

Modelling biological systems: a computational challenge

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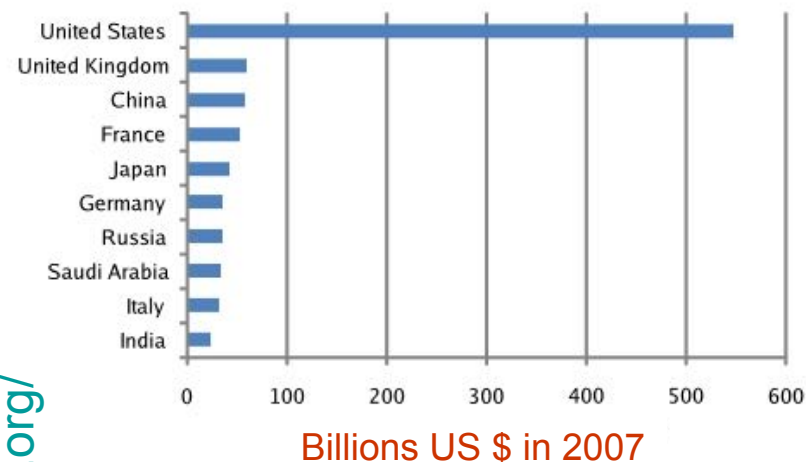
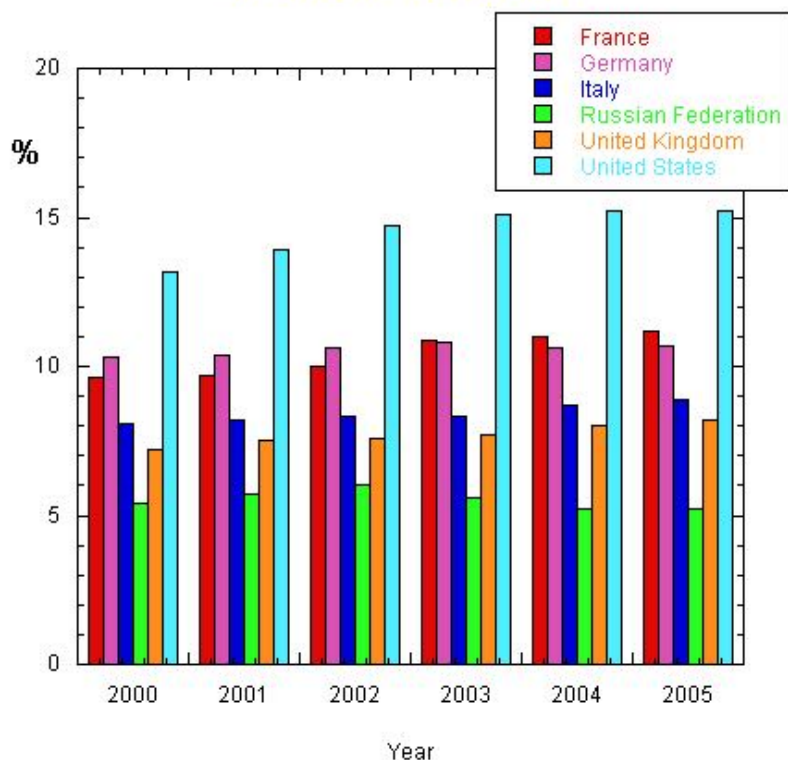
Acknowledgments --- Apologies

- I wish to thank Enrico for the opportunity he gave me to present this material
 - and all the people of the Biophysics group of ToV (especially Silvia) for ∞ -ly many discussions which are at the origin of these lectures
-
- Choice of arguments was made on the basis on my tastes, preferences and incompetence
 - The amount of underlying biological knowledge behind most of the arguments I will touch is essentially unlimited and well beyond my competence
 - Thus, I will try to convey you rather than a fully detailed biological information, some general description of certain broad classes of systems and problems on which one can probably say something interesting and useful
 - I hope you'll find some of these problems intellectually appealing and exciting, not less than High Energy Physics (HEP) or Astrophysics, if not for their dramatic impact on our everyday life

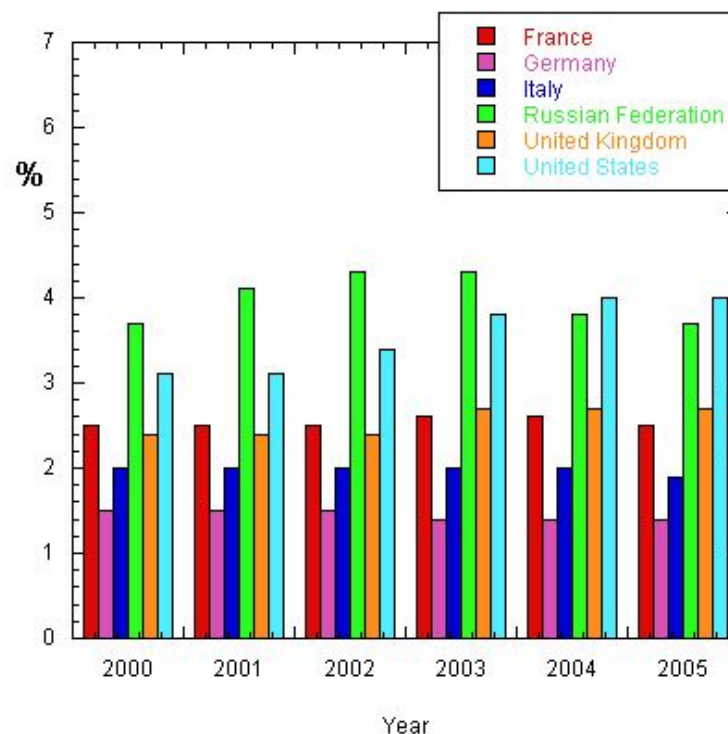
Outline?

The field of health care and biomedical sciences is where the **action** is (in developed countries)

Total expenditure on health as percentage of gross domestic product



Military expenses as percentage of gross domestic product



<http://milexdata.sipri.org/>

<http://www.who.int/en/>

Outline

- I. Reductionism vs complexity
- II. Data, (physical) models and (mathematical) tools
- III. What we would like to know and/or to do
- IV. What we can actually do and/or are really doing
- V. Conclusions and outlook

From concepts to action

I. Reductionism vs complexity

- A bit of philosophy
- A bit of phenomenology

Biology vs Physics

(the viewpoint of a theoretical physicist)

✚ Compare and contrast the situation in the study of Biological systems

- “Complex” structures governed by (as yet) unknown macro-laws
- Powerful and cheap experimental techniques
- Huge amount of data
- Inadequate models: poor understanding of “micro” to “macro” transition

✚ and, at the other extreme, of Elementary Particle Physics

- Supposedly “simple” systems governed by “elegant” known micro-laws
- Very complicated and expensive experiments
- Very few new experimental data (LHC is coming!)
- Rather good models (almost “theories”)

Physics (until very recently) has always found its way by progressively moving towards more and more elementary structures

matter → atoms → nucleons → quarks → ???

guided by the “**radical reductionism**” paradigm according to which

FUNDAMENTAL LAWS GOVERN ELEMENTARY OBJECTS

This attitude has been very fruitful in the “paradigmatic” case of **HEP**, but it is not obviously being employed in other emerging fields of investigation

- **Dynamical** systems

{	Weather forecasting
	Catalytic reactions
	Fluidodynamics (turbulence)

key words: non-linearity, chaos

- **Disordered** systems Glasses, Spin glasses

key-words: frustration, disorder

- **Biological** systems

key-words: complexity, and perhaps all of the above

1 - There are implications for the notion of **modelling** and the nature of **physical laws**

- Even in **Fundamental Physics** what we usually call

Relativity	}	Theories
Field		
String		

are actually **Models**, formulated in the language of **Mathematics**, from which they borrow the necessary internal **logical consistency**

- Complications of everyday life (like friction in Mechanics) are considered (conceptually) irrelevant (up to a certain point - airplanes, cars,...!)
- **Theories** become progressively simpler in the process of understanding
- For **Biosystems**, **Models** (nobody would call them theories) tend to become more and more complicated, as they develop (not simpler!), with a **limit**: the model shouldn't become **as** complicated **as** the system itself!
- The key questions about modelling in **Biology** are then
 - ⇒ When do we decide that we have “**understood**”?
protein folding
functional behaviour of the cell
 - ⇒ What kind of **knowledge/predictions** will we be happy with?

2 - There are implications for the notions of **experiment** and **reproducibility**

- The **Central Dogma** of Physics

Theories (models) are validated through reproducible experiments

- In many biological instances the situation is somewhat more complicated. For instance, to put it in a provocative way

“The experiment of testing **in vivo** the effectiveness of a drug (working **in vitro**), would certainly not be considered a failure if, say, only **30%** of ill people recover”

- Can we somehow understand this situation?

1. Biological experiments may not give reproducible results because not all the **relevant** dof's are/can be kept under control \Rightarrow **# dof's $\gg 1$**
2. On the other hand, in most cases (but, see later) it is not of any interest to be able to predict the properties of the final state of a biological system, or process, in its finest details \Rightarrow **disorder & redundancy**
3. Models are very crude (when they exist at all) and most often overwhelming complicated \Rightarrow **need for some intrinsically new concept?**

The systems of interest

- Elementary is an **object** characterized by a small # of properties
- All **elementary objects** of a given kind are alike (**electrons**)
- Simple physical laws (**theories**) apply to **elementary** objects
- Strict **determinism** and **experimental reproducibility** follow

- **Complex systems** have many dof's and many functionally relevant components

- One should talk of **classes** of systems, e.g.
 - the class of nervous cells, the class of liver cells
 - or, more generally, the class of nucleated cells

Classes are defined by identifying the common properties of the constituent systems

- **Models** yield a mathematical description of common features of systems belonging to a given **class** in terms of **probability distribution functions (PDF)**

- **Class averages** are computed and compared to results coming from **averages** over sets of **experiments**

3 - There are implications for the **amount** and the **nature** of the **possible information output**

Key point

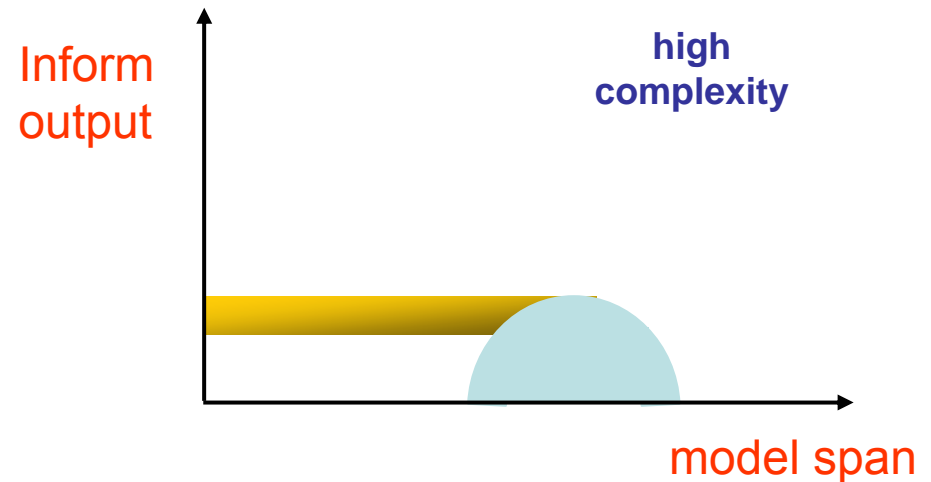
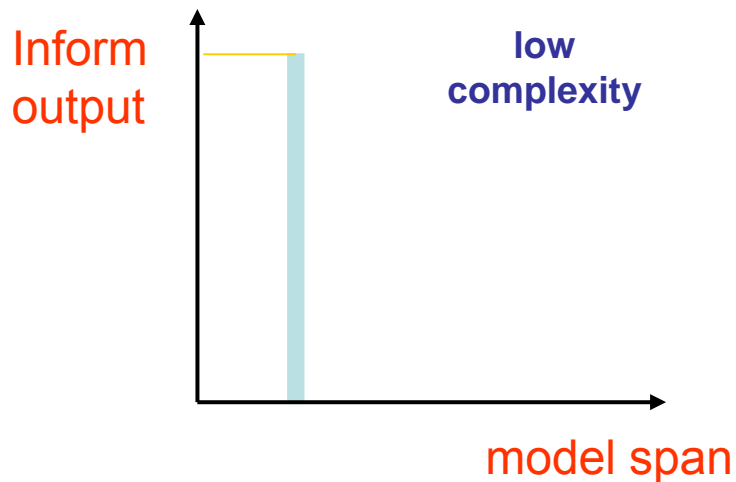
is the accuracy by which a class of homogeneous objects can be defined

The **more accurate** (**looser**) the definition of the objects belonging to a certain class

the **simpler** (**more complicated**) the model

the **sharper** (**more involved**) its mathematical description

the **more precise** (**fuzzier**) the information output



Key questions at this point are

Q1: What is complexity?

A1: Its meaning is context dependent

Q2: Are biosystems complex objects?

A2: Looks like they are

1. Algorithmic Complexity of Kolmogorov and Chaitin

- Definition:

Given a string S of N symbols \longrightarrow $AC = \#$ of bits of a T.M. code that can produce S as an output

- Such a definition does not look interesting for us

$$AC(\text{random string}) \gg AC(\pi)$$

$$\begin{cases} AC(\text{random string}) \sim N \\ AC(\pi) \sim \log N \end{cases} \quad [\text{actually the digits of } \pi \text{ are totally random}]$$

2. Logical depth of Bennett

- Definition:

Given a string S of N symbols \longrightarrow $LD = \text{time } (\# \text{ of operation}) \text{ for a T.M. to run the shortest code that can produce } S \text{ as an output}$

- A somewhat more interesting definition

$$\begin{cases} LD(\text{random string}) \propto \text{time to read } S \sim N \\ LD(\pi) \propto \text{time to generate } \pi \sim N \end{cases}$$

Biological Complexity

- is not **randomness**

Box of molecules
with random velocities

- is not **entropy**

Box of molecules
with all parallel velocities

$S = \text{large}$



$S = 0$

- is not **logical depth**

Life emerged from a very short
(random) program, but it took 10^9 y
to run the code: very high **LD**!
What about running the code today?

Then what is it?

Necessary conditions

- many variables
- many relevant dof's

Here a bit of “phenomenology” starts

**N
u
m
e
r
o
s
i
t
y**

	# of elementary constituents (atoms)
ATOM	1
AMINO ACID	10
PROTEIN	10^3 - 10^5
CELL	10^{10}
.	
.	
.	
HUMAN BODY	5×10^{28} (nucleons)

- Proteins

$10^2 - 10^3$ amino acids
 $10^3 - 10^5$ atoms \longrightarrow (only $\sim 10^7$ expressed)
 20^{300} different possible sequences!

- Immune system

10^6 actual repertoire of Ab's
 10^7 available repertoire
 10^8 lymphocytes

- Brain

10^{10} neurons
 $\times 10^3 \sim 10^4$
 $10^{13} - 10^{14}$ synapses

- Genoma

3×10^9 bases (human DNA)
 4^n with $n = 3 \times 10^9$ possible genomes
 (only 10^{60} expressed @ 1 mut/sec) Eigen

2-3 nm helix x 2 m long
 2x23 chromosomes $V \sim (1.5 \mu\text{m})^3$

It is not so much the number of “elementary” objects
 that is important (gas), but rather the existence of a large
 number of “functionally” relevant distinct components

- There is a lot of disorder in Biosystems

They have ($\sim \infty$ -ly) many randomly distributed microscopic variables and few (still very many!) mesoscopic variables controlling the system

Not every detail can be encoded in DNA,
nor every Genoma has been tried

No optimal evolution

- There is a lot of redundancy in Biosystems

They can exist in very many “equilibrium/metastable” states

{ Individuals
Organs
Immune system states
Proteins

Microscopically different organs (harts, brains,...)
equally well accomplish their task

High degeneracy

Complexity:

here is a sort of “phenomenological” definition

The more one can say about a class of systems,
the more the systems of that class are complex

Complexity is complexity of classification

1. Sequences of random numbers

Not much can be said

all instances belong to the same class



It is a very simple class of systems

2. Equilibrium states of a system of spins at $H = 0$, $T \sim 0$

Only two states: spin up, spin down



It is a simple system

3. Class of sequences of symbols giving rise to “books”

Many things can be said

Language	⇒	English, Italian, German, ...
Style	⇒	Poem, Tragedy, ...
Plot	⇒	Love story, Detective story, ...
...	⇒	...

Many “description levels” or tasks	⇒	Various possible “types of classification”
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It is a complex class of systems

4. Set of painters

We could learn a lot, if we could establish

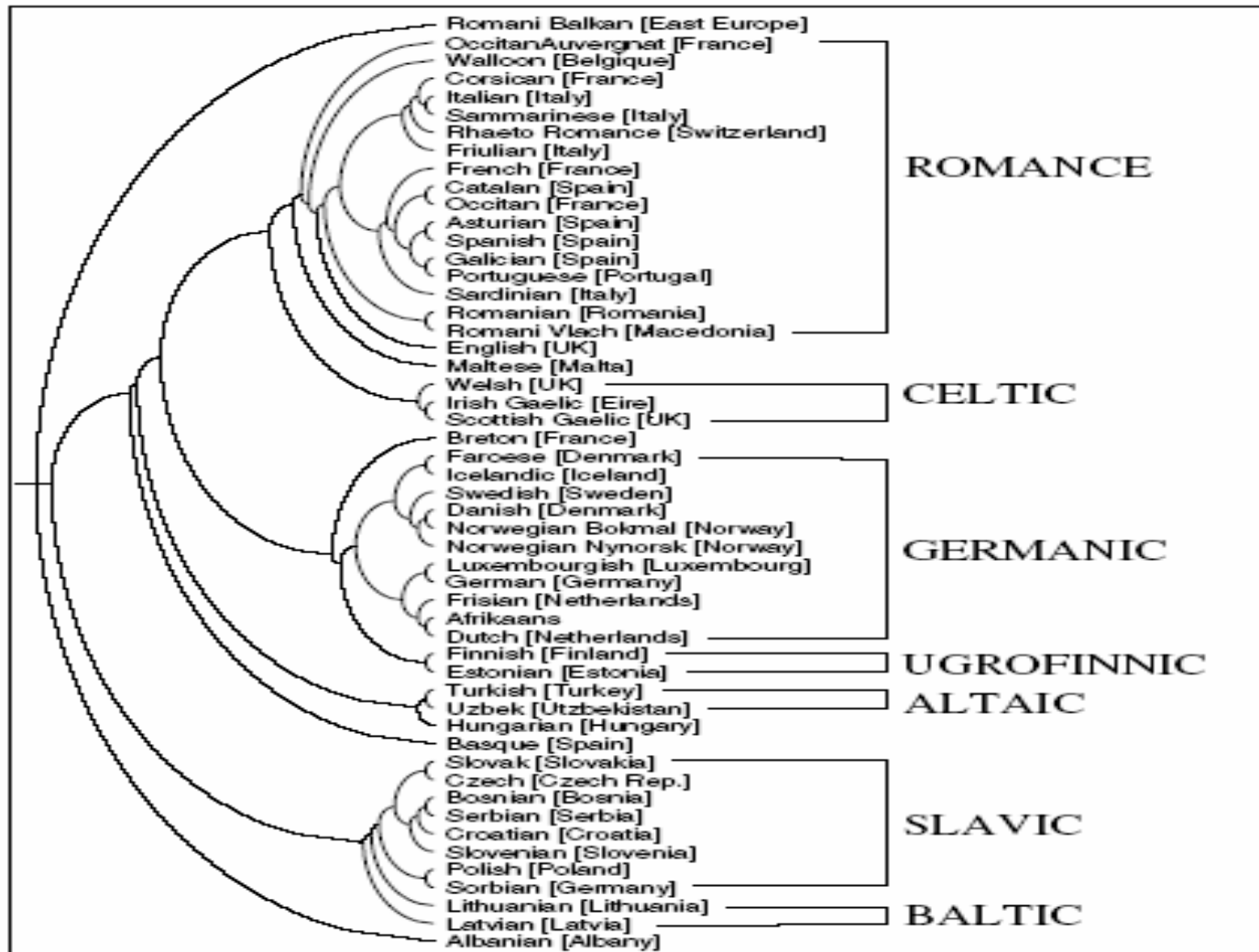
When they were active	⇒	Date of birth
Where they were active	⇒	Place of birth
Their style	⇒	Relative influence
...	⇒	...

Many “description levels” or tasks	⇒	Various possible “types of classification”
---------------------------------------	---	---



It is a complex class of systems

5. The class of human languages is a complex system



Evolutionary tree

Correlating Genetic Tree and Linguistic Phyla

Cavalli Sforza
Piazza Menozzi
Mountain

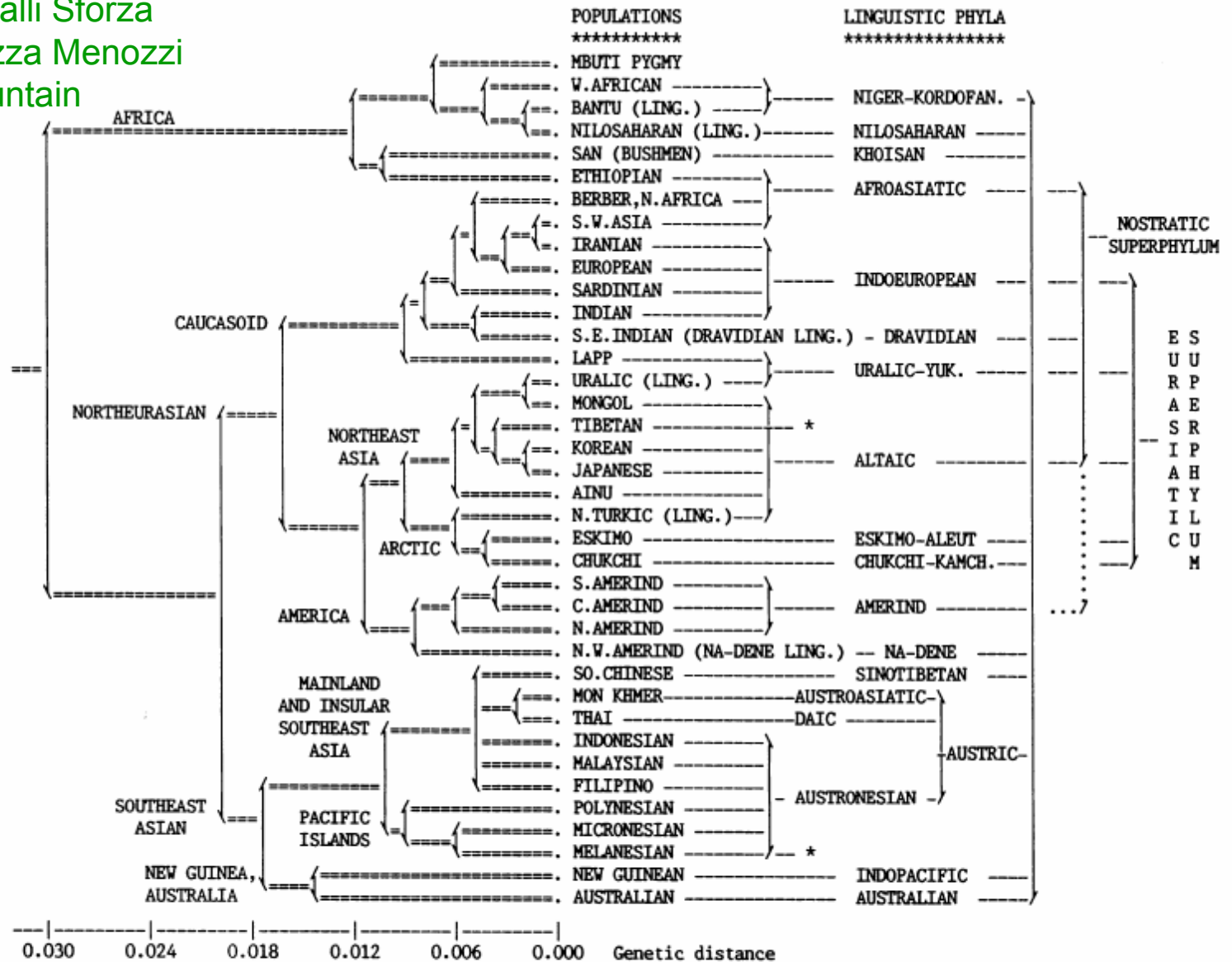


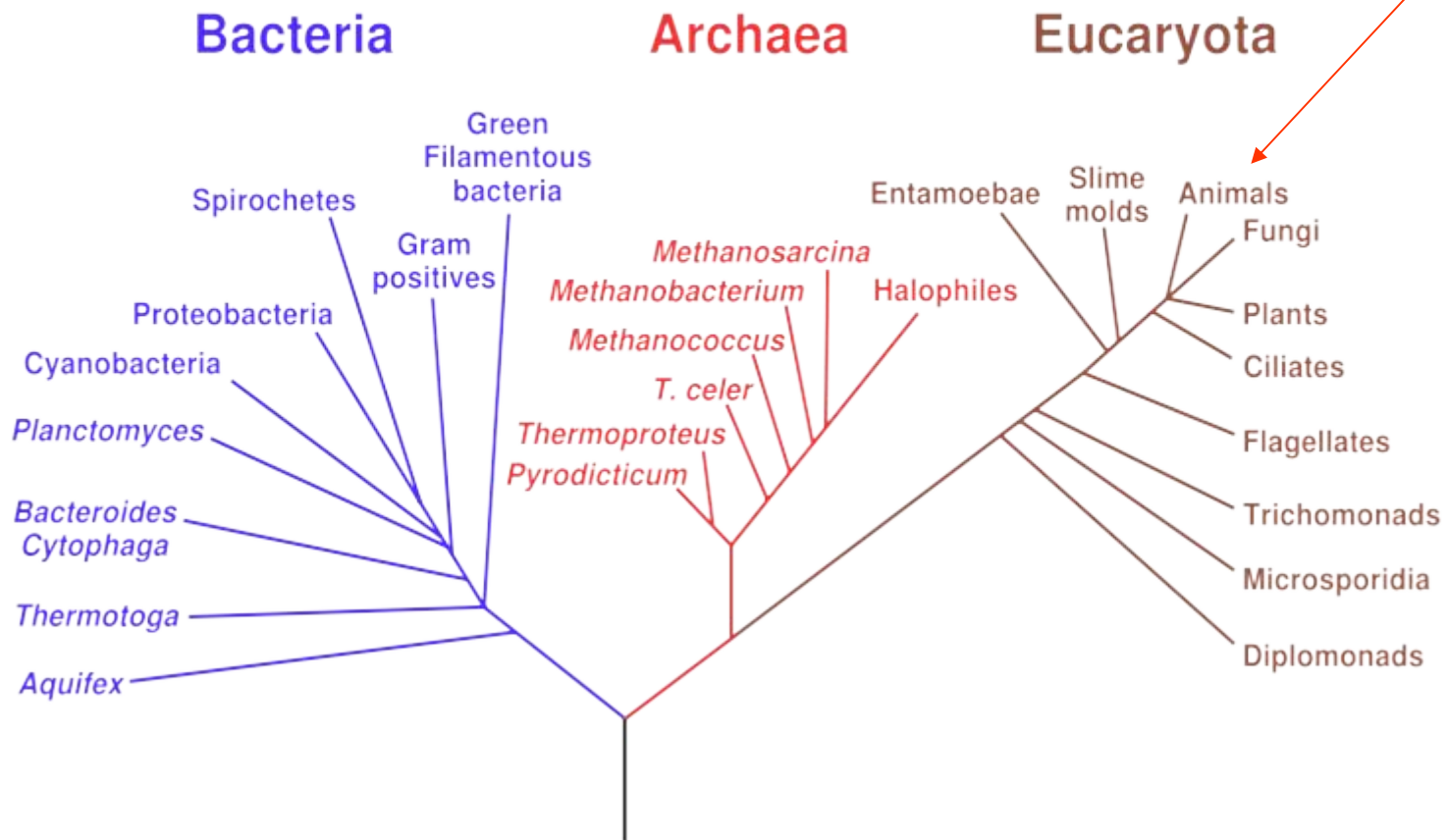
FIG. 1. Comparison of genetic tree and linguistic phyla. See text for details. (Ling.) indicates populations pooled on the basis of linguistic classification. The tree was constructed by average linkage analysis of Nei's genetic distances. Distances were calculated based on 120 allele frequencies from the following systems: *A1A2BO*, *MNS*, *RH*, *P*, *LU*, *K*, *FY*, *JK*, *DI*, *HP*, *TF*, *GC*, *LE*, *LP*, *PEPA*, *PEPB*, *PEPC*, *AG*, *HLAA* (12 alleles), *HLAB* (17 alleles), *PI*, *CP*, *ACP*, *PGD*, *PGM1*, *MDH*, *ADA*, *PTC*, *EI*, *SODA*, *GPT*, *PGK*, *C3*, *SE*, *ESD*, *GLO*, *KM*, *BF*, *LAD*, *E2*, *GM*, and *PG*.

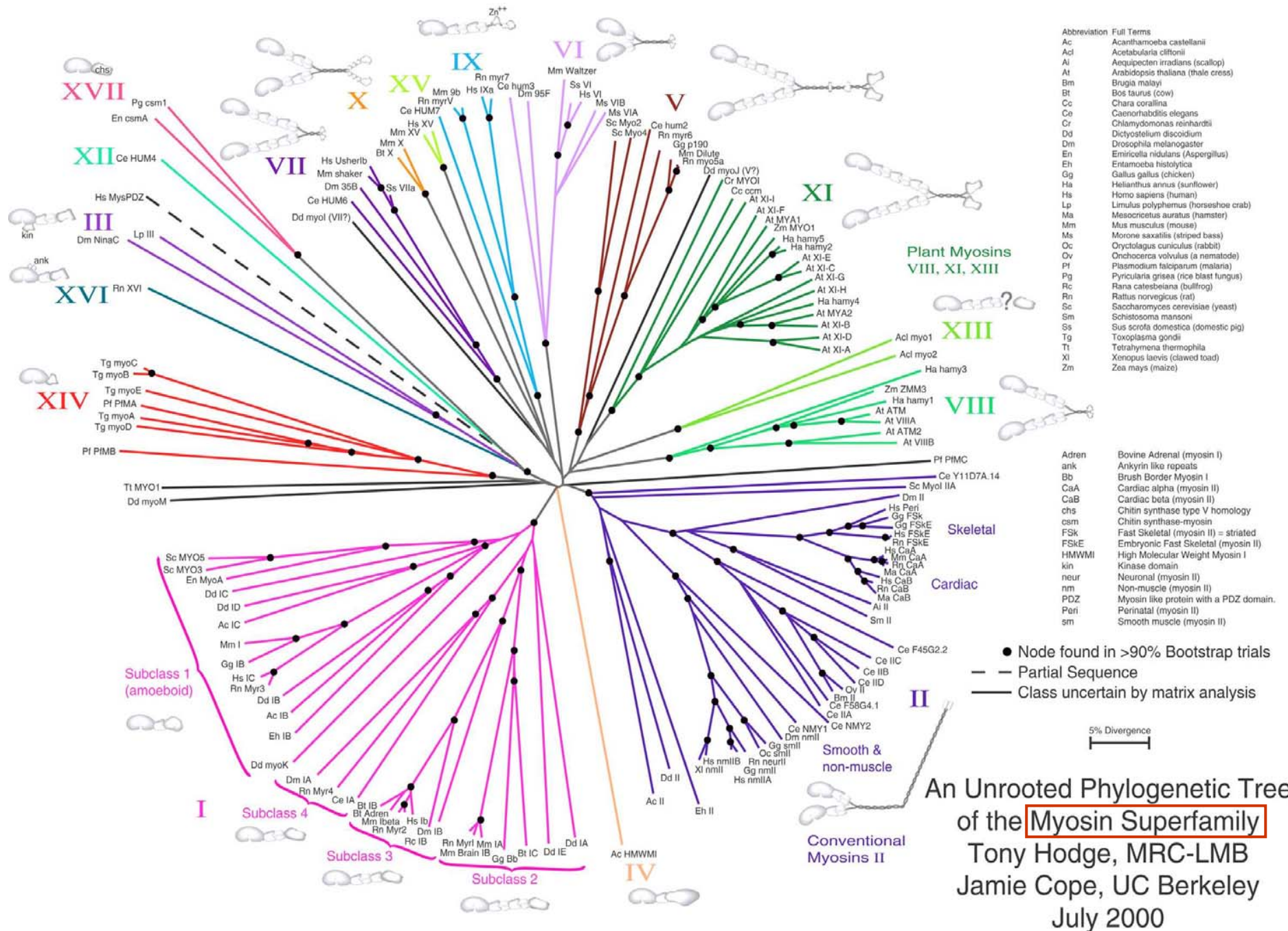
6. The set of living organisms on the Earth is a complex system

Phylogenetic Tree of Life

we are here

temporal evolutive tree





Biological systems and Spin glasses

Biosystems

Disorder

very many random variables,
few dynamical (relevant) dof's

Degeneracy

can exist in very many “equilibrium” states

Spin glasses

Disorder

random coupling among spins

Frustration

within triplets of spins

Complexity of classification



Spin glasses: a suggestive paradigm for biosystems

- Protein folding (see below)
- Associative memory
- Scaling laws in taxonomy
- Immune system memory and stability
- ...

Iori Marinari Parisi
Hopfield
Mezard Parisi Virasoro
Parisi

A Spin glass Primer

- N individuals interacts pairwise with couplings

$$\begin{array}{lll} J_{AB}=+1 & \text{if} & A \text{ likes } B \\ J_{AB}=-1 & \text{if} & A \text{ dislikes } B \end{array}$$

- Given 3 individuals, there is **frustration** if

$$J_{AB} J_{BC} J_{CA} = -1$$

- The N individuals are asked to separate in 2 fields so as to minimize in each field the number of pairs of “enemies”
- Given a **J-PDF** and an initial subdivision, “equilibrium” is reached by asking each individual to decide to change field if the move lowers the frustration
- If many pairs are frustrated $\left\{ \begin{array}{l} \text{system is highly } \mathbf{unstable} \\ \downarrow \\ \text{many possible equally good } \mathbf{subdivisions} \end{array} \right.$

A **locally** optimal state is reached in **polynomial** time

A **globally** optimal state (if it can be reached at all) generically requires an **exponential** time (**NP-problem**)

An illuminating example

- M likes M W likes W
 M dislikes W W dislikes M 

For any triplet $J^3=+1$
No frustration

⇒ Optimal state: 2 separate groups, [M] and [W]

- M dislikes M W dislikes W
 M likes W W likes M 

For any triplet $J^3=-1$
Maximal frustration

⇒ Optimal state: any subdivision with equal number of M and W

Further examples of interesting physical systems

- Alloys, like $\text{Fe}_x \text{Au}_{100-x}$, with small x % → $H = \sum_{ik} \sigma_i J(|x_i - x_k|) \sigma_k$
 $J(|x_i - x_k|)$ very rapidly oscillating with $|x_i - x_k|$, almost a random function
- Electrons moving in a metallic glass, containing various types of atoms, located at fixed but random positions

⇒ We expect the electron conductivity not to depend on the detailed positions of the impurities (for not too small samples)

$$H_{\text{SG}} = \sum_{ik} \sigma_i J_{ik} \sigma_k, \text{ with some PDF for the } J_{ik}$$



Basic Mathematics

Sherrington
Kirkpatrick
Parisi

- Hamiltonian

$$H_J [\sigma] = \sum_{ik} \sigma_i J_{ik} \sigma_k \quad J_{ik} = J_{ki}, J_{ii} = 0$$

- J_{ik} are random variables with PDF $\Rightarrow P(J)$

- Partition Function and Free Energy at fixed $P(J)$

$$Z_J = \sum_{[\sigma]} \exp -\beta H_J [\sigma] \quad \beta = 1/KT$$

$$F_J = -\frac{1}{\beta N} \log Z_J$$

- N is the number of spins

- We want to compute the **quenched** average

$$F = \sum_J P(J) F_J = -\frac{1}{\beta N} \sum_J P(J) \log Z_J$$

and not the **annealed** average

$$F_{An} = -\frac{1}{\beta N} \log Z_{An} \quad Z_{An} = \sum_J P(J) \sum_{[\sigma]} \exp -\beta H_J [\sigma]$$

- time scale of J -dynamics \gg time scale of σ -dynamics

The Replica Method

$$Z_n \equiv \sum_J P(J) (Z_J)^n$$

$$\Rightarrow \lim_{n \rightarrow 0} F_n = F$$

$$F_n = -\frac{1}{\beta N} \frac{1}{n} \log Z_n$$

the replica index

A simple proof

$$\begin{aligned} \lim_{n \rightarrow 0} -\frac{1}{\beta N} \frac{1}{n} \log Z_n &= \lim_{n \rightarrow 0} -\frac{1}{\beta N} \frac{1}{n} \log [\sum_J P(J) (Z_J)^n] = \\ &= \lim_{n \rightarrow 0} -\frac{1}{\beta N} \frac{1}{n} \log [\sum_J P(J) (1 + n \log Z_J + \dots)] = \\ &= \lim_{n \rightarrow 0} -\frac{1}{\beta N} \frac{1}{n} \log [1 + n \sum_J P(J) \log Z_J + \dots] = \\ &= -\frac{1}{\beta N} \sum_J P(J) \log Z_J = F \end{aligned}$$

looks OK, except that n is an integer...

Typical $P(J)$'s

Gaussian: $P(J) \propto \exp[-(J-J_0)^2/2\sigma_J^2]$

Uniform: $P(J=+1) = P(J=-1) = 1/2$

Phase structure

Edwards
Anderson

$$m_i(J) = \langle \sigma_i \rangle = \sum_{[\sigma]} \sigma_i \exp -\beta H_J [\sigma]$$

$$q(J) = \frac{1}{N} \sum_i [m_i(J)]^2 = \sum_J P(J) [m_i(J)]^2 = q$$

self-averaging

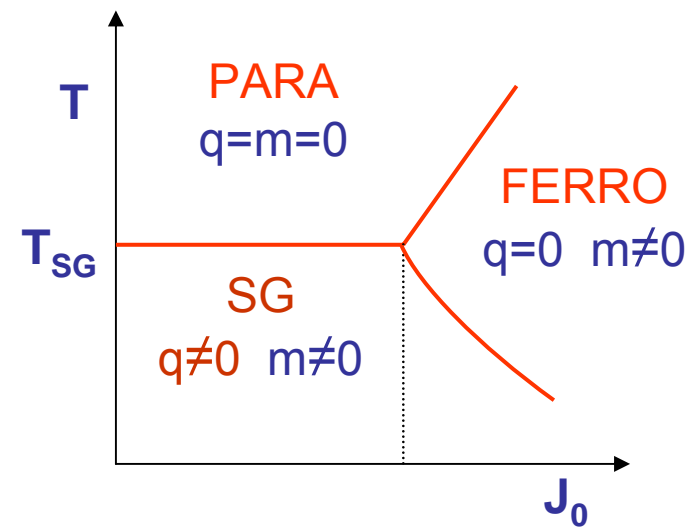
High temperature $m_i(J) = 0 \Rightarrow q = 0$

Low temperature $\left\{ \begin{array}{l} m_i(J) \neq 0 \text{ for some } i \\ \text{with } \sum_i [m_i(J)] = 0, \text{ but} \\ q(J) = \frac{1}{N} \sum_i [m_i(J)]^2 \neq 0 \end{array} \right.$

self-averaging

Order parameters $\left\{ \begin{array}{l} q = \frac{1}{N} \sum_i [m_i(J)]^2 \\ m = \frac{1}{N} \sum_i [m_i(J)] \end{array} \right.$

The whole game is to compute $P(q)$



Few further numbers

dimensions

times

weights

chemical events

chemical events

Human body: $\sim 7 \times 10^{27}$ atoms:
99% C, H, O and N; 87% are either H or O;
but 41 different elements

Estimated Atomic Composition of a lean 70 kg Male Human Body

Element	Sym	# Atoms	Element	Sym	# Atoms	Element	Sym	# Atoms			
Hydrogen	H	1	4.22 x 10 ²⁷	Rubidium	Rb	37	2.2 x 10 ²¹	Zirconium	Zr	40	2 x 10 ¹⁹
Oxygen	O	8	1.61 x 10 ²⁷	Strontium	Sr	38	2.2 x 10 ²¹	Cobalt	Co	27	2 x 10 ¹⁹
Carbon	C	6	8.03 x 10 ²⁶	Bromine	Br	35	2 x 10 ²¹	Cesium	Cs	55	7 x 10 ¹⁸
Nitrogen	N	7	3.9 x 10 ²⁵	Aluminum	Al	13	1 x 10 ²¹	Mercury	Hg	80	6 x 10 ¹⁸
Calcium	Ca	20	1.6 x 10 ²⁵	Copper	Cu	29	7 x 10 ²⁰	Arsenic	As	33	6 x 10 ¹⁸
Phosphorus	P	15	9.6 x 10 ²⁴	Lead	Pb	82	3 x 10 ²⁰	Chromium	Cr	24	6 x 10 ¹⁸
Sulfur	S	16	2.6 x 10 ²⁴	Cadmium	Cd	48	3 x 10 ²⁰	Molybdenum	Mo	42	3 x 10 ¹⁸
Sodium	Na	11	2.5 x 10 ²⁴	Boron	B	5	2 x 10 ²⁰	Selenium	Se	34	3 x 10 ¹⁸
Potassium	K	19	2.2 x 10 ²⁴	Manganese	Mn	25	1 x 10 ²⁰	Beryllium	Be	4	3 x 10 ¹⁸
Chlorine	Cl	17	1.6 x 10 ²⁴	Nickel	Ni	28	1 x 10 ²⁰	Vanadium	V	23	8 x 10 ¹⁷
Magnesium	Mg	12	4.7 x 10 ²³	Lithium	Li	3	1 x 10 ²⁰	Uranium	U	92	2 x 10 ¹⁷
Silicium	Si	14	3.9 x 10 ²³	Barium	Ba	56	8 x 10 ¹⁹	Radium	Ra	88	8 x 10 ¹⁰
Fluorine	F	9	8.3 x 10 ²²	Iodine	I	53	5 x 10 ¹⁹				
Iron	Fe	26	4.5 x 10 ²²	Tin	Sn	50	4 x 10 ¹⁹				
Zinc	Zn	30	2.1 x 10 ²²	Gold	Au	79	2 x 10 ¹⁹				
								TOTAL			6.71x10 ²⁷

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 1																	He 2
2	Li 3	Be 4											B 5	C 6	N 7	O 8	F 9	Ne 10
3	Na 11	Mg 12											Al 13	Si 14	P 15	S 16	Cl 17	Ar 18
4	K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36
5	Rb 37	Sr 38	Y 39	Zr 40	Nb 41	Mo 42	Tc 43	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50	Sb 51	Te 52	I 53	Xe 54
6	Cs 55	Ba 56	* 72	Hf 73	Ta 74	W 75	Re 76	Os 77	Ir 78	Pt 79	Au 80	Hg 81	Tl 82	Pb 83	Bi 84	Po 85	At 86	Rn 86
7	Fr 87	Ra 88	** 104	Rf 105	Db 106	Sg 107	Bh 108	Hs 109	Mt 110	Uun 111	Uuu 112	Uub						
			*	La 57	Ce 58	Pr 59	Nd 60	Pm 61	Sm 62	Eu 63	Gd 64	Tb 65	Dy 66	Ho 67	Er 68	Tm 69	Yb 70	Lu 71
			**	Ac 89	Th 90	Pa 91	U 92	Np 93	Pu 94	Am 95	Cm 96	Bk 97	Cf 98	Es 99	Fm 100	Md 101	No 102	Lr 103

Element Groups (Families)

Alkali Earth	Alkaline Earth	Transition Metals
Rare Earth	Other Metals	Metalloids
Non-Metals	Halogens	Noble Gases

Estimated Molecular Content of a Typical 20-micron Human Cell

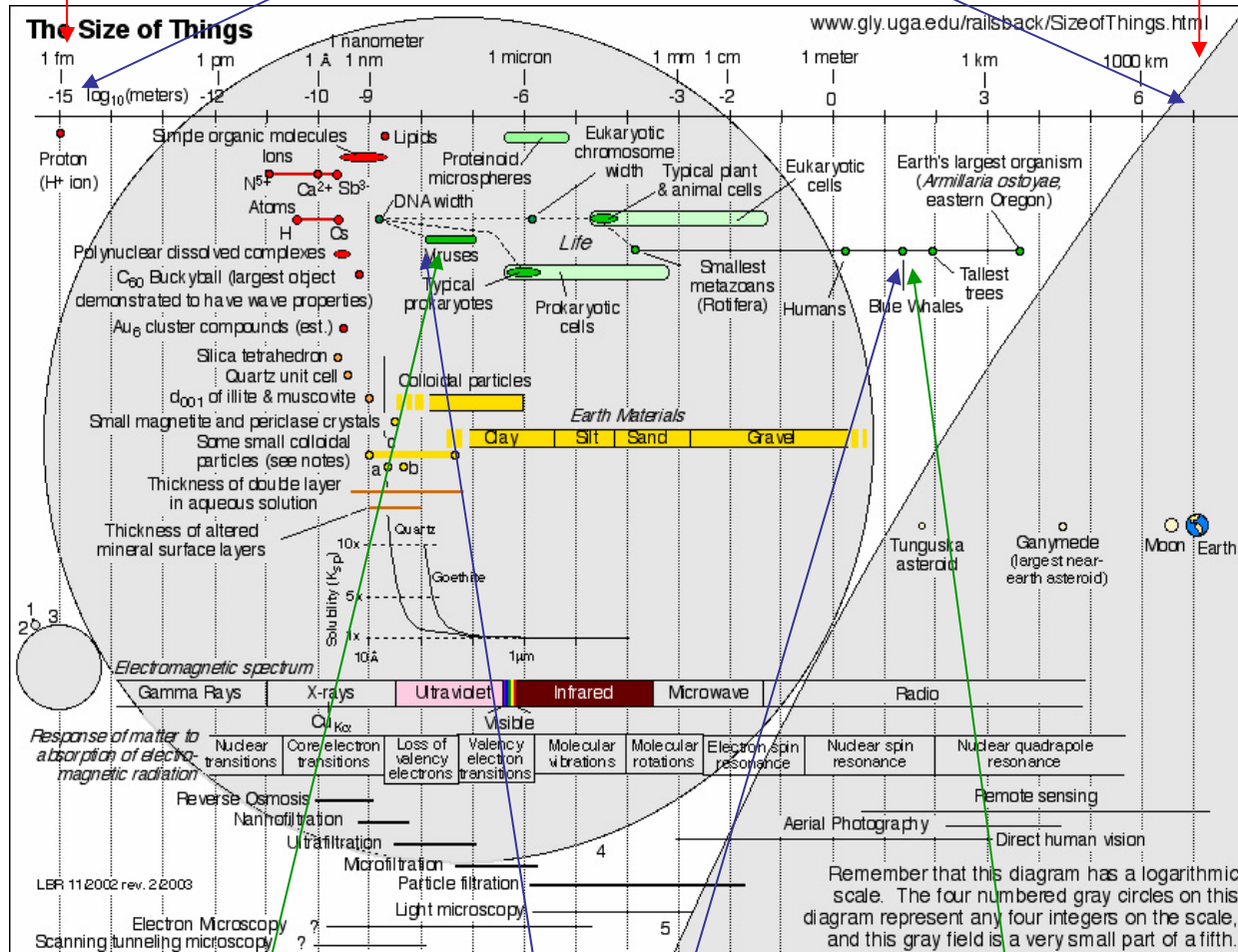
Molecule	Mass %	<MW> (Daltons)	# Molecules	Molecule %	# of Types
Water	65%	18	1.74×10^{14}	98.73 %	1
Other Inorganic	1.5%	55	1.31×10^{12}	0.74 %	20
Lipid	12%	700	8.4×10^{11}	0.475 %	50
Other Organic	0.4%	250	7.7×10^{10}	0.044 %	~200
Protein	20%	50,000	1.9×10^{10}	0.011 %	~5,000
RNA	1.0%	1×10^6	5×10^7	3×10^{-5} %	----
DNA	0.1%	1×10^{11}	46	3×10^{-11} %	----
TOTALS	100%	----	1.76×10^{14}	100%	----

1 Da (Dalton) = 1 atomic unit = $m_a(^{12}\text{C}) / (12 \times 1,660540 \times 10^{-27} \text{ kg} \sim \text{hydrogen mass})$
dimensionless unit

Proton

10^{22}

Earth



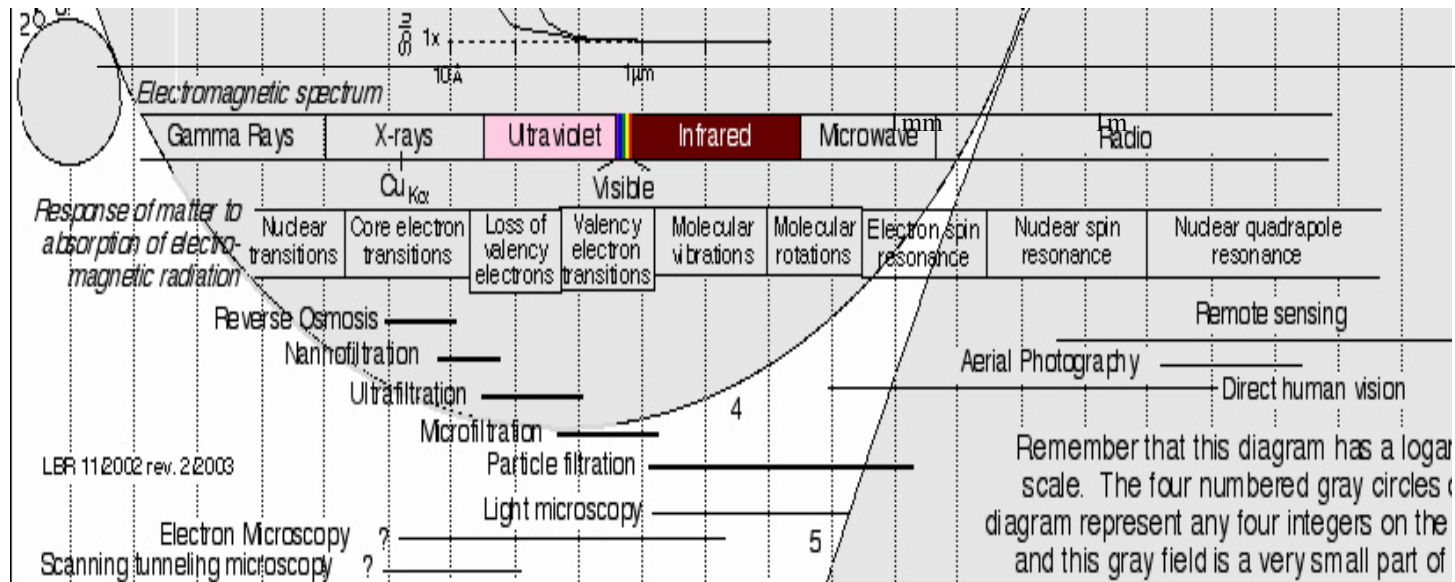
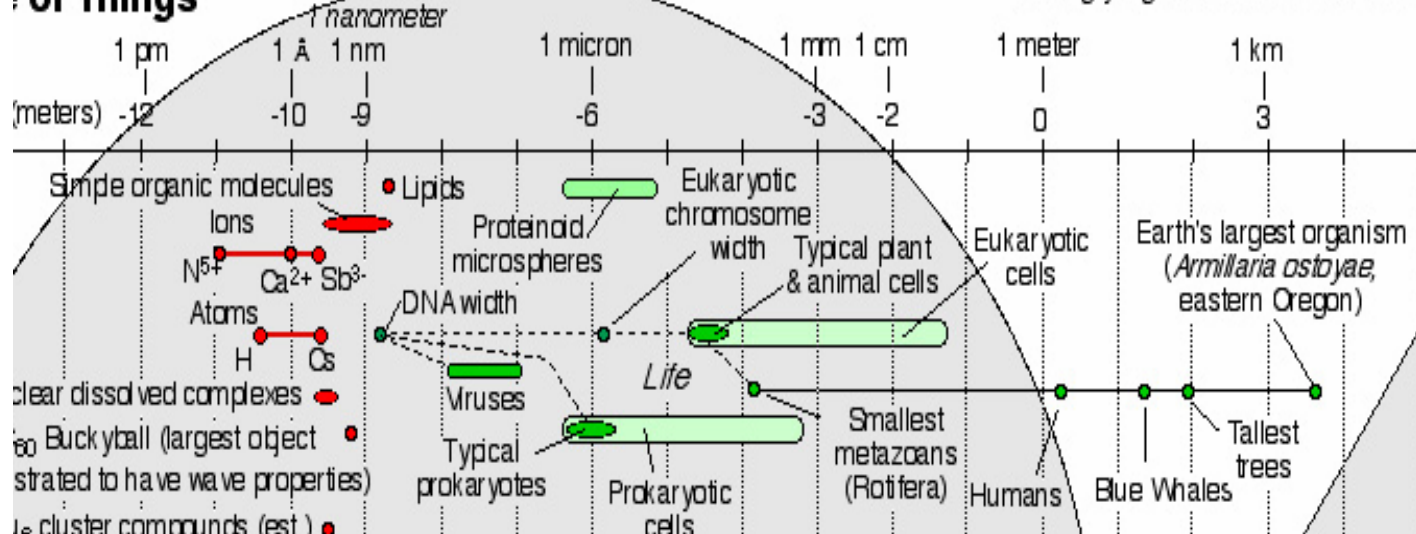
Virus

10^9

Blue whale

Scale of Things

www.gly.uga.edu/railsback/Size



LBR 11/2002 rev. 2/2003

The **largest** and **smallest** cells in the human body
are the gametes or the sex cells

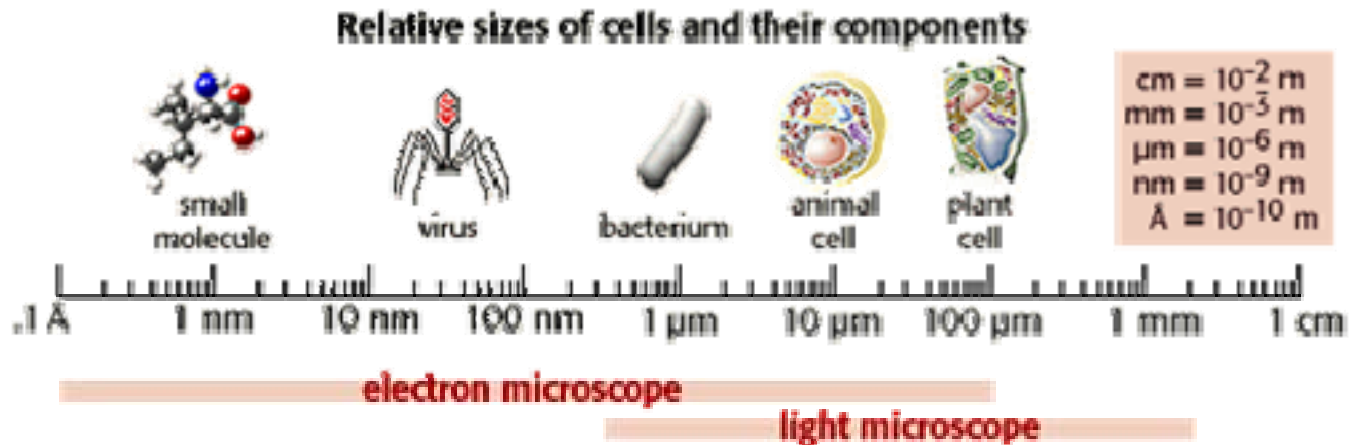
♀ **female** = oocyte: $\varnothing \approx 35 \mu\text{m}$ (almost visible with the naked eye)

♂ **male** = spermatozoon: $\varnothing \approx 3 \mu\text{m}$

The **smallest** known organism capable
of independent growth and reproduction

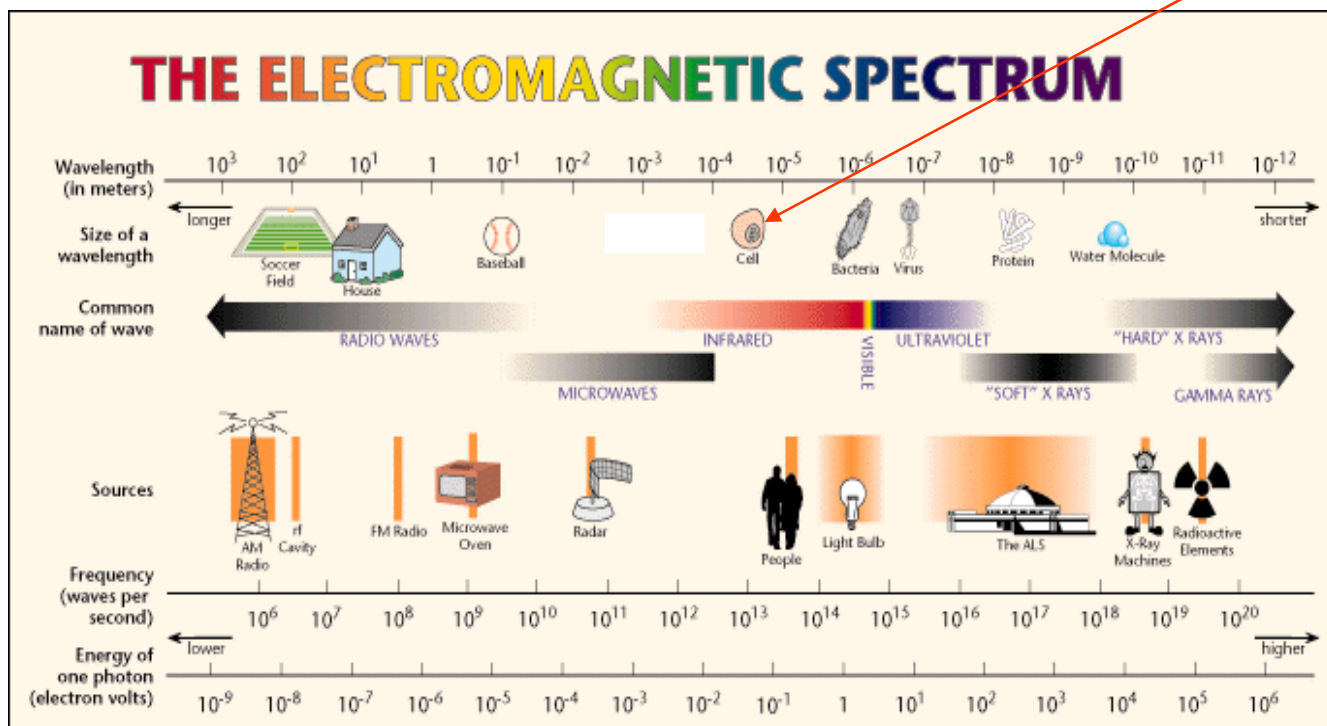
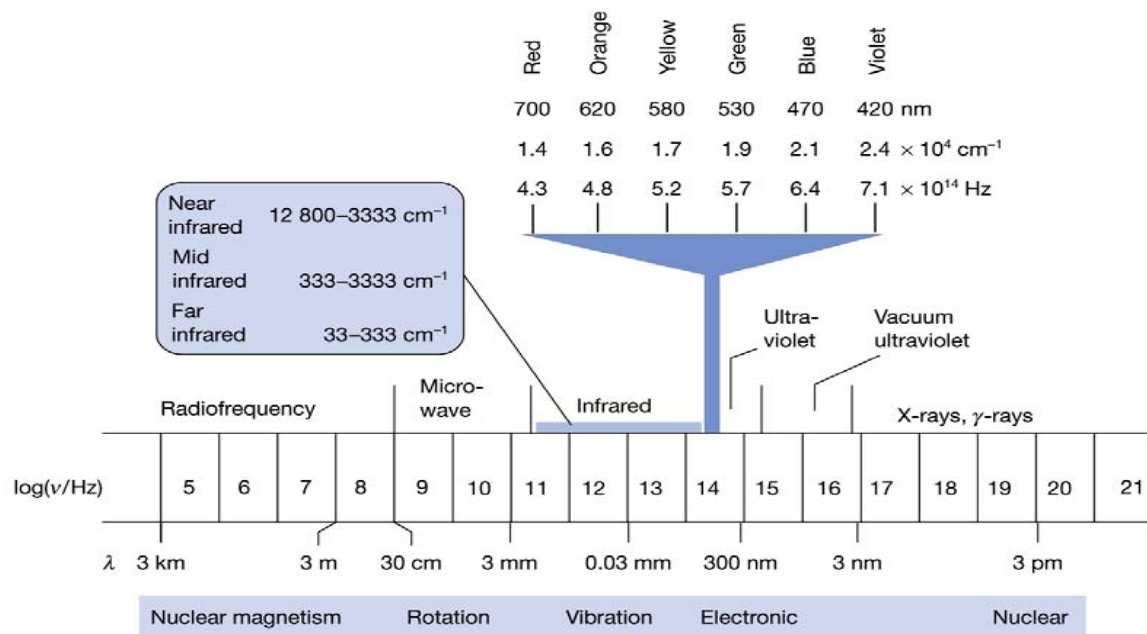
Mycoplasma genitalium: $\varnothing \approx 0.2 - 0.3 \mu\text{m}$

The smallest “**theoretical**” bacterium: $\varnothing \approx 0.17 \mu\text{m}$

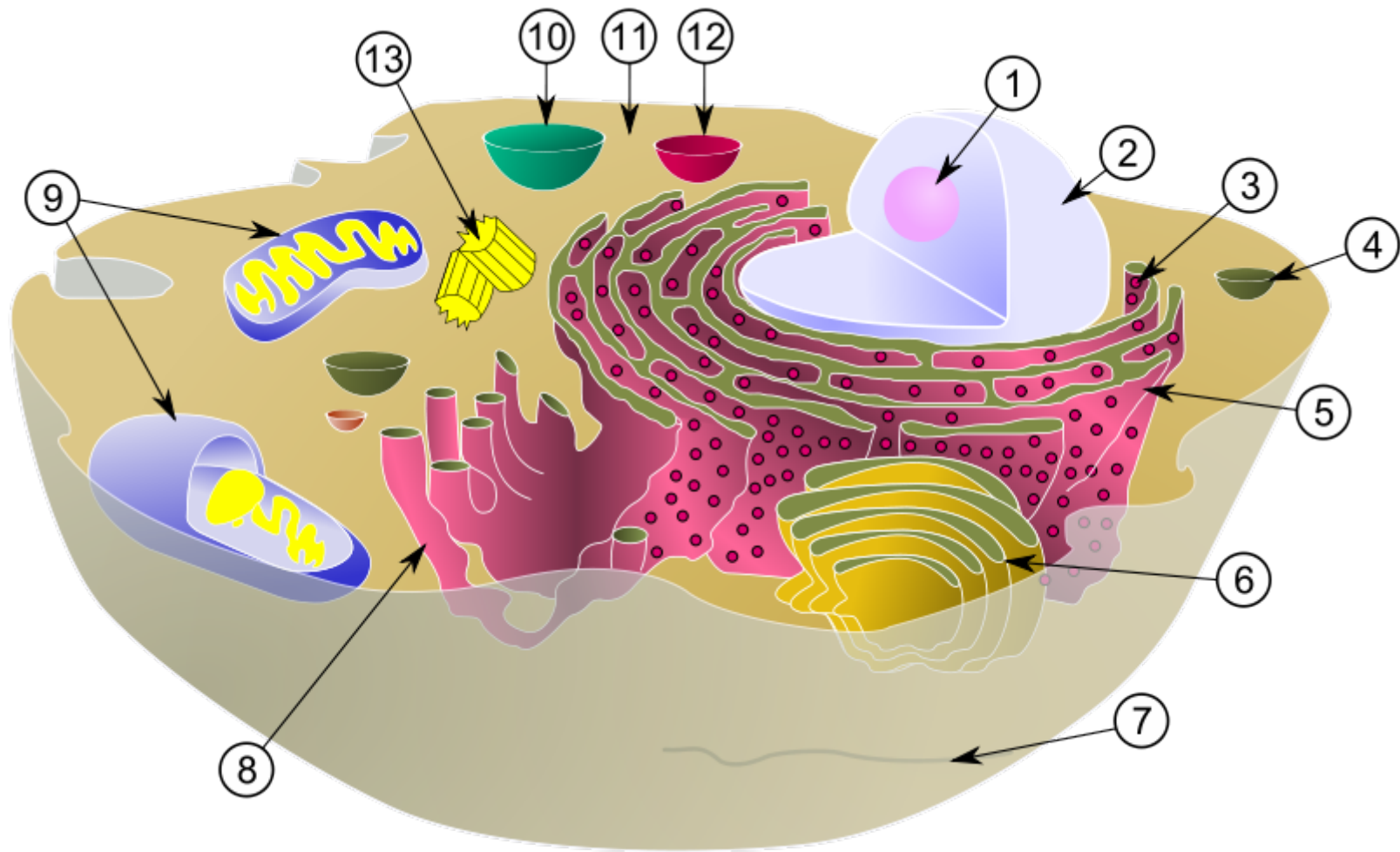


<Average bacterium>: rod shape $V \approx 1 \mu\text{m}^2 \times 3 \mu\text{m}$

<Average human cell>: spherical shape $\varnothing \approx 25 \mu\text{m}$

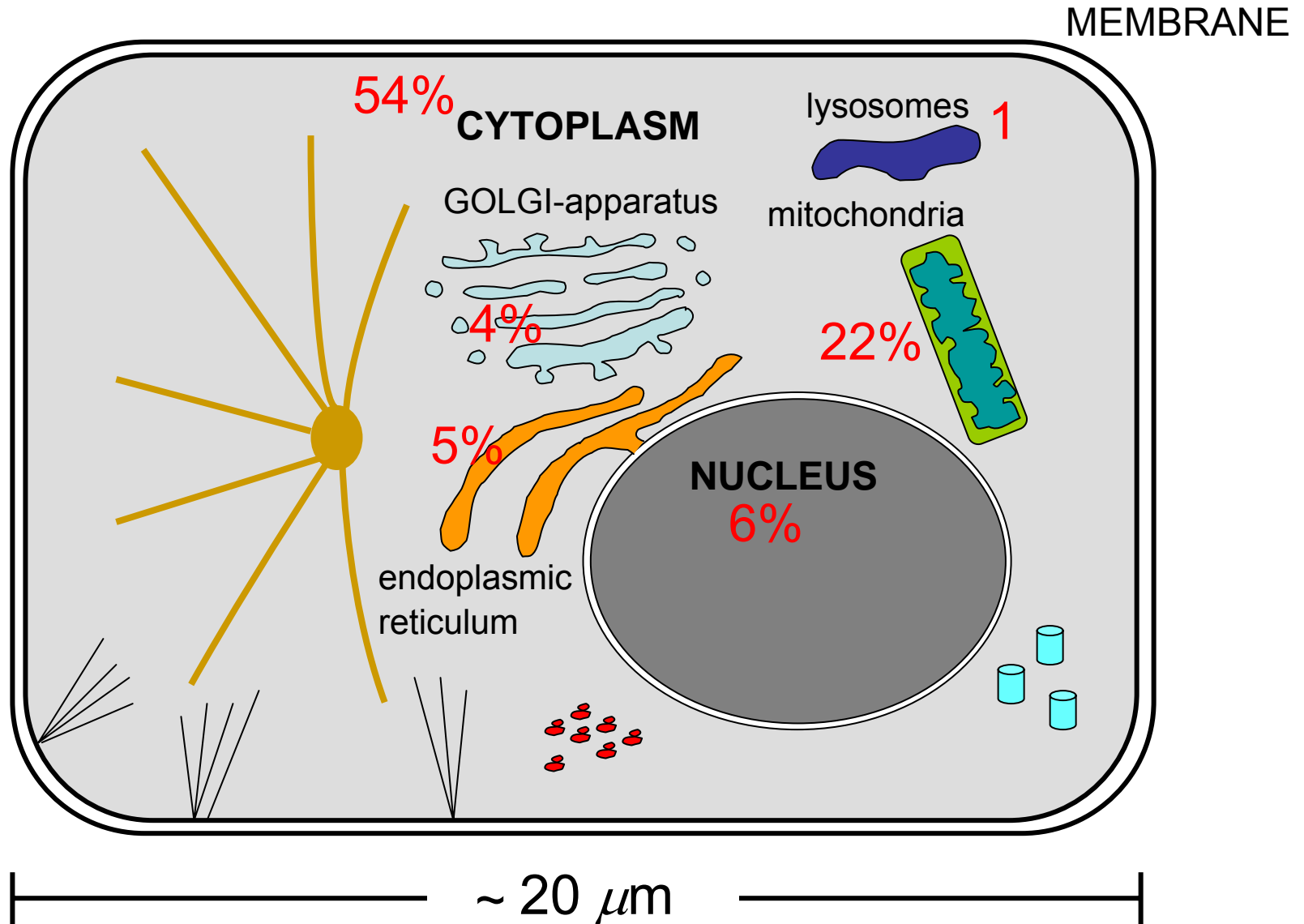


A eukaryotic cell artistic view

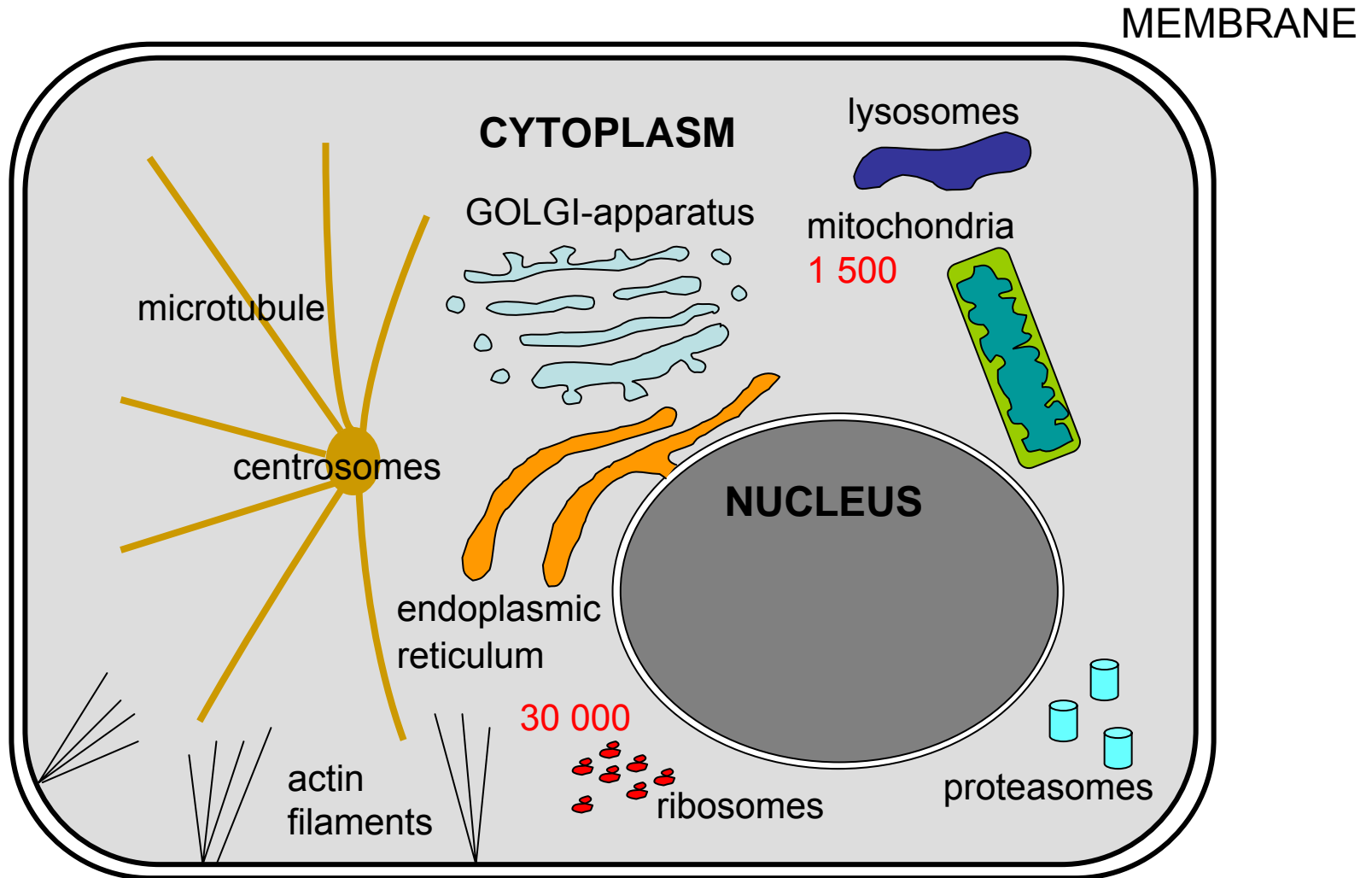


1. Nucleolus
2. Nucleus
3. Ribosome
4. Vesicle
5. Rough endoplasmic reticulum
6. Golgi apparatus
7. Cytoskeleton
8. Smooth endoplasmic reticulum
9. Mitochondrion
10. Vacuole
11. Cytosol
12. Lysosome
13. Centriole

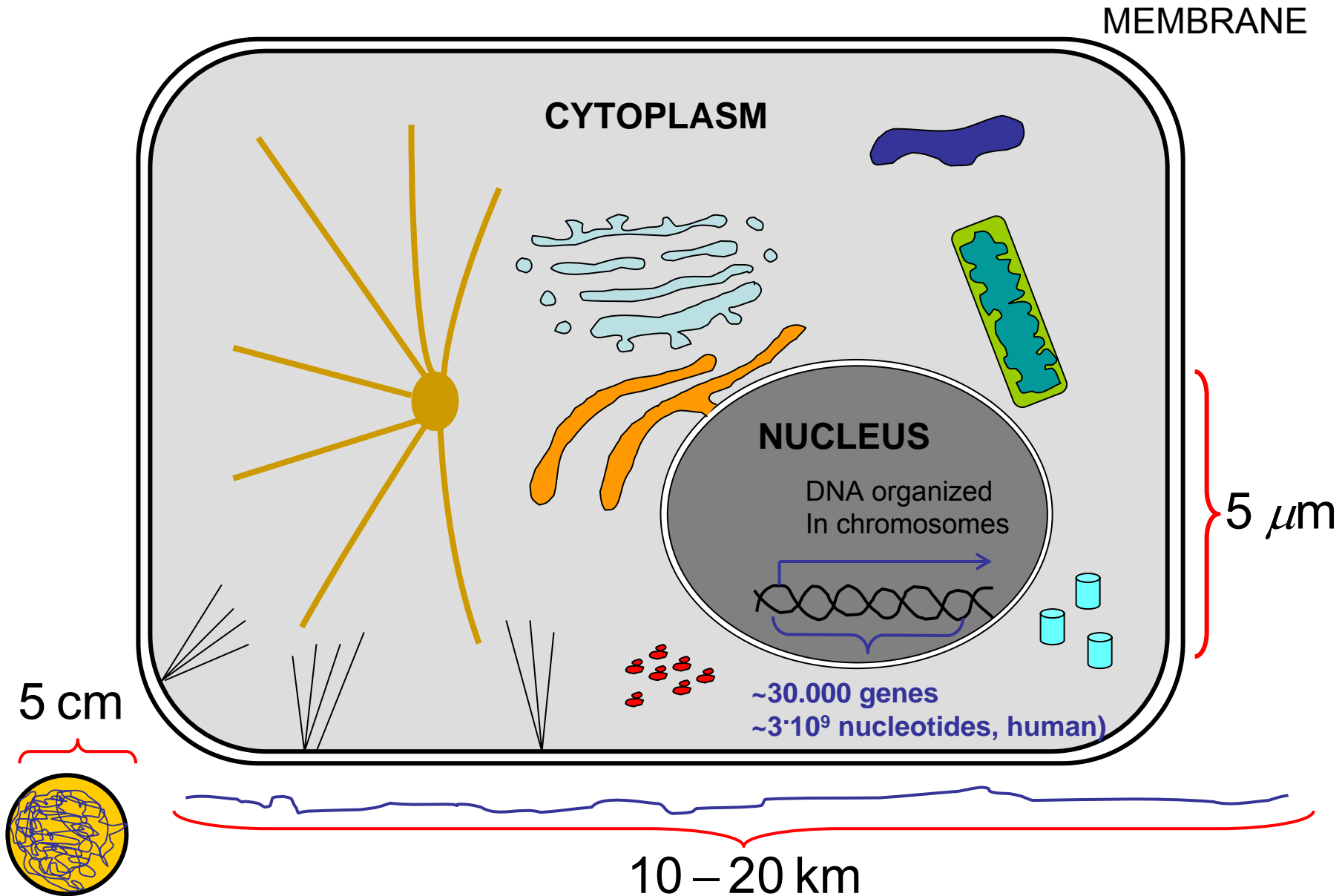
SCHEMATIC STRUCTURE OF LIVING EUKARYOTIC CELLS



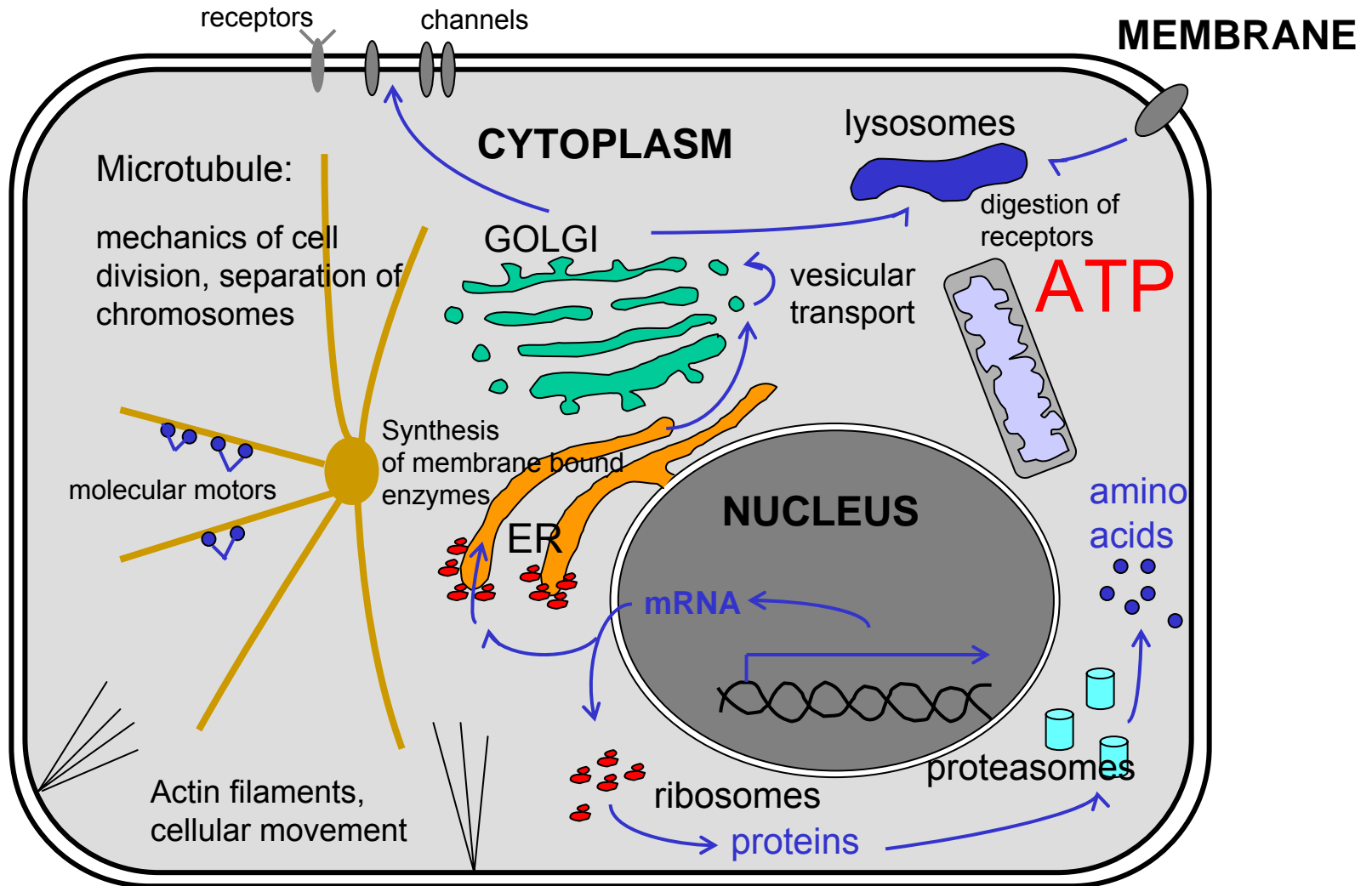
SCHEMATIC STRUCTURE OF LIVING EUKARYOTIC CELLS



SCHEMATIC STRUCTURE OF LIVING EUKARYOTIC CELLS



SCHEMATIC STRUCTURE OF LIVING EUKARYOTIC CELLS



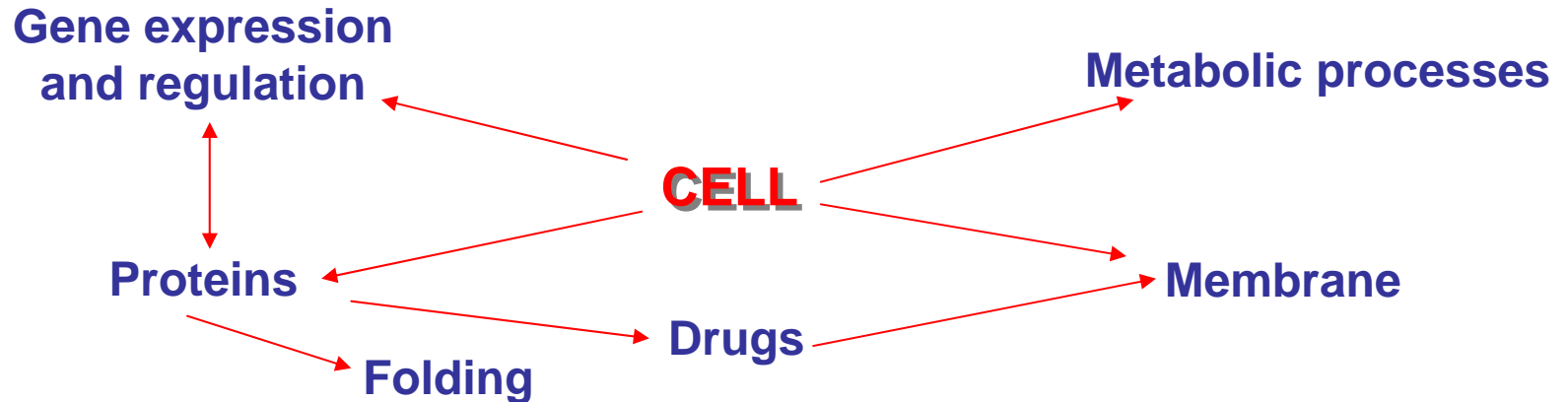
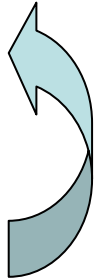
Comparison of features of prokaryotic and eukaryotic cells

	Prokaryotes	Eukaryotes
Typical organisms	bacteria , archaea	protists , fungi , plants , animals
Typical size	~ 1-10 µm	~ 10-100 µm (sperm cells , apart from the tail, are smaller)
Type of nucleus	nucleoid region ; no real nucleus	real nucleus with double membrane
DNA	circular (usually)	linear molecules (chromosomes) with histone proteins
RNA-/protein-synthesis	coupled in cytoplasm	RNA-synthesis inside the nucleus protein synthesis in cytoplasm
Ribosomes	50S+30S	60S+40S
Cytoplasmatic structure	very few structures	highly structured by endomembranes and a cytoskeleton
Cell movement	flagella made of flagellin	flagella and cilia containing microtubules ; lamellipodia and filopodia containing actin
Mitochondria	none	one to several thousand (though some lack mitochondria)
Chloroplasts	none	in algae and plants
Organization	usually single cells	single cells, colonies, higher multicellular organisms with specialized cells
Cell division	Binary fission (simple division)	Mitosis (fission or budding) Meiosis

II. Data, (physical) models and (mathematical) tools

II. Data, (physical) models and (mathematical) tools

- **Genome/Protein sequencing**
genome sequence reconstruction is an NP-hard problem
- **Annotation**
elucidation and description of biologically relevant features in the sequence and relations with other data
- **Identification of gene regulation and metabolic pathways**
reaction constants and chemical affinities
-



- How to make a model
- Analytical methods
- Numerical approaches and simulations



D.D. Shoemaker et al.
15 February 2001
Vol. **409**, pp. 745-964

**Human Genome Project
HGP**



J. Craig Venter et al.
16 February 2001
Vol. **291**, pp. 1304-1351

CELERA

Genome Overview

Table 11. Genome overview.

Size of the genome (including gaps)	2.91 Gbp
Size of the genome (excluding gaps)	2.66 Gbp
Longest contig	1.99 Mbp
Longest scaffold	14.4 Mbp
Percent of A+T in the genome	54
Percent of G+C in the genome	38
Percent of undetermined bases in the genome	9
Most GC-rich 50 kb	Chr. 2 (66%)
Least GC-rich 50 kb	Chr. X (25%)
Percent of genome classified as repeats	35
Number of annotated genes	26,383
Percent of annotated genes with unknown function	42
Number of genes (hypothetical and annotated)	39,114
Percent of hypothetical and annotated genes with unknown function	59
Gene with the most exons	Titin (234 exons)
Average gene size	27 kbp
Most gene-rich chromosome	Chr. 19 (23 genes/Mb)
Least gene-rich chromosomes	Chr. 13 (5 genes/Mb), Chr. Y (5 genes/Mb)
Total size of gene deserts (>500 kb with no annotated genes)	605 Mbp
Percent of base pairs spanned by genes	25.5 to 37.8 [±]
Percent of base pairs spanned by exons	1.1 to 1.4 [±]
Percent of base pairs spanned by introns	24.4 to 36.4 [±]
Percent of base pairs in intergenic DNA	74.5 to 63.6 [±]
Chromosome with highest proportion of DNA in annotated exons	Chr. 19 (9.33)
Chromosome with lowest proportion of DNA in annotated exons	Chr. Y (0.36)
Longest intergenic region (between annotated + hypothetical genes)	Chr. 13 (3,038,416 bp)
Rate of SNP variation	1/1250 bp

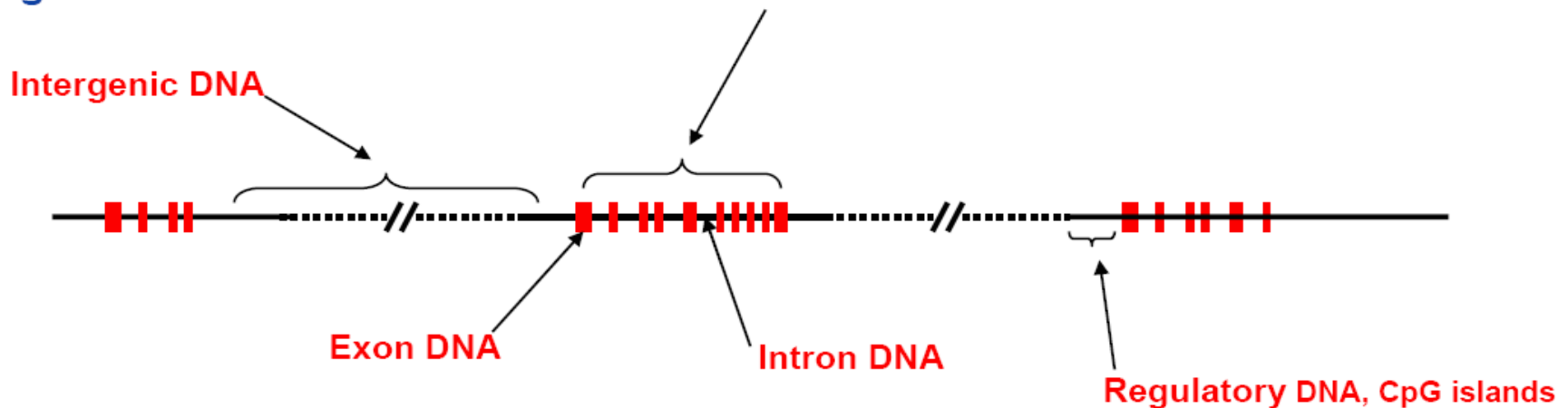
*

In these ranges, the percentages correspond to the annotated gene set (26, 383 genes) and the hypothetical + annotated gene set (39,114 genes), respectively.

Genetic structure of human chromosomes

Each human chromosome contains a single long DNA molecule.

A gene is a locus of co-transcribed exons



Intergenic DNA	75%
Intron DNA	24%
Exon DNA	1.1%

There are approximately **38,000** human genes predicted from analysis of the human genome sequence

Human chromosome 9



NCBI Map Viewer

PubMed

Entrez

BLAST

OMIM

Taxonomy

St

Search

Find

Find in This View

Advanced Search

Human genome
overview page (Build
36.3)
Human genome
overview page (Build
35.1)

Map Viewer Home

Map Viewer Help
Human Maps Help
FTP

Data As Table View

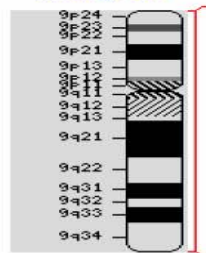
Maps & Options

Compress Map ☐

Region Shown:

Go

out
zoom
in
You are here:
Ideogram



☒ default
☐ master

Homo sapiens (human) Build 36.3 (Current)

BLAST The Human Genome

Chromosome: [1](#) [2](#) [3](#) [4](#) [5](#) [6](#) [7](#) [8](#) [9](#) [10](#) [11](#) [12](#) [13](#) [14](#) [15](#) [16](#) [17](#) [18](#) [19](#) [20](#) [21](#) [22](#) [X](#)

Master Map: Genes On Sequence

Summary of Maps

Maps

Region Displayed: 0-140M bp

Download/View Sequence

Ideogram ☒

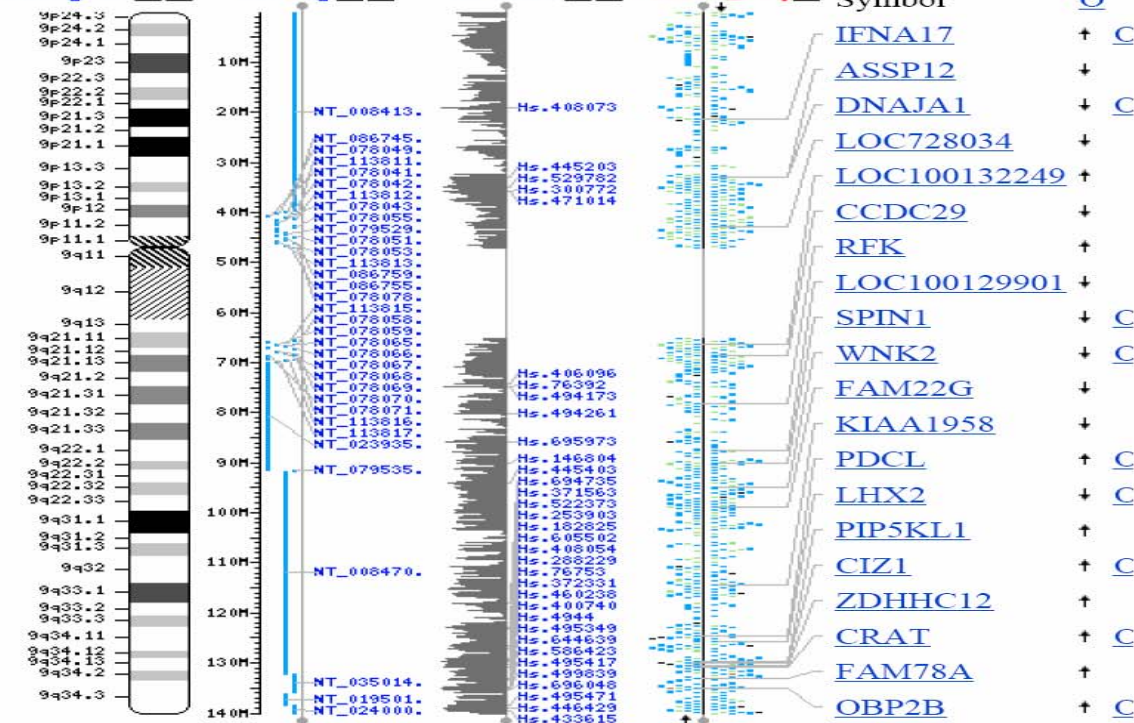
Contig ☒

Hs UniG ☒

Genes_seq ☒

Symbol

O



Summary of Maps:

Map 1: Ideogram

Region Displayed: 9pter-9qter

Map 2: Contig

Region Displayed: 0-140M bp

Total Contigs On Chromosome: 39 [7 not localized]

Contigs Labeled: 34 Total Contigs in Region: 39

Map 3: Homo sapiens UniGene Clusters

Region Displayed: 0-140M bp

Total Transcript alignments On Chromosome: 4925 [29 not localized]

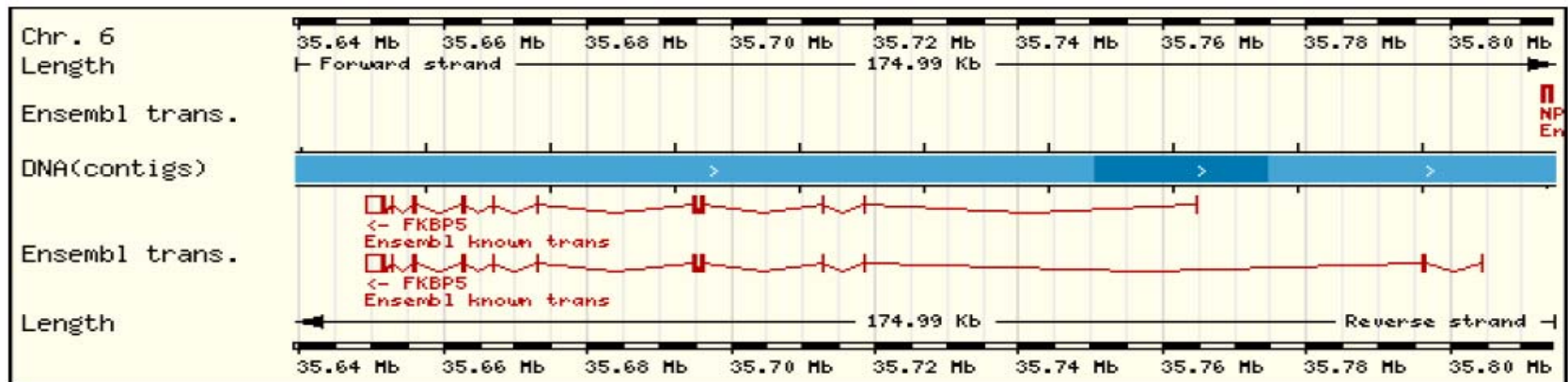
UniGene Clusters Labeled: 34 Total Transcript alignments in Region: 4925

Histogram Data: Tick Width=282,809bp/pixel, Max Height=5099 transcripts

Download/View S

Download/View S

Gene Report for ENSG00000096060, FKBP5

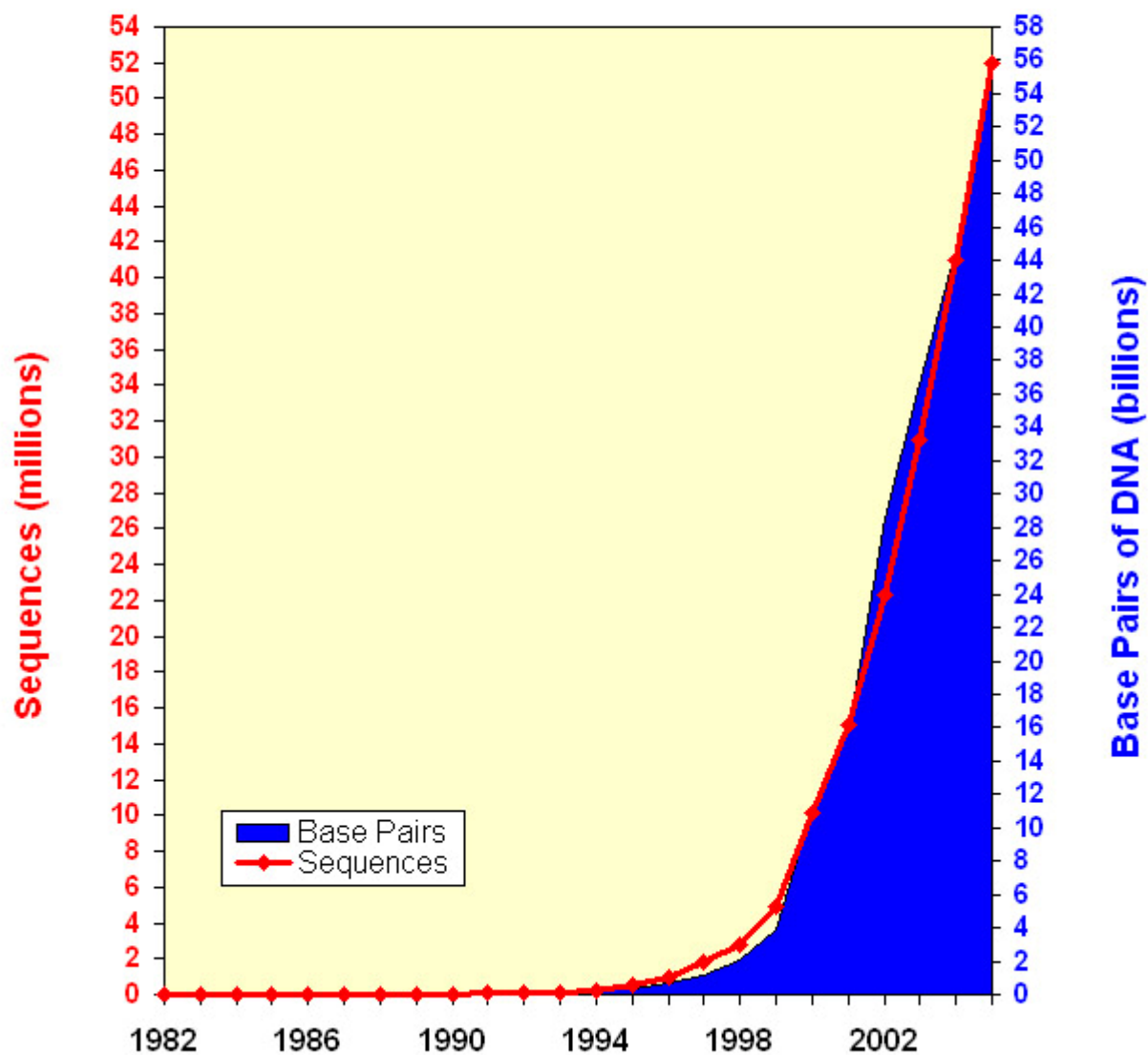


Nucleotide Sequence (1374 nt):

ATGACTACTGATGAAGGTGCCAAGAACAATGAAGAAAGCCCCACAGCCACTGTTGCTGAGCAGGGAGAGG
ATATTACCTCCAAAAAAGACAGGGGAGTATTAAAGATTGTCAAAGAGTGGGGAATGGTGAGGAAACGCC
GATGATTGGAGACAAAGTTTATGTCCATTACAAAGGAAAATTGTCAAATGGAAAGAAGTTTGATTCCAGT
CATGATAGAAATGAACCATTTGTCTTTAGTCTTGGCAAAGGCCAAGTCATCAAGGCATGGGACATTGGGG
TGGCTACCATGAAGAAAGGAGAGATATGCCATTTACTGTGCAAACCAGAATATGCATATGGCTCGGCTGG
CAGTCTCCCTAAAATTCCCTCGAATGCAACTCTCTTTTTTTGAGATTGAGCTCCTTGATTTCAAAGGAGAG
GATTTATTTGAAGATGGAGGCATTATCCGGAGAACCAAACGGAAAGGAGAGGGATATTCAAATCCAAACG
AAGGAGCAACAGTAGAAATCCACCTGGAAGGCCGCTGTGGTGGAAAGGATGTTTGACTGCAGAGATGTGGC
ATCACTGTGGGCGAAGGAGAAGACCACGACATTCGAATTGGAATTGACAAAGCTCTGGAGAAAATGCAG
CGGGAAGAACAATGTATTTTATATCTTGGACCAAGATATGGTTTTTGGAGAGGCAGGGAAGCCTAAATTTG
GCATTGAACCTAATGCTGAGCTTATATATGAAGTTACACTTAAGAGCTTCGAAAAGGCCAAAGAATCCTG
GGAGATGGATACCAAAGAAAAAATTGGAGCAGGCTGCCATTGTCAAAGAGAAGGGAACCGTATACTTCAAG
GGAGGCAAATACATGCAGGCGGTGATTCAGTATGGGAAGATAGTGTCCTGGTTAGAGATGGAATATGGTT
TATCAGAAAAGGAATCGAAAGCTTCTGAATCATTTCTCCTTGCTGCCTTTCTGAACCTGGCCATGTGCTA
CCTGAAGCTTAGAGAATACACCAAAGCTGTTGAATGCTGTGACAAGGCCCTTGGACTGGACAGTGCCAAT
GAGAAAGGCTTGTATAGGAGGGGTGAAGCCCAGCTGCTCATGAACGAGTTTGAGTCAGCCAAGGGTGACT
TTGAGAAAGTGCTGGAAGTAAACCCCCAGAATAAGGCTGCAAGACTGCAGATCTCCATGTGCCAGAAAAA
GGCCAAGGAGCACACGAGCGGGACCGCAGGATATACGCCAACATGTTCAAGAAGTTTGCAGAGCAGGAT
GCCAAGGAAGAGGCCAATAAAGCAATGGGCAAGAAGACTTCAGAAGGGGTCACTAATGAAAAAGGAACAG
ACAGTCAAGCAATGGAAGAAGAGAAACCTGAGGGCCACGTATGA

Growth of GenBank

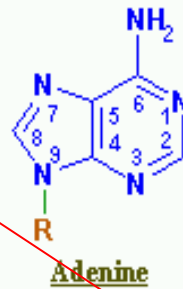
(1982 - 2005)



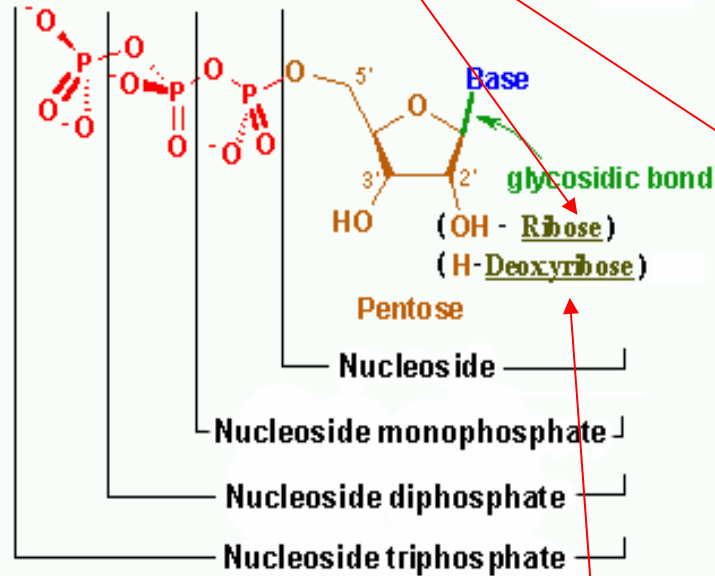
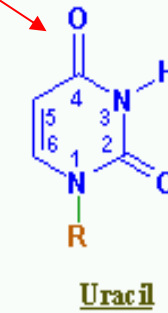
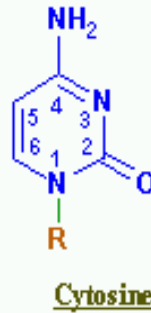
RNA



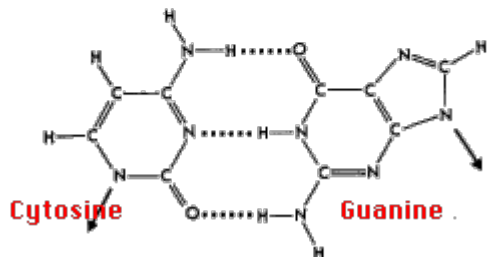
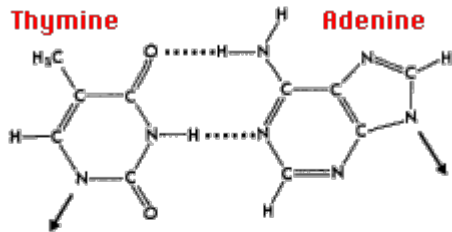
Purines



Pyrimidines



DNA



Complementary bases, paired via 2 (T-A) or 3 (C-G) hydrogen bonds

PDB

DNA sequencing

- Cut long DNA strands in short fragments using Restriction Enzymes
- Expand short DNA fragments (e.g. by PRC) Polymerase Chain Reaction
- Two strategies

1) Maxam-Gilbert method

- mark (radioactively by ^{32}P) at the 5' end DNA fragments
- cut out chemically the 3' end from each basis onward
- keep only the marked fragments

2) Sanger method

- single strand fragments are let to polymerize in 4 kinds of different environments, i.e. in the presence of A, T, C, G, plus either A', or T', or C', or G', respectively
- when either A', or T', or C', or G' is incorporated copying is blocked
- primer that let the polymerization start or dideoxynucleotides A', T', C', G' are marked (e.g. with a fluorescent dye)

- Electrophoresis and radiography

From marked positions one gets all possible locations of each of the four bases along the fragment from which its sequence is reconstructed

The Maxam-Gilbert method

marked sequence \Rightarrow 5'-³²P-GCTACGTA-3'

Resulting radioactive fragments

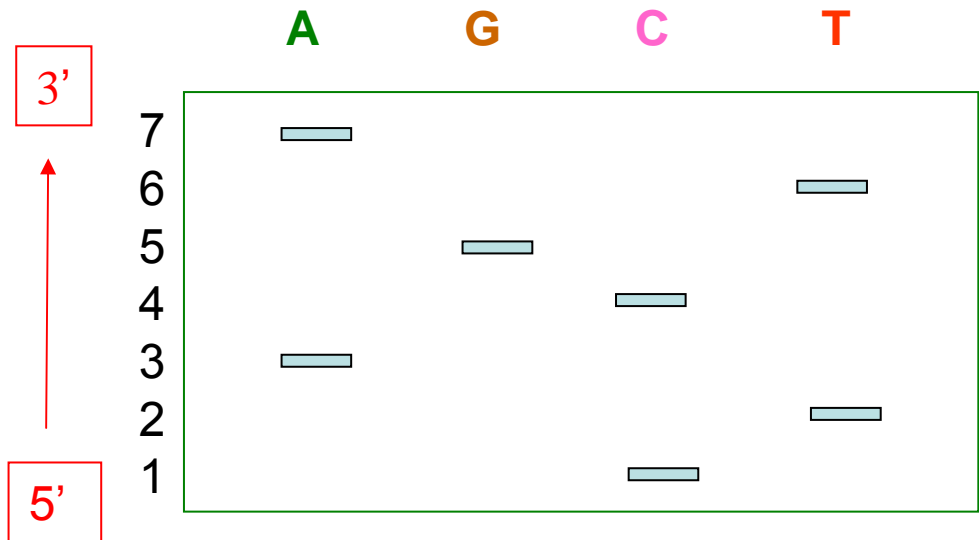
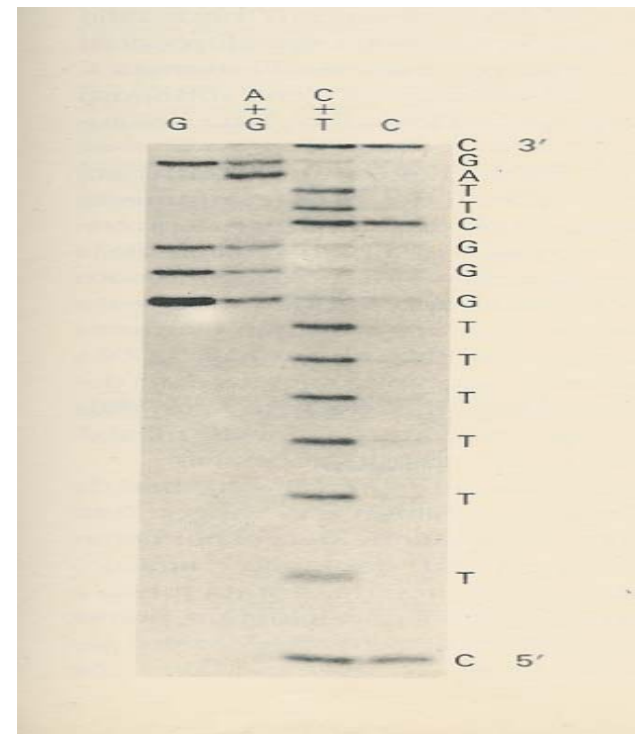
Cleavage @ **A** ³²P-GCT
³²P-GCTACGT

Cleavage @ **G** ³²P-GCTAC

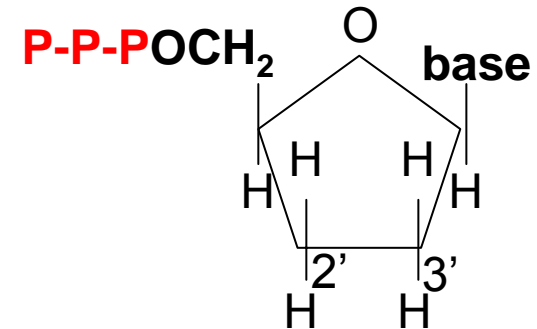
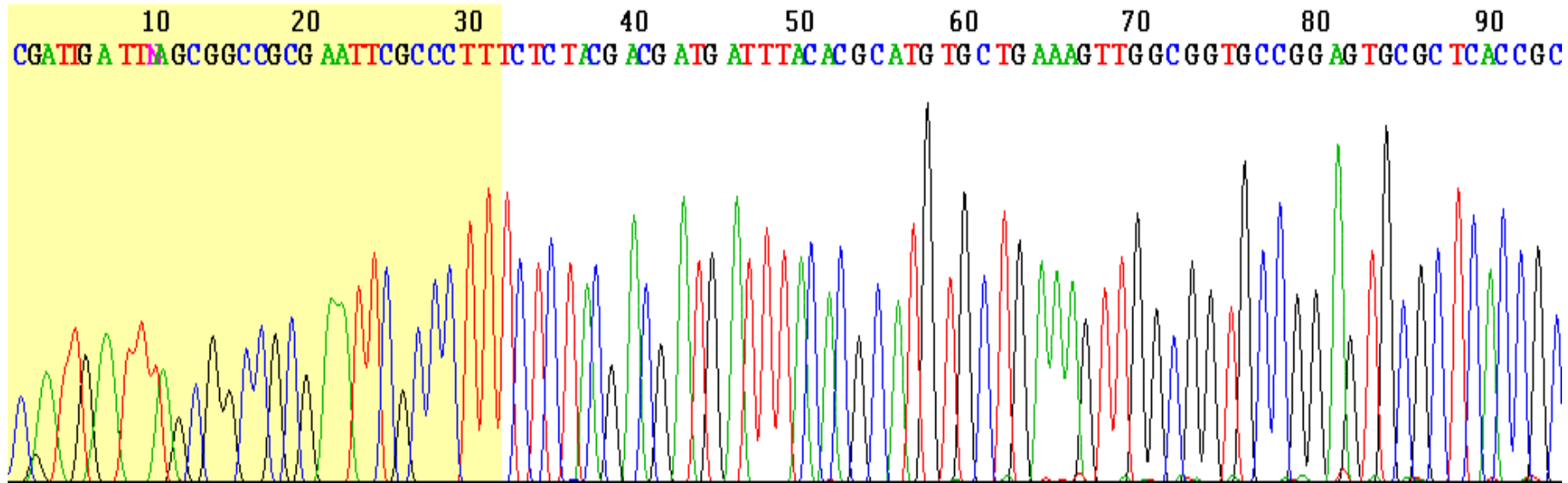
Cleavage @ **C** ³²P-G
³²P-GCTA

Cleavage @ **T** ³²P-GC
³²P-GCTACG

Radiography



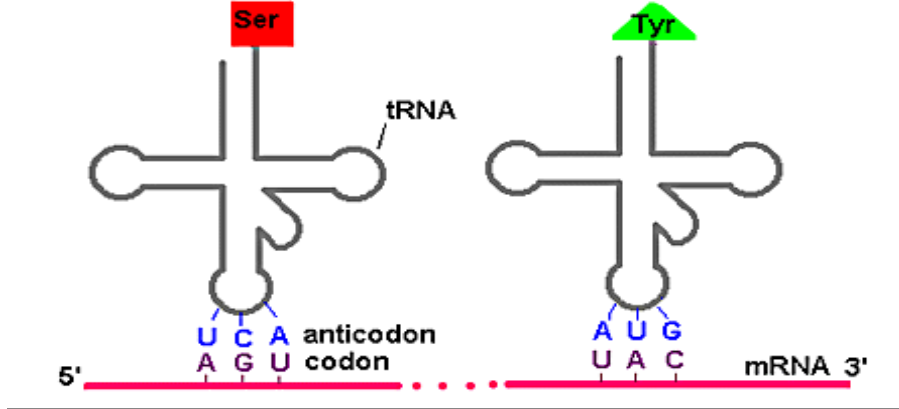
The Sanger method



dideoxynucleotides

A' → ddAATP
G' → ddGATP
C' → ddCATP
T' → ddTATP

chain-terminating nucleotides, lacking both 3'-OH groups required for the formation of a phosphodiester bond between two nucleotides during DNA strand elongation



20 amino acids plus a stop
(beginning is Methionine)

What's the reason
for these numbers?
Perhaps QC? Patel

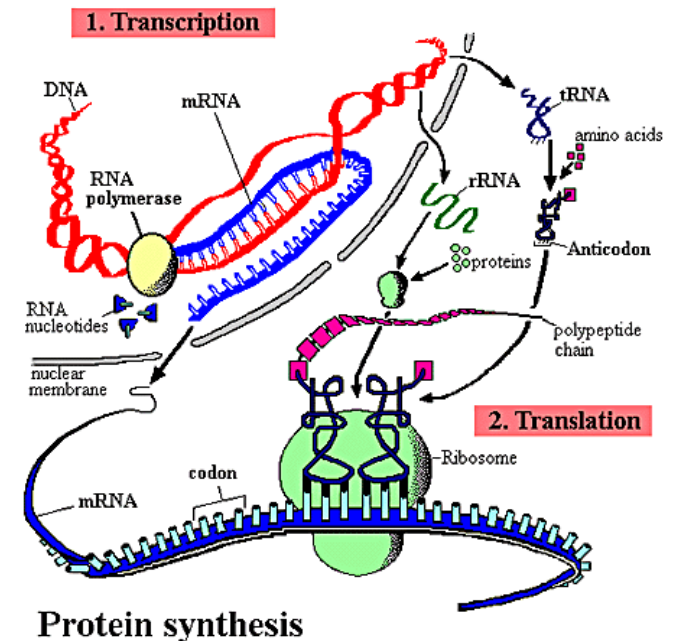
		2nd base in codon				3rd base in codon
1st base in codon		U	C	A	G	
	U	Phe Phe Leu Leu	Ser Ser Ser Ser	Tyr Tyr STOP STOP	Cys Cys STOP Trp	U C A G
C	C	Leu Leu Leu Leu	Pro Pro Pro Pro	His His Gln Gln	Arg Arg Arg Arg	U C A G
	A	Ile Ile Ile Met	Thr Thr Thr Thr	Asn Asn Lys Lys	Ser Ser Arg Arg	U C A G
G	G	Val Val Val Val	Ala Ala Ala Ala	Asp Asp Glu Glu	Gly Gly Gly Gly	U C A G

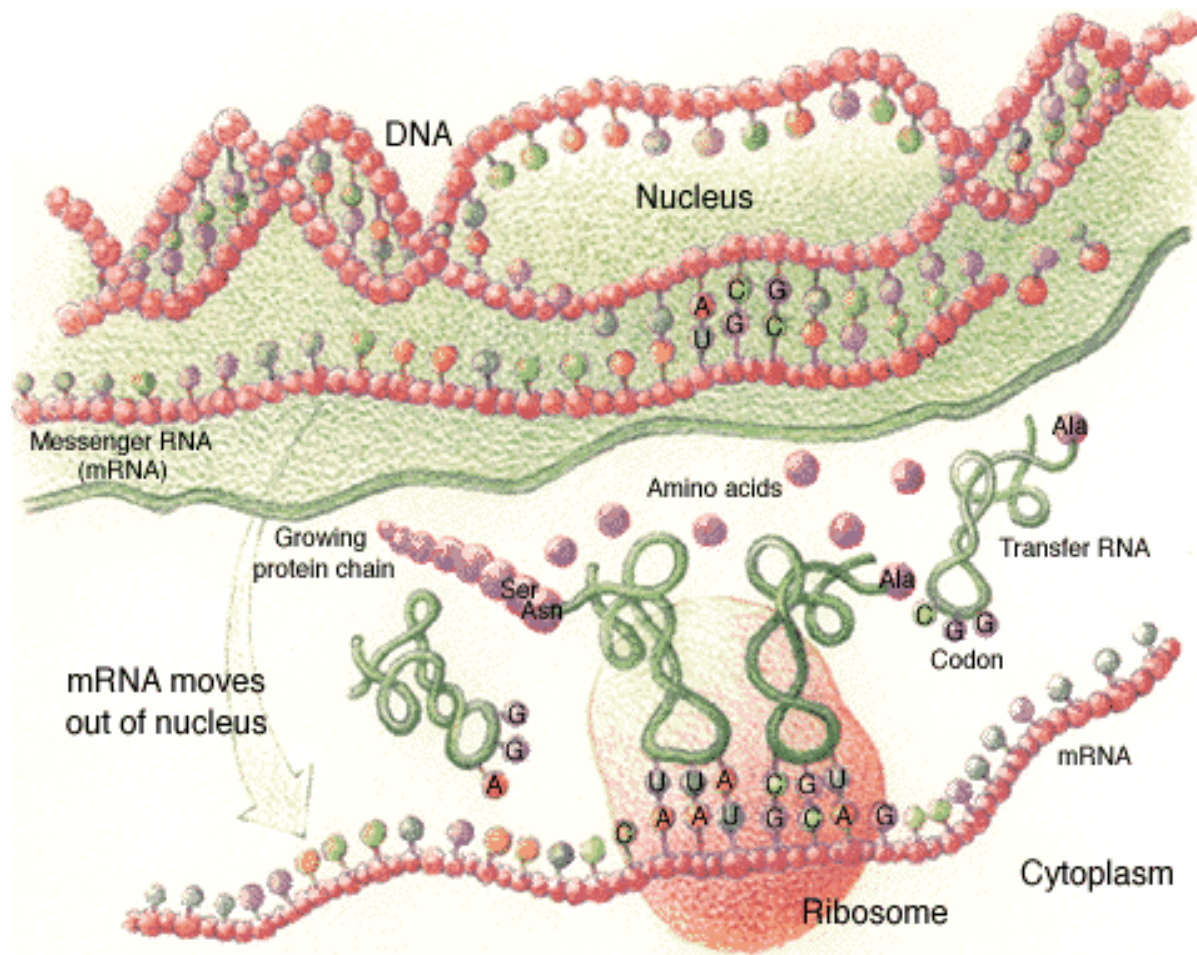
The Genetic Code

Three bases make a codon

4 bases are A T G C

Adenosine Thymine Guanine Cytosine
Uracyle

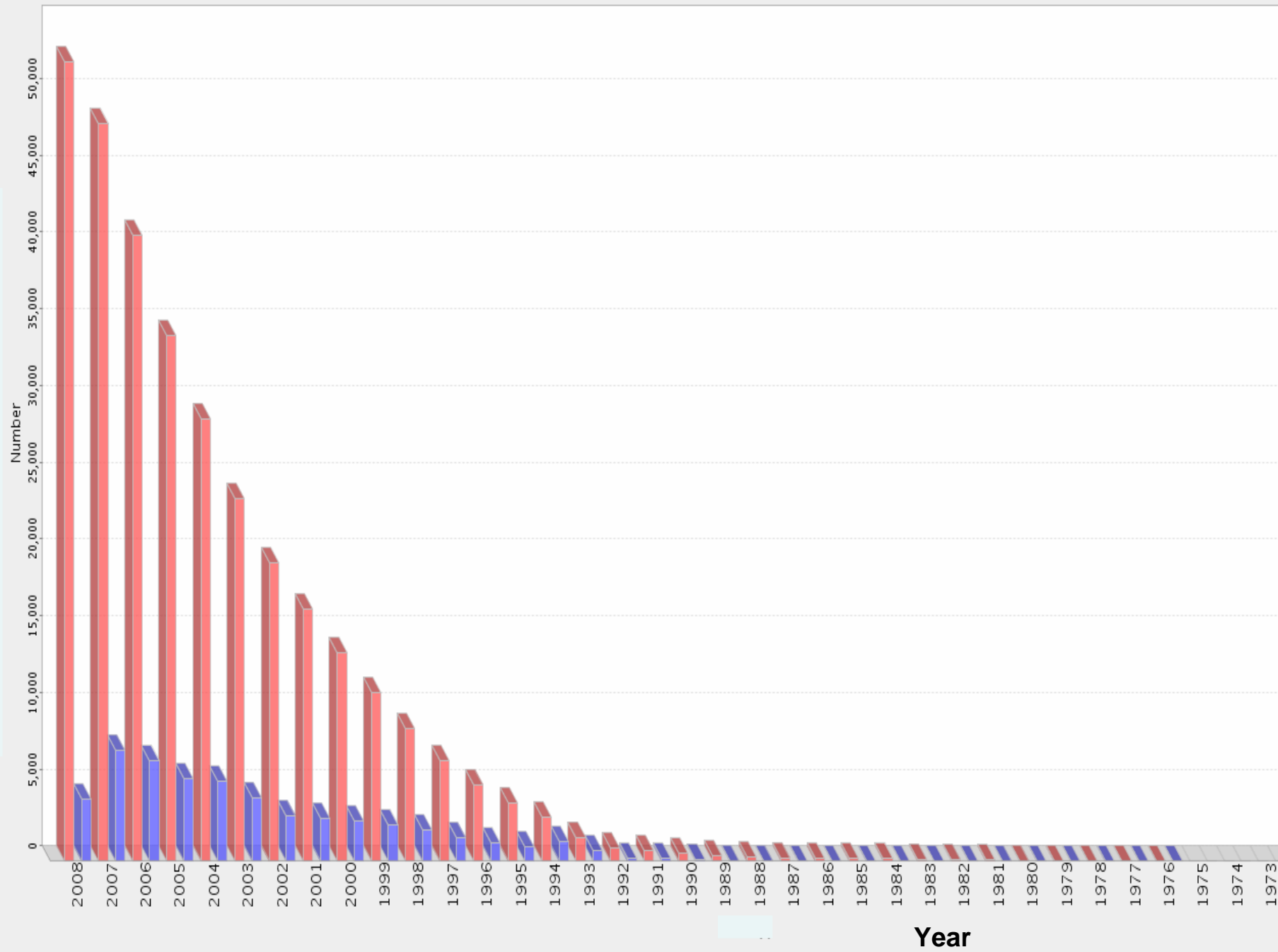




PDB Current Holdings Breakdown

		Molecule Type				
		Proteins	Nucleic Acids	Protein/NA Complexes	Other	Total
Exp. Method	X-ray	<u>41431</u>	<u>1058</u>	<u>1902</u>	<u>24</u>	<u>44415</u>
	NMR	<u>6447</u>	<u>814</u>	<u>138</u>	<u>7</u>	<u>7406</u>
	Electron Microscopy	<u>125</u>	<u>11</u>	<u>47</u>	<u>0</u>	<u>183</u>
	Other	<u>89</u>	<u>4</u>	<u>4</u>	<u>2</u>	<u>99</u>
	Total	<u>48092</u>	<u>1887</u>	<u>2091</u>	<u>33</u>	<u>52103</u>

Yearly Growth of Total Structures



Data

● Data types

- Amino acid sequences (proteins)
- Genomic sequences (DNA, RNA,...)
- 3D-structures of macromolecules
- Biochemical/Physiological
- Medical/Epidemiological
-

There exist about 60
DataBases, each
containig **trillions** of bits

<http://expasy.org/>

● Data handling

- Collecting/Recording
- Releasing/Validating/Curing
- Updating/Maintaining
- Mining
- Analyzing/Organizing

IN

OUT

- Metabolic pathways
- **siRNA/RNAi**
- Peptide antigens
- Protein interactions
- Kinase-Phosphate
- Transcription factors
- Disease Genes
- Protein database
-

● Data availability

- Need easy, standardized, free access to DataBanks
- Patent laws and regulations

Most important aspect in the production of antibodies or drugs is the design of peptide-antigens. A **peptide-antigen** is a small segment (15-18 amino acids) of the protein sequence of interest. These peptide-antigens can be used for immunization in order to produce antibodies against the protein or they can be used as a basis for small-molecule/drug targeting.

The **Peptide-Antigen** database <http://www.proteinlounge.com/> contains antigenic peptide targets **against all known protein** sequences throughout a variety of organisms.

An example



Homo sapiens

24225 Genes with 119846 Peptide Sequences



Mus musculus

39995 Genes with 212168 Peptide Sequences



Rattus norvegicus

19245 Genes with 95044 Peptide Sequences



Bos taurus

1169 Genes with 5809 Peptide Sequences



Danio rerio

1139 Genes with 5651 Peptide Sequences



Drosophila melanogaster

17673 Genes with 87744 Peptide Sequences



Anopheles gambiae

15297 Genes with 75342 Peptide Sequences



Caenorhabditis elegans

26843 Genes with 216976 Peptide Targets



Arabidopsis thaliana

15276 Genes with 75626 Peptide Sequences



Saccharomyces cerevisiae

5655 Genes with 28138 Peptide Sequences

New words for new concepts and needs, like

Proteomics

Genomics

Metabolomics

Reactomics

...

come into play.

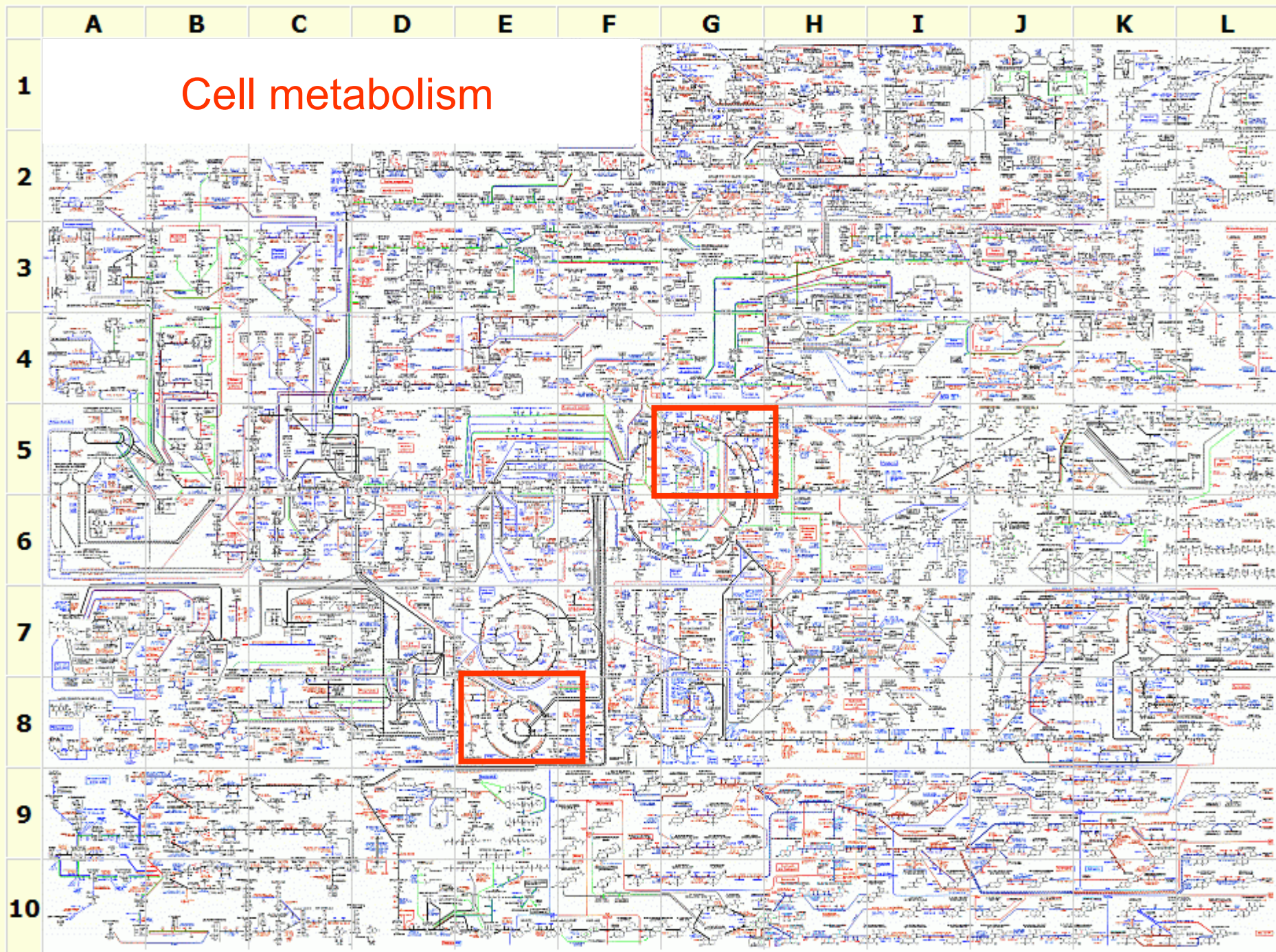
The suffix “**omics**” is alluding to the fact that are not just the single objects of each class (proteins, genoma, metabolic reactions,...) that matter, but their relations and interconnections

Cell reactome

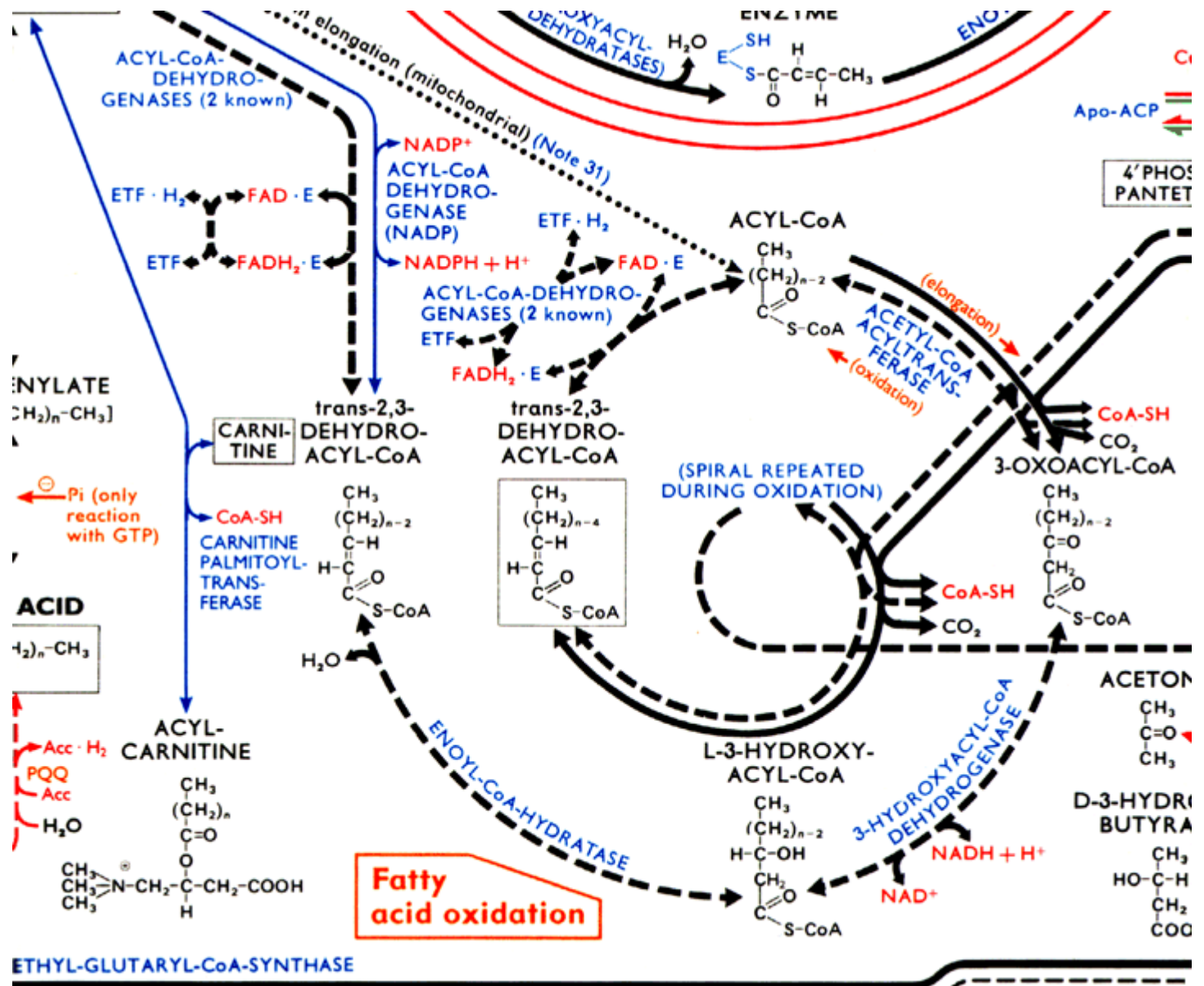
<http://www.reactome.org/>

<u>Apoptosis</u>	<u>Biological oxidations</u>	<u>Botulinum neurotoxicity</u>	<u>Cell Cycle Checkpoints</u>
<u>Cell Cycle, Mitotic</u>	<u>DNA Repair</u>	<u>DNA Replication</u>	<u>Electron Transport Chain</u>
<u>Gap junction trafficking and regulation</u>	<u>Gene Expression</u>	<u>HIV Infection</u>	<u>Hemostasis</u>
<u>Influenza Infection</u>	<u>Integration of energy metabolism</u>	<u>Lipid and lipoprotein metabolism</u>	<u>Membrane Trafficking</u>
<u>Metabolism of amino acids</u>	<u>Metabolism of carbohydrates</u>	<u>Metabolism of nitric oxide</u>	<u>Metabolism of non-coding RNA</u>
<u>Metabolism of vitamins and cofactors</u>	<u>Nucleotide metabolism</u>	<u>Porphyrin metabolism</u>	<u>Pyruvate metabolism and TCA cycle</u>
<u>Post-translational protein modification</u>	<u>Regulation of beta-cell development</u>	<u>Regulatory RNA pathways</u>	<u>Signaling by BMP</u>
<u>Signaling by EGFR</u>	<u>Signaling by FGFR</u>	<u>Signaling in Immune system</u>	<u>Signaling by Insulin receptor</u>
<u>Signalling by NGF</u>	<u>Signaling by Notch</u>	<u>Signaling by Rho GTPases</u>	<u>Signaling by TGF beta</u>
<u>Signaling by VEGF</u>	<u>Signaling by Wnt</u>	<u>Synaptic Transmission</u>	<u>Telomere Maintenance</u>
<u>Transcription</u>	<u>Translation</u>	<u>mRNA Processing</u>	

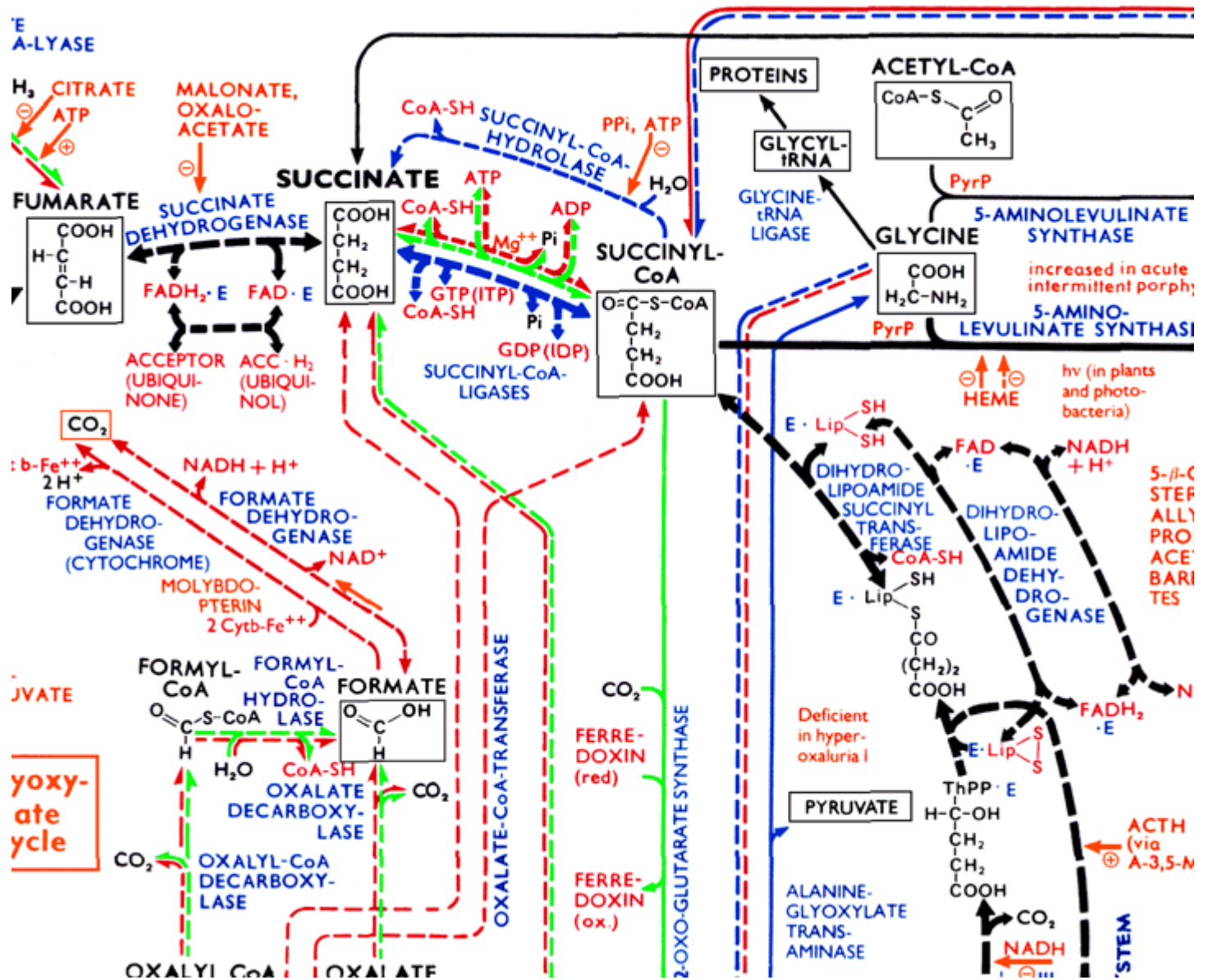
Cell metabolism



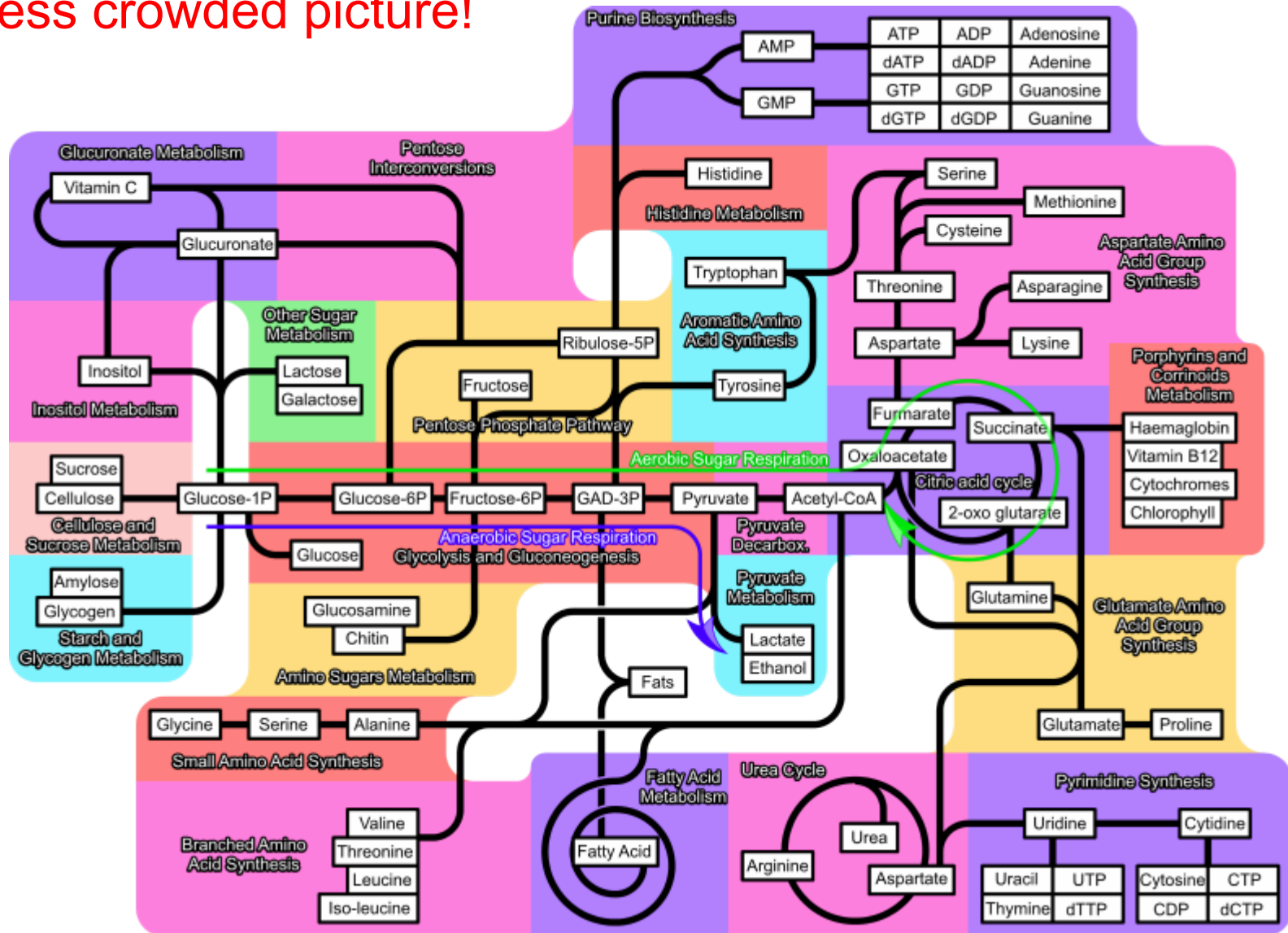
E8



G5



A less crowded picture!



METABOLIC NETWORKS

Metabolism of eukaryotic cells

~5000-6000 enzymatic reactions

~3000 metabolites

most simple:

Red blood cells

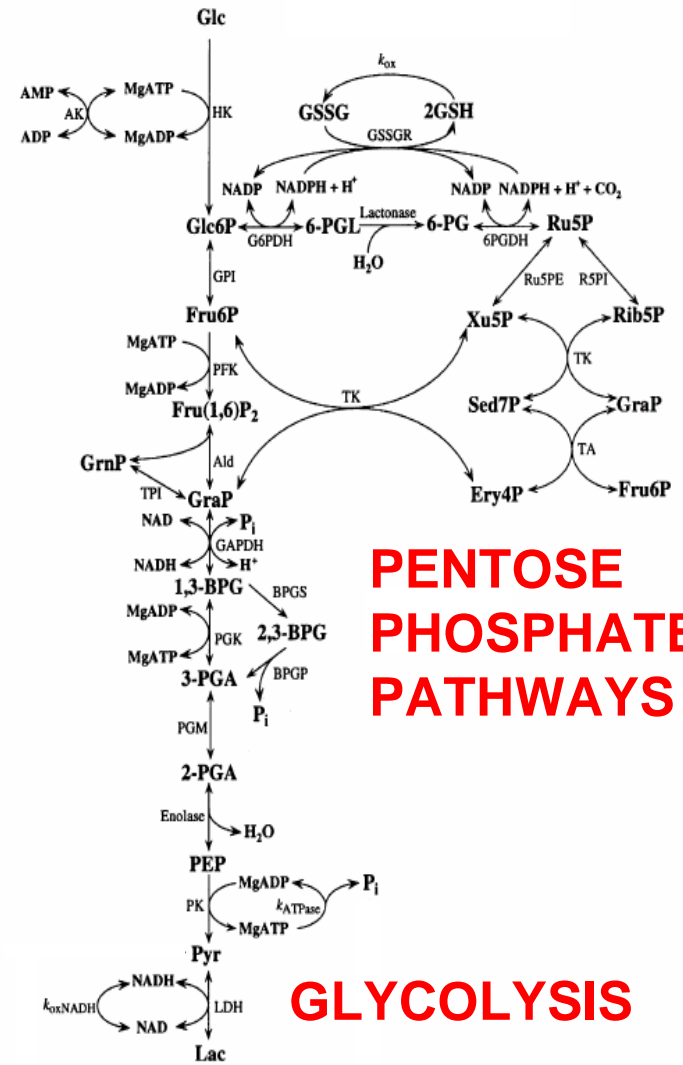
Model system for

Calculation of dynamical properties of whole pathways based on the kinetic properties of single enzymes

metabolite concentration

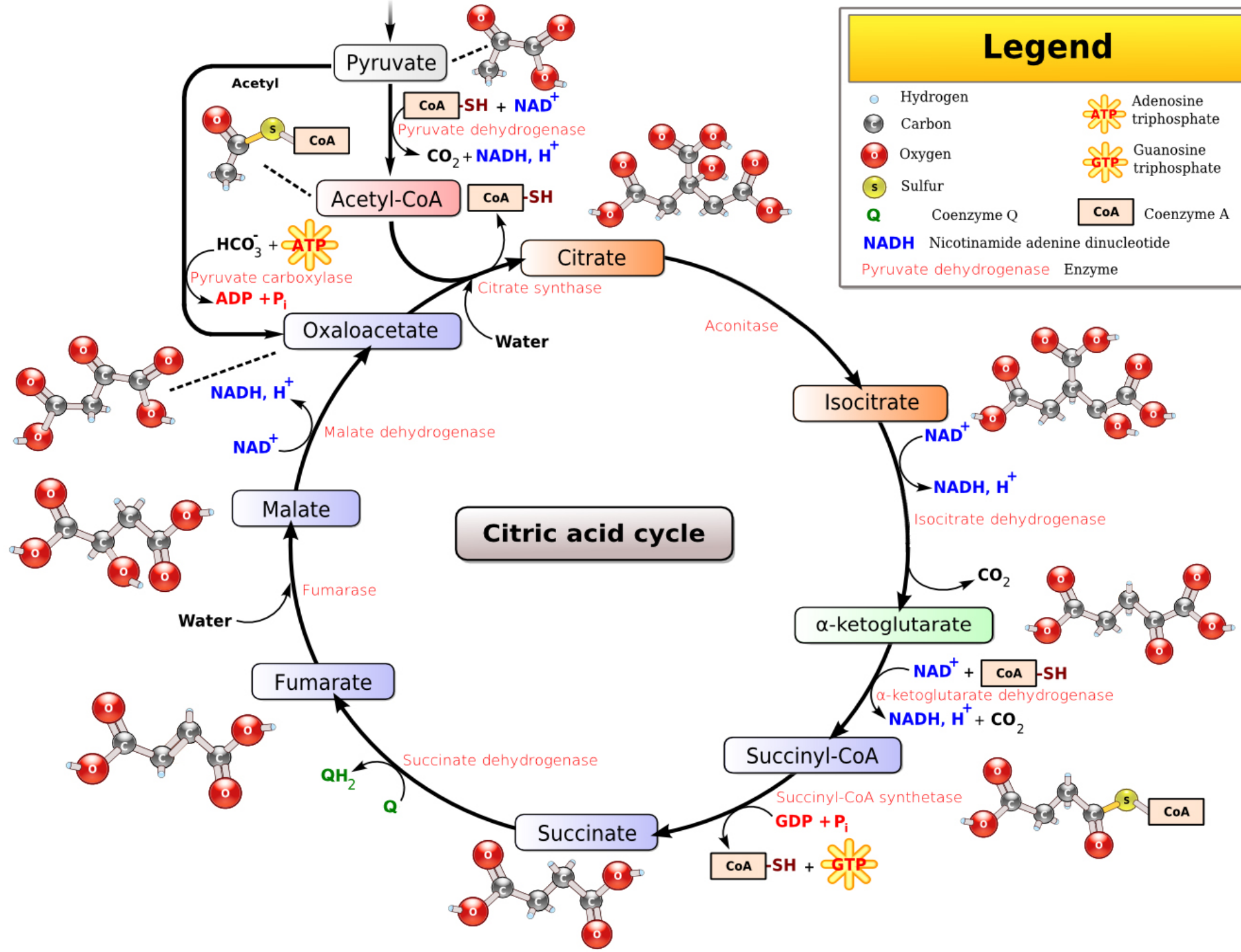
~ 1 μ M: $10^8 - 10^9$ molecules/cell

for most substances

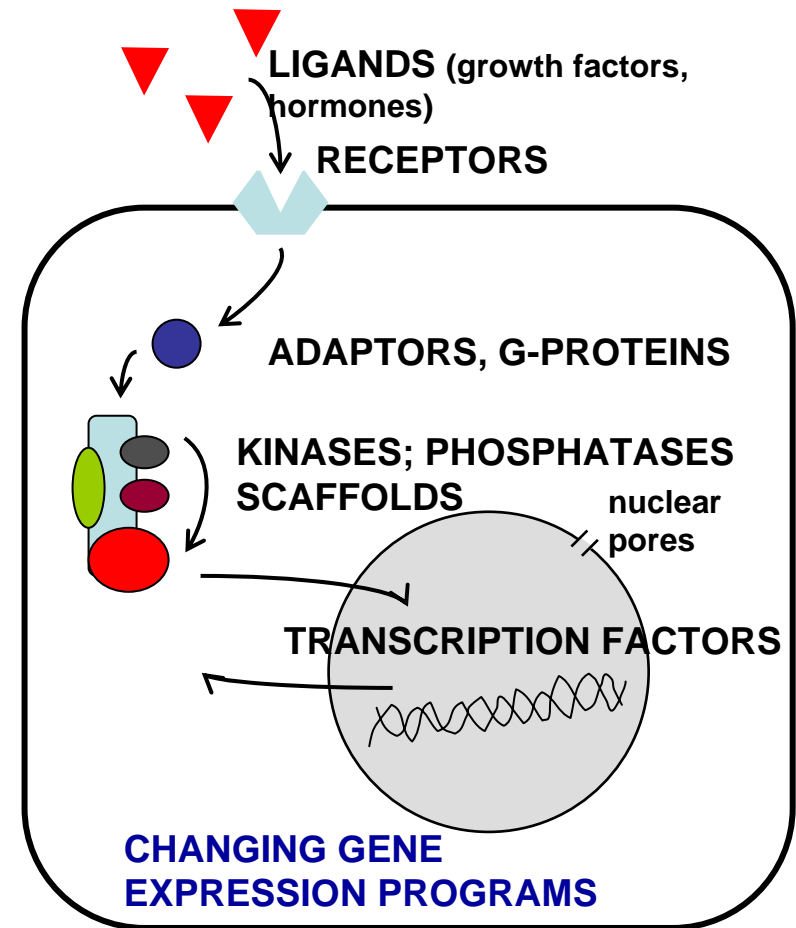
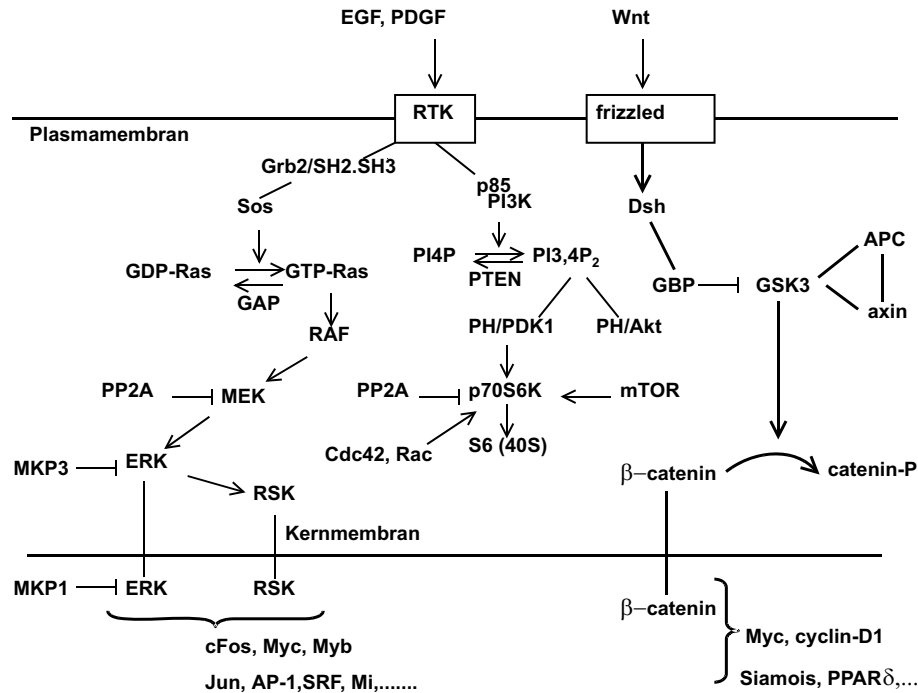


Legend

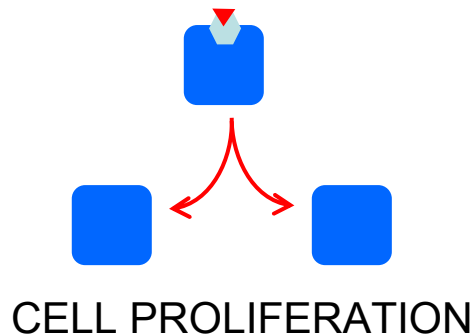
	Hydrogen		Adenosine triphosphate
	Carbon		Guanosine triphosphate
	Oxygen		Coenzyme A
	Sulfur		
	Coenzyme Q		
	Nicotinamide adenine dinucleotide		
	Pyruvate dehydrogenase		Enzyme



SIGNAL TRANSDUCTION

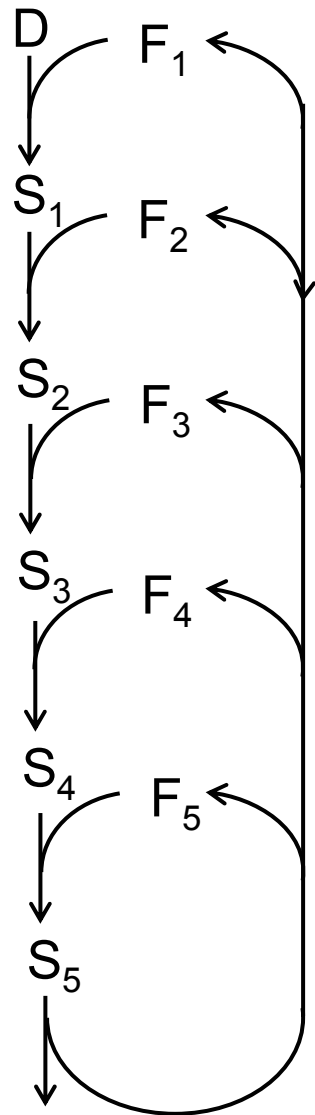


concentrations of signaling molecules: ~100 nM
 $10^4 - 10^5$ molecules/cell



DNA-REPAIR

Nucleotide Excision Repair (NER) Global Genom Repair Pathway

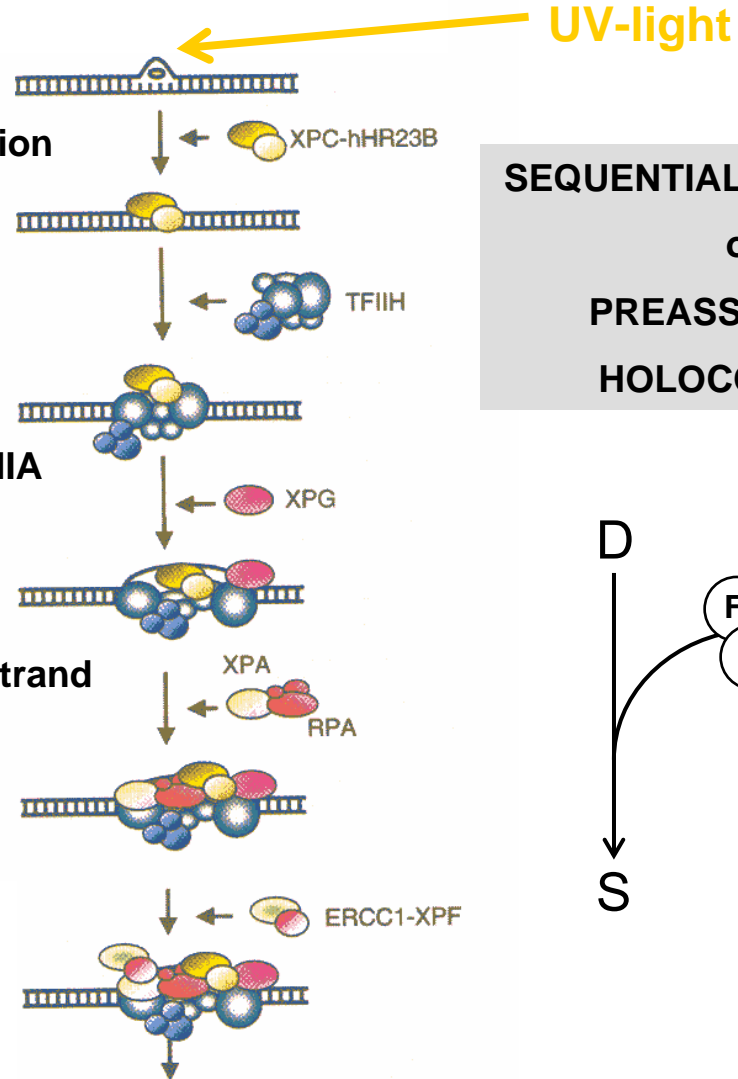


damage recognition
by XPC

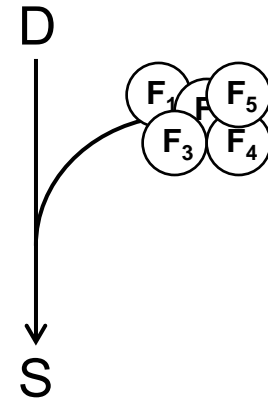
DNA helix
unwinding by TFIIA

Cutting of DNA-strand
and removing of
damaged region

Repair: Synthesis of new DNA strand, ~ 10 nucleotides long

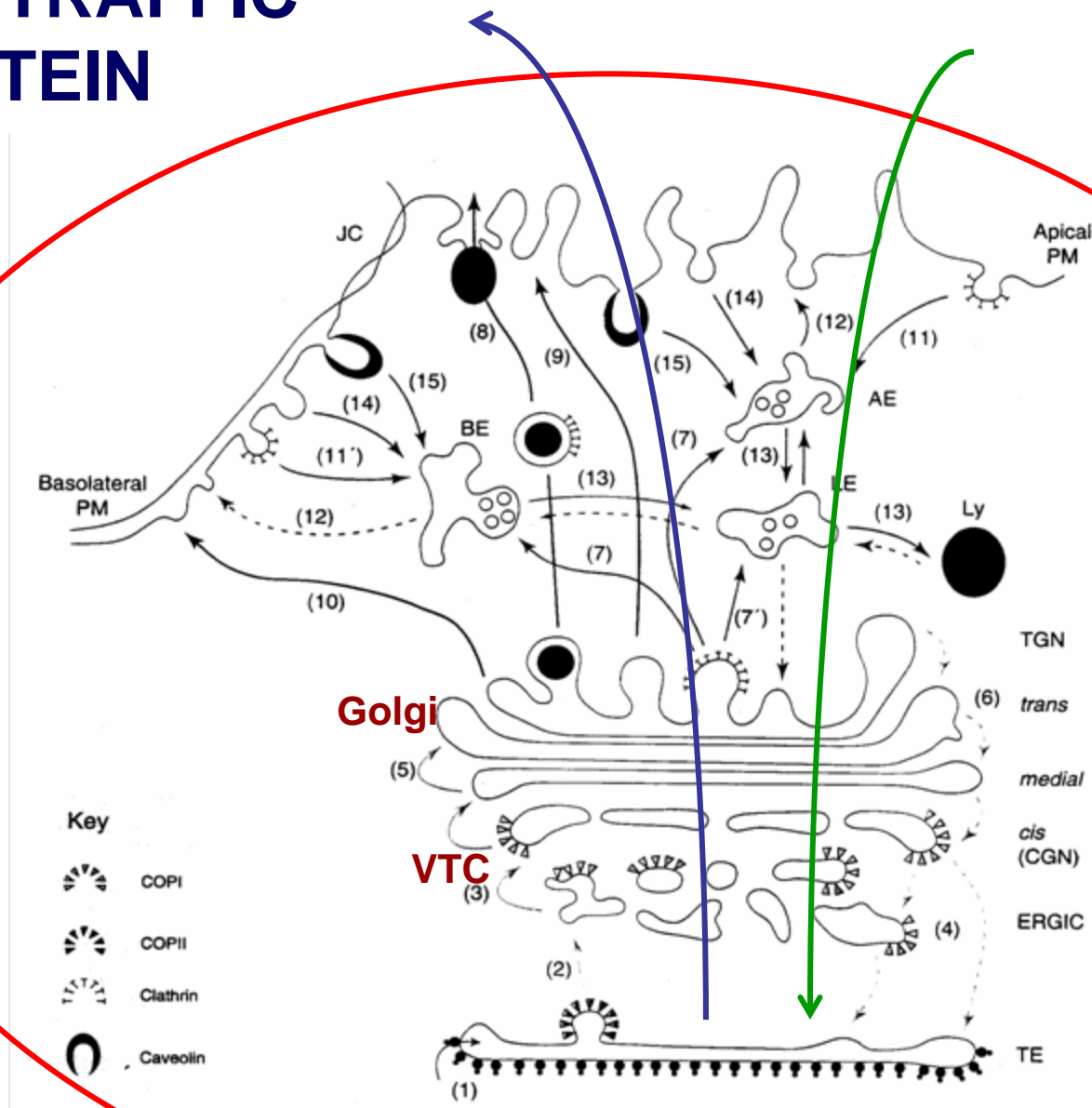


SEQUENTIAL MECHANISM
or
PREASSEMBLED
HOLOCOMPLEX



Mutations in NER-proteins: photosensitivity and sunlight-induced skin cancer

PROTEIN TRAFFIC AND PROTEIN SORTING



Dynamics of Cell Reactions

Network of very many interconnected
sub-networks of related biochemical reactions

Barabasi

- Non-linear diffusive (of the heat type) PDE's
- Small number of some of the involved molecular species 10^2 - 10^5 /cell
 - large number-fluctuations
 - competition with thermal fluctuations
- Events are discrete with a certain degree of randomness
- Multiplicity of time scales

Gillespie

Realistic (?) Single Cell Simulation

Even for the smallest living organism, *Mycoplasma Genitalium*

100 genes

500 proteins

100 regulatory elements

10 cellular compartments



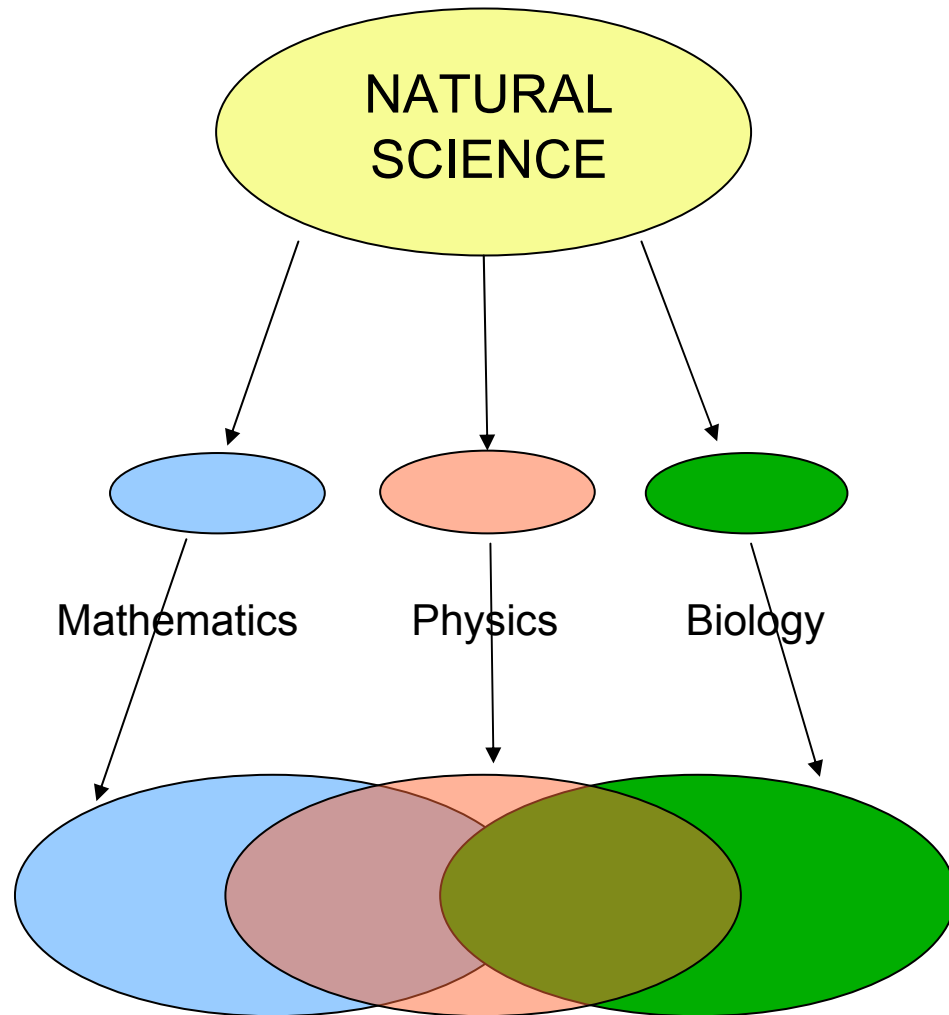
The **E-Cell Simulation Environment** is an object-oriented software suite for modelling, simulation, and analysis of large scale complex systems such as biological cells.

Cellular processes and typical computational approaches.

Process type	Dominant phenomena	Typical computation schemes
Metabolism	Enzymatic reaction	DAE, S-Systems, FBA
Signal transduction	Molecular binding	DAE, stochastic algorithms (StochSim and Gillespie, for example), diffusion-reaction
Gene expression	Molecular binding, polymerization, degradation	OOM, S-Systems, DAE, Boolean networks, stochastic algorithms
DNA replication	Molecular binding, polymerization	OOM, DAE
Cytoskeletal	Polymerization, depolymerization	DAE, particle dynamics
Cytoplasmic streaming	Streaming	Rheology, finite-element method
Membrane transport	Osmotic pressure, membrane potential	DAE, electrophysiology

DAE—differential-algebraic equations (rate equation-based systems), FBA—flux balance analysis, and OOM—object-oriented modeling (includes E-Cell's substance-reactor model, or SRM).

Can we hope to attack such fantastically complicated problems?
Probably yes,
looking back at the development of Natural Science



**FROM PROGRESSIVE
SEPARATION**




**TO OVERLAPPING
AREAS OF INTEREST**




- Similar Mathematical Description and Algorithms

 Cross-fertilization among nearby Research Fields

Statistical Physics Quantum Field Theory  Structure of Macro-molecules Econo-physics

Stochastic Methods

Meteorology Fluidodynamics  Metabolic networks Turbulence Chaos

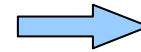
PDE & Stability Analysis

- Dealt with by Numerical Tools

 Advances in Computer Developments

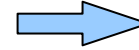
Numerical Simulations  Dedicated Computers

New Architectures

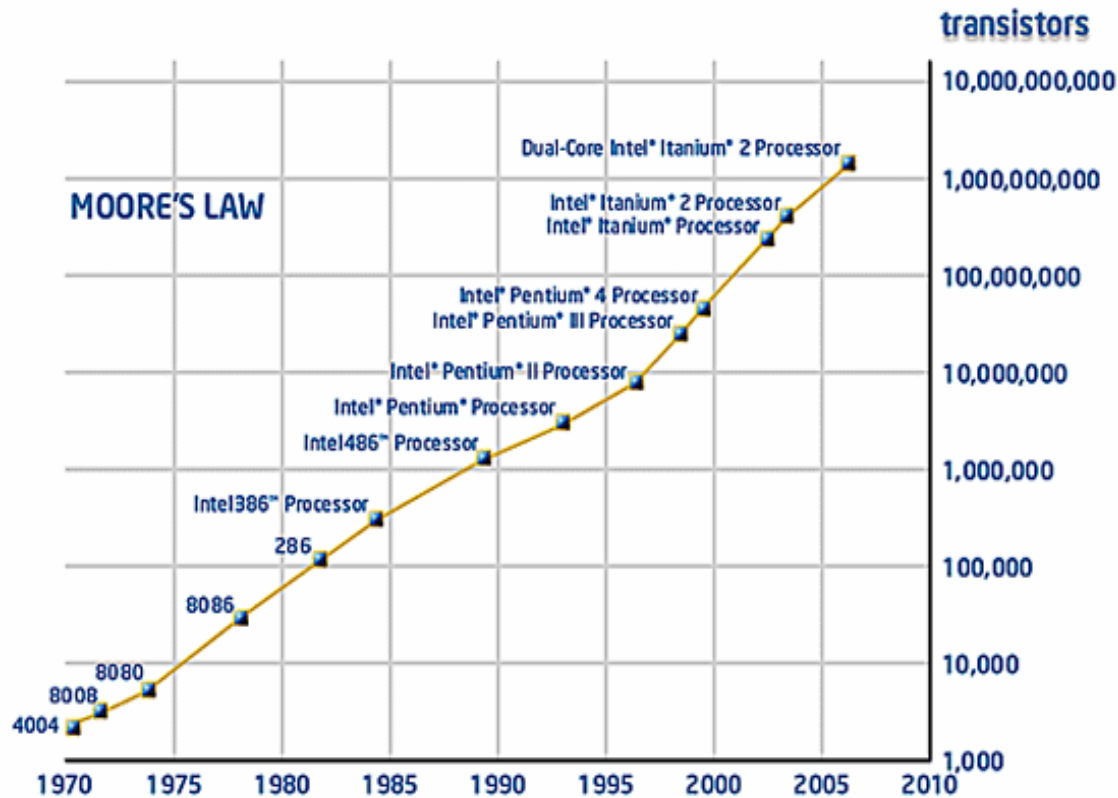


Parallel Platforms
PC-Clusters
GRID

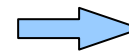
Exponential Increase of



CPU Time
Memory
Storing Capacity



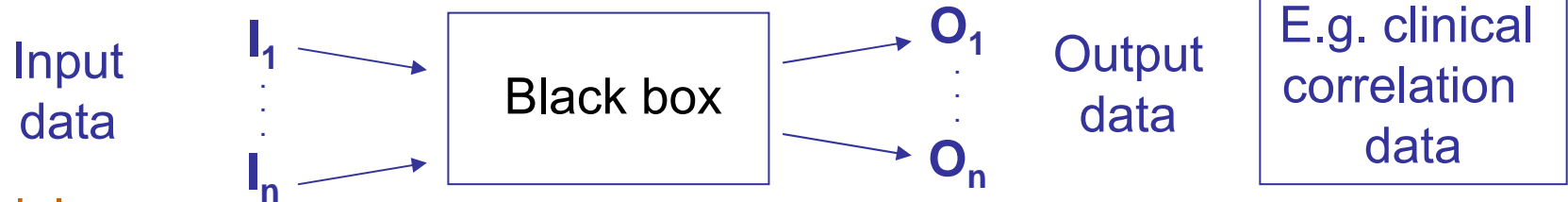
Wide Spectrum of Applications



Lattice QCD
Computational Astrophysics
Weather Forecasting
Genome Project
Computational Biology

Models and modellization Strategies

- Statistical Correlation



- Modular

Isolating functional modules, well separated

in time: protein folding **vs** cell duplication

in space: ribosomal protein synthesis **vs** cell translocation

chemically: metabolic pathways **vs** DNA transcription

logically: neuronal network **vs** electrical transmission along the axon

- Comprehensive (holistic)

Full simulations of a living cell

Degrees of freedom

Positions and velocities

Concentrations / reaction constants

Order parameters

Transmembrane potential / ionic current

Physiological / epidemiological data

.....

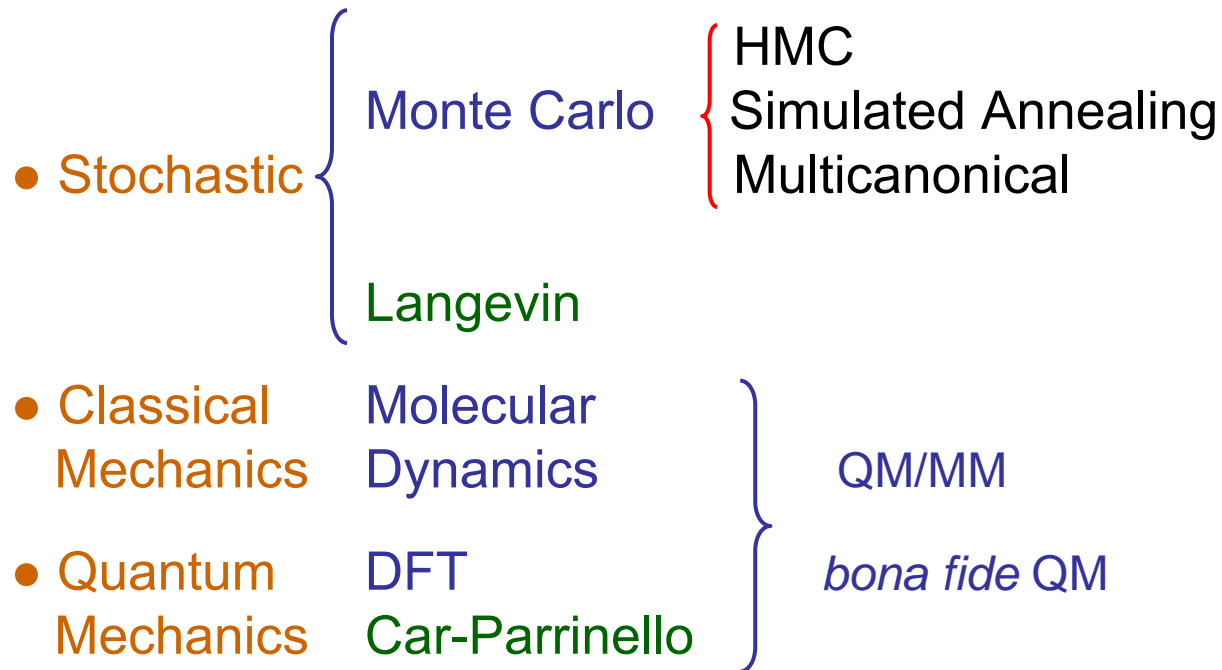
Mathematical Tools

- Differential equations

- Ordinary
- Partial

Metabolic processes
Regulatory processes

- Statistical-Mechanics-Inspired Algorithms



Fluids
Proteins/SG
Folding/Phase trans.

Cell growth

Protein dynamics
Cell membrane

Local recognition
Cell membrane

- Non-equilibrium Statistical Mechanics?

Einstein, Onsager, Tauschek
Gallavotti, Jona-Lasinio

Open, almost
stationary systems

III. What we would like to know
and/or to do

III. What we would like to know
and/or to do

Here is a (partial) list of wishes

- Protein folding and functioning
- Protein docking and recognition
- Immunological recognition
- Gene expression and regulation
- Metabolic networks
- System biology
- etc.
- Protein/DNA interactions
- Amyloid aggregation
- Memory and networking
- miRNA/siRNA
- Signal transduction
- Nano-bio devices
- etc.

**Not to talk about the ultimate goal,
of curing all possible diseases**

■ What we would like to know about METABOLIC NETWORKS

A

Structure → Dynamics

Network topology, kinetic properties, enzyme amounts

Steady states, transitions, oscillations, chaos

time scale of a living organism
 $10^{-3} - 10^2$ years

B

Physical constraints

Free energy changes, upper limits for concentrations

Biological function

ATP-production, special chemical conversions (e.g. hexoses into pentoses)
fitness properties

Structure

Network topology, kinetic properties, enzyme amounts

evolutionary time scale
 $10^8 - 10^9$ years

SIMULATION MODELS

metabolite concentrations

stoichiometric coefficients

reaction rates

$$\frac{dS_i}{dt} = \sum_{j=1}^r n_{ij} v_j$$
$$\frac{dS}{dt} = N \cdot v$$

$V = (V_1, \dots, V_r)$

$v = v(S, k)$

Attractors, Chaos (?)

Robustness

Recovery of function

Kinetic parameters

large number **10-1000** of variables

large number **10-1000** of equations

non-linearity

regulatory loops

separation of time scales

natural selection of kinetic parameters

■ What we would like to know about PROTEINS

+ primary structure → folding → function
linear 3D conform. switches

- predict geometry and dynamics of folding and conformational changes
3D times e.g. heme, rhodopsin
- predict function
motif conservation, structural similarity

+ evolution/selection → $\#10^7$ among $(\#10^2)^{20}$ possibilities
folding vs aggregation?

- understand mis-folding and aggregation
Mad cow (Prion), Amyloidosis (e.g. β -amyloid in Alzheimer disease)

+ recognition/docking
Ab vs Ag, ..., transcription factors, promoters, ...

- characterize macromolecules binding
- clarify molecular mimicry and auto-immune reactions

even tiny atomic displacements matter

SIMULATION MODELS

Coarse grained models

▣ how general is folding?

- Geometrical considerations
- Lattice models
- Statistical Mechanics

spin glasses

Atomistic models

▣ classical

- Minim. of config. energy (no entropy)
- Canonical/micro-canonical simulations
- Multi-canonical simulations

▣ QM/MM

“right” ensemble?
“right” thermodynamic variables?

▣ QM

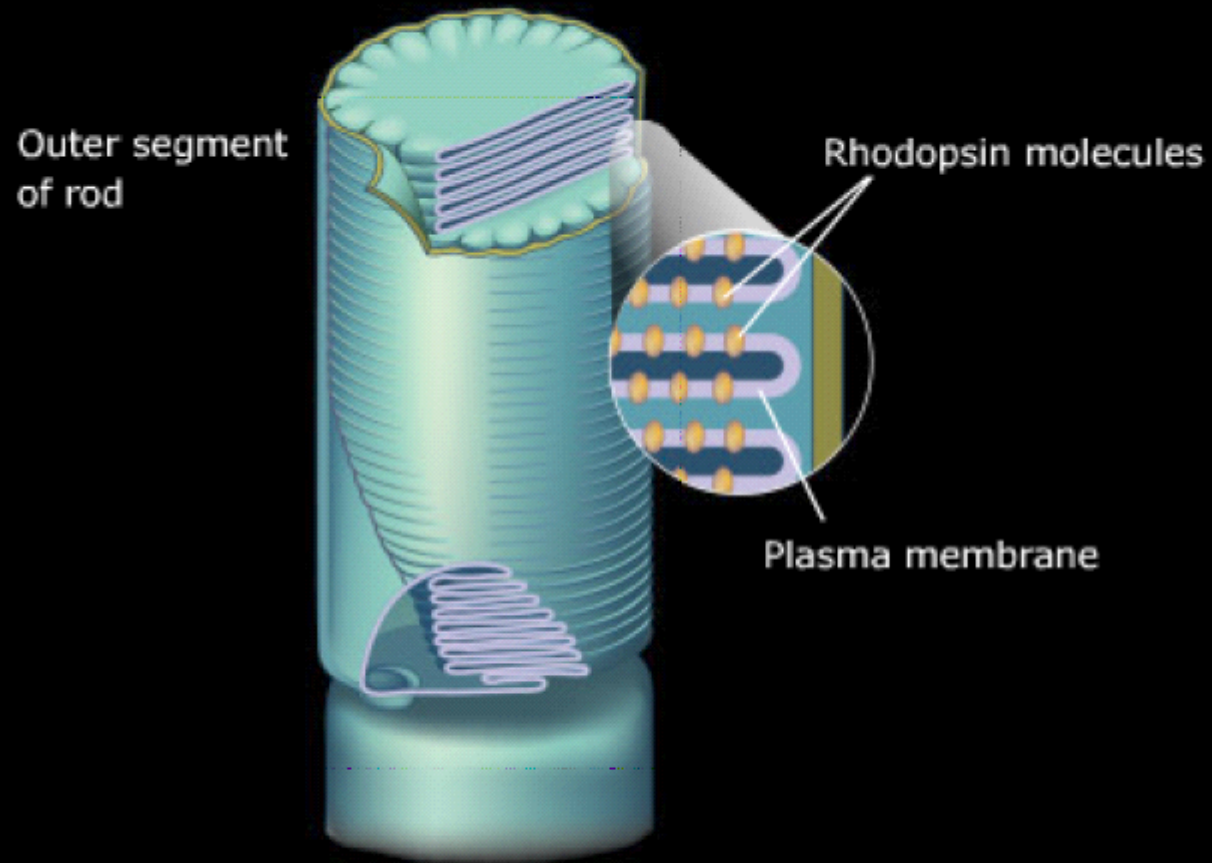
- Quantum Chemistry
- DFT
- Car-Parrinello dynamical simulations

even tiny atomic
displacements matter

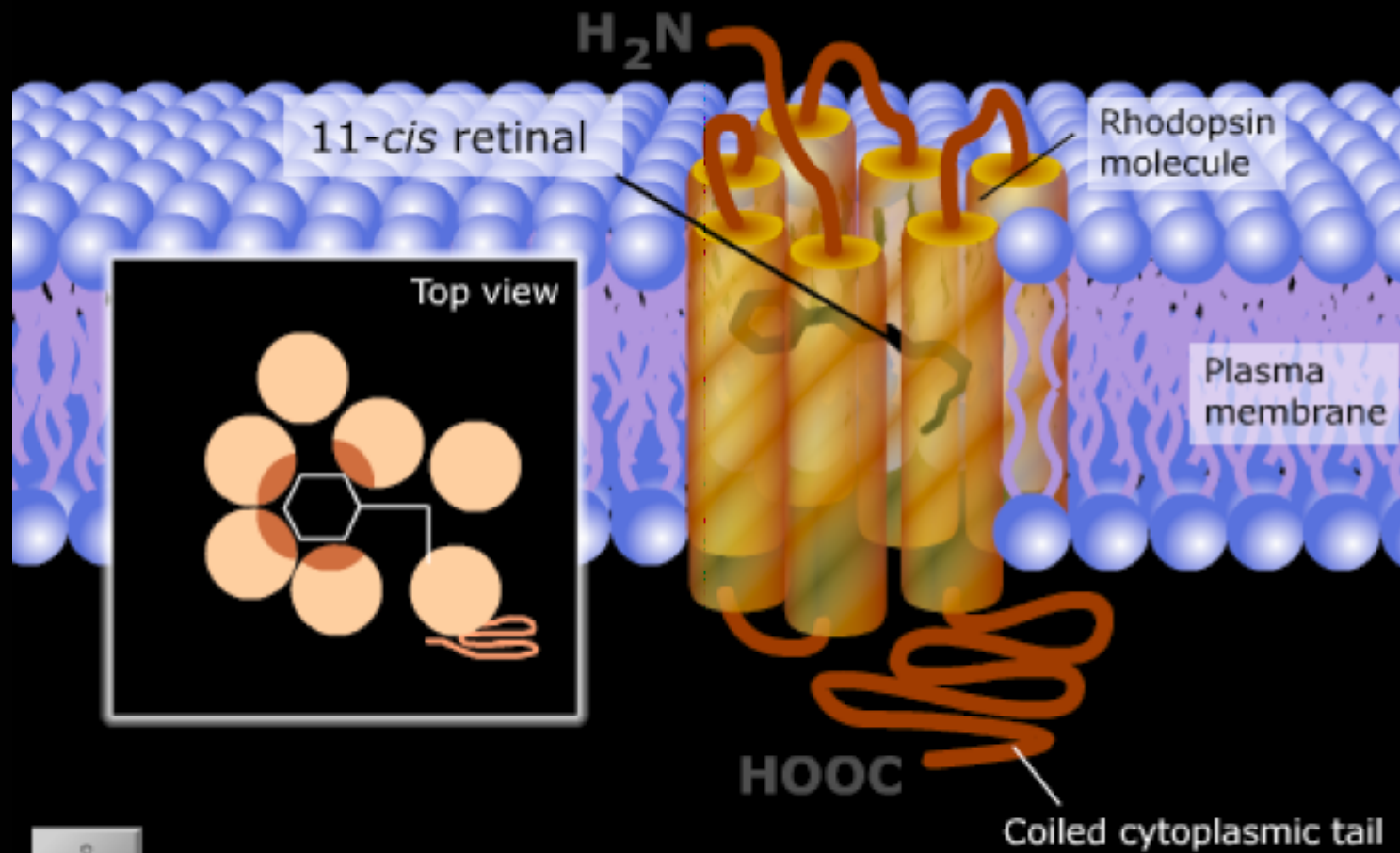
Two examples, among ∞ -ly many

- I. Cis \rightarrow Trans isomerization of 11-cis retinal
- II. Hemoglobin “breathing”

Photoisomerization of rhodopsin

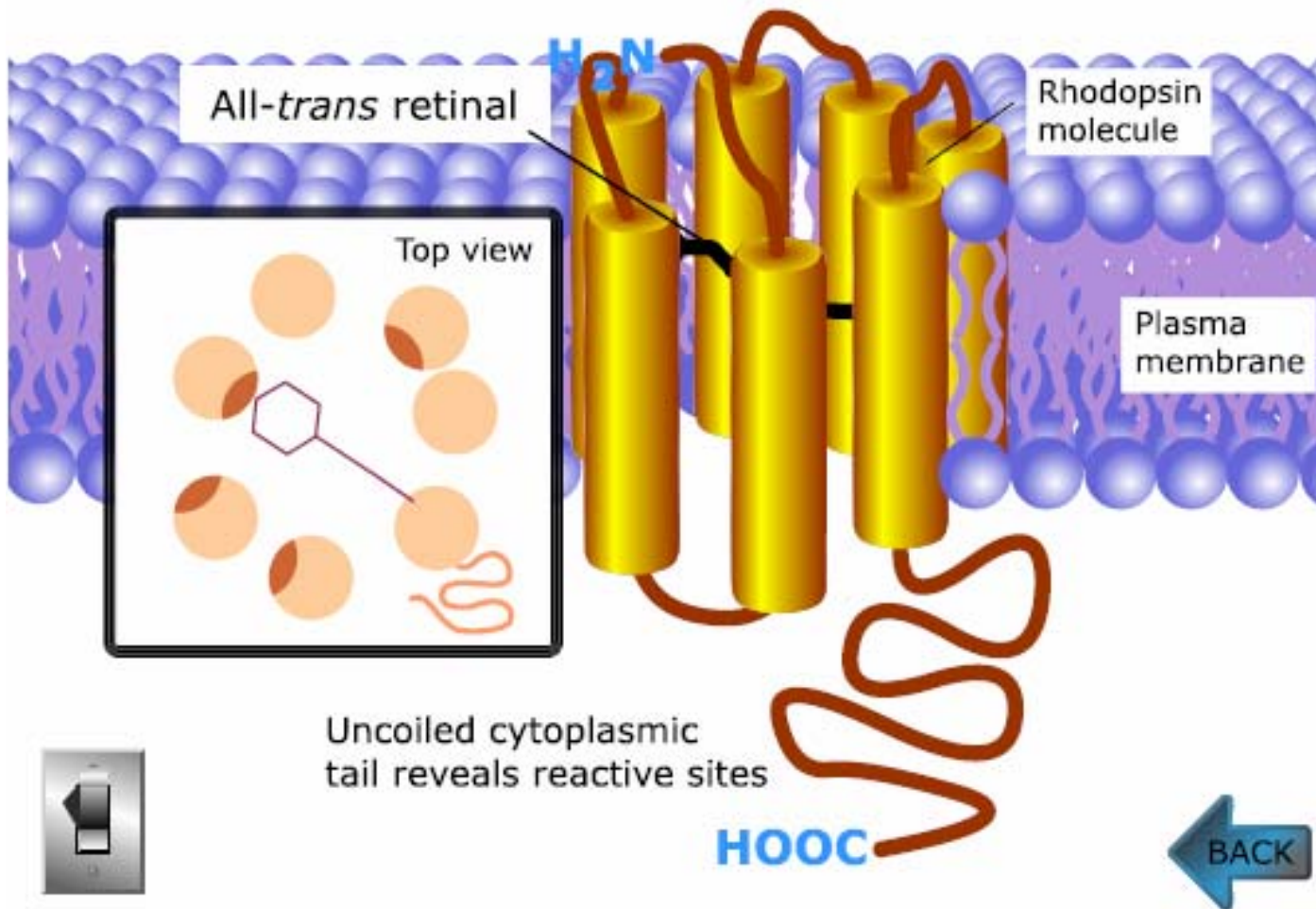


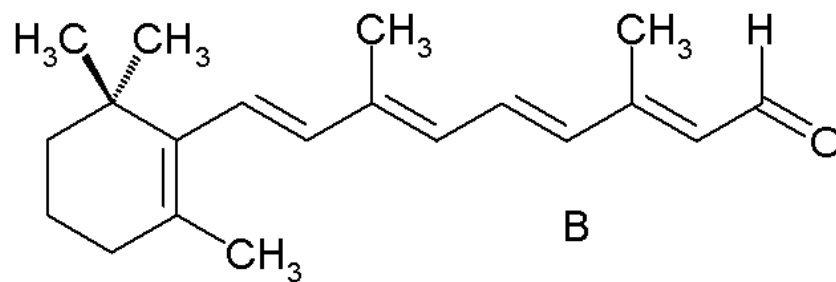
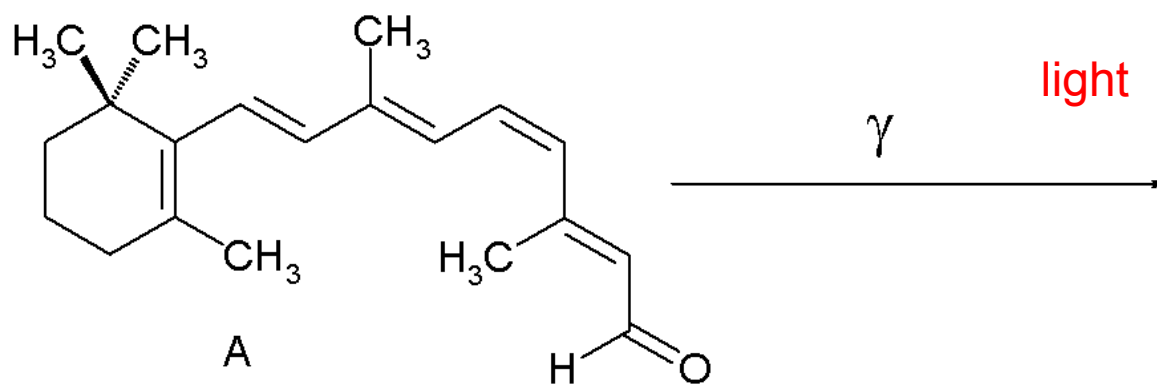
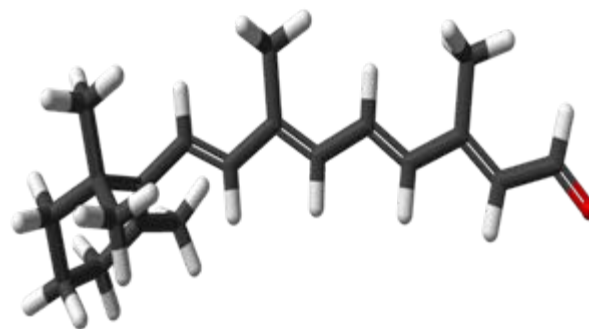
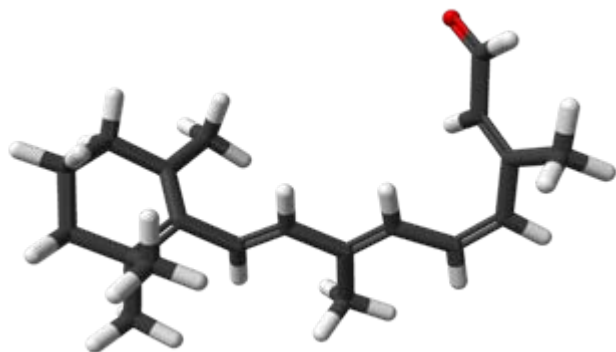
Photoisomerization of rhodopsin



BACK

Photoisomerization of rhodopsin



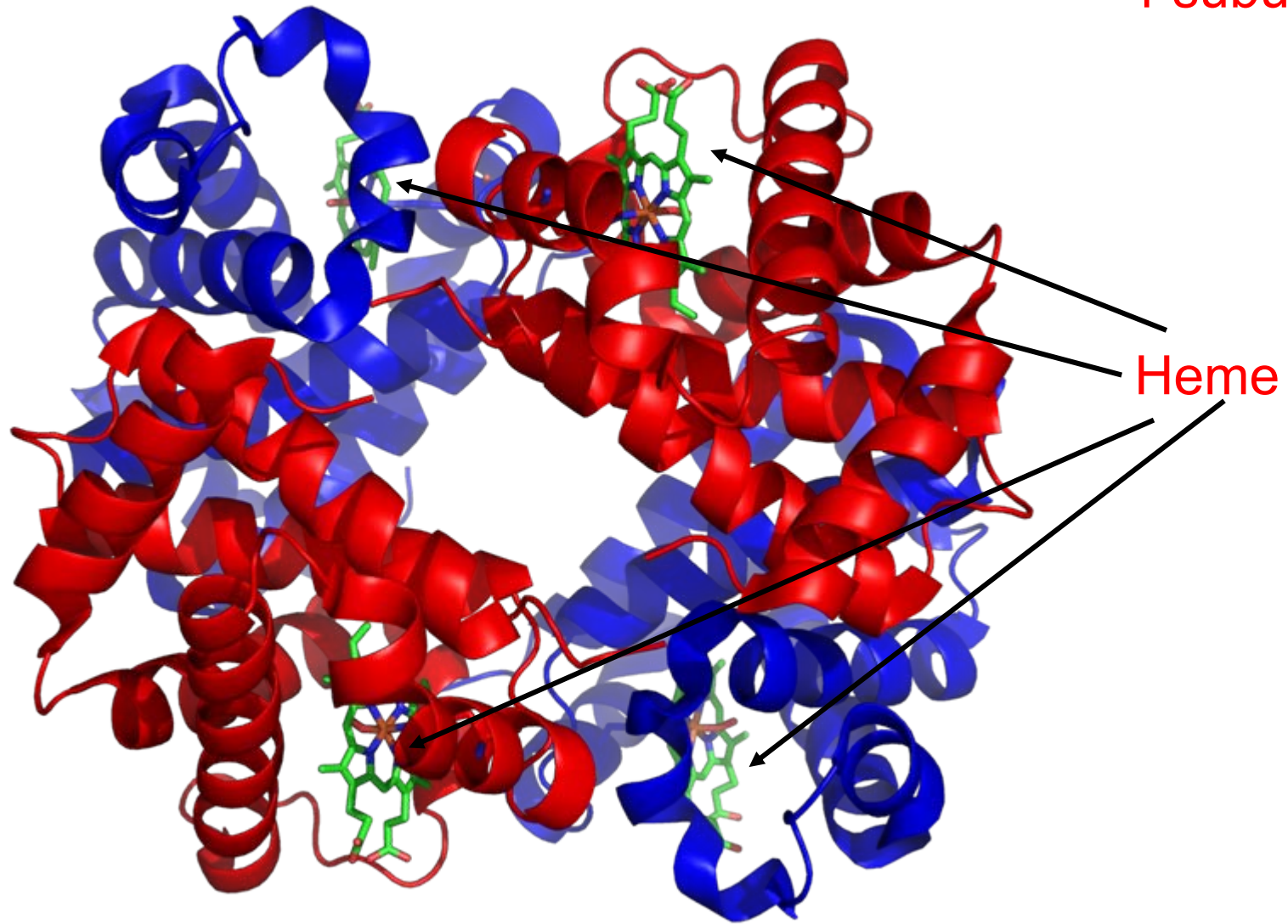


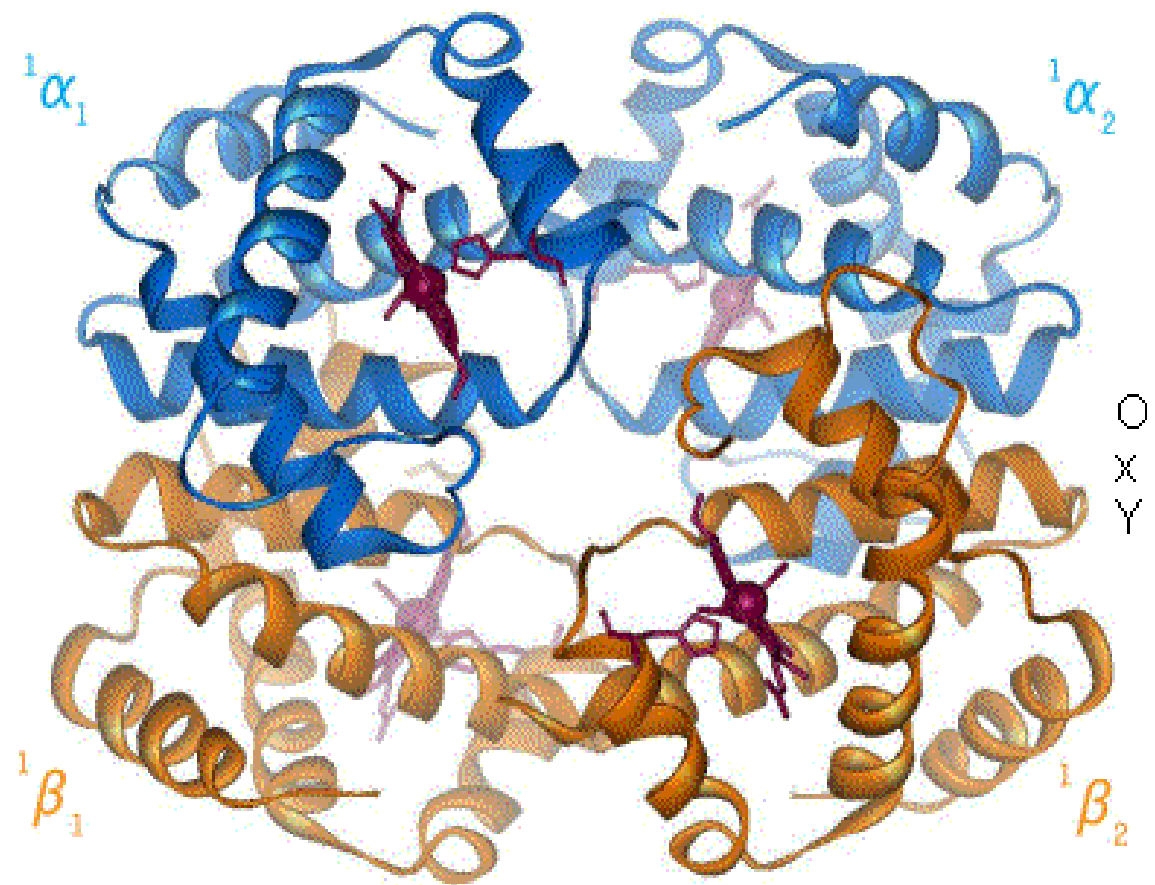
11-cis retinal

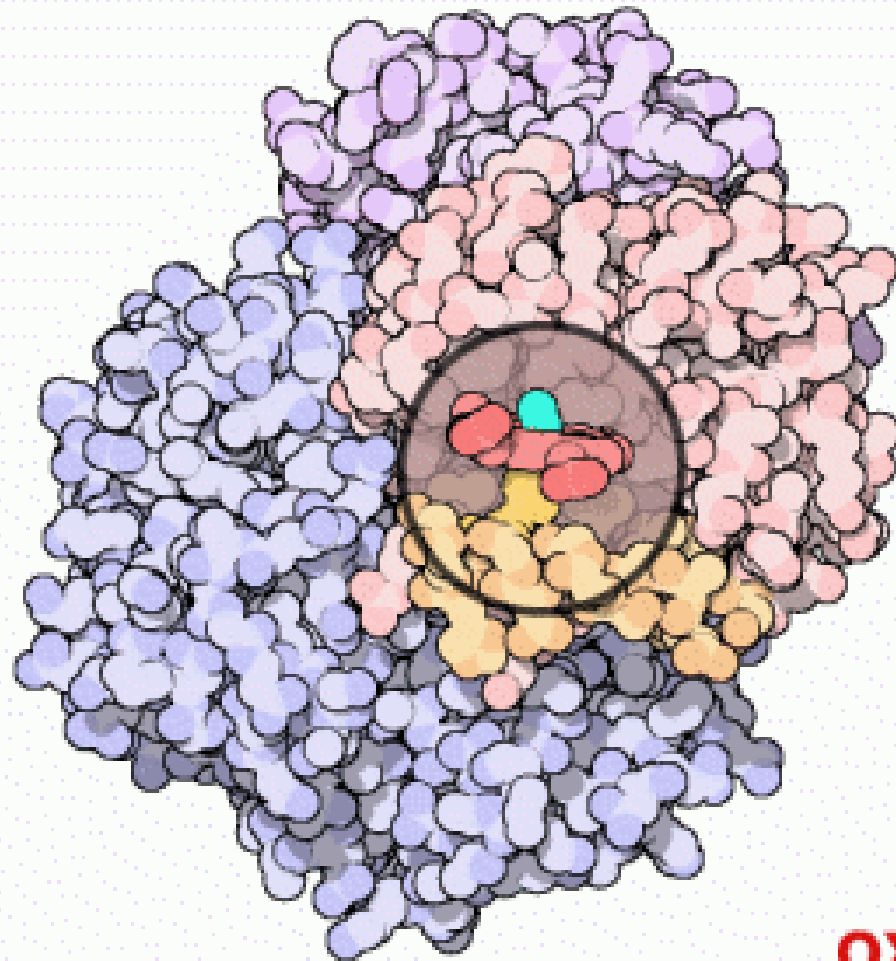
all-trans retinal

Hemoglobin

4 subunits

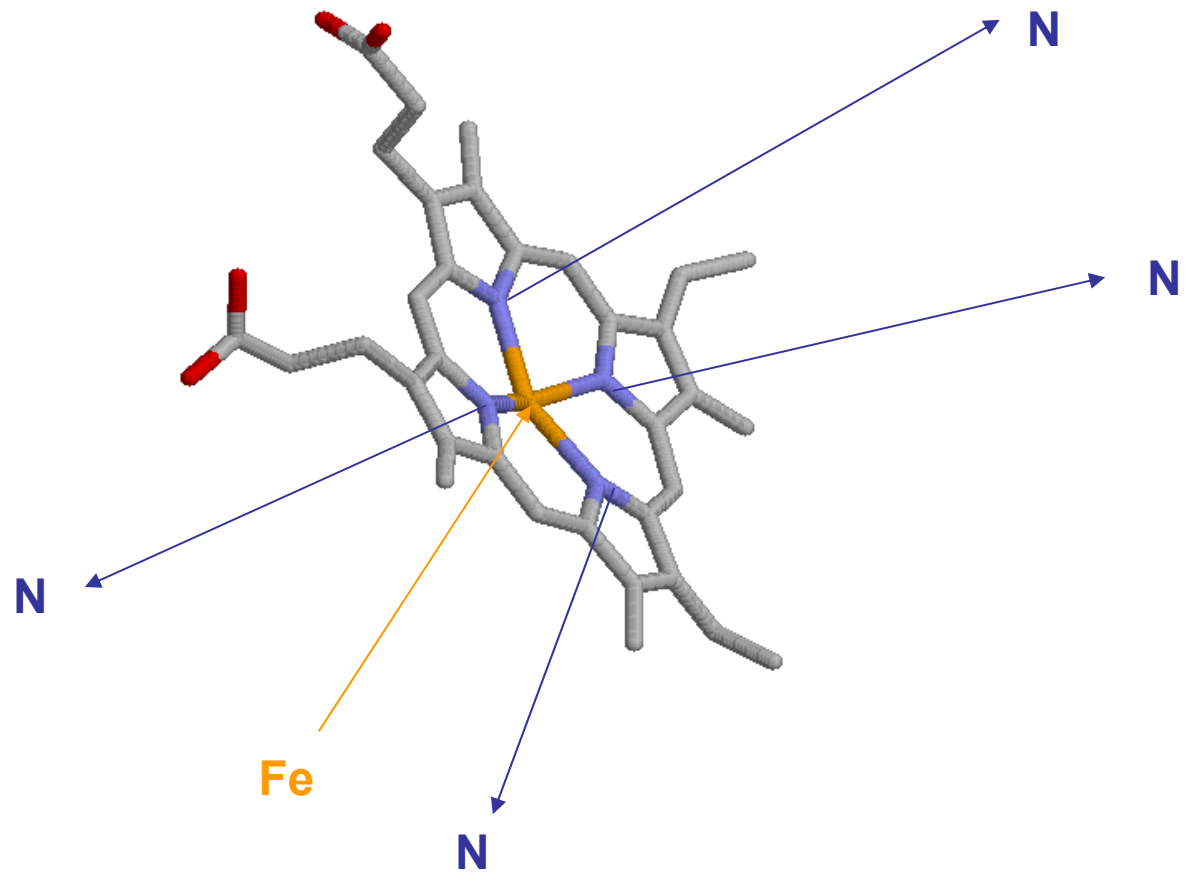




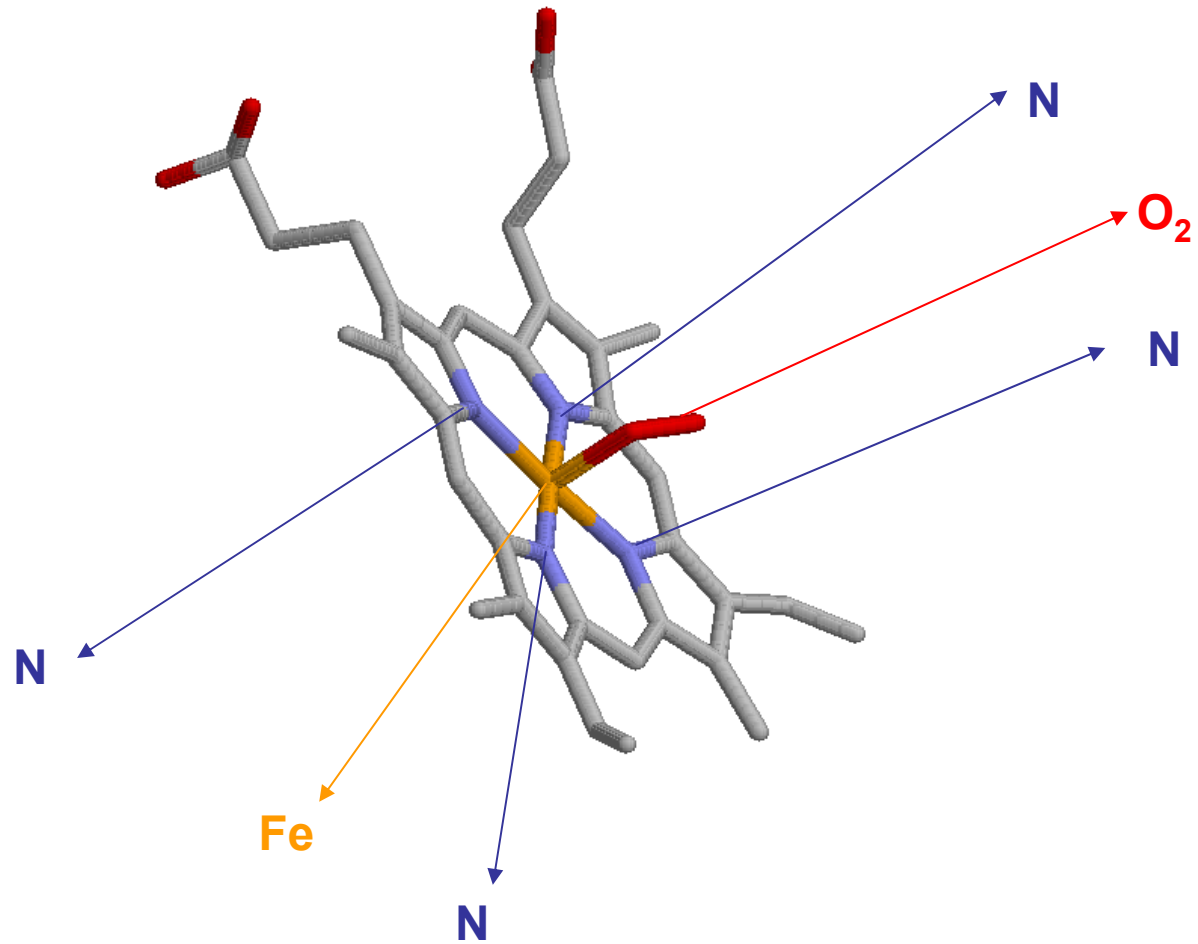


oxy

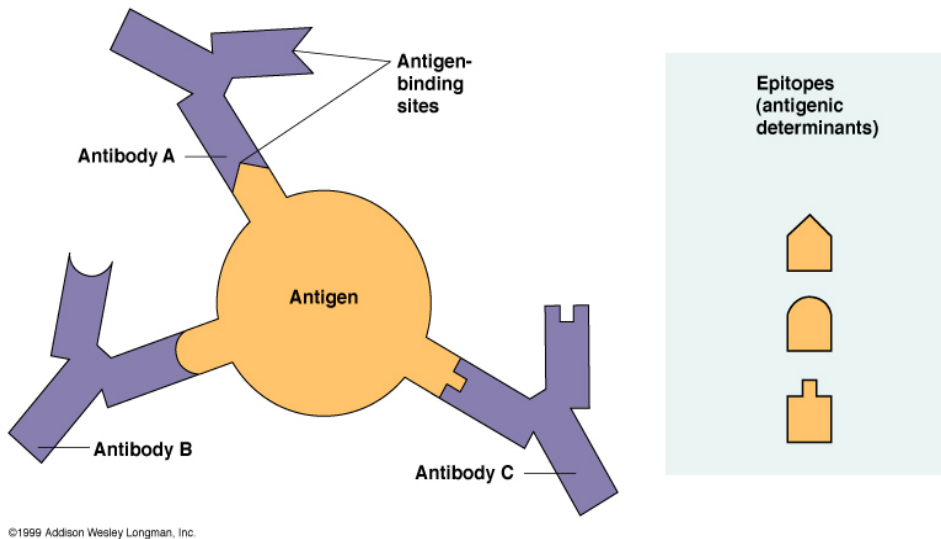
Deoxy-hemoglobin



Oxi-hemoglobin



Antigen-functionalized Nanotube for disease diagnosis



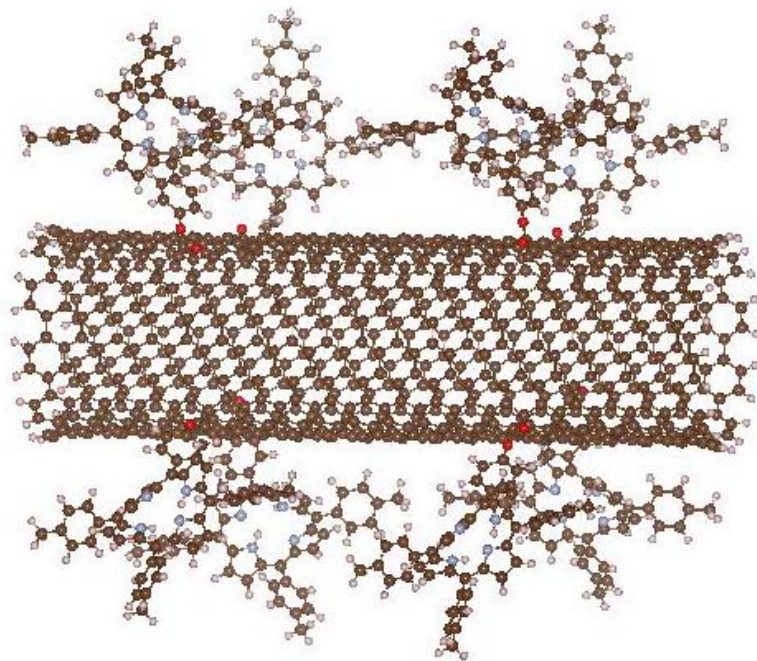
- Specific Antibodies **Ab** are produced in response to an external Antigen **Ag** (like a viral or bacterial protein)
- If you have been infected by **Ag_A**, you will produced **Ab_A**, detectable in your blood
- One would like to functionalize a nanotube with the **Ag** we wish to detected

• Questions

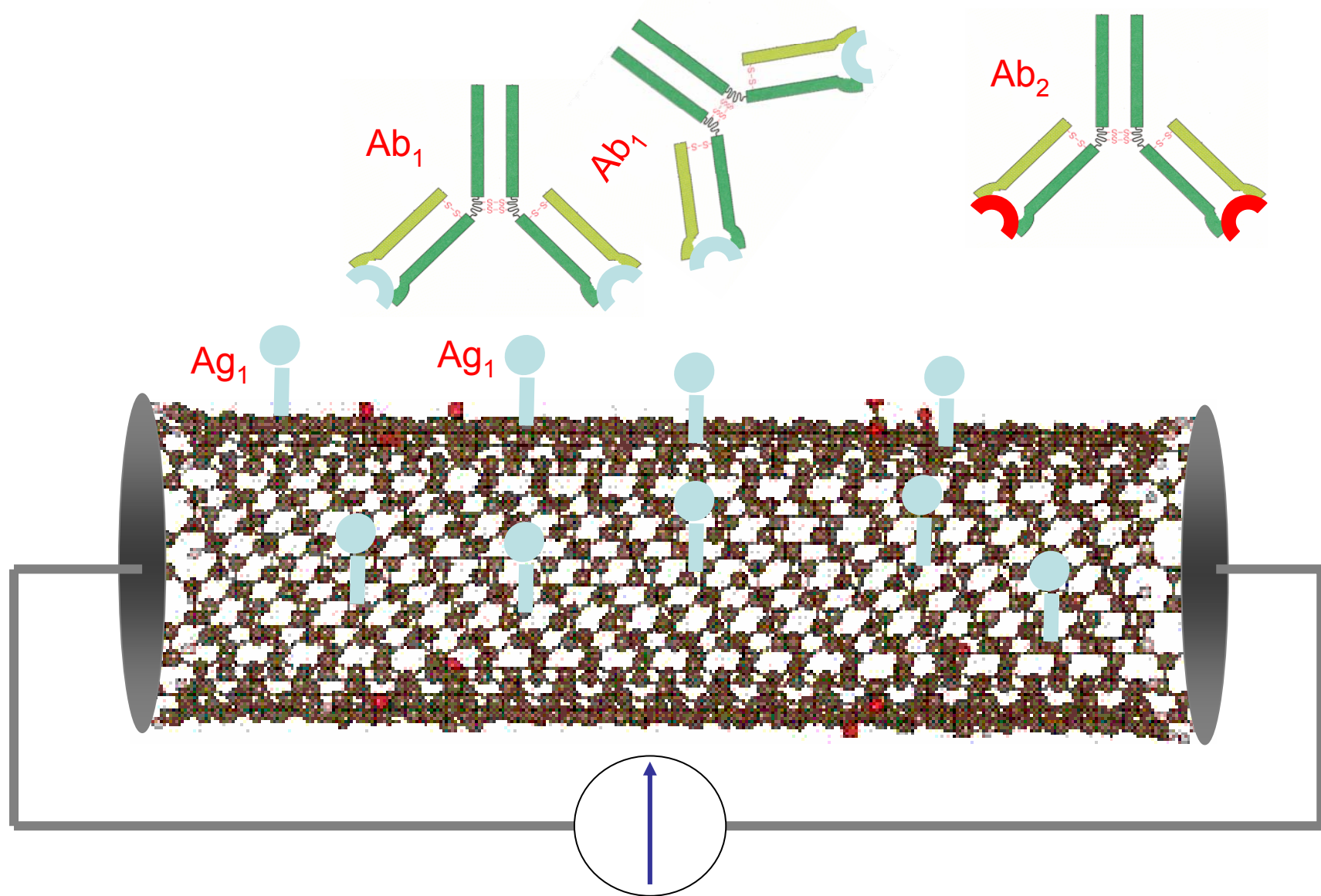
- can all this be done?
- will it work (specificity)?
- can one detect a signal upon **Ab_A** binding the **Ag_A**?
- can simulations be of help?

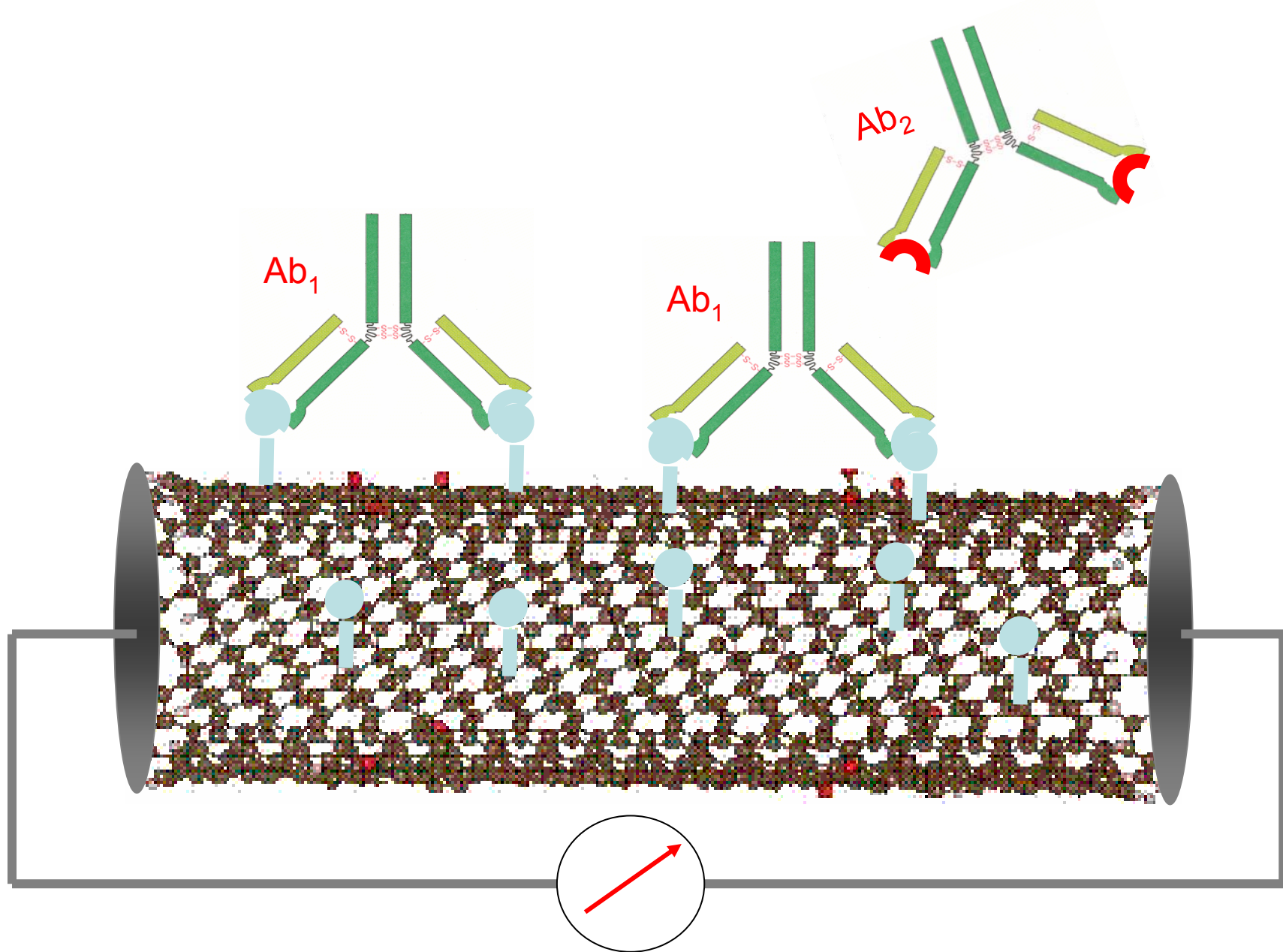
Porphyrin Functionalized Nanotube

- New materials for **solar energy** applications
- Relatively simple, synthetically feasible (at ORNL-UT) mimics of light-harvesting antenna units
- **Porphyrin** molecules are the light absorbing antenna and the **nanotube** may provide a conducting channel
- Key research questions to address are:
 - How does porphyrin attach to the nanotube?
 - How does the electronic structure change as porphyrin molecules are added to the nanotube (up to 22 % in weight)?
 - How is the conductance affected by surface orientation and composition?
- Problem size **1500** (~ 60 Å) to **5000 atoms** (202 Å by 60 Å)
10 times more **electrons**



a case for
numerical simulations





IV. What we can actually do and/or are really doing

Two examples

IVa Metabolic networks

IVb Protein folding and aggregation

IV. What we can actually do and/or are really doing

Two examples

IVa Metabolic networks

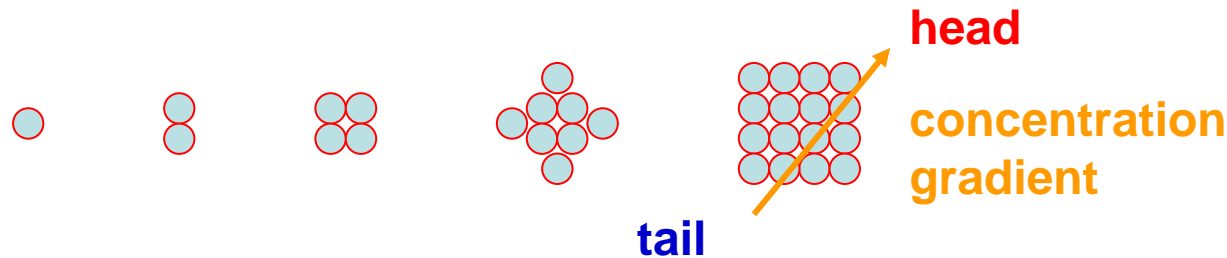
IVb Protein folding and aggregation

■ Metabolic networks

- The case of the WNT pathway
the context and the problem
- Modelization
data and approximations
- Results
some numerics
- Outlook
understanding cancer onset (?)

● A paradigmatic case: the WNT pathway

- Morphogenes are proteins that specify the different cell fate in a **concentration** dependent way
- WNT, Hh, BMP, ... regulator proteins that (during **embryogenesis**) provide positional information and organize embryonic patterning



- WNT-signalling mechanism is much studied, because **defects** in its regulation ultimately lead to **cancer**
- Normally WNT regulates the level of β -catenine in the cell

- 1) In the **absence** of a **WNT** signal, a multi-component destruction complex, containing **GSK3**, **Axin**, **ACP**,... promotes **Phosphorylation** of **β -catenine**, making it ready for **degradation** by **β -TRCP** (an **E3 Ubiquitin** ligase)
- 2) In the **presence** of a **WNT** signal, the activity of the **destruction** complex is **inhibited**, and the level of cytoplasmatic **β -catenine** **risers**

β -catenine becomes complexed with the transcription factor **TCF** and **activate** **TCF**-target genes (**c-myc**, **cyclinD1**, **tcf-1**,...), which directly influence cell development processes



Accumulation of **β -catenine** in the cell and/or **deregulation** of the **TCF/ β -catenine** activity can promote **carcinogenesis** in many tissues

- **Mutations** in the **β -catenine** gene **CTNNb1** with consequent protein alterations (mostly in the region **S29-K49**)
- **Defects** in the **WNT pathway**, resulting in a deregulation of the cytoplasmatic **β -catenine** level

● Modeling the canonical **WNT** pathway

Lee Salic Krueger Heinrich Kirschner
PloS Biology, 1 (2003) 116

MAIN COMPONENTS

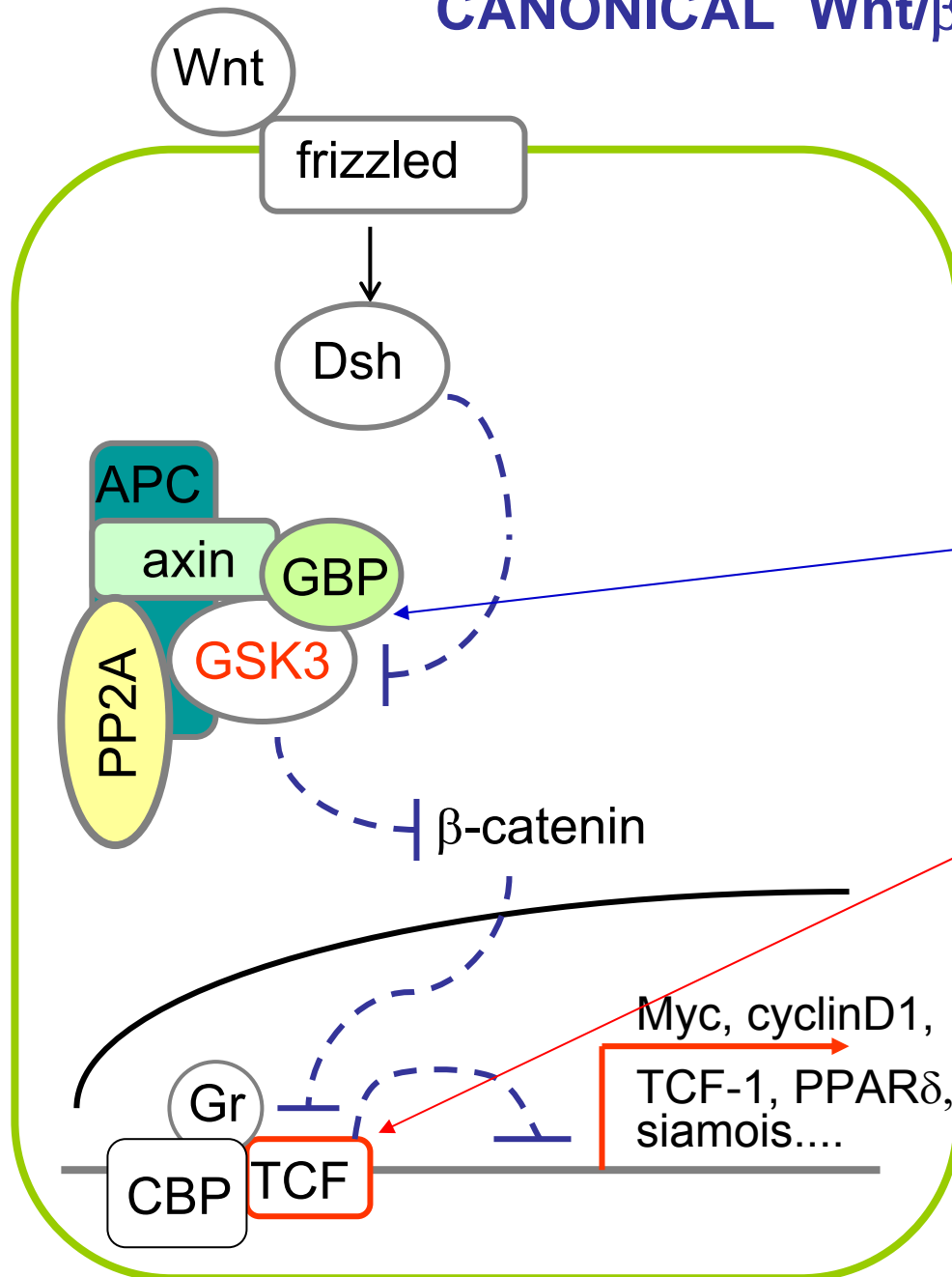
WNT (ligand)
FRIZZLED (receptor)
DISHEVELLED
AXIN (scaffold)
APC (scaffold)
GSK3 (Kinase)
GBP (GSK3 binding protein)
PHOSPHATASE (PP2A)
CASEINE KINASE
 β -CATENINE (transcription coactivator)
TCF (transcription factor)

MUTATIONS IN **APC** PLAY A
PARTICULARLY IMPORTANT ROLE
IN COLORECTAL CANCER

APC: ADENOMATOUS POLYPOSIS
COLIPROTEIN

and many, many more ...

CANONICAL Wnt/ β -CATENINE PATHWAY



LOGIC OF THE PATHWAY

Either the destruction complex forms

or β -catenin gets complexed with TCF



WITH ACTIVATION OF
WNT-TARGET GENES

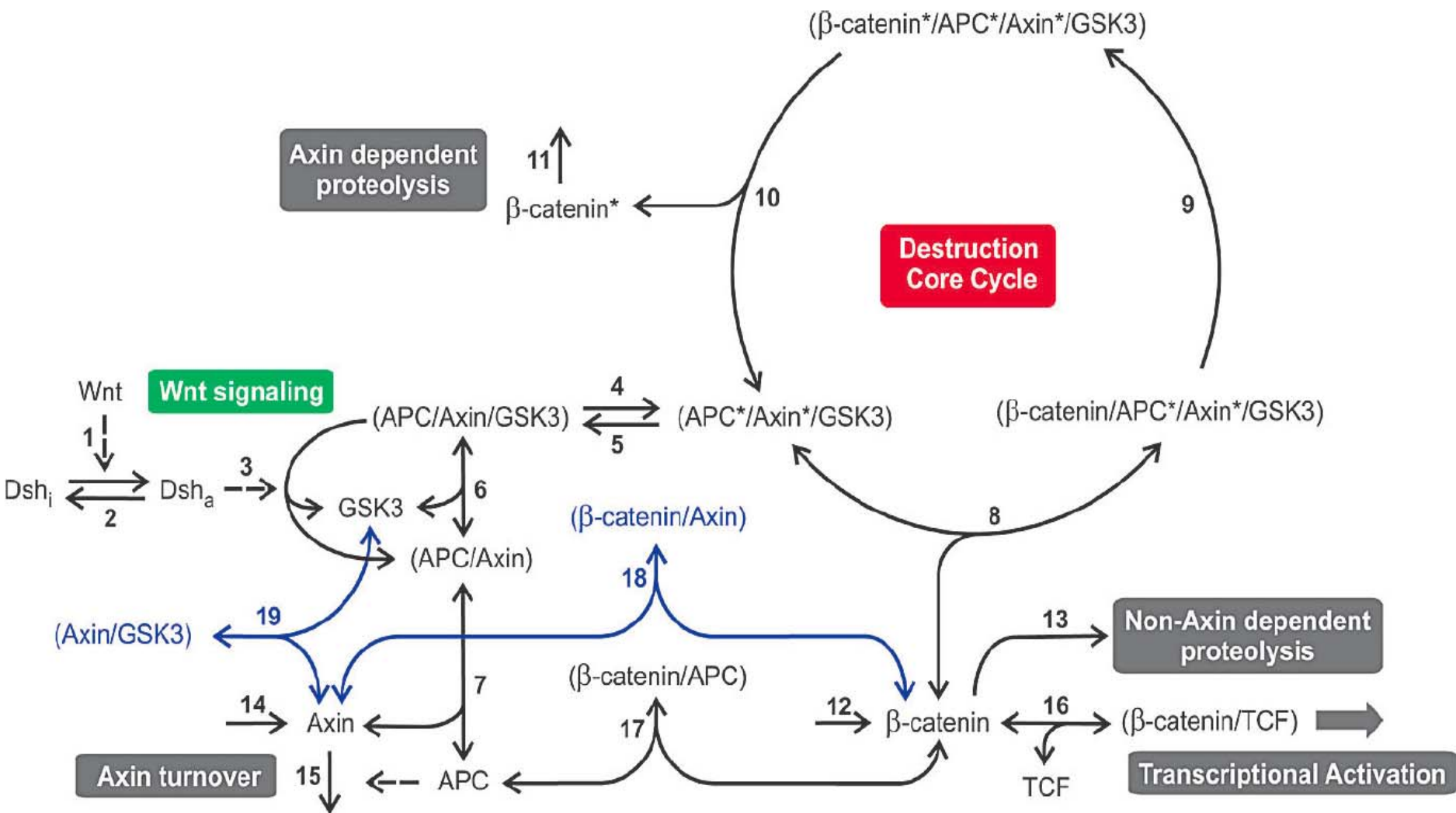


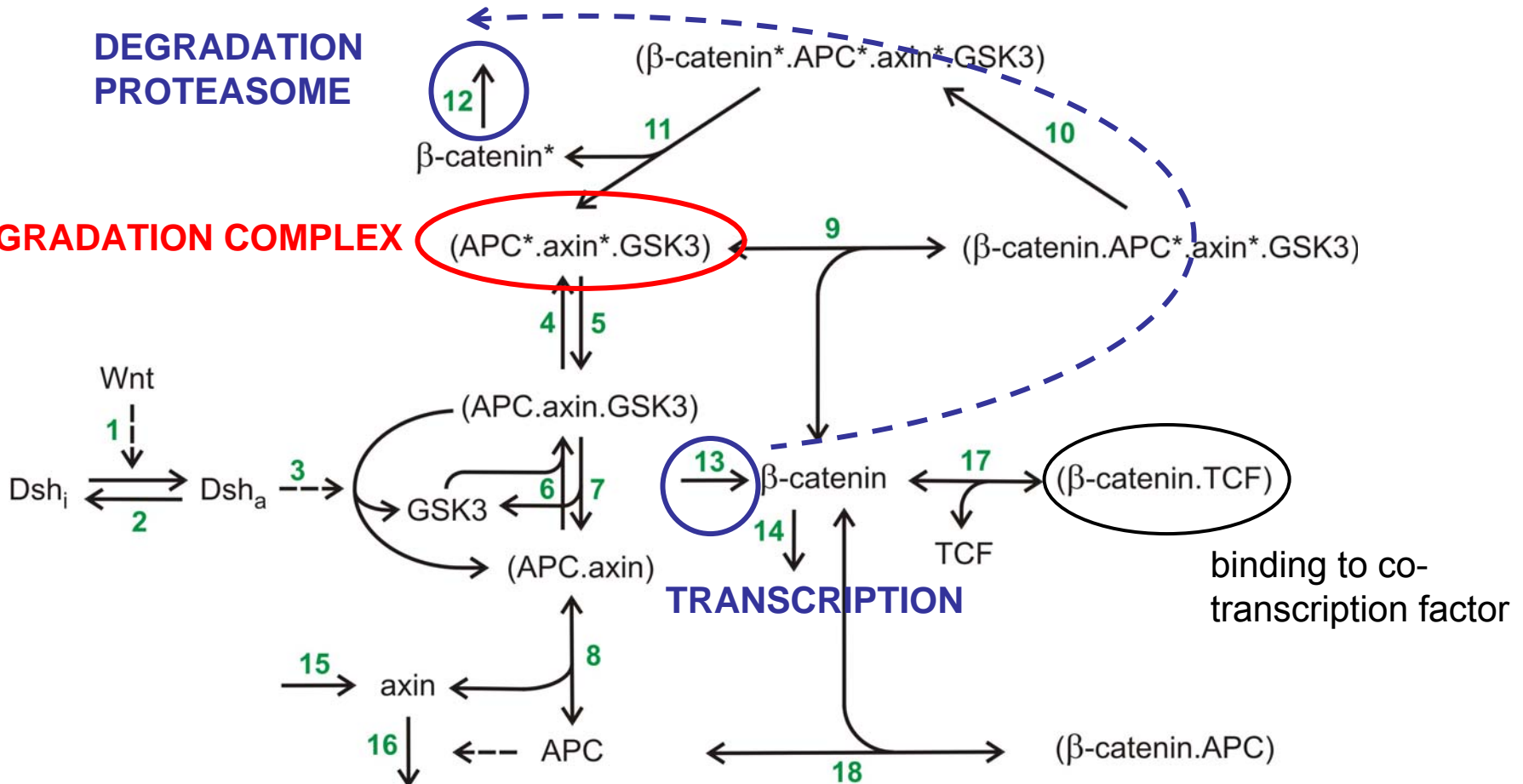
Figure 1. Reaction Scheme for Wnt Signaling

The reaction steps of the Wnt pathway are numbered 1 to 19. Protein complexes are denoted by the names of their components, separated by a slash and enclosed in brackets. Phosphorylated components are marked by an asterisk. Single-headed solid arrows characterize reactions taking place only in the indicated direction. Double-headed arrows denote binding equilibria. Blue arrows mark reactions that have only been taken into account when studying the effect of high axin concentrations. Broken arrows represent activation of Dsh by the Wnt ligand (step 1), Dsh-mediated initiation of the release of GSK3β from the destruction complex (step 3), and APC-mediated degradation of axin (step 15). The broken arrows indicate that the components mediate but do not participate stoichiometrically in the reaction scheme. The irreversible reactions 2, 4, 5, 9–11, and 13 are unimolecular, and reactions 6, 7, 8, 16, and 17 are reversible binding steps. The individual reactions and their role in the Wnt pathway are explained in the text.

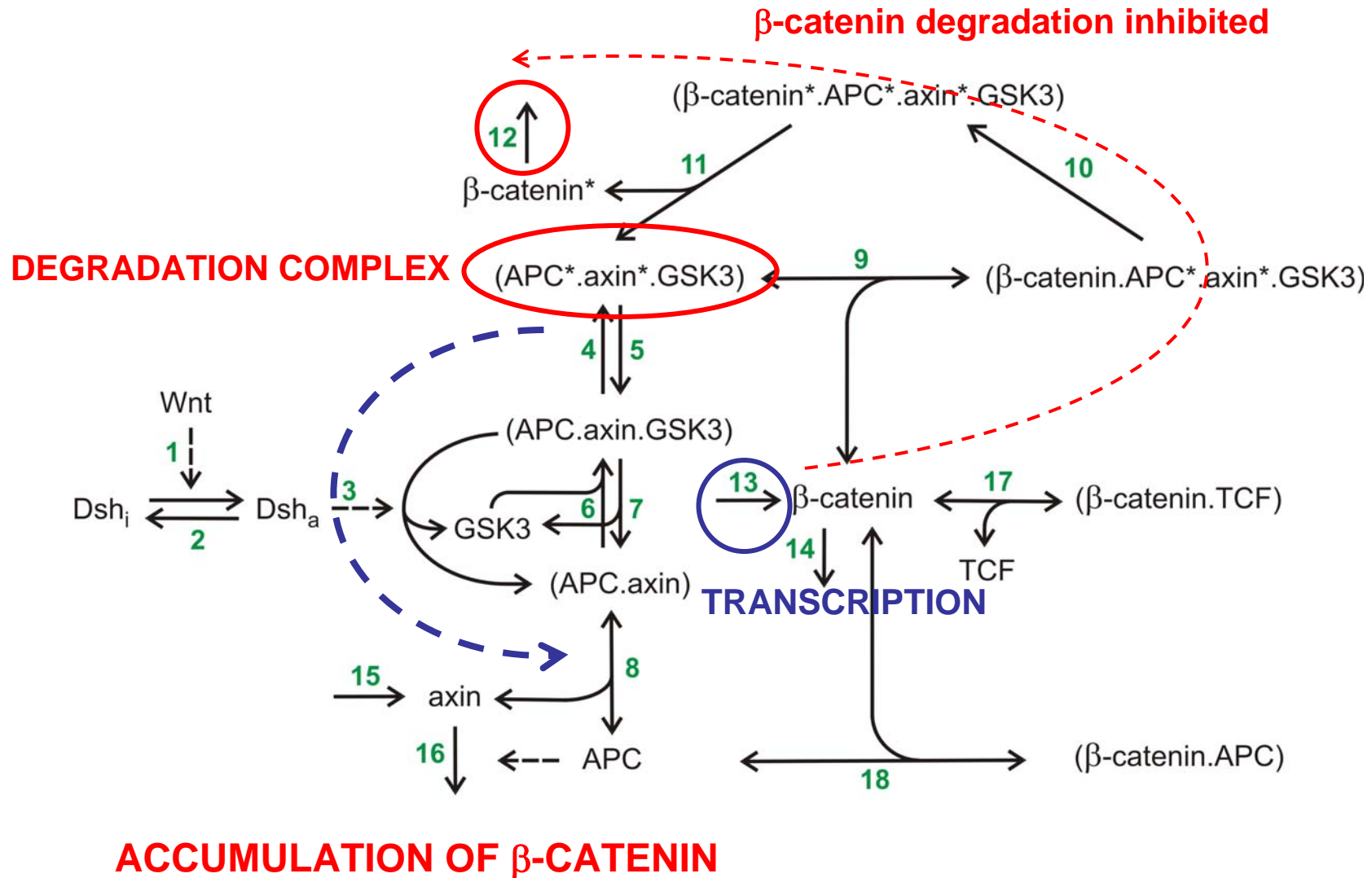
Unstimulated reference state Absence of Wnt

DEGRADATION
PROTEASOME

DEGRADATION COMPLEX



Effect of Wnt-stimulation



MAIN INPUT DATA OF THE MODEL

CONCENTRATIONS

total Dsh	100 nM
total APC	100 nM
total TCF	15 nM
total GSK3	50 nM
total axin	0.02 nM
total β -catenin	35 nM
free phosphorylated β -catenin	1 nM

DISSOCIATION CONSTANTS

binding of GSK3 to (APC.axin)	10 nM
binding of APC to axin	50 nM
binding of β -catenin to (APC.axin.GSK)	120 nM
binding of β -catenin to TCF	30 nM
binding of β -catenin to APC	1200 nM

FLUXES

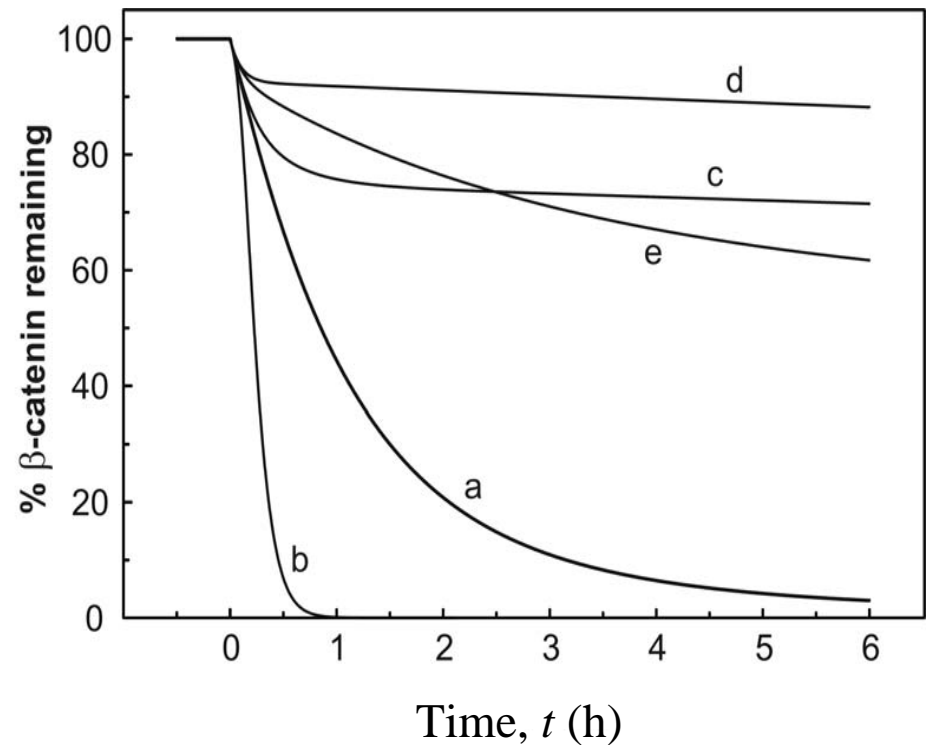
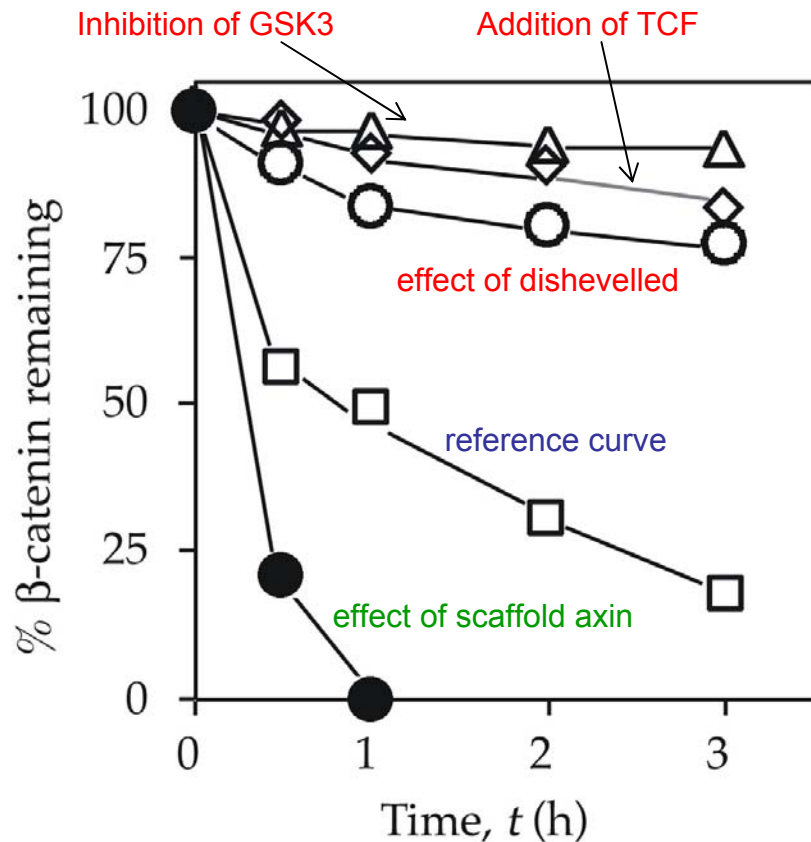
degradation flux of β -catenin via the proteasome	25 nM/h
Share of degradation of β -catenin via unphosphorylated form	1.5 %

CHARACTERISTIC TIMES

phosphorylation/dephosphorylation of APC and axin	2.5 min
GSK3 association/dissociation	1 min
Axin degradation	6 min

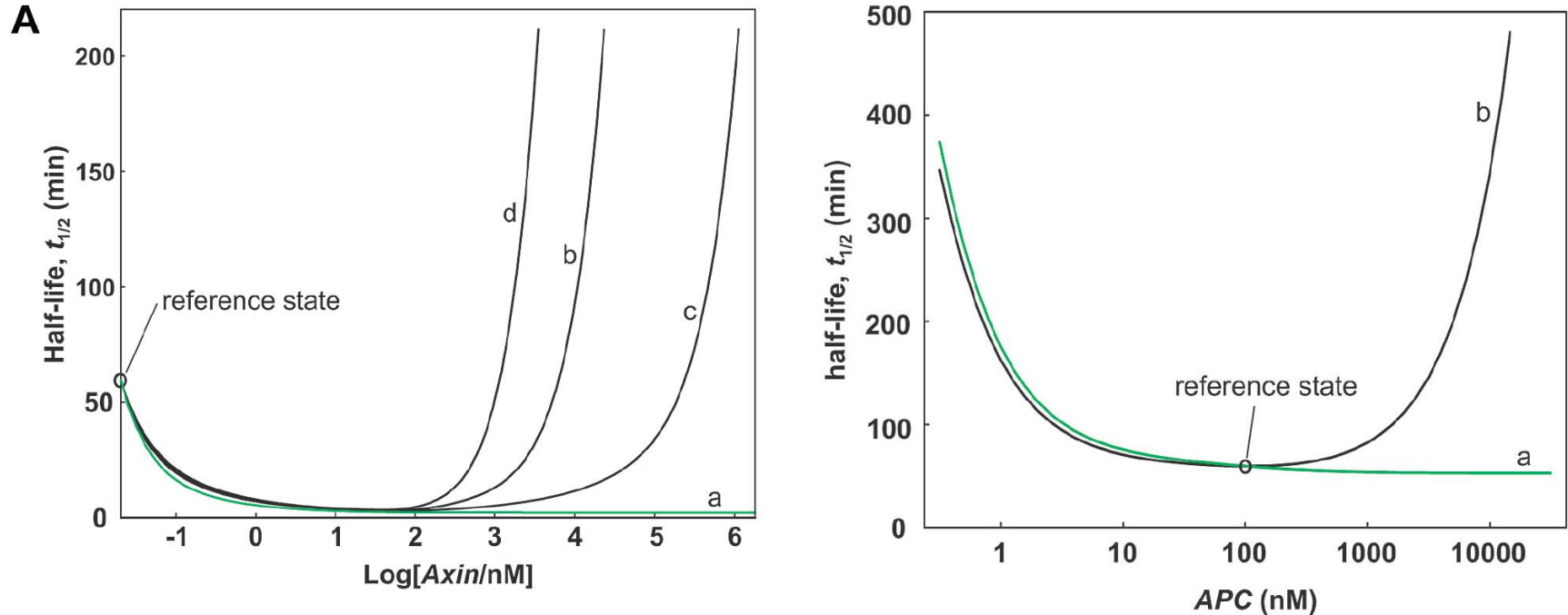
● Results

β -catenin degradation,
simulations and comparison with experimental data



● Outlook

Tumor suppressor role of Axin and/or APC?



Very complicated to devise a winning strategy (non-linear dynamics)

- Axin degradation is APC dependent
- Axin and APC both involved in the β -catenin destruction complex

HUMBOLDT-UNIVERSITY

Reinhart Heinrich

SIGNAL TRANSDUCTION

Thomas Höfer

Roland Krüger

Holger Nathansen

METABOLIC NETWORKS

Oliver Ebenhöf

Edda Klipp

Stefan Schuster

Jana Wolf

HARVARD MEDICAL SCHOOL, BOSTON

Marc Kirschner

Leon Murphy

Benjamin G. Neel

Tom A. Rapoport

Adrian Salic

Ethan Lee

Stefanie Schalm

BIOCENTRUM AMSTERDAM

Hans Westerhoff

Roel van Driel

Martijn Moné



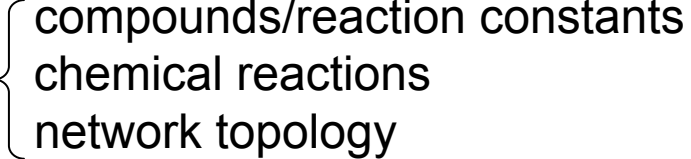
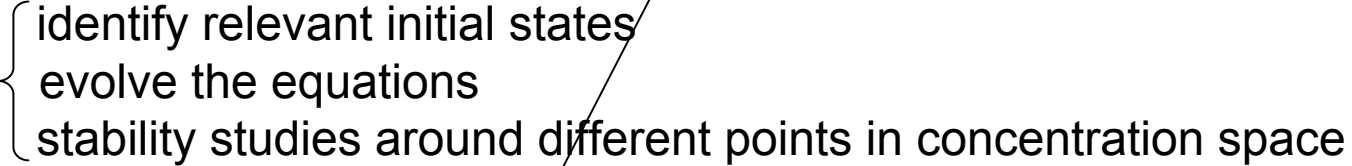
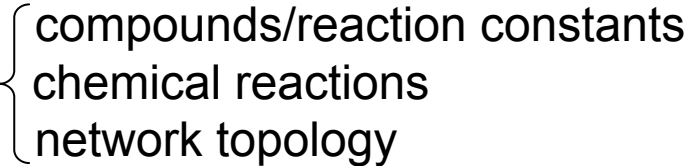
UNIVERSITY BORDEAUX II

Jean-Pierre Mazat

Christine Reder

Summary

what can be/was done about metabolic networks

- Bio-chemical **data** suggest the set of relevant 
 - compounds/reaction constants
 - chemical reactions
 - network topology
- Construct the **set** of (non-linear) diff. eqs (time and space) for concentrations
- **Solution** 
 - identify relevant initial states
 - evolve the equations
 - stability studies around different points in concentration space
- Devise **experiments** and compare
- Identify the **key features** of the system 
 - compounds/reaction constants
 - chemical reactions
 - network topology
- hence what is needed to **correct** what goes wrong

(like accumulation of **β -catenin** in adult cell, as it would promote unwanted expression of the silenced **TRC β** gene)

■ Protein folding and aggregation

- Generalities
- Universality vs natural selection
the case of random hetero-polymers
- Folding vs aggregation
the case of the Prion protein (PrP)
and the role of Cu
- XAS (NMR, EPR) experiments
data analysis and EXAFS theory
- QM calculations
DFT and Car-Parrinello dynamics

● Generalities

Protein is a complex
(and complicated) system

➤ Many degrees of freedom

protein: ~ 300 a.a.'s $\times 10$ atoms = ~ 3000 atoms
solvent: ~ 1000 atoms



3 to 4 times more
“active” electrons

➤ Large range of folding times

from $\mu\text{sec}'\text{s}$ to $\text{sec}'\text{s}$

the **Levinthal's** paradox $\left\{ \begin{array}{l} \text{too fast for an exhaustive search} \\ \text{too slow for a straight descent to absolute minimum} \end{array} \right.$

➤ Interaction is not short-range

choice of a phenomenologically acceptable potential in MD
a **Q.M.** treatment (**DFT**, **Car-Parrinello**) is often needed

➤ Free-energy landscape looks very corrugated

many hierarchically organized local minima, separated by high barriers

➤ System is not living at thermodynamic equilibrium

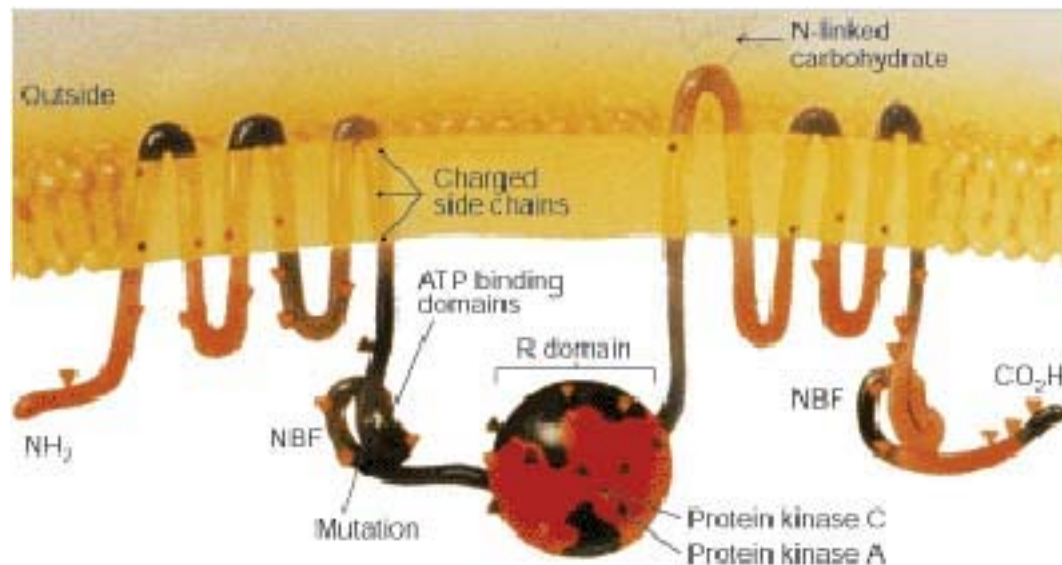
flux of energy and matter

➤ Even single mutations matter

though not always

The CFTR gene is found at the q31.2 locus of chromosome 7, is 230 000 base pairs long, and creates a protein that is **1,480 amino acids** long. The most common mutation, $\Delta F508$ is a deletion (Δ) of three nucleotides that results in a **loss of the amino acid phenylalanine F at the 508th position** on the protein. This mutation accounts for two-thirds of CF cases worldwide and 90 percent of cases in the United States, however, there are over 1,400 other mutations that can produce CF.

There are several mechanisms by which these mutations cause problems with the CFTR protein. $\Delta F508$, for instance, creates a protein that does not fold normally and is degraded by the cell. Several mutations, which are common in the Ashkenazi Jewish population, result in proteins that are too short because production is ended prematurely. Less common mutations produce proteins that do not use energy normally, do not allow chloride to cross the membrane appropriately, or are degraded at a faster rate than normal. Mutations may also lead to fewer copies of the CFTR protein being produced.



Even a **single** mutation (deletion) can be fatal

Cystic Fibrosis

The protein cannot be crystallized.
No full resolution of the critical
a.a. **508** region → **simulations**?

We expect numerical approaches to be difficult

- . Which atoms are going to be bound?

structure of the potential is not *a priori* known (QM)

- . Force computation time grows like $N \times N$

two-body potential

- . The system is very heterogeneous

the problem is not “embarrassingly” parallel

- . Dynamics time step is of the order of a *femtosec*

the system can be followed for very short times

- . The system gets easily trapped in metastable states

the exploration of the system phase-space is far from ergodic

- . Energy may not be a good label of the states of the system

states with largely different 3D-structures can have similar energies

states with only slightly different 3D-structures can have very different energies

Countless number of approaches

- Geometrical approaches
- Simulated annealing
- Molecular Dynamics
- Monte Carlo simulations
- Simulated tempering and variations thereof
- Multi-canonical simulations
- Effective free-energy profile evaluation
- ...

Different levels of description

- Systems with discretized degrees of freedom
- String of beads
- Detailed atomistic description
 - with effective interaction potentials
 - with *ab initio* potentials
- ...

• Universality vs natural selection

Iori Marinari
Parisi Struglia

Self-interacting random hetero-polymers

- ♦ The complexity of the system is encoded in a certain amount of **randomicity** of the **Hamiltonian**

- $H = \sum_{i=1}^N \sum_{i>j} E_{ij}, \quad N \geq 30$



- $E_{ij} = k\delta_{i,j+1}r_{ij}^2 + \frac{B}{r_{ij}^{12}} + \frac{\eta_{ij} - A}{r_{ij}^6}, \quad r_{ij}^2 = |\vec{x}_i - \vec{x}_j|^2$

binding

repulsive
($B = 2$)

it depends on
the sign of $\varepsilon - A$


- η_{ij} uncorrelated **random** gaussian variables

$$\langle \eta_{ij} \rangle = 0$$

$$\langle \eta_{ij}^2 \rangle = \varepsilon$$

- ♦ The system is brought to equilibrium at $\beta = 1/k_B T$
under the **Boltzmann** probability distribution $\propto \exp [-\beta H]$

- ◆ During the evolution the shape of the chain is continuously monitored and various interesting features are revealed

{	Coil (open)	$\delta \gg \rho, \lambda$
	Unshaped globule	$\delta \geq \lambda$
	Frozen well-shaped structures	$\delta < \rho, \lambda$
	 ~ folding?	

- $\delta_{\alpha\beta}^2 = \frac{1}{N} \sum_{i=1}^N | \vec{x}_i^{(\alpha)} - \vec{x}_i^{(\beta)} |^2 \rightarrow$ "distance" between $\{\vec{x}_i^{(\alpha)}\}$ and $\{\vec{x}_i^{(\beta)}\}$

\swarrow configurations
 \searrow
- $\rho = \frac{1}{N_{conf}} \sum_{\alpha} \frac{1}{N} \sum_{i=1}^N | \vec{x}_i^{(\alpha)} - \langle \vec{x}_i^{(\alpha)} \rangle | \rightarrow$ average giration radius
- $\lambda = \frac{1}{N_{conf}} \sum_{\alpha} \frac{1}{N-1} \sum_{i=1}^{N-1} | \vec{x}_i^{(\alpha)} - \vec{x}_{i+1}^{(\alpha)} | \rightarrow$ average link length

I. $\varepsilon = 0$, no randomness \rightarrow homo-polymer

- phase transition at $A \approx 2$

coil (open) \rightarrow un-shaped globule (closed)

$P(\delta^2)$ peaked at large $\delta^2 \rightarrow$ small δ^2

II. $\varepsilon \neq 0$, some random interaction \rightarrow hetero-polymer

- new phase beyond a critical $\varepsilon_c > A$

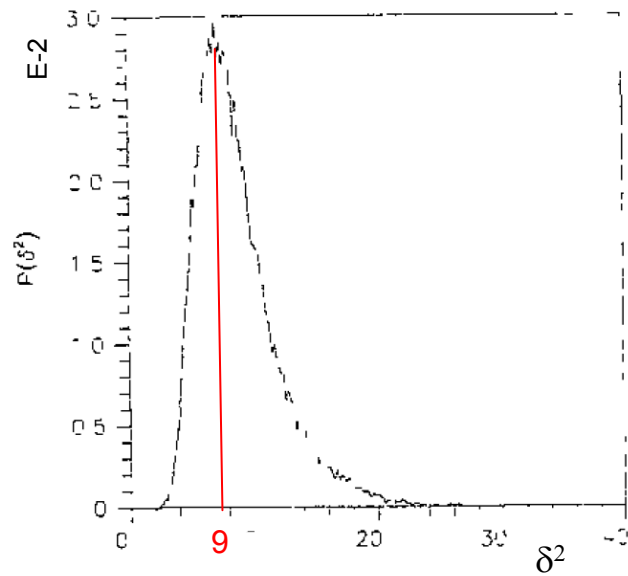
well-shaped globule (\sim glassy phase in SG ?)

$P(\delta^2)$ is endowed with a lot of structure

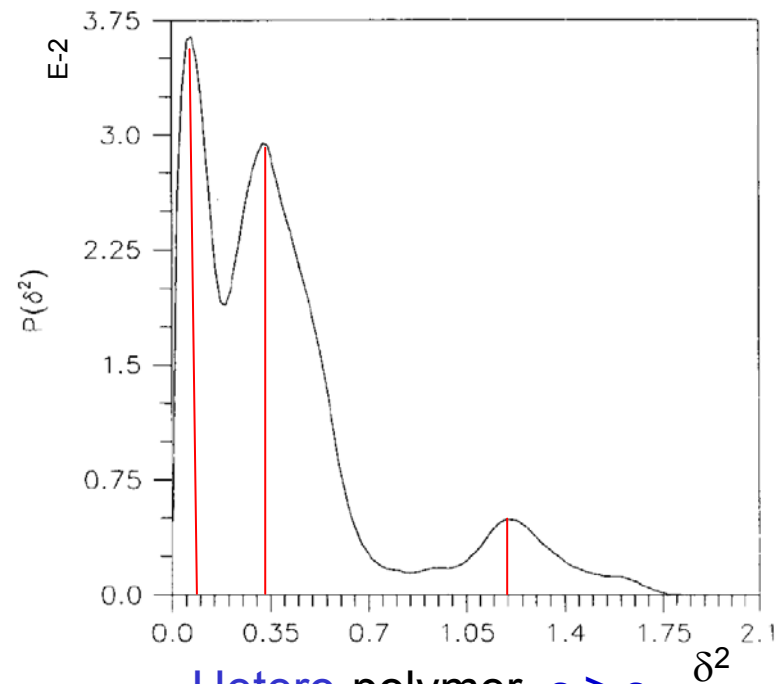
Main result \rightarrow

Sufficiently random hetero-polymers generically fold

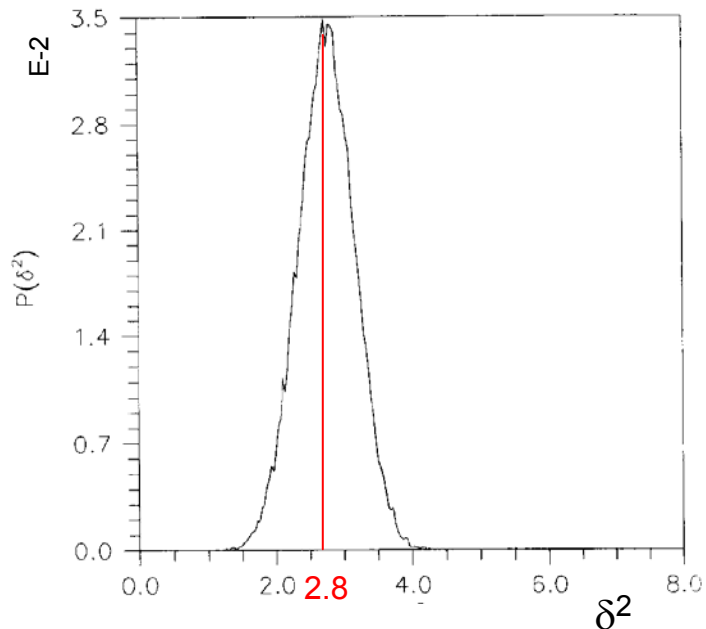
Speculation \rightarrow Perhaps (all the) other a.a. sequences do not fold.
Do they rather aggregate?



Homo-polymer, $\varepsilon = 0$
open phase $A = 1.6$



Hetero-polymer, $\varepsilon > \varepsilon_c$
"folded" phase

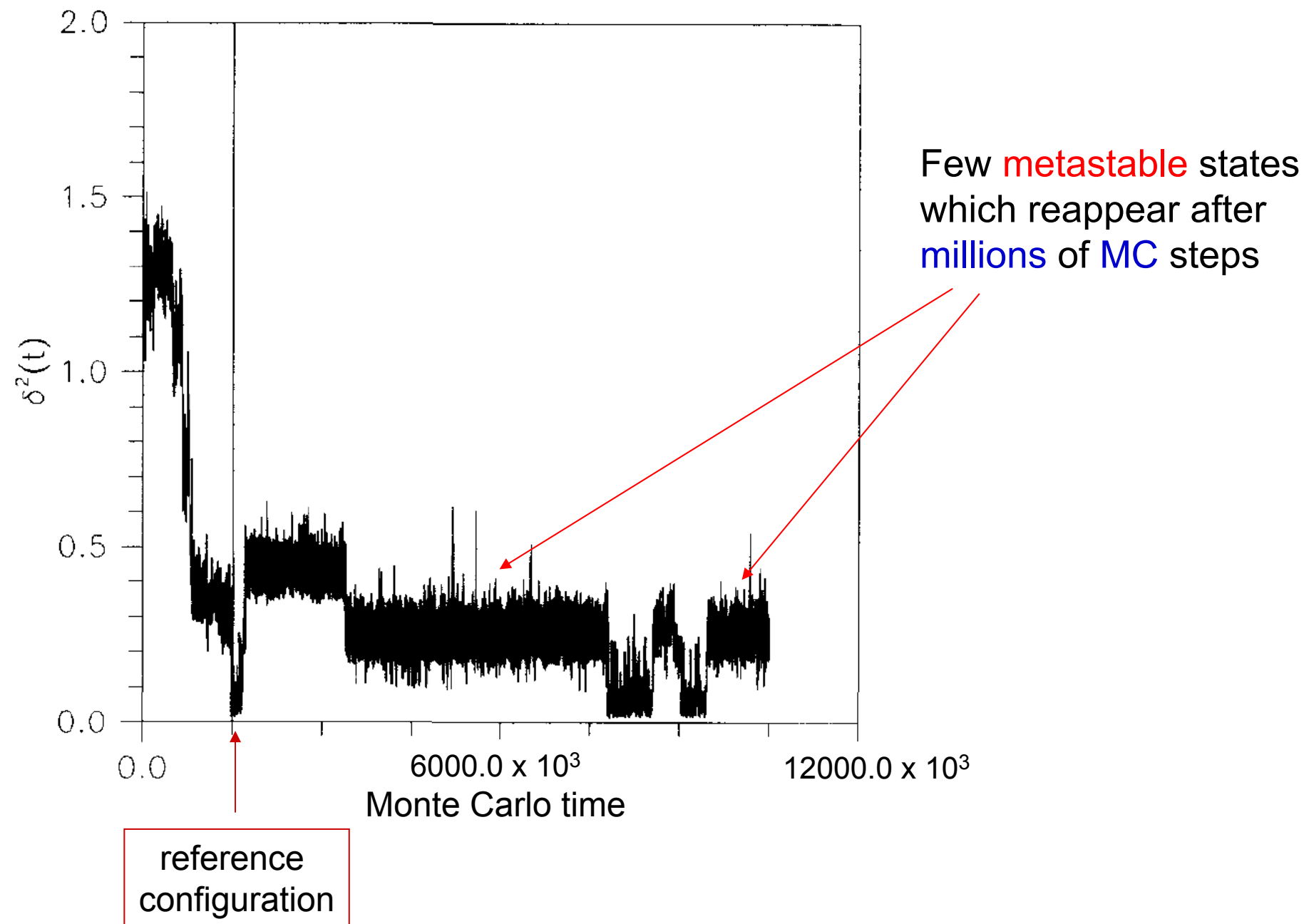


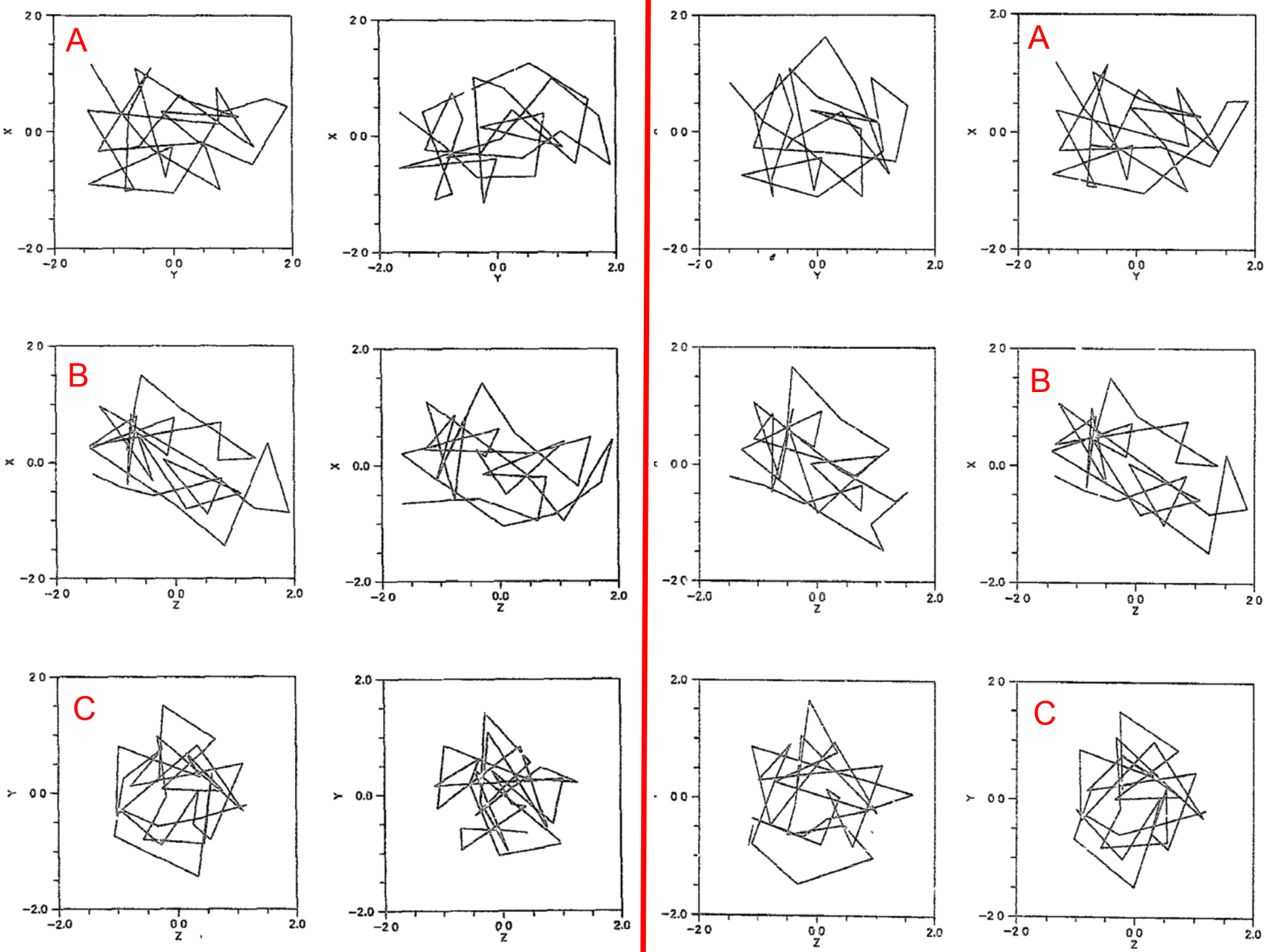
Homo-polymer, $\varepsilon = 0$
globular phase $A = 3.8$

Peaks \rightarrow macroscopically different folded states

Not δ -functions \rightarrow many (only) microscopically different states

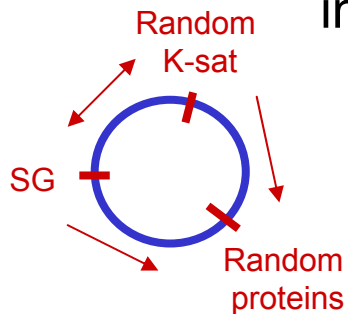
Macrostates are very long living (see next figures)





Comments

- In the "folded" phase the situation displays analogies with what one finds in the glassy phase of SG
 - Many long living, hierarchically organized states at sufficiently large randomness (frustration)
 - Very long (actually not well defined) correlation times (stretched exponentials: $\propto \exp[-(t/\tau)^\alpha]$, $\alpha < 1$, "aging")
 - Complexity of protein folding is reflected in the NP-completeness of SG
- Can one make the SG analogy more stringent and useful?
 - Perhaps yes, taking inspiration from results in K-sat problem theory



- Random K-sat problems can be mapped to SG
- Alg's borrowed from SG can help solving Random K-sat problems in polynomial time with probability ~ 1
- Can a random protein be folded in polynomial time?

- Should we instead move to a more reductionist point of view?

K-sat problems and SG

- **K-sat problem**: M constraints among N boolean variables, p_1, p_2, \dots, p_N
- **Constraint**: clause among K variables (or their negation, \neg)
e.g. $(p_1 \vee \neg p_2) \wedge (p_2 \vee p_3) \wedge (\neg p_1 \vee \neg p_3) \rightarrow$
 $[p_1 = \text{t}, p_2 = \text{t}, p_3 = \text{f}]$ or $[p_1 = \text{f}, p_2 = \text{f}, p_3 = \text{t}]$
- $K \geq 3 \Rightarrow$ NP-complete problem

Conjunctive Normal
Form (CNF)

K-sat problems

- $p_i = \text{true/false}$
- clause among a set of p_i
- negated / non-negated variables
- clauses satisfied / violated
- # of violated clauses
- 2^N possible ansatz's


Spin systems

- spin $\Rightarrow \sigma_i +1/-1$
- interaction among a set of spins
- coupling $J = -1 / +1$
- energy = 0 / 1
- total energy H
- $s = 1, 2, \dots, 2^N$ possible states

$$P(\sigma, \beta) \propto \exp [-\beta H(\sigma)]$$

- minimal # of violated clauses
- minimum of $H \rightarrow$ SM at $\beta \rightarrow \infty$ ($T = 0$)

Random K-sat problem: building the a -th clause, C_a ($a = 1, 2, \dots, M$)


- 
- 1) $p_{i1}, p_{i2}, \dots, p_{iK}$ ($K \geq 3$) are picked up with uniform probability among the N variables p_1, p_2, \dots, p_N
 - 2) variables $p_{i1}, p_{i2}, \dots, p_{iK}$ are **randomly** negated

Spin Glass: building the a -th interaction term, E_a ($a = 1, 2, \dots, M$)

- 1) $\sigma_{i1}, \sigma_{i2}, \dots, \sigma_{iK}$ ($K \geq 3$) are picked up with uniform probability among the N variables $\sigma_1, \sigma_2, \dots, \sigma_N$
- 2) coupling is $J_a = J_{i1} J_{i2} \dots J_{iK}$ with $J_{ir} = -1$ or $J_{ir} = +1$, according to whether p_{ir} was **randomly** negated or not.

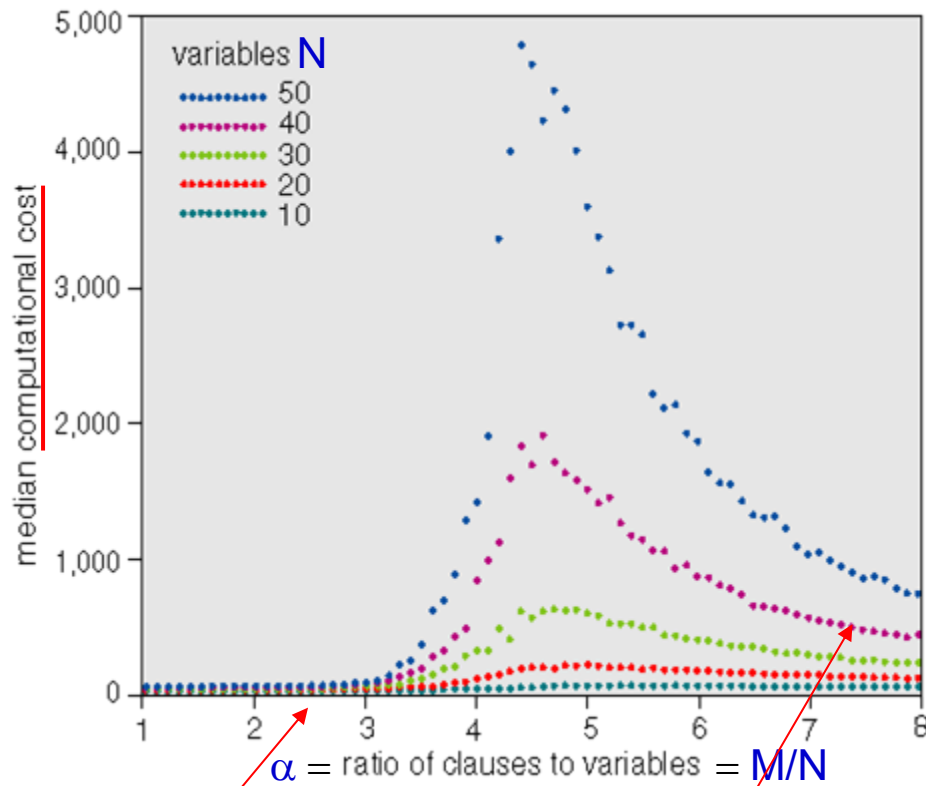
Mézard Monasson
Parisi Zecchina

Some interesting result

- 1) Emergence of a phase transition as $N \rightarrow \infty$, at a critical value of $\alpha = M/N$
 - 2) Methods developed in **SG** theory can be used to solve hard **K-sat** problems (**cavity method**, **decimation alg.**, ...)
 - 3) The average random (not the worst) case can be solved in polynomial time with probability ~ 1
- 

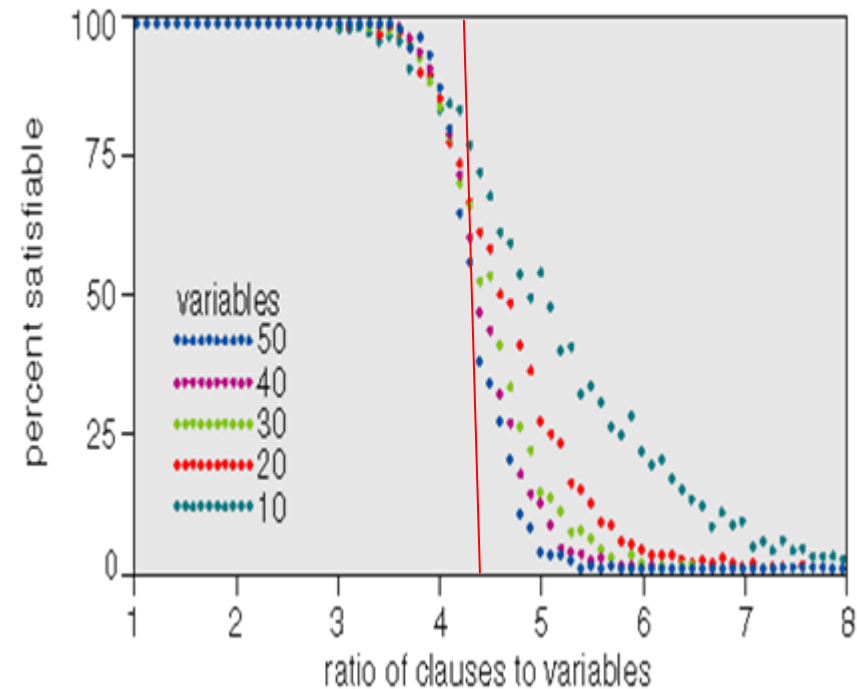
Mitchell Levesque Selman

Hardest problems
around $\alpha_c \approx 4.3$,
where **SAT** propositions
tend to become **UNSAT**



many variables
few clauses
under-constrained
 \Downarrow
SATisfiable

not so many variables
compared to # of clauses
over-constrained
 \Downarrow
UNSATisfiable



Phase transition \rightarrow
the jump becomes sharper
as N gets larger

■ Protein folding and aggregation

- Generalities
- Universality vs natural selection
the case of random hetero-polymers
- Folding vs aggregation
the case of the Prion protein (PrP)
the role of Cu
- XAS (NMR, EPR) experiments
data analysis and EXAFS theory
- Q.M. simulations
DFT and Car-Parrinello dynamics

● Folding vs aggregation

A test case: Prion Protein - PrP

(A bit of phenomenology)

- PrP is a cell membrane glycoprotein (highly expressed in the central nervous system of many mammals), whose physiological role is unclear
- It is, however, known to selectively bind copper, Cu
- Mature PrP has a flexible, disordered, N-terminal (23-120) and a globular C-terminal (121-231)
- **Misfolding** of PrP is held responsible for brain plaque formation and the development of Transmissible Spongiform Encephalopathies (TSE)
- The N-terminal domain contains four (in humans) copies (repeats) of the octa-peptide PHGGGWGQ, each capable of binding Cu
- Experiments, more specifically, indicate that the Cu binding site is located within the HGGG tetra-peptide
- Cu seems to play a crucial role
- Cries for (Car-Parrinello) *ab initio* simulations

Quantum Chemistry
in the BO approx
K. Wilson

- **Alzheimer's disease**

- **Transmissible Spongiform Encephalopathies (TSEs)**

in humans: Creutzfeldt-Jakob Disease

sporadic

familial

iatrogenic

variant

in sheep: Scrapie

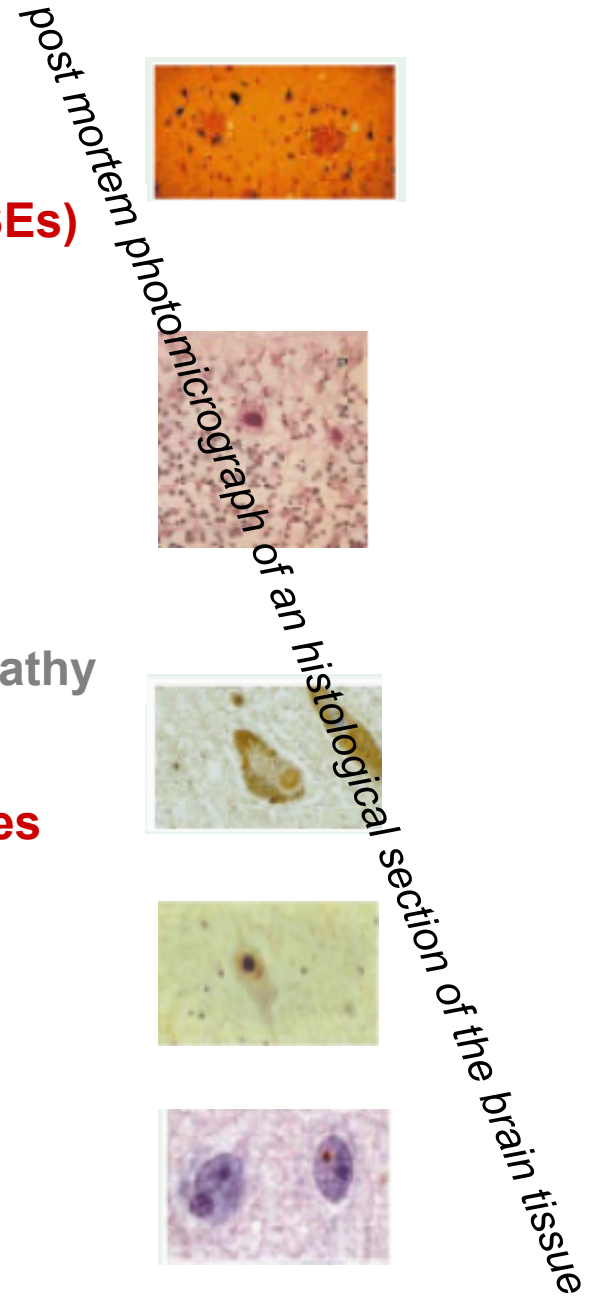
in cattle: Bovine Spongiform Encephalopathy

- **Parkinson's disease; Dementia with Lewy bodies**

- **Amyotrophic Lateral Sclerosis**

- **Huntington's Disease**

in vivo diagnosis by Positron Emission Tomography, PET



DISEASE	AGGREGATING PROTEINS
Alzheimer's disease	Amyloid β -peptide
Transmissible Spongiform Encephalopathies	Full-length prion protein or fragments
Hereditary cerebral haemorrhage with amyloidosis	Amyloid β -peptide or Cystatin C
Parkinson's disease; dementia with Lewy bodies	α -Synuclein
Frontotemporal dementia with parkinsonism	Tau
Type II diabetes	Amylin
Medullary carcinoma of the thyroid	Procalcitonin
Atrial amyloidoses	Atrial natriuretic factor
Amyotrophic lateral sclerosis	Superoxide dismutase
Huntington's disease	Long glutamine stretches within proteins
Primary systemic amyloidosis	Intact immunoglobulin light chains or fragments
Secondary systemic amyloidosis	Fragments of serum amyloid A protein
Familial amyloidotic polyneuropathy 2	Fragments of apolipoprotein A1
Senile systemic amyloidosis	Wild-type transthyretin and fragments
Familial amyloidotic polyneuropathy 1	Mutant transthyretin and fragments
Familian Mediterranean fever	Fragments of serum amyloid A protein
Haemodialysis-related amyloidosis	β_2 -Microglobulin
Finnish hereditary amyloidosis	Fragments of mutant gelsolin
Lysozyme amyloidosis	Full-length mutant lysozyme
Insulin-related amyloid	Full-length insulin
Fibrinogen α -chain amyloidosis	Fibrinogen α -chain variants

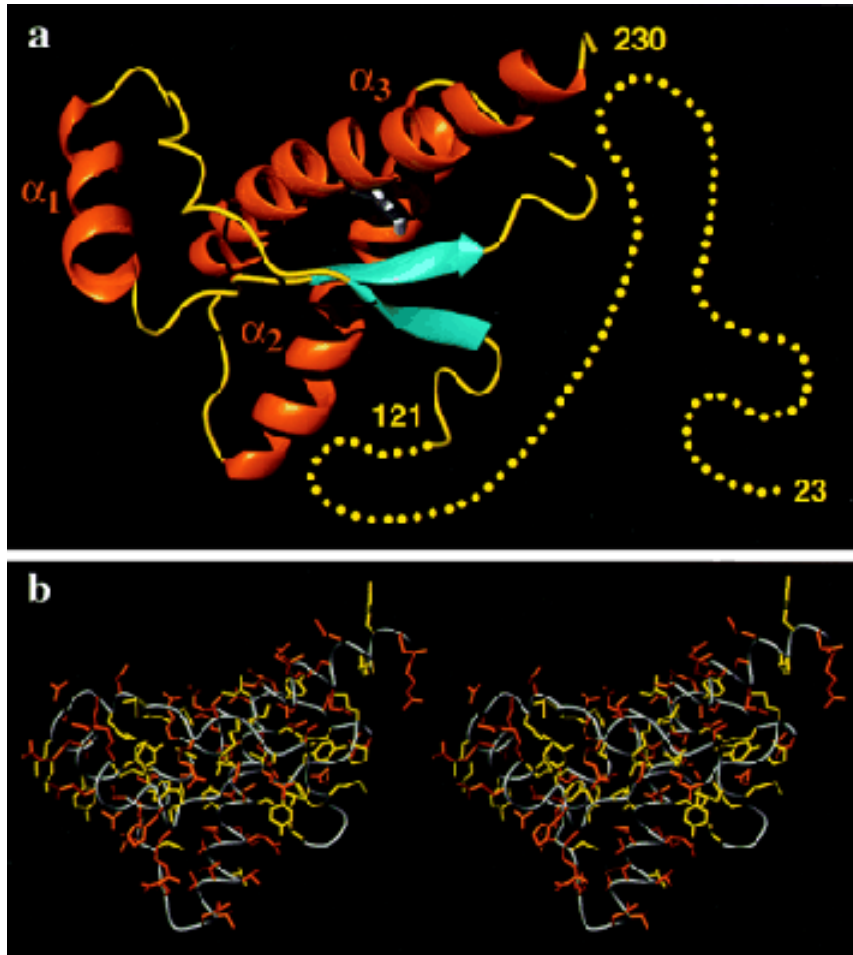
How do we go about such a complicated problem?

- 1) Hints from physiological/biological/biochemical data → **PrP accumulated data**
- 2) Make a working hypothesis and/or a model for misfolding or aggregation → **The role of Cu**
- 3) Test it against appropriately designed experiments → **EXAFS experiments**
- 4) Phenomenological interpretation of EXAFS data → **EXAFS theory**
- 5) Go to an atomic description to check 4) and interpret the model → **Ab initio calculations**

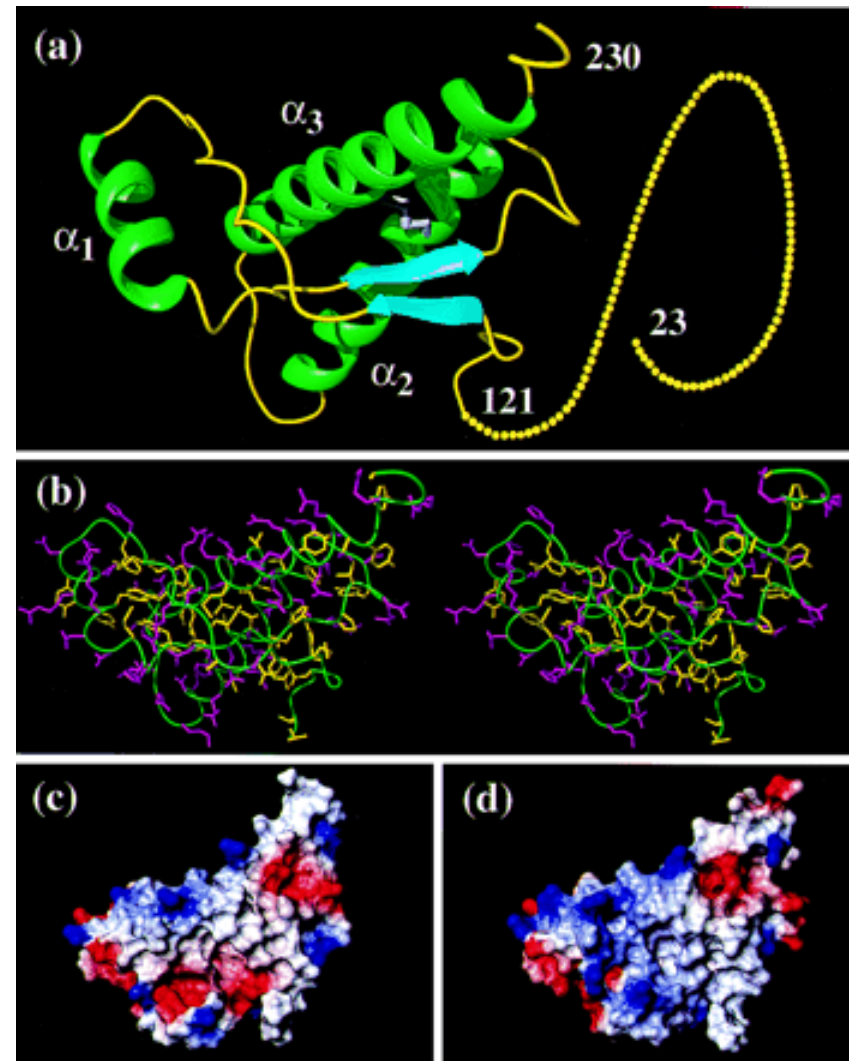
At this point, if you think you have understood something

- 6) Devise (?) an anti-aggregation strategy → **Test it *in vivo***
- 7) Most probably → **need to go back to**
- 2)

We start with some data

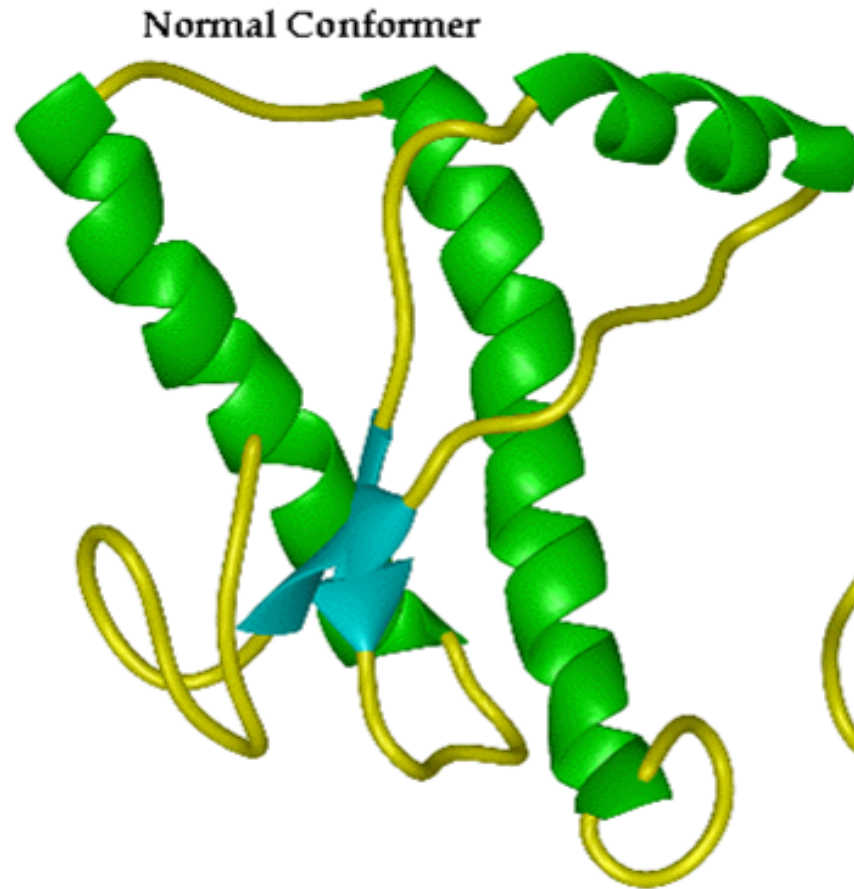


HuPrP (human)
 α -helices = orange
 β -strands = cyan
non-regular secondary structure = yellow,
flexible disordered "tail" (23-121) = yellow dots

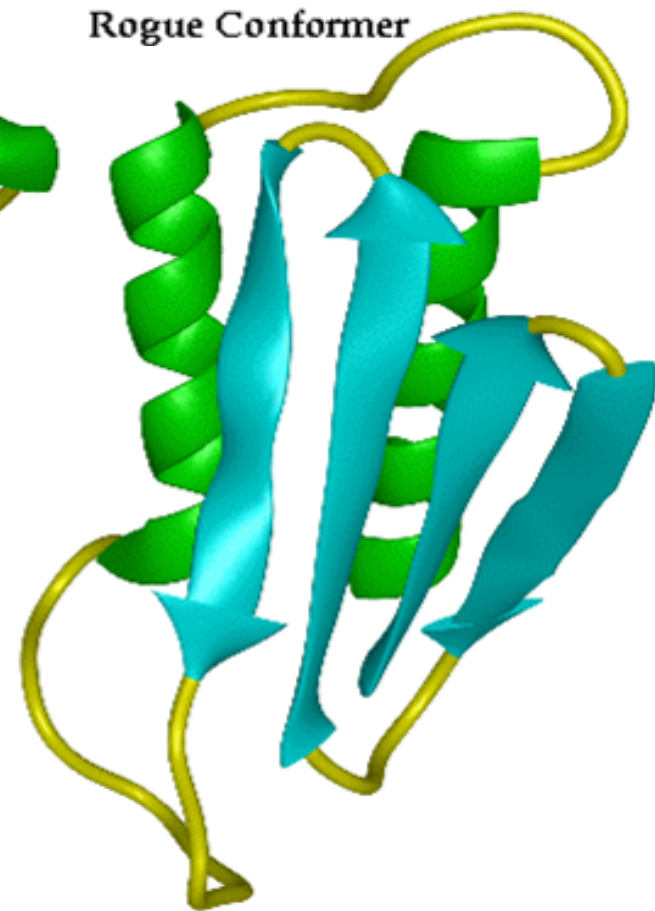


BoPrP (bovine)
 α -helices = green
 β -strands = cyan,
non-regular secondary structure = yellow
flexible disordered "tail" (23-121) = yellow dots

C-terminal part



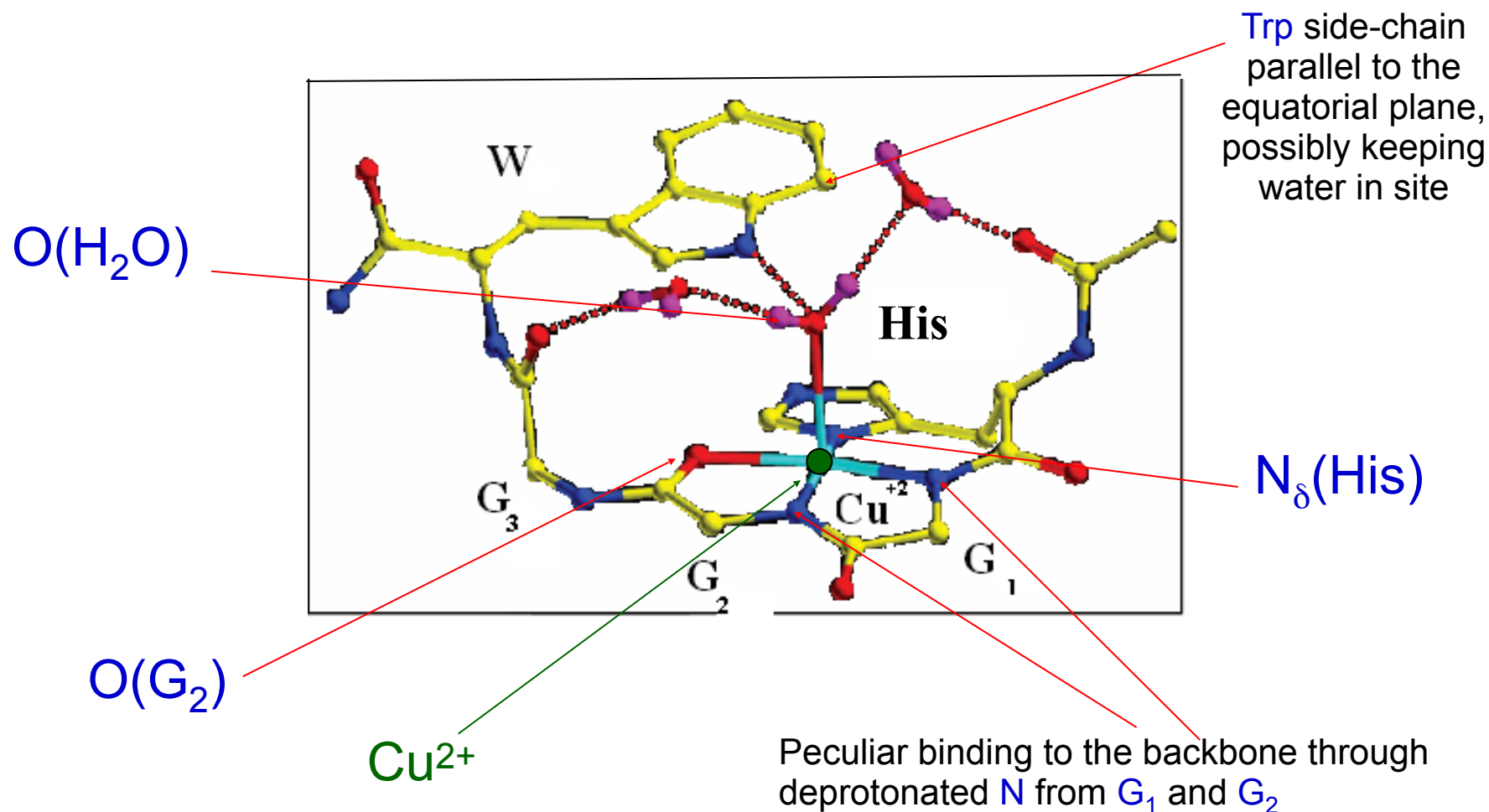
X-ray crystallography



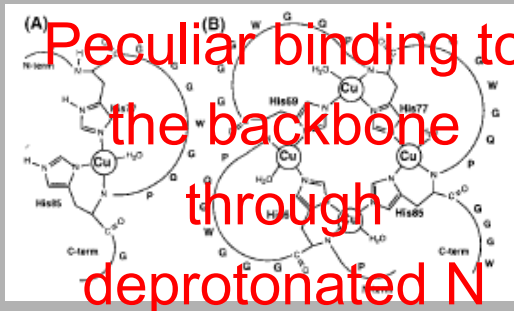
Speculative

X-crystallography of the **HGGGW-Cu⁺²** complex

Burns, et al. *Biochemistry* **41**:3991 (2002)



ESR, CD, and visible absorption spectra

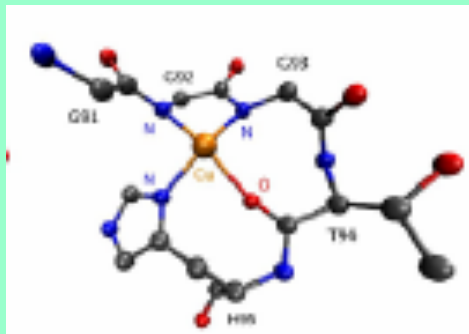


Viles et al. (1999) PNAS 96: 2042

Peculiar binding to the backbone through deprotonated N from Gly₁ and Gly₄ pH dependence of

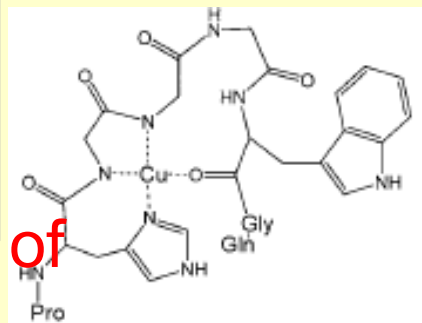
Cu binding

Ab initio computation on EPR data



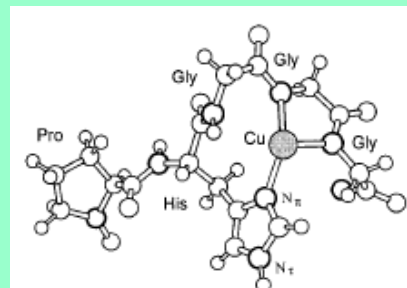
Cox et. al. (2006)
Biophys. J.

EPR



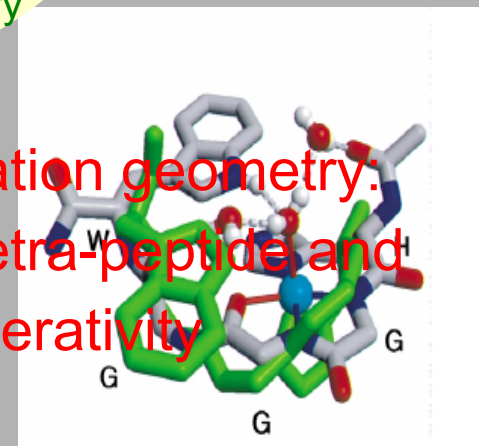
Aronoff-Spencer et al. Biochemistry (2000) 39: 13760.

Raman



Miura et al. Biochemistry (1999) 38:11560

NMR



Cu coordination geometry: di-peptide, tetra-peptide and cooperativity

Zahn (2003)
J. Mol. Biol. 334: 477

● EXAFS experiments

Synchrotron Radiation (SR)

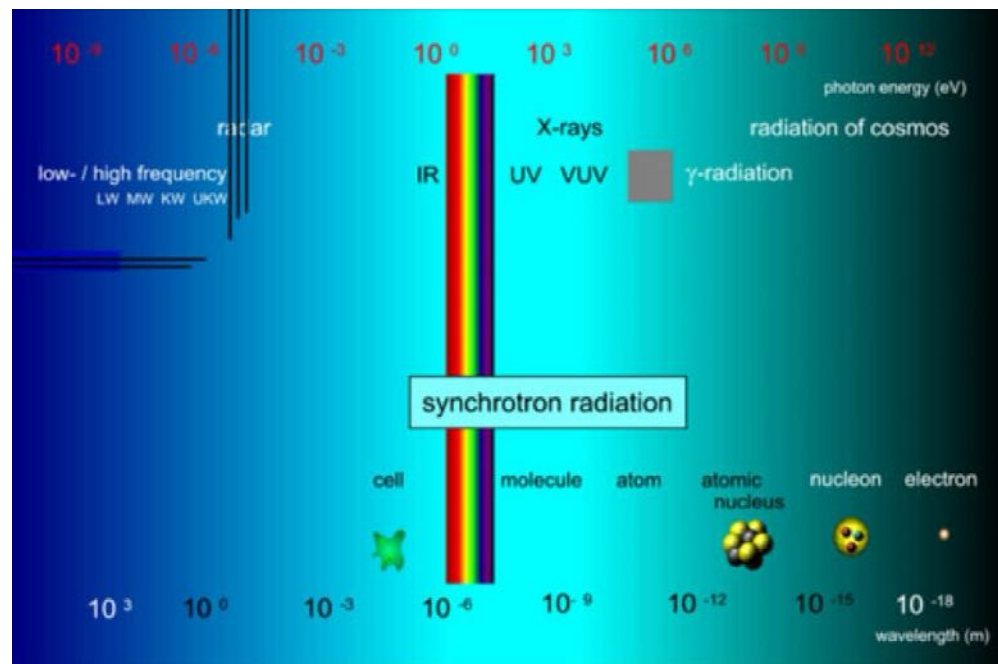
charged (electrons)
accelerated
relativistic ($E = \gamma mc^2$)

particles

radial acceleration
(e.g. deflection by a magnet)
Lorentz force
 $F = e v \times B$

SR is always emitted in the
forward direction
and is observed in a narrow cone
tangentially to the orbit

The higher the electron **kinetic energy**
the narrower the emission cone



SR spans the electromagnetic spectrum
from infrared (IR) to X-ray radiation

Synchrotron Light Sources of the World

1. [Advanced Light Source \(ALS\), Berkeley, California](#)
2. [Advanced Photon Source \(APS\), Argonne, Illinois](#)
3. [ALBA Synchrotron Light Facility \(formerly Laboratorio de Luz Sincrotrón\), Vallés, Spain](#)
4. [ANKA Synchrotron Strahlungsquelle, Karlsruhe, Germany](#)
5. [Australian Synchrotron, Melbourne, Victoria](#)
6. [Beijing Synchrotron Radiation Facility \(BSRF\), Beijing](#)
7. [Berliner Elektronenspeicherring-Gesellschaft für Synchrotronstrahlung \(BESSY\), Berlin](#)
8. [Canadian Light Source \(CLS\), Saskatoon, Saskatchewan](#)
9. [Center for Advanced Microstructures and Devices \(CAMD\), Baton Rouge, Louisiana](#)
10. [Center for Advanced Technology \(INDUS-1 and INDUS-2\), Indore, India](#)
11. [Cornell High Energy Synchrotron Source \(CHESS\), Ithaca, New York](#)
12. [diamond, Rutherford Appleton Laboratory, Didcot, England](#)
13. [Dortmund Electron Test Accelerator \(DELTA\), Dortmund, Germany](#)
14. [Electron Stretcher Accelerator \(ELSA\), Bonn, Germany](#)
15. [Elettra Synchrotron Light Source, Trieste, Italy](#)
16. [European Synchrotron Radiation Facility \(ESRF\), Grenoble, France](#)
17. [Hamburger Synchrotronstrahlungslabor \(HASYLAB\) at DESY, Hamburg, Germany](#)
18. [Institute for Storage Ring Facilities \(ISA, ASTRID\), Aarhus, Denmark](#)
19. [Laboratoire pour l'Utilisation du Rayonnement Electromagnétique \(LURE\), Orsay, France](#)
20. [Laboratório Nacional de Luz Síncrotron \(LNLS\) Sao Paolo, Brazil](#)
21. [MAX-lab, Lund, Sweden](#)
22. [National Synchrotron Light Source \(NSLS\), Brookhaven, New York](#)
23. [National Synchrotron Radiation Laboratory \(NSRL\), Hefei, China](#)
24. [National Synchrotron Radiation Research Center \(NSRRC\), Hsinchu, Taiwan, R.O.C](#)
25. [National Synchrotron Research Center \(NSRC\), Nakhon Ratchasima, Thailand](#)
26. [Photonics Research Institute, National Institute of Advanced Industrial Science and Technology \(AIST\)](#)
27. [Photon Factory \(PF\) at KEK, Tsukuba, Japan](#)
28. [Pohang Accelerator Laboratory, Pohang, Korea](#)
29. [Shanghai Synchrotron Radiation Facility, \(SSRF\),](#)
30. [Siberian Synchrotron Radiation Centre \(SSRC\), Novosibirsk, Russia](#)
31. [Singapore Synchrotron Light Source \(SSLS\), Singapore](#)
32. [SOLEIL Synchrotron, Saint-Aubin, France](#)
33. [Stanford Synchrotron Radiation Laboratory \(SSRL\), Menlo Park, California](#)
34. [Super Photon Ring - 8 GeV \(SPring8\), Nishi-Harima, Japan](#)
35. [Swiss Light Source \(SLS\), Villigen, Switzerland](#)
36. [Synchrotron Radiation Center \(SRC\), Madison, Wisconsin](#)
37. [Synchrotron Radiation Source \(SRS\), Daresbury, U.K.](#)
38. [Synchrotron Ultraviolet Radiation Facility \(SURF III\) at NIST, Gaithersburg, Maryland](#)
39. [UVSOR Facility, Okazaki, Japan](#)
40. [VSX Light Source, Kashiwa, Japan](#)

April 2005

BRIGHTNESS

(photons/(s mm² mrad²) 0.1% B.W.)

photon intensity +
spectral width +
focussing conditions =
brightness

SASE
FEL

Synchr.
sources

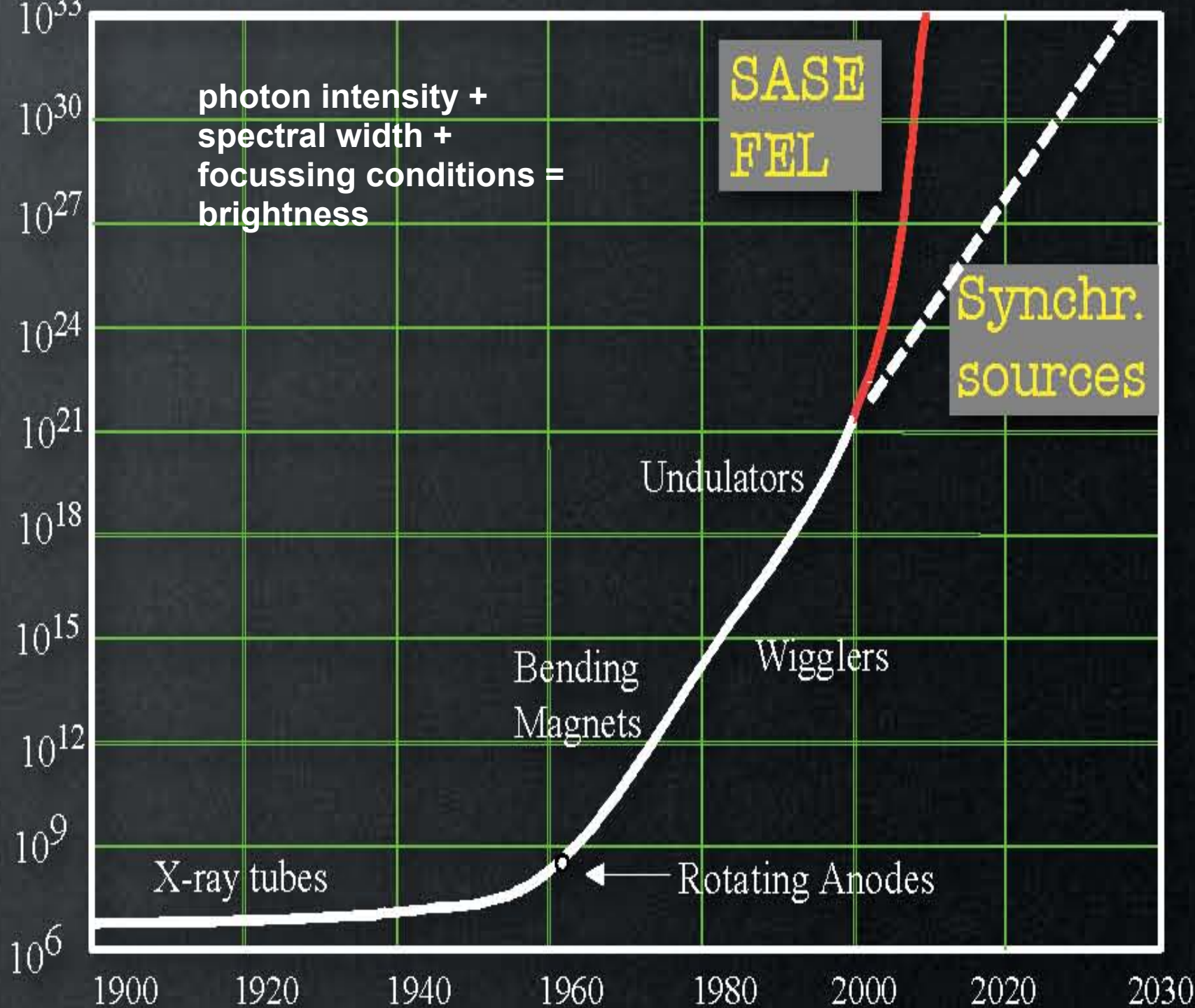
Undulators

Bending
Magnets

Wigglers

X-ray tubes

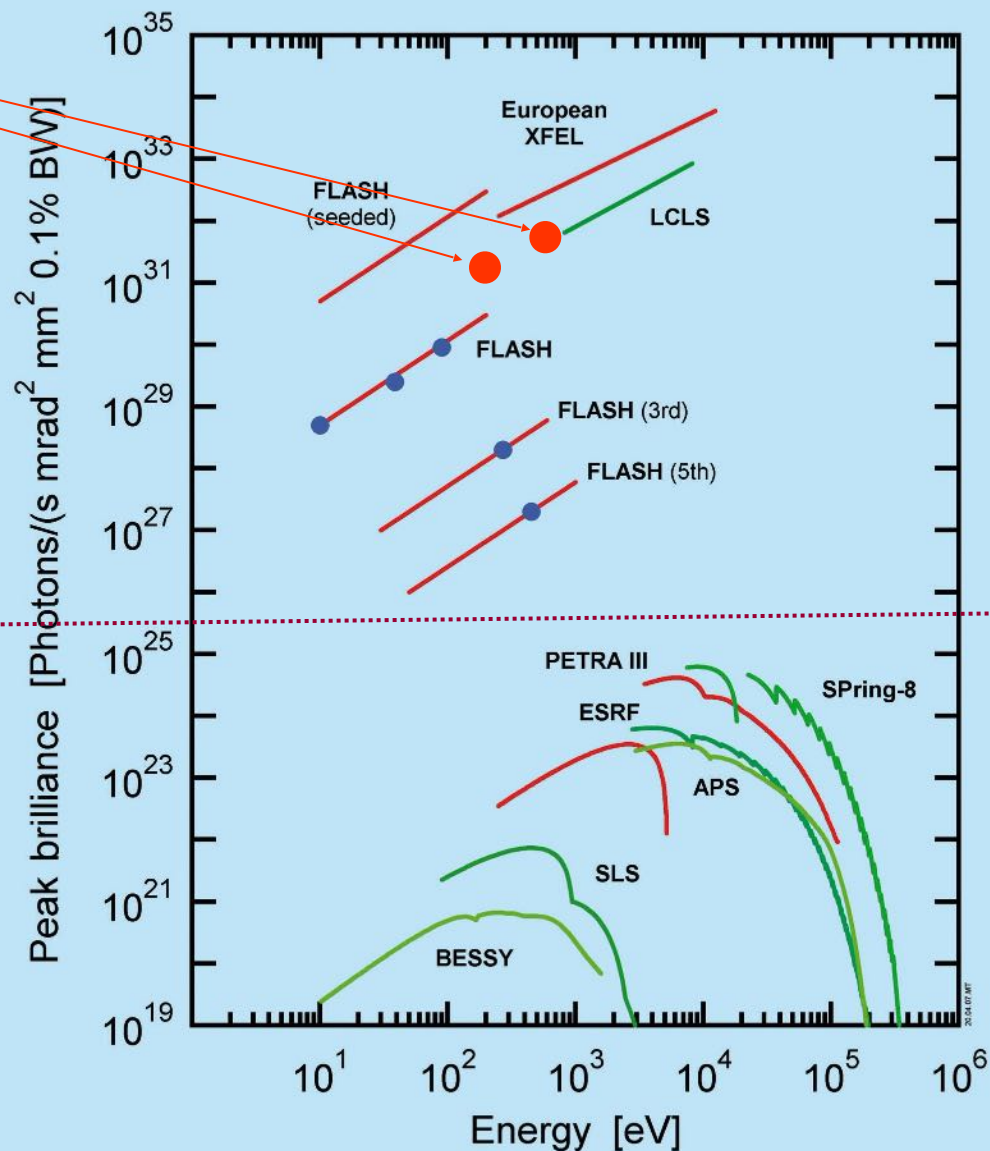
Rotating Anodes



Sparx

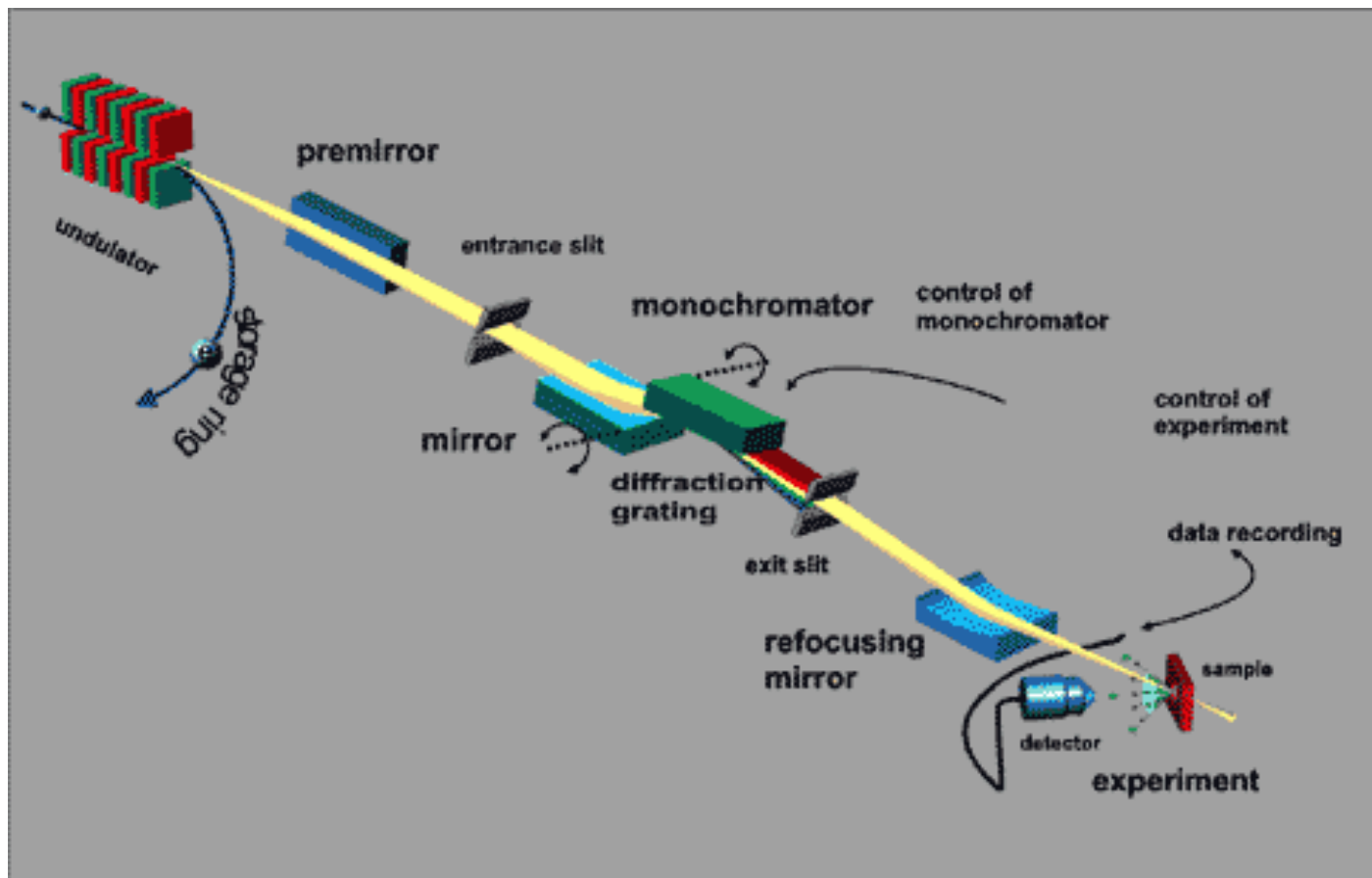
FEL Region

Synchrotron Radiation



Experimental setting

- radiation is directed by optical elements to the monochromator
- monochromator selects the desired wavelength of the spectrum
- the radiation is directed to the sample



Hard X-ray photons $\Rightarrow \lambda \sim$ inter-atomic distances in crystals

Radiation absorption \Rightarrow **photo-electric effect** $I = I_0 \exp[-\mu(E_k)d]$

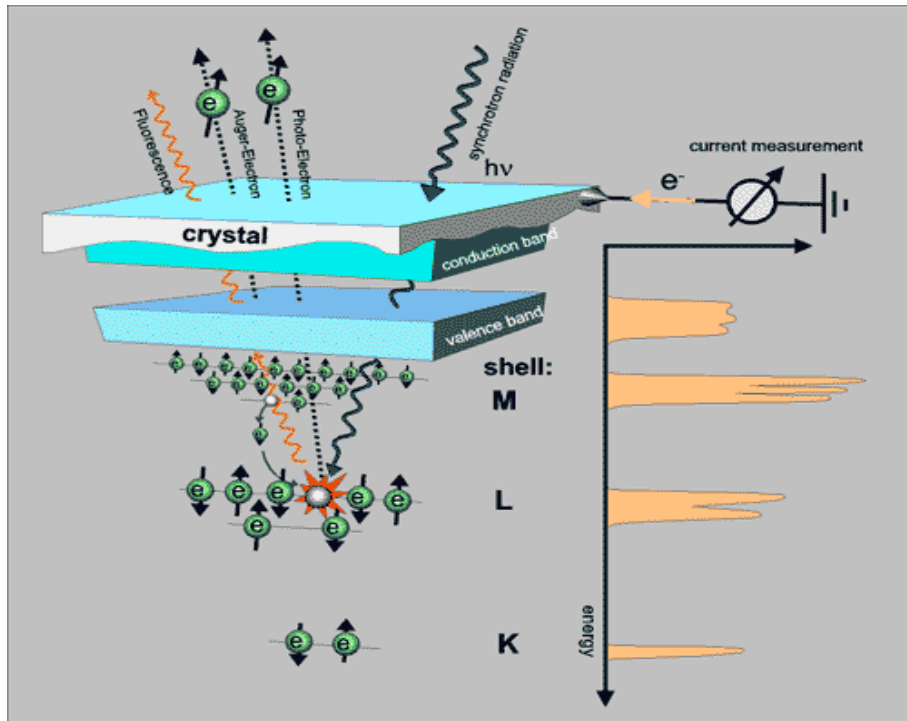
$$E_k = h\nu - E_0$$

E_k = kinetic energy of the emitted photo-electron

$h\nu$ = energy of the photon

E_0 = electron binding energy *

*characteristic of the specific material and bound state of the electron

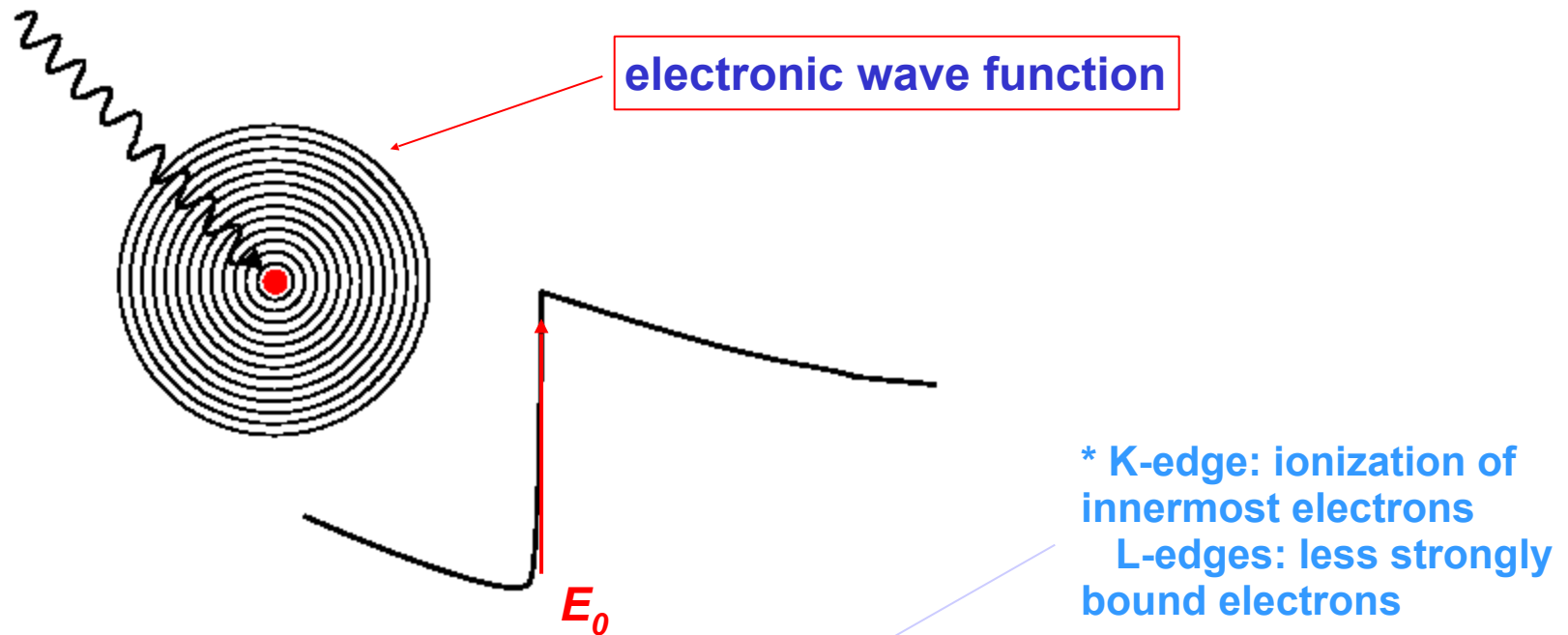


extracted photo-electron leaves
behind a positively charged
state

holes in the inner shell are filled
by electrons from outer shells

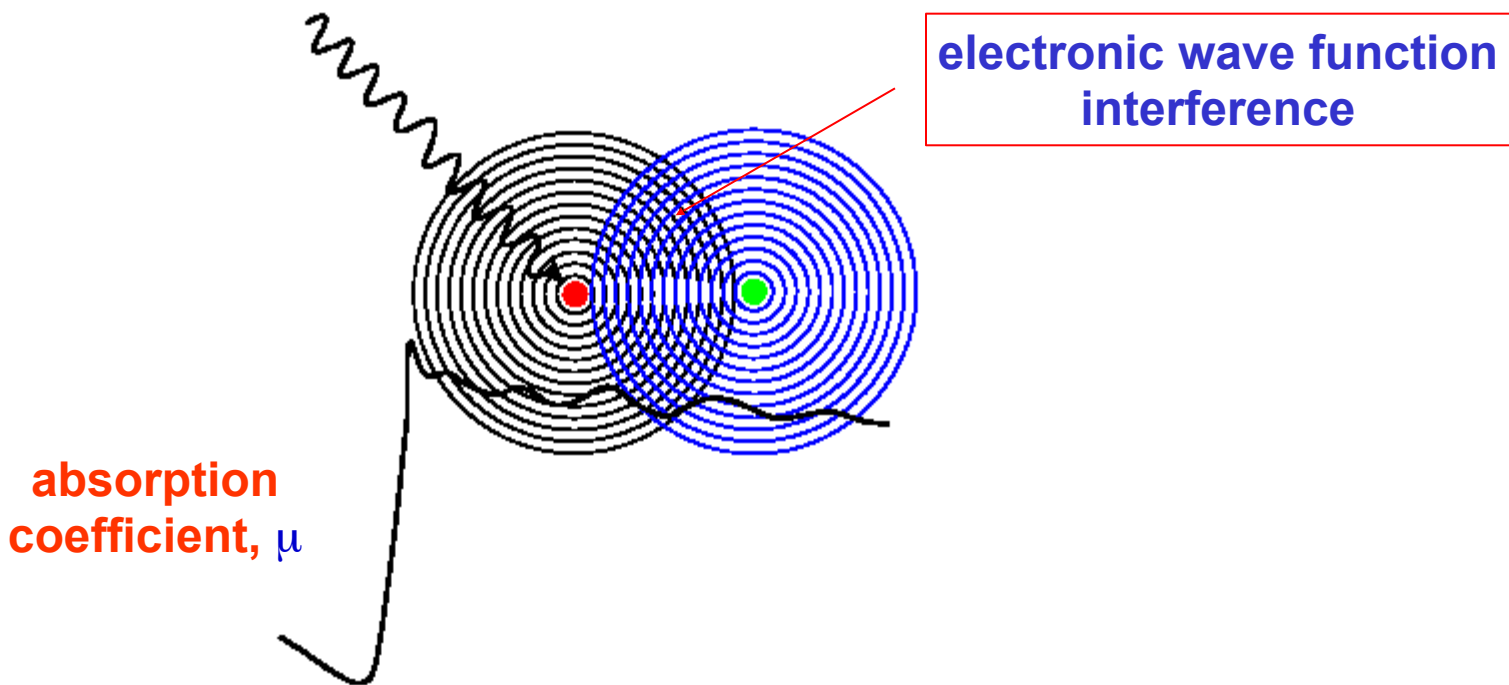
emission of photons
(**fluorescence**)
or further electrons from outer
shells (**Auger electrons**)

XAS spectrum from an isolated atom (e.g. mono-atomic gas)



- The **absorption coefficient**, μ , decreases monotonically with the incident photon energy, $h\nu$
- When $h\nu = E_0$ = photo-ionization energy of an inner electron of the absorbing atom (edge energy*), μ sharply increases.
- It then decreases monotonically soon after the edge

XAS spectrum from a non-isolated atom (e.g. a diatomic molecule)



- In a multi-atomic system μ doesn't decrease monotonically after the edge, rather it has an oscillating behaviour
- The absorber (**red dot**) emits an outgoing spherical wave (the ionized electron, *photo-electron*)
- The scatterer (**green dot**) acts as diffusion center of the backscattered wave, which interferes (*in phase or out-of-phase*) with the outgoing wave

EXAFS analysis of Cu^{++} site geometry in Prion peptide complexes

(EMBL-DESY) Hamburg

S1. (BoPrP 25-30, 60-70) KKRPKPWGQPHGGGWGQ

S2. (BoPrP 25-30, 60-78) KKRPKPWGQPHGGGWGQPHGGGWGQ

S3. (BoPrP 25-30, 60-94) KKRPKPWGQPHGGGWGQPHGGGWGQPHGGGWGQPHGGGWGQ

S4. (α BoPrP 24-242) CKKRPKPGGGWNTGGSRYPGQGSPGGNRYPPQGGGGWGQPHGGGWGQ
PHGGGWGQPHGGGWGQPHGGGWGQPHGGGGWGQGGTHGQWNKPSKPKTNMKHVAGAAAAG
 AVVGGLGGYMLGSAMSRPLIHFGSDYEDRYRENMHRYPNQVYYRPVDQYSNQNNFVHDCVNITV
 KEHTVTTTTTKGENFTETDIKMMERVVEQMCITQYQRESQAYYQRGAS

Cu⁺⁺ stoichiometry

Sample	Cu ⁺⁺ equivalence		
	N	E	E/N
S1	1	0.5	0.5
S2	2	1.5	0.75
S3a	4	3.2	0.8
S3b	4	2.0	0.5
S4	4-5	2.0	0.5/0.4

N: number of Cu^{++} coordination sites in the complex = number of octarepeats

E: $[\text{Cu}^{++}] / [\text{protein (or peptide)}]$

E/N: number of sites saturated with Cu^{++}
 $= [\text{Cu}^{++}] / [\text{octarepeat}]$

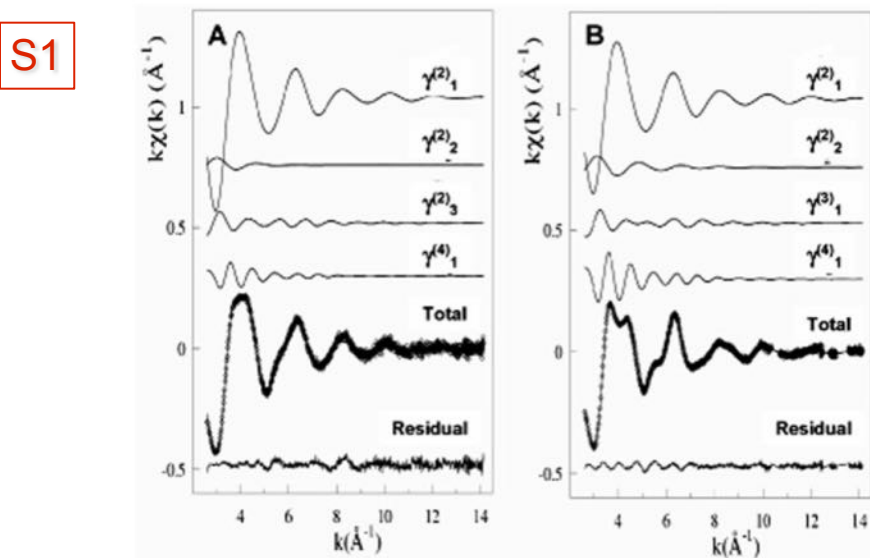


sub-stoichiometric Cu^{++} concentration

Morante et al., J. Biol. Chem. 279 (2004) 11753

EXAFS data: Single and Multiple Scattering contributions

Fitted curves are within data fluctuations

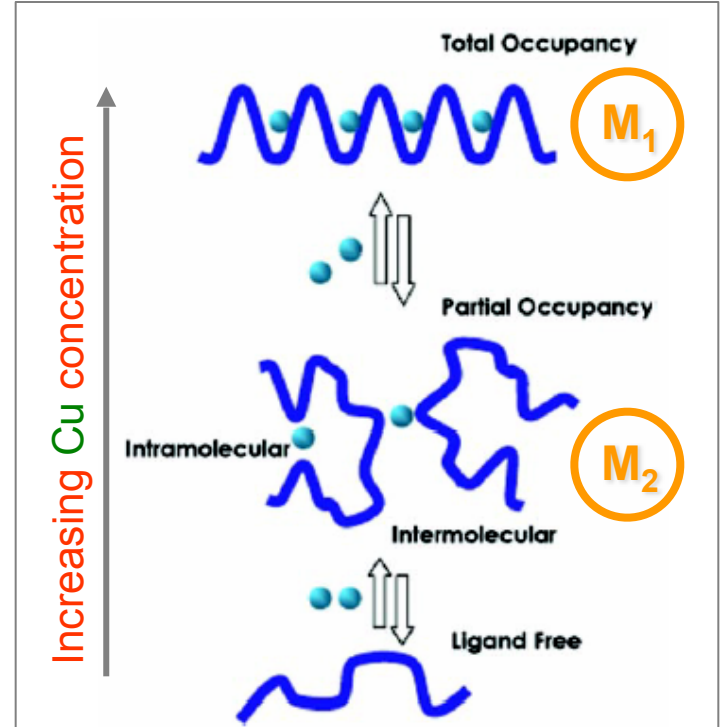
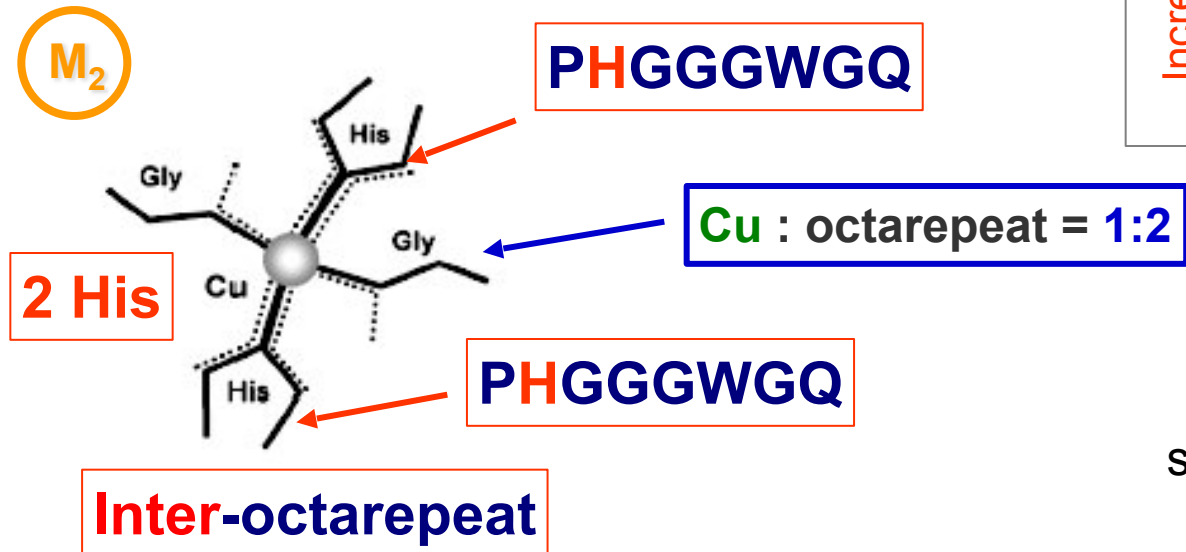
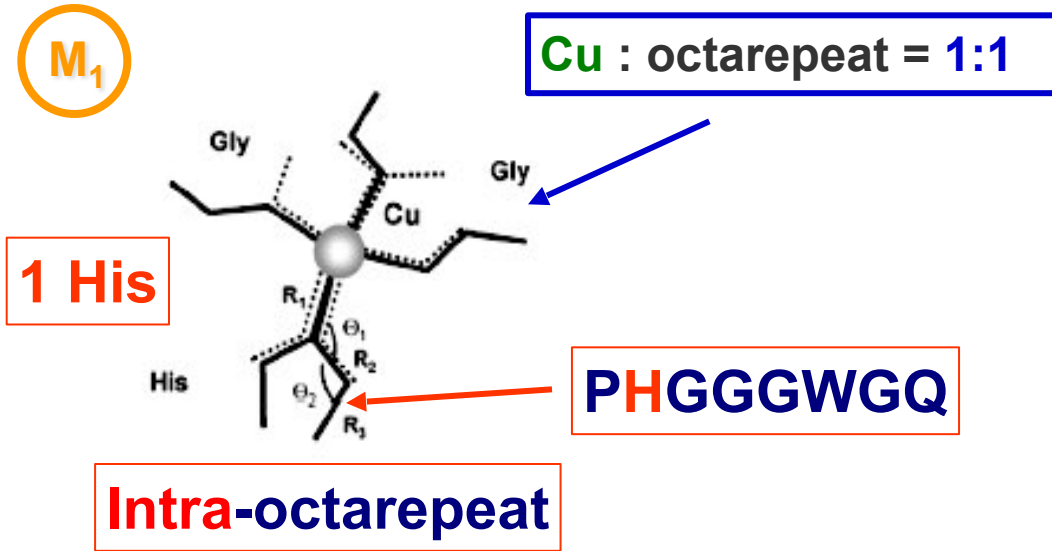


S4

His's are identified from their typical MS contributions

	Cu(II) complexes	[Cu/OR ratio]	First shell scatterers	First shell distance Å	Debye-Waller factor σ_{DW}^2 10^{-3}Å^2	First shell His ^a
S1	BoPrP-(25–30,60–70)	[0.5]	4–5 × Cu-N/O 1 × Cu-O 2.4 × Cu-C	1.97 ± 0.01 2.43 ± 0.01 3.29 ± 0.02	5.5 ± 0.4 3.2 ± 0.3 18 ± 8	1.3 ± 0.15
S2	BoPrP-(25–30, 60–78)	[0.75]	4–5 × Cu-N/O 1 × Cu-O 3 × Cu-C	1.97 ± 0.01 2.34 ± 0.01 3.29 ± 0.02	6.0 ± 0.4 3.2 ± 0.3 9 ± 3	1.5 ± 0.15
S3	BoPrP-(25–30, 60–94)	[0.5]	4 × Cu-N/O 1 × Cu-O	1.97 ± 0.01 2.34 ± 0.01	4.3 ± 0.8 5.0 ± 0.1	2.6 ± 0.3
S4	αBoPrP(24–242)	[0.5]	4 × Cu-N/O 1 × Cu-O	1.97 ± 0.01 2.35 ± 0.01	4 ± 0.5 8 ± 0.2	2.3 ± 0.2

Model interpretation of EXAFS data analysis



In the actual experimental situation no aggregates form

• EXAFS theory

Extracting structural information from EXAFS data

Data $I = I_0 \exp[-\mu(k)d]$ are expressed and analyzed in terms of

$$\chi(k) = \frac{\sigma_a - \sigma_0}{\sigma_0} = \frac{\mu(k) - \mu_0(k)}{\mu_0(k)} \quad k = \frac{\sqrt{2m(h\nu - E_0)}}{\hbar}$$

μ = **absorption coefficient**

$$\mu \propto \sigma_a$$

σ_a = **absorption cross section**

$$\sigma_a = 4\pi^2 \alpha \hbar \omega \left| \langle f | \hat{\epsilon} \cdot \vec{r} | i \rangle \right|^2 N(\omega) \quad \text{Fermi's golden rule}$$

ω

photon frequency

$N(\omega)$

density of photo-electron final state

$\hat{\epsilon}$

polarization vector of incidente radiation

$M_{fi} = \left| \langle f | \hat{\epsilon} \cdot \vec{r} | i \rangle \right|$

matrix element describing the electron transition

$|i\rangle$: initial bound state

$|f\rangle$: final “free” state

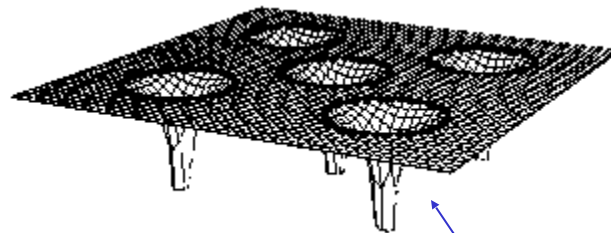
Initial, $|i\rangle$, and the final states, $|f\rangle$, are eigenfunctions of the Hamiltonian

$$H = \frac{\hbar^2}{2m} \nabla^2 - \frac{Ze^2}{r} + V(r)$$

The potential $V(r)$ is (most often) evaluated in the so-called **muffin tin (MT)** approximation



The **MT** potential consists of non overlapping spherical regions



In the interstitial regions the potential is set to a constant

Computing the transition matrix element, M_{fi}

❑ **Electron initial state:** one neglects V

The **Schrödinger** equation for the **innermost** (K) electron is

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + \frac{Ze}{r} \right) |i\rangle = E|i\rangle \quad \text{whose normalized solution is}$$

$$\psi_i(r) = \langle r|i\rangle = \pi^{-1/2} \left(\frac{Z}{a_0} \right)^{3/2} \exp(-Zr/a_0) \equiv \psi_0(r)$$

$n=1, l=0$ eigenfunction of hydrogen atom

❑ **Electron final state:** one neglects the **Coulomb** potential

The **Schrödinger** equation for the **outgoing** electron is

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) |f\rangle \equiv (H_0 + V) |f\rangle = E|f\rangle$$

H_0 is the free Hamiltonian, V is the potential due to the presence of scatterers

Iterative solution: let's write $|f\rangle = |k\rangle + |rest\rangle$

$|k\rangle$ **wave function of a free electron of momentum** \vec{k}

$$\langle r|k\rangle = N \exp \frac{i\vec{k} \cdot \vec{r}}{\hbar}$$

and satisfies the equation $H_0|k\rangle = E|k\rangle \rightarrow (E - H_0)|k\rangle = 0$

we have

$$(H_0 + V)|f\rangle = E|f\rangle \rightarrow (E - H_0)|f\rangle = V|f\rangle$$

inserting the definition of $|f\rangle \Rightarrow$

$$\bullet (E - H_0)|k\rangle + (E - H_0)|rest\rangle = V|f\rangle \Rightarrow |rest\rangle = (E - H_0)^{-1}V|f\rangle$$

$$\begin{aligned} \bullet |f\rangle &= |k\rangle + (E - H_0)^{-1}V|f\rangle = \\ &= |k\rangle + (E - H_0)^{-1}V|k\rangle + (E - H_0)^{-1}V(E - H_0)^{-1}V|k\rangle + \dots \end{aligned}$$

Introducing the Green function $G_0 = (E - H_0)^{-1}$

where, we recall

$$(E - H_0) \langle \vec{r} | G_0 | \vec{r}' \rangle = \delta^{(3)}(\vec{r} - \vec{r}') \quad \langle \vec{r} | G_0 | \vec{r}' \rangle = -\frac{m}{2\pi\hbar^2} \frac{e^{i\vec{k}(\vec{r}-\vec{r}')/\hbar}}{|\vec{r} - \vec{r}'|}$$

one obtains

$$M_{fi} = \langle f | \hat{\epsilon} \cdot \vec{r} | i \rangle = \langle k | \hat{\epsilon} \cdot \vec{r} | i \rangle + \langle k | G_o V \hat{\epsilon} \cdot \vec{r} | i \rangle + \langle k | G_o V G_o V \hat{\epsilon} \cdot \vec{r} | i \rangle + \dots \equiv A_0 + A_1 + A_2 + \dots$$

- Stopping the expansion after the first term (single scattering events), one gets

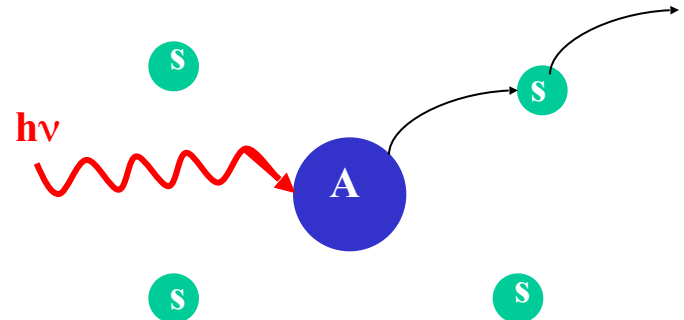
$$\left| \langle f | \hat{\epsilon} \cdot \vec{r} | i \rangle \right|^2 = |A_0|^2 + |A_1|^2 + 2 \operatorname{Re}(A_0 A_1^*) + 2 \operatorname{Re}(A_0 A_2^*)$$

atomic absorption contribution
(isolated atom)

oscillations of the EXAFS signal

- Including further terms (multiple scattering events), one gets

$$\sigma_a = \sigma_0 + \sum_i \sigma_i + \sum_{ij} \sigma_{ij} + \sum_{ijk} \sigma_{ijk} + \dots$$

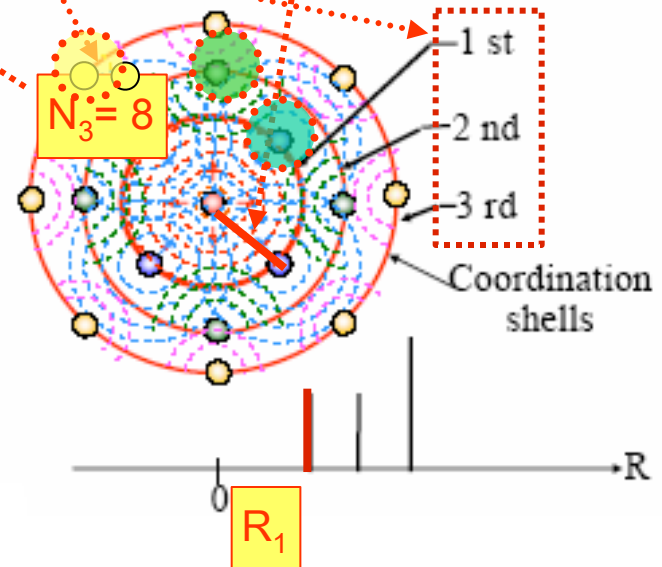


In Single Scattering approximation ($\hbar\omega \gg E_0$)

Boland Crane Baldeschieler, JPC 77, 142 (1982)

$$\chi(k) = S_0^2 \sum_i \frac{N_i}{kR_i^2} |f_i(k, \pi)| e^{-2k^2 \sigma_i^2} e^{-2R_i/\lambda(k)} \sin[2kR_i + \Phi_i(k)]$$

N_i, R_i, σ_i are fit parameters

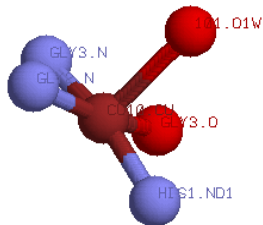


Single scattering approximation

Information on type, number and mean distance of scatterers

BUT

Copper ligands in the Prion peptide

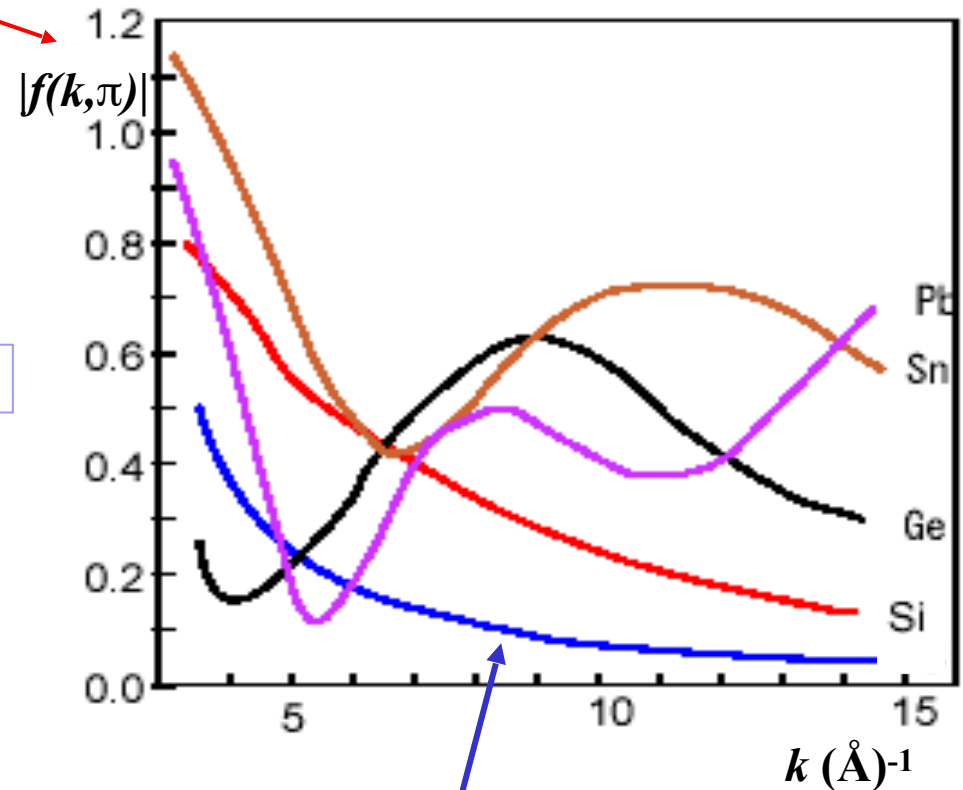


nitrogen

oxygen

Need Multiple Scattering
terms in σ_a to disentangle
C, O and N contributions

Backscattering Amplitude



Light atoms: C, O and N

Need to know

- the position of atoms in the vicinity of **Cu**, as the whole analysis of **EXAFS** data rests on this knowledge
- which are the actual metal ligands
- how the rest of the peptide is structurally organized

Ab initio calculations are necessary

- Quantum Mechanics to determine the atomic force field (in the **Born-Oppenheimer** approximation)
- Electrons are dealt with by **DFT** (Density Functional Theory)
 - **Schroedinger** equation is solved *à la* **Kohn-Sham**
- Atoms are treated classically
- **Car-Parrinello** simulations especially useful
 - Atomic Molecular Dynamics
 - Some dynamics helps in understanding stability

The DFT method

STEP 1

Decoupling of atomic and electronic dof's (v_A 's $\ll v_{el}$'s \Rightarrow BOA)

STEP 2

At fixed atomic coordinates, compute the electronic ground-state w.f. with the help of DFT (Schroedinger eq \Rightarrow Kohn-Sham eq's)

STEP 3

"Optimize" atomic coordinates to adapt them to the currently computed inter-atomic potential

STEP 4

Iterate STEP 2 and STEP 3 until you get consistency

The Car-Parrinello idea

Update atomic coordinates while solving Kohn-Sham eq's

Faster convergence \Rightarrow CPU-time

Control over configuration stability

Atomic and KS eq's are made to both look Newtonian (2nd order in time)

In Formulae

Starting point is the **Schroedinger** equation

electronic
coordinates

atomic
coordinates

$$\bullet i\hbar \frac{\partial}{\partial t} \Phi[\{\vec{r}\}, \{\vec{R}\}; t] = H \Phi[\{\vec{r}\}, \{\vec{R}\}; t]$$

$$\bullet H = - \sum_I \frac{\hbar^2}{2M_I} \nabla_I^2 + V_A[\{\vec{R}\}] + H_e[\{\vec{r}\}, \{\vec{R}\}]$$

$$\bullet V_A[\{\vec{R}\}] = \sum_{I < J} \frac{Z_I Z_J e^2}{|\vec{R}_I - \vec{R}_J|}$$

$$\bullet H_e[\{\vec{r}\}, \{\vec{R}\}] = - \frac{\hbar^2}{2m_e} \sum_i \nabla_i^2 + \sum_{i < j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} - \sum_{i, I} \frac{Z_I e^2}{|\vec{R}_I - \vec{r}_i|}$$

STEP 1

BO Molecular Dynamics

$$\bullet M_I \frac{d^2 \vec{R}_I(t)}{dt^2} = -\vec{\nabla}_I \left(\langle \Psi_0 | H_e | \Psi_0 \rangle + V_A[\{\vec{R}\}] \right)$$

$$H_e | \Psi_0 \rangle = E_0 | \Psi_0 \rangle$$

$$E_0 = E_0[\{\vec{R}\}]$$

$$\langle \{\vec{r}\} | \Psi_0 \rangle = \Psi_0[\{\vec{r}\}, \{\vec{R}\}]$$

Atoms move **classically** in the **Quantum Mechanical** potential generated by the electrons living in their ground-state w.f., Ψ_0

Difficulties

Schroedinger eq. should be solved over and over again at each atomic MD step

Contributions of excited states should be taken into account

One does not really know how to solve the electronic **Schroedinger** eq.

A **useful** approximation is **Hartree-Fock**

- Ψ_0 is written as a **Slater** determinant (**Pauli** principle) of N_e single particle trial w.f.'s, $\{\psi_i(\mathbf{r}_i)\}$
- The latter are determined by minimizing the total electronic energy

$$\min_{\{\psi\}} \left\langle \Psi_0[\{\psi\}] \left| H_e^{\text{HF}} \right| \Psi_0[\{\psi\}] \right\rangle \bigg|_{\langle \psi_i | \psi_j \rangle = \delta_{ij}} \quad \langle \{\vec{r}\} | \Psi_0[\{\psi\}] \rangle = \det_{ij} [\{\psi_i(\vec{r}_j)\}]$$

$$H_e^{\text{HF}} = -\frac{\hbar^2}{2m_e} \sum_i \nabla_i^2 - \sum_{i,I} \frac{Z_I e^2}{|\vec{R}_I - \vec{r}_i|} + W^{\text{dir}}[\{\psi\}] + W^{\text{exch}}[\{\psi\}]$$

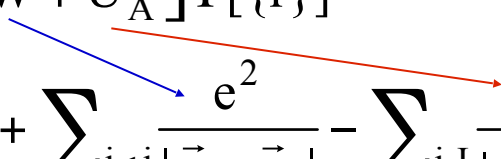
$$W^{\text{dir}}[\{\psi\}] = \sum_j \left[\int d\vec{r}' \psi_j^*(\vec{r}') \frac{1}{|\vec{r}' - \vec{r}|} \psi_j(\vec{r}') \right] \psi_i(\vec{r})$$

$$W^{\text{exch}}[\{\psi\}] = \sum_j \left[\int d\vec{r}' \psi_j^*(\vec{r}') \frac{1}{|\vec{r}' - \vec{r}|} \psi_i(\vec{r}') \right] \psi_j(\vec{r})$$

- H_e^{HF} is a one-body Hamiltonian
- which depends **non-locally** and **non-linearly** on all $\{\psi\}$

STEP 2

DFT → provides a way to systematically map the **many-body** problem (with electron self-interaction, W)

$$\begin{aligned} \bullet H_e \Psi[\{\vec{r}\}] &= [T + W + U_A] \Psi[\{\vec{r}\}] = \\ &= \left[-\frac{\hbar^2}{2m_e} \sum_i \nabla_i^2 + \sum_{i < j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} - \sum_{i,I} \frac{Z_I e^2}{|\vec{R}_I - \vec{r}_i|} \right] \Psi[\{\vec{r}\}] = E[\{\vec{R}\}] \Psi[\{\vec{r}\}] \end{aligned}$$


into a **single-body** problem (without electron self-interaction, W)

DFT → is based on the Hohenberg-Kohn (Phys. Rev **136** (1964) 864) →

Theorem

“There exists a one-to-one mapping between the set of U_A potentials and the set of (admissible) **ground-state electronic densities**”

$$\bullet \{n\} \leftrightarrow \{U_A\} \text{ where } n(\vec{r}) = N \int \prod_{i=1}^N d\vec{r}_i \Psi_0^*(\vec{r}, \vec{r}_2, \dots, \vec{r}_N) \Psi_0(\vec{r}, \vec{r}_2, \dots, \vec{r}_N)$$

Lemma 1

Since U_A fixes $H_e \rightarrow \Psi_0$ is in turn a unique functional of n , hence of U_A

$$\bullet \{n\} \leftrightarrow \{U_A\} \leftrightarrow \{\Psi_0\} \quad \longleftarrow \quad \text{HK mapping}$$

Lemma 2

- $F_{\text{HK}}[n] = \langle \Psi[n] | T + W | \Psi[n] \rangle$

is a well-defined, universal functional of the (admissible) electronic density

Lemma 3

The functional

- $E_{u_A}[n] = F_{\text{HK}}[n] + \int d\vec{r} u_A(\vec{r})n(\vec{r}), \quad u_A(\vec{r}) = \sum_I \frac{Z_I e^2}{|\vec{R}_I - \vec{r}|}$

- 1) attains its **minimum** when $n = n_{u_A}(\vec{r})$, i.e. when the electronic density equals the value which is in correspondence with U_A in the **HK** mapping
- 2) at the minimum it equals the total electronic energy

Corollary

We can compute the ground-state electronic density, hence all the ground-state observables, from the **minimum** equation

- $\frac{\delta E_{u_A}[n]}{\delta n(\vec{r})} = 0 = \frac{\delta F_{\text{HK}}[n]}{\delta n(\vec{r})} + u_A(\vec{r}) \quad (\heartsuit)$

except that **we do not know** the **HK**-functional, F_{HK}

Kohn and Sham have proposed a way to go around this problem

The Kohn-Sham equations

- The key observation is that the HK mapping exists, even if we set the electronic self-interaction term to zero in all the above equations, $W \equiv 0$
- in this situation the many-body electronic Schroedinger equation separates into N decoupled one-body equations
- furthermore for any given electronic density, n , there exists a u_A^{NSI} such that one can represent n as the sum of the moduli square of the solutions of the one-body Schroedinger equation

$$\left[-\frac{\hbar^2}{2m_e} \nabla^2 + u_A^{\text{NSI}}[n; \vec{r}] \right] \varphi_i(\vec{r}) = \varepsilon_i \varphi_i(\vec{r}) \quad i = 1, 2, \dots, N$$
$$n(\vec{r}) = \sum_{i=1}^N |\varphi_i(\vec{r})|^2$$

$u_A \leftrightarrow n \leftrightarrow u_A^{\text{NSI}}$

Kohn-Sham equations

■ We are done if we can find the relation between u_A and u_A^{NSI}

- Ψ_0 is exactly the Slater determinant of the $\{\varphi_i\}$

- the NSI HK-functional is simply the kinetic energy

$$\bullet F_{\text{HK}}^{\text{NSI}}[n] \equiv T_{\text{HK}}^{\text{NSI}}[n] = \langle \Psi_0[n] | T | \Psi_0[n] \rangle = -\frac{\hbar^2}{2m_e} \sum_{i=1}^N \int d\vec{r}_i \varphi_i^*(\vec{r}_i) \nabla^2 \varphi_i(\vec{r}_i)$$

- and satisfies the equation

$$\bullet \frac{\delta T_{\text{HK}}^{\text{NSI}}[n]}{\delta n(\vec{r})} + u_A^{\text{NSI}} = 0 \quad (\clubsuit)$$

■ We now rewrite $E_{u_A}[n] = F_{\text{HK}}[n] + \int d\vec{r} u_A(\vec{r})n(\vec{r})$ in the form

$$\bullet E_{u_A}[n] = T_{\text{HK}}^{\text{NSI}}[n] + \int d\vec{r} u_A(\vec{r})n(\vec{r}) + \frac{e^2}{2} \int d\vec{r} d\vec{r}' \frac{n(\vec{r})n(\vec{r}')}{|\vec{r} - \vec{r}'|} + E^{\text{exch}}[n]$$

$$\bullet E^{\text{exch}}[n] \equiv F_{\text{HK}}[n] - T_{\text{HK}}^{\text{NSI}}[n] - \frac{e^2}{2} \int d\vec{r} d\vec{r}' \frac{n(\vec{r})n(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

■ Minimizing $E_{u_A}[n]$ and using equations (\heartsuit) and (\clubsuit), we get

$$\bullet u_A^{\text{NSI}}(\vec{r}) = u_A(\vec{r}) - e^2 \int d\vec{r}' \frac{n(\vec{r}')}{|\vec{r} - \vec{r}'|} + \frac{\delta E^{\text{exch}}[n]}{\delta n(\vec{r})}$$

- Inserting back u_A^{NSI} in the **KS equations** one ends up with

$$\bullet \left[-\frac{\hbar^2}{2m_e} \nabla^2 + u_A(\vec{r}) - e^2 \int d\vec{r}' \frac{n(\vec{r}')}{|\vec{r} - \vec{r}'|} + \frac{\delta E^{\text{exch}}[n]}{\delta n(\vec{r})} \right] \varphi_i(\vec{r}) = \varepsilon_i \varphi_i(\vec{r}) \quad (\spadesuit)$$

- **formally** identical to the **HF** equations, but for
- ε_i are **Lagrange** multipliers enforcing $\langle \varphi_i | \varphi_j \rangle = \delta_{ij}$

- On the solution the total energy reads

$$\bullet E_0^{\text{HK}} = \sum_{i=1}^N \varepsilon_i + \frac{e^2}{2} \int d\vec{r} d\vec{r}' \frac{n_0(\vec{r}) n_0(\vec{r}')}{|\vec{r} - \vec{r}'|} + E^{\text{exch}}[n_0] - \int d\vec{r} \frac{\delta E^{\text{exch}}[n]}{\delta n(\vec{r})} \bigg|_{n_0} n_0(\vec{r})$$

- it is a function of the atomic positions
- it plays the role of **inter-atomic potential** in **MD** simulations

- We need an expression for $E^{\text{exch}}[n]$ and $\frac{\delta E^{\text{exch}}[n]}{\delta n(\vec{r})}$

- for the **Free Electron Gas**

$$\bullet T_{\text{FEG}}[n] = \frac{3}{10} \int d\vec{r} (3\pi^2 n)^{2/3} n \quad E_{\text{FEG}}^{\text{exch}}[n] = -\frac{3}{4\pi} \int d\vec{r} (3\pi^2 n)^{1/3} n$$

- **LDA** / **GGA** / ...

$$\longrightarrow E_{\text{LDA/GGA}}^{\text{exch}}[n] = c \int d\vec{r} \eta_{\text{LDA/GGA}}^{\text{exch}}[n] n$$

STEP 3 → STEP 4

“Optimization” of atomic coordinates can be achieved in various ways

1) Solve the **classical** eqs of motion

$$\bullet M_I \frac{d^2 \vec{R}_I(t)}{dt^2} = -\vec{\nabla}_I \left(E^{\text{HK}}[\{\vec{R}\}] + V_A[\{\vec{R}\}] \right)$$

but, need to know $E^{\text{HK}}[\{\mathbf{R}\}]$ for all values of $\{\mathbf{R}\}$

2) Solve simultaneously **classical** eqs of motion for **atoms** and the **KS** eqs for **electrons**

It can be elegantly done by introducing the effective **Lagrangian**

$$\bullet L_{\text{CP}} = \frac{1}{2} \sum_I M_I \left(\frac{d\vec{R}_I}{dt} \right)^2 + \frac{1}{2} \sum_i \mu_i \int d\vec{r} \frac{d\varphi_i^*(\vec{r}, t)}{dt} \frac{d\varphi_i(\vec{r}, t)}{dt} +$$

$$+ \sum_{I < J} \frac{Z_I Z_J e^2}{|\vec{R}_I - \vec{R}_J|} - E^{\text{HK}}[\{\varphi\}, \{\vec{R}\}] + \sum_i \Lambda_{ij} \int d\vec{r} \varphi_i^*(\vec{r}, t) \varphi_i(\vec{r}, t)$$

$\{\mathbf{R}\}$ and $\{\varphi\}$ are **Lagrangian** coordinates, $n(\mathbf{r}) = \sum_i \varphi_i^*(\mathbf{r}) \varphi_i(\mathbf{r})$ and

$$\bullet E^{\text{HK}}[\{\varphi\}, \{\vec{R}\}] = \sum_{i=1}^N \varepsilon_i + \frac{e^2}{2} \int d\vec{r} d\vec{r}' \frac{n(\vec{r}) n(\vec{r}')}{|\vec{r} - \vec{r}'|} + E^{\text{exch}}[n] - \int d\vec{r} \frac{\delta E^{\text{exch}}[n]}{\delta n(\vec{r})} n(\vec{r})$$

- Rather than the minimum equation (\spadesuit), we get for the electronic w.f., the 2nd order equation in the (fictitious) time

$$0 =$$

ε_i eigenvalues
of Λ_{ij}

$$\mu_i \frac{d^2 \varphi_i(\vec{r}, t)}{dt^2} = \left[-\frac{\hbar^2}{2m_e} \nabla^2 + u_A(\vec{r}) - e^2 \int d\vec{r}' \frac{n(\vec{r}', t)}{|\vec{r} - \vec{r}'|} + \frac{\delta E^{\text{exch}}[n]}{\delta n(\vec{r})} \right] \varphi_i(\vec{r}, t) - \Lambda_{ij} \varphi_j(\vec{r}, t)$$

- A unique time step for atomic MD and KS eqs, $\Delta t \approx \text{femtosecond}$
- We need to solve the KS eqs by adiabatically lowering the electronic “kinetic energy”
 - “total electronic energy” is (almost) conserved
we have a Lagrangian system
little energy transfer between atoms and electrons
 - by progressively lowering $T_e \rightarrow 0$, the system will end in the minimum of the “potential”
 - where the force, hence the acceleration is zero

CP dynamics is implemented in a number of codes,
among which Quantum ESPRESSO and CPMD

<http://www.quantum-espresso.org/>

<http://www.cpmd.org/>

- Quantum ESPRESSO is an initiative of the DEMOCRITOS National Simulation Center (Trieste) and of its partners.
- In collaboration with
 - CINECA, the Italian National Supercomputing Center in Bologna
 - Ecole Polytechnique Fédérale de Lausanne
 - Princeton University
 - Massachusetts Institute of Technology
 - Many other individuals...
- Integrated computer code suite for electronic structure calculations and materials modeling at the nanoscale
 - Released under a free license (GNU GPL)
 - Written in Fortran 90, with a modern approach
 - Efficient, Parallelized (MPI), Portable
- Suite components
 - PWscf (Trieste, Lausanne, Pisa): self-consistent electronic structure, structural relaxation, BO molecular dynamics, linear-response (phonons, dielectric properties)
 - CP (Lausanne, Princeton): (variable-cell) Car-Parrinello molecular dynamics

The Quantum-ESPRESSO Software Distribution

- Car-Parrinello variable-cell molecular dynamics with Ultrasoft PP's.
- Developed by A. Pasquarello, K. Laasonen, A. Trave, R. Car, N. Marzari, P. Giannozzi, C. Cavazzoni, G. Ballabio, S. Scandolo, G. Chiarotti, P. Focher.
- Verlet dynamics with mass preconditioning
- Temperature control: Nosé thermostat for both electrons and ions, velocity rescaling
- Variable-cell (Parrinello-Rahman) dynamics
- Damped dynamics minimization for electronic and ionic minimization
- Modified kinetic functional for constant-pressure calculations
- “Grid Box” for fast treatment of augmentation terms in Ultrasoft PP's
- Metallic systems: variable-occupancy dynamics
- Nudged Elastic Band (NEB) for energy barriers and reaction paths
- Dynamics with Wannier functions

A *first principle* study
of the Cu-HGGG interactions

A - the monomer

B - the dimer

Furlan, La Penna, Guerrieri,
Morante, GCR, JBIC **12(4)**
(2007) 571

A - Initial Cu⁽⁺²⁾ HG⁽⁻⁾G⁽⁻⁾G configuration

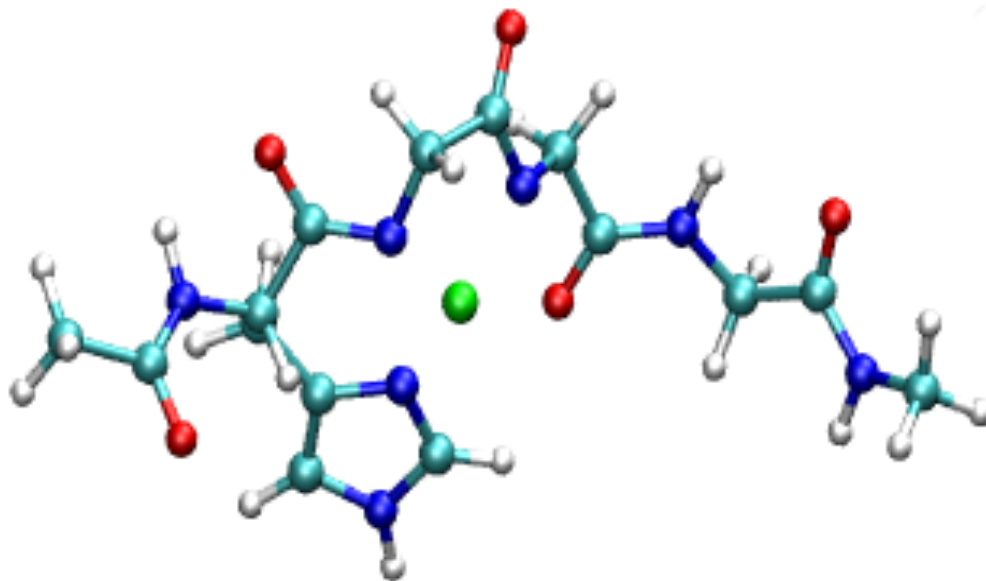
Cu⁽⁺²⁾

O

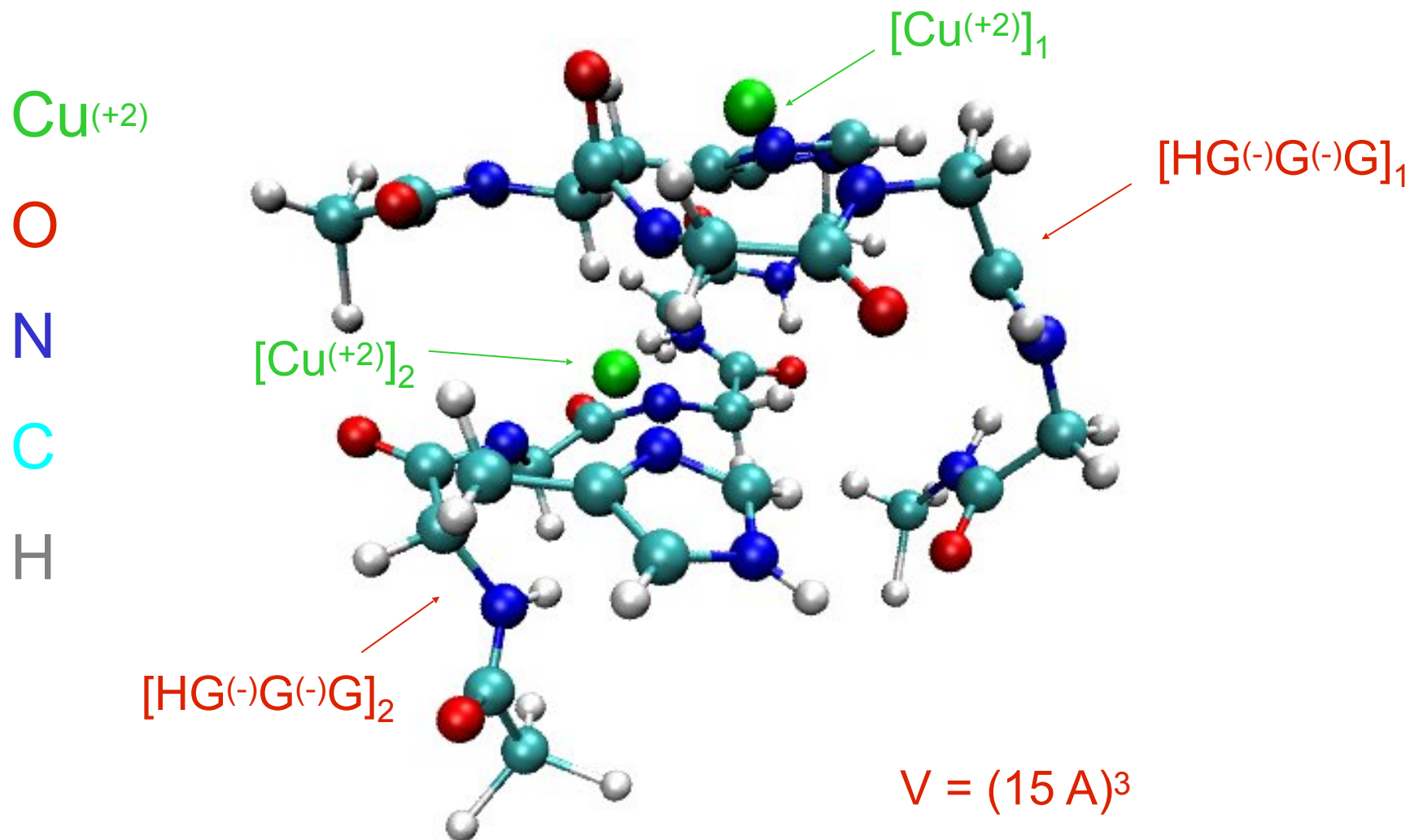
N

C

H

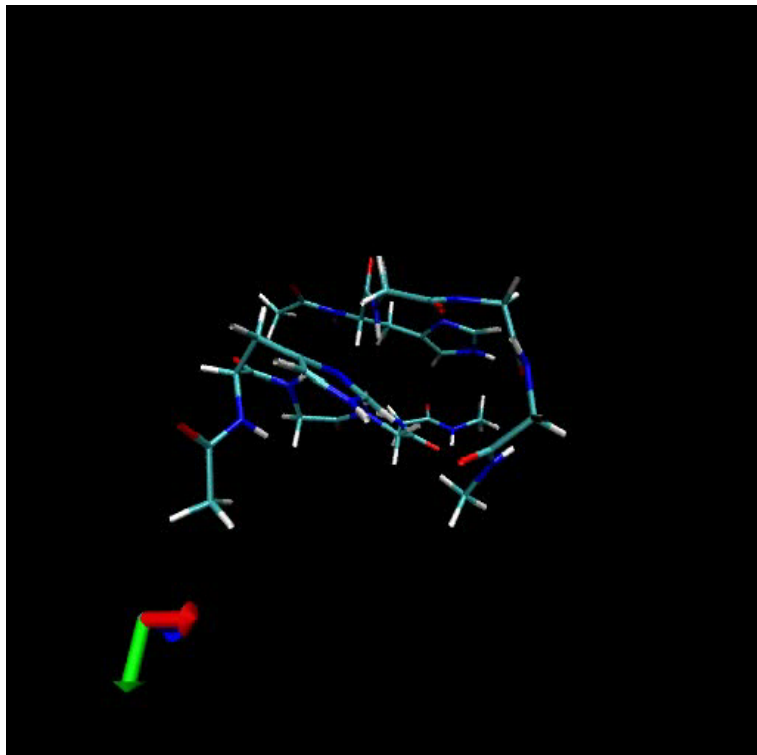


B - Initial 2 x [Cu⁽⁺²⁾ HG^{(-)G^{(-)G}] configuration}

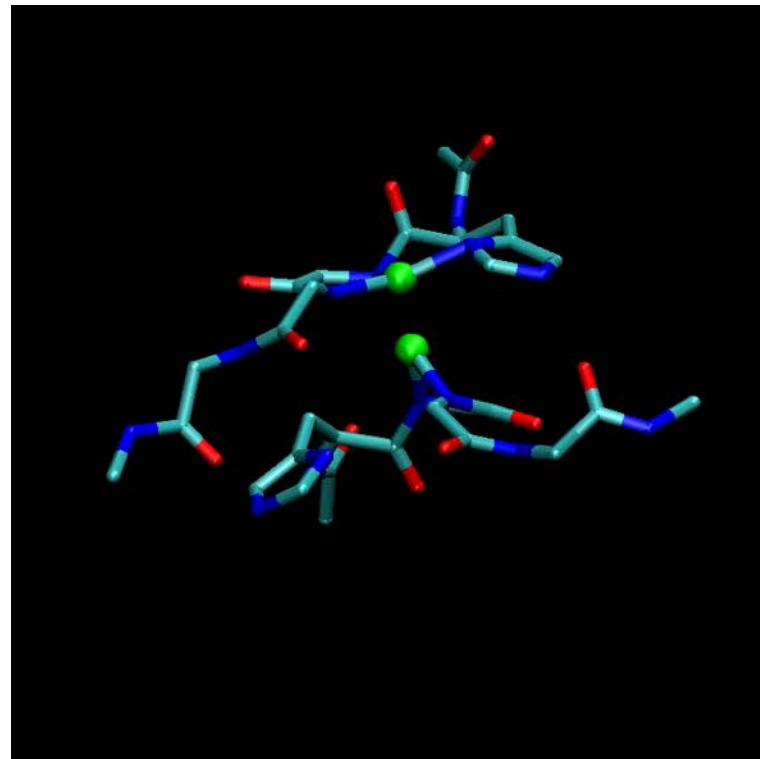


1.8 ps trajectory @ 300K

no Cu → no binding



Cu bonds with Gly and His are dynamically formed and destroyed



Cu

O

N

C

Quantum Mechanics at work

Car-Parrinello ab initio simulations

A *first principle* study of the
influence of pH on the
geometry of the Cu binding
site in the
HGGG + H(Im) peptide

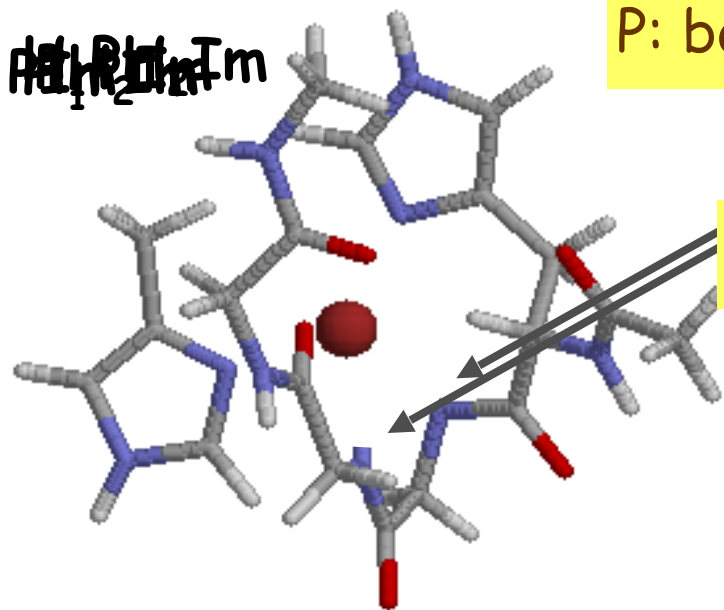
Furlan, La Penna, Guerrieri, Morante, GCR, JBIC

System 1, S1: $\text{Cu}^{2+}(\text{HisG}_1\text{-G}_2\text{-G}_3) + \text{Im} + 83 \text{ (H}_2\text{O)}$

System 2, S2: $\text{Cu}^{2+}(\text{HisG}_1\text{G}_2\text{-G}_3) + \text{Im} + 105 \text{ (H}_2\text{O)}$

System 3, S3: $\text{Cu}^{2+}(\text{HisG}_1\text{-G}_2\text{G}_3) + \text{Im} + 92 \text{ (H}_2\text{O)}$

System 4, S4: $\text{Cu}^{2+}(\text{HisG}_1\text{G}_2\text{G}_3) + \text{Im} + 92 \text{ (H}_2\text{O)}$



P: both Gly_1 and Gly_2 deprotonated

PH₂: only Gly_2 protonated
PH₁: only Gly_1 protonated

S1: HisG₁⁻G₂⁻G₃ + Im

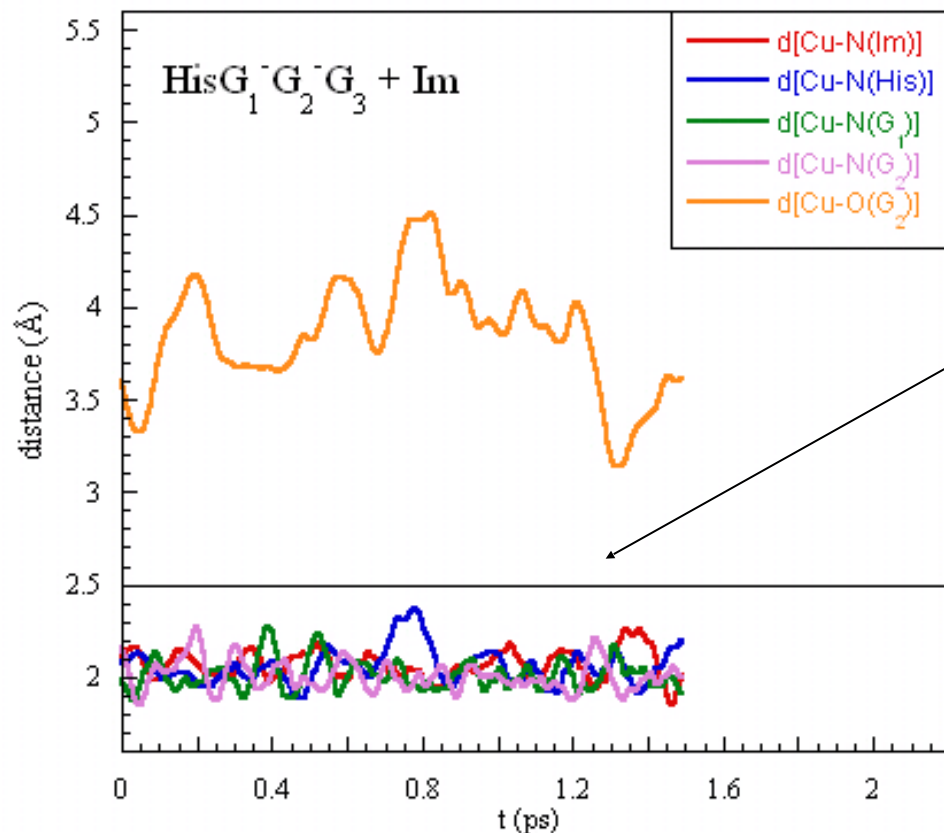
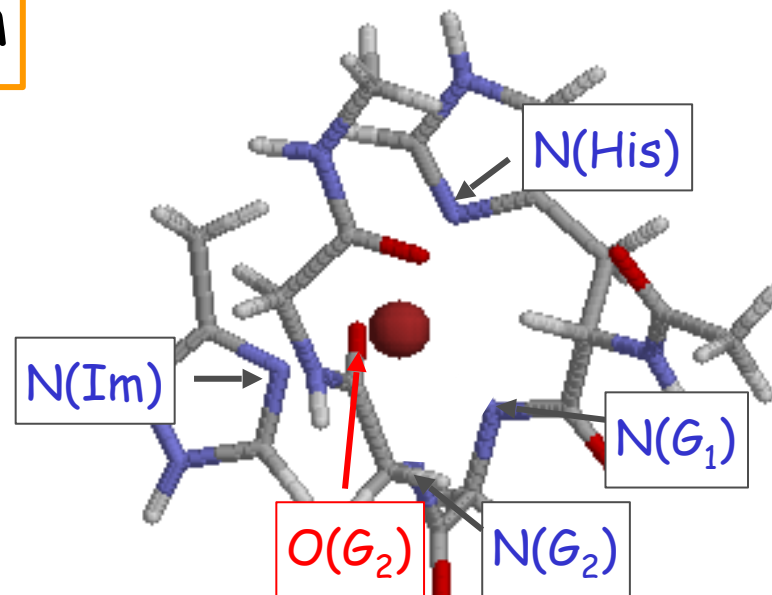
N_δ of isolated imidazole → N(Im)

N_δ of His → N(His)

N of first Gly → N(G₁)

N of second Gly → N(G₂)

Carbonyl O of second Gly → O(G₂)



Atom	<d> (Å)	σ (Å)
line of "coordination sphere" .08		
N(His)	2.10	0.10
N(G ₁)	2.01	0.08
N(G ₂)	2.01	0.08
O(G ₂)	3.80	0.30

S2: HisG₁G₂⁻G₃ + Im

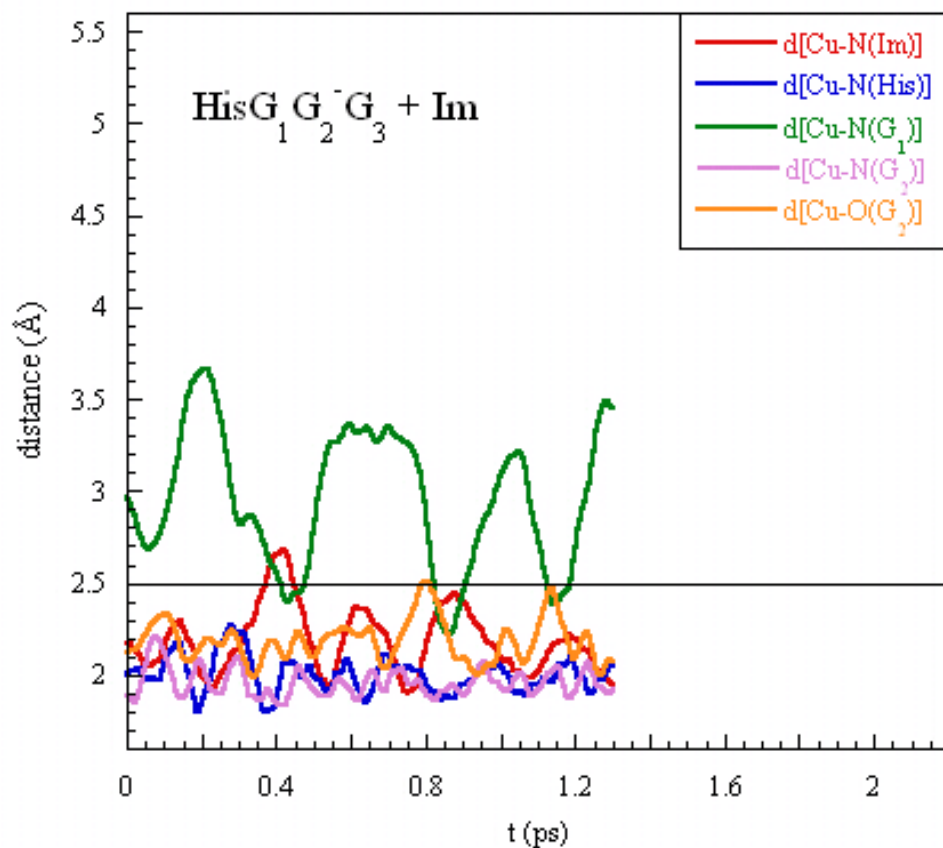
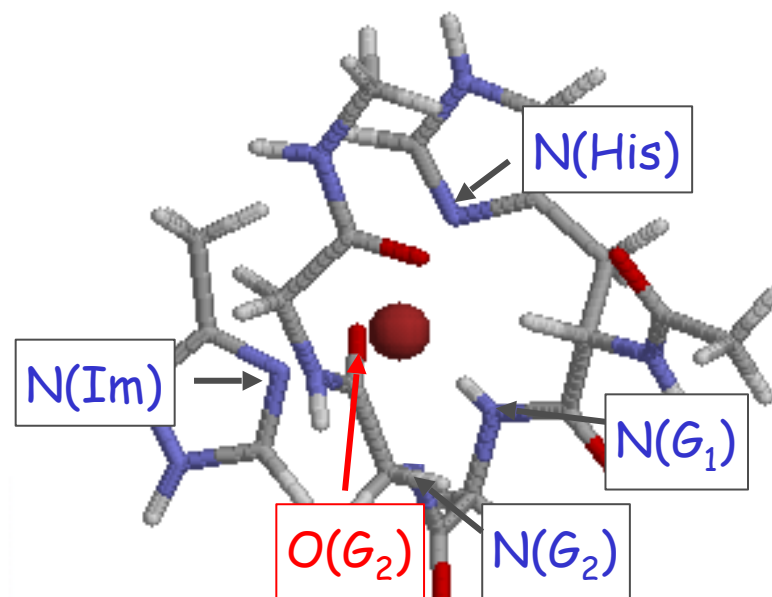
N_δ of isolated imidazole → N(Im)

N_δ of His → N(His)

N of first Gly → N(G₁)

N of second Gly → N(G₂)

Carbonil O of second Gly → O(G₂)



Atom	<d>(Å)	σ (Å)
N(Im)	2.20	0.20
N(His)	2.00	0.10
N(G ₁)	3.00	0.40
N(G ₂)	1.96	0.07
O(G ₂)	2.20	0.10

S3: HisG₁⁻G₂G₃ + Im

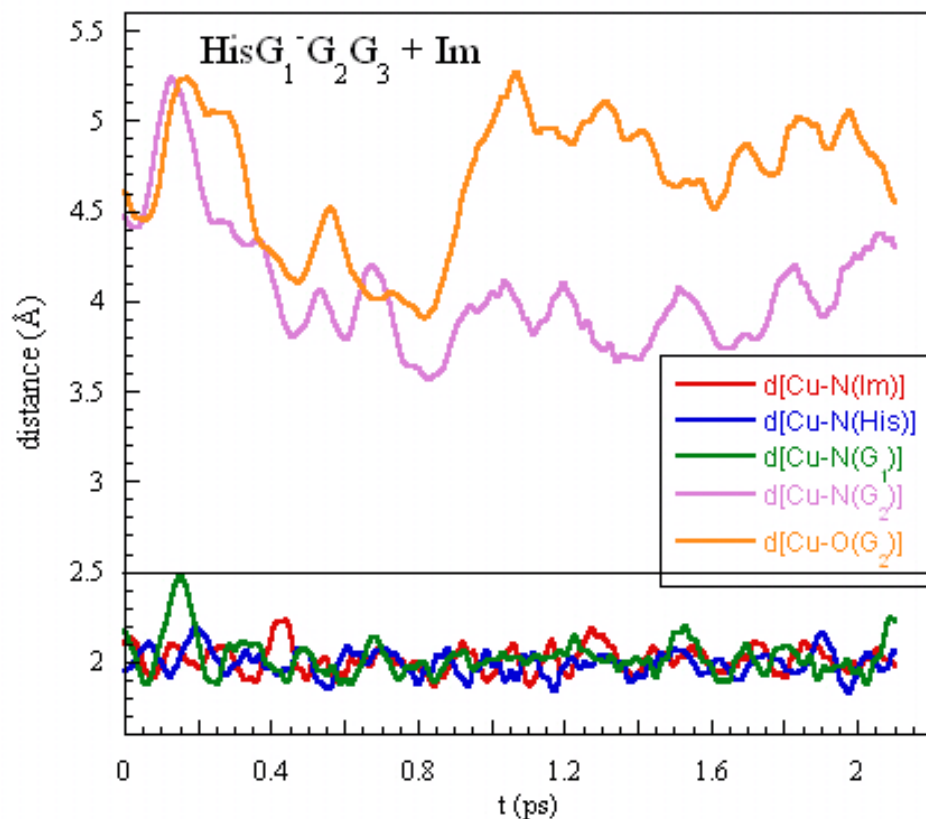
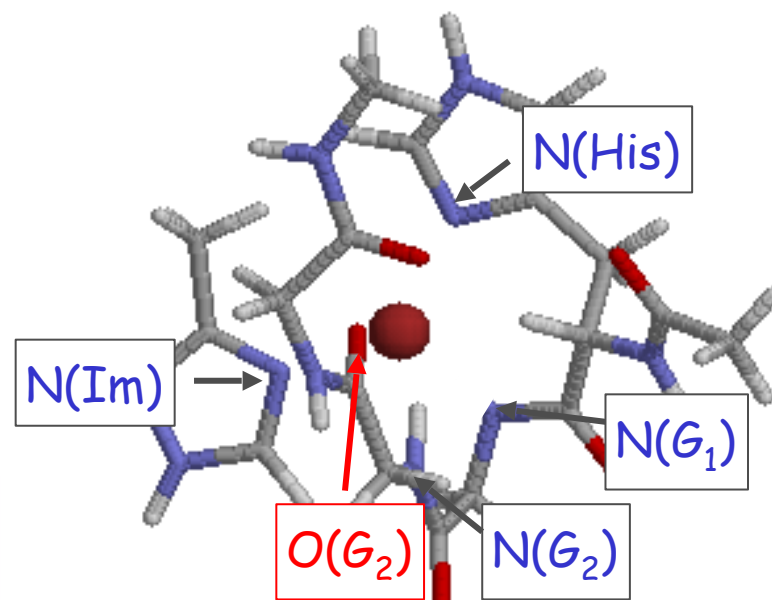
Nδ of isolated imidazole → N(Im)

Nδ of His → N(His)

N of first Gly → N(G₁)

N of second Gly → N(G₂)

Carbonil O of second Gly → O(G₂)



Atom	<d>(Å)	σ (Å)
N(Im)	2.01	0.07
N(His)	1.99	0.07
N(G ₁)	2.00	0.10
N(G ₂)	4.10	0.30
O(G ₂)	4.70	0.40

S4: HisG₁G₂G₃ + Im

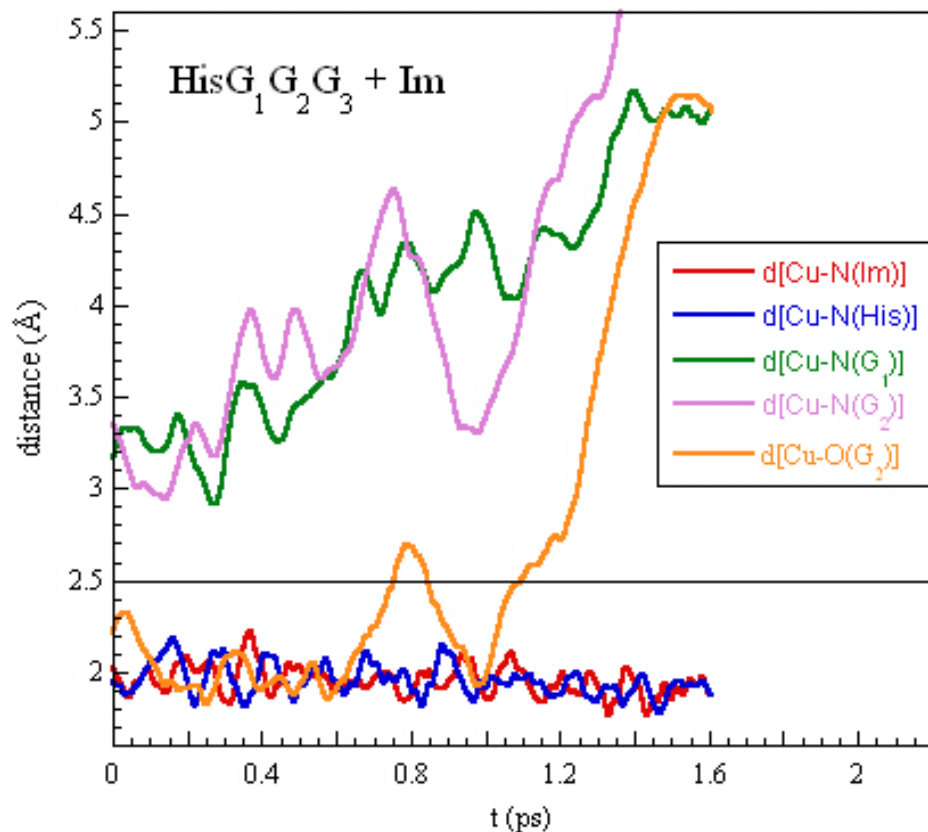
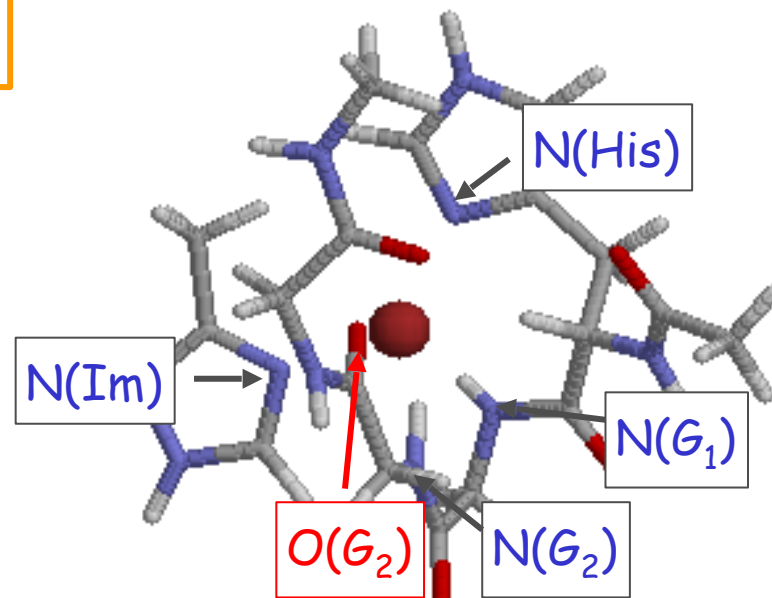
Nδ of isolated imidazole → N(Im)

Nδ of His → N(His)

N of first Gly → N(G₁)

N of second Gly → N(G₂)

Carbonil O of second Gly → O(G₂)



Atom	$\langle d \rangle$ (Å)	σ (Å)
N(Im)	1.95	0.08
N(His)	1.95	0.08
N(G ₁)	---	---
N(G ₂)	---	---
O(G ₂)	---	---

Gly₂ protonated →
low coordination number

3N

2N

BUT...

Atom	S1 HisG ₁ -G ₂ -G ₃		S2 HisG ₁ G ₂ -G ₃		S3 HisG ₁ -G ₂ G ₃		S4 HisG ₁ G ₂ G ₃	
	<d>	°	<d>	°	<d>	°	<d>	°
N(Im)	2.07	0.08	2.00	0.20	2.01	0.07	1.95	0.08
N(His)	2.10	0.10	2.00	0.10	1.99	0.07	1.95	0.08
N(G ₁)	2.01	0.08	3.00	0.40	2.00	0.10	---	---
N(G ₂)	2.01	0.00	---	0.07	4.10	0.30	---	---
O(G ₂)	3.80	0.30	2.20	0.10	4.70	0.40	---	---

3N1O

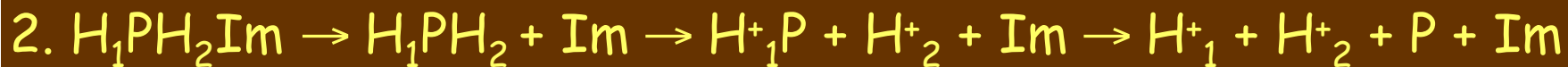
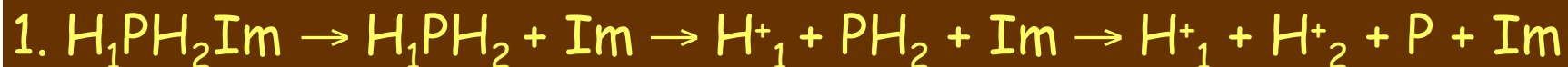
imidazole ring is **always** in Cu coordination sphere
independently of Gly's protonation state

Gly protonation state and Imidazole binding

A stability study

Is the dimeric (two octarepeats) compound more/less stable than the monomeric one?

Compute energies of products of the virtual chemical reactions:



P: both Gly₁ and Gly₂ deprotonated

H₁P: only Gly₂ deprotonated

PH₂: only Gly₁ deprotonated

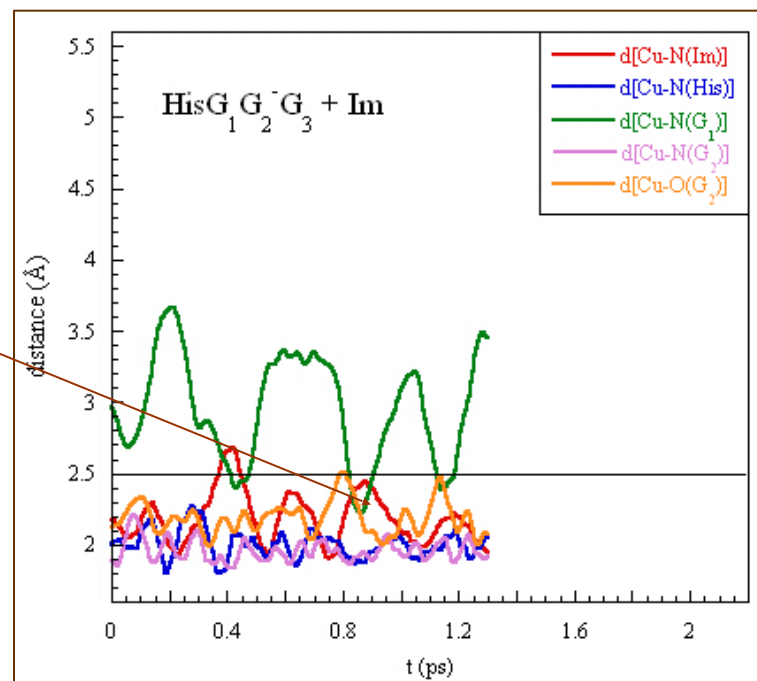
H₁PH₂: both Gly₁ and Gly₂ protonated

Two types of Conclusions

Methodological

we have seen the power of using CP-MD
in combination with DFT optimization

“unstable” structures can be
recognized and, if needed,
discarded

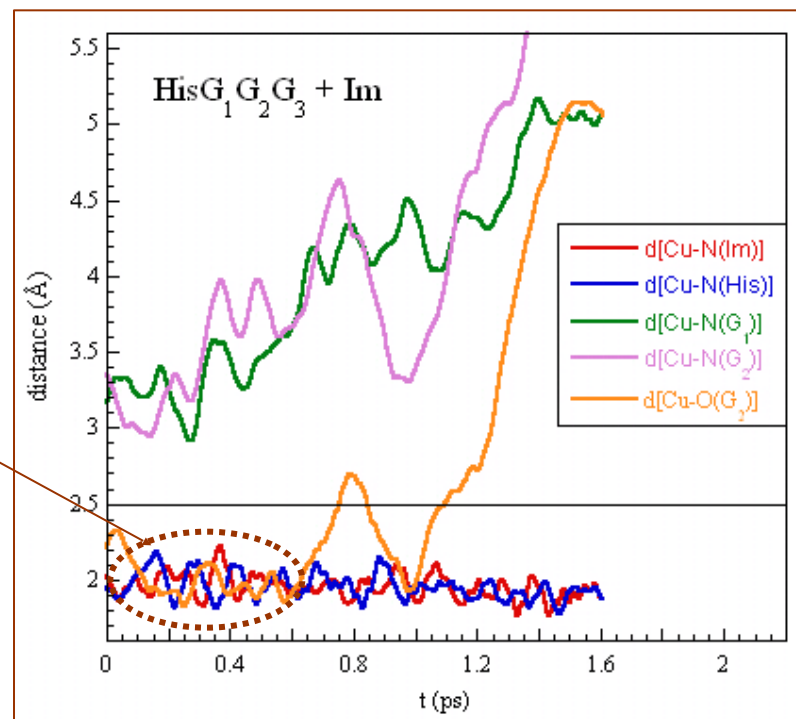


Two types of Conclusions

Methodological

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Biological

Multiple Histidine coordination can occur in the presence of deprotonated Glycines



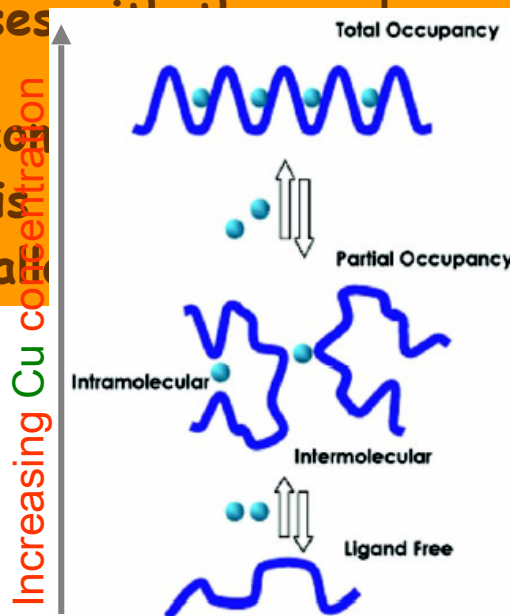
The hypothesis that low copper concentration favors

inter-repeat binding is confirmed



The binding energy decreases with increasing concentration of deprotonated Glycines

The energy of the conformational change is lowered when the peptide's nitrogens are deprotonated, P, is the free energy of the conformational change



VI. Conclusions and outlook

Conclusions

Very many difficult problems

But there is hope to successfully attack some of them

Extremely exciting research field

An arena where biology, mathematics, physics, computer science meet

Amazing experimental methods are being developed

Fantastic applications are in view

New positions are foreseeable!

Conclusions

Very many difficult problems

But there is hope to successfully attack some of them

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Fantastic applications are in view

New positions are foreseeable!

Outlook?

Outlook

But for today

This is not the end.

It is not even the beginning of the end.

But it is, perhaps, the end of the beginning



Thank you all for listening!