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GRAVITATION AND COSMOLOGY

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1. CLASSICAL BLACK HOLES AS DISSIPATIVE BRANES
2. EXPERIMENTAL TESTS OF RELATIVISTIC GRAVITY
3. GRAVITATIONAL WAVES FROM COSMIC (SUPER)STRINGS
4. GRAVITATIONAL WAVES FROM COALESCING BLACK HOLES
5. CHAOS AND SYMMETRY IN 'STRING COSMOLOGY'

1. CLASSICAL BLACK HOLES AS DISSIPATIVE BRANES

- AIM: DERIVE THE VALUE OF THE (SURFACE) SHEAR VISCOSITY OF BLACK HOLES:

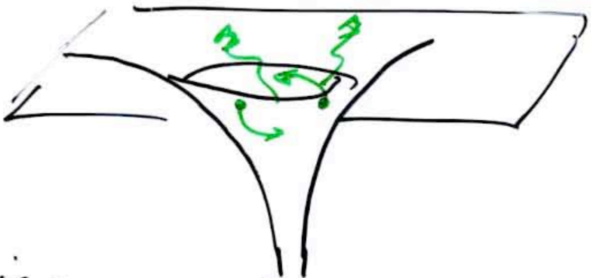
$$\boxed{\eta_{BH} = \frac{1}{16\pi G}} \quad (\text{WITH } c = 1)$$

$$\Rightarrow \frac{\eta_{BH}}{s_{BH}} = \frac{1/(16\pi G)}{1/(4\hbar G)} = \frac{\hbar}{4\pi} \leftarrow$$ OF RECENT INTEREST IN CONNECTION WITH AdS/CFT (Kovtun, Son, Starinets)

ENTROPY DENSITY $= \frac{1}{4}\frac{1}{\hbar G}$

- EVOLVING VIEWS OF BLACK HOLES

PASSIVE GRAVITATIONAL WELLS



PHYSICAL OBJECTS: GLOBAL DYNAMICS : $M, \vec{J}, Q, \delta M, M_{irr} \dots$

LOCAL DYNAMICS OF HORIZON : ~ 'MEMBRANE' WITH DISSIPATIVE PROPERTIES

RESISTIVITY = 377 OHM = 4π

SHEAR VISCOSITY $= \frac{1}{16\pi G}$

QUANTUM OBJECTS : QUANTUM INSTABILITIES, PAIR CREATION

MICROSCOPIC ORIGIN OF BH ENTROPY

SPHERICALLY SYMMETRIC BLACK HOLES $(D = 3+1)$

$$ds^2 = -A(r)\,dT^2 + B(r)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$A(r) = \frac{1}{B(r)} = 1 - \frac{2GM}{r} + G\frac{Q^2}{r^2} \qquad (c=1)$$

Reissner-Nordstrøm

TOTAL MASS M — TOTAL ELECTRIC CHARGE

SOLUTION OF $\quad R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu{}^\sigma F_{\nu\sigma} - \text{trace}\right)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$:

$$-A_0 = +V = +\frac{Q}{r}$$

$G=1$

$$A(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{r^2 - 2Mr + Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2}$$

(OUTER) HORIZON

$$\boxed{r_+ = M + \sqrt{M^2 - Q^2}} \qquad r_- = M - \sqrt{M^2 - Q^2}$$

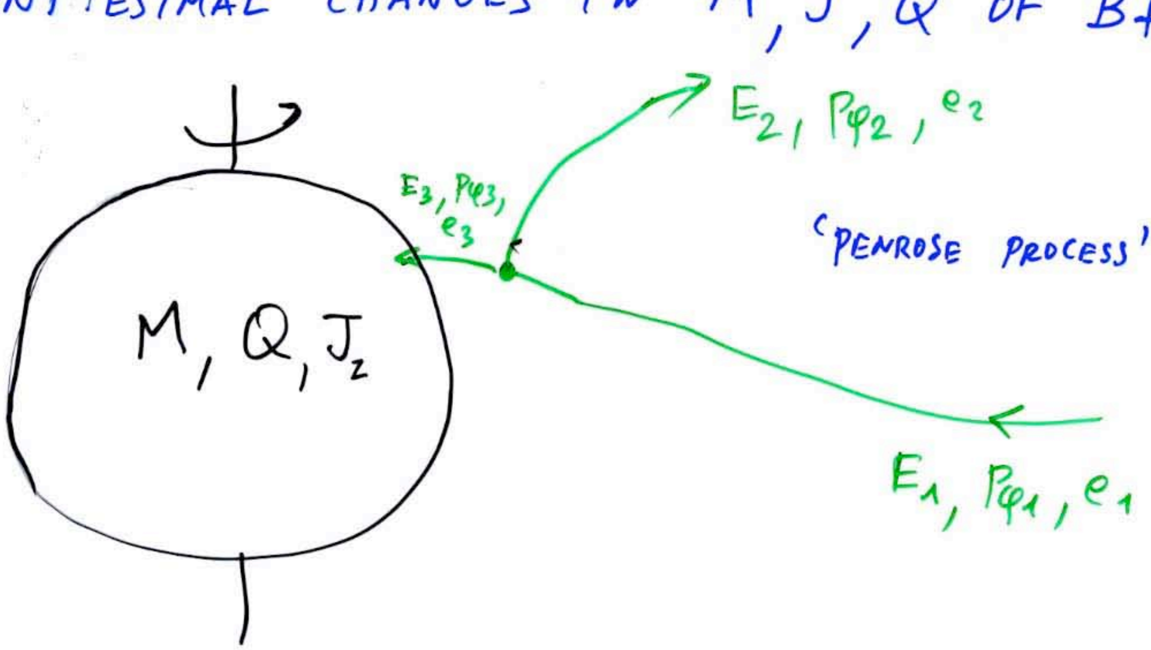
TOTAL ANGULAR MOMENTUM

Kerr - Newman BH: $\quad M, J, Q$

$$r_+ = M + \sqrt{M^2 - Q^2 - a^2}$$

$$a \equiv J/M$$

INFINITESIMAL CHANGES IN M, J, Q OF BH



TEST-PARTICLE OF REST-MASS μ IN BH BACKGROUND

GEODESIC DYNAMICS: $S = -\int \mu \, ds$

HAMILTON-JACOBI EQ FOR $p_\mu = \frac{\partial S}{\partial x^\mu}$:

$$g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = -\mu^2$$

ELECTRIC CHARGE OF TEST PARTICLE

SYMMETRIES OF BH Background $\rightarrow$ CONSERVED QUANTITIES

CONSERVED ENERGY (OF TEST PARTICLE): $E = -p_T = -p_0$

CONSERVED Z-COMPONENT OF ANGULAR MOMENTUM: p_φ

+ CONSERVED ELECTRIC CHARGE e

PENROSE PROCESS:

IN and OUT at ∞ — AT and AFTER SPLITTING

CHANGE IN TOTAL ENERGY OF SPACE-TIME

$$\delta M = E_1 - E_2 = E_3$$

$$\delta J = p_{\varphi 1} - p_{\varphi 2} = p_{\varphi 3}$$

$$\delta Q = e_1 - e_2 = e_3$$

SOLVE THE MASS-SHELL CONDITION

$$A_0 = -\frac{Q}{r}$$

$$-\mu^2 = g^{\mu\nu}(p_\mu - eA_\mu)(p_\nu - eA_\nu) = -\frac{(p_0 - eA_0(r))^2}{A(r)} + A(r)\, p_r^2 + \frac{1}{r^2}\left(p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}\right)$$

$$p^r \equiv g^{rr} p_r = A(r)\, p_r$$

CONSERVED $L^2 = p_\theta^2 + \frac{p_\varphi^2}{\sin^2\theta}$

$$\boxed{-p_0 = E = \frac{eQ}{r} + \sqrt{(p^r)^2 + A(r)\left(\mu^2 + \frac{L^2}{r^2}\right)}}$$

CHRISTODOLOU, CHRISTODOULOU-RUFFINI '71

INFINITESIMAL VARIATION OF BH MASS: APPLY TO E_3 ON THE HORIZON

$E = E_3 = \delta M$; $e = e_3 = \delta Q$

$$\boxed{\delta M = \frac{Q\,\delta Q}{r_+(M,Q)} + |p^r|_{r_+} \geq \frac{Q\,\delta Q}{r_+(M,Q)}}$$

INEQUALITY $\Rightarrow$ IRREVERSIBILITY IN BH PHYSICS

CHRISTODOULOU-RUFFINI

WHEN ADDING ANGULAR MOMENTUM:

$$\delta M - \frac{a\,\delta J + r_+ Q\,\delta Q}{r_+^2 + a^2} = \frac{r_+^2 + a^2\cos^2\theta}{r_+^2 + a^2}\,|p^r| \geq 0$$

$\rightarrow$ CHRISTODOULOU-RUFFINI MASS FORMULA

$$\boxed{M^2 = \left(M_{IRR} + \frac{Q^2}{4M_{IRR}}\right)^2 + \frac{J^2}{4M_{IRR}^2}}$$

$$\boxed{\delta M_{IRR} \geq 0}$$

M_{IRR} = INTEGRATION CONSTANT ALONG 'REVERSIBLE' TRANSFORMATIONS:

$$\delta M = \frac{a\,\delta J + r_+ Q\,\delta Q}{r_+^2 + a^2}$$

$\rightarrow$ CLASSICALLY EXTRACTABLE ENERGY OF A BH : $M - M_{IRR}$

- UP TO 29% M IN ROTATIONAL ENERGY
- UP TO 50% M IN COULOMB EN.

GIVEN INEQUALITY

$$a^2 + Q^2 \leq M^2$$

GEOMETRICAL MEANING OF M_{IRR} : $\boxed{A \equiv \underset{\text{HORIZON}}{\text{AREA}} = 16\pi M_{IRR}^2}$

GENERAL RESULT OF HAWKING '71

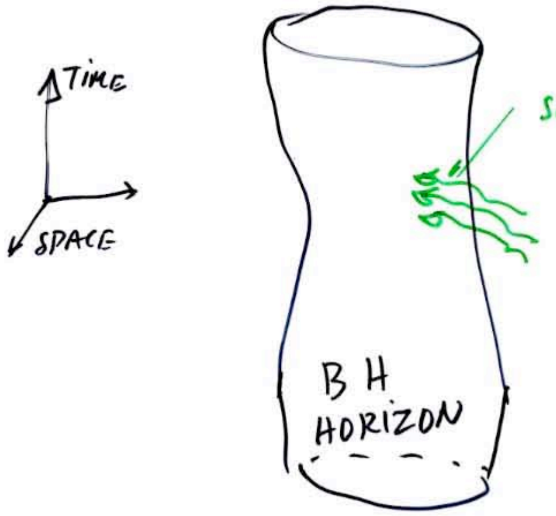
$$\boxed{\delta A \geq 0}$$

VARIOUS FACETS OF BH IRREVERSIBILITY:

CLASSICALLY $\delta A \nearrow$ CAN BE INTERPRETED AS : BH ~ DISSIPATIVE MEMBRANE

QUANTUM MECHANICALLY BEKENSTEIN SUGGESTED $A \propto$ BH ENTROPY

LET US STUDY MORE IN DETAIL WHAT HAPPENS WHEN ONE IS 'THROWING STUFF IN A BLACK HOLE':

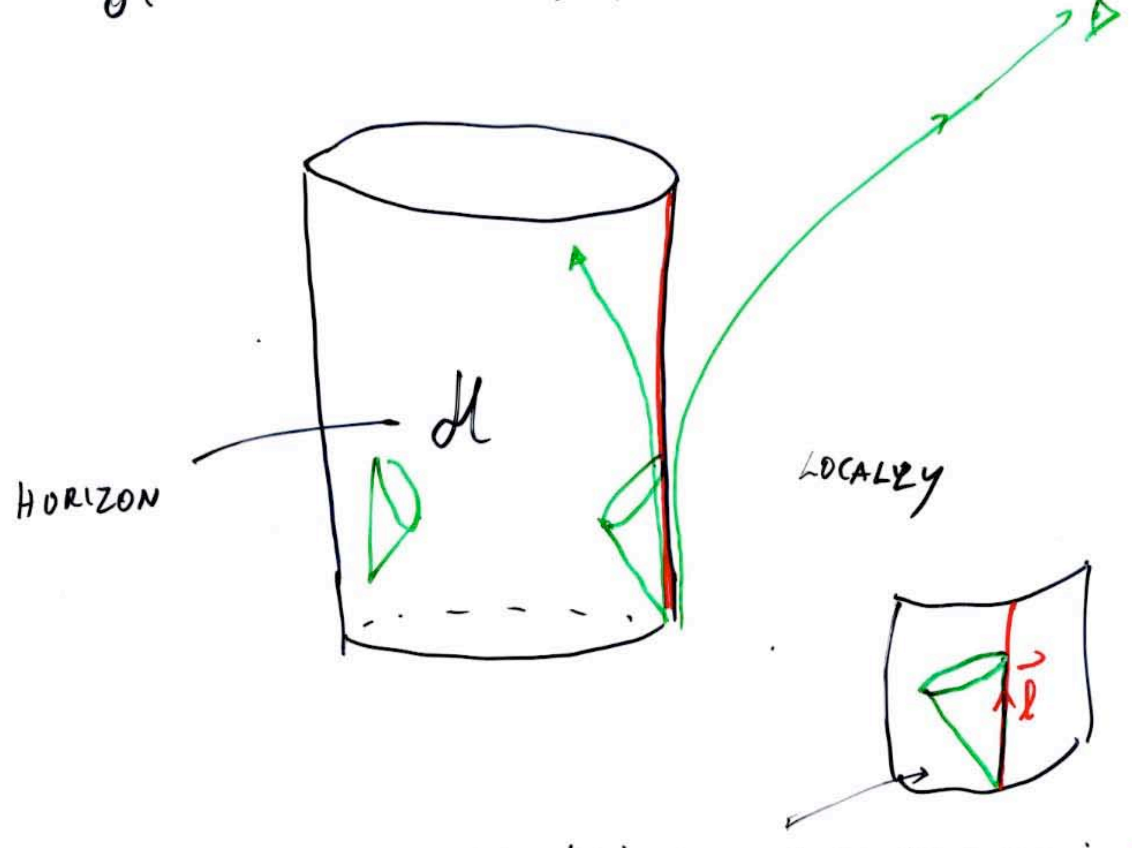


$\longrightarrow$ DERIVE AN EQUATION FOR THE DYNAMICS OF THE BH 'SURFACE' WHICH IS CLOSELY ANALOGOUS TO THE NAVIER-STOKES EQ. OF A VISCOUS FLUID (Damour '79)

(1) DEFINITION OF BH HORIZON (OR BH 'SURFACE')
IN A GENERAL TIME-DEPENDENT SITUATION

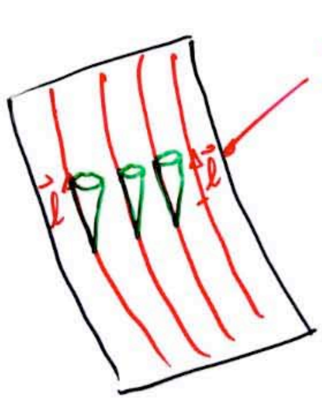
PENROSE, HAWKING, HAWKING-ELLIS

BH HORIZON $\mathcal{H}$ = BOUNDARY BETWEEN LIGHT SIGNALS THAT ESCAPE TO ∞ AND LIGHT SIGNALS THAT DO NOT ESCAPE



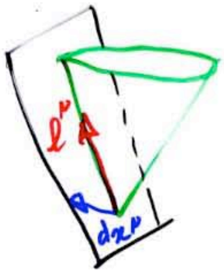
ON HORIZON THE LOCAL LIGHT CONE IS TANGENT TO THE HORIZON

$\rightarrow$ HORIZON = NULL HYPERSURFACE



$\vec{\ell}$ = PARTICULAR LIGHT-LIKE DIRECTION ALONG WHICH $\mathcal{H}$ IS TANGENT TO LIGHTCONE

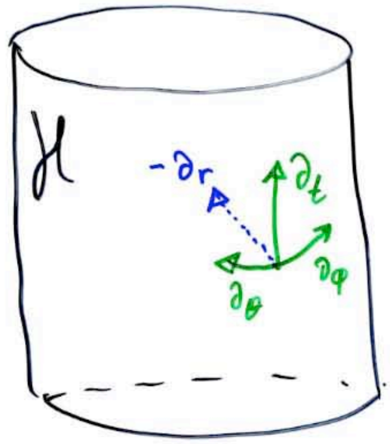
NB $\vec{l} = l^\mu \frac{\partial}{\partial x^\mu}$ IS BOTH TANGENT AND NORMAL TO $\mathcal{H}$



$l_\mu l^\mu = 0$ $\quad$ $l_\mu dx^\mu = 0$

NULL $\quad$ WITHIN $\mathcal{H}$

USE AN ADAPTED COORDINATE SYSTEM WHICH IS REGULAR ON $\mathcal{H}$



i.e. SIMILAR TO USUAL 'INGOING EDDINGTON-FINKELSTEIN COORDINATES' OF SCHWARZSCHILD (OR REISSNER-NORDSTRØM)

SINGULAR ON $\mathcal{H}$: $A(r_+) = 0$

$$ds^2 = -A(r)\,dT^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

INTRODUCE 'TORTOISE' r_*

$$= -A(r)\left[dT^2 - \left(\frac{dr}{A(r)}\right)^2\right] + r^2 d\Omega^2$$

$$\boxed{r_* \equiv \int \frac{dr}{A(r)}}$$

$$= -A(r)\,(dT - dr_*)(dT + dr_*) + r^2 d\Omega^2$$

AND THE NEW TIME

$$= -A(r)\,(dt - 2dr_*)\,dt + r^2 d\Omega^2$$

$$\boxed{t = T + r_*}$$

$$\boxed{ds^2 = -A(r)\,dt^2 + 2\,dt\,dr + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)}$$

$A(r) = \frac{(r-r_+)(r-r_-)}{r^2}$ VANISHES ON $\mathcal{H}$ BUT ds^2 REMAINS REGULAR IN COORDS (t, r, θ, φ)

$\mathcal{H}$: r = const: $\frac{\partial}{\partial r}$ IS TRANSVERSE, WHILE $\frac{\partial}{\partial t}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \varphi}$ ARE TANGENT TO $\mathcal{H}$

FOR ARBITRARY, TIME-DEPENDENT HORIZON LET US CHOOSE COORDINATES S.T.

- $\mathcal{H}$ IS LOCATED AT $x^1 = 0$ (i.e. $x^1 \sim r - r_+$)
- THE 3-d HYPERSURFACE $\mathcal{H}$ IS PARAMETRIZED BY x^0, x^2, x^3 $(\sim t, \theta, \varphi)$

$\partial_1 \equiv \frac{\partial}{\partial x^1}$ IS TRANSVERSE TO $\mathcal{H}$, WHILE $\partial_0 = \frac{\partial}{\partial x^0}, \partial_2, \partial_3$ ARE TANGENT

- NORMALIZE $\vec{l} = l^\mu \partial_\mu$ S.T. $\underset{\text{ON } \mathcal{H}}{\boxed{\vec{l} = \partial_0 + v^A \partial_A}}$, $A = 2,3$

EXPLOIT $g_{\mu\nu} l^\mu l^\nu = \vec{l} \cdot \vec{l} = 0$ (ON $\mathcal{H}$)

$\rightarrow$ FOR ANY TANGENT VECTOR $\vec{t}$ TO $\mathcal{H}$ $\quad 0 = \nabla_{\vec{t}} (\vec{l} \cdot \vec{l}) = 2 \vec{l} \cdot \underbrace{\nabla_{\vec{t}} \vec{l}}$

NORMAL TO $\mathcal{H}$ MUST BE TANGENT TO $\mathcal{H}$

ONE DEFINES THE FOLLOWING SPECIAL QUANTITIES

$\vec{e}_A \equiv \partial_A \equiv \frac{\partial}{\partial x^A}$

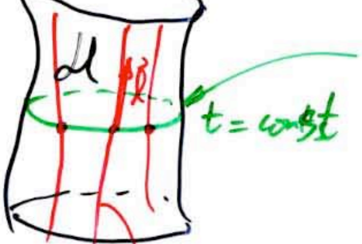
$$\boxed{\begin{aligned} \nabla_{\vec{l}} \vec{l} &= g \, \vec{l} \\ \nabla_{\vec{e}_A} \vec{l} &= -8\pi G \pi_A \, \vec{l} + D_A^{\ B} \, \vec{e}_B \end{aligned}}$$

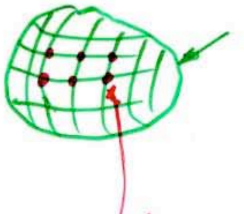
g is called SURFACE GRAVITY $\quad g^{\text{Schwarzschild}} = \dfrac{GM}{r_+^2}$

$\mathcal{D}_A{}^B$ = DEFORMATION TENSOR OF the horizon geometry:

$$ds^2|_{\mathcal{H}} = \gamma_{AB}(t, x^A)(dx^A - v^A dt)(dx^B - v^B dt)$$

METRIC TENSOR OF A $t = \text{const}$ SLICE OF $\mathcal{H}$





'2-brane'
= horizon at some 'time' t

'FLUID PARTICLES'
defined by the 'generators'
i.e. the lines tangent to $\vec{l}$

'CONVECTIVE DERIVATIVE' OF γ_{AB}:

$$\mathcal{D}_{AB} \equiv \frac{1}{2}\frac{D}{dt}\gamma_{AB} = \frac{1}{2}\mathcal{L}_{\vec{l}}\,\gamma_{AB} = \frac{1}{2}\left(\partial_t \gamma_{AB} + v^C \partial_C \gamma_{AB} + \partial_A v^C \gamma_{CB} + \partial_B v^C \gamma_{AC}\right)$$

$$\mathcal{D}_{AB} = \gamma_{BC}\,\mathcal{D}_A{}^C \qquad = \frac{1}{2}\left(\partial_t \gamma_{AB} + v_{A|B} + v_{B|A}\right)$$

AS IN USUAL FLUID: RATE OF DEFORMATION OF FLUID ELEMENTS

AS USUAL, DECOMPOSE

$$\boxed{\mathcal{D}_{AB} = \sigma_{AB} + \theta\,\gamma_{AB}}$$

SHEAR TENSOR: trace-free part

EXPANSION RATE

$$\theta = \mathcal{D}_A{}^A = \frac{1}{2}\gamma^{AB}\partial_t \gamma_{AB} + v^A{}_{|A}$$

? MEANING OF π_A AND EXPLANATION OF COEFFICIENT $\chi = 8\pi G$

TOTAL ANGULAR MOMENTUM OF ANY AXISYMMETRIC SPACETIME.

KILLING VECTOR $m^\mu \partial_\mu = \partial/\partial\varphi$

$$J_\infty = -\frac{1}{\chi}\oint_{S_\infty} \frac{1}{2}\nabla^\nu m^\mu d^2S_{\mu\nu} = -\frac{1}{\chi}\oint_{\mathcal{H}} \frac{1}{2}\nabla^\nu m^\mu d^2S_{\mu\nu} + \int m^\mu T^\nu{}_\mu d^3\Sigma_\nu$$

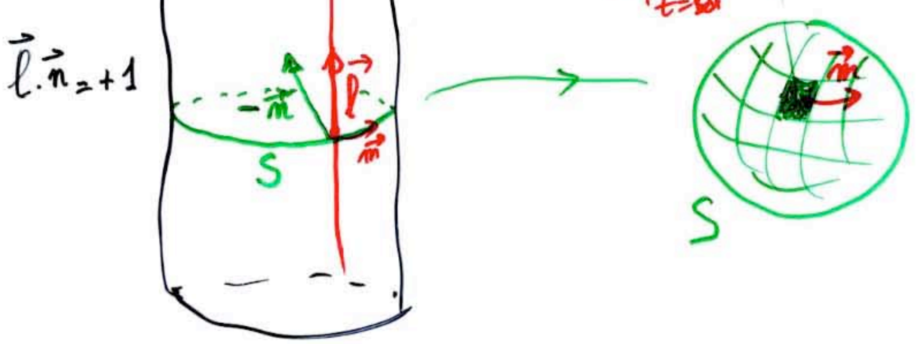
DEFINES THE ANG. MOMENTUM OF B.H.

ANG. MOMENTUM OF MATTER, AFTER USING $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi T_{\mu\nu}$

USING $d^2S^{\mathcal{H}}_{\mu\nu} = (n_\mu l_\nu - n_\nu l_\mu)dA$ AND $l^\nu\nabla_\nu m^\mu = m^\nu \nabla_\nu l^\mu$

$$\boxed{J_{BH} = -\frac{1}{8\pi G}\oint_{\mathcal{H}} n_\mu m^\nu \nabla_\nu l^\mu dA = \oint_S m^A \pi_A \, dA = \oint_S \pi_\varphi \, dA}$$

AREA ELEMENT $dA = \sqrt{\det \gamma_{AB}}\, dx^2 \wedge dx^3$ ON $\mathcal{H}|_{t=\text{const}}$



$\rightarrow$ $\boxed{\pi_A = \text{'SURFACE DENSITY OF LINEAR MOMENTUM OF B.H.'}}$

NAVIER-STOKES-LIKE EQUATION OF BH (Damour '79)

BY PROJECTING EINSTEIN'S EQS $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ ALONG $\ell^\mu e_A^\nu$ (USING THE FACT THAT (g, π_A, D_A^B) GENERALIZES THE 2^{ND} FUNDAMENTAL FORM (Weingarten map), AND USING A NULL generalization of the Gauss-Codazzi eqs) ONE FINDS

$$\boxed{\frac{D}{dt}\pi_A = -\frac{\partial}{\partial x^A}\left(\frac{g}{8\pi G}\right) + \frac{1}{8\pi G}\sigma_{A|B}^{B} - \frac{1}{16\pi G}\partial_A \theta - \ell^\mu T_{\mu A}}$$

CONVECTIVE ∂, i.e. Lie derivative $\frac{D\pi_A}{dt} = (\partial_t + \theta)\pi_A + v^B \pi_{A|B} + v^B_{|A}\pi_B$

SHEAR RATE

EXPANSION RATE $\theta = \frac{\partial_t \sqrt{\gamma}}{\sqrt{\gamma}} + v^A_{|A}$

RATE OF FLUX OF MOMENTUM THROUGH THE HORIZON

NEARLY IDENTICAL TO USUAL, NON-RELATIVISTIC NAVIER-STOKES EQ. FOR A (2-d) VISCOUS FLUID

$$\boxed{\frac{D'}{dt}\pi_i = -\frac{\partial}{\partial x^i} p + 2\eta\, \sigma_{i,k}^{k} + \zeta\, \partial_i \theta + f_i}$$

Newtonian convective ∂ $(\partial_t + \theta)\pi_i + v^k \pi_{i,k}$

PRESSURE

SHEAR VISCOSITY

BULK VISCOSITY

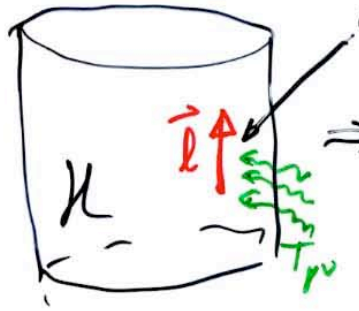
FORCE DENSITY

$$\Rightarrow \quad \boxed{p_{BH} = \frac{g}{8\pi G}} \quad \boxed{\eta_{BH} = \frac{1}{16\pi G}} \quad \boxed{\zeta_{BH} = -\frac{1}{16\pi G}}$$

GIVES $\frac{\eta_{BH}}{s_{BH}} = \frac{(16\pi G)^{-1}}{(4\hbar G)^{-1}} = \frac{\hbar}{4\pi}$

Bekenstein-Hawking

IRREVERSIBLE THERMODYNAMICS OF BH's



'NORMAL' THROUGH HORIZON is $\vec{l}$

$\Rightarrow$ FLUX OF ENERGY-MOMENTUM THROUGH $\mathcal{H}$

is $l^\mu T_{\mu\nu}$

NAVIER-STOKES FOLLOWS FROM $\nu = A$: "SPATIAL AND TANGENT TO $\mathcal{H}$"

ANOTHER USEFUL RESULT FOLLOWS FROM "$\nu = 0$" 'TIME AND TANGENT'

NAMELY $l^\mu l^\nu T_{\mu\nu}$ ('Raychauduri eq.')

FOLLOWING BEKENSTEIN - HAWKING ATTRIBUTE AN

ENTROPY $s = \hat{\alpha}\, dA$ TO EACH BH SURFACE ELEMENT

$$\boxed{\frac{d}{dt} s - \tau \frac{d^2}{dt^2} s = \frac{1}{T_{BH}}\Big[2\eta_{BH}\,\sigma_{AB}\sigma^{AB} + \zeta_{BH}\,\theta^2 + \rho_{BH}\underbrace{(\vec{K} - \sigma_H \vec{v})^2}_{\text{"JOULE's LAW"}}\Big] dA}$$

$\tau = \frac{1}{g}$ $\qquad T_{BH} = \frac{g}{8\pi G \hat{\alpha}}$

BH SURFACE RESISTIVITY $4\pi = 377\,$OHM

ALSO, THERE $\exists$ BH OHM's LAW (Damour, Znajek '78)

$$\boxed{\vec{E} + \vec{v} \times \vec{B} = \rho_{BH}(\vec{K} - \sigma_H \vec{v})}$$

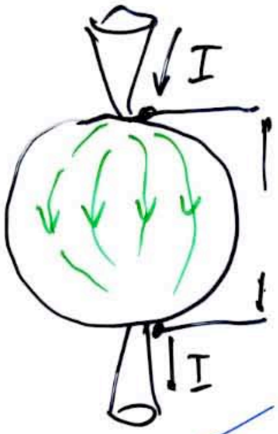
THIS RESULT CONFIRMS THE CONSISTENCY OF INTERPRETING A BH AS A VISCOUS MEMBRANE, ENDOWED WITH ELECTRICAL RESISTIVITY ρ_{BH}

$V = R_{BH}\, I$

WITH R_{BH} COMPUTED FROM ρ_{BH}

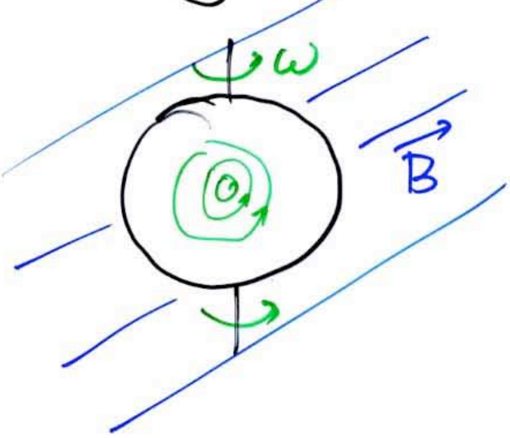
EXAMPLES OF DISSIPATIVE PROCESSES

CONSEQUENCES OF $\rho_H = 4\pi = 377\Omega \neq 0$



$V = RI$ AND $\frac{dQ}{dt} = T\frac{dS}{dt} = RI^2$

$R \sim 30\,\Omega$

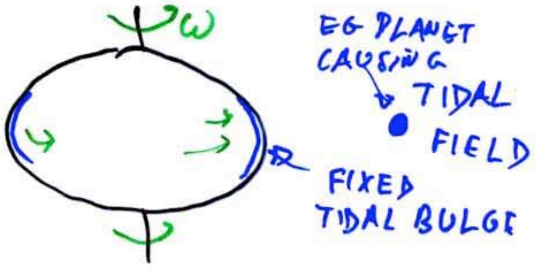


$\rightarrow$ EDDY CURRENTS $\rightarrow$ DISSIPATION AND TORQUE ALIGNING $\vec{\omega}$ TOWARD $\vec{B}$

CONSEQUENCES OF $\eta_{BH} = \frac{1}{16\pi G} \neq 0$

B.H. EQUILIBRIUM STATES : $\sigma_{AB} = \theta = 0$, $\partial_t = 0 \rightarrow$ UNIFORM 'PRESSURE' $\Rightarrow$ g UNIFORM ON $\mathcal{H}$

TIDAL BULGE $\rightarrow$ DISSIPATION (Hartle)



$\rightarrow$ DISSIPATION $\rightarrow$ $\frac{d\omega}{dt} < 0$

+ VALIDITY OF 'MINIMUM ENTROPY PRODUCTION PRINCIPLE' à la Prigogine

BEKENSTEIN's VIEW OF WHY $T_{BH} = \frac{g}{8\pi G \hat{\alpha}}$

SURFACE GRAVITY

RECALL CHRISTODOULOU-RUFFINI (SPHERICAL SYMM.)

$$\delta M = \underbrace{\frac{Q\,\delta Q}{r_+(M,Q)}}_{\text{REVERSIBLE (WORK)}} + \underbrace{|g^{rr} p_r|_{r_+}}_{\text{IRREVERSIBLE (HEAT)}}$$

$$\delta E = \delta W + T\delta S$$

TO REACH REVERSIBILITY ONE WOULD NEED

TO FIX BOTH $r = r_+$ AND $p_r = 0$ SIMULTANEOUSLY

$\not\!\!Z$ CONTRARY TO HEISENBERG's $\delta r\, \delta p_r \geqslant \frac{1}{2}\hbar$

$$\Rightarrow \quad (r - r_+)\, p_r \geqslant \frac{1}{2}\hbar$$

USING $\quad g^{rr} = A(r) = \frac{(r-r_+)(r-r_-)}{r^2} \simeq \left(\frac{\partial A}{\partial r}\right)_{r_+}(r-r_+)$

$$g^{rr} p_r = A(r)\, p_r \simeq \left(\frac{\partial A}{\partial r}\right)_{r_+} \underbrace{(r-r_+)\, p_r}_{\text{Heisenberg}} \geqslant \frac{1}{2}\hbar\left(\frac{\partial A}{\partial r}\right)_{r_+}$$

$$\Rightarrow \quad T\delta S \geqslant \frac{1}{2}\hbar\left(\frac{\partial A}{\partial r}\right)_{r_+}$$

WHEN ABSORBING ONE PARTICLE

ONE CHECKS $\boxed{\left.\dfrac{\partial A(r)}{\partial r}\right|_{r_+} = 2g}$

EG FOR SCHWARZSCHILD $A(r) = 1 - 2\dfrac{GM}{r}$

$$\Rightarrow \frac{\partial A}{\partial r} = 2\frac{GM}{r^2}$$

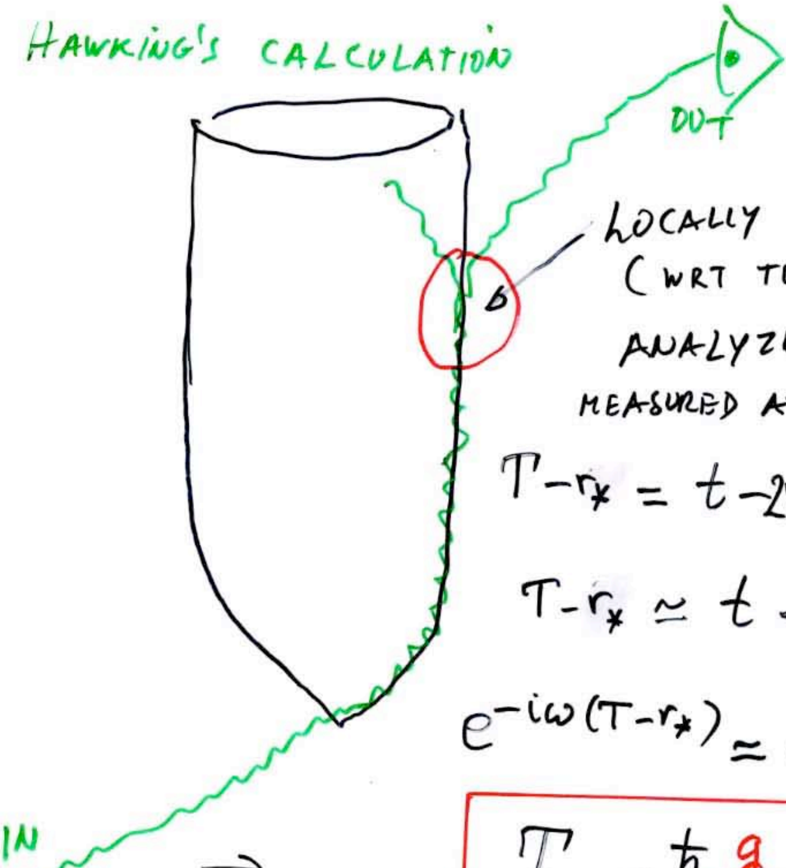
SO THAT

$$T\,\delta S \geq \hbar g$$ UPON ABSORBING ONE PARTICLE

BEKENSTEIN ARGUED $\delta S \gtrsim 1$ ('ONE BIT OF INFORMATION LOST')

$$\Rightarrow \quad T_{BH} \sim \hbar g$$

HAWKING'S CALCULATION



LOCALLY NEGATIVE-FREQUENCY MODE (WRT TO LOCALLY REGULAR COORDS t, r, θ, φ) ANALYZED WRT GLOBAL TIME MEASURED AT ∞,

$A(r) \simeq 2g(r - r_+)$

$$T - r_* = t - 2r_* = t - 2\int \frac{dr}{A(r)}$$

$$T - r_* \simeq t - \frac{1}{g}\ln(r - r_+)$$

$$e^{-i\omega(T - r_*)} \simeq e^{-i\omega t}\exp\left(i\frac{\omega}{g}\ln(r - r_+)\right)$$

$$\rightarrow \quad \boxed{T_{BH} = \hbar\frac{g}{2\pi}}$$

3

GRAVITATIONAL WAVES FROM COSMIC (SUPER)STRINGS

VARIOUS COSMOLOGICAL SCENARIOS

STANDARD COSMOLOGICAL
- "SCENARIO" TO EXPLAIN WHY UNIVERSE SO LARGE, SO HOMOGENEOUS, + $\frac{\delta\rho}{\rho} \sim 5\times 10^{-5}$

GR IS VALID

$\exists$ 'INFLATON' φ WITH $V(\varphi)$ VERY FLAT

$\epsilon \sim M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1$ AND $\eta \sim M_P^2 \frac{V''}{V} \ll 1$

QUANTUM FLUCT. $\hat{\delta\varphi} \Rightarrow$ ADIABATIC GAUSSIAN $\frac{\delta\rho}{\rho}$

$\frac{\delta\rho}{\rho} \sim 5\times 10^{-5} \Leftrightarrow \exists$ SMALL PARAMETER

$V(\varphi) = \lambda\varphi^4$ or $\frac{1}{2} m^2\varphi^2$ $\quad \lambda \sim 10^{-13}$, $m^2 \sim 10^{-12} m_P^2$

- STRING THEORY CHALLENGES

- FIND A NATURAL CANDIDATE FOR THE INFLATON FIELD φ

E.G. DILATON (Veneziano, Gasperini ...)

SEPARATION OF D-BRANES (Dvali, Tye; Burgess...Quevedo; KKLT, KKLMMT; ...)

- GET GR WITHOUT DILATON-MODULI EFFECTS WHICH "KILL" INFLATION BY INTRODUCING "STEEP" DIRECTIONS IN $V(\varphi, \Phi, \dots)$

E.G. WARPED FLUX COMPACTIFICATIONS (Giddings, Kachru, Polchinski; Kachru et al...

- ARRANGE EXISTENCE OF SLOW-ROLL REGIONS OF $V(\varphi)$

E.G. LARGE BRANE SEPARATION: (Dvali, Tye; ...; KKLMMT

OR SYMMETRIC CONFIGURATIONS (Trivedi, ...

OR HIGH-DERIVATIVE TERMS $\sim -g(\phi)\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} - V(\phi)$ (Silverstein, Tong)

à la k-inflation (Armendariz-Picon, Damour, Mukhanov; Garriga, Mukhanov)

- TUNE-IN SOME SMALL PARAMETER ($\lambda \sim 10^{-13}$) TO ARRANGE $\frac{\delta\rho}{\rho} \sim 5\times 10^{-5}$
- INCORPORATE STANDARD MODEL, AND ARRANGE FOR REHEATING
- ? ARRANGE INITIAL CONDITIONS, OR USE "ANTHROPIC-LIKE" ARGUMENTS

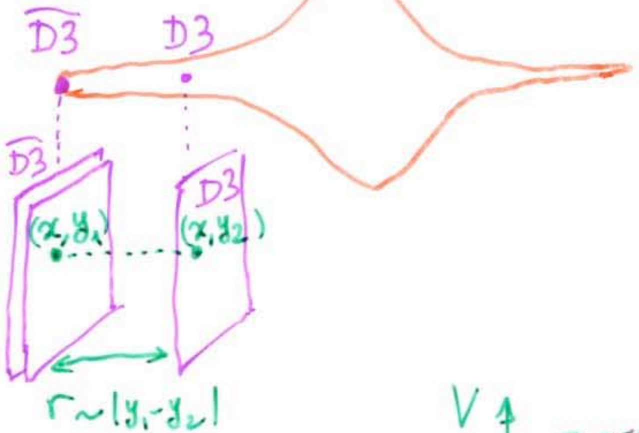
COSMIC SUPERSTRINGS?

Witten '85; ... Dvali, Tye; Tye, ...; KKLMMT; Copeland, Myers, Polchinski; Dvali, Vilenkin

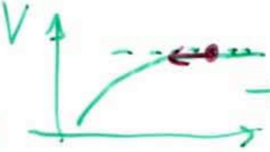
10 dim spacetime:

$$X^M = (x^\mu, y^a)$$

4 , 6 COMPACT



$$V(r) = A - \frac{B}{r^4}$$

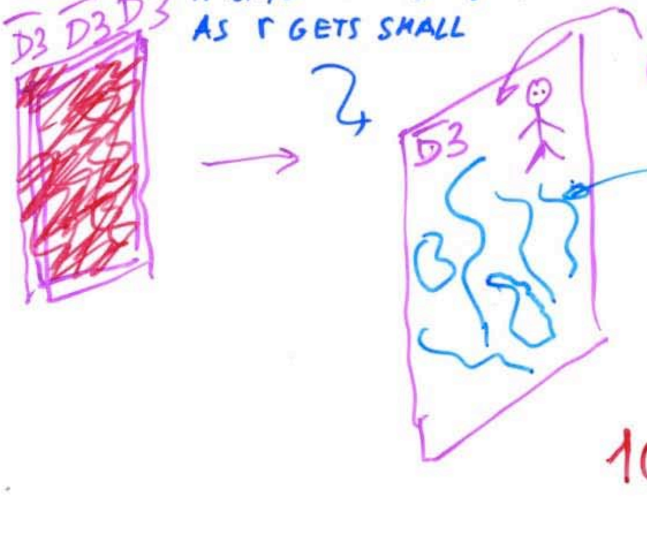


SLOW ROLL INFLATION

$\varphi(x) = r(x)$

TACHYONIC MODE $-\partial T \partial \bar{T} + |m^2| \bar{T} T$

AS r GETS SMALL



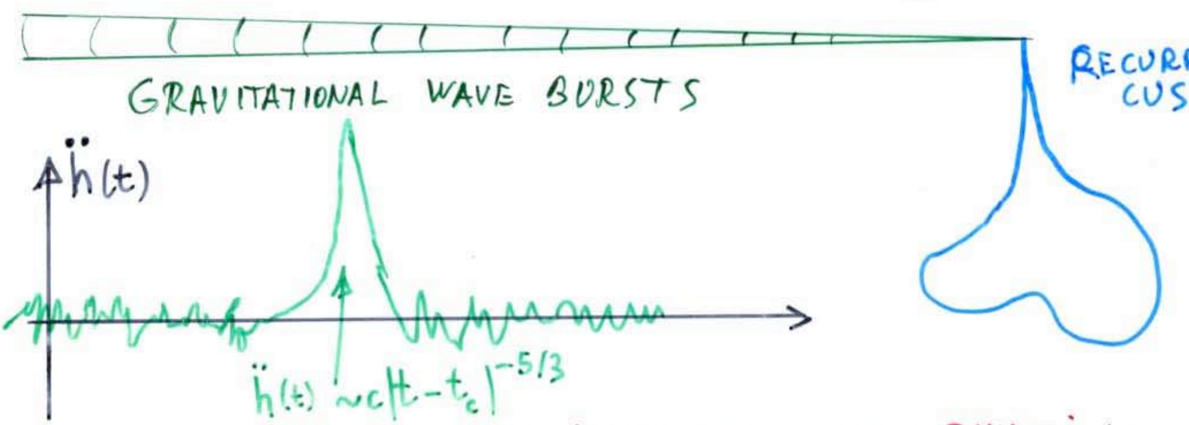
HEAT OF HOT BIG BANG

COSMOLOGICAL NETWORK OF MASSIVE (F OR D) STRINGS

WITH STRING TENSION

$$10^{-11} \lesssim G\mu \lesssim 10^{-6} \quad \text{Tye}$$

$$G\mu \sim 10^{-8} - 10^{-9} \quad \text{KKLMMT, Copeland MP}$$



POTENTIALLY DETECTABLE IN LIGO/VIRGO/...; LISA; PULSARTIMING

Damour, Vilenkin

STRING DYNAMICS: $X^\mu(\tau,\sigma)$

$$S_{\text{NAMBU}} = -\mu \int d\tau\, d\sigma \sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2 (X')^2}$$

$\partial_\tau X^\mu$ $\quad$ $\partial_\sigma X^\mu$ $\quad$ $\dot{X}\cdot X' \equiv g_{\mu\nu}(X)\dot{X}^\mu X'^\nu$

$$S_{\text{POLYAKOV}} = -\frac{1}{2}\mu \int d\tau\, d\sigma\, \sqrt{h}\, h^{ab} g_{\mu\nu}(X)\, \partial_a X^\mu \partial_b X^\nu$$

CONFORMAL GAUGE: $\sqrt{h}\, h^{ab} = \eta^{ab}$

$\Rightarrow$ CONSTRAINTS $\quad \dot{X}^2 + X'^2 = 0 \quad ; \quad \dot{X}\cdot X' = 0$

EQ. OF MOTION: $\boxed{\ddot{X}^\mu - X''^\mu + \Gamma^\mu_{\alpha\beta}(X)\,(\dot{X}^\alpha \dot{X}^\beta - X'^\alpha X'^\beta) = 0}$

IN FLAT SPACE: $\quad \ddot{X}^\mu - X''^\mu = 0$

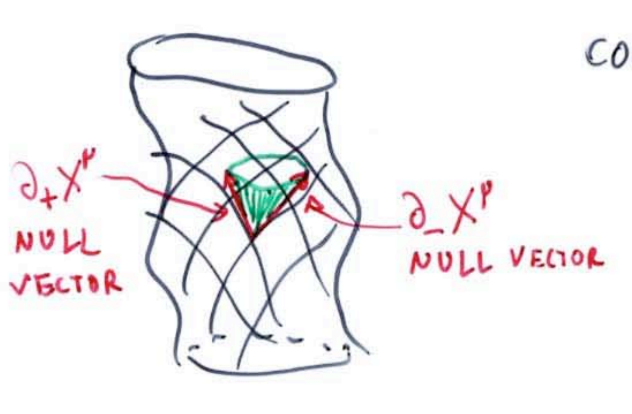
NULL WORLD-SHEET COORDS:

$$\sigma_\pm = \tau \pm \sigma$$

EOM: $\quad \dfrac{\partial}{\partial\sigma_+}\dfrac{\partial}{\partial\sigma_-} X^\mu = 0$

LEFT-MOVING $\quad$ RIGHT-MOVING

$$\boxed{X^\mu(\tau,\sigma) = \frac{1}{2}\left[X^\mu_+(\sigma_+) + X^\mu_-(\sigma_-) \right]}$$



CONSTRAINTS: $\quad \boxed{\begin{array}{c} (\partial_+ X^\mu_+)^2 = 0 \\ (\partial_- X^\mu_-)^2 = 0 \end{array}}$

CUSPS

TIME GAUGE: $X^0(\tau,\sigma) = \tau = \frac{1}{2}[\sigma_+ + \sigma_-]$

IN CENTER-OF-MASS FRAME: $X^i_\pm(\sigma_\pm) = \text{PERIODIC} \Rightarrow \langle \dot{X}^i_\pm(\sigma_\pm)\rangle = 0$

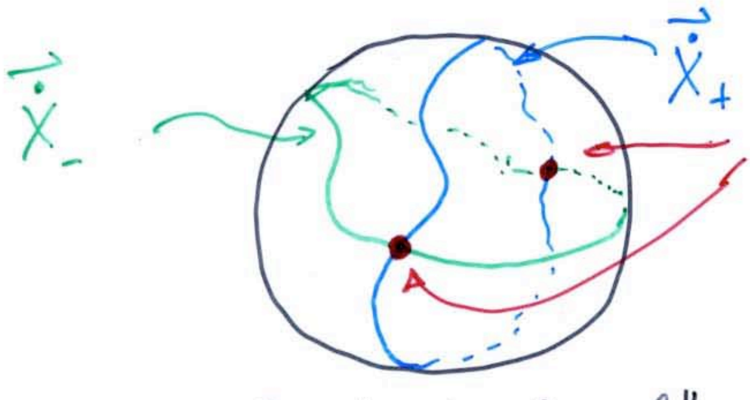
CONSTRAINTS: $$\left(\partial_\pm X^\mu_\pm\right)^2 = -\big(\partial_\pm \underbrace{X^0_\pm}_{\sigma_\pm}\big)^2 + \left(\partial_\pm X^i_\pm\right)^2 = 0$$

$$-1 + \left(\dot{X}^i_\pm\right)^2 = 0$$

$$\left(\vec{\dot{X}}_+\right)^2 = 1 = \left(\vec{\dot{X}}_-\right)^2$$

$\vec{\dot{X}}_+$ AND $\vec{\dot{X}}_-$ ARE PERIODIC (WITH ZERO AVERAGE) ON UNIT SPHERE

Kibble, Turok '82



GENERICALLY EXPECT

$\exists$ 2 INTERSECTIONS

Turok '84

INTERSECTION: $\partial_+ X^\mu_+ = \partial_- X^\mu_- = \ell^\mu$

LIGHT-CONE TANGENT TO WORLD-SHEET

IN SPACE:

GW BURSTS FROM CUSPY STRINGS

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$

$\partial^\nu \bar{h}_{\mu\nu} = 0$

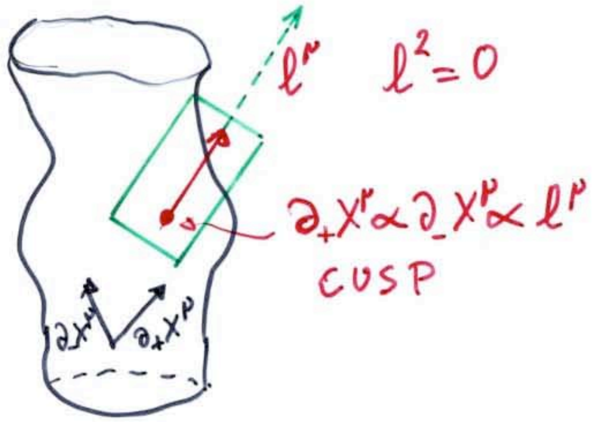
HARMONIC GAUGE

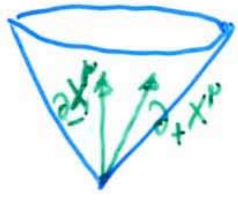
STRING STRESS-ENERGY TENSOR

$$T^{\mu\nu}(x^\lambda) = \mu \int d\tau d\sigma \left(\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta^{(4)}(x^\lambda - X^\lambda(\tau,\sigma))$$

- USE LEFT – RIGHT DECOMPOSITION : $\boxed{\sigma_\pm \equiv \tau \pm \sigma}$

$$\boxed{X^\mu(\tau,\sigma) = \frac{1}{2}\left[X_+^\mu(\sigma_+) + X_-^\mu(\sigma_-) \right] \qquad (\partial_+ X_+^\mu)^2 = 0 \qquad (\partial_- X_-^\mu)^2 = 0}$$





- USE FOURIER TRANSFORM

$$\boxed{T^{\mu\nu}(k^\lambda) = \frac{\mu}{T_\ell} \int_{\Sigma_\ell} d\tau d\sigma \, \dot{X}_+^{(\mu} \dot{X}_-^{\nu)} e^{-\frac{i}{2} k \cdot (X_+ + X_-)}}$$

- STAY POINCARÉ COVARIANT

GW AMPLITUDE FROM STRINGS

$$\boxed{\bar{h}_{\mu\nu}(t,\vec{x}) = \frac{\kappa_{\mu\nu}(t-r,\vec{n})}{r} + O\left(\frac{1}{r^2}\right)}$$

$$\kappa_{\mu\nu}(t-r,\vec{n}) = \sum_{\omega = \pm m\frac{2\pi}{T_\ell}} 4G e^{-i\omega(t-r)} T_{\mu\nu}(\omega, \vec{k}=\omega\vec{n})$$

$T_\ell = \frac{\ell}{2}$; ℓ invariant length $\ell = \frac{E_0}{\mu}$

$$\boxed{T^{\mu\nu}(\vec{k}_m,\omega_m) = \frac{\mu}{\ell} I_+^{(\mu} I_-^{\nu)}}$$

$$\boxed{I_\pm^\mu \equiv \int_0^\ell d\sigma_\pm \, \dot{X}_\pm^\mu(\sigma_\pm)\, e^{-\frac{i}{2} k_m \cdot X_\pm}}$$

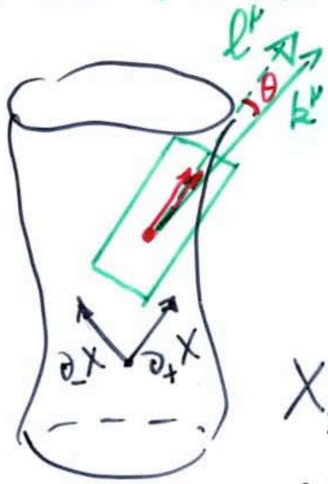
LOGARITHMIC FOURIER TRANSFORM OF WAVEFORM (HIGH-FREQ PART)

$$\kappa^{\mu\nu}(f,\vec{n}) \equiv |f| \int dt \, e^{2\pi i f t} \kappa^{\mu\nu}(t-r,\vec{n})$$

$$\boxed{\kappa^{\mu\nu}(f,\vec{n}) = 2G\mu |f| \, I_+^{(\mu}(\omega,\omega\vec{n})\, I_-^{\nu)}(\omega,\omega\vec{n})}$$

LEFT-RIGHT FACTORIZATION OF GW AMPLITUDE $\kappa^{\mu\nu}(k)$

GW AMPLITUDE FROM CUSPS



AT CUSP

$$\partial_+ X^\mu \propto \partial_- X^\mu \propto l^\mu$$ NULL VECTOR

NEAR CUSP

$$X^\mu_\pm(\sigma_\pm) = X^\mu_c + l^\mu \sigma_\pm + \frac{1}{2}\ddot{X}^\mu_\pm \sigma_\pm^2 + \frac{1}{6}\dddot{X}^\mu_\pm \sigma_\pm^3 + \dots$$

$$\dot{X}^\mu_\pm(\sigma_\pm) = l^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \frac{1}{2}\dddot{X}^\mu_\pm \sigma_\pm^2 + \dots$$

$$\kappa^{\mu\nu}(f) \propto I_+^{(\mu} I_-^{\nu)} \qquad \omega_l = \frac{2\pi}{T_l} = \frac{4\pi}{l}$$

$$I^\mu_\pm = \int_{\sigma_0}^{\sigma_0 + l} d\sigma_\pm \left(l^\mu + \ddot{X}^\mu_\pm \sigma_\pm + \dots\right) e^{+\frac{i}{12} m \omega_l \ddot{X}^2_\pm \sigma_\pm^3 + \dots}$$

CAN BE GAUGED AWAY (WHEN $\theta = 0$)

$m \to \pm\infty$

$\sim \theta \sigma_\pm^2 + \theta^2 \sigma_\pm$ WHEN $\theta \neq 0$

$$\boxed{\kappa^{\mu\nu}(f, \vec{n}_{cusp}) = -C \frac{G\mu}{(2\pi |f|)^{1/3}} e^{2\pi i f t_c} A_+^{(\mu} A_-^{\nu)} + \text{GAUGE}}$$

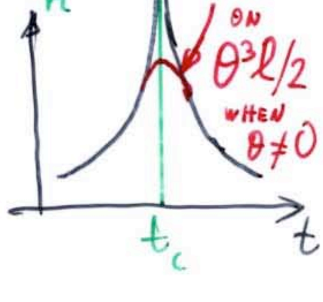
i.e. FOR $\theta = 0$

$$C = 4\pi (12)^{4/3} / [3\Gamma(\tfrac{1}{3})]^2 \qquad A^\mu_\pm = \ddot{X}^\mu_\pm / |\ddot{X}_\pm|^{4/3}$$

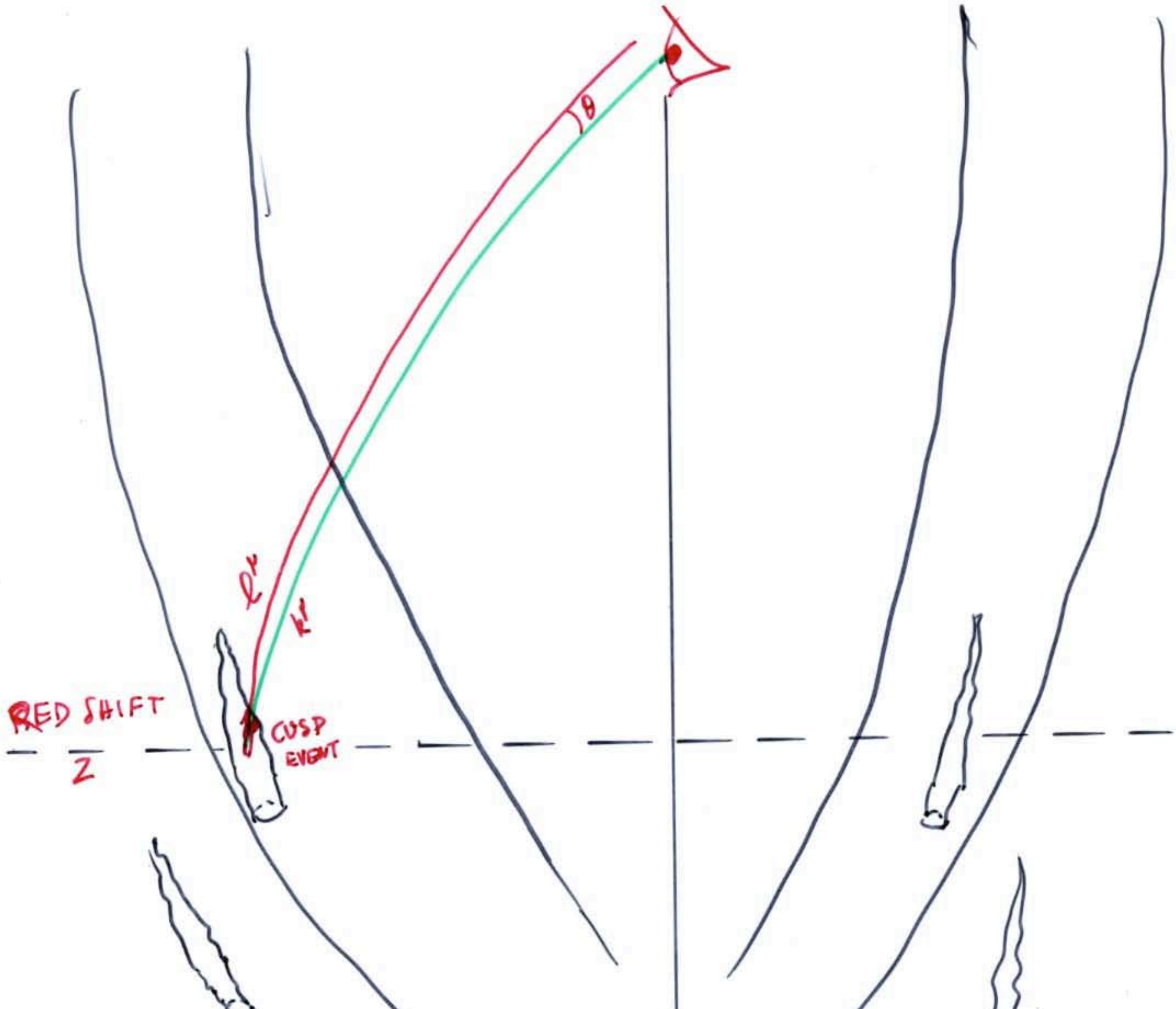
TIME DOMAIN

SIGNAL ROBUST UNDER $\exists$ SMALL-SCALE WIGGLES (Siemens, Olum '03)

$$\boxed{\begin{aligned} \kappa(t) &\propto |t - t_c|^{1/3} \\ \ddot{\kappa}(t) &\propto |t - t_c|^{-5/3} \end{aligned}}$$



GW BURSTS FROM COSMOLOGICAL STRING NETWORK



- EFFECT OF COSMOLOGICAL EXPANSION ON $\tilde{h}_{\mu\nu}(f)$ < ON FREQUENCY / ON AMPLITUDE
- NUMBER OF CUSP EVENTS PER UNIT SPACE-TIME VOLUME

$$\boxed{\nu(t) \sim c\, n_\ell(t)/(\ell/2)}$$

$c \equiv$ # cusp events per loop period

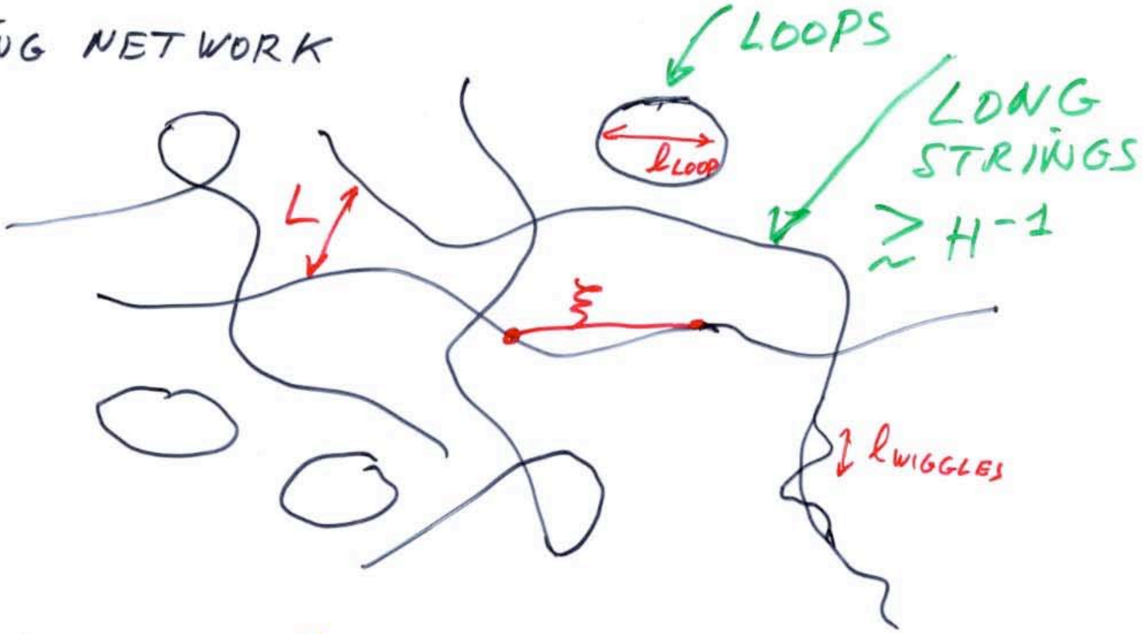
$n_\ell(t)$ = loop density

- BEAMING FRACTION WITHIN $\boxed{\theta_m \equiv \left[(1+z) f \ell/2\right]^{-1/3}}$

LOOP NUMBER DENSITY $n_\ell(t)$?

SEVERAL COMPETING PHENOMENA AT WORK:

- STRING NETWORK



COSMOLOGICAL EXPANSION: $\longrightarrow$ TENDS TO STRAIGHTEN OUT STRINGS

STRING INTERACTIONS: INTERSECTIONS, RECONNECTIONS (WITH PROBA. P), SELF-RECONNECTIONS
$\longrightarrow$ CREATES LOOPS AND SMALL-SCALE STRUCTURE

- VARIOUS SCALES: ξ, L, ℓ_{LOOP}, $\ell_{WIGGLES}$

- NUMERICAL SIMULATIONS INDICATE SCALING BUT
$\exists$ LARGE UNCERTAINTY IN FINAL SCALING STATE AND PARAMETERS

PROBABLY: $\xi(t) \sim t$, $\boxed{L(t) \sim p^{1/2}\, t}$, $\boxed{\ell_{WIGGLES} \sim \ell_{LOOP\ JUST\ FORMED} \sim \alpha\, t}$

GOOD NEWS $p \ll 1$
INCREASES THE DENSITY
OF LONG STRINGS AND LOOPS

CRUCIAL
DIMENSIONLESS PARAMETER α
IS VERY UNCERTAIN

α: 50 $G\mu$? ... 0.1 ?

α DETERMINES $\ell_{LOOP} \sim \alpha t$ → LOOP LIFE-TIME $\tau \sim \frac{\alpha}{50 G\mu} t$

JUST FORMED

redshifts $z \lesssim 1$

high-redshift

⇒ LOOP DENSITY $\boxed{n_\ell \sim \frac{1}{p 50 G\mu t^3} + n_\ell^{z>1}}$

DOMINATES IF $\alpha \lesssim 50 G\mu$ (considered by Damour Vilenkin 05)

DOMINATES IF $\alpha \gg 50 G\mu$ (considered by Hogan06)

∃ LARGE RANGE OF VALUES OF $G\mu$ WHERE GW BURSTS WOULD BE OBSERVABLE BY LIGO OR LISA. IN ADDITION, AT LOW GW FREQUENCIES CONFUSION NOISE → STOCHASTIC GW BACKGROUND OBSERVABLE BY PULSAR TIMING

OBSERVABLE RANGE OF $G\mu$ VALUES IS LARGER BUT THE POPULATION OF HIGH z LOOPS CREATE A LARGER CONFUSION NOISE (i.e. STOCHASTIC BACKGND) WHICH MAKES MORE DIFFICULT TO SEE INDIVIDUAL BURST EVENTS

LIGO COULD DETECT $G\mu \gtrsim 10^{-12}$

LISA COULD DETECT $G\mu \gtrsim 10^{-14}$

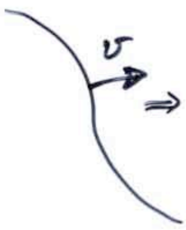
LISA COULD DETECT $G\mu \gtrsim 10^{-15}$

BUT, PULSAR TIMING GIVE ALREADY STRINGENT LIMITS ON THE EXISTENCE OF A STOCHASTIC BACKGRND OF GW's (Jenet et al. 06) WHICH (PROBABLY) ALREADY SETS SEVERE LIMITS ON COSMIC (SUPER) STRINGS

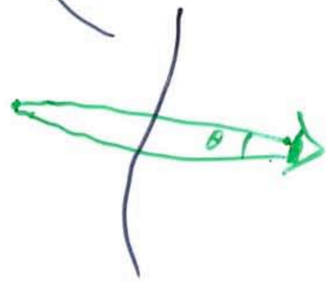
? $G\mu \lesssim 10^{-9}$ OR EVEN $G\mu \lesssim 10^{-10}$?

! WHICH IS ALREADY ~ THE KKLMMT LEVEL !

NOTE THE VARIOUS POSSIBLE OBSERVABLE SIGNALS FROM COSMIC (SUPER)-STRINGS



CMB $\Rightarrow \frac{\delta T}{T} \sim 8\pi\, G\mu v \gamma$ CURRENT BOUND $G\mu \lesssim 5\times 10^{-7}$



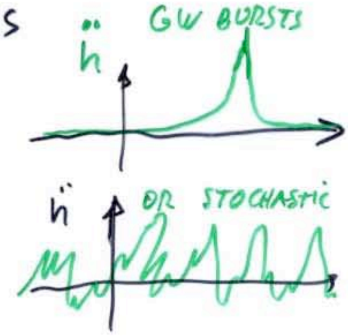
LENSING ONLY SENSITIVE TO $G\mu \gtrsim 10^{-7}$

SEVERAL GW DETECTORS

LIGO / VIRGO / GEO

ADVANCED LIGO

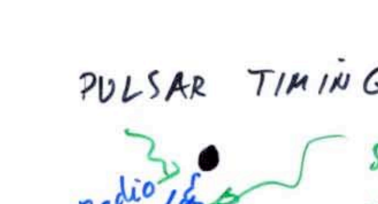
LISA



POTENTIALLY SENSITIVE DOWN TO $G\mu \gtrsim 10^{-12}$

$G\mu \gtrsim 10^{-14}$ OR 10^{-15}

PULSAR TIMING



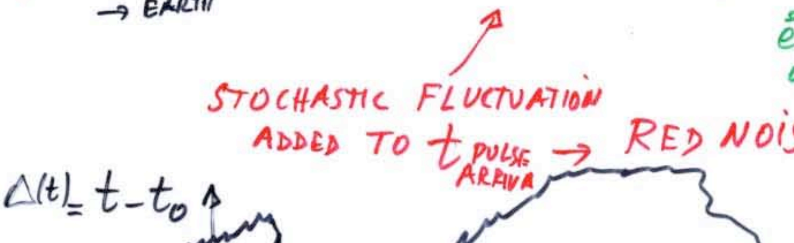
$$H_{ij}(t) = \int dt\, h_{ij}(t)$$

$\vec{m}$ = DIRECTION OF GW

$$\Rightarrow t_{\substack{\text{PULSAR}\\ \to \text{EARTH}}} = t_0 + \frac{1}{2}\frac{1}{1-\vec{m}\cdot\vec{e}}\, e^i e^j \left[H_{ij}(t) - H_{ij}\left(t-(1-\vec{m}\cdot\vec{e})t_0\right)\right]$$

$\vec{e}$ = DIRECTION OF ELM WAVE

STOCHASTIC FLUCTUATION ADDED TO $t_{\text{PULSE ARRIVAL}}$ $\rightarrow$ RED NOISE



FREQUENCY ANALYSIS OF $\Delta(t)$: $$\Delta(f) = \frac{H_0^2}{8\pi^4}\, \frac{\Omega_{GW}(f)}{f^4}$$

ENERGY DENSITY OF GW

- EXCITING POSSIBILITY OF DETECTING COSMIC (SUPER) STRINGS
- FRUSTRATING UNCERTAINTIES IN NETWORK PROPERTIES (see Polchinski)

Parma

4

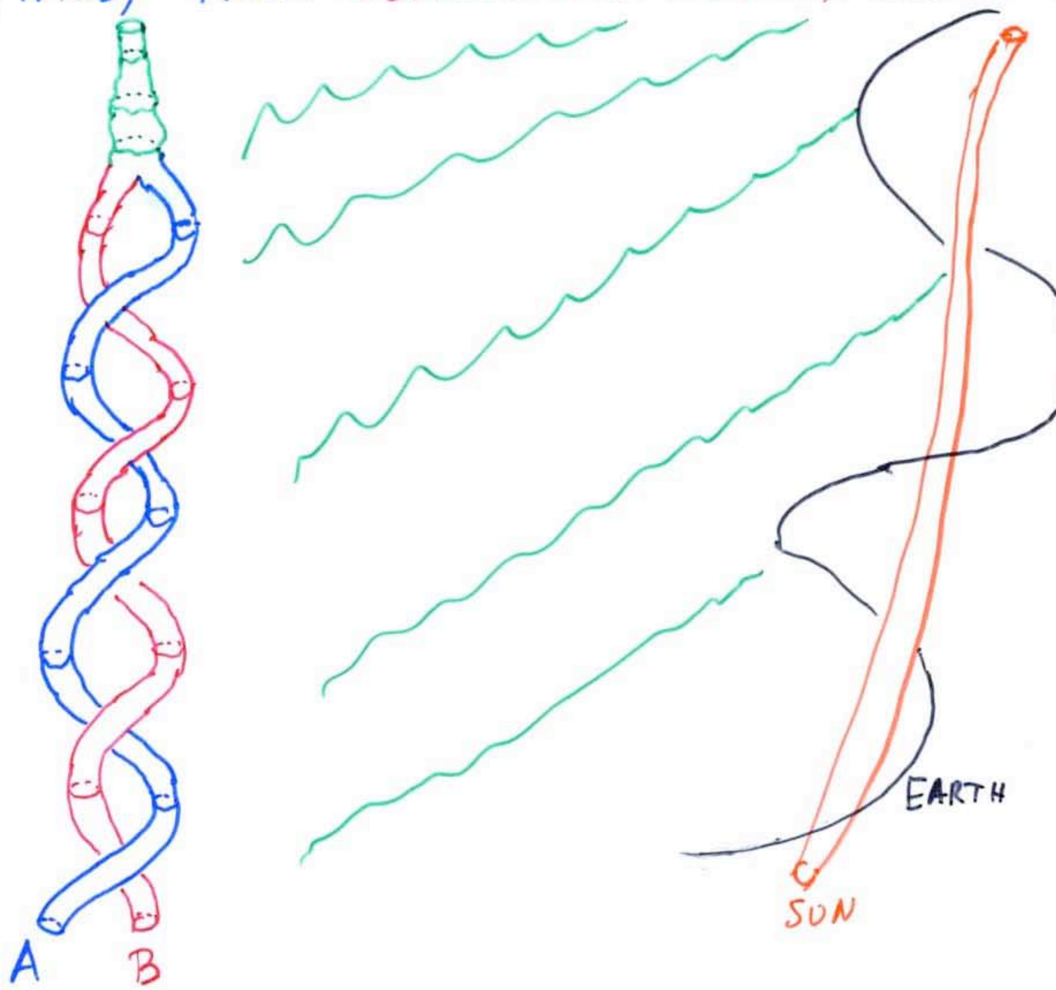
GRAVITATIONAL WAVES

FROM

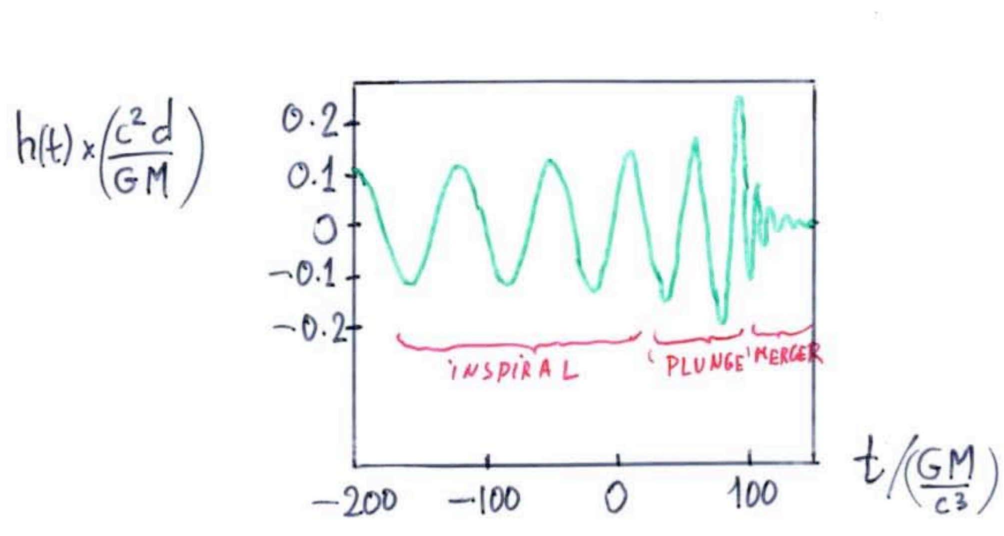
COALESCING BLACK HOLES

GRAVITATIONAL WAVES:

NOTABLY FROM COALESCING BINARY BLACK HOLES



GRAVITATIONAL WAVE SIGNAL EXPECTED ON EARTH



CHALLENGES

ANALYTICAL APPROACHES

- NEED EQS OF MOTION TO VERY HIGH PERTURBATION ORDER
- NEED GW GENERATION FORMALISM TO HIGH PERTURBATION ORDER
- NEED TO PACKAGE THIS PERTURBATIVE INFORMATION IN A FORM WHICH REMAINS VALID DURING BOTH INSPIRAL AND PLUNGE, AND WHICH CAN CONNECT WITH MERGER AND RINGDOWN
- (QUASI-) ANALYTICAL DESCRIPTION IS NEEDED FOR COVERING THE FULL PARAMETER SPACE: $m_1, m_2, \vec{S}_1, \vec{S}_2$, ECCENTRICITY, ...

NUMERICAL APPROACHES

- NEED STABLE CODES
- NEED TO DESCRIBE B. HOLES
- NEED TO EXTRACT GW WAVEFORMS
- LIMITED TO A FEW ORBITS BEFORE MERGER
- LIMITED TO EXPLORING A FEW SAMPLE POINTS IN THE PARAMETER SPACE
- GIVES CRUCIAL NON PERTURBATIVE INFORMATION THAT GOES BEYOND WHAT CAN BE ANALYTICALLY COMPUTED

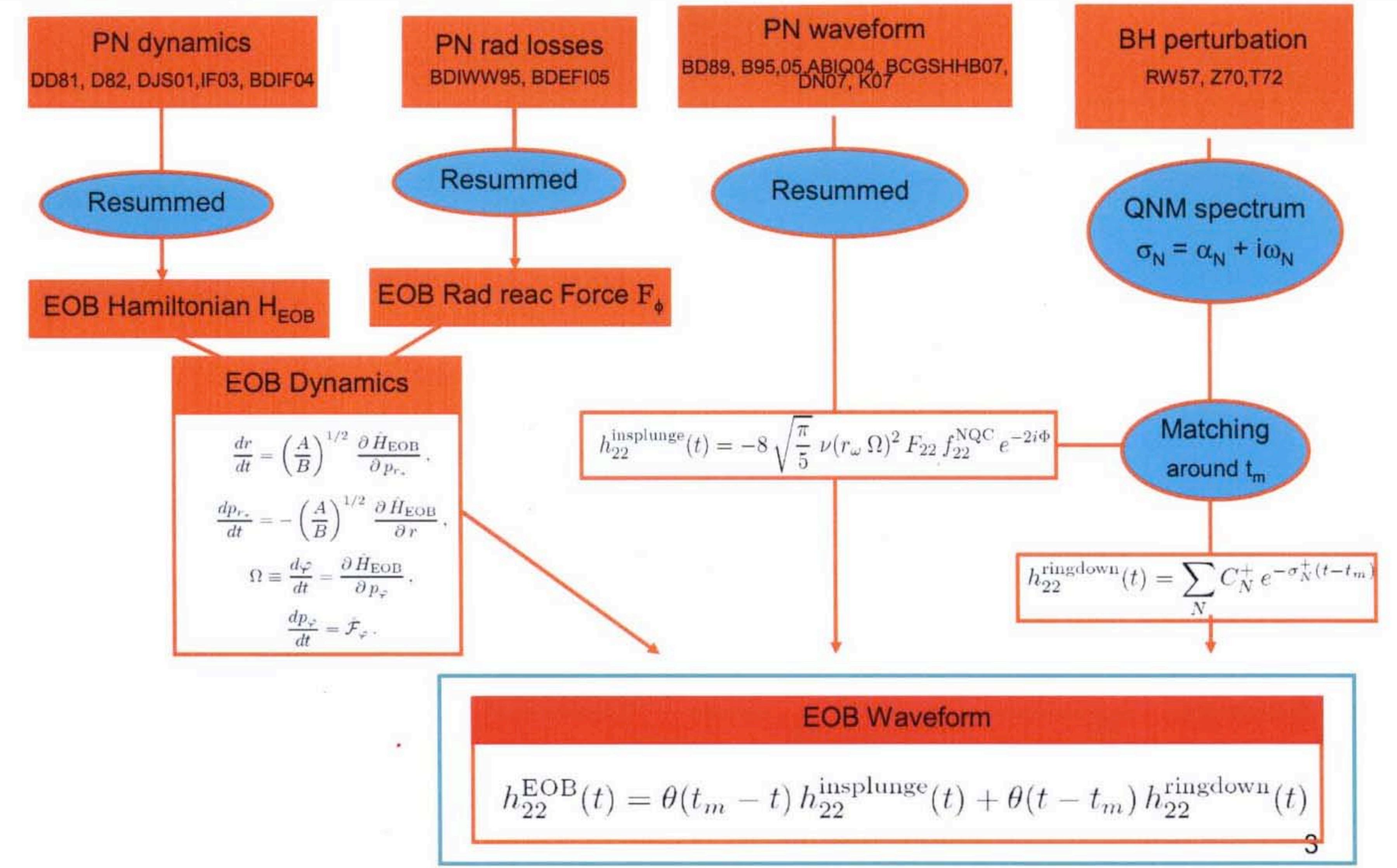
COMPLEMENTARITY

AIM: TAKE ADVANTAGE OF FLEXIBILITY OF ANALYTICAL METHODS, NOTABLY OF RESUMMED (EFFECTIVE ONE BODY + PADÉ) PN METHODS, TO CONSTRUCT GW TEMPLATES THAT HAVE A GOOD PHASING ALL OVER INSPIRAL + PLUNGE + MERGER + RINGDOWN

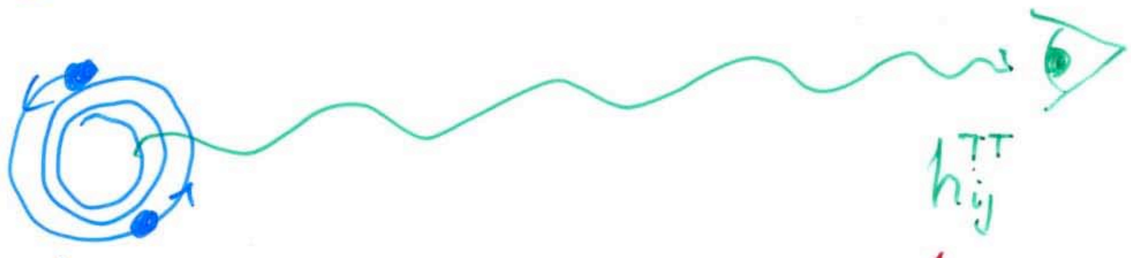
TOOL: COMPARISON BETWEEN EOB-LIKE TEMPLATES/PREDICTIONS AND NUMERICAL RESULTS

- $\nu \equiv \frac{m_1 m_2}{(m_1+m_2)^2} \sim \frac{1}{4}$ COMPARABLE MASSES
- OR $\nu \ll 1$ EXTREME MASS RATIO

Structure of EOB formalism



WHAT IS NEEDED FOR DESCRIBING COALESCING BINARIES



$$\text{EoM} \quad \frac{d^2\vec{x}_a}{dt^2} = \vec{a}_a^{\,CONS} + \vec{a}_a^{\,RR} \qquad r h_{ij}^{TT} = \left[U_{ij} + U_{ijk}\frac{N^k}{c} + \dots \right]^{TT}$$

$a = 1, 2$

NEED RADIATION REACTION TO HIGHEST POSSIBLE ACCURACY

$$a^{RR} = \frac{GM}{r^2}\Big[\frac{v^5}{c^5} + \frac{v^7}{c^7} + \underbrace{\frac{v^8}{c^8} + \frac{v^9}{c^9} + \frac{v^{10}}{c^{10}} + \frac{v^{11}}{c^{11}} + \frac{v^{12}}{c^{12}}}_{\text{HEURISTICALLY OBTAINED BY ASSUMING BALANCE OF } E,\ \vec{J}} \Big]$$

WAVE FORM :

$$U_{ij} = \mu x^i x^j \left\{ 1 + \frac{v^2}{c^2} + \frac{v^3}{c^3} + \frac{v^4}{c^4} + \frac{v^5}{c^5} + \frac{v^6}{c^6} + \frac{v^7}{c^7} \right\}$$

\+ NEED CONSERVATIVE DYNAMICS TO HIGHEST POSSIBLE ACCURACY

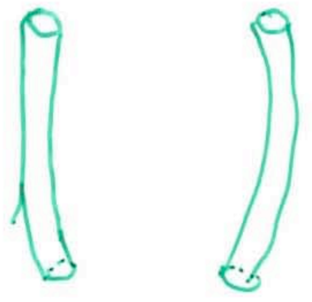
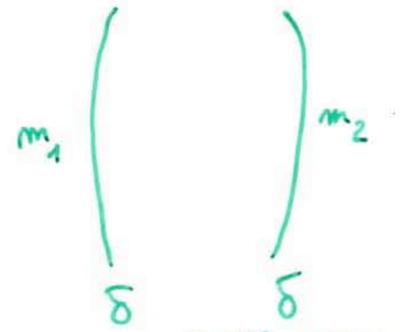
$$a^{CONS} = \frac{GM}{r^2}\left[1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} \right]$$

OR

$$H[x, p] = H_0 + \frac{1}{c^2} H_2 + \frac{1}{c^4} H_4 + \frac{1}{c^6} H_6$$

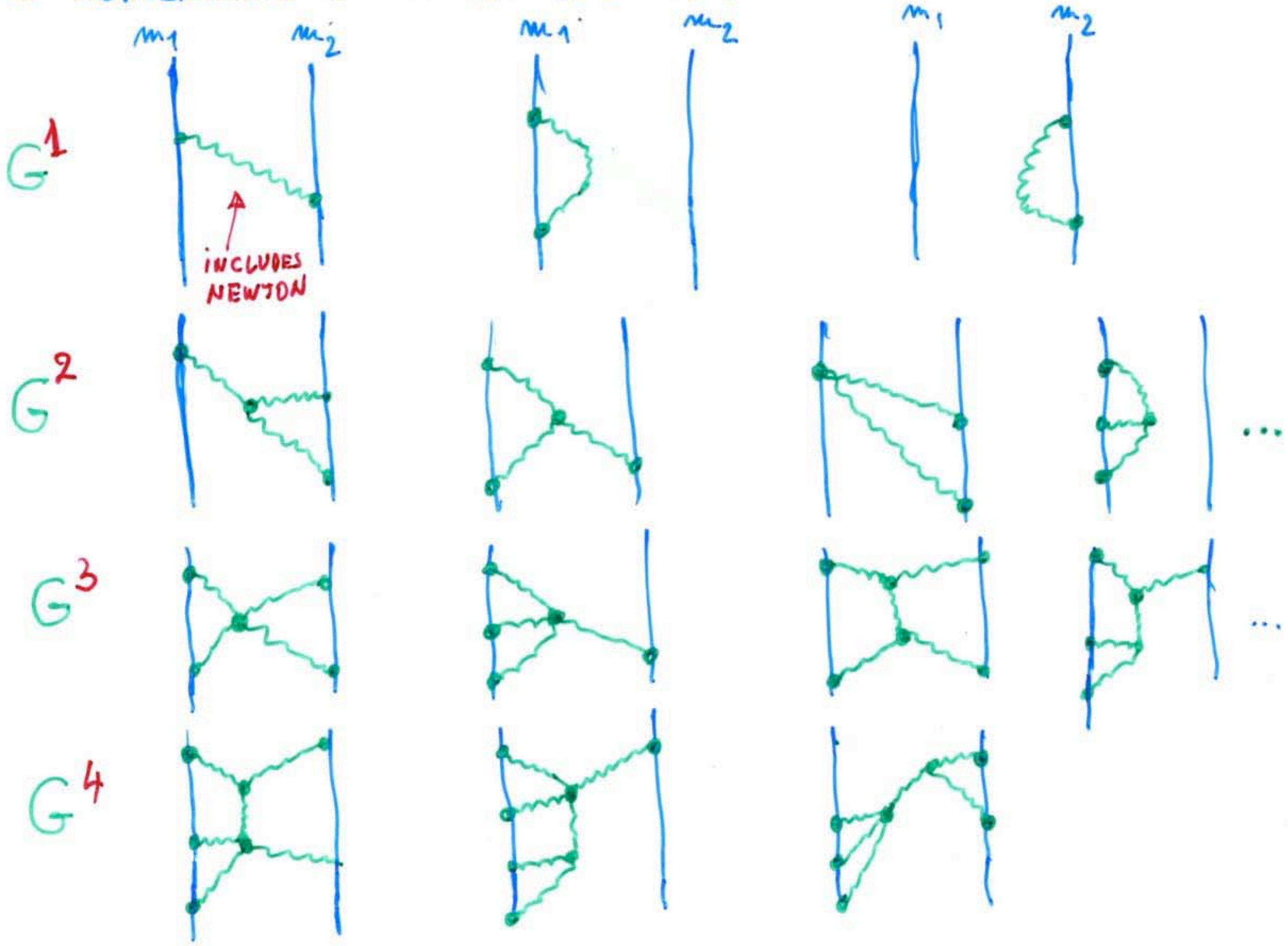
ST THOMAS

HIGH-PERTURBATION ORDER CALCULATIONS OF EQS. OF MOTION 6

- REPLACE  BY 

BECAUSE OF EFFACEMENT OF INTERNAL STRUCTURE UP TO $\left(\frac{v}{c}\right)^{10} \sim$ 5PN ORDER
(Damour, '82)

- CONSERVATIVE PART OF HAMILTONIAN :



- NEED DIMENSIONAL REGULARIZATION ('t Hooft Veltman '72) TO COMPUTE G^4 (3PN ~ 3 LOOP) (Damour Jaranowski Schäfer '00, Blanchet Damour Esposito F. '04)
in ADM FORMALISM HARMONIC GAUGE

2-body Taylor-expanded 3PN Hamiltonian [JS98, DJS00,01]

$$H(\mathbf{x}_a,\mathbf{p}_a)=\sum_a m_a c^2+H_{\rm N}(\mathbf{x}_a,\mathbf{p}_a)+\frac{1}{c^2}H_{\rm 1PN}(\mathbf{x}_a,\mathbf{p}_a)+\frac{1}{c^4}H_{\rm 2PN}(\mathbf{x}_a,\mathbf{p}_a)+\frac{1}{c^6}H_{\rm 3PN}(\mathbf{x}_a,\mathbf{p}_a)+\mathcal{O}\left(\frac{1}{c^8}\right)$$

$$H_{\rm N}(\mathbf{x}_a,\mathbf{p}_a)=\sum_a\frac{\mathbf{p}_a^2}{2\,m_a}-\frac{1}{2}\sum_a\sum_{b\neq a}\frac{G\,m_a\,m_b}{r_{ab}}$$

$$H_{\rm 1PN}(\mathbf{x}_a,\mathbf{p}_a)=-\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3}+\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left[-12\frac{\mathbf{p}_1^2}{m_1^2}+14\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2}+2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right]+\frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}}+(1\longleftrightarrow 2).$$

1PN

$$\begin{aligned}H_{\rm 2PN}(\mathbf{x}_a,\mathbf{p}_a)={}&\frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5}+\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left[5\frac{(\mathbf{p}_1^2)^2}{m_1^4}-\frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2}-\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}+5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right.\\&\left.-6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2}-\frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right]\\&+\frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left[m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2}+19\frac{\mathbf{p}_2^2}{m_2^2}\right)-\frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2)+6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right]\\&-\frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2+5m_1m_2+m_2^2)}{r_{12}^2}+(1\longleftrightarrow 2).\end{aligned}$$

2PN

$$\begin{aligned}H_{\rm 3PN}^{\rm reg}(\mathbf{x}_a,\mathbf{p}_a)={}&-\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7}+\frac{1}{32}\frac{Gm_1m_2}{r_{12}}\left[-14\frac{(\mathbf{p}_1^2)^3}{m_1^6}+4\frac{\left((\mathbf{p}_1\cdot\mathbf{p}_2)^2+4\mathbf{p}_1^2\mathbf{p}_2^2\right)\mathbf{p}_1^2}{m_1^4m_2^2}+\frac{\left(\mathbf{p}_1^2\mathbf{p}_2^2-2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\right)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3m_2^3}\right.\\&-10\frac{\left(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2+\mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\right)\mathbf{p}_1^2}{m_1^4m_2^2}+24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4m_2^2}+2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3m_2^3}\\&+\frac{\left(7\mathbf{p}_1^2\mathbf{p}_2^2-10(\mathbf{p}_1\cdot\mathbf{p}_2)^2\right)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3m_2^3}+6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4m_2^2}\\&\left.+15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3m_2^3}-18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3m_2^3}+5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3m_2^3}\right]\\&+\frac{G^2m_1m_2}{r_{12}^2}\left[\frac{1}{16}(m_1-27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4}-\frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3m_2}+\frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2+371\,\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2}\right.\\&+\frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3}-\frac{1}{8}m_1\frac{\left(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)+11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)\right)(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3m_2}+\frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3}\\&-\frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3m_2}+\frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2}+\frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\\&\left.-\frac{1}{48}(220m_1+193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right]+\frac{G^3m_1m_2}{r_{12}^3}\left[-\frac{1}{48}\left(466\,m_1^2+\left(473-\frac{3}{4}\pi^2\right)m_1m_2+150\,m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2}\right.\\&+\frac{1}{16}\left(77(m_1^2+m_2^2)+\left(143-\frac{1}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2}+\frac{1}{16}\left(61\,m_1^2-\left(43+\frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2}\\&\left.+\frac{1}{16}\left(21(m_1^2+m_2^2)+\left(119+\frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right]\\&+\frac{1}{8}\frac{G^4m_1m_2^3}{r_{12}^4}\left[\left(\frac{227}{3}-\frac{21}{4}\pi^2\right)m_1+m_2\right]+(1\longleftrightarrow 2).\qquad(12)\end{aligned}$$

3PN

Defining H_{EOB} by thinking quantum-mechanically (Wheeler)

Real 2-body system (m_1, m_2) (in the c.o.m. frame)

an effective particle of mass μ in some effective metric $g_{\mu\nu}^{eff}(M)$

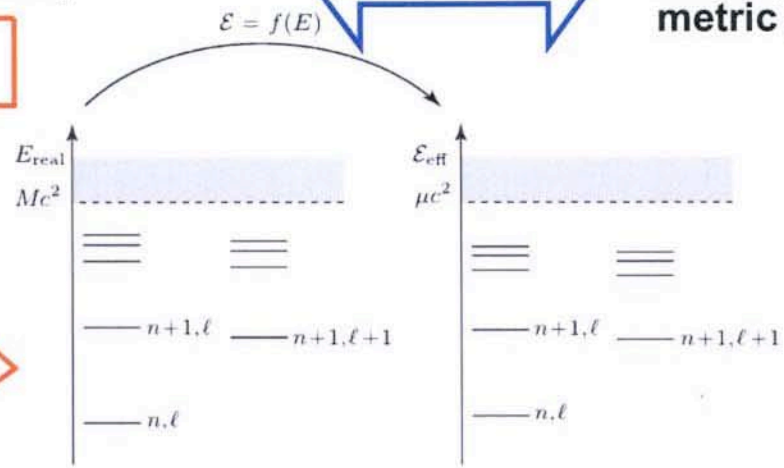


Figure 1: Sketch of the correspondence between the quantized energy levels of the real and effective conservative dynamics. n denotes the 'principal quantum

Sommerfeld "Old Quantum Mechanics":

$$\boxed{\begin{aligned} J &= \ell\hbar = \frac{1}{2\pi}\oint p_\varphi\, d\varphi \\ N &= n\hbar = I_r + J \\ I_r &= \frac{1}{2\pi}\oint p_r\, dr = I_r + J \end{aligned}}$$

$\boxed{H^{classical}(q,p)}$ — $\boxed{H^{classical}(I_a)}$ $\longrightarrow$ $\boxed{H^{quantum}(I_a = n_a h)}$

Damour, Schäfer '88
Damour, Jaranowski, Schäfer, '00

SOMMERFELD-TYPE (DELAUNAY!) HAMILTONIAN

$\vec{q} = \vec{q}_1 - \vec{q}_2 \,;\; \vec{p} = \vec{p}_1 = -\vec{p}_2$

- CENTER-OF-MASS FRAME $H(\vec{q}_1 - \vec{q}_2, \vec{p}_1, \vec{p}_2) \rightarrow H(\vec{q}, \vec{p}, -\vec{p})$
- $H^{CoM}(\vec{q}, \vec{p}) = H_0(\vec{q}, \vec{p}) + \frac{1}{c^2} H_2(qp) + \frac{1}{c^4} H_4(qp) + \frac{1}{c^6} H_6(q,p)$

$$H_0(qp) = \frac{\vec{p}^{\,2}}{2\mu} + \frac{GM\mu}{|\vec{q}|}$$

SEPARATION OF VARIABLES $(\varphi, p_\varphi) \,;\, (\theta, p_\theta) \,;\, (r, p_r)$

ACTION VARIABLES (e.g. for $\theta = \frac{\pi}{2}$)

$$J = \ell \hbar = \frac{1}{2\pi} \oint p_\varphi \, d\varphi$$

$$I_r = \frac{1}{2\pi} \oint p_r \, dr$$

$$N = n\hbar = I_r + J$$

$\alpha \equiv \frac{G m_1 m_2}{\hbar}$

$$\Rightarrow \quad \boxed{H_0(n,\ell) = -\frac{1}{2}\mu \left(\frac{GM\mu}{I_r + J}\right)^2 = -\frac{1}{2}\mu \frac{\alpha^2}{n^2}}$$

PRINCIPAL QUANTUM NUMBER

WHEN CONSIDERING $H_0 + \frac{1}{c^2} H_2 + \frac{1}{c^4} H_4 + \frac{1}{c^6} H_6$, MORE COMPLICATED CALCULATIONS, BUT SAME LOGIC:

$$\boxed{H^{2\text{-BODY}}(n,\ell) = -\frac{1}{2}\mu \frac{\alpha^2}{n^2}\left[1 + \frac{\alpha^2}{c^2}\left(\frac{c_{11}}{n\ell} + \frac{c_{20}}{n^2}\right) + \frac{\alpha^4}{c^4}\left(\frac{c_{13}}{n\ell^3} + \frac{c_{22}}{n^2\ell^2} + \frac{c_{31}}{n^3\ell} + \frac{c_{40}}{n^4}\right) + \dots\right]}$$

SOMMERFELD HAMILTONIAN FOR 'EFFECTIVE ONEBODY' PROBLEM

TO START WITH, CONSIDER ONE BODY OF MASS μ MOVING IN SOME 'EXTERNAL' METRIC

$$g^{ext}_{\mu\nu}(x)\, dx^\mu dx^\nu = -\underbrace{A(R)}_{\text{UNKNOWN FUNCTIONS, TO BE DETERMINED}} c^2 dT^2 + \underbrace{B(R)}_{\text{UNKNOWN FUNCTIONS, TO BE DETERMINED}} dR^2 + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

UNKNOWN FUNCTIONS, TO BE DETERMINED

WRITE

$$A(R) = 1 + a_1 \frac{GM}{c^2 R} + a_2 \left(\frac{GM}{c^2 R}\right)^2 + a_3 \left(\frac{GM}{c^2 R}\right)^3 + \dots$$

$$B(R) = 1 + b_1 \left(\frac{GM}{c^2 R}\right) + b_2 \left(\frac{GM}{c^2 R}\right)^2 + \dots$$

THEN CONSIDER HAMILTON-JACOBI EQ FOR EOB DYNAMICS

$$g^{\mu\nu}_{eff} \frac{\partial S_{eff}}{\partial x^\mu} \frac{\partial S_{eff}}{\partial x^\nu} + \mu^2 c^2 = 0$$

- SEPARATE VARIABLES

$$S_{eff} = -\mathcal{E}_{eff} T + J_{eff}\, \varphi + S_{eff}(R)$$

- COMPUTE $I_R = \frac{1}{2\pi} \oint P_R\, dR$ AND THEN $H_{eff}(J_{eff}, N_{eff})$

$$\boxed{H^{eff}(n_{eff}, l_{eff}) = \mu c^2 - \frac{1}{2}\mu \frac{\alpha^2}{n_{eff}^2}\left[1 + \frac{\alpha^2}{c^2}\left(\frac{c_{11}^{eff}}{n_{eff}\, l_{eff}} + \frac{c_{20}^{eff}}{n_{eff}^2}\right) + \frac{\alpha^4}{c^4}(\dots) + \dots\right]}$$

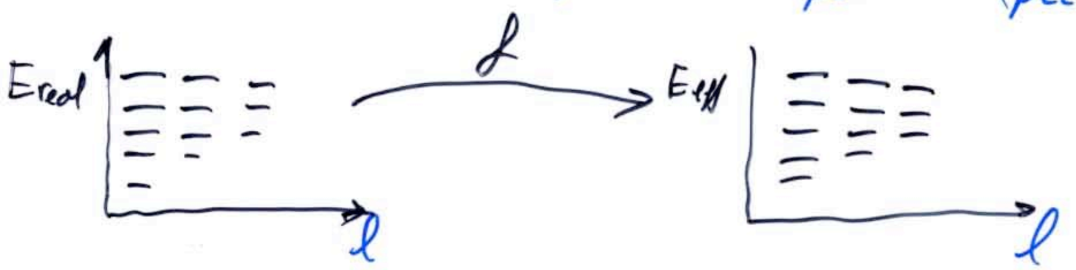
DICTIONARY BETWEEN REAL AND EFFECTIVE DYNAMICS

$$H^{real}(n,\ell) \longleftrightarrow H^{eff}(n_{eff},\ell_{eff})$$

- IDENTIFY QUANTIZED INTEGERS : $\boxed{\begin{array}{l} n = n_{eff} \\ \ell = \ell_{eff} \end{array}}$

- LOOK FOR ANOTHER FUNCTION REALIZING ENERGY CORRESPONDENCE

$$\frac{E_{eff}}{\mu c^2} - 1 = f\left(\frac{E_{real}}{\mu c^2}\right) = \frac{E_{real}}{\mu c^2}\left(1 + \alpha_1 \frac{E_{real}}{\mu c^2} + \alpha_2 \left(\frac{E_{real}}{\mu c^2}\right)^2 + \dots\right)$$



MANY EQUATIONS FOR MANY UNKNOWNS $(a_1, a_2, a_3 \dots, b_1, b_2 \dots, \alpha_1, \alpha_2 \dots)$

- IF ONE ASSUMES $a_1 = -2$, $b_1 = 2$ (LINEARIZED SCHWARZSCHILD), ONE FINDS A UNIQUE SOLUTION FOR $a_2, a_3, b_2, \alpha_1, \alpha_2$ AT 2PN $(1/c^4)$ WHICH CAN THEN BE EXTENDED @ 3PN IF ONE ADDS SOME $\vec{p}^{\,4}$ TERMS IN HAMILTON-JACOBI EQ.

VERY SIMPLE f

$$\boxed{\frac{E_{eff}}{\mu} = \frac{\mathcal{E}_{real}^2 - m_1^2 - m_2^2}{2 m_1 m_2} = \frac{s - m_1^2 - m_2^2}{2 m_1 m_2}}$$

Explicit form of the effective metric

The effective metric at 3PN + a 4PN correction

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

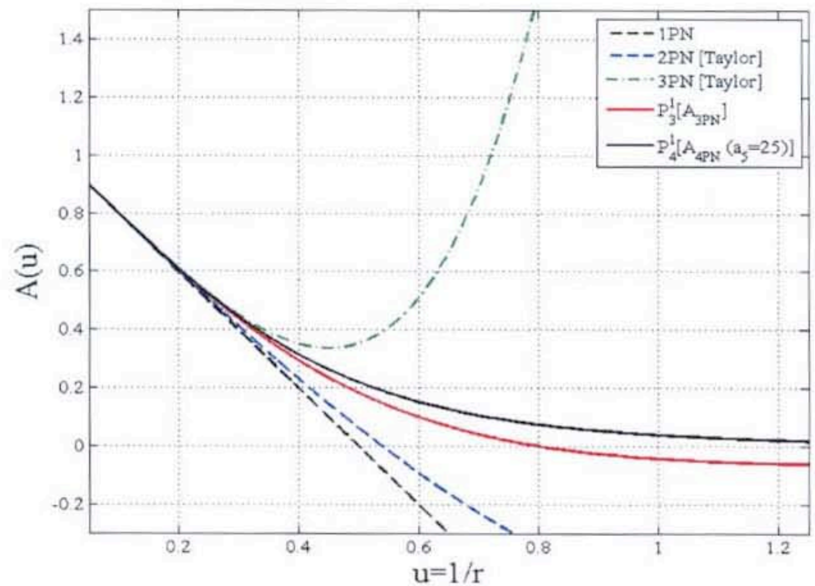
where the coefficients are a ν-dependent "deformation" of the Schwarzschild ones:

$$(BA)^{3\text{PN}}(r) \equiv D^{3\text{PN}}(r) \equiv 1 - \frac{6\nu}{r^2} + 2(3\nu - 26)\frac{\nu}{r^3}.$$

$$A^{\text{Taylor}}(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + a_5 \nu u^5 \qquad u = 1/r$$

$$+ \mathcal{O}(\nu u^6),$$

- Extremely compact representation of PN dynamics
- Bad behaviour at 3PN. Padé resummation of A(r) is needed to ensure that an effective horizon exists.
- Impose, by continuity with the Schwarzschild case, that A(r) has a simple zero at r~2.
- The a_5 constant parametrizes (yet) uncalculated 4PN corrections



The EOB Hamiltonian

The effective Hamiltonian (+quartic-in-momenta non-geodesic contribution)

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A\left(1 + \frac{p_\varphi^2}{r^2} + \boxed{z_3 \frac{p_{r_*}^4}{r^2}}\right)}.$$

$$\frac{dr_*}{dr} = \sqrt{\frac{B}{A}}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

The real EOB Hamiltonian of the binary system (from the energy map)

$$\hat{H}^{\rm EOB} = \frac{1}{\nu}\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$$

The Hamiltonian (and the related dynamics) depends, through the "potential" A(u), on the 4PN parameter a_5. ***a_5 is a "free" parameter that needs to be fixed via comparisons with NR simulations***.

$$M = m_1 + m_2$$

$$\nu = m_1 m_2/(m_1 + m_2)^2$$

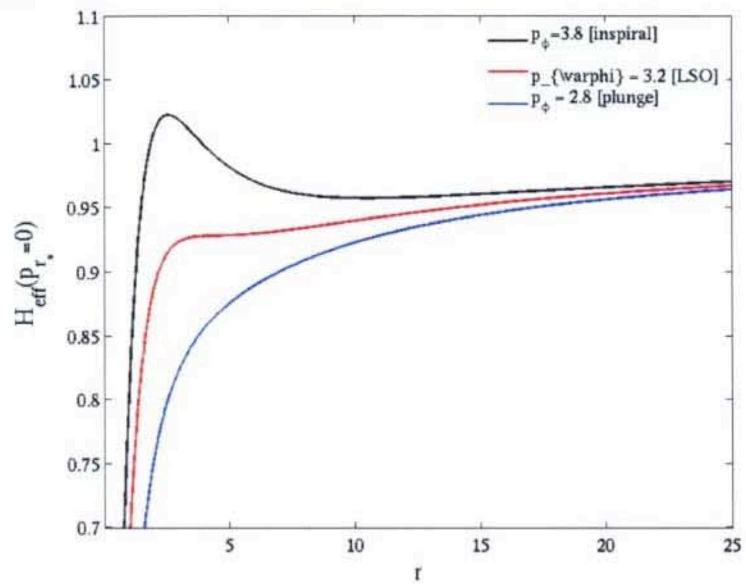
Hamilton's equation + radiation reaction

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$

$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$



Angular momentum loss due to GW emission: start from the PN expression for *radiation reaction* that is explicitly known during the quasi-circular adiabatic inspiral (3.5PN + 4PN correction)

$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\,\nu\,\Omega^5\,r_\omega^4\,\hat{F}^{\rm Taylor}(v_\varphi)$$

$$\begin{aligned}\hat{F}^{\rm Taylor}(v) &= 1 + A_2(\nu)\,v^2 + A_3(\nu)\,v^3 + A_4(\nu)\,v^4 + A_5(\nu)\,v^5 \\ &\quad + A_6(\nu,\log v)\,v^6 + A_7(\nu)\,v^7 + A_8(\nu=0,\log v)\,v^8\end{aligned}$$

Needs resummation of energy flux!

The PN expansions are non-uniformly and non-monotonically convergent in the strong-field regime.
One needs to "resum" them in some form in order to extend their validity during the late-inspiral and plunge

- Factorize a simple pole in the GW energy flux

- Resum using near-diagonal Padé approximants (DIS98)

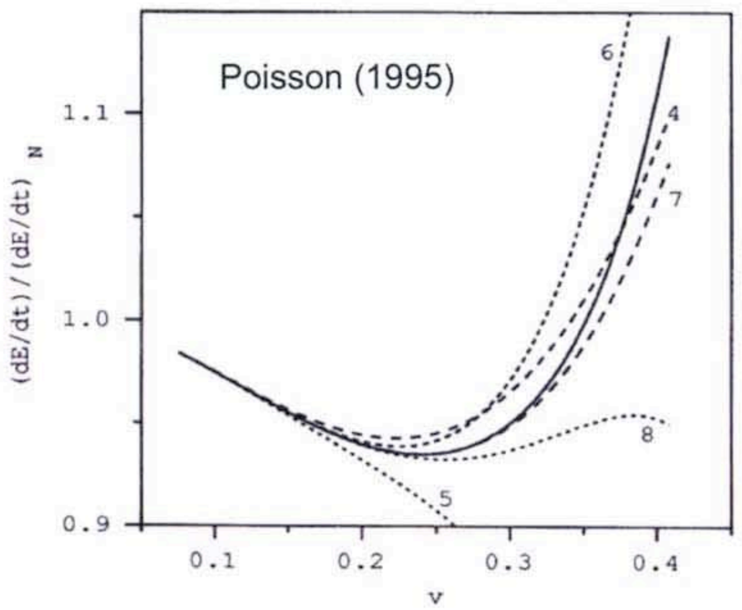


FIG. 1. Various representations of $(dE/dt)/(dE/dt)_N$ as a function of orbital velocity $v = (M/r)^{1/2} = (\pi M f)^{1/3}$. The solid curve represents the exact result $P(v)$, as calculated numerically. The various broken curves represent the post-Newtonian approximations $P_n(v)$, for $n = \{4, 5, 6, 7, 8\}$. The smallest value of v corresponds to an orbital radius r of $175M$; the largest value of v corresponds to $r = 6M$, the innermost stable circular orbit.

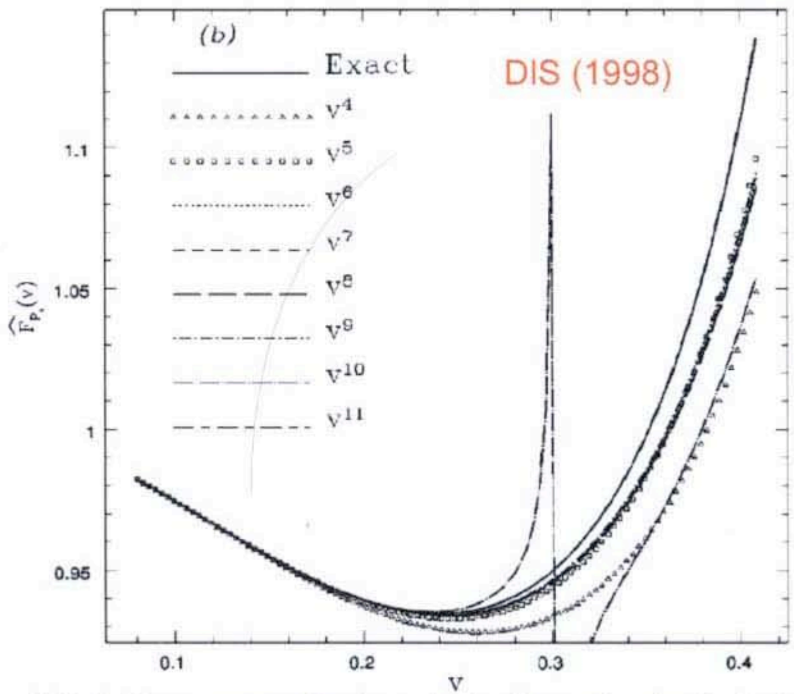


FIG. 3. Newton-normalized gravitational wave luminosity in the test particle limit: (a) T-approximants and (b) P-approximants.

Resumming radiation reaction

Padé resummation of $\hat{\mathcal{F}}_{\varphi}^{\rm Taylor} = -\frac{32}{5}\,\nu\,\Omega^5\,r_\omega^4\,\hat{F}^{\rm Taylor}(v_\varphi)$

Note prefactor
à la DG06

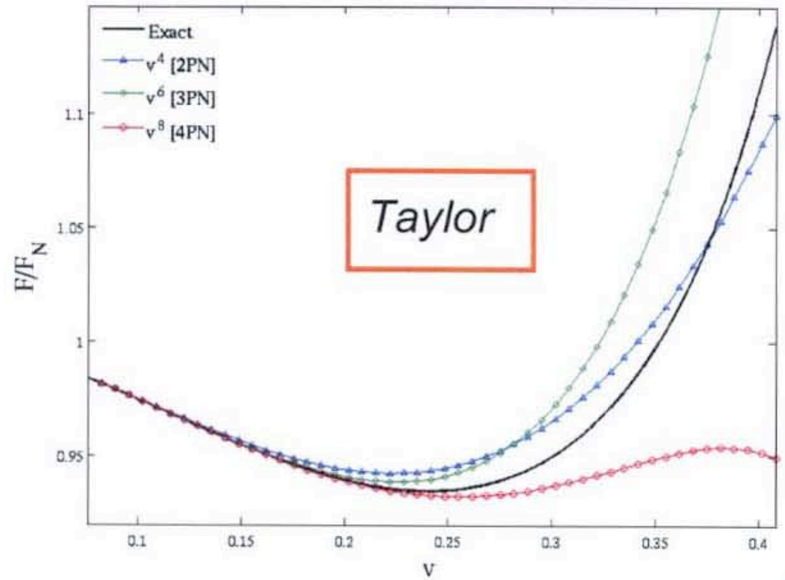
- factorize a pole parametrized by v_{pole}
- consider logarithms as coefficients
- use comparable-mass 3.5PN+test-mass 4PN flux
- choose P^4_4 which has no spurious poles
- add non-quasi-circular correction parametrized by a^{RR} [with ε=0.12]
- choose argument $v=r\Omega\psi^{1/3}$

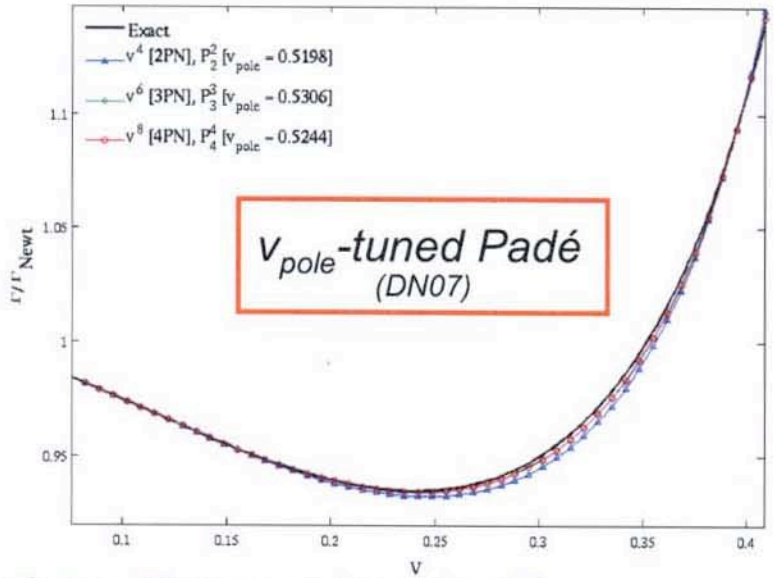
$$\boxed{\hat{F}^{\rm resummed}(v_\varphi) = \left(1-\frac{v_\varphi}{v_{\rm pole}}\right)^{-1} P_4^4\left[\left(1-\frac{v_\varphi}{v_{\rm pole}}\right)\hat{F}^{\rm Taylor}(v_\varphi)\right]\left(1+\bar{a}^{\rm RR}\frac{p_{r_*}^2}{(r\Omega)^2+\epsilon}\right)^{-1}}$$

3.5PN + test-mass 4PN contribution

v_{pole}, a^{RR} are (in addition to a5 in H_{EOB})
"free" parameters that need to be fixed
via comparisons with NR simulations.

Comparing Taylor and (tuned) Padé in test-mass case





- Maximum difference on interval v<0.4:

Taylor(2PN): 0.039	Padé(2PN): 0.0069
Taylor(3PN): 0.130	Padé(3PN): 0.0033
Taylor(4PN): 0.189	Padé(4PN): 0.0035

Henri Padé, 1863-1953

Which one is the most "effective" ?

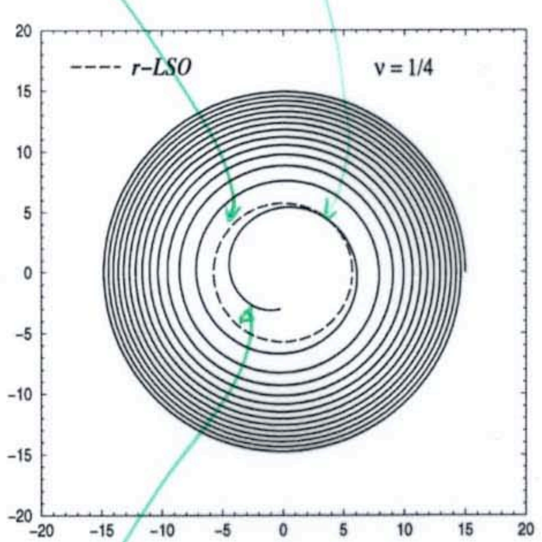
(RESUMMED) EFFECTIVE ONE BODY DYNAMICS Nr 4.247 T8/5-b

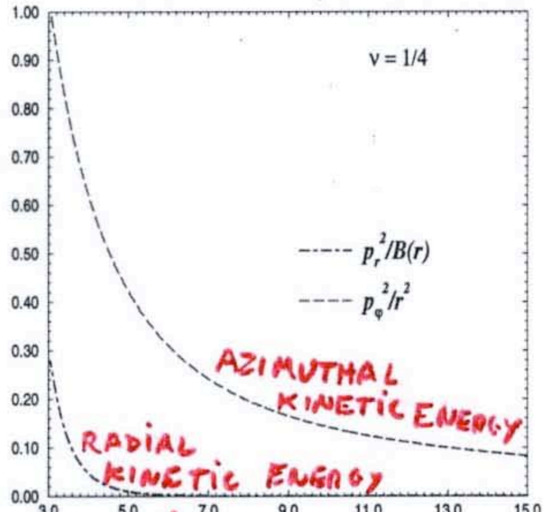
+ RESUMMED RADIATION REACTION (QUASI-CIRCULAR ORBITS)

TRANSITION INSPIRAL → PLUNGE WITH ARBITRARY MASS RATIO

① YIELDS INITIAL DYNAMICAL DATA $(q_1 q_2 P_1 P_2)$ AT BEGINNING OF PLUNGE: 0.6 ORBIT LEFT

GOOD IN VIEW OF STATE OF THE ART NUMERICAL SIMULATIONS (Pretorius 05)





AZIMUTHAL KINETIC ENERGY

RADIAL KINETIC ENERGY

LIGHT-RING

LSO

REMAINS QUASI-CIRCULAR DURING THE WHOLE PLUNGE

② FIRST ESTIMATE OF FULL WAVEFORM: "6M" → "3M" ≃ MERGER

Resummed EOB *metric* gravitational waveform: inspiral+plunge

- Zerilli-Moncrief normalized (even-parity) waveform (*Real part gives h_+ & imaginary part gives h_x*).
- Multipolar decomposition (expansion on spin-weighted spherical harmonics) here, *l=m=2*.

$$\left(\frac{c^2}{GM}\right)\Psi_{22}^{\rm insplunge}(t) = -4\sqrt{\frac{\pi}{30}}\nu(r_\omega\Omega)^2 f_{22}^{\rm NQC} F_{22} e^{-2i\Phi}$$

New *PN-resummed (3^{+2}PN) correction factor* (*DN07a, 07b*): 3PN comparable mass + up to 5PN test-mass

$$F_{22}(t) = \hat{H}_{\rm eff} T_{22} f_{22}(x(t)) e^{i\delta_{22}(t)}$$

- H_{eff}: resums an infinite number of binding energy contributions
- $$T_{\ell m} = \frac{\Gamma(\ell+1-2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi\hat{\hat{k}}} e^{2i\hat{\hat{k}}\log(2kr_0)}$$ *resums* an infinite number of leading logarithms in tail effects (both amplitude and phase) obtained from exact solution of Coulomb wave problem
- $$f_{22}(x;\,\nu) = P_2^3\left[f_{22}^{\rm Taylor}(x;\,\nu)\right]$$ Padé-resummed remaining PN-corrected amplitude [flexibility in choice of argument *x(t)*]
- δ_{22}: computed at 3.5PN

Non-quasi-circular corrections to waveform amplitude and phase:

$$f_{22}^{\rm NQC} = \left[1 + a\frac{p_{r_*}^2}{(r\Omega)^2+\epsilon}\right]\exp\left(+ib\frac{p_{r_*}}{r\Omega}\right)$$

b=0; a is fixed by requiring that the maximum of the modulus of the waveform coincides with the maximum of the orbital frequency

EOB *metric* gravitational waveform: merger and ringdown

EOB approximate representation of the merger (DRT1972 inspired) :

- sudden change of description around the "EOB light-ring" $t=t_m$ (maximum of orbital frequency)
- "match" the insplunge waveform to a superposition of QNMs of the final Kerr black hole
- matching on a 5-teeth comb (*found efficient in the test-mass limit, DN07a*)
- comb of width 7M centered on the "EOB light-ring"
- use 5 positive frequency QNMs (found to be near-optimal in the test-mass limit)
- Final BH mass and angular momentum are computed from a fit to NR ringdown (*5 eqs for 5 unknowns*)

$$\Psi_{22}^{\rm ringdown}(t) = \sum_N C_N^{+} e^{-\sigma_N^{+} t}$$

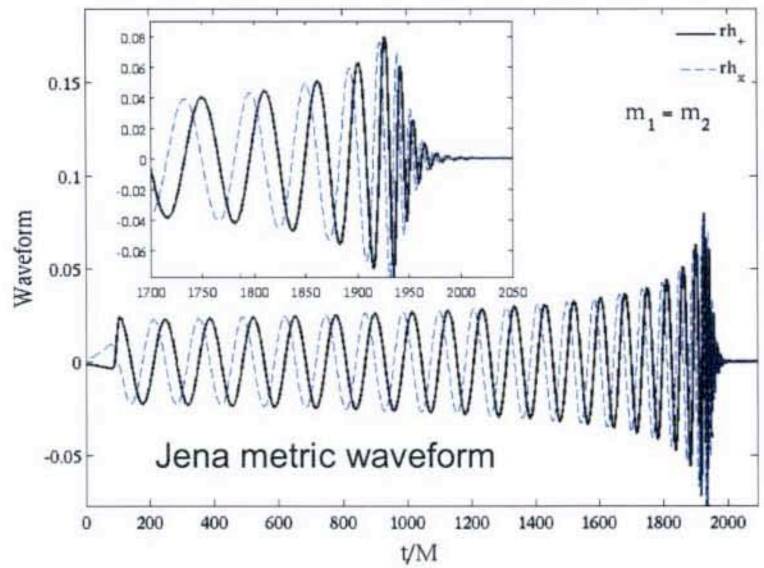
Total EOB waveform covering inspiral-merger and ringdown

$$h_{22}^{\rm EOB}(t) = \theta(t_m - t)\, h_{22}^{\rm insplunge}(t) + \theta(t - t_m)\, h_{22}^{\rm ringdown}(t)$$

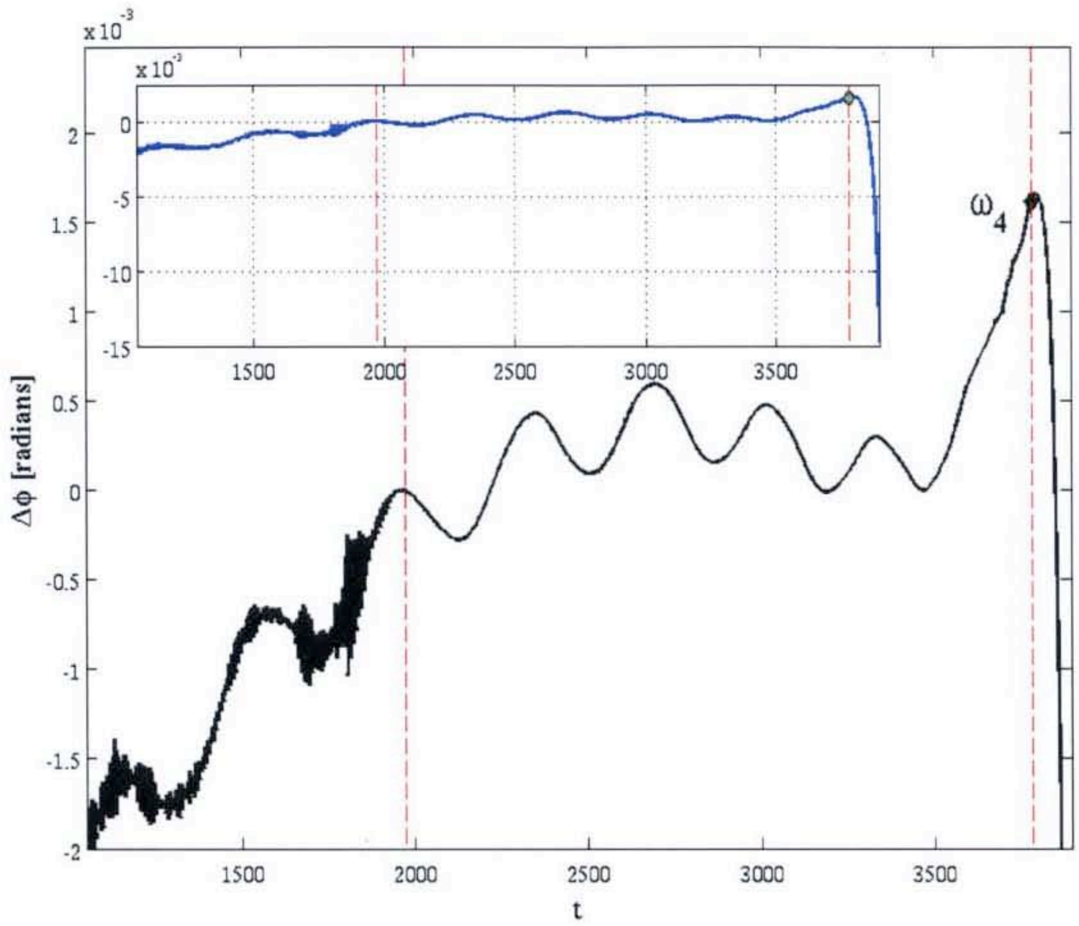
Accurate EOB-NR comparisons (and calibration)

NR, reduced eccentricity data used (non-spinning black holes only):

- very accurate *inspiral only* data ($m_1=m_2$), 30 GW-cycles *$r\psi_4$ curvature waveform* Caltech-Cornell [Boyle et al. 07] used up to GW frequency 0.1
- Albert-Einstein-Institute, 12 GW-cycles *metric (Zerilli) waveform*, inspiral+merger data ($m_1=m_2$) [DNDPR,08]
- Jena, about 20 GW-cycles *$r\psi_4$ curvature waveform*, inspiral+merger data ($m_1=m_2$; $m_1=2m_2$; $m_1=4m_2$) [DNHHB,08]
- Getting the *metric waveform* by twice integrating the curvature waveform and subtracting linear floors.

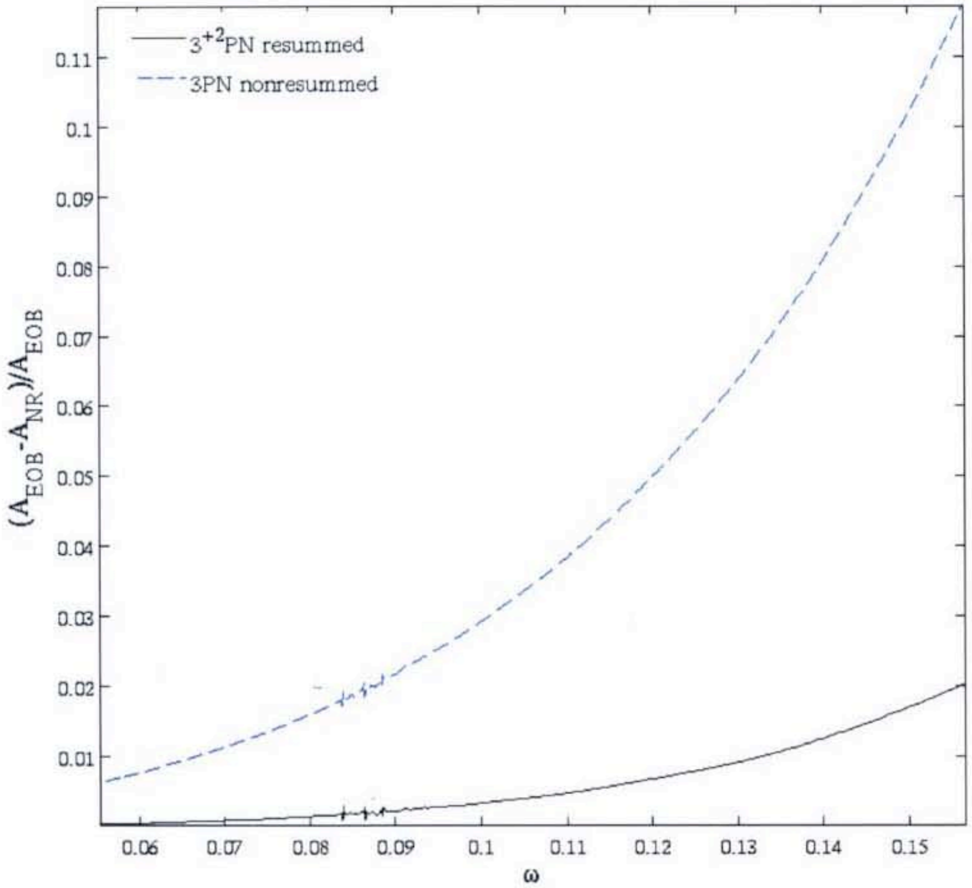


Curvature-waveform phase difference EOB-CC (actual data) for a_5=25



- Close up (black): maximum phase difference 0.002 radians up to GW frequency 0.1 (OK with DN07b, which used published data)
- Full range (blue, inset): maximum phase difference 0.015 radians accumulated between frequency 0.1 and 0.156
- Two "pinching-times" indicated

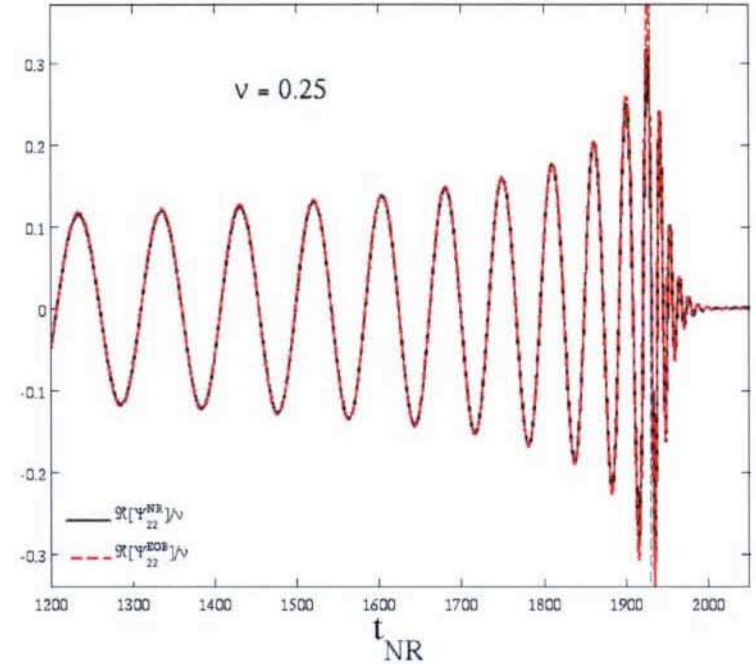
(Fractional) curvature amplitude difference EOB-CC (actual data) for a_5=25

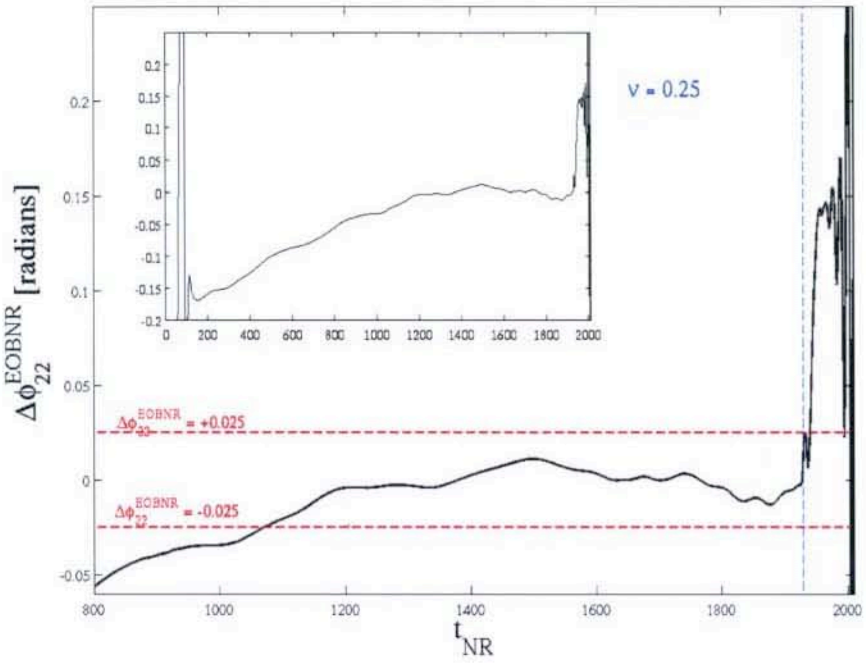


- Nonresummed: fractional differences start at the 1% level and build up to more than 10%
- New resummed EOB amplitude: fractional differences start at the 0.04% level and build up to only 2%
- *Resummation: factor ~20 improvement!*

Which one is the most "effective"?

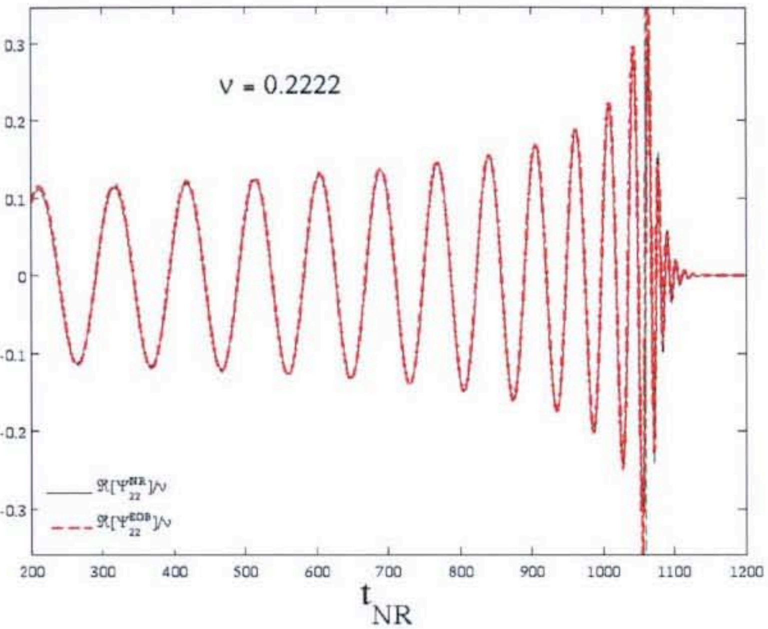
Comparing EOB-NR *metric* waveforms 1:1 case: Jena data

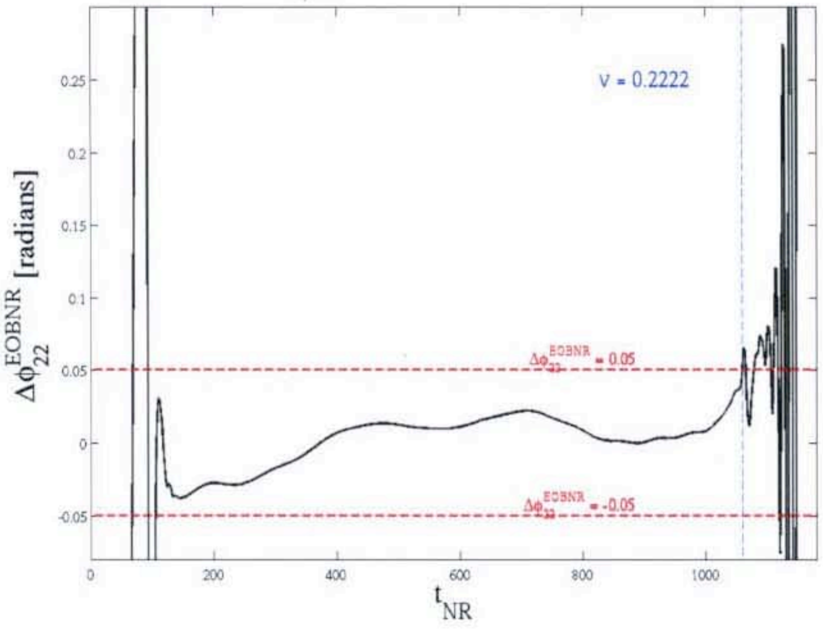




- Metric waveforms from double time-integration of NR curvature waveforms
- $-0.025<\Delta\phi_{22}<+0.025$ radians (=0.004 GW cycles) over 730 M [1200M-1930M]
- At merger, phase jump of only 0.15 radians [=0.02 GW cycles].
- We use the same values of flexibility parameters for CC and Jena data: consistency achieved!

Comparing EOB-NR *metric* waveforms 2:1 case: Jena data





- Metric waveforms from double time-integration of NR curvature waveforms
- $-0.05<\Delta\phi_{22}<+0.05$ radians (=0.008 GW cycles) over 957M [143M-1100M]
- At merger, phase jump of only 0.06 radians [= 0.009 GW cycles].
- We use the same values of flexibility parameters for CC and Jena data: consistency achieved!

PARMA 5

5

CHAOS AND SYMMETRY

IN

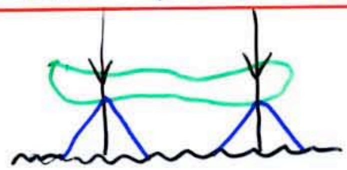
'STRING COSMOLOGY'

COSMOLOGICAL SINGULARITIES

- NOT AS A MODEL OF THE EARLY UNIVERSE

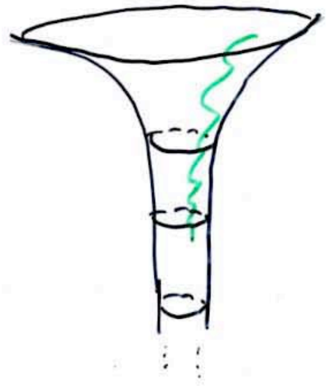
- BUT AS A TOOL FOR PROBING THE STRUCTURE OF M-THY, AND, IN PARTICULAR, FOR SEARCHING FOR HIDDEN SYMMETRIES

NEAR SPACELIKE SINGULARITY LIMIT



? $\updownarrow$

NEAR HORIZON LIMIT



$\downarrow$

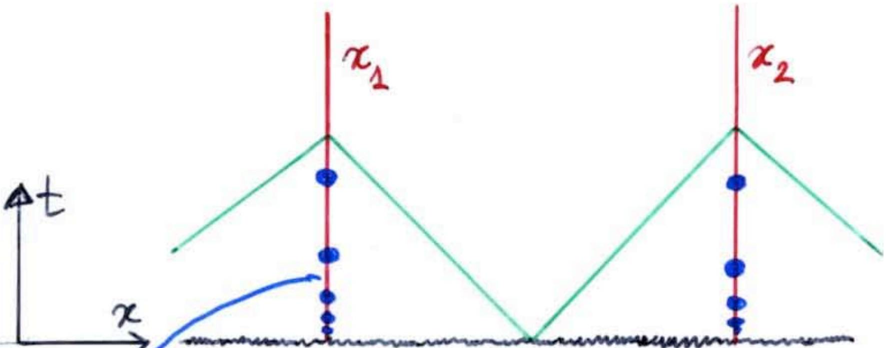
HOLOGRAPHIC CORRESPONDENCE

STRINGS ON $AdS_5 \times S_5 \longleftrightarrow CFT_4$

CHAOS IN (SUPER) GRAVITY

- Belinskii, Khalatnikov, Lifshitz (BKL) 1969

infinitely 'oscillatory' behaviour of generic inhomogeneous solution of $R_{\mu\nu}=0$ near $t=0$ in $D=4$



BKL expansion: $\partial_x \ll \partial_t$

$$ds^2 \simeq -dt^2 + \sum_a A_a^2(t)\left(e^a_i(x)\,dx^i\right)^2$$

SUCCESSIVE KASNER EPOCHS: $A_a(t) \sim t^{p_a}$

- CHAOTIC BEHAVIOUR:
  - OF SUCCESSIVE p_a's : Lifshitz, Lifshitz, Khalatnikov '71; Khalat, Lif., Khanin, Schur, Sinai '85
  - OF ASSOCIATED HAMILTONIAN BILLIARD: Misner, Chitre, Kirillov,...

- Demaret, Henneaux, Spindel 1985

  BKL chaos disappears in $D \geqslant 11$! monotonic Kasner-like

  T.D., Henneaux, Rendall, Weaver '02

- T.D., Henneaux 2000

  BOSONIC SECTORS OF $D=11$ SUPERGRAVITY, AS WELL AS $D=10$ STRING THEORIES (I, IIA, IIB, HO, HE) ARE ALL BKL CHAOTIC

HAMILTONIAN APPROACH TO BKL BEHAVIOUR

Misner, Chitre, Kirillov, Kirillov-Melnikov, TD Henneaux Nicolai

$$\text{ADM: } S_{ADM} = \int d^d x \Big[\underset{\substack{\uparrow \\ \text{CONJUGATE MOMENTUM}}}{\pi^{ij}} \underset{\substack{\uparrow \\ \text{SPATIAL METRIC}}}{\dot{g}_{ij}} (+\pi^{ijk}\dot{A}_{ijk} + \ldots) - \tilde{N} \underset{\substack{\uparrow \\ \text{HAMILTONIAN CONSTRAINT}}}{\mathcal{H}} - \cdots \Big]$$

LOCAL + IWASAWA DECOMPOSITION

$$\boxed{g_{ij}(t,\vec{x}) = \sum_a e^{-2\beta^a(t,\vec{x})} \mathcal{N}^a_i(t,\vec{x}) \mathcal{N}^a_j(t,\vec{x})}$$

LOGARITHMS OF "SCALE FACTORS"

$$A_a(t,\vec{x}) \equiv e^{-\beta^a}$$

UPPER TRIANGULAR "OFF DIAGONAL" METRIC COMPONENTS

$$\mathcal{N}^a_i = \begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix}$$

NEW CANONICAL VARIABLES: $\left(\beta^a(t\vec{x}), \pi_a(t\vec{x}) \right) ; \left(\mathcal{N}^i_a(t\vec{x}), \mathcal{P}^a_i(t\vec{x}) \right) \ldots$

SIMPLIFICATION OF DYNAMICS AS $t \to 0$

ALL "OFF-DIAGONAL" VARIABLES FREEZE: $\mathcal{N}^i_a, \mathcal{P}^a_i, A_{abc}, \pi^{abc}, \ldots$

ONLY "DIAGONAL" ONES HAVE A COMPLICATED DYNAMICS : $(\beta^a, \pi_a) + (\varphi, \pi_\varphi)$ if $\exists$

WHICH IS APPROXIMATELY GOVERNED BY ODE's $\boxed{\text{: "TODA-LIKE" SYSTEM}}$

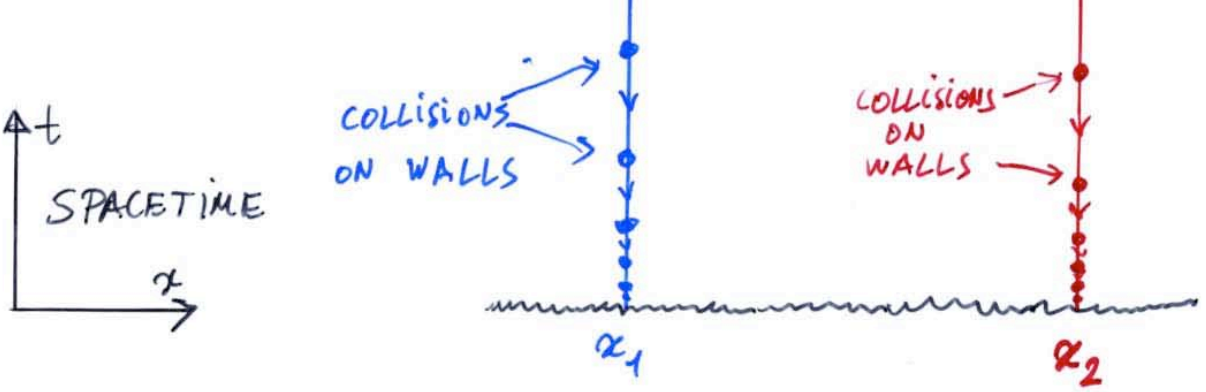
$$\boxed{\mathcal{H}_{RED} \simeq G_{ab}\pi^a\pi^b + \sum_A c_A e^{-2w_A(\beta)}}$$

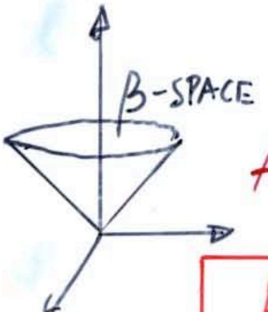
EXPONENTIAL WALLS

$c_A(\mathcal{N}, \mathcal{P}, A_{abc}, \pi^{abc} \ldots) \to$ LIMIT

COSMOLOGICAL BILLIARDS

Chitre, Misner, Kirillov, Kirillov-Melnikov, TD Henneaux Nicolai

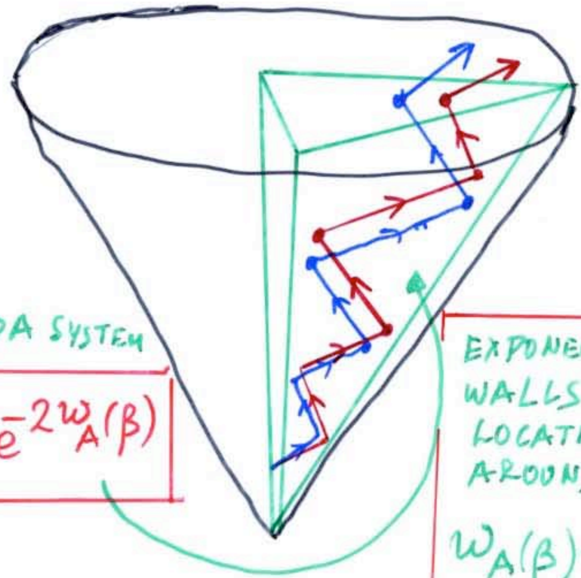




$A_a \equiv e^{-\beta^a}$

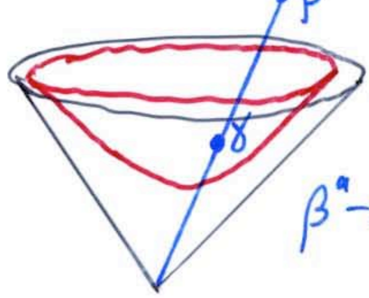
ASYMPTOTIC TODA SYSTEM

$$\boxed{\mathcal{H}_{RED} \simeq G_{ab}\pi^a\pi^b + \sum_A c_A e^{-2w_A(\beta)}}$$



EXPONENTIAL WALLS LOCATED AROUND

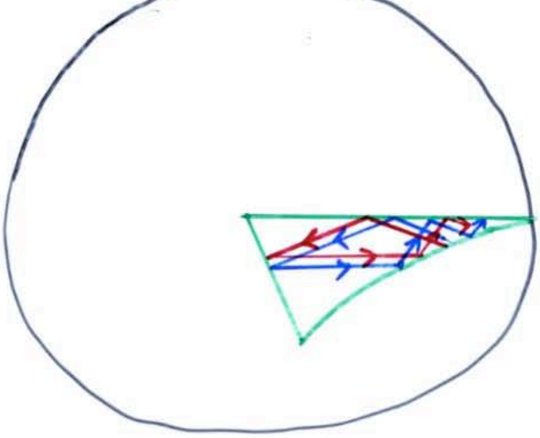
$w_A(\beta) = 0$



HYPERBOLIC SPACE

$$\beta^a \to \gamma^a = \frac{\beta^a}{\sqrt{-G_{bc}\beta^b\beta^c}}$$

$$G_{bc}\gamma^b\gamma^c = -1$$



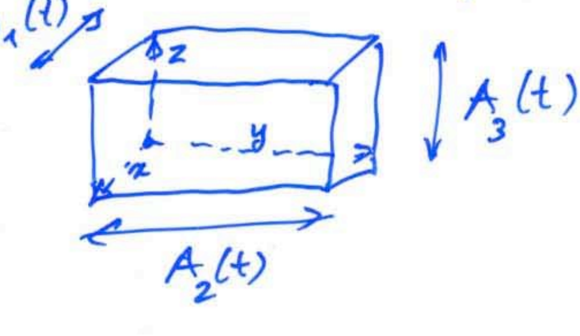
ESI

ANISOTROPIC SPATIAL GEOMETRY

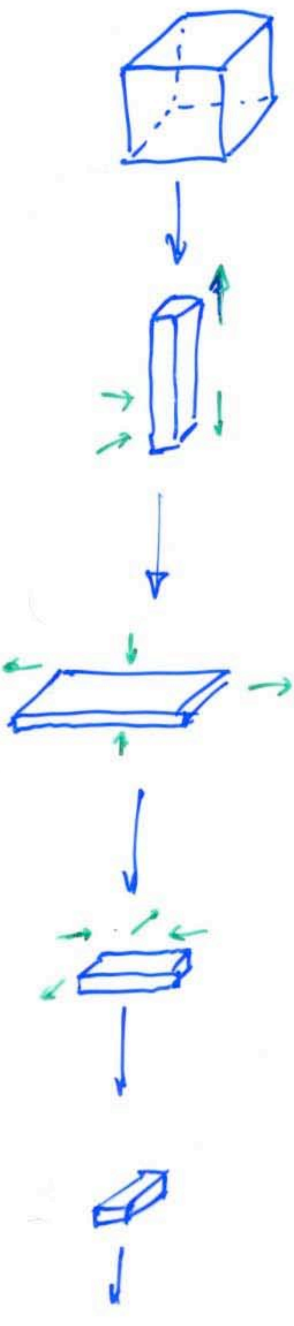
e.g. $ds^2 = A_1^2(t,x)\,\omega_1^2 + A_2^2(t)\,\omega_2^2 + A_3^2(t)\,\omega_3^2$

(LIMITING) IWASAWA FRAME

"BOX"



AS $t \to 0$



SYMMETRY IN GRAVITY

- Ehlers 1959 $GR_{D=4}$ WITH 1 KILLING VECTOR $k = \frac{\partial}{\partial x^3}$

$\Rightarrow$ CONTINUOUS SYMMETRY

$SL(2,\mathbb{R})_E$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R})$

$$\boxed{Z' = \frac{aZ+b}{cZ+d}}$$

$ad-bc=1$

$a,b,c,d \in \mathbb{R}$

$$Z = B + i\Delta \qquad \Delta \equiv g_{33} = k^2$$

NON-LOCAL DUAL TO $A_\mu = g_{3\mu}/g_{33}$

$$\varepsilon_{\mu\nu\lambda} \nabla^\lambda B = \Delta^2 (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

- Matzner, Misner 1967 GR_4 WITH 2 COMMUTING KILLING V. k_1, k_2

$\Rightarrow$ CONTINUOUS SYMMETRY

$SL(2,\mathbb{R})_{MM}$

LINKED TO FREEDOM IN $k'_a = \Lambda_a^b k_b$

LOCAL IN GRAVITY VARIABLES

- Geroch 1972 GR_4 WITH 2 COMMUTING K.V. $\oplus$ $\epsilon_{\mu\nu\rho\sigma} \overset{1}{k}{}^\mu \overset{2}{k}{}^\nu \nabla^\rho \overset{a}{k}{}^\sigma = 0$

INTERPLAY OF $SL(2,\mathbb{R})_E \times SL(2,\mathbb{R})_{MM}$: INFINITE DIMENSIONAL LIE GROUP

- Julia 1981 ; Breitenlohner-Maison '84, '87 ; Belinskii-Zakharov ...

GEROCH GROUP $= \widehat{SL(2,\mathbb{R})} = A_1^{(1)}$ 

AFFINE KAC-MOODY EXTENSION OF $SL(2,\mathbb{R})$

SYMMETRY IN SUPERGRAVITY

- Cremmer, Julia 1979 SUGRA $_{D=11}$ WITH

- 7 COMMUTING KILLING VECTORS : SYMMETRY E_7
  $D=11 \rightarrow D=4$

- 8 COMM. KILLING V. E_8
  $D=11 \rightarrow D=3$ Cremmer-Julia Marcus-Schwarz '83

- 9 COMM. KILLING V. $E_9 = E_8^{(1)}$
  $D=11 \rightarrow D=2$ Nicolai '87

- ? 10 KV $\overset{?}{\Longrightarrow}$? E_{10}
  ? $D=11 \rightarrow D=1$
  Julia
  Nicolai
  Mizoguchi
  Ganor

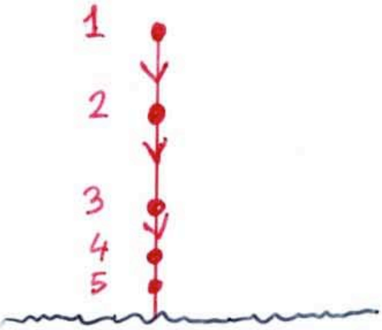
- ?? $D=11 \rightarrow D=0$?? $\overset{??}{\Longrightarrow}$ E_{11}
  Nicolai
  West
  Kleinschmidt. Schnakenburg West
  Englert Houart

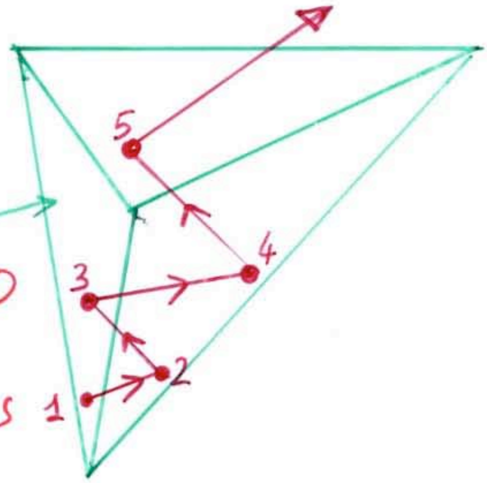
HIDDEN SYMMETRY IN BKL CHAOS

- T.D., Henneaux 2001

∃ HIDDEN SYMMETRY STRUCTURE IN THE COSMOLOGICAL CHAOS OF $SUGRA_{11}$, AS WELL AS THE VARIOUS $D=10$ STRING THEORIES

EFFECT OF 'BKL COLLISION' $\longleftrightarrow$ REFLECTION IN HYPERPLANE ORTHOGONAL TO 'SIMPLE ROOT' OF A HYPERBOLIC KAC-MOODY (LIE) ALGEBRA





WALLS

EXPONENTIAL WALLS $e^{-2 w_A(\beta)}$ $w_A(\beta)=0$

$\updownarrow$

SIMPLE ROOTS $\alpha_a(\beta)$

BKL-LIKE CHAOTIC OSCILLATIONS $\longleftrightarrow$ CHAOTIC BILLIARD MOTION WITHIN WEYL CHAMBER OF KAC-MOODY ALG.

$D=11$ SUGRA $\longleftrightarrow$ Weyl group of E_{10}

$D=n+1$ GR $\longleftrightarrow$ Weyl group of AE_n

T.D. Henneaux, Julia, Nicolai '01

USUAL $D=4$ GR $\longleftrightarrow$ KAC-MOODY $AE_3 = \widehat{\widehat{SL(2,\mathbb{R})}} = \widehat{GEROCH}$

HYPERBOLIC

FINITE-DIMENSIONAL (SIMPLE) LIE ALGEBRAS

$SU(2) \sim SO(3)$

$$[J_x , J_y] = i\, J_z$$
$$[J_y , J_z] = i\, J_x$$
$$[J_z , J_x] = i\, J_y$$

OR, BETTER, $\boxed{J_\pm \equiv J_x \pm i J_y}$

'RAISING GENERATOR'

$$[J_z , J_+] = + J_+$$
$$[J_z , J_-] = - J_-$$

'LOWERING GENERATOR'

TO BE DIAGONALIZED: 'CARTAN GENERATOR'

$SU(3)$ $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$\lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8 = \begin{pmatrix} 1/\sqrt{3} & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & -2/\sqrt{3} \end{pmatrix}$

- CAN DIAGONALIZE TWO GENERATORS $h_1, h_2 \sim \lambda_3, \lambda_8$
- AND THEN FIND ANALOGS OF 'RAISING' AND 'LOWERING' GENERATORS

$$\begin{cases} [h_1 , E_1^+] = + E_1^+ \\ [h_2 , E_1^+] = 0 . E_1^+ \end{cases} \begin{cases} [h_1 , E_2^+] = -\frac{1}{2} E_2^+ \\ [h_2 , E_2^+] = \frac{\sqrt{3}}{2} E_2^+ \end{cases} \begin{cases} [h_1 , E_3^+] = +\frac{1}{2} E_3^+ \\ [h_2 , E_3^+] = \frac{\sqrt{3}}{2} E_3^+ \end{cases}$$

GENERAL (SIMPLE) LIE ALGEBRA

$\exists$ CARTAN SUBALGEBRA : LINEAR SPACE $\mathbb{R}^r$ ← RANK

$[h', h''] = 0 \qquad \mathfrak{h} = \{ \beta^a h_a \, ; \; a = 1, 2, \cdots, r \}$

COORDINATES IN CARTAN SPACE $\qquad h = \sum_{a=1}^{r} \beta^a h_a \qquad$ r independent Cartan gators

$\exists$ TRIANGULAR DECOMPOSITION $\mathfrak{g} = \mathfrak{n}^- \oplus \mathfrak{h} \oplus \mathfrak{n}^+$

LOWERING GATORS F_α $\qquad$ CARTAN $\qquad$ RAISING GATORS E_α

degeneracy index

$$\boxed{[h, E_\alpha^{(s)}] = \alpha(h) \, E_\alpha^{(s)}}$$

Cartan $\qquad$ Raising $\qquad$ ROOT α

$\equiv$ EIGENVALUE OF ad_h

AS A LINEAR FORM OF $h \in \mathfrak{h}$

$$h = \beta^a h_a \Rightarrow \alpha(h) = \alpha_a \beta^a = \alpha(\beta)$$

$$[h, F_\alpha^{(s)}] = -\alpha(h) \, F_\alpha^{(s)}$$

$$[E_\alpha^{(s)}, E_\beta^{(t)}] = c_{\alpha\beta}^{(stu)} \, E_{\alpha+\beta}^{(u)}$$

'SIMPLE ROOTS' $\alpha_1, \dots, \alpha_r$

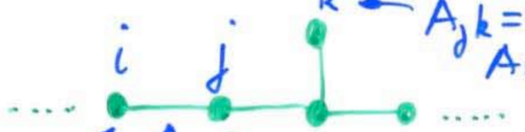
ANY ROOT $\alpha = \sum_{j=1}^{r} n_j \alpha_j$ ← INTEGER (n_j)

IE ANY RAISING $E_\alpha = [E_{\alpha_1}[E_{\alpha_1}[\dots[E_{\alpha_2}[\dots[E_{\alpha_3} \dots]]]]]]$

CARTAN MATRIX

$\alpha_j(h_i) \equiv A_{ij} \in \mathbb{Z}$

DYNKIN DIAGRAM



$A_{jk} = 0$, $A_{ik} = 0$

$A_{ii} = 2$, $A_{jj} = 2$

IF $A_{ij} = -1$

QUADRATIC FORMS

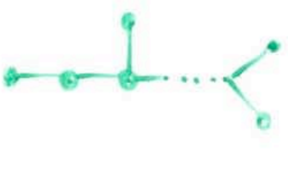
CARTAN $A_{ij} \to \langle h, h \rangle \to \langle x, x \rangle$

KAC-MOODY ALGEBRAS

GIVEN CARTAN MATRIX A_{ij}

$i, j = 1, 2, \dots, r$

↕ ENCODED IN



$$[h_i, h_j] = 0$$
$$[h_i, e_j] = A_{ij} e_j$$
$$[h_i, f_j] = -A_{ij} f_j$$
$$[e_i, f_j] = \delta_{ij} h_j$$
$$\mathrm{ad}_{e_i}^{1-A_{ij}} e_j = 0$$
$$\mathrm{ad}_{f_i}^{1-A_{ij}} f_j = 0$$

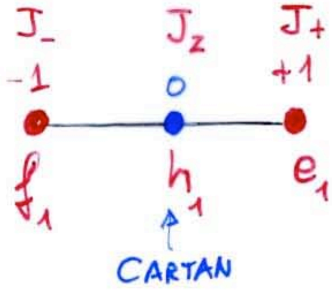
+ JACOBI IDENTITIES

SU(2)

RANK = 1

1 DIM

ROOT DIAGRAM



CARTAN MATRIX

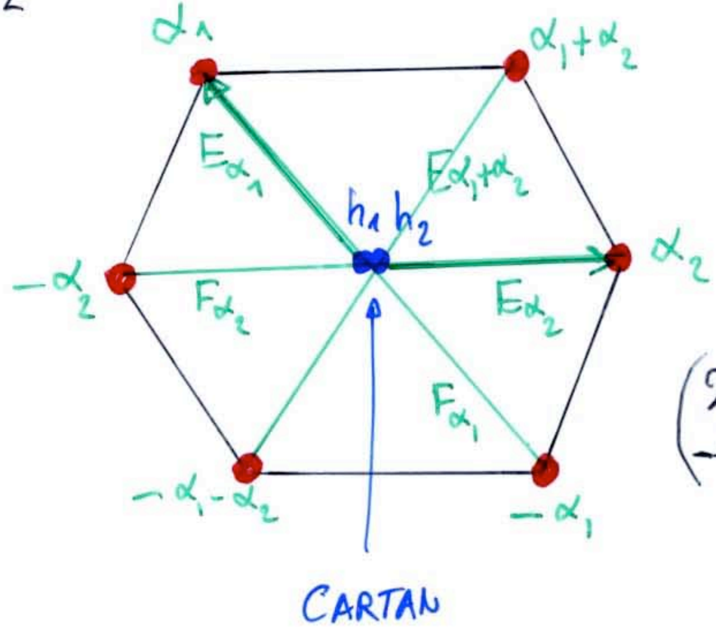
$$\begin{pmatrix} 2 \end{pmatrix}$$

DYNKIN DIAGRAM

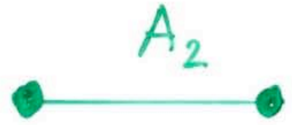
SU(3)

RANK = 2

2 DIM



$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$



$i,j = 1 \dots r$

$$[h_i, h_j] = 0$$
$$[h_i, e_j] = A_{ij}\, e_j$$
$$[h_i, f_j] = -A_{ij}\, f_j$$

+ Jacobi identities

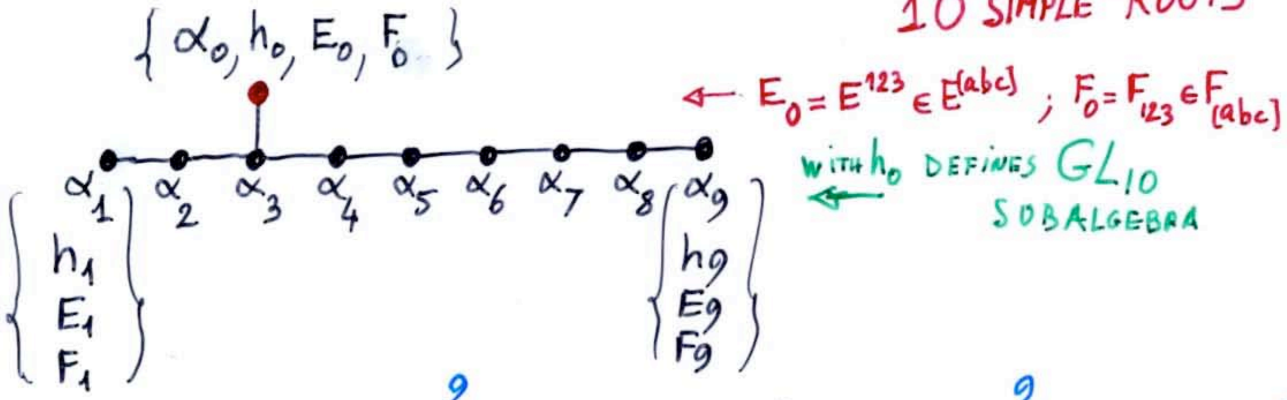
$$[e_i, f_j] = \delta_{ij} h_j$$

$$\left.\begin{aligned} \mathrm{ad}_{e_i}^{1-A_{ij}}\, e_j &= 0 \\ \mathrm{ad}_{f_i}^{1-A_{ij}}\, f_j &= 0 \end{aligned}\right\} \text{Serre relations}$$

E_{10}

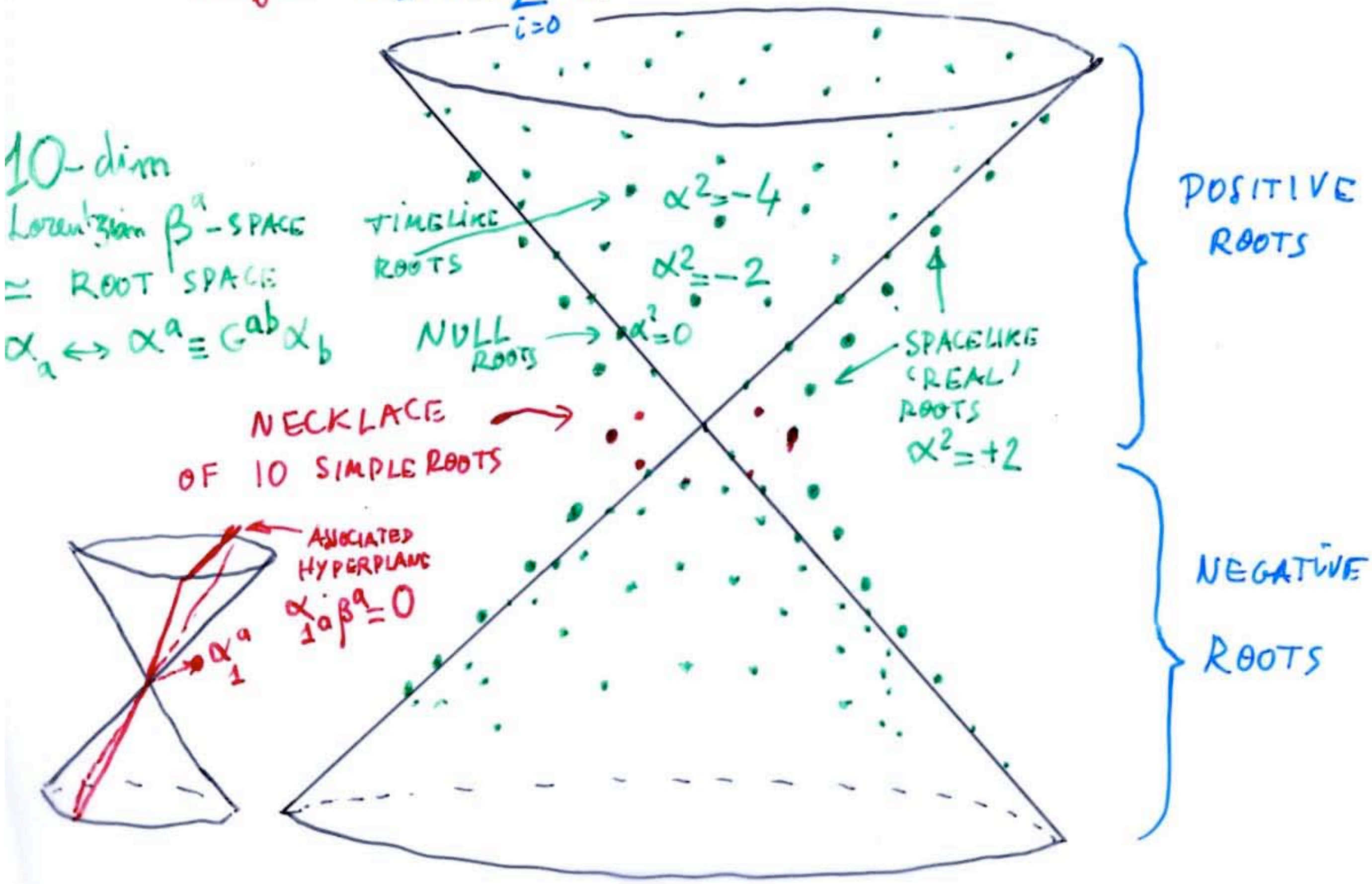
rank 10 : $\dim \mathfrak{h} = 10$ AND $\exists$ 10 basic raising gators E_{α_i}

10 SIMPLE ROOTS



$$\{\text{ALL ROOTS}\} = \underbrace{\left\{\alpha = \sum_{i=0}^{9} n^i \alpha_i \, ; n^i \in \mathbb{N}\right\}}_{\text{POSITIVE ROOTS}} \cup \underbrace{\left\{\alpha = -\sum_{i=0}^{9} n^i \alpha_i \, ; n^i \in \mathbb{N}\right\}}_{\text{NEGATIVE ROOTS}}$$

height $ht[\alpha] = \sum_{i=0}^{9} n_i$



KAC-MOODY ALGEBRAS

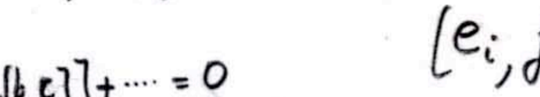
Lie algebra $\mathfrak{g} \equiv \mathfrak{g}(A)$

$r \times r$ integer-valued Cartan matrix
$$\begin{cases} a_{ii} = +2 \\ -a_{ij} \in \mathbb{N}\,;\ i \neq j \\ a_{ij} = 0 \Rightarrow a_{ji} = 0 \end{cases}$$

rank

eg
$$A_{E_{10}} = \begin{bmatrix} 2 & -1 & & & & & & & & \\ -1 & 2 & -1 & & & & & & & \\ & -1 & 2 & -1 & & & & & & \\ & & -1 & 2 & -1 & & & & & \\ & & & -1 & 2 & -1 & & & & \\ & & & & -1 & 2 & -1 & & & \\ & & & & & -1 & 2 & -1 & & -1 \\ & & & & & & -1 & 2 & -1 & \\ & & & & & & & -1 & 2 & \\ & & & & & & -1 & & & 2 \end{bmatrix}$$

DYNKIN DIAGRAM :



Chevalley-Serre generators : $\{h_i, e_i, f_i\}_{i=1\ldots r}$ $\sim (J_z, J_+, J_-)$ for $SU(2) \sim A_1$

$[h_i, h_j] = 0$ ⌐ Cartan subalgebra

$[e_i, f_j] = \delta_{ij}\, h_j$

$[h_i, e_j] = A_{ij}\, e_j$

$[h_i, f_j] = -A_{ij}\, f_j$

+ Jacobi $[a,[b,c]] + \cdots = 0$

+ Serre relations

$$\mathrm{ad}_{e_i}^{\,1-A_{ij}}\, e_j = 0 = \mathrm{ad}_{f_i}^{\,1-A_{ij}}\, f_j$$

FIRST FACT

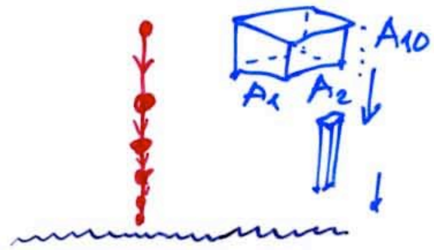
BOSONIC SUGRA$_{11}$

$D=11$ $\qquad$ $F_4 = dA_3$

$$S=\int d^Dx\,\sqrt{-G}\left[R(G)-\frac{1}{2\cdot 4!}F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho}\right]+\frac{1}{(12)^4}\int A_3\wedge F_4\wedge F_4$$

$\Downarrow$

CHAOTIC BEHAVIOUR
NEAR A COSMOLOGICAL SING.



$\Downarrow$

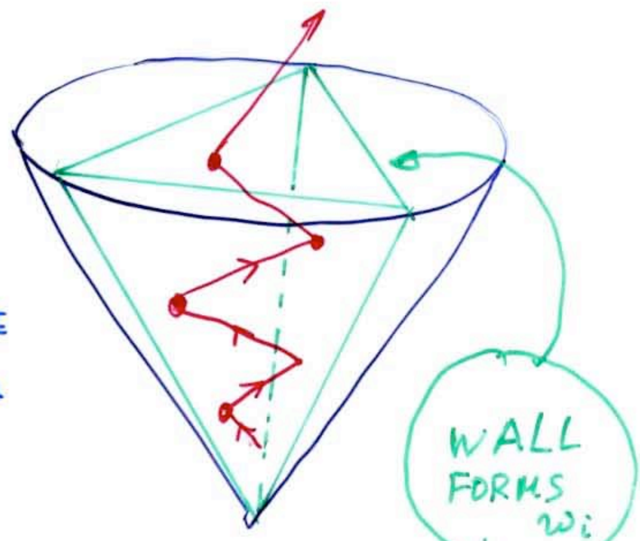
BILLIARD DESCRIPTION
OF THIS CHAOS

$\Downarrow$

TODA-LIKE SYSTEM $\quad$ IN β-SPACE

$A_a \equiv e^{-\beta^a}$

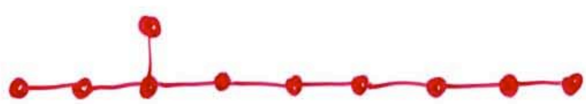
$$\mathcal{H} = G_{ab}\pi^a\pi^b + \sum_A C_A\, e^{-2w_A(\beta)}$$



GEOMETRY OF 'COSMOLOGICAL BILLIARD'

RESCALED GRAM MATRIX $\quad 2\,\dfrac{w_i\cdot w_j}{w_i\cdot w_i}$

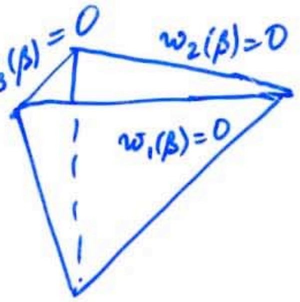
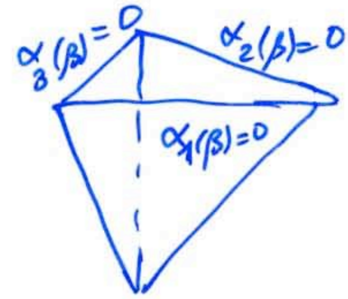
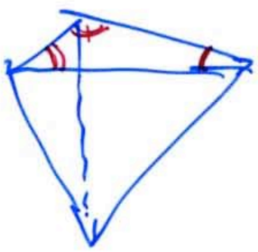
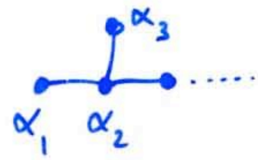
$\Longleftrightarrow$

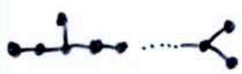


$\equiv$ E_{10} DYNKIN DIAGRAM

COSMOLOGICAL SINGULARITIES AND KAC-MOODY ALGEBRAS

DICTIONARY

GRAVITY		KAC-MOODY	
LOGARITHMIC SCALE FACTORS	$\beta^a = -\ln A_a$	CARTAN ELEMENT	$h = \sum_{a=1}^{r} \beta^a h_a$
DOMINANT COSMOLOGICAL WALLS	$e^{-2w_i(\beta)}$	SIMPLE ROOTS	$\alpha_i(\beta)$
COSMOLOGICAL BILLIARD		WEYL CHAMBER	
GEOMETRY OF COSMO. BILLIARD		DYNKIN DIAGRAM	
RESCALED GRAM MATRIX	$2\,\dfrac{w_i \cdot w_j}{w_i \cdot w_i}$	CARTAN MATRIX	$A_{ij} = \dfrac{2\alpha_i \cdot \alpha_j}{\alpha_i \cdot \alpha_i} = \begin{pmatrix} 2 & -1 & 0 & \\ -1 & 2 & \cdots & -1 \\ \vdots & & 2 & \\ & -2 & & \ddots \end{pmatrix}$

GRAVITY		KAC-MOODY
PURE GRAVITY IN $D = d+1$		$AE_d = A^{\wedge\wedge}_{d-2}$
SUGRA$_{11}$; IIA_{10} , IIB_{10}		E_{10}
I_{10} ; HET_{10}		BE_{10}
BOSONIC STRING$_D$		DE_D

HIDDEN SYMMETRY GROUP ?

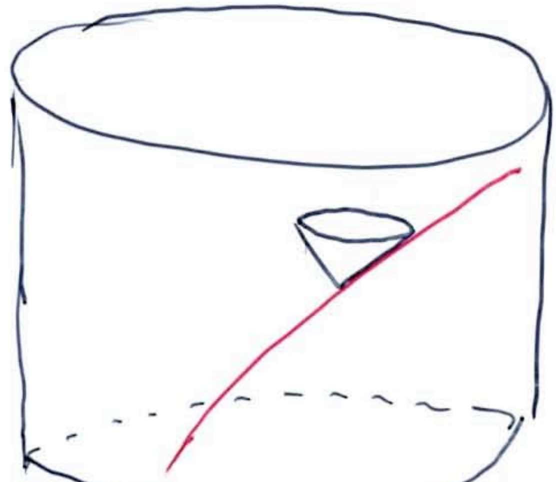
SOME EVIDENCE FOR A HOLOGRAPHIC CORRESPONDENCE

$$\mathrm{SUGRA}_{11} \longleftrightarrow$$ MASSLESS SPINNING PARTICLE ON COSET $E_{10}/K(E_{10})$

$G_{\mu\nu}(t,\vec{x})$

$A_{\mu\nu\lambda}(t,\vec{x})$ $\longleftrightarrow$

$\psi_\mu(t,\vec{x})$



$$S_{11} = \int d^{11}x \Big\{ \frac{E}{4} R(G) - \frac{E}{48}(dA_3)^2 + \frac{2}{(12)^4} F_4 \wedge F_4 \wedge A_3 - \frac{i}{2} \bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{i}{96} \left(\bar{\psi}_\mu \Gamma^{\mu\alpha\beta\gamma\delta\nu} \psi_\nu + 12 \bar{\psi}^\alpha \Gamma^{\gamma\delta} \psi^\beta \right) F_{\alpha\beta\gamma\delta} + \dots \Big\}$$

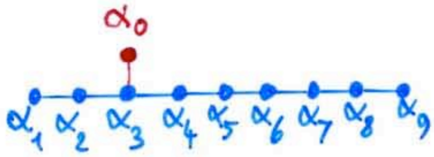
$\longleftrightarrow$

$$S_1^{\mathrm{COSET}} = \int dt \Big\{ \frac{1}{4n} \langle P(t) | P(t) \rangle - i \left(\Psi(t) | \mathcal{P}^{vs} \Psi(t) \right)_{vs} + \left(\chi(t) | P(t) \odot \Psi(t) \right)_s \Big\}$$

+ LOOP CORRECTIONS

T.D., Henneaux, Nicolai '02; T.D., Kleinschmidt, Nicolai '06; de Buyl, Henneaux, Paulot '06

DECOMPOSING E_{10} W.R.T. $GL(10)$ SUBALGEBRA



← $SL(10) = A_9$ SUBALGEBRA

"LEVEL" ℓ : $\quad \alpha = \ell\,\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j$

$\ell = 0$ GL(10) GENERATORS $K^a{}_b$ $\quad [K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b$

$\ell = \pm 1$ $\quad E^{[a_1 a_2 a_3]}, \; F_{[a_1 a_2 a_3]}$

3 INDICES

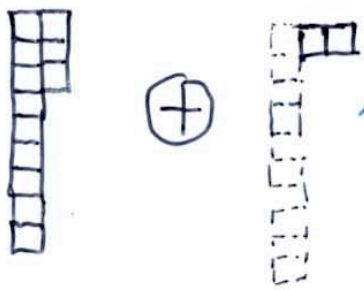
$\ell = \pm 2$ $\quad E^{[a_1 \cdots a_6]}, \; F_{[a_1 \cdots a_6]}$

6 INDICES

$\ell = \pm 3$ $\quad E^{[a_0 | a_1 \cdots a_8]}, \; F_{[a_0 | a_1 \cdots a_8]}$

9 INDICES

$\ell = \pm 4$

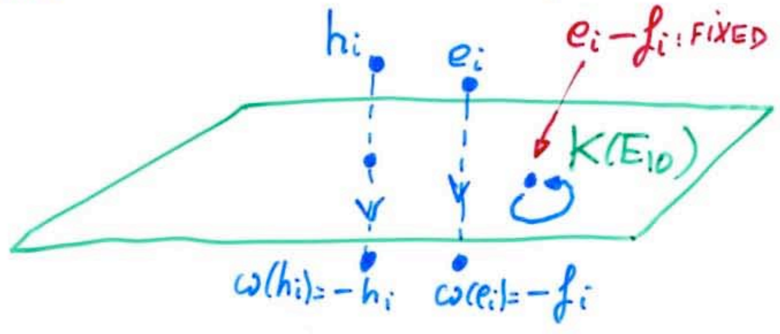


12 INDICES

•
•
•

$K(E_{10})$: MAXIMAL COMPACT SUBGROUP OF THE CANONICAL REAL FORM OF E_{10}

FIXED SET OF CHEVALLEY INVOLUTION ω



$GL(10)$ DECOMPOSITION OF $K(E_{10})$

$J^{ab} = K^a{}_b - K^b{}_a$: $SO(10)$ $\quad [J^{ab}, J^{cd}] = 4\,\delta^{[b}_{[c}\, J^{a]}{}_{d]}$

$J^{a_1a_2a_3} = E^{a_1a_2a_3} - F_{a_1a_2a_3}$ $\quad [J^{a_1a_2a_3}, J^{b_1b_2b_3}] = J^{a_1a_2a_3b_1b_2b_3} - 18\,\delta^{a_1b_1}\delta^{a_2b_2}J^{a_3b_3}$

$J^{a_1a_2\cdots a_6} = E^{a_1\cdots a_6} - F_{a_1\cdots a_6}$ $\quad [J^3, J^6] = J^9 + J^3$

$J^{a_0|a_1\cdots a_8} = E^{a_0|a_1\cdots a_8} - F_{a_0|a_1\cdots a_8}$ $\quad [J^3, J^9] = J^{12} + J^6$

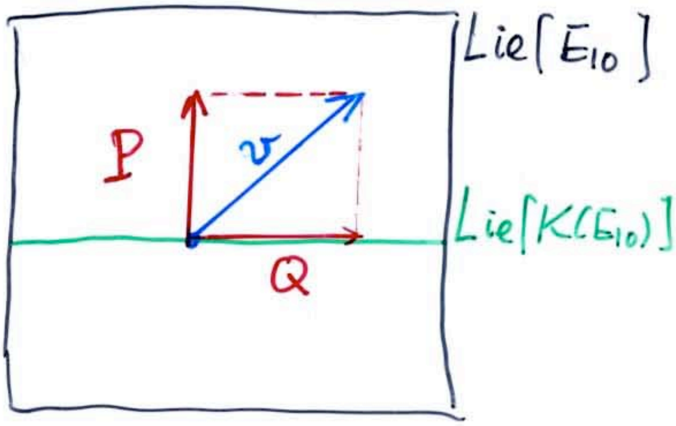
$\vdots$

$D=1$ $\quad E_{10}/K(E_{10})$ COSET MODEL

GROUP ELEMENT $\quad g(t) \in E_{10}$

$Lie[E_{10}]$ 'VELOCITY': $\boxed{v \equiv \frac{dg}{dt}\, g^{-1}}$

DECOMPOSING v



$$\underset{Lie(E_{10})}{\underset{\cap}{v}} \equiv \underset{Lie(K(E_{10}))^{\perp}}{\underset{\cap}{P}} + \underset{Lie(K(E_{10}))}{\underset{\cap}{Q}}$$

'VERTICAL' $\qquad$ 'HORIZONTAL'

COSET ACTION

BOSONIC PART

'VERTICAL' PART OF VELOCITY $v = \dot{g} g^{-1}$

$$\boxed{S_{1\,\mathrm{BOS}}^{\mathrm{COSET}} = \int dt \, \frac{1}{4n} \langle P(t) \mid P(t) \rangle}$$

UNIQUE INVARIANT QUADRATIC FORM OF $\mathrm{Lie}(E_{10})$

SIGNATURE: $\underbrace{-+++++++++}_{\mathrm{CARTAN}}$; $\underbrace{+++++\cdots\cdots}_{K(E_{10})^{\perp}}$; $\cancel{\underbrace{-----\cdots}_{K(E_{10})}}$

SYMMETRY: $g(t) \longrightarrow \underbrace{k(t)}_{\text{LOCAL } K(E_{10})} g(t) \underbrace{g_0}_{\text{GLOBAL } E_{10}}$

$$P(t) \longrightarrow k(t) \, P(t) \, k(t)^{-1}$$

$$Q(t) \longrightarrow k(t) \, Q(t) \, k^{-1}(t) + \partial_t k \, k^{-1}$$

HORIZONTAL VELOCITY: $K(E_{10})$ CONNECTION

FERMIONIC PART

$$\boxed{S_{1\,\mathrm{FERM}}^{\mathrm{COSET}} = -\frac{i}{2} \int dt \, \big(\Psi(t) \mid \overset{\mathrm{VS}}{\mathcal{D}} \Psi(t) \big)_{\mathrm{VS}} + \int dt \, \big(\chi(t) \mid P(t) \circ \Psi(t) \big)_{S}}$$

'VECTOR-SPINOR REPRES. OF $K(E_{10})$': $\Psi = (\psi_a, \cdots ? \cdots)$

EXPLICIT PARAMETRIZATION OF $E_{10}/K(E_{10})$

$$\boxed{g(t) = e^{h^a{}_b(t) K^b{}_a} \; e^{\frac{1}{3!} A_{a_1a_2a_3}(t) E^{a_1a_2a_3} + \frac{1}{6!} A_{a_1\cdots a_6} E^{a_1\cdots a_6} + \frac{1}{9!} A_{a_0|a_1\cdots a_8} E^{a_0|a_1\cdots a_8} + \cdots}}$$

$GL(10): K^a{}_b$

h^a_b

$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c$

$A_{a_1a_2a_3}$ 

$A_{a_1\cdots a_6}$

$A_{a_0|a_1\cdots a_8}$

$+\cdots$

indices raised by g^{ab}

$$\boxed{S_1^{E_{10}/K(E_{10})} = \int \frac{dt}{n(t)} \left[\frac{1}{4} \left(g^{ac} g^{bd} - g^{ab} g^{cd} \right) \dot{g}_{ab} \dot{g}_{cd} + \frac{1}{2}\frac{1}{3!} \dot{A}_{a_1a_2a_3} \dot{A}^{a_1a_2a_3} + \frac{1}{2}\frac{1}{6!} DA_{a_1\cdots a_6} DA^{a_1\cdots a_6} + \frac{1}{2}\frac{1}{9!} DA_{a_0|a_1\cdots a_8} DA^{a_0|a_1\cdots a_8} + \cdots \right.}$$

$$DA_{a_1\cdots a_6} = \dot{A}_{a_1\cdots a_6} + 10\, A_{[a_1\cdots a_3} \dot{A}_{a_4\cdots a_6]}$$

$$DA_{a_0|a_1\cdots a_8} = \dot{A}_{a_0|a_1\cdots a_8} + 42\, A_{\langle a_1\cdots a_3} \dot{A}_{a_4\cdots a_9\rangle} - 42\, \dot{A}_{\langle a_1\cdots a_3} A_{a_4\cdots a_9\rangle} + 280\, A_{\langle a_1\cdots a_3} A_{a_4\cdots a_6} \dot{A}_{a_7\cdots a_9\rangle}$$

$\langle \ldots \rangle$ = projection on Young

M24'
ADI'16

CORRESPONDENCE $E_{10}/K(E_{10})$ COSET $\Longleftrightarrow$ SUGRA$_{11}$

$$\mathcal{L}_1^{E_{10}} \sim (g^{-1}\dot g)^2 + (\dot A_3)^2 + (\dot A_6 + A_3 \dot A_3)^2 + (\dot A_9 + A_6 \dot A_3 + A_3 A_3 \dot A_3)^2 + \dots$$

$$\Downarrow$$

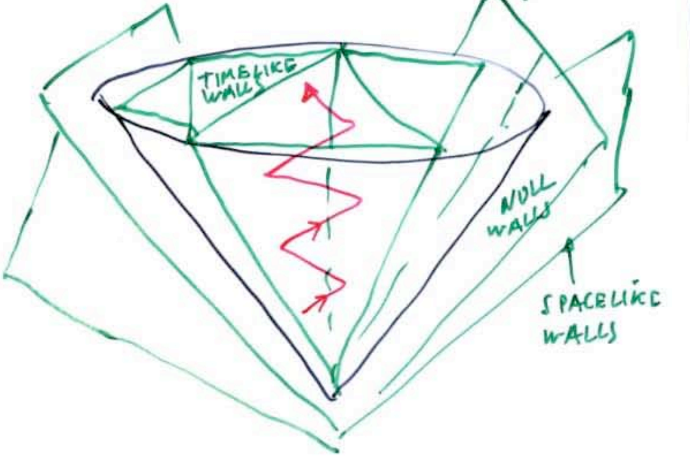
BILLIARD WITH INFINITE NUMBER OF EXPONENTIAL WALLS FOR CARTAN ELEMENT $\beta^{a}(t)$

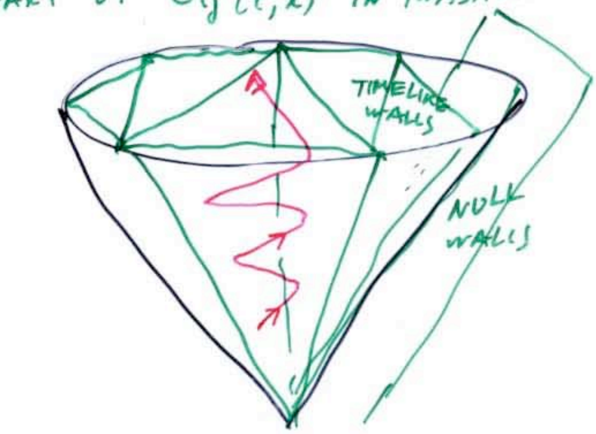
$\mathcal{F}_4 = d\mathcal{A}_3$

$$\mathcal{L}_{11}^{SUGRA} = \int d^{11}x \left[\sqrt{-G}\left[R(G) - \frac{(d\mathcal{A}_3)^2}{48}\right] + \frac{1}{(12)^4}\mathcal{F}_4 \wedge \mathcal{F}_4 \wedge \mathcal{A}_3\right]$$

$$\Downarrow$$

BILLIARD WITH LARGE BUT FINITE # OF EXPONENTIAL WALLS FOR $\beta^a(t,x)$, DIAGONAL PART OF $G_{ij}(t,x)$ IN IWASAWA DECOMP.





$$\mathcal{H}_1 = \frac{1}{2} G^{\mu\nu}\pi_\mu \pi_\nu + \sum C_\alpha(q,p)\, e^{-2\alpha(\beta)}$$

$$\alpha(\beta) = \sum_i m^i \alpha_i(\beta) \qquad m^i \in \mathbb{N}$$

SIMPLE ROOTS

$$\mathcal{H}_{11} = \frac{1}{2} G^{\mu\nu}\pi_\mu \pi_\nu + \sum_A C_A(Q, P, \partial_x\beta, \partial_x Q, ..) \times e^{-2 w_A(\beta)}$$

$$w_A(\beta) = \sum_i m^i w_i(\beta)$$

DOMINANT WALLS

$\boxed{\text{DICTIONARY}}$

$$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c = G^{ab}(t, \vec{x}_0)$$

WRT A SPECIAL FRAME

$$\dot A_{a_1 a_2 a_3}(t) = \mathcal{F}_{0 a_1 a_2 a_3}(t, \vec{x}_0) \qquad \theta^a(x) = e^a_i(x)\, dx^i$$

$$DA^{a_1 \dots a_6}(t) = g^{a_1 a_1'} \cdots g^{a_6 a_6'}[\dot A_{a_1' \dots a_6'} + 10 A_{[3}\dot A_{3]}] = -\frac{1}{4!}\varepsilon^{a_1 \dots a_6 b_1 \dots b_4}\mathcal{F}_{b_1 \dots b_4}(t, \vec{x}_0)$$

$$DA^{b|a_1 \dots a_8}(t) = g^{\cdot\cdot}[\dot A_9 + 42 A_3 \dot A_6 + 280 A_3 A_3 \dot A_3] = +\frac{3}{2}\varepsilon^{a_1 \dots a_8 b_1 b_2} C^b{}_{b_1 b_2}(\vec{x}_0)$$

$$d\theta^a = \frac{1}{2} C^a{}_{bc}\, \theta^b \wedge \theta^c$$

THE CORRESPONDENCE WORKS FOR ALL TERMS OF HEIGHT ≤ 29

$$\sum_i m^i \leq 29 \qquad \qquad \sum_i m^i \leq 29$$

HIGHER-ORDER M-THEORY CORRECTIONS AND E_{10}

$$S_{\text{M-THEORY}} = \int \frac{d^{11}x}{\ell_P^9} \left[R - F^2 + A \wedge F \wedge F \right]$$

$$+ \int \frac{d^{11}x}{\ell_P^3} \left[t_8 t_8 R^4 + \frac{2 \cdot 1}{4} \varepsilon_8 \varepsilon_8 R^4 - 4 \varepsilon_{11} A_3 [t_8 R^4 \ldots] + \ldots \right. \\ \left. R (DF)^2 + \ldots + F^8 \ldots \right]$$

Green Schwarz '82, Sakai Tanii '87, Deser Seminara '99, Green Vanhove '97 ... Tseytlin
Green Gutperle Vanhove ... Duff, Liu Minasian

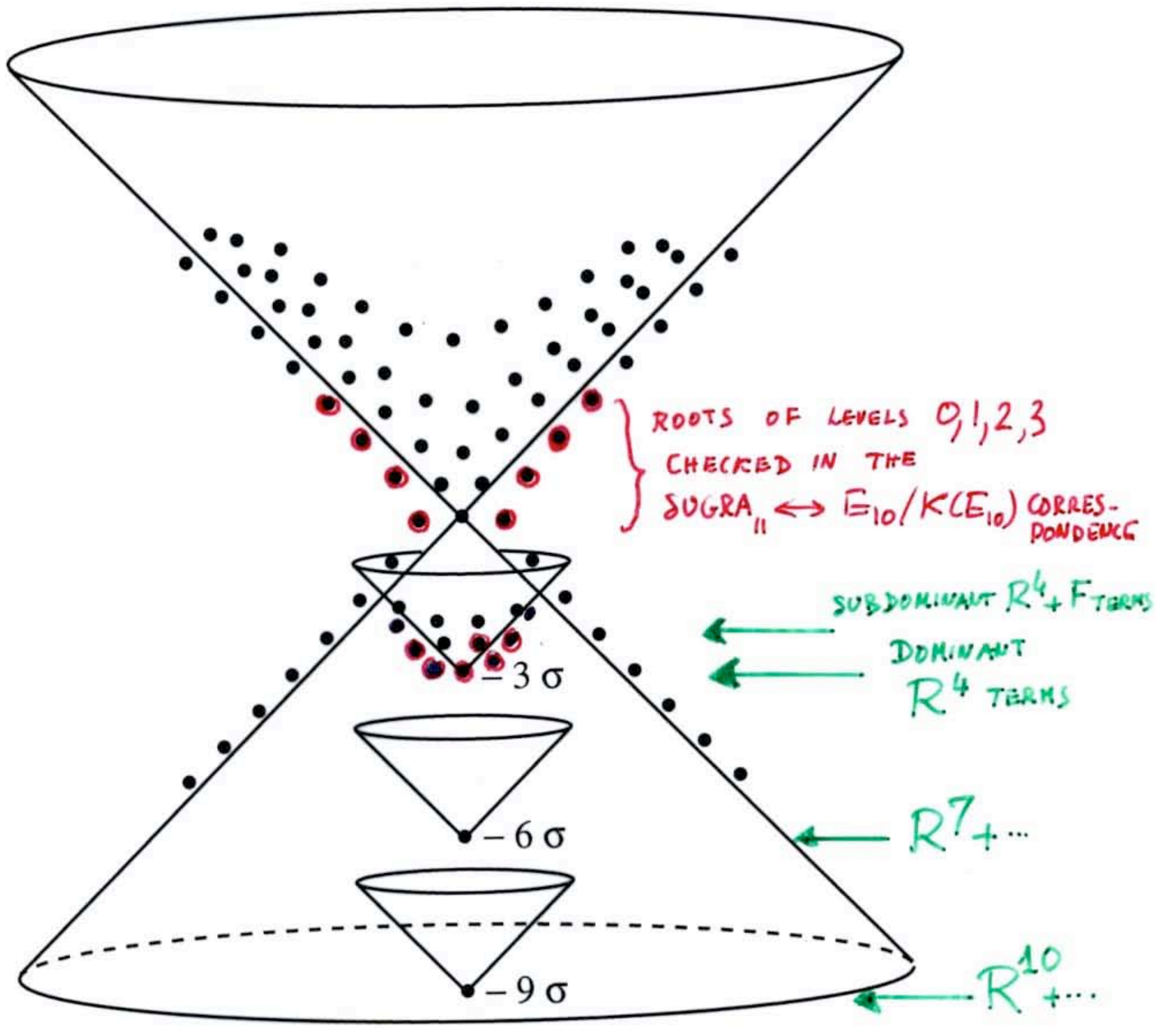
? EFFECT OF $R^4 + \ldots$ ON COSMOLOGICAL BILLIARD ?
(IN INTERMEDIATE ASYMPTOTICS)

$\Rightarrow$ ADD NEW WALLS, WHICH ARE FOUND TO STILL CORRESPOND TO SOME ROOTS OF E_{10}

(T.D., Nicolai, 2005)

AEI
19
Euros 13

ROOTS OF E_{10}



CORRESPONDENCE AND FERMIONS

T.D., Kleinschmidt, Nicolai '06 ; de Buyl, Henneaux, Paulot '06

$$S^{COSET}_{1\ FERM} = -\frac{i}{2}\int dt\, \big(\Psi(t) \,|\, \overset{VS}{\mathcal{D}}\Psi(t)\big)_{VS} + \int dt\, \big(\chi(t) \,|\, P(t) \odot \Psi(t)\big)_S$$

$$S^{SUGRA}_{11\ FERM} = \int d^{11}x \Big\{ -\frac{i}{2}\bar{\psi}_\mu(t,x)\Gamma^{\mu\nu\rho}D_\nu\psi_\rho - \frac{i}{96}\big(\bar{\psi}_\mu \Gamma^{\mu\alpha\beta\gamma\delta\nu}\psi_\nu + 12\,\bar{\psi}^\alpha\Gamma^{\alpha\beta}\psi^\gamma\big)F_{\alpha\beta\gamma\delta} + \dots \Big\}$$

EXTENDED DICTIONARY

COSET		SUGRA
$e^i{}_a \equiv (\exp h)^i{}_a$	$\longleftrightarrow$	$\theta^i_m\, e^m_{(10)\,a}$
VECTOR-SPINOR REP. OF KE_{10} $\Psi(t) = (\psi_a, \dots)$ ψ_a	$\longleftrightarrow$	$\psi^{(11)}_\alpha = E_{(11)}{}^\mu{}_\alpha\, \psi^{(11)}_\mu(t, \vec{x}_0)$ IN GAUGE $\psi^{(11)}_0 = \Gamma_0 \Gamma^a \psi^{(11)}_a$

$\exists$ KE_{10} INVARIANT VECTOR-SPINOR QUADR. FORM

$$(\Psi \,|\, \Phi)_{VS} = \psi_a^T\, \Gamma^{ab}\, \phi_b$$

VECTOR-SPINOR KE_{10}-INVARIANT CONNECTION $\overset{VS}{\mathcal{D}}\Psi(t) = (\partial_t - \overset{VS}{Q})\Psi(t)$ $\longleftrightarrow$ SUGRA FERMIONIC EOM

$$\hat{\mathcal{E}}_A := \Gamma^B\big[(D_A + \mathcal{F}_A)\psi^{(11)}_B - (D_B + \mathcal{F}_B)\psi^{(11)}_A\big] = 0$$

WITH $\mathcal{F}_A = +\frac{1}{144}\big[\Gamma_A{}^{BCDE} - 8\,\delta_A^B\,\Gamma^{CDE}\big] \times F^{(11)}_{BCDE}$

CONSTRAINTS

(Damour, Kleinschmidt, Nicolai '07)

THE CORRESPONDENCE

$$SUGRA_{11} \longleftrightarrow \text{MASSLESS PARTICLE ON } E_{10}/K(E_{10})$$

CONCERNS THE (10+1)-SPLIT, GAUGE-FIXED EVOLUTION EQS OF SUGRA

IN GAUGE: $\tilde{N} \equiv \frac{N}{\sqrt{g}} = 1$, $N_i \equiv g_{0i} = 0$, $A_{0ij} = 0$

NEED TO SUPPLEMENT EVOLUTION EQS BY CONSTRAINTS

$$\boxed{S_{SUGRA} = \int d^d x \left[\pi^{ij} \dot{g}_{ij} + \pi^{ijk} \dot{A}_{ijk} - \tilde{N} \mathcal{H} - N^i \mathcal{H}_i - A_{0ij} \mathcal{G}^{ij} \right]}$$

HAMILTONIAN CONSTRAINT MOMENTUM (SPATIAL DIFFEOS) GAUSS

+ BIANCHI CONSTRAINTS FOR $R_{\mu\nu\rho\sigma}$ und $F_{\mu\nu\rho\sigma}$

$$\boxed{\text{SUGRA CONSTRAINTS} \overset{?}{\longleftrightarrow} \text{COSET CONSTRAINTS}}$$

$\mathcal{H}(x) \approx 0$, $\mathcal{H}_i(x) \approx 0$, ...

?

∞ # OF ∂_x^k CONDNS

HAMILTONIAN $\leftrightarrow$ NULL GEODESIC

$\mathcal{H} \approx 0 \leftrightarrow \langle P, P \rangle \approx 0$

OK MODULO SOME $\ell=3$ TERMS

? OTHER CONSTRAINTS

CONSTRAINTS AND THE E_{10} COSET MODEL

TD, Kleinschmidt, Nicolai /07

GRAVITY BOSONIC CONSTRAINTS

$$e_i(t,x_0)\,,\ e^{ij}(t,x_0)\,,\ e_{i_1\cdots i_5}(t,x_0)\,,\ e^{\ell_0}_{[ijk]}(t,x_0)\,,\ \cdots$$

COSET BOSONIC CONSTRAINTS (REDEFINED)

$$\overset{(-3)}{\mathcal{L}}{}^{m_1\cdots m_9} = 28\,\overset{(-1)}{J}{}^{m_1 m_2 m_3}\,\overset{(-2)}{J}{}^{m_4\cdots m_9} + \overset{(0)}{J}{}^{m_1}{}_p\,\overset{(-3)}{J}{}^{p|m_2\cdots m_9}$$

$$\overset{(-4)}{\mathcal{L}}{}^{m_1\cdots m_{12}} = \overset{(-2)}{J}\cdots\overset{(-2)}{J}\cdots + \overset{(-1)}{J}\cdots\overset{(-3)}{J}\cdots$$

$$\overset{(-5)}{\mathcal{L}}{}^{m_1\cdots m_{15}} = \overset{(-2)}{J}\cdots\overset{(-3)}{J}\cdots$$

$$\overset{(-6)}{\mathcal{L}}{}^{m_1\cdots m_{18}} = \overset{(-3)}{J}\cdots\overset{(-3)}{J}\cdots$$

SUGAWARA-LIKE
$$\boxed{\overset{(-\ell)}{\mathcal{L}}{}^{n_1\cdots n_{3\ell}} = \sum_m \overset{(-\ell+m)}{J}\cdots\overset{(-m)}{J}\cdots}$$

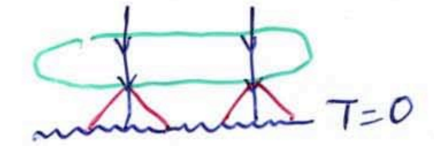
WHERE
$$\boxed{J = g^{-1}\,P\,g \in Lie(E_{10})}$$

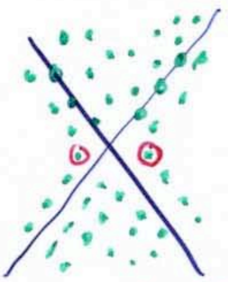
IS THE CONSERVED 1-d E_{10} NOETHER CURRENT (OR CHARGE)

- $\overset{(-\ell)}{\mathcal{L}}$ ARE STRONGLY CONSERVED
- $\overset{(-\ell)}{\mathcal{L}}$ IS A LINEAR REPRESENTATION OF E_{10}^{+} — PARABOLIC SUBGROUP $GL_{10} \oplus E^{\alpha>0}$
- GENERALIZES SUGAWARA CONSTR. $L_m \propto \sum_n k_{ab}\, J^a_{m-n}\, J^b_n$ — VIRASORO — AFFINE KAC-MOODY
- $\overset{(-\ell)}{\mathcal{L}}$ IS NOT A REPRESENTATION OF FULL E_{10}

CONCLUSIONS

- THE 'NEAR COSMOLOGICAL SINGULARITY LIMIT' SUGGESTS $\exists$ HIDDEN $E_{10}(\mathbb{R})$ SYMMETRY OF $SUGRA_{11}$ (AND M-THEORY) $[+ AE_m(\mathbb{R})$ FOR GR_{m+1}?$]$

- 

$\longleftrightarrow$


BKL GRADIENT EXPANSION

$$\partial_{x^1}^{k_1}\partial_{x^2}^{k_2}\cdots\partial_{x^{10}}^{k_{10}} \ll \partial_T^{k_1+k_2+\cdots+k_{10}}$$

HEIGHT EXPANSION IN HYPERBOLIC KAC-MOODY ALGEBRA

ROOT: $\alpha = m_0\alpha_0 + m_1\alpha_1 + \cdots + m_9\alpha_9$

- SUGGESTS $\exists$ A 'HOLOGRAPHIC' (OR 'PHOTOGRAPHIC' (POLYAKOV)) CORRESPONDENCE

GRAVITY IN D=11 DYNAMICS $\longleftrightarrow$ D=1 PARTICLE DYNAMICS ON (∞-DIM) COSET SPACE

- OPTIMISTICALLY

$\rightarrow$ BACKGROUND-INDEPENDENT FORMULATION OF (A SECTOR OF) M-THEORY

$\rightarrow$? NEW DESCRIPTION OF THE (QUANTUM) NATURE OF SPACE-(TIME) AT PLANCK SCALE VIA A 'DE-EMERGENCE' OF SPACE NEAR A SINGULARITY

'SPACE' I.E. $\left\{\begin{matrix} G_{\mu\nu}(t,x) \\ A_{\mu\nu\lambda}(t,x) \\ \psi_\mu(t,x) \end{matrix}\right\} \longrightarrow$ INFINITE TOWER OF LIE-ALGEBRAIC VARIABLES $\left\{\begin{matrix} g(t) \in E_{10}/K(E_{10}) \\ \underline{\Psi}(t) \end{matrix}\right.$

OPEN ISSUES

- HOW TO GO BEYOND HEIGHT 29 IN TESTING THE CONJECTURE ?
- SUGAWARA-LIKE CONSTRAINTS $\mathcal{L}^{(-n)} \sim J \otimes J$?

ARE THEY DESCRIBING THE 'GAUGE SYMMETRY OF M-THEORY'

AS A VAST GENERALIZATION OF THE USUAL SUGAWARA CONSTRUCTION:

$$L_m \sim J_{\text{AFFINE}} \otimes J_{\text{AFFINE}} \longleftrightarrow \text{CONFORMAL SYMMETRY}$$

- MEANING OF BREAKING $E_{10} \to E_{10}^{+}$?

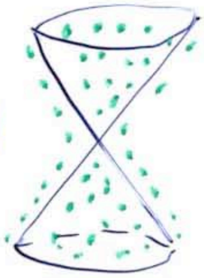
? SIMILAR TO STRING: $L_n = \frac{1}{2} \sum_m \eta_{\mu\nu} J^{\mu}_{n-m} J^{\nu}_m$

SPECTRUM GENERATING ALGEBRA

AUXILIARY SYMMETRY OF GAUGE-FIXED ACTION

MAYBE E_{10} COSET MODEL = GAUGE-FIXED VERSION OF AN UNDERLYING GAUGE-INVARIANT ACTION

- INFINITE # OF CONSTRAINTS $\mathcal{L}^{(-\ell)} = 0$ WELCOME FOR REDUCING THE # DOF TO $\simeq$ SUGRA (OR M-THY?)



- QUANTIZED COSET ACTION $\sim \Box \Psi(g) = 0$

+ TOROIDAL COMPACTIFICATION $[\Rightarrow E_{10}(\mathbb{Z})]$ (Hull, Townsend '95)

$\Rightarrow$ MODULAR FORM OVER $E_{10}(\mathbb{Z}) \backslash E_{10}(\mathbb{R}) / K(E_{10}(\mathbb{R}))$?

(Ganor '99, Brown, Ganor, Helfgott '04)