Electroweak symmetry breaking

Lecture 1: General picture

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Outline

The outline of this lecture is

Outline

- Standard Model overview
- Electroweak breaking
- Higgs and Goldstone bosons
- Fermion gauge interactions
- Yukawa interactions
- Neutral currents
- Charged currents and CKM mixing

Standard Model overview

- The Standard Model (SM) is a gauge theory based on the group

Gauge group

$$SU(3) \otimes SU(2) \otimes U(1)_Y$$

- $SU(3)$ describes the strong interactions (QCD) $\Rightarrow$ Paolo Nason's lectures
- Since the gauge interactions conserve helicity we can decompose fermions as

$$f = f_L + f_R, \quad f_L = \frac{1}{2}(1-\gamma_5)f, \quad f_R = \frac{1}{2}(1+\gamma_5)f$$

- The SM choice was to place f_L in $SU(2)$ doublets and f_R in $SU(2)$ singlets
- One can instead replace f_R by

$f_R \rightarrow f_L^c = C\bar{f}^T$, where C=charge conjugation matrix

- They appear in at least three generations

SM fermions

$$\begin{pmatrix} \nu_i \\ \ell_i^- \end{pmatrix}_L \qquad \begin{pmatrix} u_i^\alpha \\ d_i^\alpha \end{pmatrix}_L$$

$$\alpha = \text{colors}$$
$$i = \text{generations}$$
$$Q = T_3 + Y$$

$$\ell_{iR}^- \ [\ell_{iL}^+] \qquad u_{iR}^\alpha \ [u_{iL}^{c\ \alpha}] \quad d_{iR}^\alpha \ [d_{iL}^{c\ \alpha}]$$

$$(1,2)_{-1/2} + (3,2)_{1/6}$$

$$(1,1)_1 + (\bar{3},1)_{-2/3} + (\bar{3},1)_{1/3}$$

- The pure gauge boson part lagrangian is

Electroweak gauge bosons lagrangian

$$\mathcal{L}_{gauge} = -\frac{1}{4} G_{\mu\nu a} G^{\mu\nu a} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

$$G_{\mu\nu a} \equiv \partial_\mu W_{\nu a} - \partial_\nu W_{\mu a} + g \epsilon_{abc} W_{\mu b} W_{\nu c}$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

- To properly quantize the theory we need the Faddeev-Popov gauge fixing

Faddeev-Popov lagrangian (symmetric phase)

$$\mathcal{L}_{GF+FP} = \frac{1}{2\xi}(\partial^\mu W^a_\mu)^2 + \frac{1}{2\xi'}(\partial^\mu B_\mu)^2 + \bar{c}^a(-\partial^\mu D^{ab}_\mu)c^b$$

$$D^{ab}_\mu = \partial_\mu \delta^{ab} + g\epsilon^{acb} W^c_\mu$$

- The interaction of gauge bosons with fermions is achieved in the gauge invariant lagrangian

Fermion lagrangian

$$\mathcal{L}_{fer} = i\sum_{f_L} \bar{f}_L \gamma^\mu (\partial_\mu - ig\frac{\sigma_a}{2} W_{\mu a} - ig' Y_{f_L} B_\mu) f_L$$

$$+ i\sum_{f_R} \bar{f}_R \gamma^\mu (\partial_\mu - ig' Y_{f_R} B_\mu) f_R$$

Electroweak breaking

- In the Standard Model the electroweak symmetry is spontaneously broken by the Higgs mechanism where an $SU(2)_L$ doublet Higgs boson is needed

Higgs mechanism

$$H = \begin{pmatrix} \chi^+ \\ H^0 \end{pmatrix}_{1/2}$$

$$\widetilde{H} = i\sigma_2 H^* = \begin{pmatrix} \bar{H}^0 \\ -\chi^- \end{pmatrix}_{-1/2}$$

$$\mathcal{L}_{Higgs} = \left|(\partial_\mu - ig\frac{\sigma_a}{2}W_{\mu a} - ig'\frac{1}{2}B_\mu)H\right|^2 - V(H)$$

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

- By minimization of the Higgs potential one obtains the VEV

$$\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad v = \sqrt{\frac{m^2}{\lambda}}, \quad m_h^2 = 2\lambda v^2$$

- By replacing $H = \langle H \rangle + \hat{H}$ in $\mathcal{L}_{Higgs}$ one obtains

$$\frac{v^2}{8}(-g^2 W_{\mu a} W^{\mu a} + 2gg' B_\mu W^{3\mu} - g'^2 B_\mu B^\mu)$$

$$= -\frac{1}{4} g^2 v^2 W_\mu^+ W_\mu^-$$

$$-\frac{1}{4} v^2 \begin{pmatrix} W_3^\mu & B^\mu \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$W_\mu^\pm = \frac{W_\mu^1 \pm i W_\mu^2}{\sqrt{2}}$$

- The gauge boson mass spectrum is then

Gauge boson masses and relations

$$m_{W^\pm} = \frac{1}{2} g v; \quad m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v; \quad m_A = 0$$

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu; \quad A_\mu = \cos\theta_W W^3_\mu + \sin\theta_W B_\mu$$

$$\tan\theta_W = \frac{g'}{g}$$

- The mixing angle can be put in relation with gauge boson masses as

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

- The muon decay lifetime determines the relation

$$v^2 = \frac{1}{\sqrt{2} G_\mu} = (246.22 \text{ GeV})^2$$

Higgs and Goldstone bosons

- We can write the Higgs field as

$$H(x) = \begin{pmatrix} \chi_2 + i\chi_1 \\ \frac{1}{\sqrt{2}}(v+h) - i\chi_3 \end{pmatrix}$$

$$= e^{i\chi_a(x)\sigma^a/v} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h(x)) \end{pmatrix}$$

- The unitary gauge is defined as $(\chi^a \to 0)$

$$H(x) \to e^{-i\chi_a(x)\sigma^a/v} H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- In the unitary gauge the gauge boson propagators

$$\Delta^{\mu\nu}_{VV}(q) = \frac{-i}{q^2 - m_V^2 + i\epsilon}\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{m_V^2}\right]$$

- It is more convenient to work in R_ξ gauge characterized by the GF lagrangian

$$\mathcal{L}_{\rm GF} = \frac{-1}{2\xi}\left[2(\partial^\mu W_\mu^+ - i\xi m_W \chi^+)(\partial^\mu W_\mu^- - i\xi m_W \chi^-)\right.$$

$$\left. + (\partial^\mu Z_\mu - i\xi m_Z \chi^0)^2 + (\partial^\mu A_\mu)^2\right]$$

- The propagators in R_ξ gauge

R_ξ gauge

$$\Delta_{VV}^{\mu\nu}(q) = \frac{-i}{q^2 - m_V^2 + i\epsilon}\left[g^{\mu\nu} + (\xi - 1)\frac{q^\mu q^\nu}{q^2 - \xi m_V^2}\right]$$

$$\Delta_{\chi^0\chi^0}(q^2) = \frac{i}{q^2 - \xi m_Z^2 + i\epsilon}$$

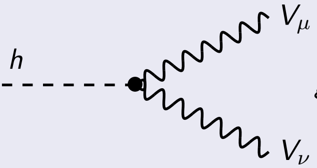
$$\Delta_{\chi^\pm\chi^\mp}(q^2) = \frac{i}{q^2 - \xi m_W^2 + i\epsilon}$$

- $\xi = 0$ is the Landau gauge
- $\xi = 1$ is the 't Hooft-Feynman gauge (the $q^\mu q^\nu$ term is absent
- $\xi \to \infty$ is the Unitary gauge.
- In gauge boson propagators the last term $(-q^\mu q^\nu / m_V^2)$ leads to very complicated cancellations in the invariant amplitudes involving the exchange of V bosons at high energies and, even worse, make the renormalization program very difficult to carry out, as the latter usually makes use of four–momentum power counting analyses of the loop diagrams.
- The Goldstone boson propagators vanish in the unitary gauge
- The Higgs propagator

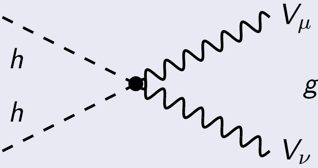
$$\Delta_{hh}(q^2) = \frac{i}{q^2 - m_h^2 + i\epsilon}$$

- The couplings of the Higgs bosons to gauge bosons

Higgs-gauge bosons



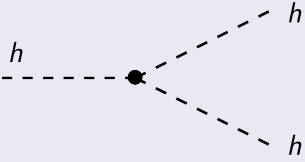
$$g_{hVV} = -i g_{\mu\nu}\, 2m_V^2/v$$



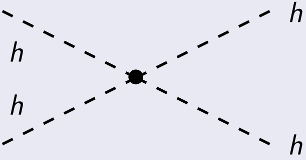
$$g_{hhVV} = -i g_{\mu\nu}\, 2m_V^2/v^2$$

- The self-couplings of the Higgs bosons

Higgs-Higgs bosons



$$g_{hhh} = i\ 3m_h^2/v$$



$$g_{hhhh} = i\ 3m_h^2/v^2$$

Fermion gauge interactions

Using the lagrangian $\mathcal{L}_{fer}$ one obtains the interaction of fermions with gauge bosons eigenfunctions in the broken phase

- The weak isospin currents of $SU(2)$ are

$$J_a^\mu = \sum_{f_L} \bar{f}_L \gamma^\mu \frac{\sigma_a}{2} f_L$$

- The hypercharge current is

$$J_Y^\mu = \sum_{f_L} \bar{f}_L \gamma^\mu Y_{f_L} f_L + \sum_{f_R} \bar{f}_R \gamma^\mu Y_{f_R} f_R$$

- They are coupled to gauge bosons (W, Z, A) as

$$g J_a^\mu W_a^\mu + g' J_Y^\mu B_\mu$$

with the decomposition

$$W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu; \; B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$$

- $W^{\pm}_{\mu}$ couple to the weak charged currents

Charged currents lagrangian

$$\mathcal{L}^{CC}_{int} = \frac{g}{\sqrt{2}}(W^{+}_{\mu} J^{\mu}_{-} + W^{-}_{\mu} J^{\mu})$$

$$J^{\mu}_{\pm} = \frac{1}{2}(J^{\mu}_{1} \pm i J^{\mu}_{2})$$

- The electromagnetic interactions are

Electromagnetic lagrangian

$$\mathcal{L}^{EM}_{int} = e J^{EM}_{\mu} A^{\mu}$$

$$J^{EM}_{\mu} = \sum_{f} [\bar{f}_L \gamma_\mu Q f_L + \bar{f}_R \gamma_\mu Q f_R]$$

$$Q = T_3 + Y; \quad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

- Z_μ couples to neutral current

Neutral current lagrangian

$$\mathcal{L}_{int}^{NC} = \sqrt{g^2 + g'^2} J_\mu^0 Z^\mu$$

$$J_\mu^0 = J_\mu^3 - \sin^2\theta_W J_\mu^{EM}$$

- Notice that the neutral currents

Neutral currents

$$\propto \ \bar{f}_{L,R}\gamma^\mu f_{L,R}$$

and charged currents

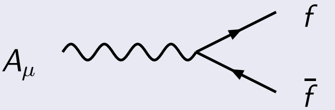
Charged currents

$$\propto \ \bar{u}_{L,R}\gamma^\mu d_{L,R}$$

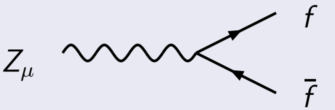
are all flavor-diagonal in the interaction basis.

Diagrammatically the Feynman rules are

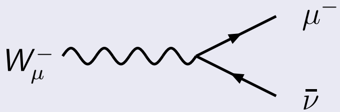
Fermion gauge interactions



$$ieQ_f\gamma_\mu$$



$$\frac{ie}{sc}\gamma_\mu\left[(T_f^3 - Q_f s^2)P_L - Q_f s^2 P_R\right]$$



$$\frac{ie}{s\sqrt{2}}\gamma_\mu P_L$$

Yukawa interactions

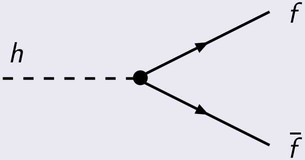
- Fermion masses and mixing appear from the Yukawa interactions

Quarks Yukawa lagrangian

$$\mathcal{L}_Y = -Y_{ij}^U (\bar{u}_L, \bar{d}_L)_i \begin{pmatrix} \bar{H}^0 \\ -\chi^- \end{pmatrix} u_{Rj}$$

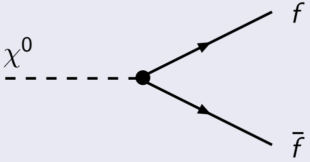
$$-Y_{ij}^D (\bar{u}_L, \bar{d}_L)_i \begin{pmatrix} \chi^+ \\ H^0 \end{pmatrix} d_{Rj} + h.c.$$

Higgs fermion interactions

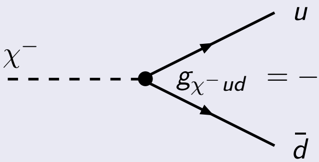


$$g_{Hff} = i\ m_f / v$$

Goldstone bosons fermion interactions



$$g_{\chi^0 ff} = -2T^3_f m_f/v$$



$$g_{\chi^- ud} = -\frac{i}{\sqrt{2}v} V_{ud}[m_d(1-\gamma_5) - m_u(1+\gamma_5)]$$

- After electroweak breaking it gives rise to the mass terms

Mass lagrangian

$$\mathcal{L}_{mass} = -\frac{v}{\sqrt{2}}\bar{u}_L^i Y_{ij}^U u_R^j + h.c.$$

$$-\frac{v}{\sqrt{2}}\bar{d}_L^i Y_{ij}^D d_R^j + h.c.$$

- We can diagonalize the bilinear mass terms by unitary transformations

$$u_{L,R} \to V^u_{L,R} u_{L,R}; \quad d_{L,R} \to V^d_{L,R} d_{L,R}$$

interaction $\longrightarrow$ mass eigenstates basis

- The mass lagrangian becomes

Mass lagrangian

$$\mathcal{L}_{mass} = -\frac{v}{\sqrt{2}}\bar{u}_L V_L^{u\dagger} Y^U V_R^u u_R + h.c.$$

$$-\frac{v}{\sqrt{2}}\bar{d}_L V_L^{d\dagger} Y^D V_R^d d_R + h.c.$$

- With

$$V_L^{u\dagger} Y^U V_R^u \propto diag(m_u, m_c, m_t)$$

$$V_L^{d\dagger} Y^D V_R^d \propto diag(m_d, m_s, m_b)$$

Neutral currents

- Neutral currents which were flavor-diagonal in the interaction basis remains flavor-diagonal in the mass eigenstate basis

Neutral currents in mass eigenstates

$$\bar{f}_{L,R}\gamma^\mu f_{L,R} \to \bar{f}_{L,R} V^{f\dagger}_{L,R}\gamma^\mu V^f_{L,R} f_{L,R} = \bar{f}_{L,R}\gamma^\mu f_{L,R}$$

- This ensures that FCNC will not be generated at tree level

Charged currents: CKM mixing

- Charged currents which were flavor-diagonal in the interaction basis do not remain flavor diagonal in the mass eigenstate basis

Charged currents in mass eigenstates

$$W^+_\mu \bar{u}_L \gamma^\mu d_L \to W^+_\mu \bar{u}_L \gamma^\mu V_L^{u\dagger} V_L^d d_L = W^+_\mu \bar{u}_L \gamma^\mu V_{CKM} d_L$$

$$V_{CKM} = V_L^{u\dagger} V_L^d$$

- V_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix defined as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A standard parametrization for the CKM matrix is

$$V_{CKM} =$$
$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

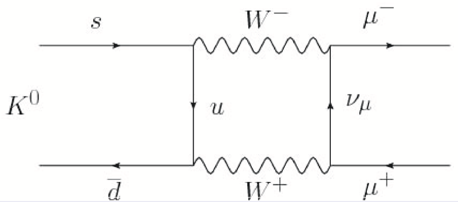
- A good approximation is

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Where $\lambda = s_{12}$, $s_{23} = A\lambda^2$, $s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$
- $\lambda \simeq \sin\theta_C = 0.23$
- The experimental values for the V_{CKM} entries can be found in RPP

- The GIM mechanism explains the smallness of processes as $K_L \to \mu^+\mu^-$ as given by the diagrams in the figure

GIM mechanism



- CKM mixing leads to the three diagrams where the vertical line is (u, c, t).
- In the limit of exact flavor symmetry the three diagrams cancel by virtue of

$$\sum_{i=u,c,t} V_{is} V_{id}^* = 0$$

- Exercise: Estimate the suppression of the previous process

ELECTROWEAK SYMMETRY BREAKING

LECTURE 2: THEORETICAL BOUNDS ON THE HIGGS

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- Unitarity bounds
- Triviality bounds
- Stability bounds
- Metastability bounds
  - Thermal corrections
  - Thermal tunneling
  - Bounds

Unitarity bounds

- The longitudinal components of the W and Z bosons give rise to interesting features
- In the gauge boson rest frame one can define the transverse and longitudinal polarization four–vectors as

$$\epsilon^{\mu}_{T_1} = (0,1,0,0)\,, \quad \epsilon^{\mu}_{T_2} = (0,0,1,0)\,, \quad \epsilon^{\mu}_{L} = (0,0,0,1)$$

- For a four–momentum $p^\mu = (E, 0, 0, |\vec{p}|)$, after a boost along the z direction, the transverse polarizations remain the same while the longitudinal polarization becomes

$$\epsilon^{\mu}_{L} = \left(\frac{|\vec{p}|}{m_V}, 0, 0, \frac{E}{m_V}\right) \overset{E \gg m_V}{\Longrightarrow} \frac{p_\mu}{m_V}$$

- Since this polarization is proportional to the gauge boson momentum, at very high energies, the longitudinal amplitudes will dominate in the scattering of gauge bosons

- In processes involving the W_L and Z_L bosons, this would eventually lead to cross sections which increase with the energy which would then violate unitarity at some stage
- We will briefly discuss this aspect in the following, taking as an example the scattering process $W^+W^- \to W^+W^-$ at high energies, which can violate the unitarity bounds
- We first decompose the scattering amplitude A into partial waves a_ℓ of orbital angular momentum ℓ

$$A = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)\, a_\ell$$

where P_ℓ=Legendre polynomials and θ=scattering angle.
- For a $2 \to 2$ process, the cross section is given by

$$\mathrm{d}\sigma/\mathrm{d}\Omega = |A|^2/(64\pi^2 s), \quad d\Omega = 2\pi d\cos\theta$$

$$\sigma = \frac{16\pi}{s} \sum_{\ell=0}^{\infty} (2\ell+1)|a_\ell|^2$$

- Unitarity implies the

Optical theorem

$$\sigma = \frac{1}{s}\,\mathrm{Im}\,[\,A(\theta=0)\,] \;=\; \frac{16\pi}{s}\sum_{\ell=0}^{\infty}(2\ell+1)|a_\ell|^2$$

- This leads to the

Unitarity condition

$$|a_\ell|^2 = \mathrm{Im}(a_\ell) \Rightarrow [\mathrm{Re}(a_\ell)]^2 + [\mathrm{Im}(a_\ell)]^2 = \mathrm{Im}(a_\ell)$$

$$[\mathrm{Re}(a_\ell)]^2 + [\mathrm{Im}(a_\ell) - \frac{1}{2}]^2 = \frac{1}{4}$$

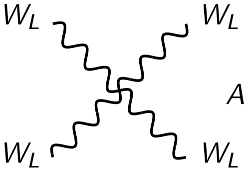
$$\Updownarrow$$

$$|\mathrm{Re}(a_\ell)| < \frac{1}{2}$$

- In particular for the $J=0$ partial wave

$$|\mathrm{Re}(a_0)| < \frac{1}{2}$$

- The unitarity condition is badly violated by the quartic W_L interactions



$$A \propto g^2 \frac{s^2}{M_W^4} \Rightarrow s \leq M_W^2$$

- This problem can be partly cured by adding the other SM gauge interactions

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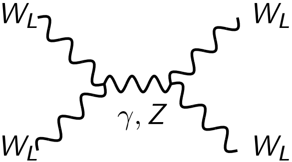
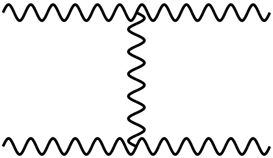
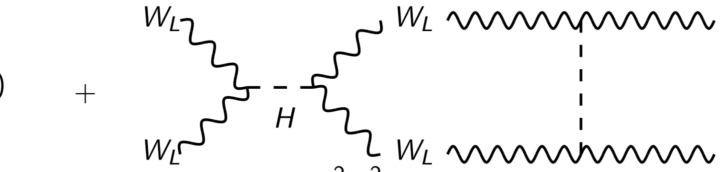




$$a_0 = \frac{g^2 s}{16\pi M_W^2} \Rightarrow \sqrt{s} \leq 1.7\, TeV$$

- The problem is fully solved by introducing the Higgs interactions



$$a_0 = \frac{g^2 m_H^2}{32\pi M_W^2} \Rightarrow m_H \le 870 \;\; GeV$$

- Channel $W_L^+ W_L^-$ considered above can be coupled with other neutral $Z_L Z_L$, HH and $Z_L H$ and charged $W_L^+ H$ and $W_L^+ Z_L$ channels. The scattering amplitude and a_0 is then given by a 6×6 matrix. The requirement that the largest eigenvalues of a_0, respects the unitarity constraint yields

$$M_H \lesssim 710 \text{ GeV}$$

- Goldstone bosons are useful tools to enforce unitarity because of the

Electroweak Equivalence Theorem

At very high energies, the longitudinal massive vector bosons can be replaced by the Goldstone bosons.

$$A(V^1 \cdots V^n \to V^1 \cdots V^{n'}) \sim A(V_L^1 \cdots V_L^n \to V_L^1 \cdots V_L^{n'})$$
$$\sim A(w^1 \cdots w^n \to w^1 \cdots w^{n'})$$

- Thus, in this limit, one can simply replace in the SM scalar potential, the W and Z bosons by their corresponding Goldstone bosons $\chi^{\pm}, \chi_0$, leading to

Higgs-Goldstones interactions

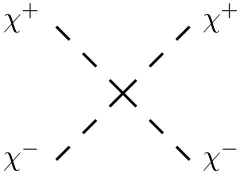
$$V = \frac{m_h^2}{2v}(h^2 + \chi_0^2 + 2\chi^+\chi^-)h + \frac{m_h^2}{8v^2}(h^2 + \chi_0^2 + 2\chi^+\chi^-)^2$$

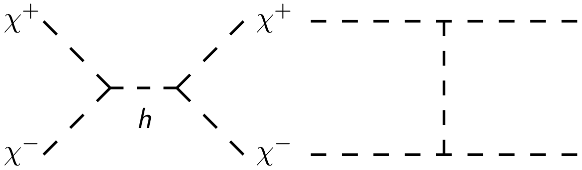
and use this potential to calculate the amplitudes

Exercise: compute a_0 as

$$a_0 = -\frac{M_H^2}{16\pi v^2}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right]$$

for the set of diagrams



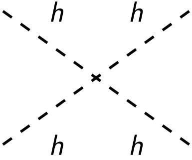


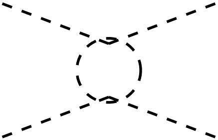
Triviality bound

- The variation of the quartic Higgs coupling with the energy scale Q is described by the Renormalization Group Equation (RGE)

RGE

$$\frac{d\lambda}{d\log Q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda h_t^2 - 3h_t^4 \right.$$
$$\left. -\frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$





- For large values of the Higgs mass (λ) the quartic coupling dominates the RGE and its solution can be written analytically

$$\lambda(Q^2) = \lambda(v^2)\left[1 - \frac{3}{4\pi^2}\,\lambda(v^2)\log\frac{Q^2}{v^2}\right]^{-1}$$

- When the energy is much higher than the weak scale, $Q^2 \gg v^2$, the quartic coupling grows and eventually becomes infinit. This point is called Landau pole
-
$$\Lambda = v\,\exp\left(\frac{4\pi^2}{3\lambda}\right) = v\,\exp\left(\frac{4\pi^2 v^2}{m_h^2}\right)$$
- The general triviality argument states that

Triviality argument

The scalar sector of the SM is a ϕ^4–theory, and for these theories to remain perturbative at all scales one needs to have a coupling $\lambda = 0$ [which in the SM, means that the Higgs boson is massless], thus rendering the theory trivial, i.e. non–interacting

- One can turn around the argument: fixing the value of m_h one can use the RGE for the quartic Higgs self–coupling to establish the energy domain in which the SM is valid, i.e. the energy cut–off Λ below which the self–coupling λ remains finite
- Alternatively, fixing Λ one can determine an upper bound on the Higgs mass for the theory to remain perturbative i.e. for self–coupling λ remains finite

Triviality bound

In the previous approximation

$$m_h^2 < \frac{4\pi^2 v^2}{\log \frac{\Lambda}{v}}$$

- If Λ is large, the Higgs mass should be small to avoid the Landau pole: for $\Lambda \sim 10^{16}$ GeV $\Rightarrow m_h \lesssim 200$ GeV
- If Λ_C is small, the Higgs boson mass can be rather large: for $\Lambda \sim 10^3$ GeV $\Rightarrow m_h \sim 1$ TeV

- In particular, if the cut–off is set at the Higgs boson mass itself, $\Lambda = m_h$, the requirement that the quartic coupling remains finite implies that $m_h \lesssim 700\ \mathrm{GeV}$
- Of course there is a caveat in this argument: when λ is too large, one cannot use perturbation theory anymore and this constraint is lost. However, from simulations of gauge theories on the lattice, where the non–perturbative effects are properly taken into account, it turns out that one obtains the rigorous bound $m_h \lesssim 640$ GeV, which is in a remarkable agreement with the bound obtained by naively using perturbation theory
- Triviality bound is an upper bound: for heavy Higgs masses.
- Next we will study the stability bounds. They are lower bounds: for light Higgs masses. Together they will make an allowed window

Stability bounds

- In the region of light Higgs there is another effect of the RGE for the quartic coupling

RGE

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\log Q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda h_t^2 - 3h_t^4 \right.$$
$$\left. -\frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

- For small values of λ the RGE is dominated by the h_t^4 coupling

$$8\pi^2 \frac{d\lambda}{d\log\Lambda} \simeq -3h_t^4$$

and λ decreases with Λ

$$\lambda(\Lambda) \simeq \lambda(v) - \frac{3}{8\pi^2} h_t^4 \log\frac{\Lambda}{v}$$

- When $\lambda(\Lambda) < 0$ the potential is unbounded from below
- For fixed Λ there is a lower bound on the Higgs mass

$$m_h^2 \geq \frac{3h_t^2 m_t^2}{2\pi^2} \log \frac{\Lambda}{v}$$

- For fixed m_h there is an upper bound on Λ

$$\Lambda \leq v \exp(2\pi^2 m_h^2 / 3h_t^2 m_t^2)$$

- A more precise bound of course requires the numerical solution to the system of couple differential RGE to find out the scale where $\lambda(\Lambda) = 0$
- Going beyond the one-loop result can be achieved by using RGE techniques to resum the effective potential as we will show next

- The SM effective potential can be written in the 't Hooft-Landau gauge and the $\overline{MS}$ renormalization scheme as $V_{\rm eff} = V_0 + V_1$

SM effective potential

$$V_0 = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{8}\lambda(t)\phi^4(t)$$

$$V_1 = \sum_{i=W,Z,t} \frac{n_i}{64\pi^2} M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2(t)} - C_i\right] + \Omega(t)$$

$$C_W = C_Z = \frac{5}{6}, \quad C_t = \frac{3}{2}, n_W = 6, \quad n_Z = 3, \quad n_t = -12,$$

$$M_i^2 = \kappa_i \phi^2(t), \quad \phi(t) = \xi(t)\phi_c$$

$$\xi(t) = \exp\left\{-\int_0^t \gamma(t')dt'\right\}, \quad \mu(t) = m_Z e^t$$

$$\kappa_W = \frac{1}{4}g^2(t), \quad \kappa_Z = \frac{1}{4}[g^2(t) + g'^2(t)], \quad \kappa_t = \frac{1}{2}h^2(t).$$

- The pole masses M_h and M_t

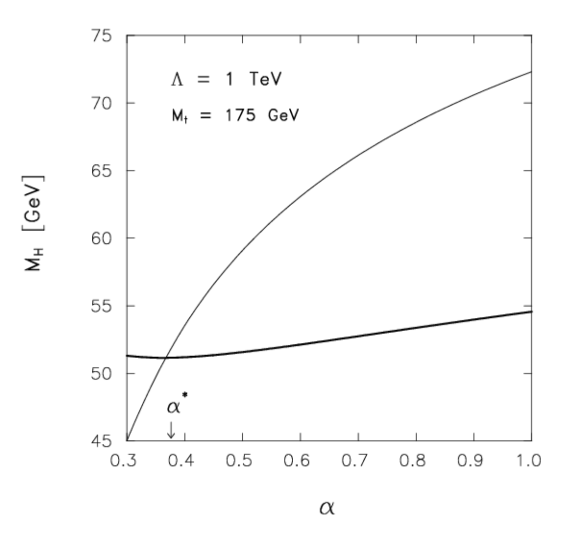
$$M_h^2 = \quad m_h^2[\mu(t)] + \mathrm{Re}\left[\Pi_{HH}(p^2 = M_h^2) - \Pi_{HH}(p^2 = 0)\right],$$

$$M_t = \quad \left[1 + \frac{4}{3}\frac{\alpha_s(M_t)}{\pi}\right] m_t[M_t].$$

- The effective potential improved by RGE is highly scale independent. This allows fixing the renormalization scale as $\mu(t) \sim \phi(t)$ in order to tame potentially dangerous logarithms at large values of the field (where the instability is expected to appear).
- In particular, fixing $\mu(t) = \alpha\phi(t)$, allows to translate the scale-independence of the (whole) effective potential into the α independence
- We can find out the optimum value α^* to study the instability region using the one-loop approximation: that for which the results are more scale-invariant

The scale independence in the appropriate region is shown in the figure

Scale (in)dependence



- We can write the potential as

$$V_{\rm eff} = -\frac{1}{2} m^2(t)\phi^2(t) + \frac{1}{8}\lambda_{\rm eff}\phi^4(t) + \Omega(t)$$

from where

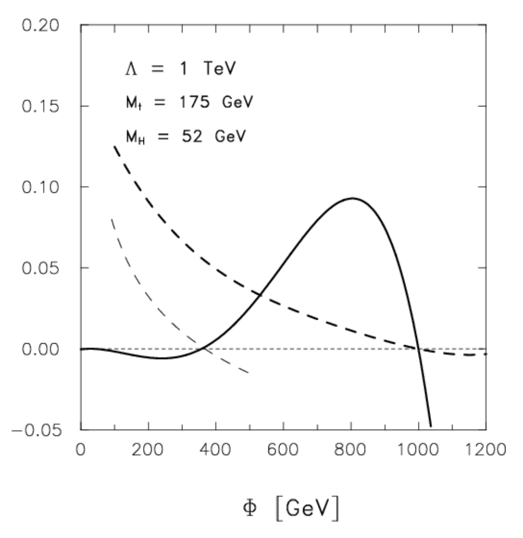
$$\lambda_{\rm eff}(t) = \lambda(t) + \sum_i \frac{n_i}{8\pi^2}\kappa_i^2 \left[\log\frac{\kappa_i}{\alpha^2} - C_i\right].$$

- The value of the scale Λ where new physics has to stabilize the SM potential is given by the value of the field ϕ where the depth of the potential equals the depth of the potential at the standard electroweak minimum
- Due to the steepness of the potential around that point, we can identify Λ with the value of the field where the potential vanishes, i.e.

$$V_{\rm eff}(\phi)|_{\phi=\Lambda} = 0 \quad ,$$

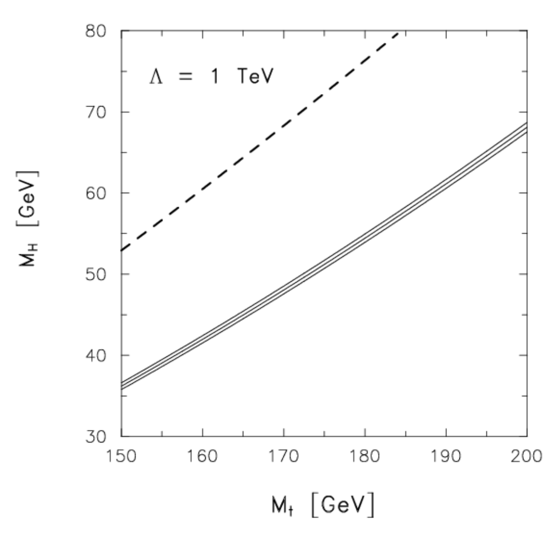
The effective potential is destabilized at a given value of the field

Effective potential



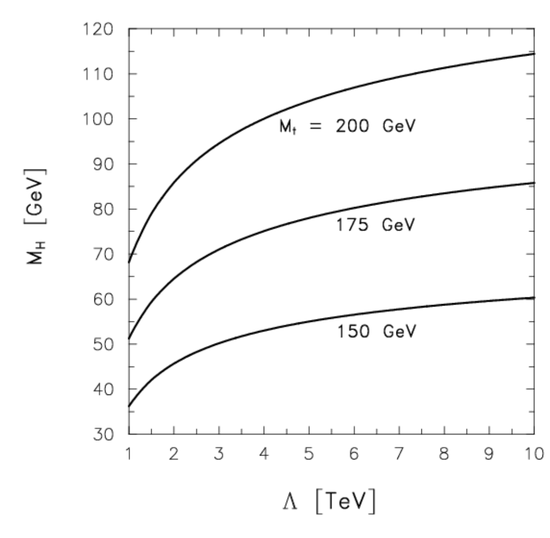
- We have plotted the lower bounds on M_h for $\Lambda = 1$ TeV as functions of M_t.

M_h vs. M_t for $\Lambda = 1$ TeV



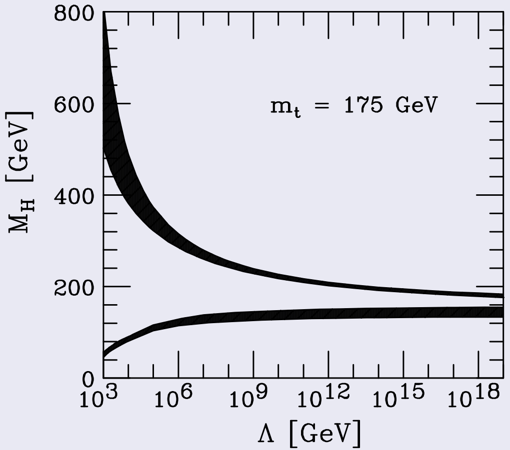
The bound as a function of the cutoff scale

M_h vs. Λ



- The summary of triviality and stability bounds

The Standard Model Window



Metastability bounds

- Even if the lower bounds on M_h arising from stability requirements are a valuable indication, they cannot be considered as absolute lower bounds in the SM since we cannot logically exclude the possibility of the physical electroweak minimum being a metastable one, provided the probability, normalized with respect to the expansion rate of the Universe, for decay to the unphysical (true) minimum, be negligibly small
- In view of the future Higgs search at LHC, it is extremely important that the bounds provided on the Higgs mass in the SM be as accurate as possible
- The main tools for that should be
  - Thermal corrections to the effective potential including plasma effects by one-loop resummation of Debye masses
  - Numerical calculation of the bounce solution and the energy of the critical bubble

Thermal corrections

- The thermal correction to the effective potential can be computed using the rules of field theory at finite temperature. Including plasma effects by one-loop ring resummation of Debye masses
- It can be written as

$$\Delta V_{\text{eff}}(\phi, T) = V_1(\phi, T) + V_{\text{ring}}(\phi, T)$$

- The one-loop thermal correction

One-loop correction

$$V_1(\phi, T) = \frac{T^4}{2\pi^2}\left[\sum_{i=W,Z} n_i J_B\left(\frac{m_i^2(\phi)}{T^2}\right) + n_t J_F\left(\frac{m_t^2(\phi)}{T^2}\right)\right]$$

- The thermal functions are given by

Thermal integrals

$$J_B(y) = \int_0^\infty dx\ x^2 \log\left[1 - e^{-\sqrt{x^2+y^2}}\right]$$

$$J_F(y) = \int_0^\infty dx\ x^2 \log\left[1 + e^{-\sqrt{x^2+y^2}}\right]$$

- Plasma effects in the leading approximation can be accounted for by the one-loop effective potential improved by the daisy diagrams

Hard thermal loops

$$V_{\rm ring}(\phi, T) = \sum_{i=W_L,Z_L,\gamma_L} n_i \left\{ \frac{m_i^3(\phi)T}{12\pi} - \frac{\mathcal{M}_i^3(\phi)T}{12\pi} \right\}$$

- Only the longitudinal degrees of freedom of gauge bosons, $\frac{1}{2}n_{W_L} = n_{Z_L} = n_{\gamma_L} = 1$, are accounted
- The thermal masses are

Debye corrected masses

$$\mathcal{M}^2_{W_L} = m^2_W(\phi) + \frac{11}{6} g^2 T^2$$

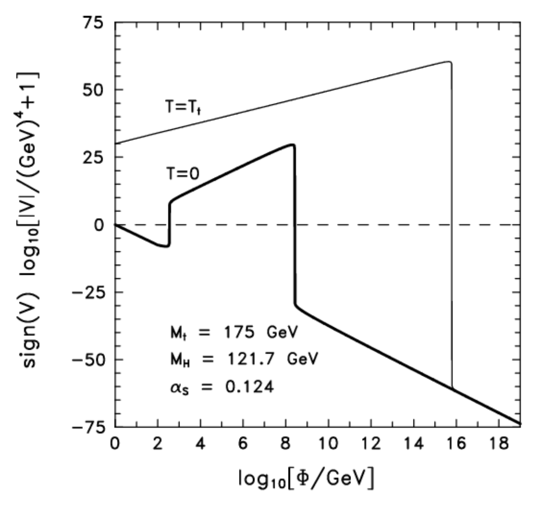
$$\mathcal{M}^2_{Z_L} = \frac{1}{2}\left[m^2_Z(\phi) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W} T^2 + \Delta(\phi, T)\right]$$

$$\mathcal{M}^2_{\gamma_L} = \frac{1}{2}\left[m^2_Z(\phi) - \frac{11}{6}\frac{g^2}{\cos^2\theta_W} T^2 + \Delta(\phi, T)\right]$$

- The discriminant is responsible for the rotation at finite temperature from the basis (W_{3L}, B_L) to the mass eigenstate basis (Z_L, γ_L)

$$\Delta^2 = m^4_Z(\phi) + \frac{11}{3}\frac{\cos^2 2\theta_W}{\cos^2\theta_W}\left[m^2_Z(\phi) + \frac{11}{12}\frac{g^2}{\cos^2\theta_W} T^2\right] T^2$$

Effective potential at $T = T_t = 2.5 \times 10^{15}$ GeV (thin solid line), for $M_t = 175$ GeV and $M_H = 122$ GeV



Thermal tunneling

- In a first-order phase transition the tunnelling probability rate per unit time per unit volume is given by

$$\frac{\Gamma}{\nu} \sim \omega T^4 e^{-E_b/T} ,$$

- E_b (the energy of a bubble of critical size) is given by the three-dimensional euclidean action S_3 evaluated at the *bounce* solution

$$E_b = S_3[\phi_B(r)]$$

- At high temperature the bounce has $O(3)$ symmetry

Euclidean action

$$S_3 = 4\pi \int_0^\infty r^2 dr \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V_{\rm eff}(\phi(r), T) \right]$$

$$r^2 = \vec{x}^2$$

- The bounce ϕ_B satisfies the Euclidean equation of motion and boundary conditions

Bounce equations

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{dV_{\rm eff}(\phi, T)}{d\phi}$$

$$\lim_{r\to\infty} \phi(r) = 0$$

$$\left.\frac{d\phi}{dr}\right|_{r=0} = 0$$

- The semiclassical picture is that unstable bubbles (either expanding or collapsing) are nucleated behind the barrier, at $\phi_B(0)$, with a probability rate given by Γ
- The actual probability P is obtained by multiplying the probability rate by the volume of our current horizon scaled back to the temperature T and by the time the Universe spent at temperature T

- The probability is then

$$\frac{dP}{d\log T} = \kappa \frac{M_{P\ell}}{T} e^{-E_b/T}$$

$$\kappa \sim 3.25 \times 10^{86}$$

- The total integrated probability is defined as

$$P(T_c) = \int_0^{T_c} \frac{dP(T')}{dT'}\, dT',$$

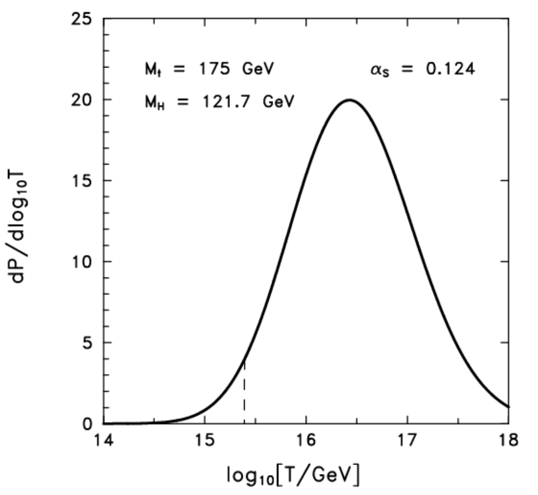
  T_c is the temperature at which the two minima of the effective potential become degenerate. In fact, when $T \to T_c$ the probability rate goes to zero, since $E_b(T) \to \infty$

- The physical meaning of the integrated probability

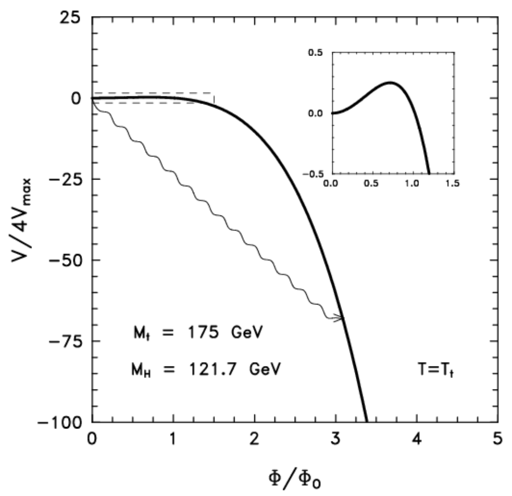
Fraction of space in the old metastable (new stable) phase

$$f_{\rm old} = e^{-P}, \quad f_{\rm new} = 1 - e^{-P}$$

Plot of $dP/d\log_{10} T$. Dashed line indicates temperature $T_t = 2.5 \times 10^{15}$ GeV at which the integrated probability is 1



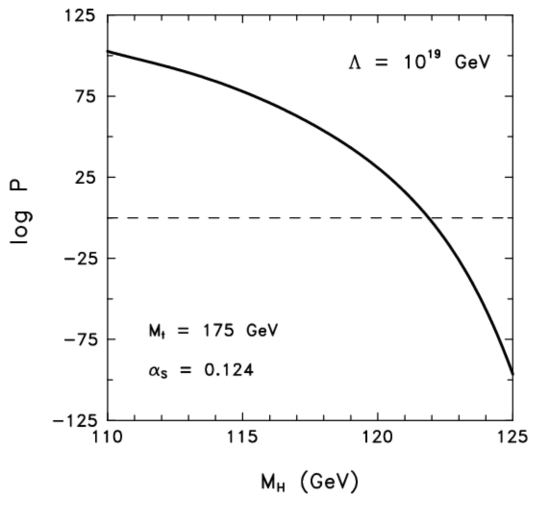
Plot of the effective potential at $T_t = 2.5 \times 10^{15}$ GeV



Bounds

- We have analyzed systematically cases with different values of M_h and different values of the cutoff Λ as e.g.

The case $\Lambda = 10^{19}$ GeV



A fit to the case $\Lambda = 10^{19}$ GeV

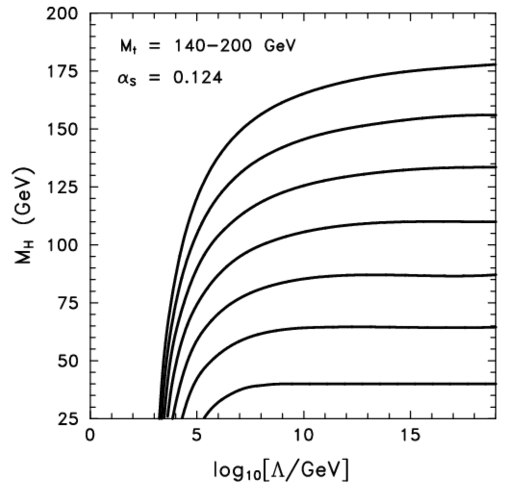
$$M_h/\mathrm{GeV} = [2.278 - 4.654\,(\alpha_S - 0.124)]\,(M_t/\mathrm{GeV}) - 277$$

A general fit

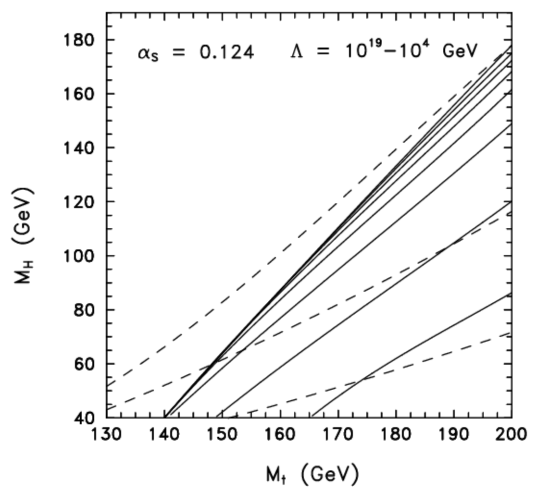
$$M_H/\mathrm{GeV} = A(\Lambda)(M_t/\mathrm{GeV}) - B(\Lambda)$$

$\log_{10}(\Lambda/\mathrm{GeV})$	$A(\Lambda)$	$B(\Lambda)$
4	1.219	157
5	1.533	186
7	1.805	212
9	1.958	230
11	2.071	245
13	2.155	258
15	2.221	268
19	2.278	277

M_h as a function of Λ



M_h for $\Lambda = 10^4$ GeV (lower solid line)–10^{19} GeV (upper solid line). The dashed lines are the absolute stability bounds for $\Lambda = 10^3$ GeV (lower dashed line), 10^4 GeV and 10^{19} GeV (upper dashed line)



ELECTROWEAK SYMMETRY BREAKING

LECTURE 3: EXPERIMENTAL BOUNDS ON THE HIGGS

PARMA INTERNATIONAL SCHOOL OF THEORETICAL PHYSICS, 2009

Mariano Quirós

Institució Catalana de Recerca i Estudis Avançats **(ICREA)**, and **IFAE** Barcelona (Spain)

September 1, 2009

Outline

The outline of this lecture is

Outline

- Standard Model observables
- Oblique corrections
- The ρ parameter
- $STU - \epsilon$ formelism
- $Zb\bar{b}$ coupling
- Indirect constraints
- Direct constraints
- Outlook: Motivation for BSM

Standard Model observables

- Observables are written with a hat on top of them
- Some observables are
  - $\hat{\alpha}$ (from Thomson limit),
  - $\hat{G}_F$ (from muon decay),
  - $\hat{m}_Z$ (Z boson mass),
  - $\hat{m}_W$ (W boson mass),
  - $\hat{\Gamma}_{l^+l^-}$ (leptonic partial width of the Z boson), and
  - $\hat{s}^2_{\rm eff}$ (effective $\sin^2\theta_W$)
- The value of $\hat{s}^2_{\rm eff}$ is defined to be the all-orders rewriting of $\hat{A}_{LR}$ as

$$\hat{A}_{LR} = \frac{\Gamma(Z\to f_L\bar{f}_L)-\Gamma(Z\to f_R\bar{f}_R)}{\Gamma(Z\to f_L\bar{f}_L)+\Gamma(Z\to f_R\bar{f}_R)} = \frac{g_L^2-g_R^2}{g_L^2+g_R^2}$$

$$\equiv \frac{(1/2-\hat{s}^2_{\rm eff})^2-\hat{s}^4_{\rm eff}}{(1/2-\hat{s}^2_{\rm eff})^2+\hat{s}^4_{\rm eff}}$$

- At tree level we need only three lagrangian parameters to compute the six observables listed above. The three parameters are v (Higgs vacuum expectation value) and
  - g ($SU(2)$ gauge coupling)
  - g' ($U(1)_Y$ gauge coupling)
- We trade these two parameters for an equivalent set
  - e (the electric charge): $g = e/s$, $g' = e/c$
  - $s(= \sin\theta_W)$
- The observables can be expressed at tree-level as

Tree-level observables and experimental values

- $\hat{\alpha} = \frac{e^2}{4\pi}$; $\quad \hat{\alpha}^{exp} = 1/137.0359895(61)$
- $\hat{G}_F = \frac{1}{\sqrt{2}v^2}$; $\quad \hat{G}_F^{exp} = 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2}$
- $\hat{m}_Z^2 = \frac{e^2 v^2}{4s^2 c^2}$; $\quad \hat{m}_Z^{exp} = 91.1876 \pm 0.0021$ GeV
- $\hat{m}_W^2 = \frac{e^2 v^2}{4s^2}$; $\quad \hat{m}_W^{exp} = 80.428 \pm 0.039$ GeV
- $\hat{s}_{\rm eff}^2 = s^2$; $\quad (\hat{s}_{\rm eff}^2)^{exp} = 0.23150 \pm 0.00016$
- $\hat{\Gamma}_{l^+l^-} = \frac{v}{96\pi}\frac{e^3}{s^3c^3}\left[\left(-\frac{1}{2} + 2s^2\right)^2 + \frac{1}{4}\right]$;
  $(\hat{\Gamma}_{l^+l^-})^{exp} = 83.984 \pm 0.086\,\mathrm{MeV}$

- The real question that a theory must answer is, *Can we reproduce all experimental results with suitable choices of our input parameters?*
- We have a set of observables $\hat{\mathcal{O}}_i^{\rm expt}$ with uncertainties $\Delta\hat{\mathcal{O}}_i^{\rm expt}$. The theory makes predictions $\mathcal{O}_i^{\rm th}$ for the observables that depend on the lagrangian parameters
- We find the best possible choices of the lagrangian parameters that fit the data by minimizing the χ^2 function

$$\chi^2(e,s,v)=\sum_i \frac{(\hat{\mathcal{O}}_i^{\rm expt}-\mathcal{O}_i^{\rm th}(e,s,v))^2}{(\Delta\hat{\mathcal{O}}_i^{\rm expt})^2}$$

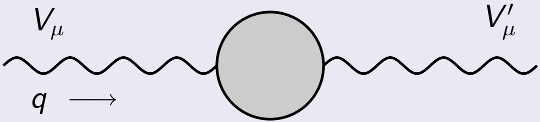
  where i sums over the observables
- The predictions of $\hat{m}_W$, $\hat{s}^2_{\rm eff}$ and $\hat{\Gamma}_{l^+l^-}$ in this particular tree-level procedure are approximately 15σ, 120σ and 10σ off from their experimentally measured values
- Should we conclude that the theory is not compatible with experiment?
- We must go to higher-order in the coupling constants to truly test the viability of the SM

Oblique corrections

- They are corrections that arise only from the self-energy corrections of the γ, $W^{\pm}$, and Z vector bosons.
- A complete analysis will all corrections explicitly computed is much more complicated but it is similar conceptually
- In BSM theories it is most common that the non-oblique corrections have a small effect compared to the oblique corrections. This is generally true in supersymmetry, with the notable exception of the $Z \to b\bar{b}$ coupling
- One main reason for the dominance of oblique corrections over non-oblique corrections is that any charged object couples to the vector bosons, whereas usually only one or two particles in a theory couple to a specific fermion species
- The sum over all contributors in self-energies wins out over the one or two diagrams that couple to an individual final state fermion

- The one-loop corrections to the vector boson self-energies

Oblique corrections



$$i[\Pi_{VV'}(q^2) g^{\mu\nu} - \Delta_{VV'}(q^2) q^\mu q^\nu]$$

- Only the $\Pi_{VV'}$ piece of the self-energies since the q^μ part of the second term is coupled with a light-fermion current and is zero by the Dirac equation

$$q^\mu J_\mu^{\text{light fermion}} \to \bar{f}\gamma^\mu q_\mu f \to \bar{f} m f \to 0.$$

- The way the self-energies are defined, they add to the vector boson masses by convention:

$$m_V^2 \to m_V^2 + \Pi_{VV}(q^2 = m_V^2)$$

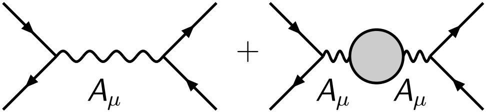
- The correction of Z and W masses is

Z and W masses

$$(\hat{m}_Z^2)^{th} = \frac{e^2 v^2}{4 s^2 c^2} + \Pi_{ZZ}(m_Z^2)$$

$$(\hat{m}_W^2)^{th} = \frac{e^2 v^2}{4 s^2} + \Pi_{WW}(m_W^2)$$

- The theory prediction for $\hat{\alpha}$ comes from

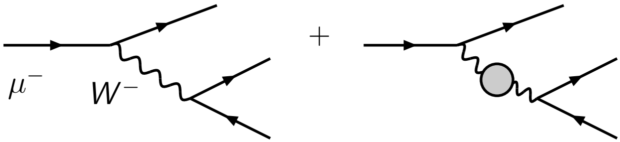


$$-i \left. \frac{4\pi\hat{\alpha}}{q^2} \right|_{q^2 \to 0} = \frac{-ie^2}{q^2} \left[1 + \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right]_{q^2 \to 0}$$

$\hat{\alpha}$

$$(\hat{\alpha})^{th} = \frac{e^2}{4\pi} \left(1 + \Pi'_{\gamma\gamma}(0)\right)$$

- $\hat{G}_F$ is computed from the lifetime of the muon



$\hat{G}_F$

$$\frac{(\hat{G}_F)^{th}}{\sqrt{2}} = \frac{g^2}{8m_W^2}\left[1 + i\Pi_{WW}(q^2)\left(\frac{-i}{q^2 - m_W^2}\right)\right]_{q\to 0}$$

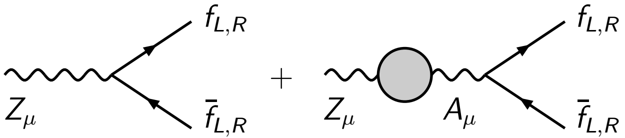
$$= \frac{1}{2v^2}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right]$$

- The definition of $\hat{s}^2_{\rm eff}$ is chosen such that observable $\hat{A}^{\ell}_{LR}$ is written in terms of $\hat{s}^2_{\rm eff}$ using the tree-level expression above with $s^2 \to \hat{s}^2_{\rm eff}$. This is an unambiguous definition

- The observable associated with $\hat{s}^2_{\rm eff}$ requires correcting

$$g_L = \frac{e}{sc}(T^3 - Qs^2) \quad \text{and} \quad g_R = -\frac{-eQs^2}{sc}$$

- We can neglect all Π_{ZZ} contributions since they will only affect the overall factor of g_L and g_R which cancels
- The $Z - A$ mixing self-energy does contribute

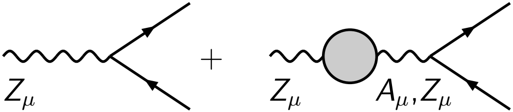


- g_L and g_R expressions are the tree-level expressions except $s^2 \to s^2 - sc\Pi_{\gamma Z}(m_Z^2)/m_Z^2$ in the numerator

$\hat{s}^2_{\rm eff}$

$$(\hat{s}^2_{\rm eff})^2 = s^2 - sc\frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}$$

- Finally for $\hat{\Gamma}_{l^+l^-}$ the relevant diagrams are



$\hat{\Gamma}_{l^+l^-}$

$$(\hat{\Gamma}_{l^+l^-})^{th} = \frac{Z_Z}{48\pi}\frac{e^2}{s^2c^2}\hat{m}_Z\left[\left(-\frac{1}{2}+2(\hat{s}^2_{\rm eff})^{th}\right)^2+\frac{1}{4}\right]$$

$$Z_Z = 1 + \Pi'_{ZZ}(\hat{m}_Z) + \text{higher order terms}$$

- $\Pi_{\gamma Z}$ had the effect of just putting $s^2 \to (\hat{s}^2_{\rm eff})^{th}$ into the numerator
- The parameter Z_Z is a wavefunction residue piece

The ρ parameter

- The relative strength of the charged and neutral currents, $J_Z^\mu J_{\mu Z}/J^{\mu+}J_\mu^-$ can be measured by

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2}$$

- It is equal to 1 in the SM. A direct consequence of the choice of the representation of the Higgs field responsible of the breaking of the electroweak symmetry
- In a model which makes use of an arbitrary number of Higgs multiplets Φ_i with isospin T_i,

$$\rho = \frac{\sum_i \left[T_i(T_i+1) - (T_i^3)^2\right] v_i^2}{2\sum_i (T_i^3)^2 v_i^2}$$

  which is also unity for an arbitrary number of doublet [as well as singlet] fields.
- This is due to the fact that in this case, the model has a custodial SU(2) global symmetry.

- The SM lagrangian has a global $SU(2)$ symmetry in the limit $g' \to 0$ and equal fermion masses of the same doublet
- This symmetry appears as follows: the field H has 4 real components and in the Higgs lagrangian there is an associated $O(4)$ symmetry broken to $O(3) \simeq SU(2)$ at the electroweak breaking
- In the SM, the custodial symmetry is broken at the loop level when fermions of the same doublets have different masses and by the hypercharge group.
- One can define an effective mixing angle and its relation with the ρ parameter as

$$\bar{s}_W^2 = 1 - \frac{M_W^2}{M_Z^2} + c_W^2 \left(\frac{\Pi_{WW}(M_W^2)}{M_W^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} \right)$$

$$\sim 1 - \frac{M_W^2}{M_Z^2} + c_W^2 \Delta\rho$$

- Because m_t is large, the contributions are approximately the same at the scale $q^2 \sim 0$ or $q^2 \sim M_V^2$; in addition the light fermion contributions to Π_{WW} and Π_{ZZ} almost cancel in the difference, $\sim \log M_W/M_Z$
- One usually writes the correction to the ρ parameter as

ρ parameter

$$\rho = \frac{1}{1-\Delta\rho} \quad , \quad \Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

- The large mass splitting between the top and bottom quark masses breaks the custodial $\mathrm{SU}(2)$ symmetry and generates a contribution which grows as the top mass squared

One-loop top quark contribution to the ρ parameter

$$\Delta\rho = \frac{3 G_\mu m_t^2}{8\sqrt{2}\pi^2} \sim 0.01$$

- Exercise: compute $\Pi_{VV}(q^2)$ from fermion loops

- At the one–loop level the Higgs boson contributes

One-loop Higgs contribution to the ρ parameter

$$(\Delta\rho)^{\rm Higgs} = -\frac{3G_\mu M_W^2}{8\sqrt{2}\pi^2} f\left(\frac{M_H^2}{M_Z^2}\right)$$

$$f(x) = x\left[\frac{\ln c_W^2 - \ln x}{c_W^2 - x} + \frac{\ln x}{c_W^2(1-x)}\right]$$

- The contribution vanishes in the limit $s_W^2 \to 0$ or $M_W \to M_Z$, i.e. when $g' \to 0$
- For a very light Higgs boson the correction vanishes

$$(\Delta\rho)^{\rm Higgs} \to 0 \quad \text{for } M_H \ll M_W$$

- For a heavy Higgs boson

$$(\Delta\rho)^{\rm Higgs} \sim -\frac{3G_\mu M_W^2}{8\sqrt{2}\pi^2}\frac{s_W^2}{c_W^2}\log\frac{M_H^2}{M_W^2}$$

- The logarithmic dependence is the "Veltman screening theorem"

STU-ϵ formalism

- It is convenient to parametrize the radiative corrections to electroweak observables in such a way that the contributions due to many kinds of New Physics beyond the SM are easily implemented and confronted with the experimental data
- If one assumes that the symmetry group of New Physics is still $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$ and that it couples only weakly to light fermions so that one can neglect all the "direct" vertex and box corrections, one needs to consider only the oblique corrections, that is, the ones affecting the γ, Z, W two–point functions and the $Z\gamma$ mixing
- If the scale of the New Physics is much higher than M_Z, one can expand the complicated functions of the momentum transfer Q^2 around zero, and keep only the constant and the linear Q^2/M^2_{NP} terms of the series which have very simple expressions in general

- The New Physics contributions can be then expressed in terms of six functions

Functions parametrizing New Physics

$$\Pi'_{\gamma\gamma}(0),\ \Pi'_{Z\gamma}(0),\ \Pi_{ZZ}(0),\ \Pi'_{ZZ}(0),\ \Pi_{WW}(0),\ \Pi'_{WW}(0)$$

QED Ward identities $\Rightarrow \Pi_{\gamma\gamma}(0) = \Pi_{Z\gamma}(0) = 0$

$$\mathcal{L}_{new} = -\frac{\Pi'_{\gamma\gamma}(0)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\Pi'_{WW}(0)}{2} W_{\mu\nu} W^{\mu\nu} - \frac{\Pi'_{ZZ}(0)}{4} Z_{\mu\nu} Z^{\mu\nu}$$

$$-\frac{\Pi'_{\gamma Z}(0)}{2} F_{\mu\nu} Z^{\mu\nu} - \Pi_{WW}(0) W^+_\mu W^{\mu -} - \frac{\Pi_{ZZ}(0)}{2} Z_\mu Z^\mu$$

- Three of these functions will be absorbed in the renormalization of the three input parameters α, G_μ and M_Z

- This leaves three variables which one can choose as being ultraviolet finite and related to physical observables
- A popular choice of the three independent variables is the STU linear combinations of self–energies introduced by Peskin and Takeuchi

STU parameters

$$\alpha S =$$
$$4 s_W^2 c_W^2 \left[\Pi_{ZZ}(0) - (c_W^2 - s_W^2)/(s_W c_W) \cdot \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right]$$
$$\alpha T = \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2$$
$$\alpha U =$$
$$4 s_W^2 \left[\Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2 s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0)\right]$$

- The variable αT is simply the shift of the ρ parameter due to the New Physics, $\alpha T = 1 - \rho - \Delta\rho|_{\rm SM}$

- Another parametrization of the radiative corrections, the ϵ approach of Altarelli and Barbieri is more directly related to the precision electroweak observables
- The three variables which parametrize the oblique corrections are defined in such a way that they are zero in the approximation where only SM effects at the tree–level, as well as the pure QED and QCD corrections, are taken into account
- Defining Δr_W and Δk as

$$M_W^2/M_Z^2\left(1-M_W^2/M_Z^2\right)=s_0^2c_0^2(1-\Delta r_W)$$

$$\sin^2\theta_{\rm eff}^{\rm lep}=(1+\Delta k)s_0^2$$

with

$$s_0^2c_0^2=\pi\alpha(M_Z)/(\sqrt{2}G_\mu M_Z^2)$$

- The variables defined by Altarelli and Barbieri are

ϵ parameters

$$\epsilon_1 = \Delta\rho$$
$$\epsilon_2 = c_0^2\Delta\rho + \frac{s_0^2}{c_0^2 - s_0^2}\Delta r_W - 2s_0^2\Delta k$$
$$\epsilon_3 = c_0^2\Delta\rho + (c_0^2 - s_0^2)\Delta k, \quad \epsilon_4 = \Delta_b$$

Experimental values of ϵ parameters

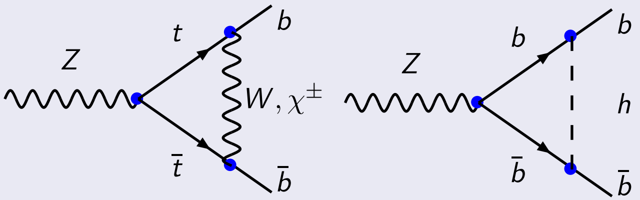
$$\epsilon_1 = -0.0009 \pm 0.0008(-0.0006)$$
$$\epsilon_2 = -0.0006 \pm 0.0009(+0.0007)$$
$$\epsilon_3 = -0.0013 \pm 0.0009(-0.0001)$$
$$M_h = 117\ (300)\ GeV$$

- Δ_b is non-oblique correction to $Z \to b\bar{b}$

$Z \to b\bar{b}$ coupling

- In the context of precision tests, the Z boson decays into bottom quarks has a special status
  1. Because of its large mass and relatively large lifetime the b quark can be tagged and experimentally separated from light quark and gluon jets allowing an independent measurement of the $Z \to b\bar{b}$ partial decay width
  2. Large radiative corrections involving the top quark and not contained in $\Delta\rho$ appear

$Z \to b\bar{b}$ one-loop diagram



- These corrections can be accounted for by shifting the reduced vector and axial–vector $Zb\bar{b}$ couplings by the amount

$$\hat{a}_b \to 2T_b^3(1+\Delta_b) \quad , \quad \hat{v}_b \to 2T_b^3(1+\Delta_b) - 4Q_b s_W^2$$

- For a heavy top quark, the correction can be cast into a rather simple form

$$\Delta_b = -\frac{G_\mu m_t^2}{4\sqrt{2}\pi^2} - \frac{G_\mu M_Z^2}{12\sqrt{2}\pi^2}(1+c_W^2)\log\frac{m_t^2}{M_W^2} + \cdots$$

  This correction is large being approximately of the same size as the $\Delta\rho$ correction
- The Higgs contribution

$$\Delta_b^{1-\mathrm{Higgs}} \propto \frac{G_\mu m_b^2}{4\sqrt{2}\pi^2}$$

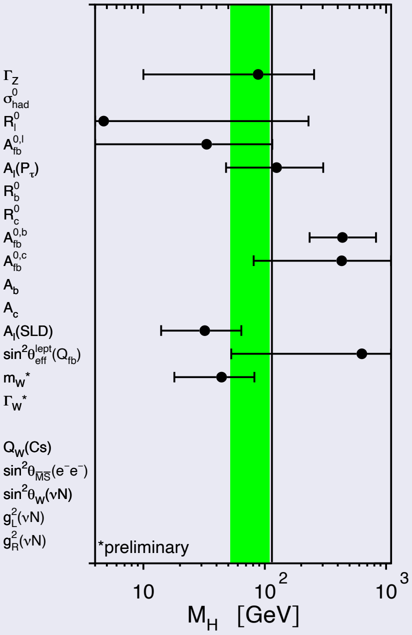
  Because the b–quark mass is very small compared to the W boson mass, $m_b^2/M_W^2 \sim 1/250$, this contribution is negligible in the SM

Indirect constraints on the Higgs mass

$\alpha(M_Z), G_\mu$ and M_Z can be used as basic input parameters. Then the other observables can be predicted as a function of the Higgs mass

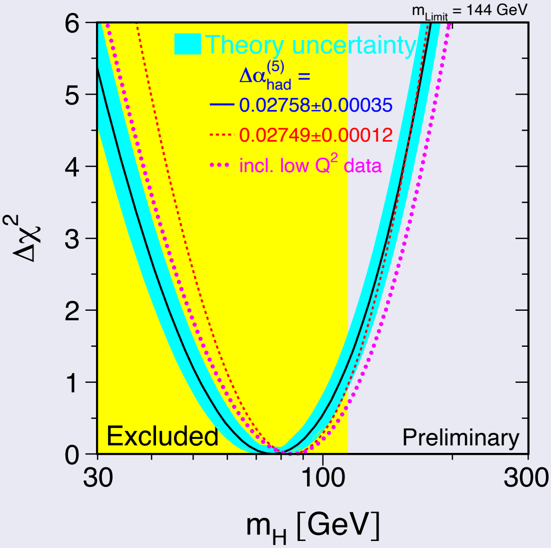
- Observables from the Z lineshape at LEP1: Γ_Z, the peak hadronic cross section $\sigma^0_{\rm had}$, $\Gamma(Z \to \ell, c, b)$ normalized to the hadronic Z decay width, $R_{\ell,c,b}$, A^f_{FB} for leptons and heavy c, b quarks, $A^\tau_{\rm pol}$;
- A^f_{LR} which has been measured at the SLC as well as the left–right forward–backward asymmetries $A^{b,c}_{LR,FB}$
- m_W and Γ_W precisely measured at LEP2
- High–precision measurements at low energies
  - The ν_μ– and $\bar{\nu}_\mu$–nucleon deep–inelastic scattering cross sections
  - The parity violation in the Cesium and Thallium atoms which provide the weak charge Q_W that quantifies the coupling of the nucleus to the Z boson

Electroweak observables



Indirect search limit

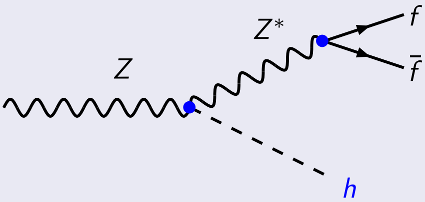
$$m_H < 144 \ GeV \ 95\% C.L.$$



Direct constraints on the Higgs mass

- The Higgs boson has been searched for at the LEP1 experiment at $\sqrt{s} \simeq M_Z$. The dominant production mode is the Bjorken process where the Z boson decays into a real Higgs boson and an off–shell Z boson which goes into two light fermions

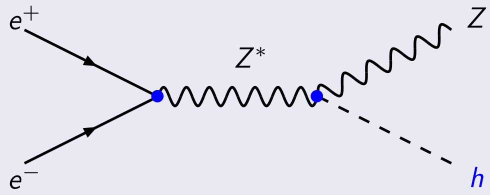
Main production mechanism for Higgs bosons at LEP1



- The Higgs boson can also be produced in the decay $Z \to H\gamma$ which occurs through triangular loops built–up by heavy fermions and the W boson

- The search for Higgs bosons has been extended at LEP2 $\sqrt{s} = 209$ GeV. The dominant production process is Higgs–strahlung where the e^+e^- pair goes into an off–shell Z boson which then splits into a Higgs particle and a real Z boson

Main production mechanism for Higgs bosons at LEP2

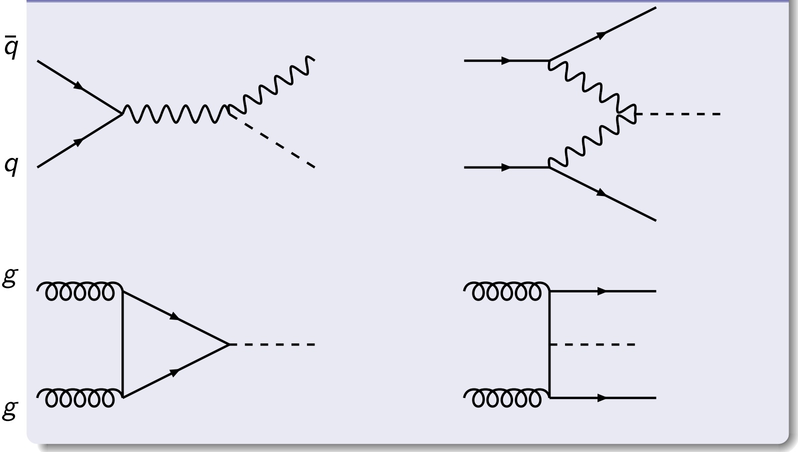


- Combining the results of the four LEP collaborations the exclusion limit

$$M_h > 114.4 \text{ GeV}$$

has been established at the 95% CL
- There is a 1.7σ excess (not significant) of events for a Higgs boson mass in the vicinity of $M_H = 116$ GeV.

Higgs production at hadron colliders



CDF and D0 have recently reported an exclusion region

$$160 \text{ GeV} < M_h < 170 \text{ GeV}$$

at 95 % CL, from $h \to WW$

Outlook: Beyond the Standard Model

Standard Model Drawbacks

- Big Hierarchy problem: The Higgs mass is sensitive to UV physics. Quantum corrections are quadratically sensitive to the cutoff Λ

$$\Delta m_H^2(F,B) = \mp \frac{n_{F,B} g_{F,B}^2}{16\pi^2}\Lambda^2$$

They are not protected by any symmetry which is enhanced when $m_H = 0$

- On the contrary fermions masses $\Delta m_F \propto \frac{m_F}{16\pi^2}\log\Lambda$ are protected by chiral symmetry for $m_F = 0$
- Electroweak symmetry breaking requires a tachyonic mass for the Higgs
- Dark Matter: there is no candidate
- There is no gauge coupling unification
- Strong CP-problem: axion required

The Little Hierarchy Problem/LEP paradox

- The leading quantum correction to the Higgs mass parameter is expected to come from the top sector as

$$\Delta m_H^2 = -\frac{3h_t^2}{8\pi^2}\Lambda^2$$

- In the absence of tuning this implies a lower bound on the cutoff scale as

$$\Lambda < 600\, GeV \left(\frac{m_H}{200\, GeV}\right)$$

- Why did LEP not detect any deviation from the SM predictions? (LEP paradox)
- In particular one can parametrize the new effects as non-renormalizable operators ($d = 6$)

$$\mathcal{L}_{eff} = \frac{c_1}{\Lambda^2}\left(\bar{e}\gamma^\mu e\right)^2 + \dots$$

- If $c_i = \mathcal{O}(1) \Rightarrow \Lambda > 10$ TeV $\Rightarrow$ tension

Possible solutions to the Higgs hierarchy problems are motivating the presence of New Physics

Hierarchy Problem ⇒ New Physics

- Supersymmetry: bosonic (fermionic) partners cancel the quadratic divergences produced by fermions (bosons) [Carlos Wagner's lectures]
- Higgs condensate that "dissolves" at high energies ⇒ strongly interacting gauge sector at TeV scales: technnicolor, top-quark condensate,... [Adam Martin's lectures], holographic Higgs [Christophe Grojean's lectures]
- Higgs as pseudoGoldstone boson: little Higgs theories and gauge-Higgs unification in higher dimensions [Christophe Grojean's lectures]
- Higgsless theories: EWSB by boundary conditions in extra dimensions [Christophe Grojean's lectures]