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ASTROPHYSICAL SOURCES OF GRAVITATIONAL WAVES

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The spectral properties of the gravitational signals emitted by stars and black holes in different astrophysical processes are reviewed.

1 Introduction

Stars and black holes emit gravitational waves in a variety of astrophysical situations. Depending on the features of the signals, these sources can be classified essentially in three cathegories: i) Sources of continuous radiation, such as binary systems or rotating stars. ii) "Impulsive" sources. These include the gravitational collapse of massive stars, the coalescence of compact stars or black holes, or perturbation processes excited, for example, by the capture or the scattering of masses by an already formed compact object. In these cases gravitational waves are emitted as a burst. iii) Stochastic sources. The cumulative effect of the radiation emitted in gravitational collapses occurring in galaxies, now and in the past, should present the characteristics of a stochastic background, the spectral energy density of which would contain information on the process of galaxy and star formation.

In this lecture I shall discuss these issues, with particular reference to impulsive and stochastic sources, and I shall show what kind of information on the generating processes the energy spectrum of the emitted gravitational radiation may contain.

In view of a possible detection of gravitational waves, the knowledge of the frequencies at which the radiation will be emitted is crucial. If the source of a burst of gravitational waves is a star or a black hole, these frequencies are associated to proper modes of vibration, said *quasi-normal modes*, because they are damped by the emission of waves. These modes are central to the theory of gravitational waves because they play an important role in several dynamical processes. For example, they are excited when an external agent, such as an infalling mass, perturbs the spacetime generated by a compact object; or during the last phases of a gravitational collapse and of the coalescence of stars or black holes, when the newborn object oscillates until its residual mechanical energy is radiated away in gravitational waves. Numerical studies have shown that in this stages the dominant contribution to the emitted radiation is due to the quasi-normal modes. The eigenfrequencies of the quasi-normal modes can be computed by studying the source-free perturbations of the equilibrium configuration, and by solving the perturbed equations with boundary conditions appropriate to the nature of the source. I shall describe this approach in next section.

2 The quasi-normal modes of compact objects.

The equations describing the perturbations of black holes and stars are obtained by writing the Einstein equations, plus the equations of hydrodynamics for stars, under the assumption that the metric functions and the fluid variables undergo small changes with respect to their equilibrium values. By retaining only the first order terms, one obtains a set of linear equations describing the perturbed configuration. If the black hole or the star are static and spherically symmetric, the perturbed equations split in two classes depending on the behaviour of the angular part of the perturbation under the transformation $\theta \rightarrow \pi - \theta$ and $\varphi \rightarrow \pi + \varphi$. In particular those that transform like $(-1)^{(\ell+1)}$ are said to be $AXIAL$, and those that transform like $(-1)^{(\ell)}$ are said to be $POLAR$. In a suitably choosen TT-gauge the axial and the polar asymptotic components of the metric tensor are respectively

$$h_{\mu\nu}^{ax} = \begin{pmatrix} (t) & (r) & (\varphi) & (\vartheta) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{\vartheta\vartheta}^{ax} & h_{\vartheta\varphi}^{ax} \\ 0 & 0 & h_{\varphi\vartheta}^{ax} & h_{\varphi\varphi}^{ax} \end{pmatrix}, \qquad h_{\mu\nu}^{pol} = \begin{pmatrix} (t) & (r) & (\varphi) & (\vartheta) \\ 0 & 0 & 0 & 0 \\ 0 & h_{rr}^{pol} & h_{r\vartheta}^{pol} & 0 \\ 0 & h_{\vartheta r}^{pol} & h_{\vartheta\vartheta}^{pol} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (1)$$

The quasi-normal modes are solutions of the perturbed equations belonging to complex eigenfrequencies $\sigma = \sigma_0 + i\sigma$ (the imaginary part is the inverse of the damping time), and satisfying the boundary conditions of a pure outgoing wave at infinity. This condition identifies physically acceptable modes, i.e. those that damp the oscillations. In addition, for a black hole one must require that the solution at the horizon reduces to a pure ingoing wave, corresponding to the requirement that nothing can escape from the horizon. Conversely, for a star all perturbed functions must have a regular behaviour at $r = 0$, and match continuously with the exterior perturbation at the surface .

2.1 The quasi-normal modes of black holes

In 1975 S. Chandrasekhar and S. Detweiler [1] computed the frequencies and the damping times of the quasi-normal modes of a Schwarzschild black hole. Those of a rotating black hole were first determined by Detweiler [2 3 4], and subsequently by Leaver [5], Seidel and Iyer [6] and Kokkotas [7]. The eigenfrequencies depend on the parameters that identify the spacetime geometry, i.e. the mass, the angular momentum and the charge. In particular the frequency of oscillation of a black hole is directly proportional to its mass M, while the damping time scales as the inverse of M. For example, for the fundamental $\ell = 2$-mode of a Schwarzschild black hole of mass $M = nM_{\odot}$

$$\nu_0 = \frac{12.1}{n} kHz, \quad \tau = n \cdot 5.5 \cdot 10^{-5} s. \tag{2}$$

In ref. 1 S. Chandrasekhar and S. Detweiler also showed that the transmission and the reflection coefficients for the axial and the polar perturbations of a Schwarzschild black hole are equal, i.e. the polar and the axial perturbations are isospectral. *This is a definite signature that gravitational waves carry on the nature of the emitting source.* In fact for stars the situation is much different.

2.2 The **polar** *quasi-normal modes of a non-rotating star*

We shall consider in the following adiabatic perturbations of stars composed by a perfect fluid. Let us analyze firstly the polar perturbations, which also exist in newtonian theory. Inside the star they are described by a set of linear equations that couple the perturbations of the fluid and the metric variables. However, it is possible to rearrange these equations and derive a set of equations that describe exclusively the metric perturbations. The thermodynamical variables can be obtained in terms of these by simple algebraic relations [8]. This decoupling is possible in general, and requires no assumption on the equation of state of the fluid.

In newtonian theory, the classification of the polar modes is based on the behaviour of the perturbed fluid, thus, it is interesting to see whether this classification survives in the relativistic approach which, conversely, focuses on the gravitational field.

An inspection of the newtonian hydrodynamical equations shows that when a star is perturbed each element of fluid moves under the competing action of two restoring forces

$$\frac{\partial \mathbf{v}}{\partial t} = \frac{\delta\rho}{\rho_0^2}\nabla p_0 - \frac{1}{\rho_0}\nabla\delta p + \delta\mathbf{f}\,, \qquad \delta\mathbf{f} = -\nabla\delta\phi, \tag{3}$$

where $\delta\phi$ is the variation of the gravitational potential. Thus, the modes of oscillations are classified according to the restoring force that is prevailing: the **g**-modes, or gravity modes, when the force is due to the eulerian change in the density $\delta\rho$, and the **p**-modes, when it is due to a change in pressure. (This classification scheme was introduced by Cowling, 1942 [9]). The two classes of modes occupy well defined regions of the spectrum, and they are separated by the **f**-mode that is the generalization of the only possible mode of oscillation of an incompressible homogeneous sphere [10]. The characteristic of this mode is that the corresponding eigenfunction has no nodes inside the star.

The relativistic approach is completely different from the newtonian approach. As mentioned earlier, the equations used to find the oscillation frequencies involve only the perturbations of the gravitational field, and the algorithm used to find them is the following. By integrating the perturbed equations for different values of the real frequency σ, one constructs the function $[\alpha^2(\sigma)+\beta^2(\sigma)]$, that is the squared amplitude of the stationary wave prevailing at infinity. It can be shown[11] that, under the hypothesis that the imaginary part of an eigenfrequency is much smaller than the real part, the values of the frequency where $[\alpha^2+\beta^2]$ has a minimum correspond to the real part of the frequencies of the quasi-normal modes. The imaginary part, i.e. the inverse of the damping time, can be obtained from the width of the parabola which fits the curve near each minimum.

For example, let us consider a star with a non-barotropic polytropic equation of state with $n = 3$, $\gamma = 5/3$, $\epsilon_0/p_0 = 5.35\cdot 10^5$ (the non-barotropic character of the equation of state is clear when we note that the chosen value of the adiabatic exponent γ is different from $4/3$). The ratio between the central energy density ϵ_0 and pressure p_0 has been chosen to coincide with that at the centre of the sun [12]. In figure 1 we show a graph of the resonance curve $log(\alpha^2+\beta^2)$ as a function of the frequency. Although the identification of the different classes of modes can be traced back to the hydrodynamical equations that generalize eqs. (3) in the relativistic case, it is impressive to see how the distinction between the **g**-modes and the **p**-modes, separated by the **f**-mode, graphically emerges from this plot, which is based on the behaviour of the metric functions at infinity. Thus, in the relativistic approach the information on the different kind of fluid modes is coded in the gravitational field. Typical values of the frequency of the **f**-mode for neutron stars are $\nu_f \approx 1-2kHz$, and for the damping times $\tau_f \approx 0.1-0.5s$.

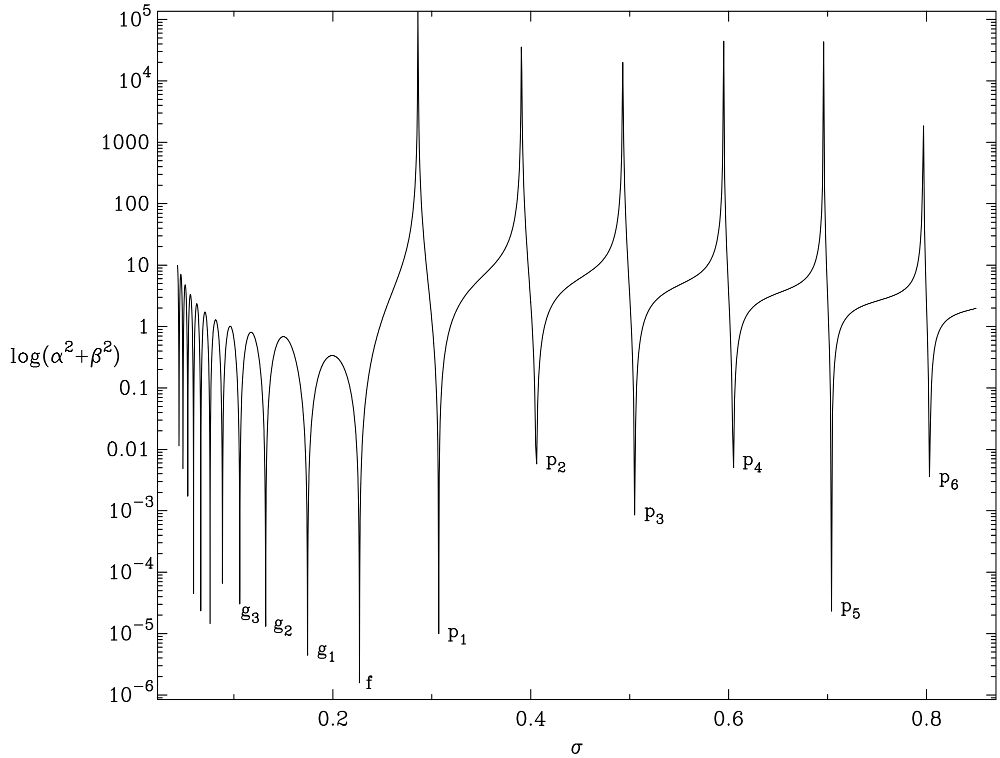

Figure 1: The resonance curve $\log(\alpha^2+\beta^2)$, is plotted versus the real frequency σ, for $\ell = 2$. σ is measured in unities of $\epsilon_0^{-1/2}$, where ϵ_0 is the energy density at the centre of the star.

From the knowledge of the eigenfrequencies of the polar quasi-normal modes one can derive interesting information. N. Andersson and K. Kokkotas [14] have determined the frequency and the damping time of the **f**-mode for several equations of state proposed in the literature for neutron stars. They fit the data with the following relations

$$\nu_f = 0.39 + 44.45\sqrt{\left(\frac{M}{R^3}\right)} \qquad \tau_f = 0.1 - \left(\frac{M}{R}\right) + 2.69\left(\frac{M}{R}\right)^2, \qquad (4)$$

where M and R are expressed in km, ν_f in kHz and τ_f in ms. These two relations provide an estimate both for M and R, good within 5% if compared with the true values.

In addition to the **g**-,**f**-,**p**-modes, that exist also in newtonian theory, in general relativity there exists a new family of modes that are essentially spacetime modes, since the corresponding motion of the fluid is negligible[13]. They are called **w**-modes, and are characterized by high frequencies $\nu_w \approx 8{-}12kHz$, and short damping times $\tau_w \approx 0.02 - 0.1ms$.

A further relation is provided by N. Andersson and K. Kokkotas[14] for the damping time of the lowest **w**-mode computed for the same neutron stars models

$$\frac{1}{\tau_{w_0}} = 0.104 - 0.063 \left(\frac{M}{R} \right). \qquad (5)$$

2.3 The **axial** *quasi-normal modes of a non-rotating star*

The radial evolution of the axial perturbations of stars is described by a Schroedinger-like equation with a potential barrier that depends on the distribution of energy and pressure in the interior of the star in the equilibrium configuration [8]. *The axial perturbations are not coupled to the oscillations of the fluid, and do not have a newtonian counterpart.* Consequently, the axial quasi-normal modes are pure spacetime modes, and they belong to two cathegories:

w-modes - highly damped and with properties similar to the polar **w**-modes,
s-modes - slowly damped[15] and related to the shape of the potential barrier. These modes appear if the star is extremely compact. For example, the potential well in the interior of homogeneous stars becomes deeper as the value of (R/M) decreases and the star shrinks. When the ratio (R/M) is sufficiently small, $(R \lesssim 3M,)$ the potential barrier outside the star has a maximum, and the potential well in the interior may become deep enough to allow for the existence of one or more quasi-stationary states, i.e. of quasi normal modes.

It is interesting to see explicitely to what extent the **s**-modes are slowly damped compared to the **w**-modes. As an illustrative example, in table 1 we show the characteristic frequencies and damping times of the first $\ell = 2$, **s**- and **w**-axial modes of homogenoeus stars, with $M = 1.35 M_\odot$, and different values of R/M. It should be stressed that the modes that one finds when the radius of the star approaches the limiting value $R/(2M) = 9/8$, are not related to the quasi-normal modes of a Schwarzschild black hole, because the boundary conditions are different. Moreover, the progressive increasing of the damping time for these modes means that they are more effectively trapped by the curvature of the star.

3 The excitation of the quasi-normal modes

It is now interesting to ask whether the quasi-normal modes can be excited in some astrophysical situations. For example one can compute the energy spectrum of the gravitational radiation emitted when a mass $m_0 << M$ is captured by a star or by a black hole of mass M, and compare the results[16]. The difficulty in the case of a star is that we do not know how the mass m_0 interacts with the fluid in the interior, and therefore the integration of the equations must be stopped when m_0 reaches the surface of the star.

Table 1: *The characteristic frequencies and damping times of the first* $\ell = 2$, **s**-*and* **w**-*axial modes of homogenoeus stars, with* $M = 1.35 M_\odot$, *and different values of* R/M.

	s-modes		**w**-modes	
$\frac{R}{M}$	ν_s in kHz	τ_s in s	ν_w in kHz	τ_w in s
2.4	8.6293	$1.52 \cdot 10^{-3}$	11.1738	$1.70 \cdot 10^{-4}$
2.3	5.6153	0.54	11.1084	$3.02 \cdot 10^{-4}$
2.28	4.4333	10.8	10.4128	$5.45 \cdot 10^{-4}$
2.26	2.6041	$5.38 \cdot 10^{3}$	10.7852	$7.60 \cdot 10^{-4}$

As a consequence of this truncation, the computed energy spectrum may be quite distorted with respect to the true spectrum, but still it will provide an indication on whether the modes are excited or not. In figure 2 the energy spectrum emitted in the axial perturbations when a mass m_0 is captured by a Schwarzschild black hole or by a star are plotted versus the frequency. m_0 starts its flight from radial infinity with a given angular momentum $\bar{L} = L/M$, and the mass of the black hole and of the star are chosen in such a way that the frequencies of their lowest quasi-normal mode, if expressed in physical units, coincide. This means that, for example, if the mass of the star is $M_S = 1.8 M_\odot$, that of the black hole will be $M_{BH} = 2.07 M_\odot$. The star is supposed to be homogeneous, with energy density ϵ, and with $R/M = 2.3$. This star possesses only two **s**-modes and several **w**-modes. Figure 2 shows that the energy spectra emitted by a black hole and by a star are morphologically very different and contain a clear signature on the nature of the source. For a black hole, figure 2a, there is only one peak at approximately the frequency of the lowest quasi-normal mode. The reason why the contribution of the different modes is not distinguishable is that, being the damping time of each mode very short, the width associated to each peak is large and its contribution cannot be isolated from the others, so that the result is the envelope. In the case of a star, figure 2b gives a clear indication that both **s**-modes are excited, though we cannot say anything definite about the relative height of the two peaks because of the truncation of the signal, as explained before. The two peaks are so well resolved because the damping times of the corresponding modes are quite large. A zoom of the spectrum at higher frequencies given in figure 3c, shows that also the **w**-modes of this star are excited.

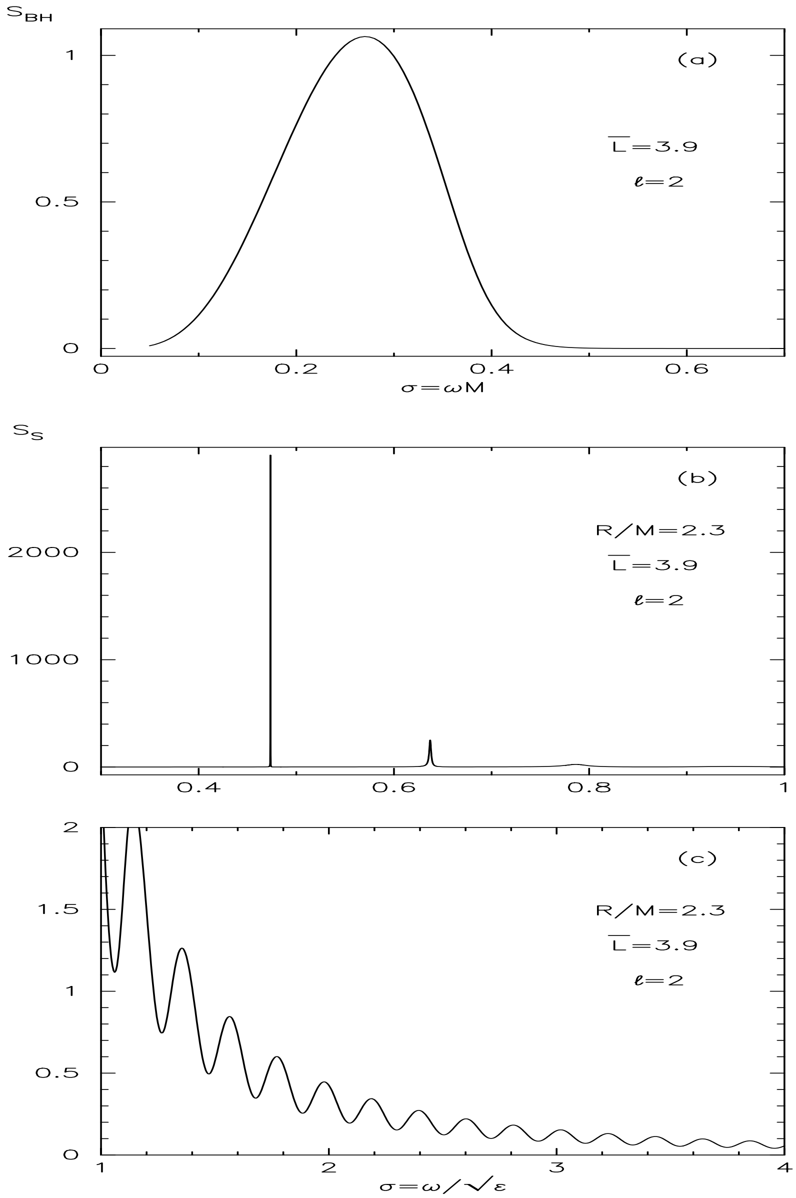

Figure 2: *The energy spectra emitted by a black hole (a) and by a homogeneous star (b,c) are plotted versus a normalized frequency.* $\left(\frac{dE}{d\omega}\right)_{BH} = M_{BH} m_0^2 S_{BH}$, *and* $\left(\frac{dE}{d\omega}\right)_S = S_S m_0^2 \epsilon^{-1/2}$.

4 A stochastic background

The evolution of sufficiently massive stars leads to gravitational collapse and to the emission of bursts of gravitational radiation. If the rate of star formation is sufficiently high, the cumulative effect of these processes produces a stochastic background of gravitational waves, the spectral features of which depend on how the process of galaxy and star formation took place.

A possible scenario for galaxies and stars formation, proposed by A.Di Fazio[17] and A. Di Fazio and Yu. Izotov[18], is the following. After radiation decoupled from matter, gravitational instabilities caused the formation of self-gravitating gaseous clouds, with primordial chemical composition. Being unstable, they collapsed and underwent fragmentation. This process recurred inside the newly formed structures originating generations of smaller fragments, up to when the first protostars were formed. The subsequent evolution of these protostars produced an intense burst of gravitational collapses, the biggest in the history of the universe, since at that time the gas available to form stars was much more than it is today. The resulting normalized mass distribution functions for galaxies, $\Psi_G(M)$, and for stars, $\Psi_S(m)$, can be modeled as follows

$$\psi_G(M) = \frac{M^{-1.87}\sqrt{1-\left(\frac{M_{min}}{M}\right)^{\frac{2}{3}}}}{\int_{\Delta M} M^{-1.87}\sqrt{1-\left(\frac{M_{min}}{M}\right)^{\frac{2}{3}}}\, dM}, \qquad \int_{\Delta M} \psi_G(M) dM = 1. \tag{6}$$

where ΔM is the protogalaxy mass range $[8 \cdot 10^8 M_\odot < M < 5 \cdot 10^{12} M_\odot]$, and

$$\psi_S(m) = \frac{m^{-1.77}\sqrt{1-\left(\frac{m_{min}}{m}\right)^{\frac{2}{3}}}}{\int_{\Delta m} m^{-1.77}\sqrt{1-\left(\frac{m_{min}}{m}\right)^{\frac{2}{3}}}\, dm}, \qquad [4M_\odot < m < 100 M_\odot]. \tag{7}$$

In the framework of this scenario we have computed[19] the rate of gravitational collapses associated to the first big burst of star formation, the expected spectral energy density and the strain amplitude. In our calculation we have made the simplifying assumption that all galaxies were formed at some redshift z_{GF}. The rate of gravitational collapses can be obtained by integrating the following expression

$$d\Re = \frac{dN_S}{(1+z_S)\Delta t_{burst}}, \tag{8}$$

over the allowed range of masses for galaxies and stars. In eq. (8) Δt_{burst} is the time interval during which the first burst of primordial collapses occurred,

z_S is the redshift of star formation which is related to z_{GF}, and dN_S is the number of protostars, with mass in the range $[m, m + dm]$, which form in primordial galaxies i.e.

$$dN_S = N_0 \Psi_G(M) dM \cdot N_S(M) \psi_S(m) dm, \tag{9}$$

where N_0 and $N_S(M)$ are respectively the total number of protogalaxies and the number of stars in each protogalaxy. We have found that the rate of gravitational collapses that led to black hole formation is $\Re \gtrsim 10^5$ events per second, depending on the values of the parameters present in our calculation, i.e.
- The value of the Hubble constant, which we write as $H_0 = 50 \frac{km}{sMpc} \cdot h$.
- The fraction of barion mass which goes into galaxies, $0.5 \lesssim \eta_G \lesssim 1$.
- The uncertainty on the value of the time interval a star stays in the main sequence before collapsing or exploding as a supernova, $\tau_{MS} \sim [2 - 3] My$ (for stars with masses in the range $25 M_\odot \lesssim m \lesssim 100 M_\odot$.)
- The redshift at which galaxy formation occurred, which we assume to be $4 \lesssim z_{GF} \lesssim 8$.

Since the sources are isotropically distributed, and due to the high rate and to the short duration of the signal generated in each event (typically a few milliseconds at the emission), the assumption that the gravitational radiation produced in these processes has the character of an isotropic, continuous stochastic background, is justified. We have considered only the collapses of those stars that gave birth to a black hole because, in this case, the energy spectra available in the literature present, quite independently from the initial conditions, some common features that can easily be modeled. As a model for a single event we have used the energy spectrum computed by R.F.Stark and T.Piran[21], who integrated by a fully relativistic computer code the equations governing the evolution of a rigidly rotating, axisymmetric polytropic configuration, with adiabatic index $\gamma = 2$. The collapse was ignited by a pressure reduction to a chosen fraction f_p of its equilibrium central pressure. The efficiency of the process was always $\frac{\Delta E_{GW}}{mc^2} < 7 \cdot 10^{-4}$.

The spectral energy-density of the stochastic background is given by

$$l_{tot}(a, \nu) = \frac{dE}{dtdSd\nu} = \int f(a, m, \nu) \cdot d\Re, \tag{10}$$

where $f(a, m, \nu)$ is the energy spectrum of the single event. It is related to the strain amplitude $\sqrt{S_h(\nu)}$ (expressed in $\frac{1}{\sqrt{Hz}}$) by the equation

$$l_{tot}(a, \nu) = \frac{\pi}{2} \frac{c^3}{G} \nu^2 S_h(\nu) \left[\frac{ergs}{cm^2 Hzs} \right]. \tag{11}$$

In Fig. 3 we plot the spectral amplitude $\sqrt{S_h(a,\nu)}$ as a function of the frequency of observation ν, for different values of the redshift of galaxy formation, and the function $\Omega_G(a,\nu) = \frac{4}{3}\frac{\pi^2}{H_0^2}\nu_o^3 S_h(a,\nu)$, related to it. In that picture all formed black holes have been assumed to have the same, and quite low, angular momentum $a = 0.5$. (The maximum value, reported in ref. [21], is $a_{crit} = 1.2 \pm 0.2$). Depending on the value of the redshift of galaxy formation, $\sqrt{S_h(a,\nu)}$ reaches its maximum values respectively in the following regions

$$
\begin{aligned}
z_{GF} &= 4 \quad 240Hz < \nu < 370Hz \\
z_{GF} &= 6 \quad 195Hz < \nu < 295Hz \\
z_{GF} &= 8 \quad 165Hz < \nu < 255Hz
\end{aligned}
$$

These data do not significantly change if we change the value of the Hubble constant. For example if we assume $H_0 = 33\frac{km}{sMpc}$ for $z_{GF} = 6$ we find that $175Hz < \nu < 270Hz$. The amplitude of $\sqrt{S_h(a,\nu)}$ scales as $\sqrt{H_0}$.

5 Concluding Remarks

Many are the problems related to the emission of gravitational waves from astrophysical sources that remain to be investigated and clarified. For example, our knowledge on the information that the energy spectrum emitted by compact sources carries on the internal structure of the source is still very limited. Furthermore, we have indications on how the rotation of a star affects its emission [22] if the rotation is slow, but much remains to be understood in the case of fast rotation. Lastly, we have a very poor understanding of the gravitational collapse, for which a fully relativistic numerical approach seems to be unavoidable. However, apart from the difficulties of modeling the physics of such catastrophic events, the computer codes designed to study the problem suffer of several problems related to the strongly non-linear regime in which they are forced to operate. Thus, there is a strong need to support the non-linear numerical approach with other techniques and semi-analytical methods that will be of great help in testing and interpreting the numerical results. In this respect, the theory of perturbations is far from being cut out of the future.

The preliminary results presented in this paper on the stochastic background of gravitational waves are only a first step towards the comprehension of a phenomenon of extreme complexity, since it is intimately related to the theory of galaxy and star formation, which is subject of debate among cosmologists. We plan to repeat our calculations in the framework of alternative theories, in order to predict from the spectral properties of the resulting grav-

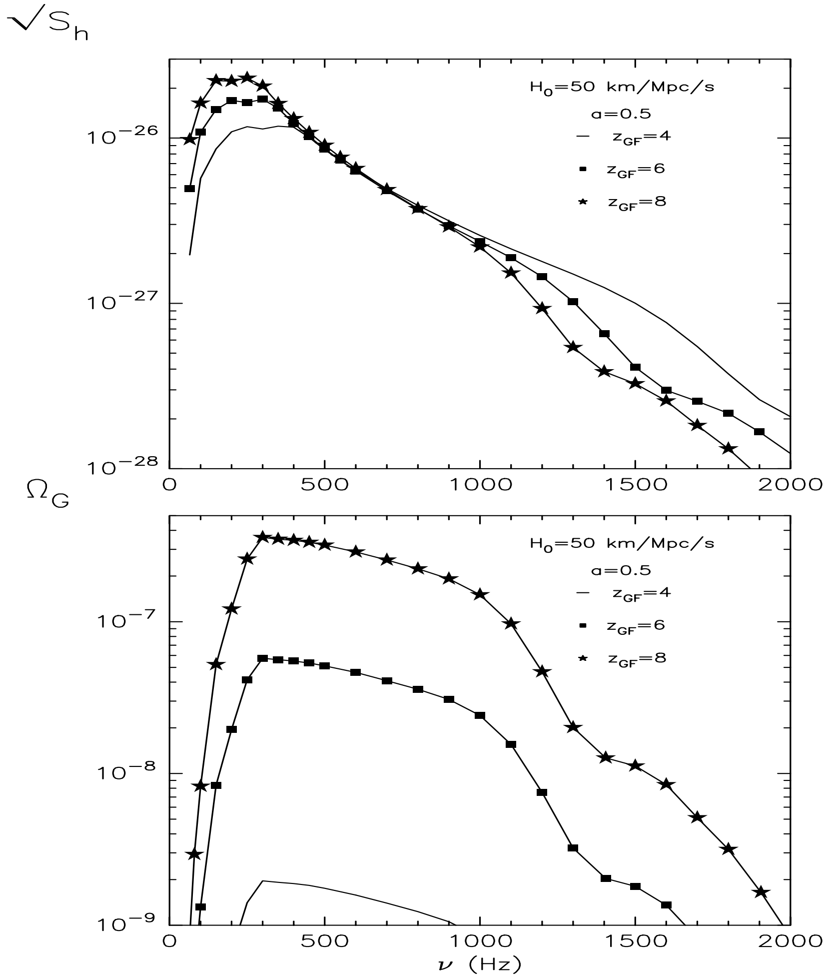

Figure 3: *The spectral amplitude* $\sqrt{S_h(a,\nu)}$, *and the spectral energy density expressed in units of the critical density,* $\Omega_G(a,\nu)$, *are plotted versus the frequency of observation, for* $a = 0.5$ *and for three different values of the redshift of galaxy formation* z_{GF}. *In these calculations we have assumed* $h = 1$, $\eta_G = 0.8$, $\tau_{MS} = 3 Myears$. $\sqrt{S_h(a,\nu)}$ *is proportional to* $\left(\frac{h\eta_G}{\beta}\right)^{1/2}$, *and* $\Omega_G(a,\nu) \sim \frac{\eta_G}{h\beta}$.

itational background, those features that may discriminate among different galaxy formation scenarios.

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