig}\right)\right] \tag{70}$$

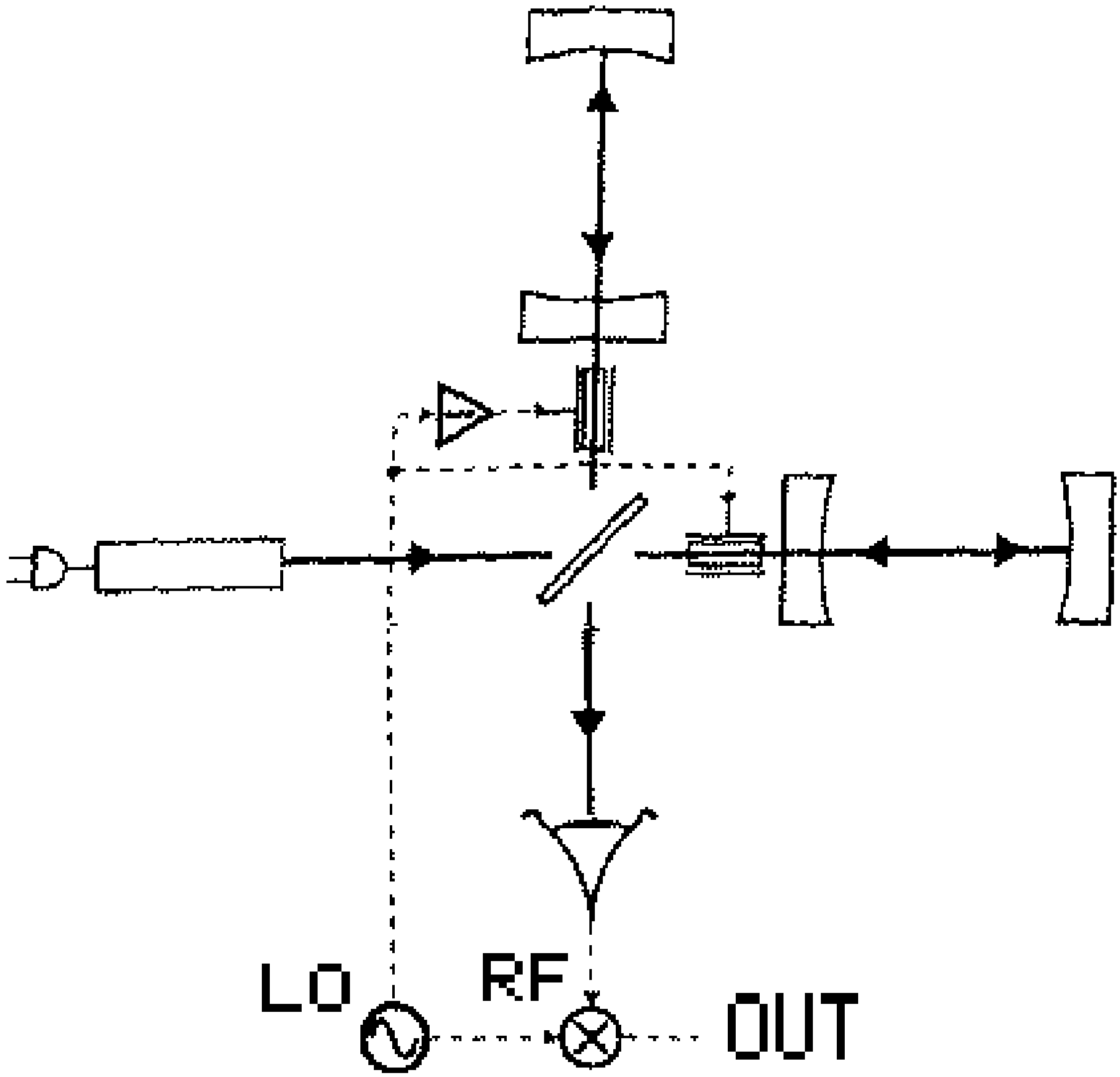
- if $\phi_0 = -\frac{\pi}{4}$ we have the "naive" operation
- if $\phi_0 = -\frac{\pi}{2}$ we are at the dark fringe.

but at the dark fringe we have

$$P_{out} = P_{in}\left[\Delta\phi\right]^2 \tag{71}$$

that is very low sensitivity to the signal!

Chopping the light



Introducing *Pockels* cells downstream the beam splitter, one can modulate the phase of the light: $1/2\varepsilon \sin \omega_{mod} t$ one cell, $-1/2\varepsilon \ldots$ the other [8].

Hence one has

$$P_{out} = \frac{P_{in}}{2} \left[1 + \cos 2 \left(\phi_0 + \phi_{sig} + \varepsilon \sin \omega_{mod} t \right) \right] \quad (72)$$

and if we assume $\varepsilon \ll 1$ we have

$$P_{out} \simeq \frac{P_{in}}{2}\left[\varepsilon^2 + 4\varepsilon\phi_{sig}\sin\omega_{mod}t - \varepsilon^2\cos 2\omega_{mod}t + \cdots\right] \tag{73}$$

This expression cures the diseases of the "naive" solution.

- The signal ϕ_{sig} is encoded as an amplitude modulation of the component at f_{mod} of the output power:

$$P_{out}(t) = 2\varepsilon P_{in}\phi_{sig}(t)\sin\omega_{mod}t \tag{74}$$

- As a good *null* instrument, when $\phi_{sig} = 0$ there is no oscillation of P_{out} at frequency f_{mod}

$$P_{out}(t) = \frac{P_{in}}{2}\varepsilon^2\left[1 - \cos 2\omega_{mod}t\right] \tag{75}$$

The signal is encoded in the same way as in AM radio signals.

If we have a sinusoidal signal $\phi_{sig}(t) = A\sin\omega_{sig}t$

$$P_{out} \propto A\left[\cos\left(\omega_{mod} - \omega_{sig}\right)t - \cos\left(\omega_{mod} + \omega_{sig}\right)t\right] \tag{76}$$

which is technically referred to as a *double-sideband suppressed-carrier amplitude modulation*

If we look for $f_{sig} \in \left[10, \cdots 10^4\right]$ Hz, we bandpass around f_{mod} (which is at several hundred MhZ) and get rid of low frequency power fluctuations.

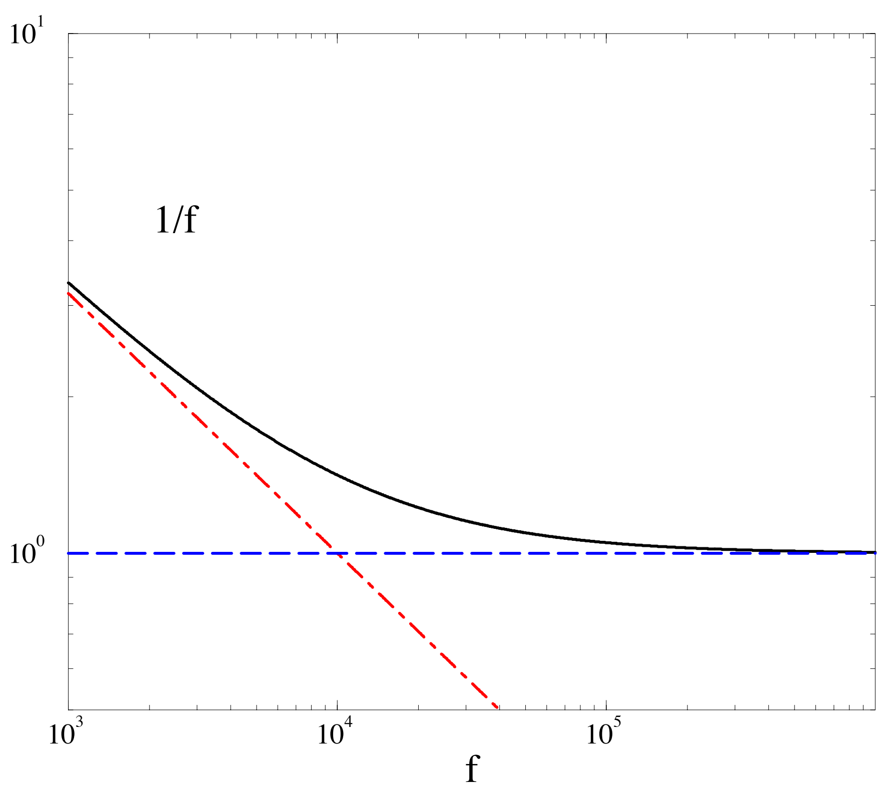
Finally the *RF mixer* allows to extract the signal: it receives as inputs the Local Oscillator *LO* and the photodiode output, multiplies and gives a voltage output

$$V_{out} = A \left[\sin \omega_{sig} t + \frac{1}{2} \sin \left(2\omega_{mod} - \omega_{sig} \right) t - \ldots \right] .$$

We have been sloppy: even in absence of signal we have

$$P_{out} = \frac{P_{in}}{2} \varepsilon^2 \left[1 - \cos 2\omega_{mod} t \right]$$

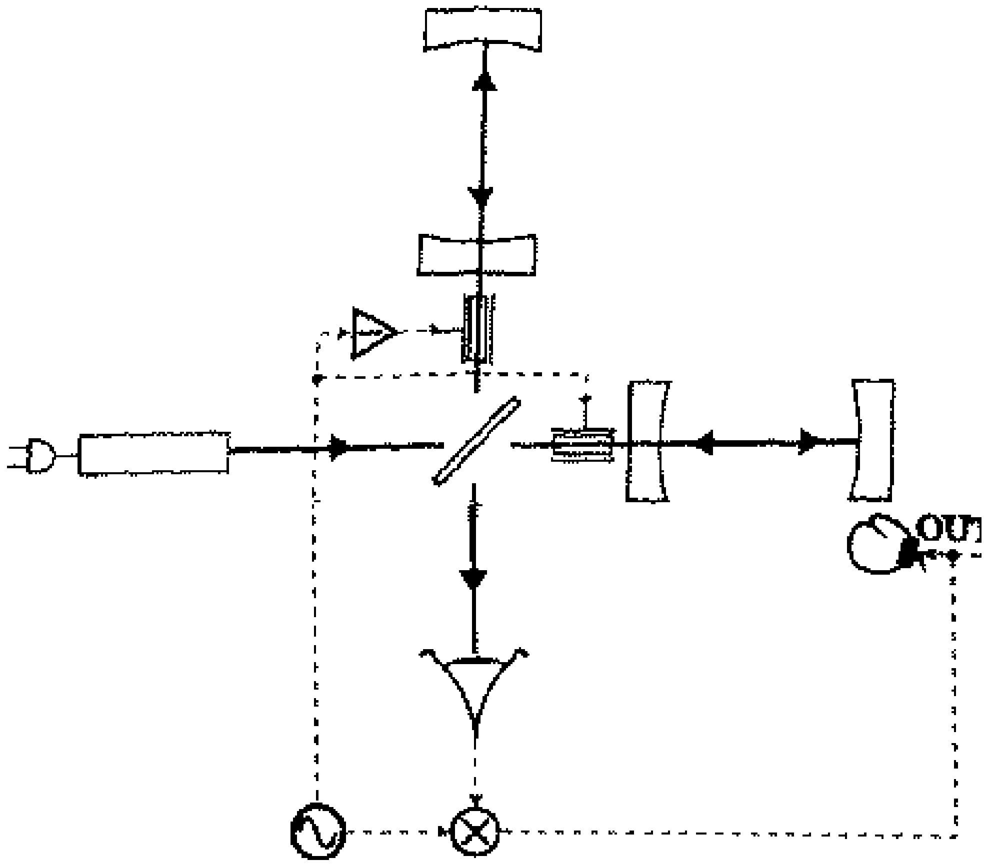
hence noise at f_{mod} in P_{in} is not rejected!



But since f_{mod} is chosen large, we avoid detecting the GW at the same *audio* frequencies where the laser noise significantly exceeds the shot noise limit.

Fringe locking

To work, the ITF needs to be kept at the dark fringe [8].



This procedure is called *locking*. But moving mirrors to set $P_{out} = 0$, don't we kill also the signal?

1. We mostly reduce oscillation induced by seisms, at low frequency, *outside* the detection band.
2. We apply a feedback, that is

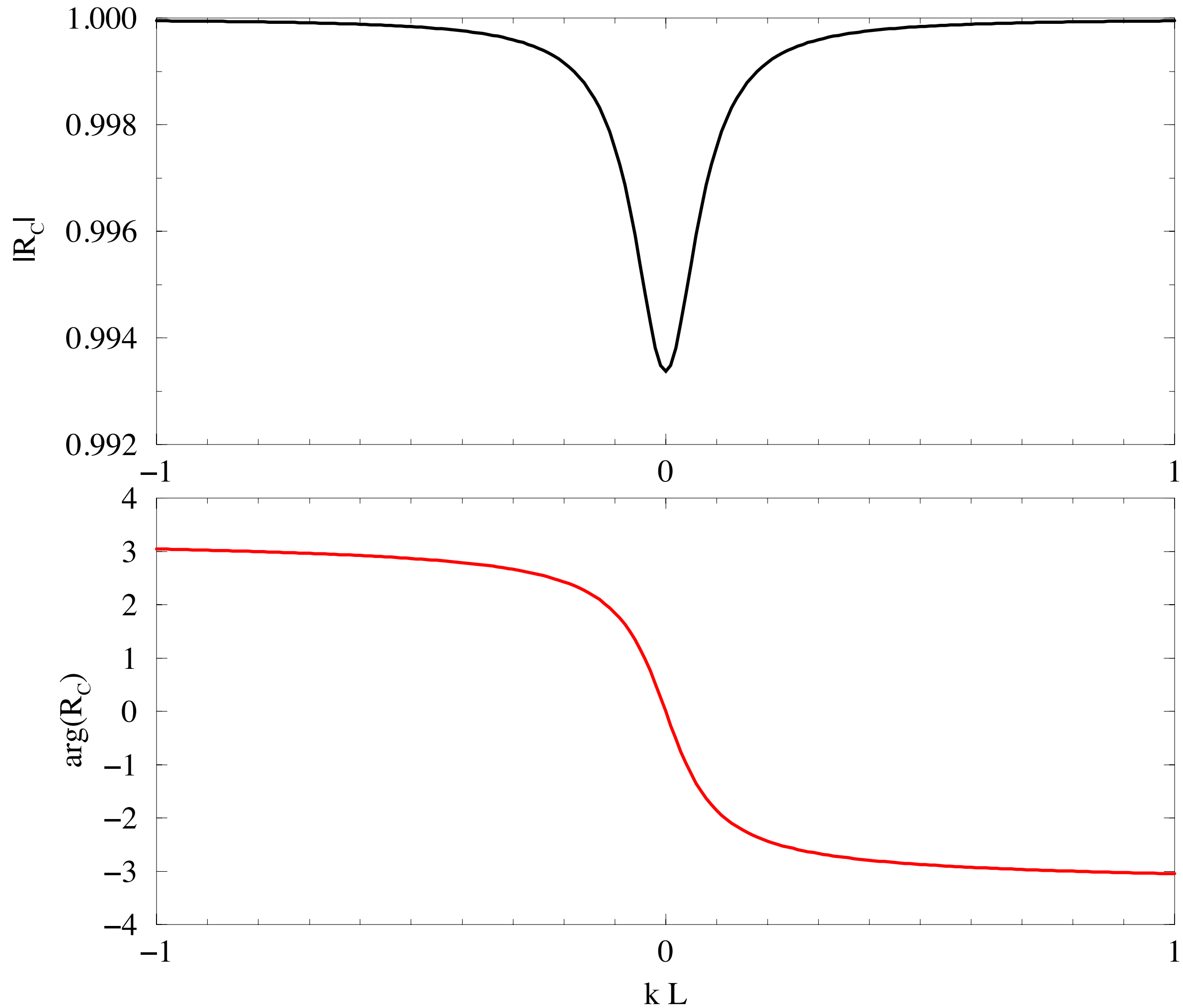
$$V_{cl} = \frac{V_{ol}}{1 + G} \; ; \tag{77}$$

signal and noise are affected in the same manner.

The single FP as an interferometer

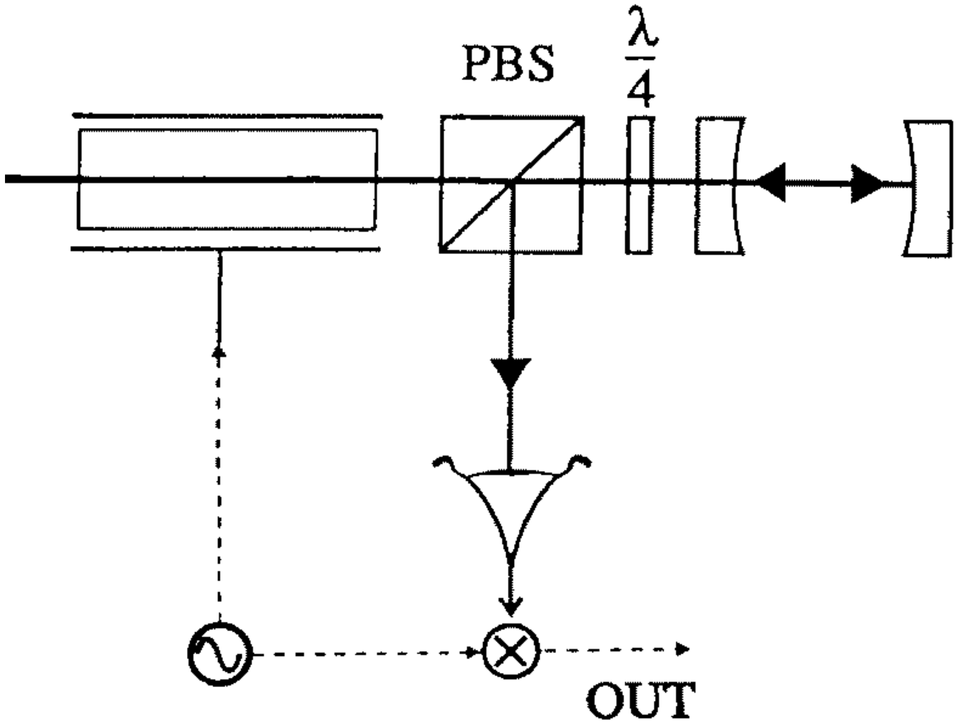
We have till now discussed locking as its only role where keeping the output on the dark fringe.

However, each FP resonator in the arms is an interferometer by itself!



The steep response to optical length changes which allows to mimick the effect of path folding is present only close to resonance: far from resonance the FP is *dead*.

Fringe lock in a FP



Again one exploits the possibility to phase modulate the light [8]. The input field is

$$
\begin{aligned}
E_{in} &= E_0 \cos\left[\omega_c t + \varepsilon \cos \omega_{mod} t\right] \qquad (78) \\
&\simeq E_0 \cos \omega_c t - \frac{\varepsilon E_0}{2} \sin\left(\omega_c + \omega_{mod}\right) t \\
&\quad - \frac{\varepsilon E_0}{2} \sin\left(\omega_c - \omega_{mod}\right) t
\end{aligned}
$$

what happens to the reflected light? If we choose ω_c close to a FP resonance, and ω_{mod} larger than the width of the resonance, we have that the carrier suffers a sign change (it is multiplied by $-R_c$), while the sidebands remain unchanged:

$$\begin{aligned} E_{refl} &= -E_0 \cos(\omega_c t + \phi_{sig}) - \varepsilon E_0 \cos(\omega_{mod} t) \sin \omega_c t \\ &= -E_0 \cos \phi_{sig} \cos \omega_c t \\ &\qquad -E_0 (\varepsilon \cos \omega_{mod} t - \sin \phi_{sig}) \underline{\sin \omega_c t} \end{aligned}$$

the quadrature term is crucial: when computing the output power, averaged over many carrier cycles, we get

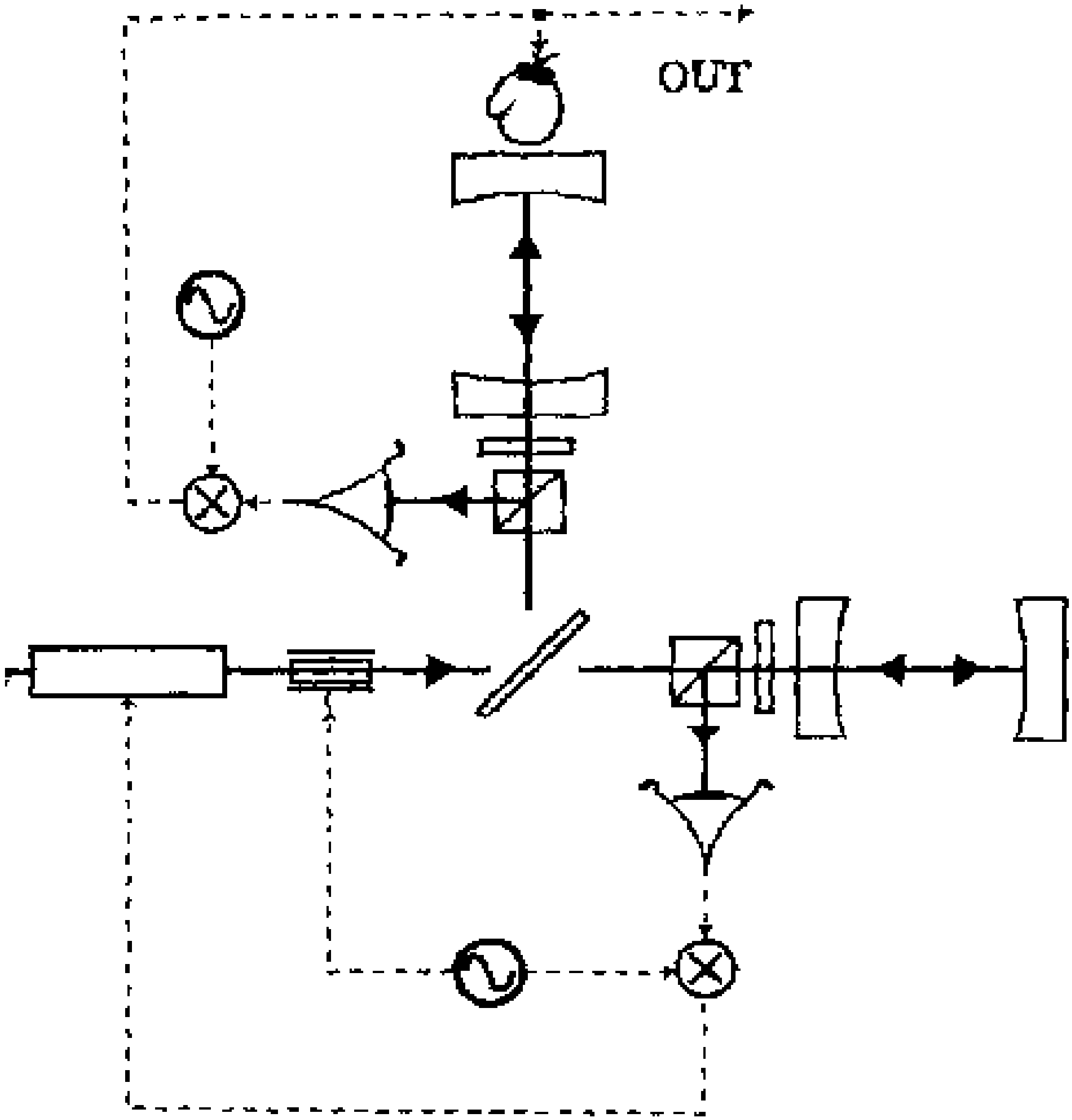
$$P_{refl} = P_0 \left[1 + \varepsilon^2 \cos^2 \omega_{mod} t - 2\varepsilon \cos \omega_{mod} t \sin \phi_{sig} \right]$$

which gives a way to measure the offset of the cavity from resonance, extracting the *error* signal ϕ_{sig}.

Once we have the error signal, we can either

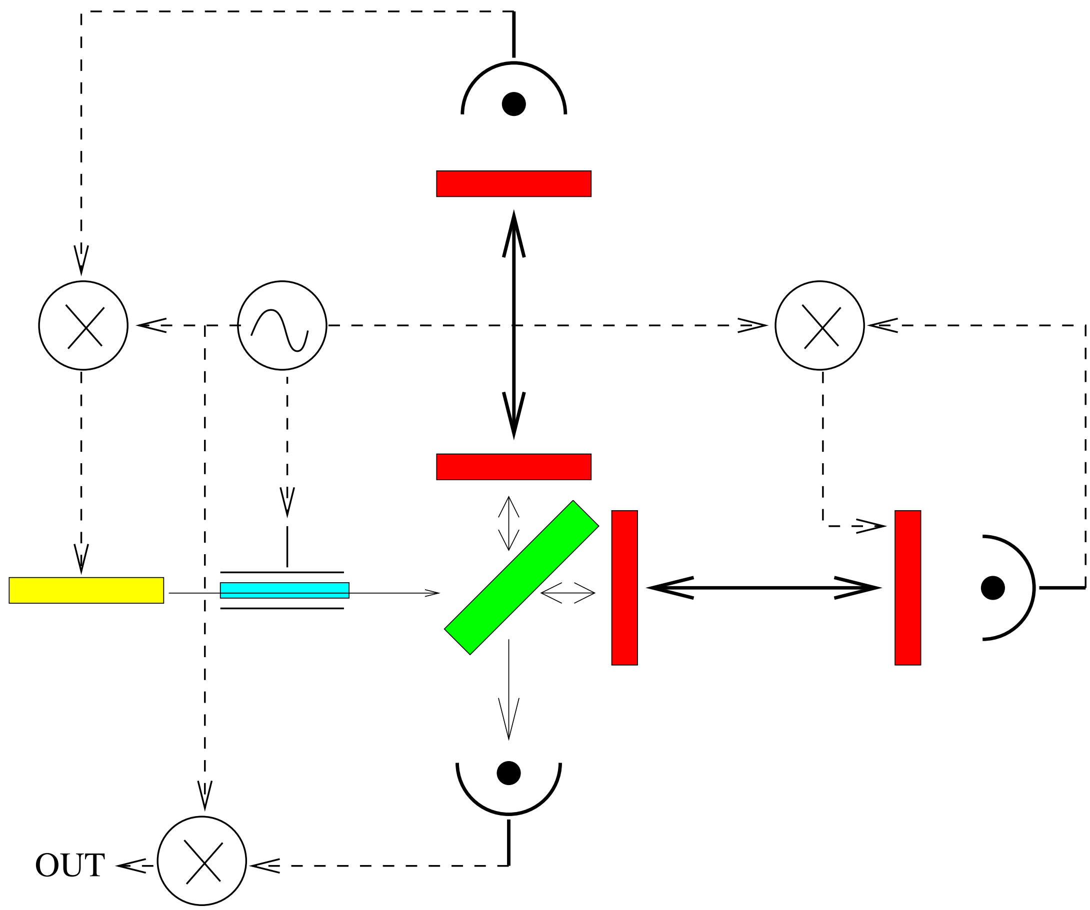
- Modulate the cavity length to change the position of the resonance and follow te frequency fluctuations of the laser, or
- we can adjust the length of the laser cavity to use the FP as a reference length. But what about the GW signal?

An example ITF [8]



1. One of the arms is used as a reference for the laser: λ increases as L increases.
2. The loop on the second arm senses an increased λ and a decreased L, hence the error signal sums the effects.

A simplified VIRGO scheme



- Input light modulated to avoid low-frequency noise.
- Laser frequency locked to one cavity.
- Second cavity locked to the first one.
- Carrier(s) interfere at dark fringe: maximal sensitivity at output port!
- Sidebands do not contain GW signal, interfere with carrier(s) and contain the GW signal as an AM of the signal at the modulation frequency.
- The signal $\phi_{sig} \cos \omega_m t$ is detected using again the *heterodyne* technique and gives a voltage $\propto$ GW.

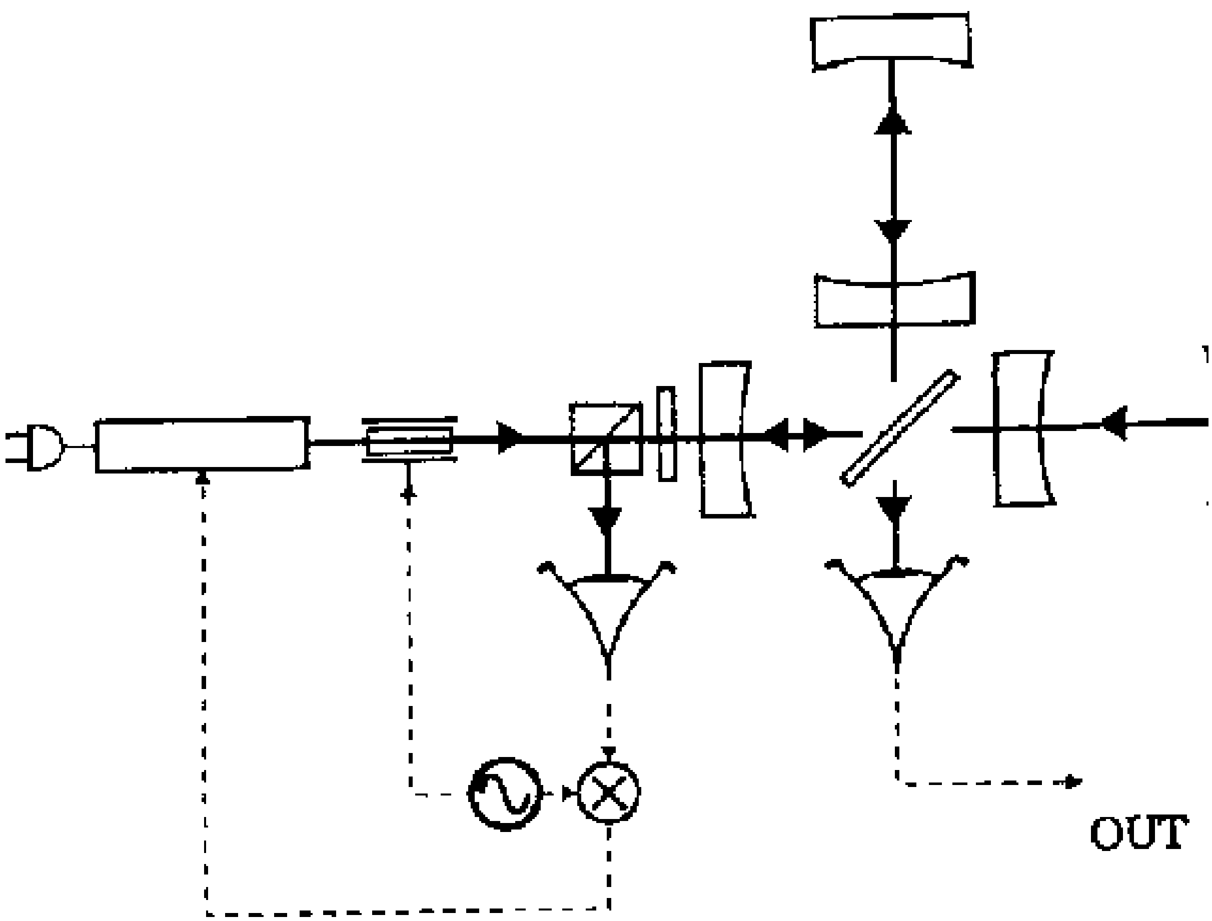
Increasing P_{in}: light recycling

We have seen that the shot noise decreases by increasing the input power in the ITF.

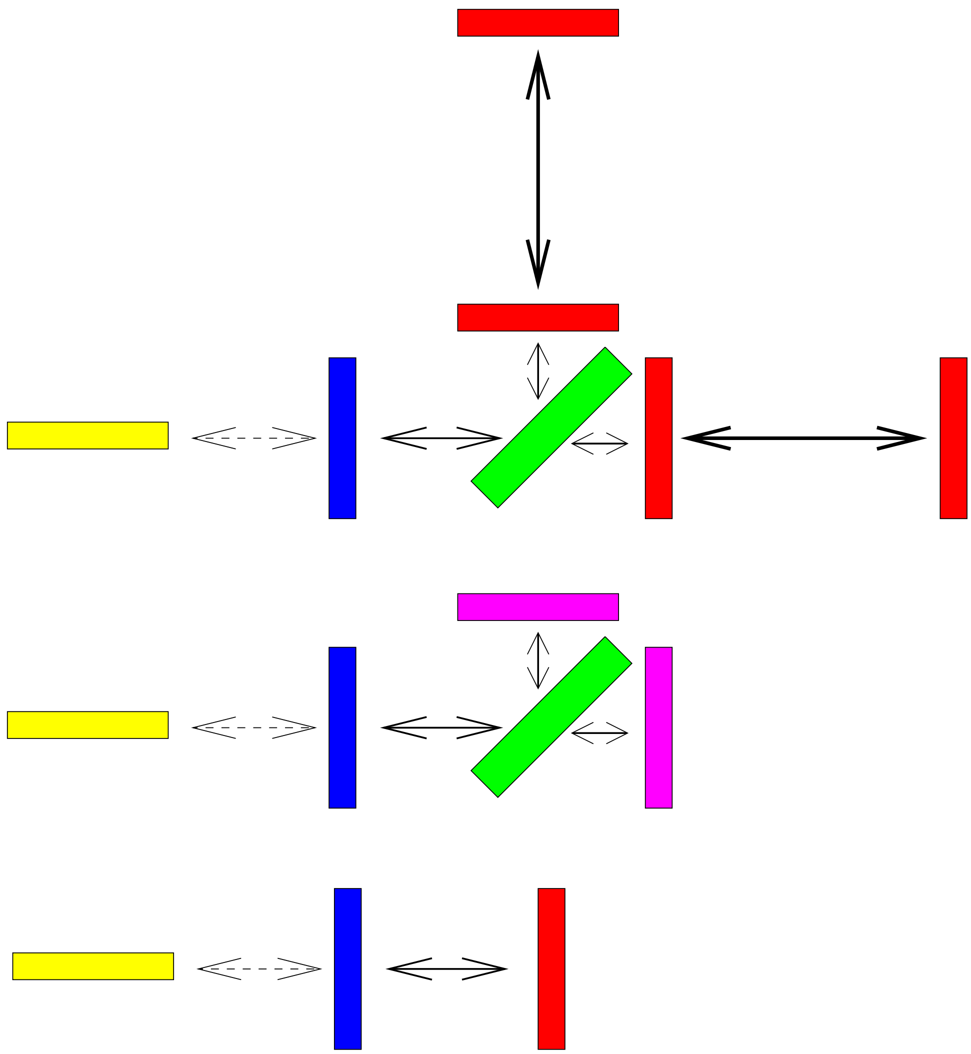
$$h_{shot} = \frac{\lambda}{4L\mathcal{F}}\sqrt{\frac{(2\pi\hbar)\,\nu}{P_{in}}} \tag{79}$$

However, the FP act as mirrors with high reflectivity $\Rightarrow$ much light is shined back toward the laser!

The idea of recycling is then simple: if a mirror is inserted on the way back of the light, one can send the light back in the Michelson in phase with the incoming light [8].



The recycling mirror encloses a new cavity, which at resonance builds up a field E_{in} larger than at input



The net result as expected is to lower the shot noise level

$$h_{shot} = \frac{\lambda}{4L\mathcal{F}}\sqrt{\frac{(2\pi\hbar)\,\nu}{P_{in}\mathcal{R}}}\sqrt{1+(4\pi f_{gw}\tau_s)^2} \qquad (80)$$

for instance VIRGO foresees $\mathcal{R} \simeq 50$ increasing the power from 10 Watt to 500 Watt.

Other sources of phase noise

Any disturbance which affects the optical path length along the two arms asymmetrically leads to extra noise. We just mention some

Index fluctuations The most expensive part of VIRGO is the vacuum enclosure!

A good vacuum is needed because fluctuations of the gas density lead to fluctuations of the refraction index: indeed

$$\phi = n_{gas} \frac{2\pi L}{\lambda} \tag{81}$$

and $n_{gas} \sim 1 + \alpha\rho$, with α polarizability and ρ density. The number of molecules involved is

$$N \simeq \rho S L_{opt} \tag{82}$$

and one expects an RMS fluctuation $\propto \sqrt{N}$, hence

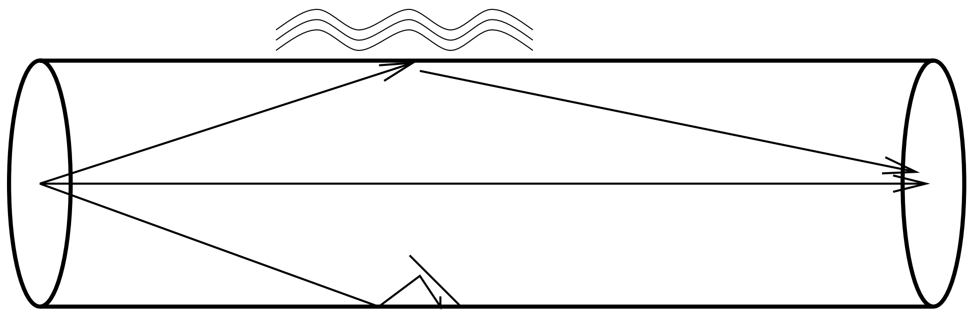
$$\sigma_\phi = \sigma_n \frac{2\pi L}{\lambda} \simeq \frac{2\pi\alpha}{\lambda} \sqrt{\frac{\rho L_{opt}}{S}} \; ; \tag{83}$$

what about the spectral distribution? It will be *white* up to some frequency cutoff which, as in the case of photon shot noise, takes into account granularity of gas. How frequently a molecule can enter or leave the beam? The time scale is $\tau_{res} \propto w_0/\bar{v}$ and leads to

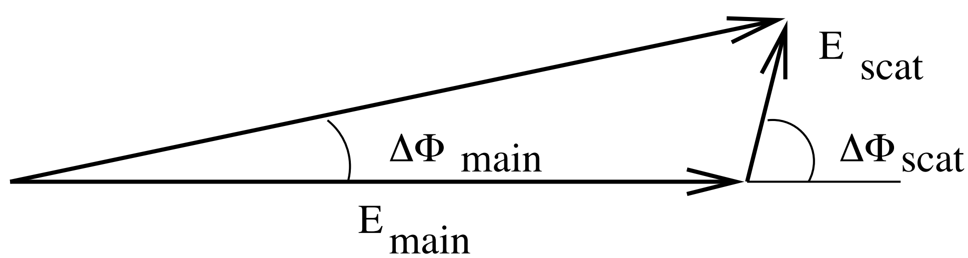
$$h(f) \sim \alpha \sqrt{\frac{\rho}{L_{opt} w_0 \bar{v}}} \tag{84}$$

introducing the numbers, one finds that ultra high vacua are needed: for instance the required partial pressure of hydrogen is 10^{-9} mbar.

Scattered light Any light which travels along paths different from the main one can acquire a different phase: if this varies it introduces noise.



For instance light bouncing off the tunnels, and modulated by seismic vibrations of the soil: a phasor sum of the fields shows



a variation in the scattered phase

$$\Delta\Phi_{main} = \frac{E_{scat}}{E_{main}} \sin \Phi_{scat} \tag{85}$$

if we want to maintain $\Delta\Phi_{main} \leq 10^{-9}$ we need $P_{scat}/P_{main} \leq 10^{-18}$!

Note an example of *up-conversion*: low frequency seismic noise in $\Delta\Phi_{scat}$ can be non-linearly converted to high frequencies [8, 9].

Light in higher modes We have always assumed that light was in the TEM_{00} mode. Why? Assume to decompone the light and take into account higher order modes of the cavities: you have

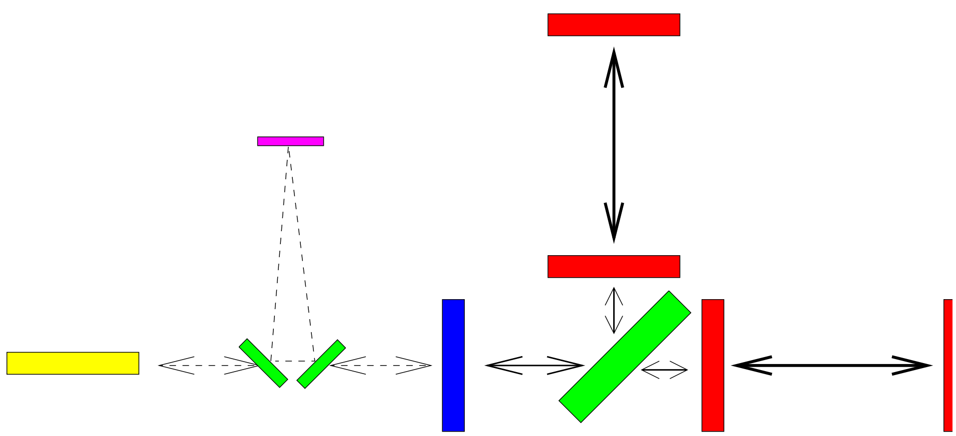
$$\begin{aligned} u_{m\,n}(x,y,z) &\propto e^{i(m+n+1)\phi} \\ \phi &= \arctan\frac{x}{\pi w_0^2/\lambda} \end{aligned} \tag{86}$$

each order acquires a different phase shift [8]!

The disease leads to the cure: in a cavity the modes resonate at different wavelengths

$$\lambda_{mnb} = 2L\left[b + \frac{2\,(m+n+1)}{\pi}\arctan\frac{L\lambda}{2\pi w_0^2}\right] \;; \tag{87}$$

hence we can shine the input light (and also the output light!) through FP cavities operated in transmission and tuned to the 00 mode, thus rejecting higher order modes. These cavities are called *mode cleaners*.



A summary

- An interferometer needs to be operated as an *active null instrument*.
- The optical components are not fixed at $t = 0$, but continuosly moved to compensate for external noises and maintain the instrument at the working point.
- A very careful *control* of the instrument is needed: this is to be fed back to proper actuators.

The suspensions of the optical parts thus have a dual role:

- Isolate from external noises.
- Allow for a very accurate positioning of the optical parts.

How these two tasks are accomplished by the suspensions is the argument of the next seminar.

Lecture 3: The dual role of the Super Attenuator

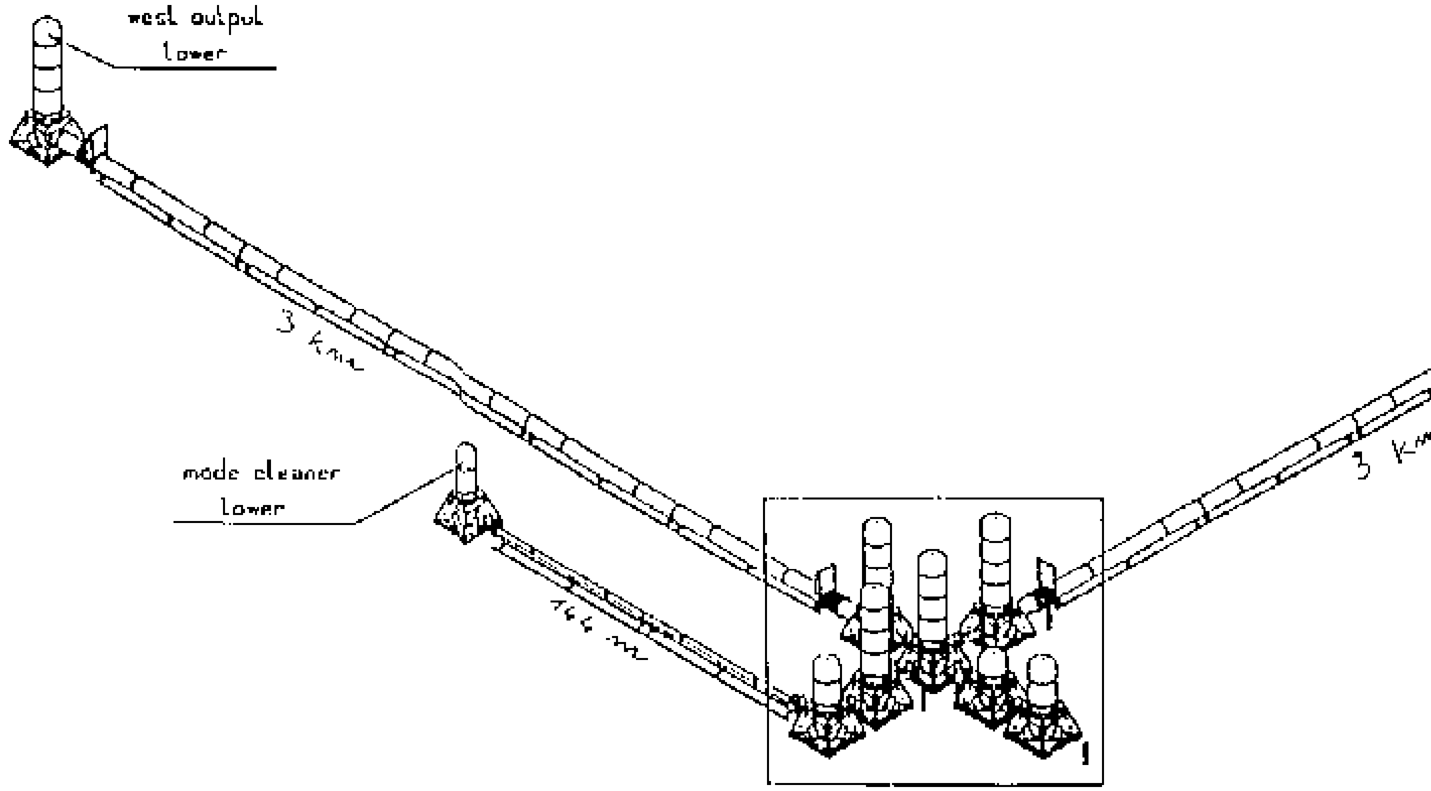
We have seen that the operation of the VIRGO interferometer requires

- A good isolation from *external* disturbances, while maintaining at a minimum the *internal* disturbances.
- The capability to accurately position the optical payloads.

We shall see that

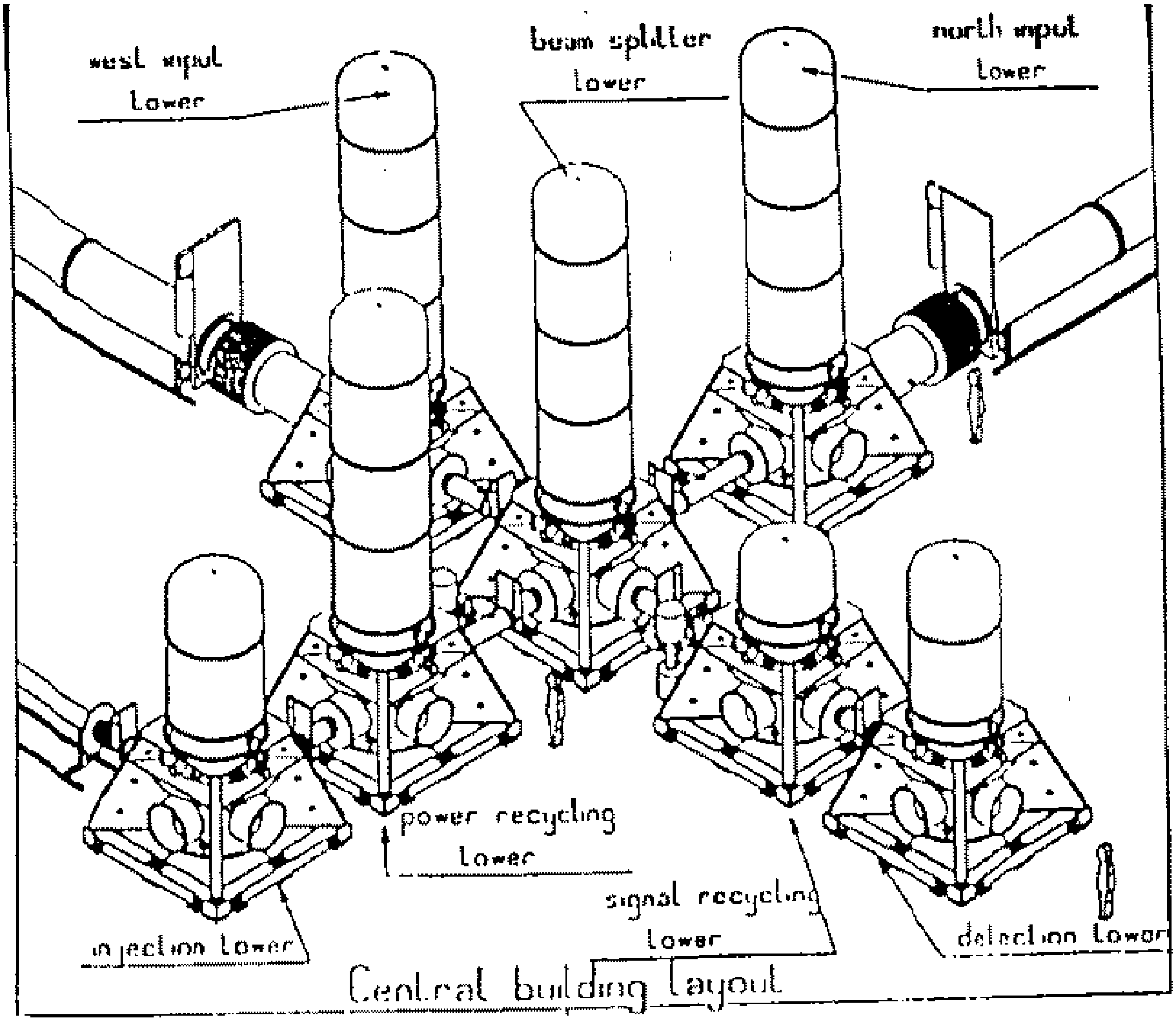
- The SA provides good isolation in 6 degrees of freedom.
- Is made of parts having low dissipations $\Rightarrow$ low thermal noise induced.
- Incorporates an *active* damping system which allows to further reduce external noise.
- Is capable of *steering* the optical components by applying tiny forces.

A suspended interferometer



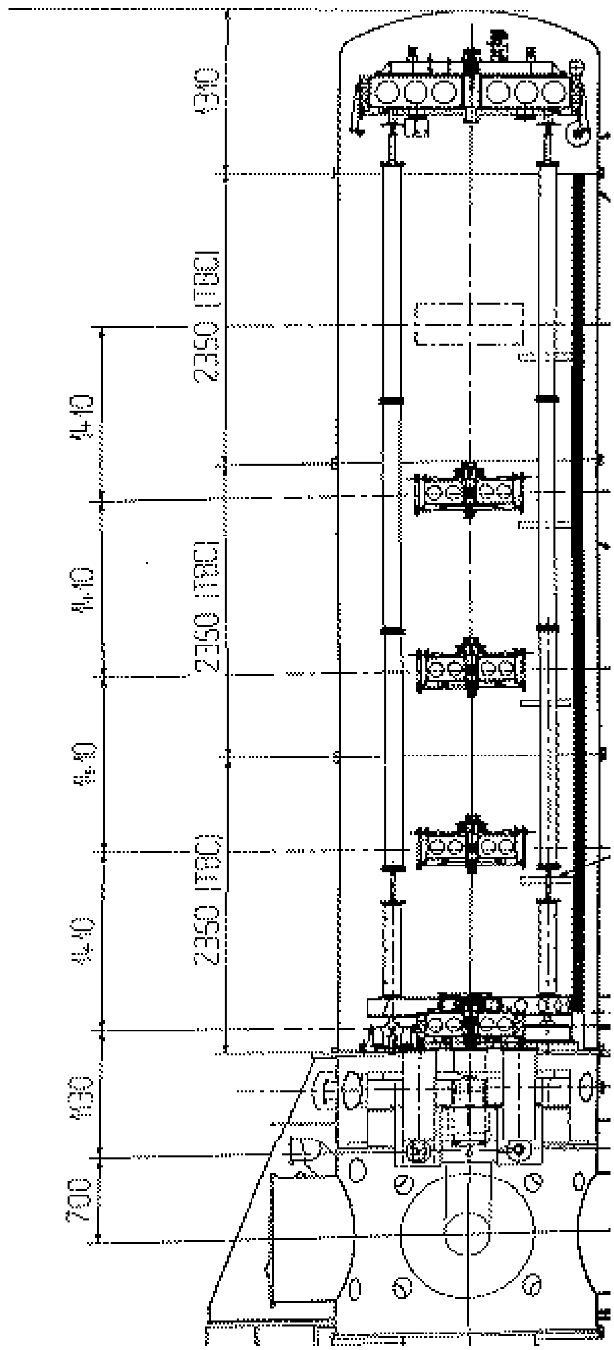
- Every optical element requires some attenuation.
- Higher towers (longer pendola) are used for the elements defining the cavities: mirrors of the Fabry-Perot, beam splitter, recycling mirror.
- Smaller towers for the elements whose *longitudinal* motion alters in the same manner the two interfering beams: the injection of the laser and the extraction of the signal, the towers of the MC, the signal recycling.

The central area expanded

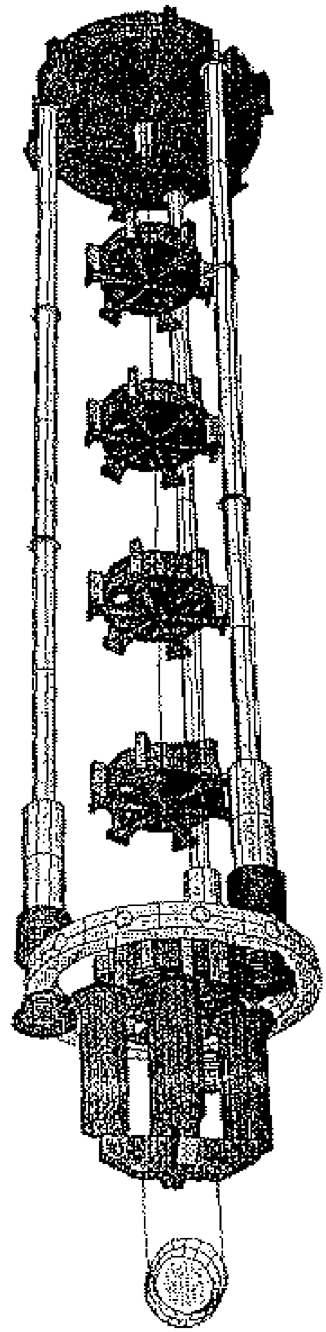


The towers crowding the central building will be realized the next year, and will form a smaller intrument, the *central area ITF*. Used for testing, it will teach how to operate the instrument.

The structure of an isolation tower



Simply a vacuum enclosure, containing the multiple pendulum.

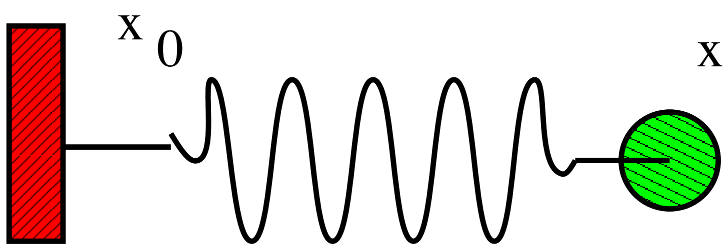


The SA is a chain of pendola suspended to a tripod which works as an *inverted* pendulum.

A structure of about 1 ton, to hold a 40 Kg mirror.

General attenuation principle

The principle of passive attenuation is elementary.



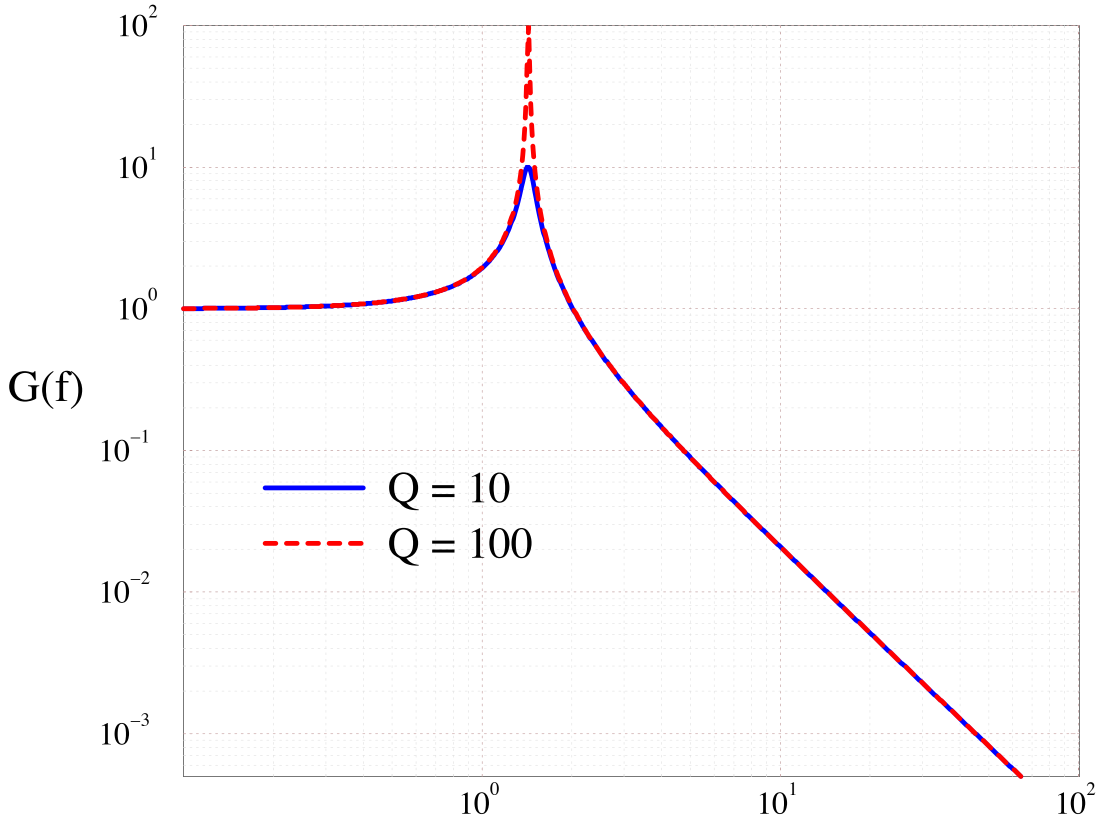
The mass connected via a spring to a source of noise x_0 is subject to the motion equation

$$\ddot{x} + \omega_0^2 x + \frac{\omega_0}{Q}\dot{x} = \omega_0^2 x_0 \quad (88)$$

hence the transfer function

$$G(\omega) = \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + \frac{i\omega\omega_0}{Q}} \quad (89)$$

The quality factor changes the TF only at resonance: at high frequencies is irrelevant: the attenuation is NOT an effect of damping!

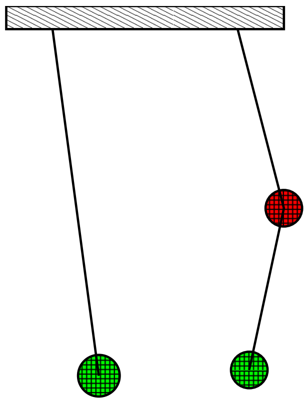


The attenuator works much similarly to the suspensions of a car.

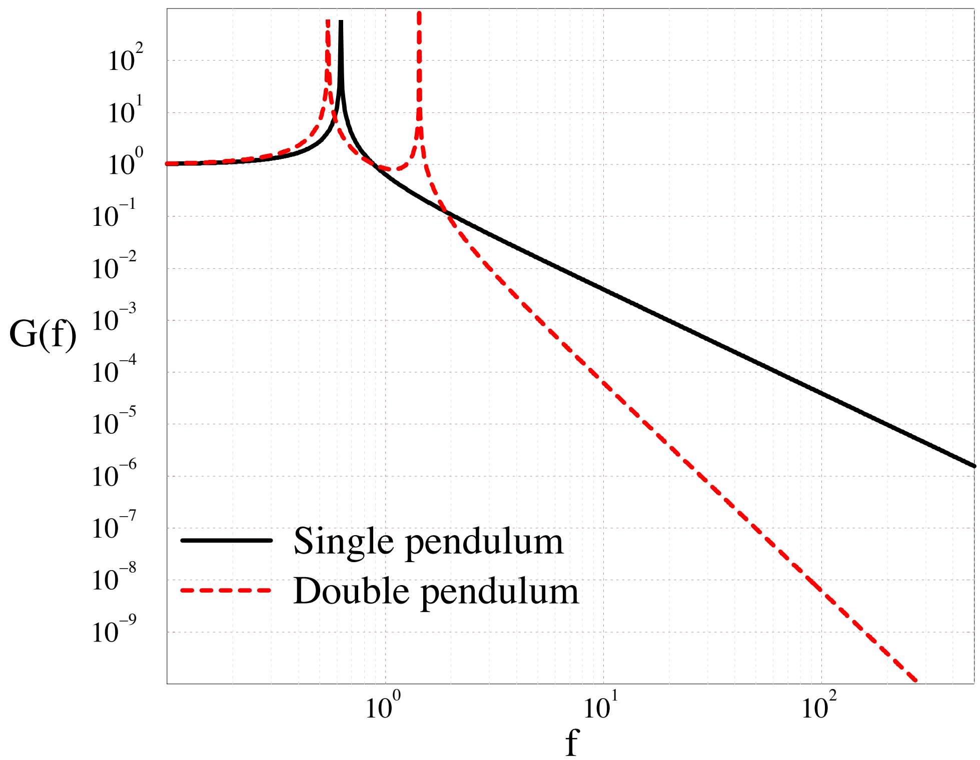
There, thermal noise is irrelevant, and the gas-dampers are used to reduce the Q of the suspensions.

Note further that US cars have smaller ω_0 than EU cars: this gives better comfort, at the price of reduced responsiveness.

Horizontal attenuator: the pendulum



A pendulum gives a resonance $\omega_0 = \frac{g}{l}$.
At fixed total length, we can introduce intermediate masses and increase the number of resonances



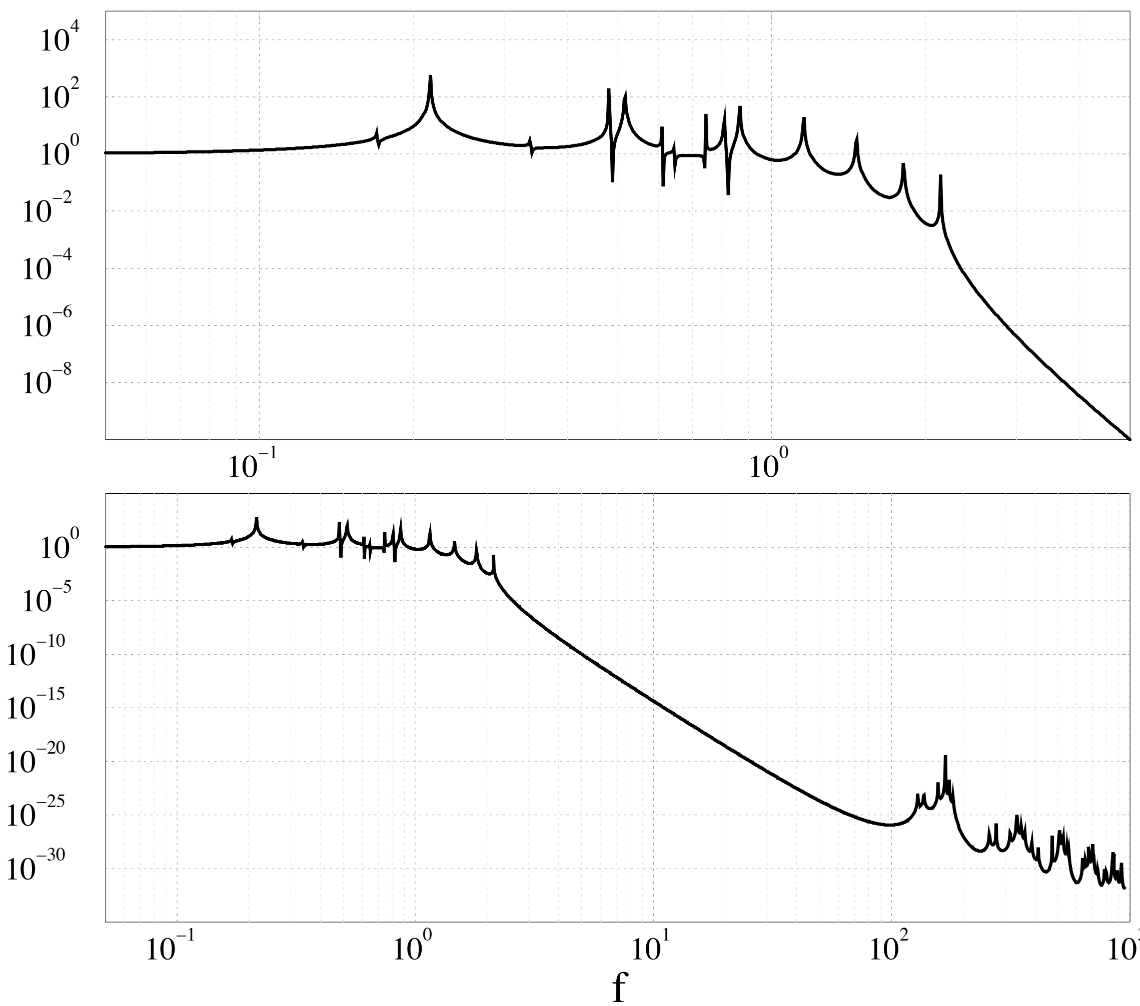
Adding masses we build a steep TF, which asymptotically goes as ω^{-2n} . However there is a limitation: masses of the wires.

Wire resonances

The wire under tension is subject to the equation [14]

$$EI\frac{\partial^4 x(z)}{\partial^4 z} - T\frac{\partial^2 x(z)}{\partial^2 z} = \rho S \omega^2 x(z) \; ; \tag{90}$$

it describes both the pendulum motion and the *violin* modes.



By choosing wires as thin as possible, one pushes the resonances to high frequencies.

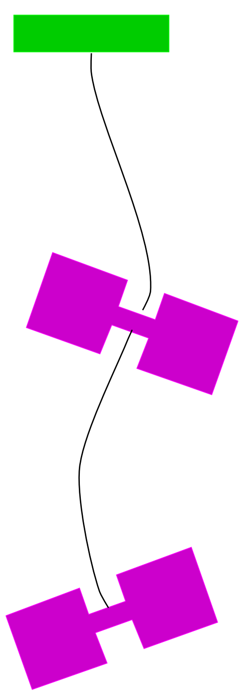
The high Q of pendola

A pendulum is described at low frequencies by a potential

$$
\begin{aligned}
U &= \frac{TL}{2}\left[\frac{x(0)-x(1)}{L}\right]^2 \\
&+ \frac{\sqrt{TEI}}{2}\Biggl\{\left[\frac{x(0)-x(1)}{L}+\theta_y(0)\right]^2+\Biggl[\frac{x(0)-x(1)}{L}+\theta_y(1
\end{aligned}
$$

Most of the recalling force comes from gravity, which is dissipationless!

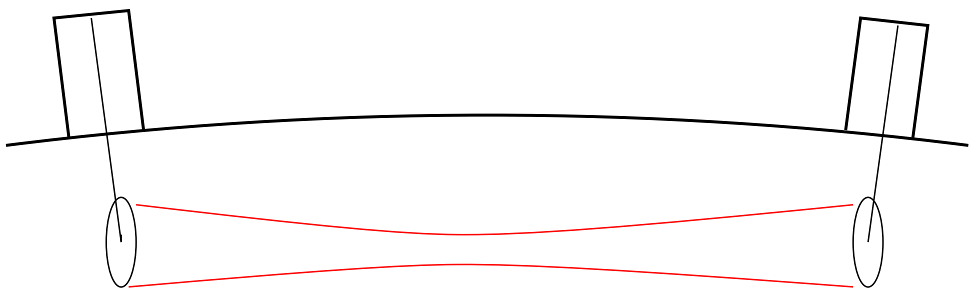
Indeed, structural dissipation can be *approximately* described by giving a small imaginary part to the Young Modulus $E \rightarrow E\,[1+i\phi(\omega)]$. It affects a term which is $O(\sqrt{EI/LT}$, which is kept small by choosing thin wires [15].



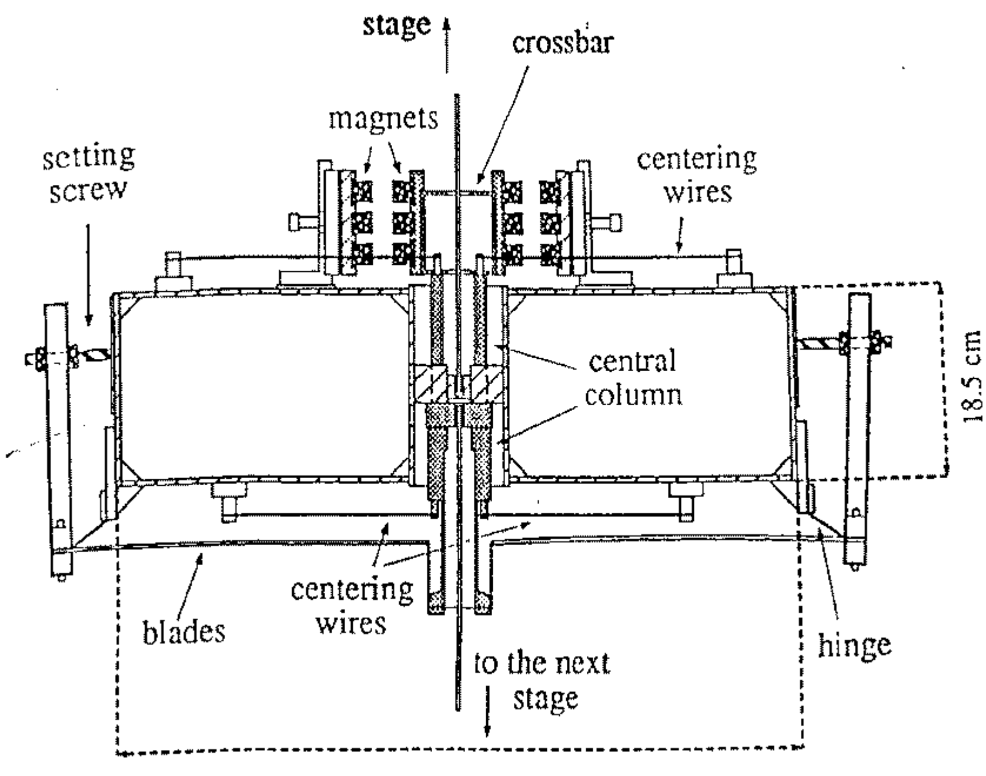
One can easily realize that the same potential gives also a small recalling force for angular motions.
Together with a design which maximizes J in the masses, it guarantees also a good attenuation for angular motions.
Indeed all the degrees of freedom are coupled!

The need to attenuate also in vertical

Assume perfect construction of the SA. Even so, horizontal and vertical motions are coupled at the level of 3×10^{-4} by the Earth curvature!

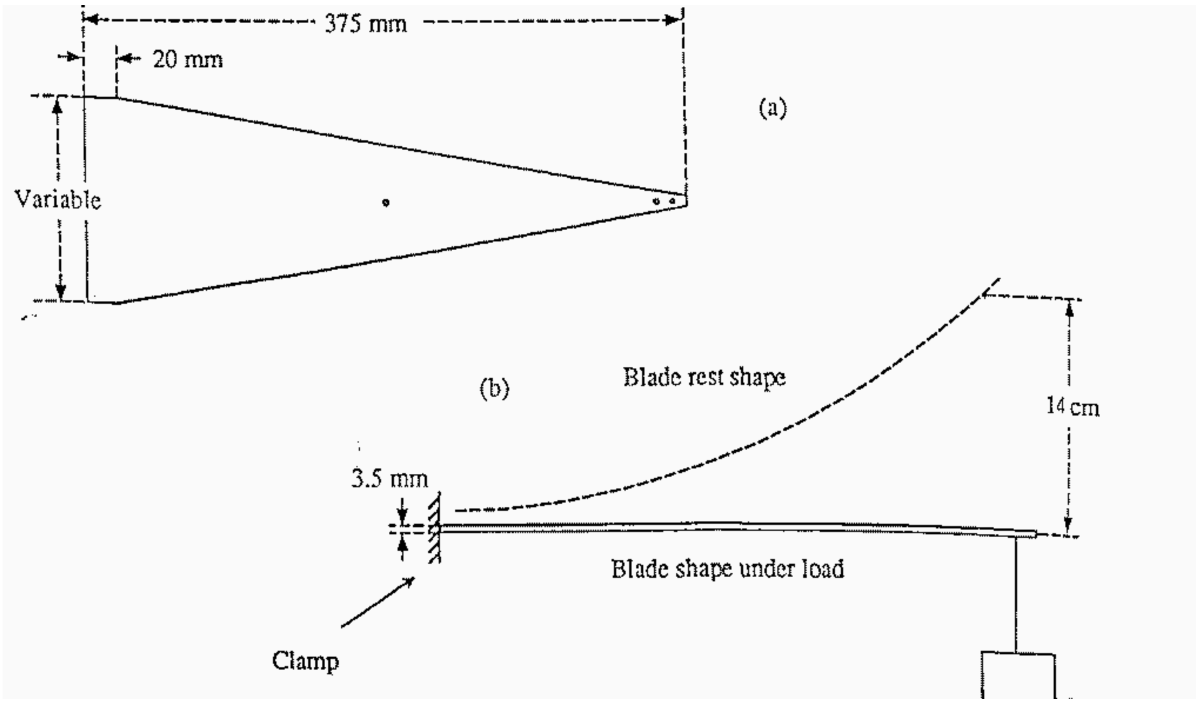


This means that also a good isolation in the vertical direction is needed. But here we do not have a gravity force!



The seismic filter is made of two parts, connected by a spring which gives the attenuation [10, 9].

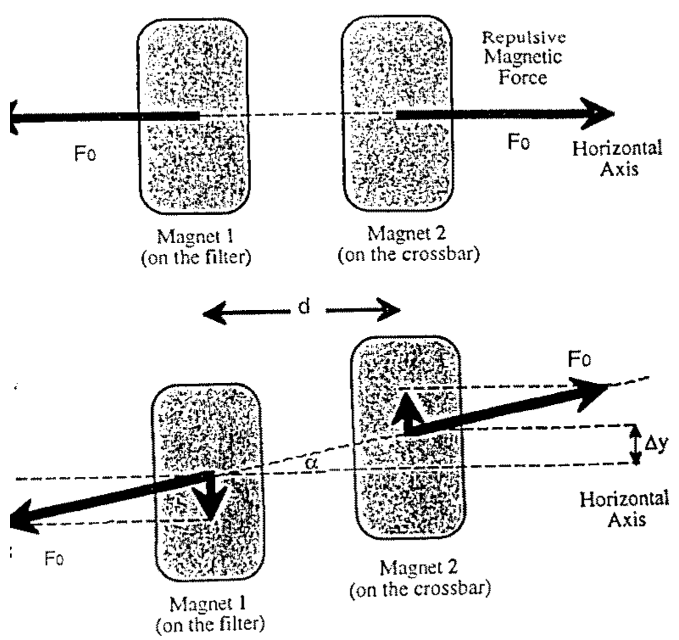
The vertical spring



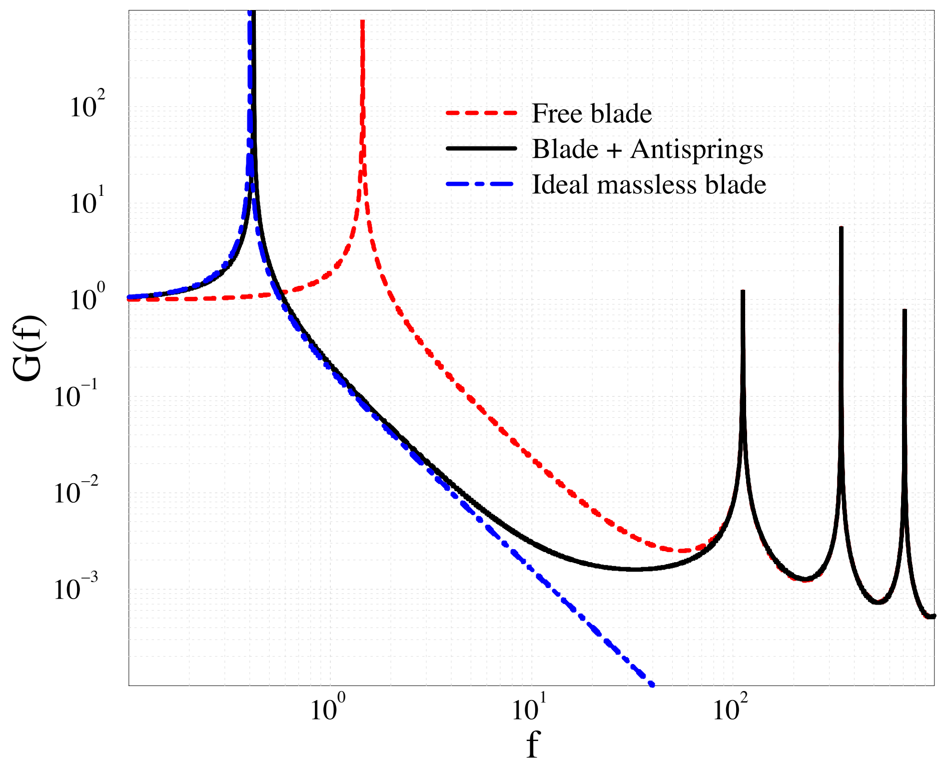
A special kind of spring is used: a cantilever *blade*.

At fixed *stress* in the metal (dangerous for the *creep* problem) a blade allows to hold a load with a low spring constant.

However, one can do better, by adding in parallel magnetic *anti-springs*



Blade resonances



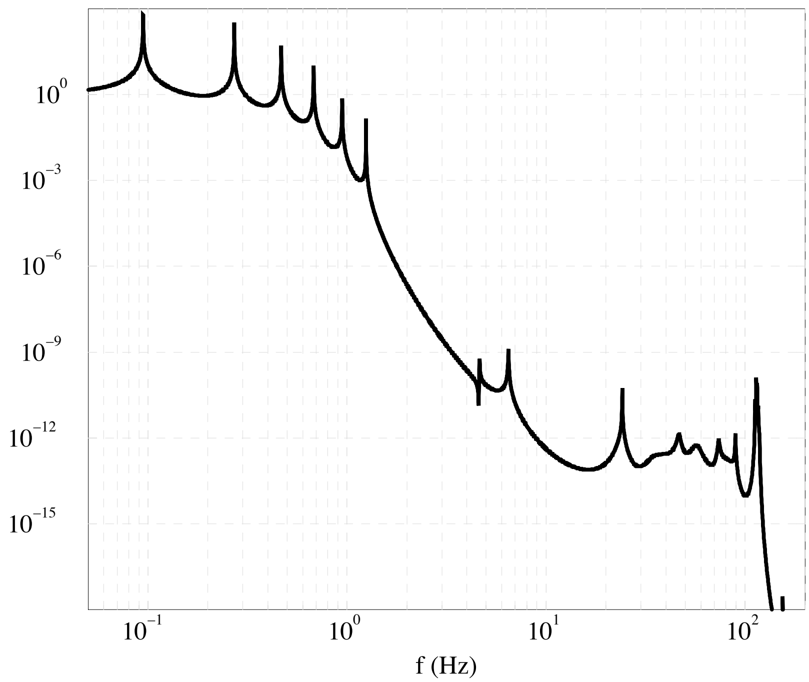
The antisprings allow to lower the fundamental frequency, to get closer to the ones of the horizontal motion.

Analogous to wire resonances, the blade equations [14]

$$
\begin{aligned}
M\ddot{z}(L) - \gamma \frac{d}{d\,x}\left[w(x) z''(x)\right]|_{x=L} &= \kappa_M \left(z(L) - z(0)\right) \\
\rho(x)\ddot{z}(x) + \gamma \frac{d^2}{d\,x^2}\left[w(x) z''(x)\right] &= 0 \\
\gamma &= \frac{E h^3}{12\left(1-\sigma^2\right)}\,, \quad (92)
\end{aligned}
$$

lead to resonances which limit the total attenuation.

A full vertical transfer function



At high frequencies, also the wires connecting the filters start to contribute to the attenuation.

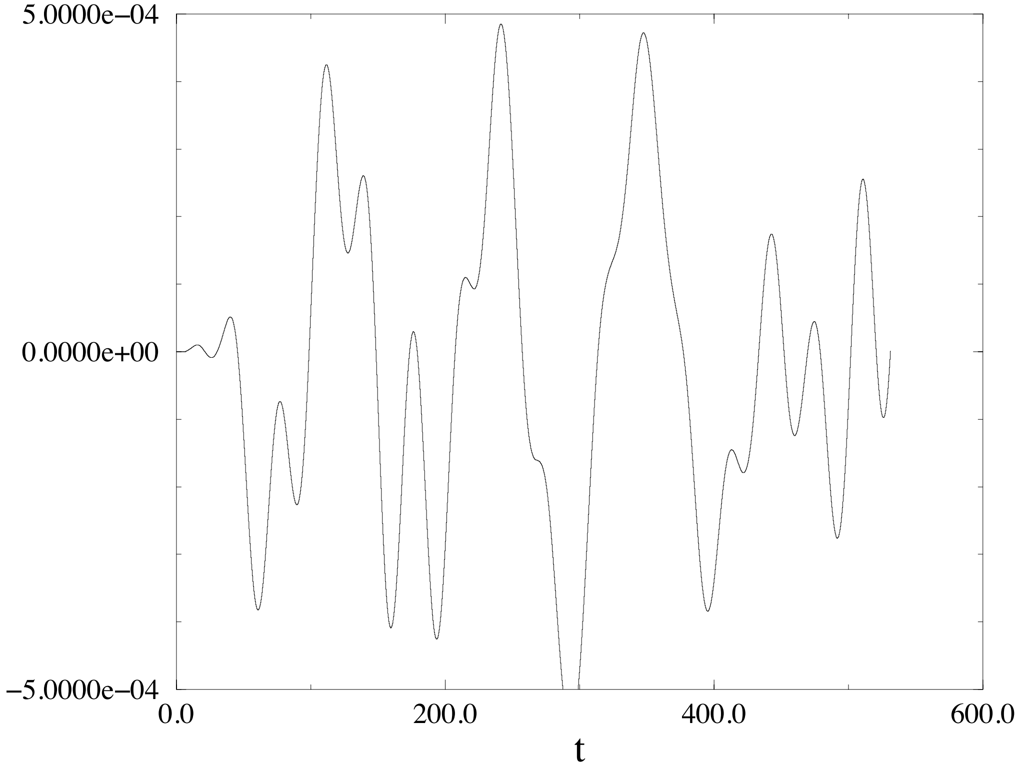
The critical point is that vertical and horizontal noises need to be maintained at similar levels: otherwise the *cross-talk* between modes can seriously hamper the overall performance.

Dangerous resonances are attenuated by introducing appropriate *dampers*, which reduce the Q where this does not introduce dangerous thermal noise.

Is the high Q armless?

Remember that the ITF works only if the optical cavities are kept at resonance.

It is not sufficient to have small residual motion in the detection range: low frequency motion swings the mirrors across many fringes!



The low frequency noise have high Q, and one has

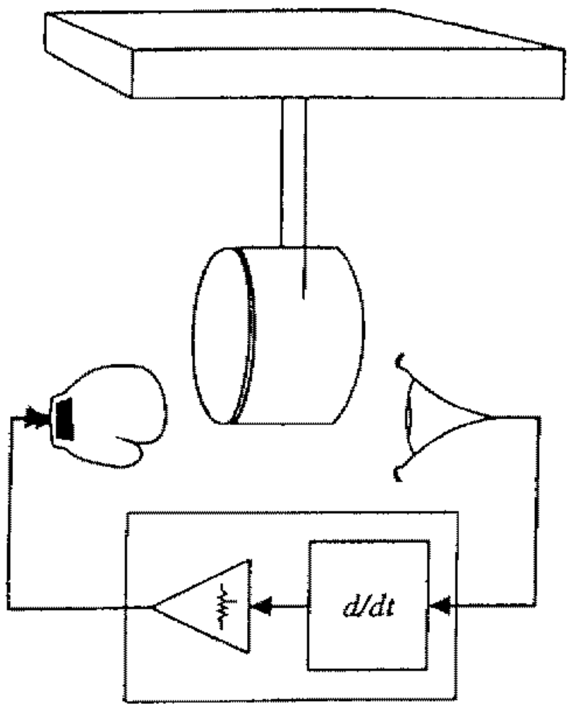
$$x_{rms} \simeq \sqrt{Q f_0} x_{ground}(f_0) \quad : \tag{93}$$

having high Q at low frequencies, where x_{ground} is large, worsens the situation.

Damping

Reducing the Q of the suspensions is not a solution: it would increase the thermal noise level.

A solution is to actively reduce the residual motions, by sensing them and counteract.



But if the ITF is not working yet, which "eye" can I use to sense the position of the mirror?

I have to reduce preliminarily the excitations reaching the mirror: this is done by *damping* the motion of the upper stages of the suspensions, where the noise is injected.

Accelerometers as absolute sensors

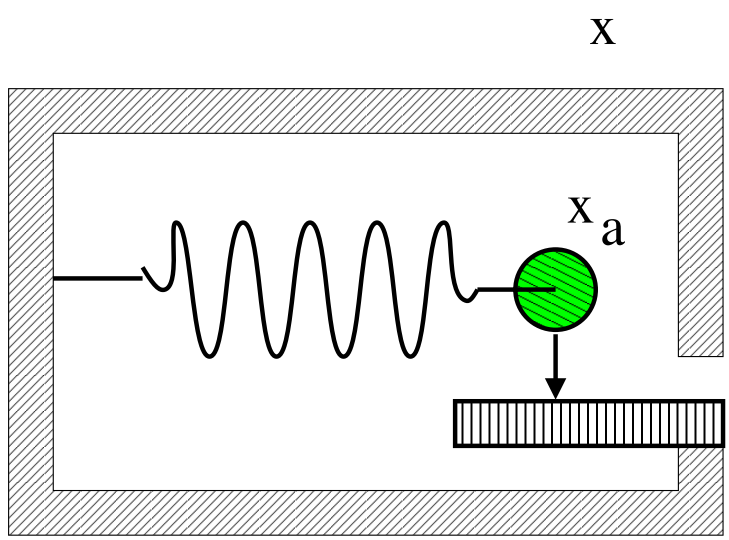
The question is: damping with respect to what? If I measure the speed $\dot{x} - \dot{x}_{ref}$ with respect to a reference, I can modify my motion equation and introduce a damping term,

$$\ddot{x} + \omega_0^2 x + \frac{s\omega_0}{Q}(x - x_{ref}) = \omega_0^2 x_0 \qquad (94)$$

but then I am limited by the noise in the reference! Actually, for small Q I am locking x to the reference . . .

$$x = \frac{\omega_0^2 x_0 + \frac{s\omega_0}{Q} x_{ref}}{(\omega_0^2 + s^2) + \frac{s\omega_0}{Q}} \, . \qquad (95)$$

The solution is to avoid references, using *accelerometers* attached to the object we want to damp



the motion equation is

$$s^2 x_a + \omega_0^2 (x_a - x) + \frac{s\omega_0}{Q}(x_a - x) = 0 \qquad (96)$$

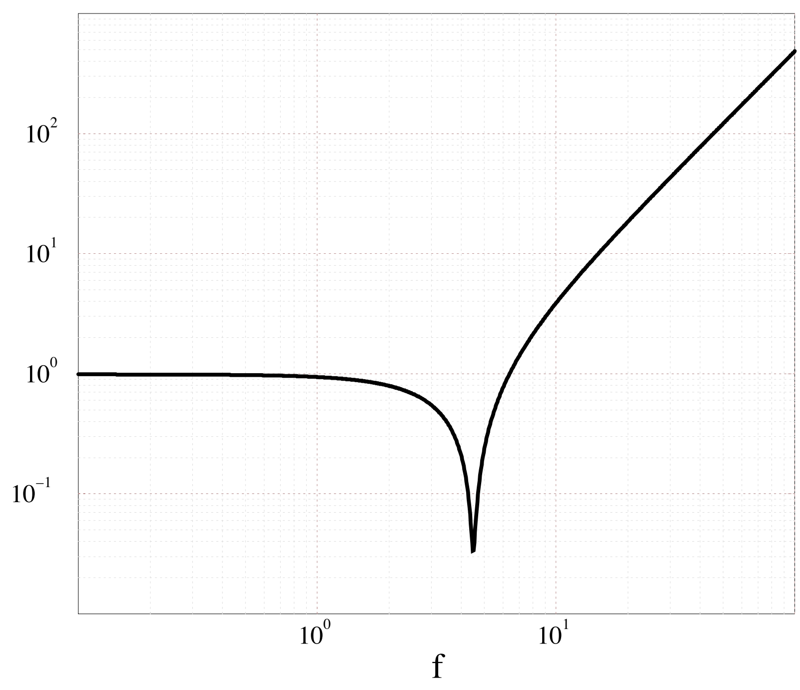
a device not shown allows to monitor $x - x_a$, and we thus have

$$s^2 x = -\left[\left(s^2 + \omega_0^2\right) + \frac{s\omega_0}{Q}\right](x_a - x) \equiv G(s)^{-1}(x_a - x) \tag{97}$$

which means that by using appropriate derivation and summation circuits we can obtain $\ddot{x}$ from the measured $x_a - x$.

Of course the position measure is noisy: if it is gaussian, with constant σ_{pos}, we have an acceleration noise

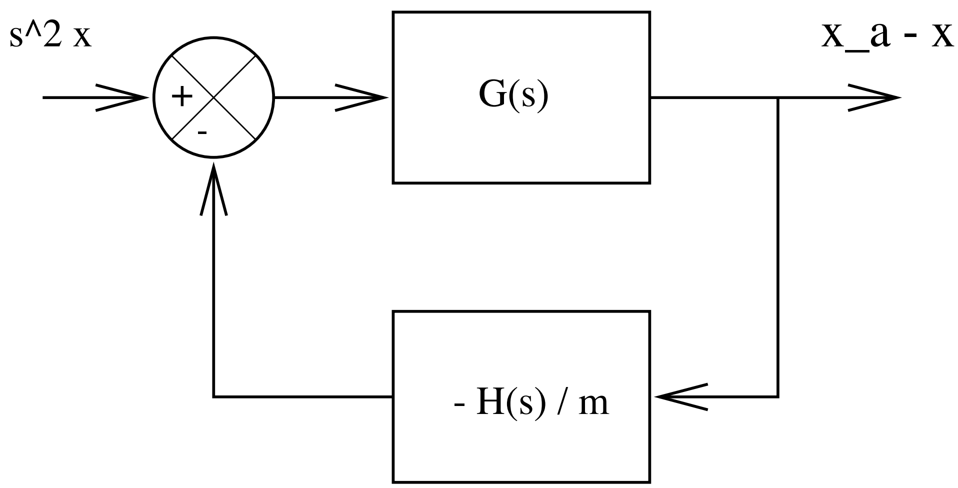
$$S_a(\omega)^{1/2} = \left|\left(\omega^2 - \omega_0^2\right) - \frac{i\omega\omega_0}{Q}\right|\sigma_{pos} \tag{98}$$



Again practice is much different from theory: the system works much better if fitted with a feedback force F acting on the mass x_a (I shift to $s = -i\omega$

$$s^2 x_a + s\frac{\omega_0}{Q}(x_a - x) + \omega_0^2 (x_a - x) = \frac{F}{m} \tag{99}$$

$$F(s) = -H(s)\left(x_a(s) - x(s)\right) \ . \tag{100}$$



We have two situations:

- In *open loop*, the feedback circuit reads a signal depending on the acceleration through an open loop TF:

$$\begin{aligned}
\frac{F^{\mathrm{open}}(s)}{m} &= G_{\mathrm{op}}(s) s^2 x(s) \\
G_{\mathrm{op}}(s) &= -\frac{H(s)}{m} G(s) \\
&= \frac{H(s)}{m}\left[s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right]^{-1} \quad ; (101)
\end{aligned}$$

- In *closed loop*, we have an output signal

$$x_a(s) - x(s) = \frac{G(s)}{1 + G_{\rm op}(s)} s^2 x(s) \qquad (102)$$

and a force signal

$$\frac{F_f(s)}{m} = \frac{G_{\rm op}(s)}{1 + G_{\rm op}(s)} s^2 x(s) \; ; \qquad (103)$$

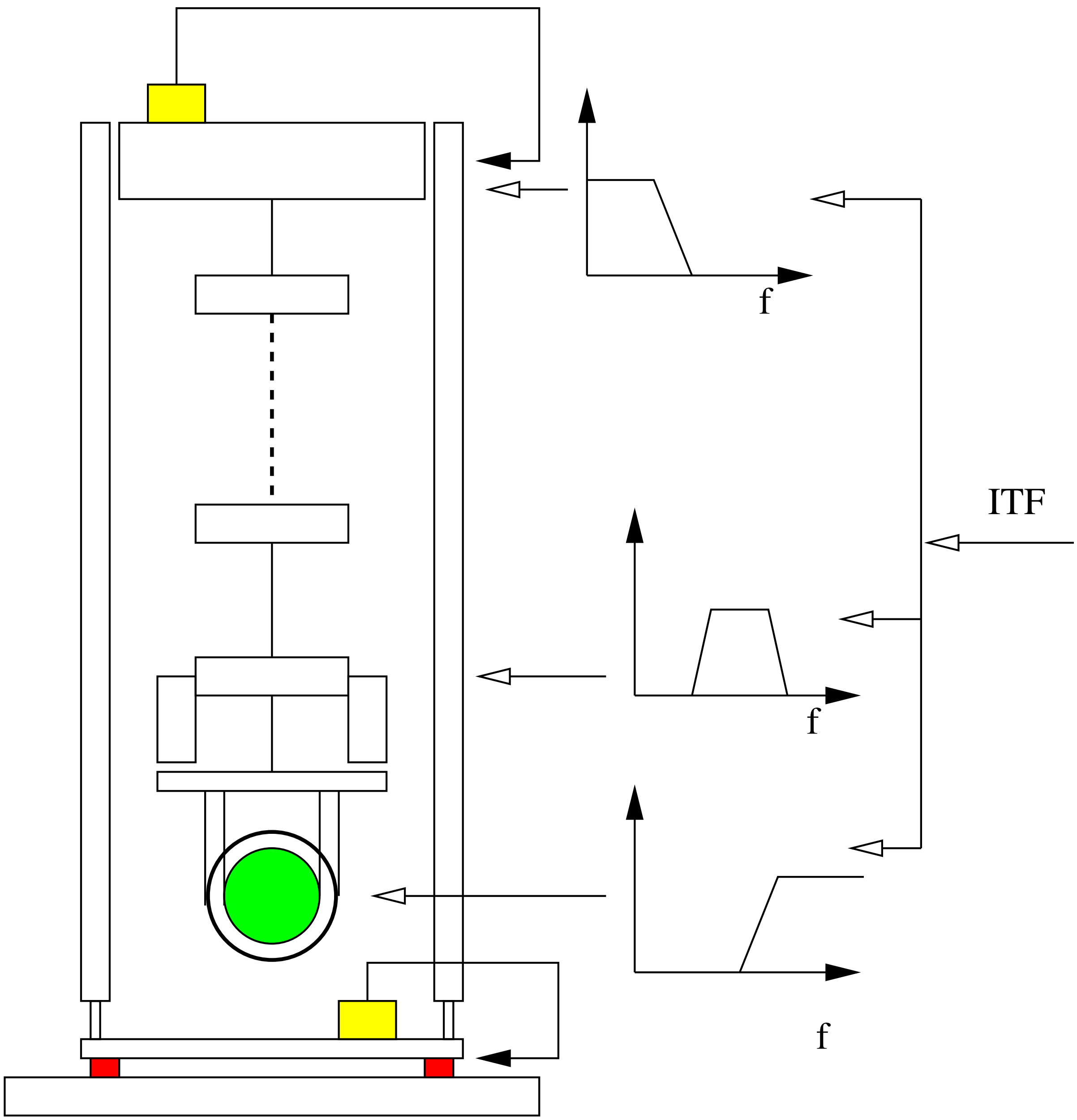
for large *gains* it gives directly the acceleration.

Where is the beef? For large gain, there is no need to know the transfer function: apart a normalization, it cancels!

Moreover, the *compensation filter* $H(s)$ can be chosen so as to modify the dynamics of the accelerometer, for instance increasing the resonance frequency and thus extending the sensitivity range.

Once we have measured the acceleration of the object, we fit it in an external loop (for instance an integrator) and we are able to reduce the motion x with respect to fixed stars!

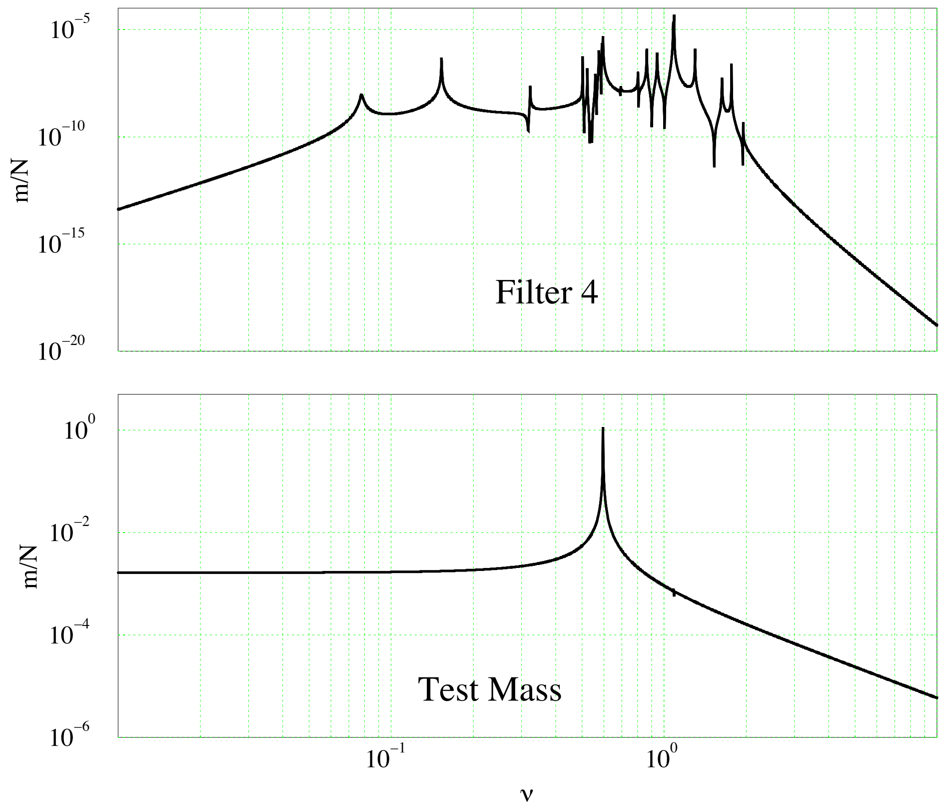
Damping and control in VIRGO



Once accelerometer have damped the residual motion, the ITF may start working (albeit intermittently) and allow the system to *lock*.

Fast control: reference mass action

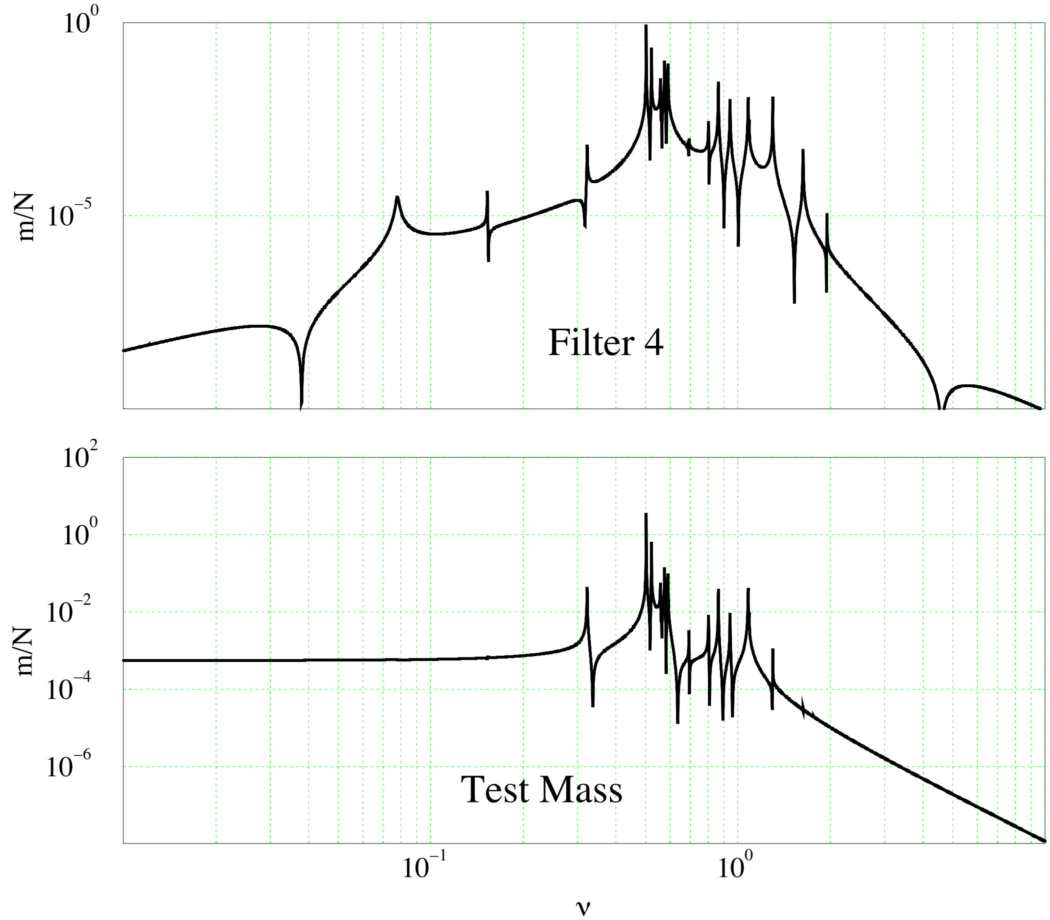
longitudinal mechanical impedence



A very simple transfer function: unfortunately, only small forces can be exerted, to avoid introducing noise due to eddy currents induced by the actuating coils.

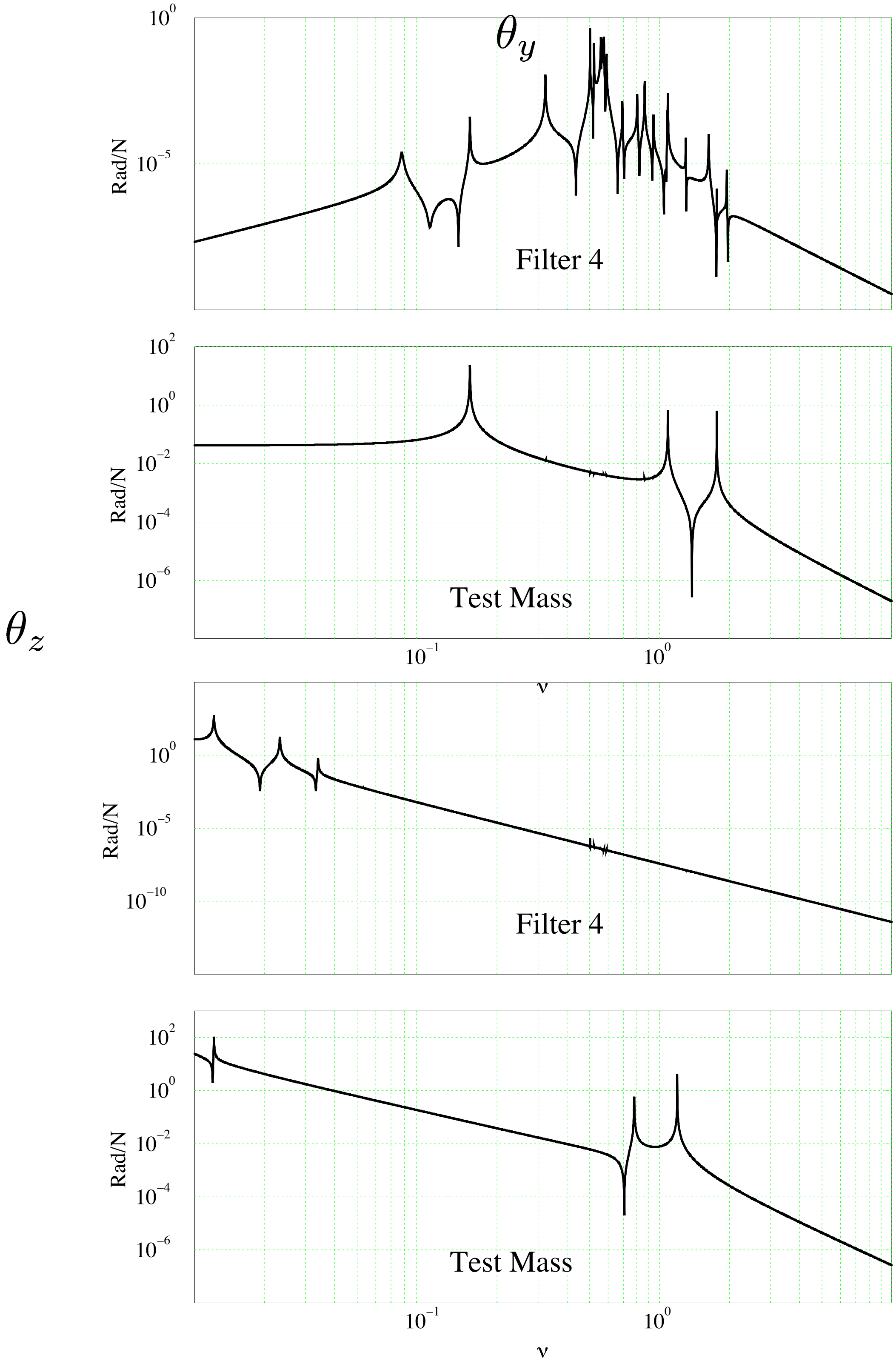
Slower control: *marionetta* action

longitudinal mechanical impedence



A much more complicated transfer function: it will require a good *compensation filter* to avoid inducing spurious oscillations.

Also angles θ_y (elevation) and θ_z (direction) need to be controlled! Not only to maintain the ITF at the working point, but to compensate the extra effects of the longitudinal steering.



θ_y
Rad/N
Filter 4
Rad/N
Test Mass
θ_z
ν
Rad/N
Filter 4
Rad/N
Test Mass
ν

Conclusions

- VIRGO is a complex experiment !
- Optics, mechanics, control: in many aspects it requires a state of the art technology.
- Even if GW were not detected, obtaining the declared sensitivity curve would already be a great success.
- For instance we have not stressed enough the difficulty of the control of the whole ITF. Matching optical transfer functions, electronics, mechanical transfer functions: fighting again non linearities, spikes; it will probably require some time before a *robust* control strategy will be available.
- Maybe theorists are interested only in the final output, the so called reconstructed h. But as in the case of HEP experiments, a closer collaboration with experimentalists is possibly important.

References

[1] The VIRGO collaboration. Virgo central web page `http://www.pg.infn.it/virgodoc/`.

[2] Kip S. Thorne. Gravitational radiation. In S. W. Hawking and W. Israel, editors, *Three Hundres Years of Gravitation*, pages 330–458. Cambridge University Press, 1987.

[3] S. Bonazzola and E. Gourghoulhon. Gravitational waves from pulsars:emission by the magnetic field induced distorsion. *Astronomy and Astrophysics*, 312:675–690, 1996.

[4] The LIGO collaboration. Ligo web page `http://www.ligo.caltech.edu/`.

[5] The GEO600 collaboration. Geo600 web page `http://www.geo600.uni-hannover.de/`.

[6] The TAMA collaboration. Tama web page `http://tamago.mtk.nai.ac.jp/`.

[7] C. W. Misner, K. S. Thorne, and J. A. Wheeler. *GRAVITATION*. Freeman and Company, New York, 1970.

[8] Peter R. Saulson. *Fundamentals of interferometric gravitational wave detectors*. World Scientific, Singapore, 1994.

[9] S. Braccini. *Seismic Isolation in VIRGO*. PhD thesis, Scuola Normale Superiore, Pisa, 1996.

[10] R. Flaminio. *La sospensione degli specchi ed il controllo di un interferometro per la rivelazione di onde gravitazionali*. PhD thesis, Universitá degli Studi di Pisa, 1994.

[11] François Bondu. *Étude du bruit thermique et stabilisation en fréquence du laser du détecteur interférométrique d'on-*

des gravitationalles VIRGO. PhD thesis, Université Paris XI Orsay, 1996.

[12] G Hernandez. *Fabry-Perot interferometers*, volume 3 of *Cambridge studies in modern optics*. Cambridge University Press, Cambridge, 1986.

[13] P. Fritschel. *Techniques for Laser Interferometric Gravitational Wave Detectors*. PhD thesis, MIT, 1992.

[14] L. D. Landau and E. M. Lifsits. *Teoria dell'elasticità*. Edizioni Mir, 1979.

[15] Leonard Meirovitch. *Principles and techniques of vibrations*. Prentice-Hall, 1997.