Event Generator Physics

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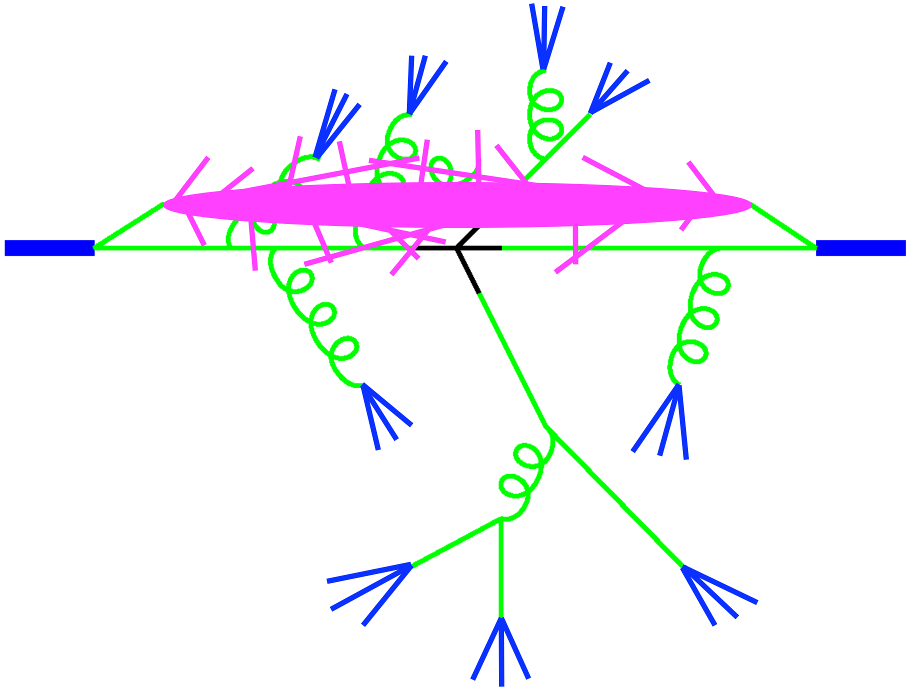
Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event



Lecture 2: Parton Showers

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.

gluons also charged.

→ cascade of partons.

= parton shower.

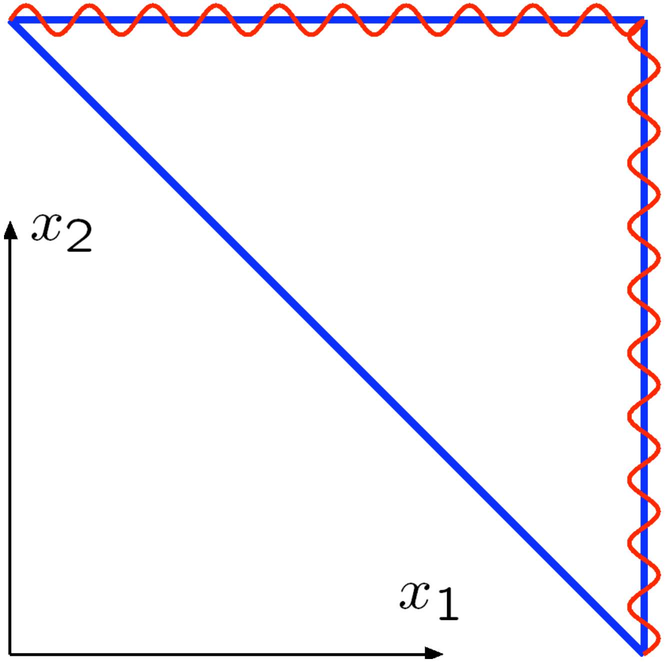
1. e^+e^-annihilation to jets.
2. Universality of collinear emission.
3. Sudakov form factors.
4. Universality of soft emission.
5. Angular ordering.
6. Initial-state radiation.
7. Hard scattering.
8. Heavy quarks.
9. The Colour Dipole Model.

e^+e^- annihilation to jets

Three-jet cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0\, C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

singular as $x_{1,2} \to 1$



Rewrite in terms of quark-gluon opening angle θ and gluon energy fraction x_3 :

$$\frac{d\sigma}{d\cos\theta\, dx_3} = \sigma_0\, C_F \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right\}$$

Singular as $\sin\theta \to 0$ and $x_3 \to 0$.

can separate into two independent jets:

$$\frac{2\, d\cos\theta}{\sin^2\theta} = \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta}$$
$$= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar{\theta}}{1-\cos\bar{\theta}}$$
$$\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}$$

jets evolve independently

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

Exactly same form for anything $\propto \theta^2$

eg transverse momentum: $k_\perp^2 = z^2(1-z)^2\, \theta^2\, E^2$

invariant mass: $q^2 = z(1-z)\, \theta^2\, E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}$$

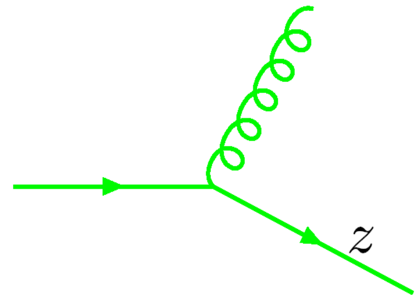
Collinear Limit

Universal:

$$d\sigma = \sigma_0 \, \frac{\alpha_s}{2\pi} \, \frac{d\theta^2}{\theta^2} \, dz \, P(z,\phi) d\phi$$

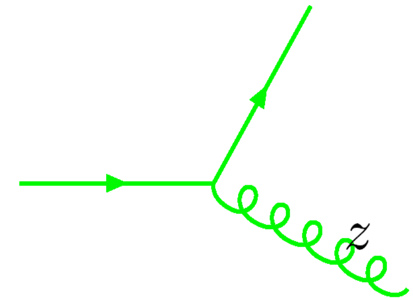
$$P(z,\phi) =$$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour and spin



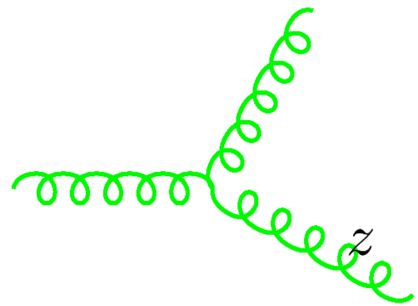
$q \to qg$

$$C_F \frac{1+z^2}{1-z}$$



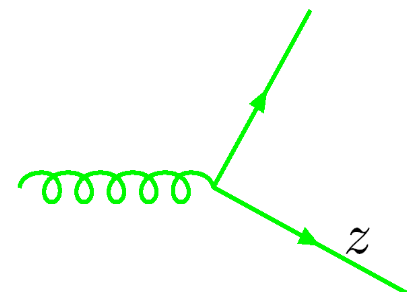
$q \to gq$

$$C_F \frac{1+(1-z)^2}{z}$$



$g \to gg$

$$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$$



$g \to q\bar{q}$

$$T_R \left(z^2 + (1-z)^2 \right)$$

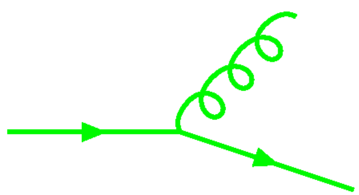
Resolvable partons

What is a parton?

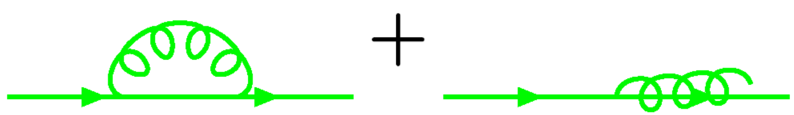
Collinear parton pair $\longleftrightarrow$ single parton

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable real emission



Resolvable emission: Finite



Virtual + Unresolvable emission: Finite

Unitarity: P(resolved) + P(unresolved) = 1

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Gives evolution equation

$$-\frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\mathcal{P}}{dq^2}$$

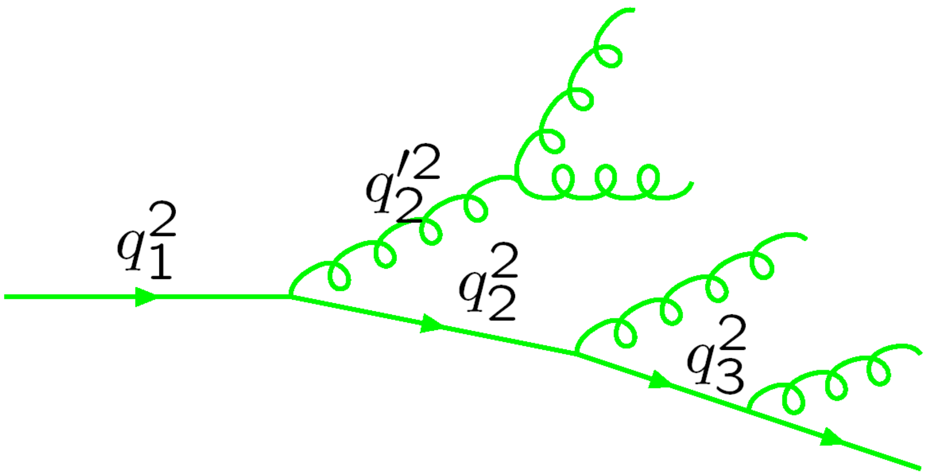
$$\Rightarrow \Delta(Q^2,q^2) = \exp -\int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

$\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor factor

=Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp -C_F\frac{\alpha_s}{2\pi}\log^2\frac{Q^2}{Q_0^2}$$

Multiple emission



$$q_1^2 > q_2^2 > q_3^2 > \ldots$$
$$q_1^2 > q_2'^2 \ldots$$

But initial condition? $q_1^2 < ???$

Process dependent

Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2,q^2)$$

By choosing $0 < \rho < 1$ uniformly:

If $\rho < \Delta(Q^2)$ no resolvable radiation, evolution stops.

Otherwise, solve $\rho = \Delta(Q^2,q^2)$

for q^2 = emission scale

Considerable freedom:

Evolution scale: $q^2/k_\perp^2/\theta^2$?

z: Energy? Light-cone momentum?

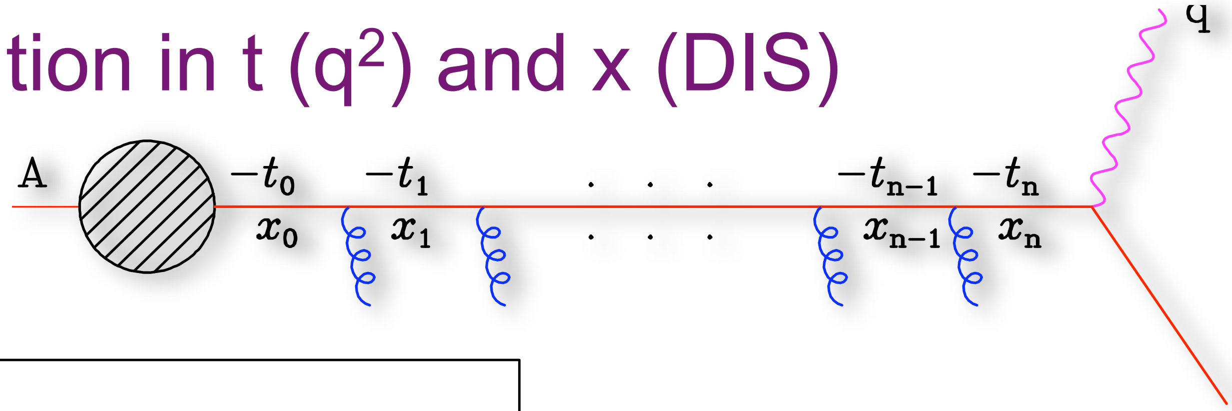
Massless partons become massive. How?

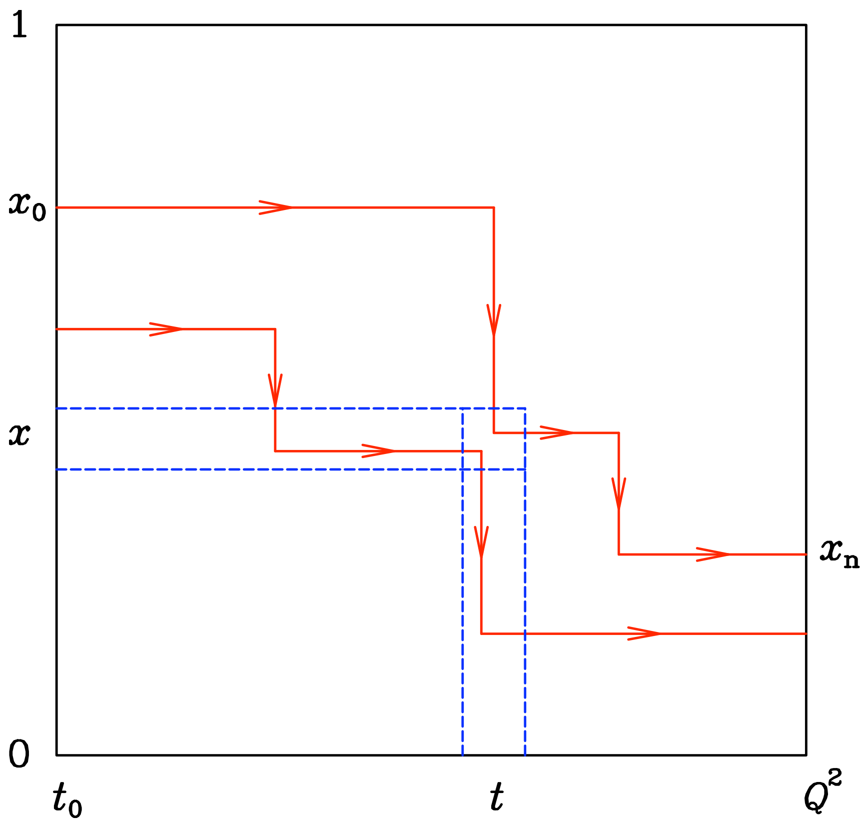
Upper limit for q^2?

} Equivalent at this stage, but can be very important numerically

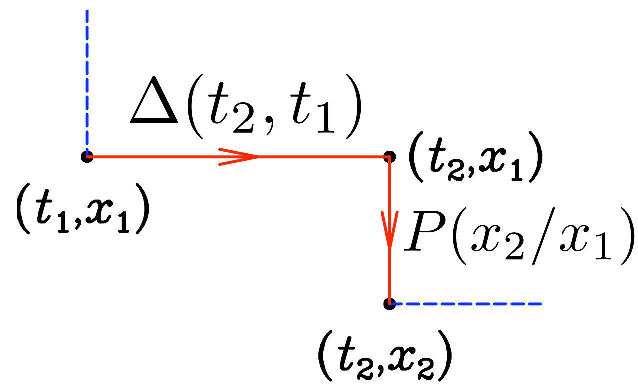
Parton Shower

- Evolution in t (q^2) and x (DIS)





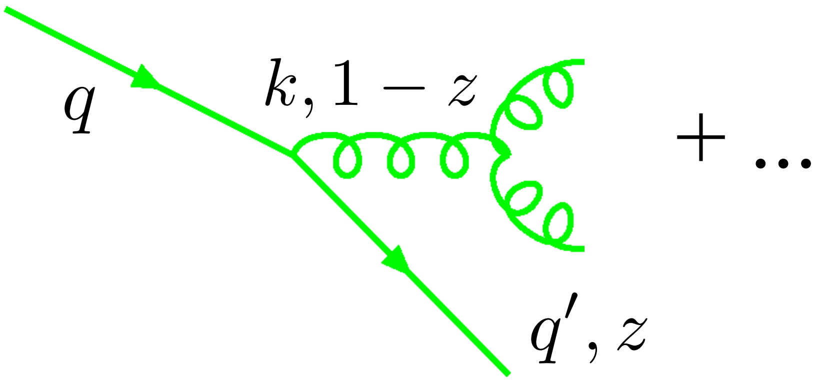
Basic 2-step:



e^+e^-: same formula, opposite direction!

Running coupling

Effect of summing up higher orders:



Scale is set by maximum virtuality of emitted gluon

$$k^2_{\max} = (1-z)q^2$$

Similarly in $g \to gg'$, scale is set by

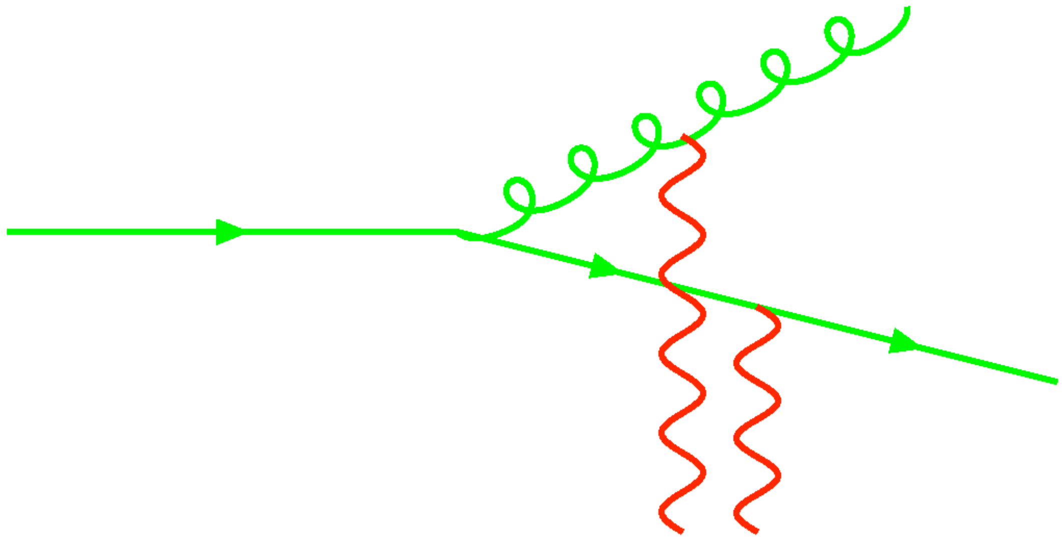
$$\min\{k^2_{\max}, k'^2_{\max}\} = \min\{z,(1-z)\}q^2 \simeq z(1-z)q^2 \equiv k_T^2$$

Scale change absorbed by replacing $\alpha_S(q^2)$ by $\alpha_S(k_T^2)$

➡ Faster parton multiplication

Soft limit

Also universal. But at amplitude level…



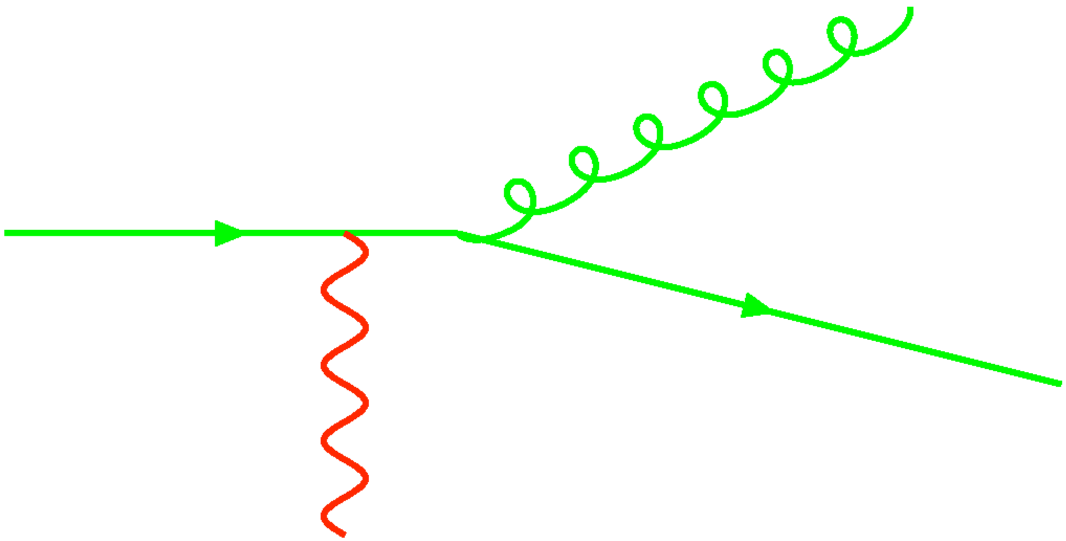
soft gluon comes from everywhere in event.

→ Quantum interference.

Spoils independent evolution picture?

Angular Ordering

NO:



outside angular ordered cones, soft gluons sum coherently: only see colour charge of whole jet.

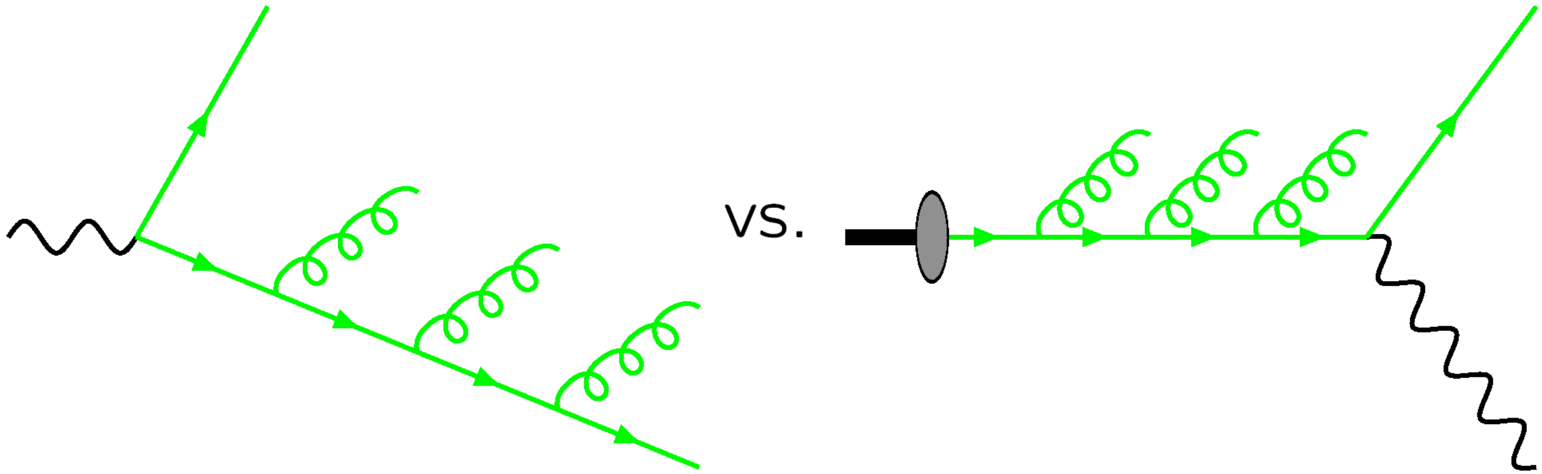
Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering

First gluon not necessarily hardest!

Initial state radiation

In principle identical to final state (for not too small x)

In practice different because both ends of evolution fixed:



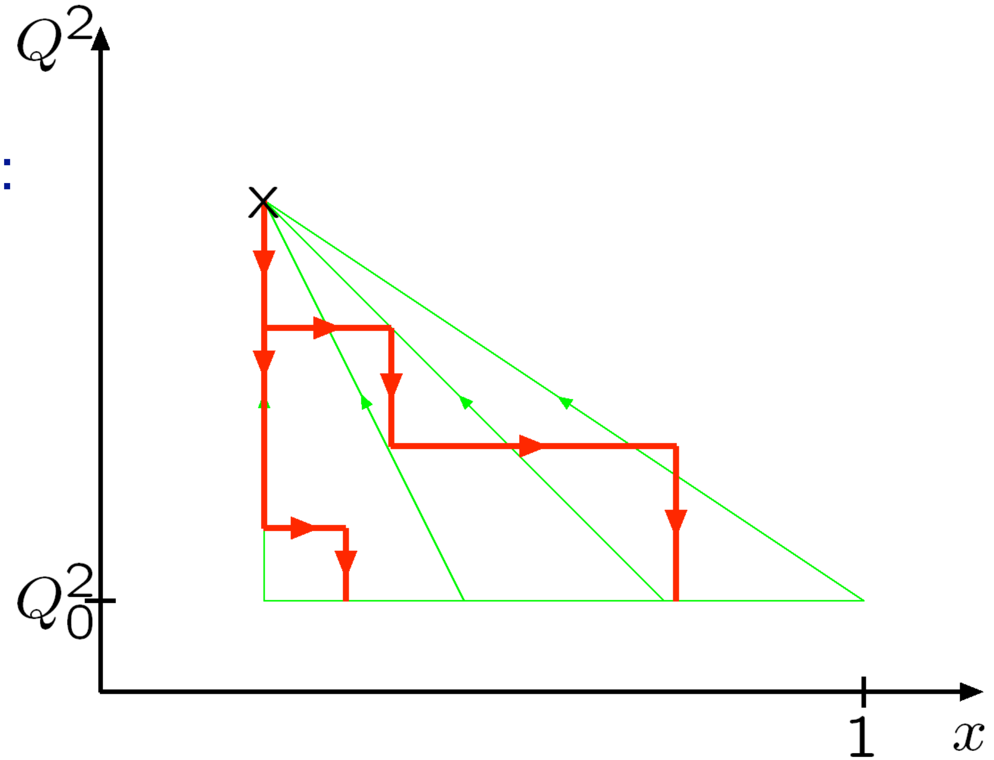
Use approach based on evolution equations…

Backward Evolution

DGLAP evolution: pdfs at (x, Q^2) as function of pdfs at $(> x, Q_0^2)$:

Evolution paths sum over all possible events.

Formulate as backward evolution: start from hard scattering and work down in q^2, up in x towards incoming hadron.



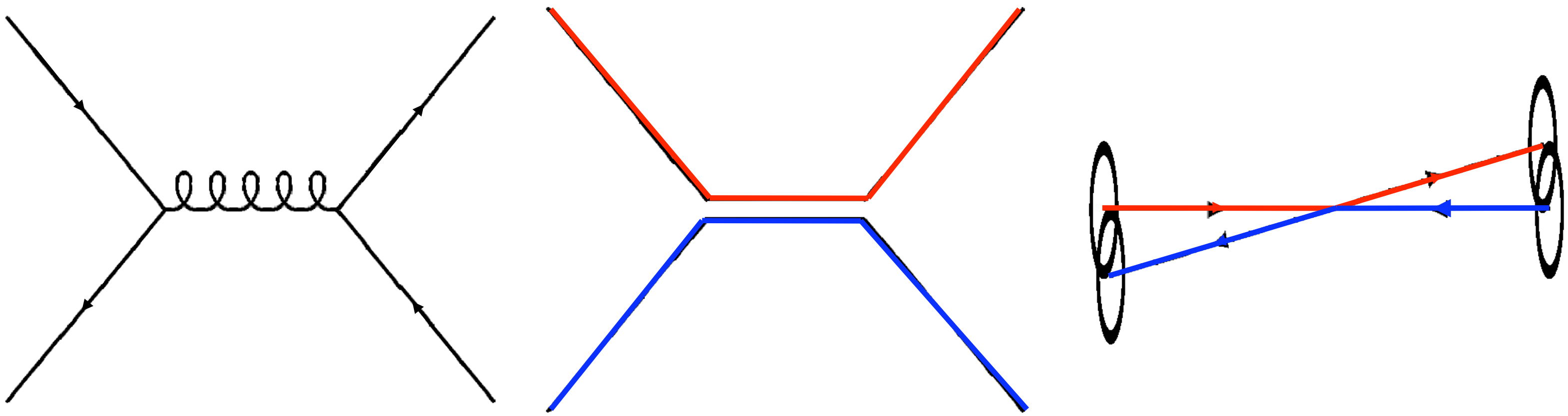
Algorithm identical to final state with $\Delta_i(Q^2, q^2)$ replaced by

$$\Delta_i(Q^2, q^2)/f_i(x, q^2).$$

Hard Scattering

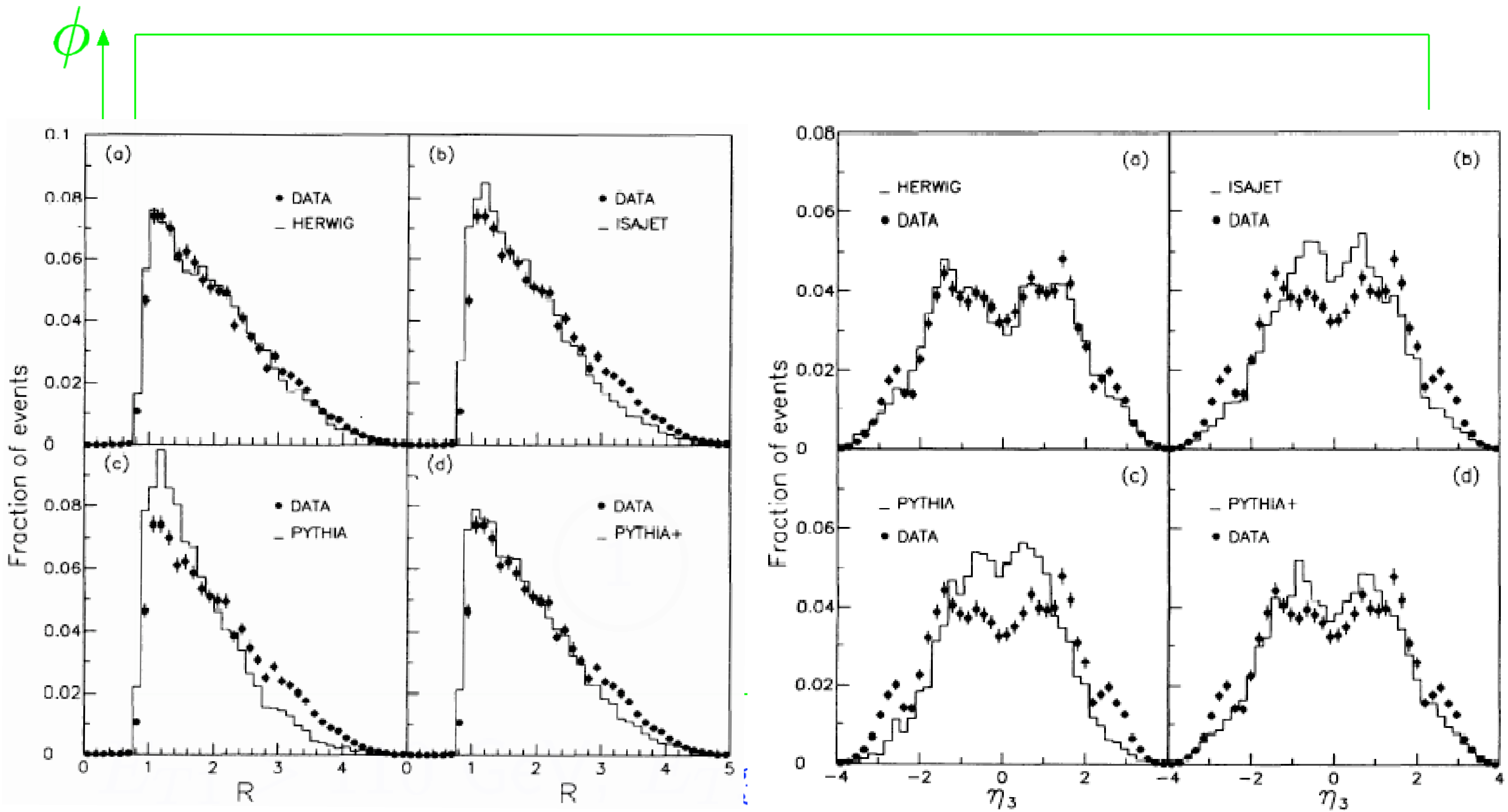
Sets up initial conditions for parton showers.

Colour coherence important here too.



Emission from each parton confined to cone stretching to its colour partner

Essential to fit Tevatron data…



Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial

The Colour Dipole Model

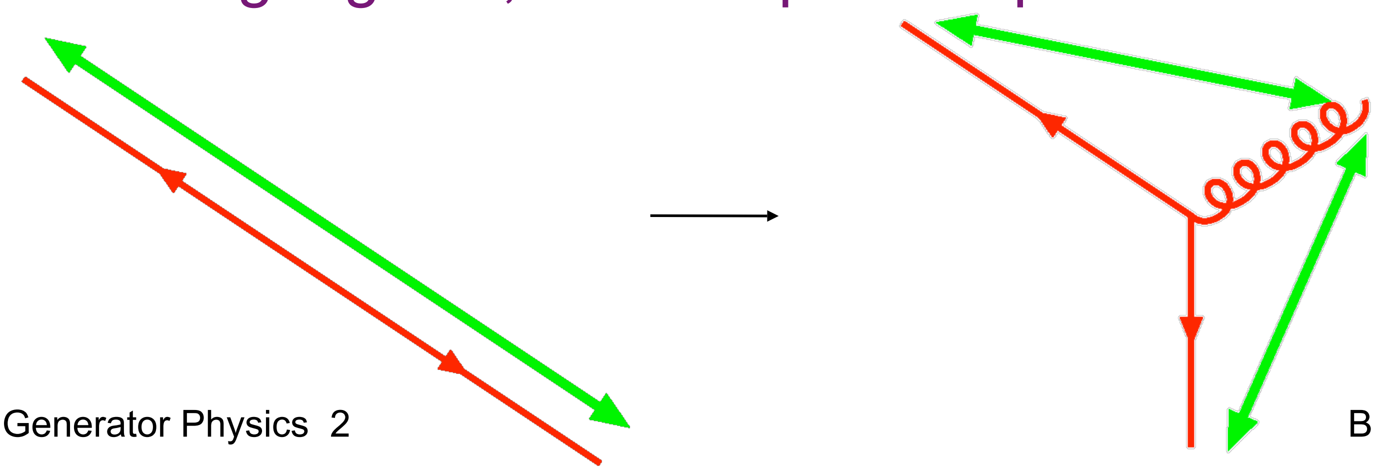
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \text{rapidity} = \log \tan \theta/2$$

After emitting a gluon, colour dipole is split:

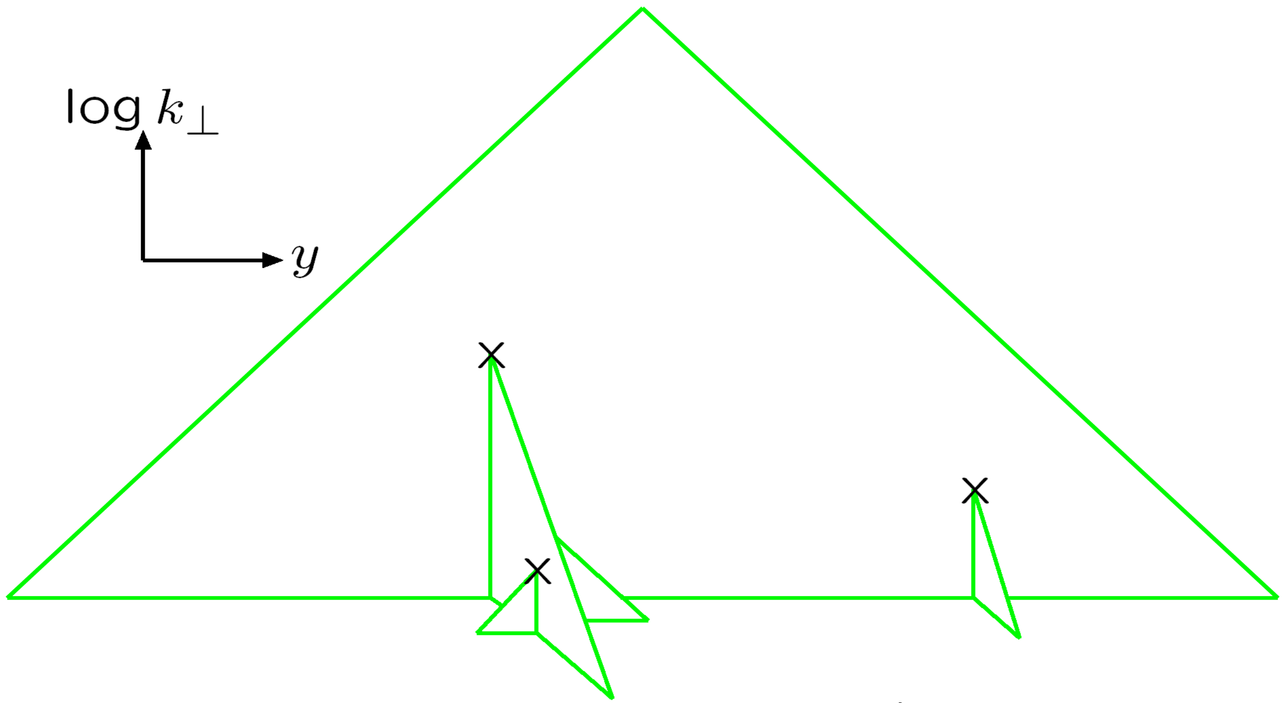


Subsequent dipoles continue to cascade

c.f. parton shower: one parton → two

CDM: one dipole → two = two partons → three

Represented in ‘origami diagram’:



Similar to angular-ordered parton shower for e^+e^- annihilation

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Summary

- Accelerated colour charges radiate gluons.

  Gluons are also charged → cascade.

- Probabilistic language derived from factorization theorems of full gauge theory.

  Colour coherence → angular ordering.

- Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models…

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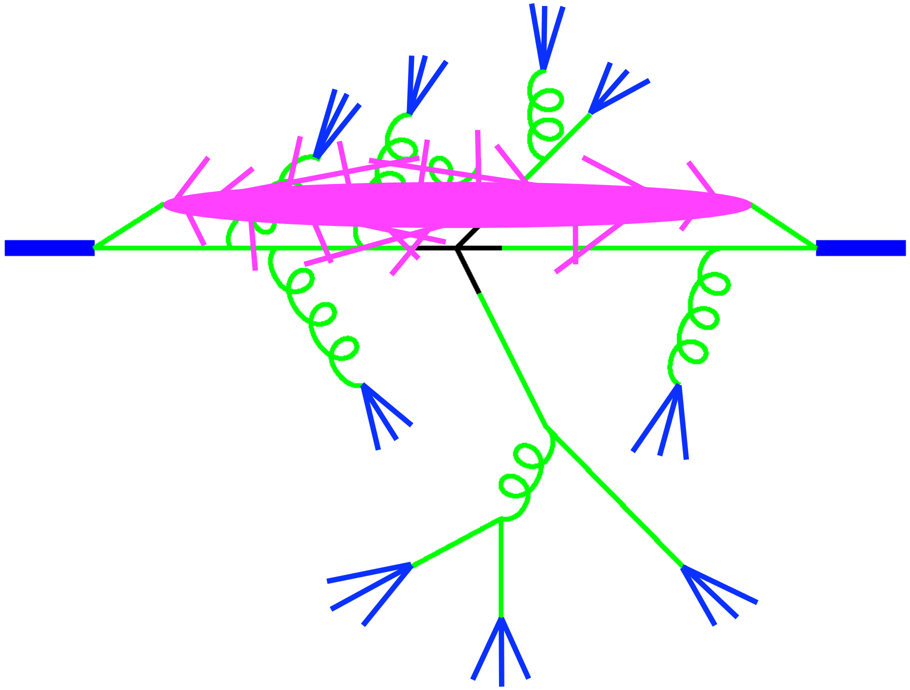


Lecture 3: Hadronization

Partons are not physical particles: they cannot freely propagate.

Hadrons are.

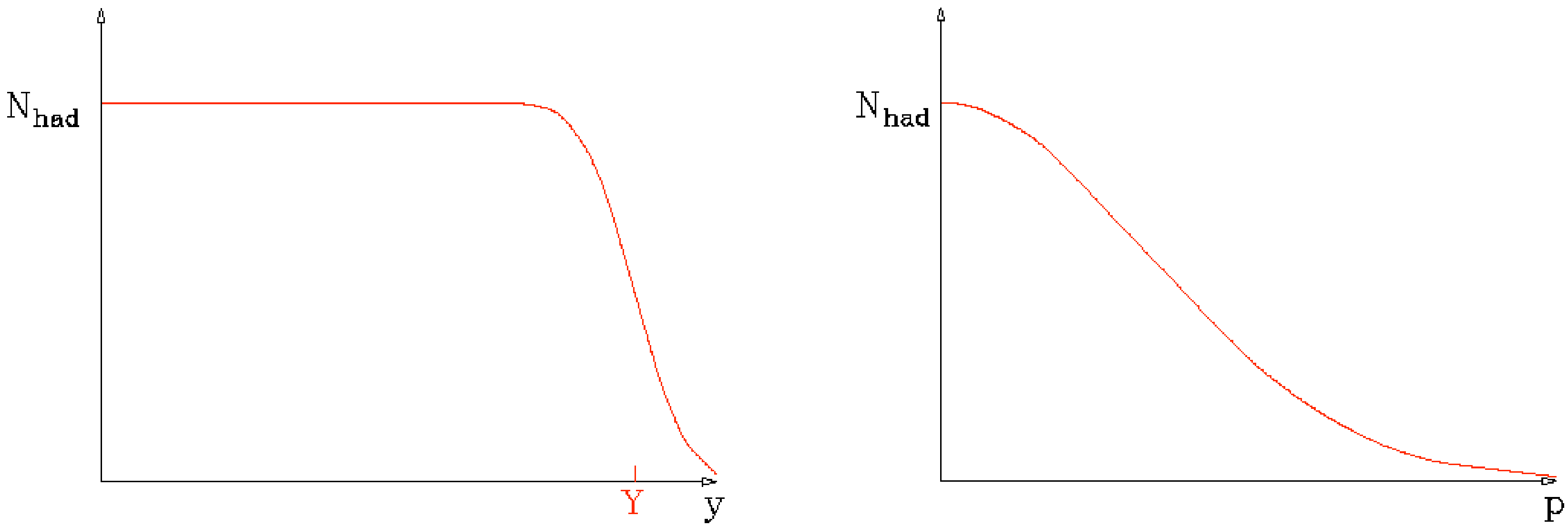
Need a model of partons' confinement into hadrons: hadronization.

1. Phenomenological models.
2. Confinement.
3. The string model.
4. Preconfinement.
5. The cluster model.
6. Underlying event models.

Phenomenological Models

Experimentally, $e^+e^- \longrightarrow$ two jets:

Flat rapidity plateau and limited p_t, $\rho(p_t^2) \sim e^{-p_t^2/2p_0^2}$



Estimate of Hadronization Effects

Using this model, can estimate hadronization correction to perturbative quantities.

Jet energy and momentum:

$$E = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t \cosh y = \lambda \sinh Y$$

$$P = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t \sinh y = \lambda(\cosh Y - 1) \sim E - \lambda,$$

with $\lambda = \int d^2p_t\, \rho(p_t^2)\, p_t,$ mean transverse momentum.

Estimate from Fermi motion $\lambda \sim 1/R_{had} \sim m_{had}.$

Jet acquires non-perturbative mass: $M^2 = E^2 - P^2 \sim 2\lambda E$

Large: ~ 10 GeV for 100 GeV jets.

Independent Fragmentation Model ("Feynman—Field")

Direct implementation of the above.

Longitudinal momentum distribution = arbitrary fragmentation function: parameterization of data.

Transverse momentum distribution = Gaussian.

Recursively apply $q \to q' + \mathrm{had}.$

Hook up remaining soft q and $\bar{q}$.

Strongly frame dependent.

No obvious relation with perturbative emission.

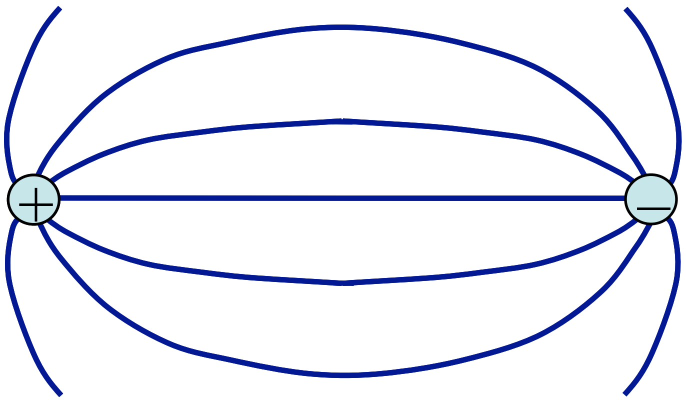
Not infrared safe.

Not a model of confinement.

Confinement

Asymptotic freedom: $Q\bar{Q}$ becomes increasingly QED-like at short distances.

QED:



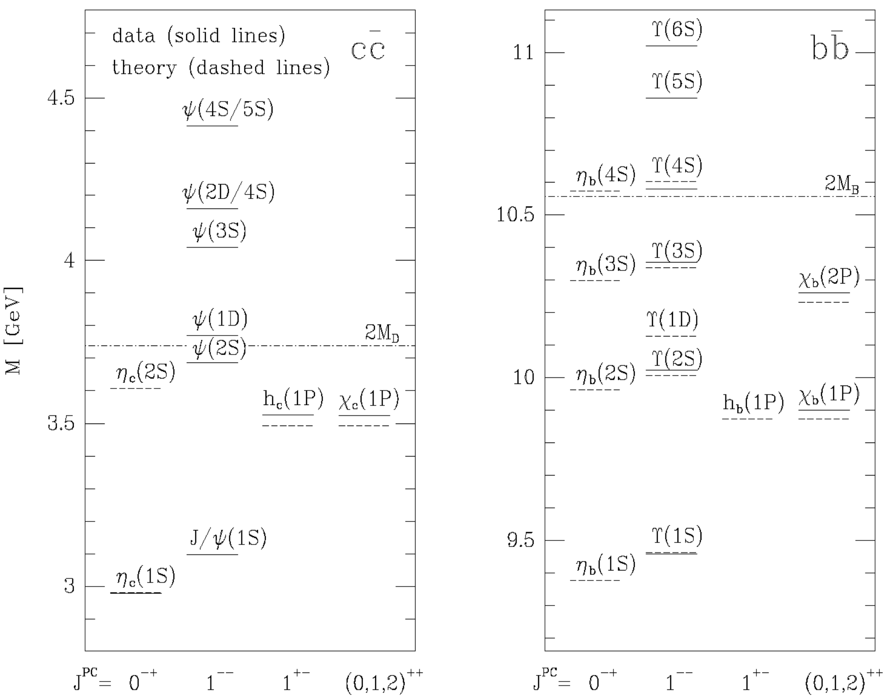
but at long distances, gluon self-interaction makes field lines attract each other:

QCD:

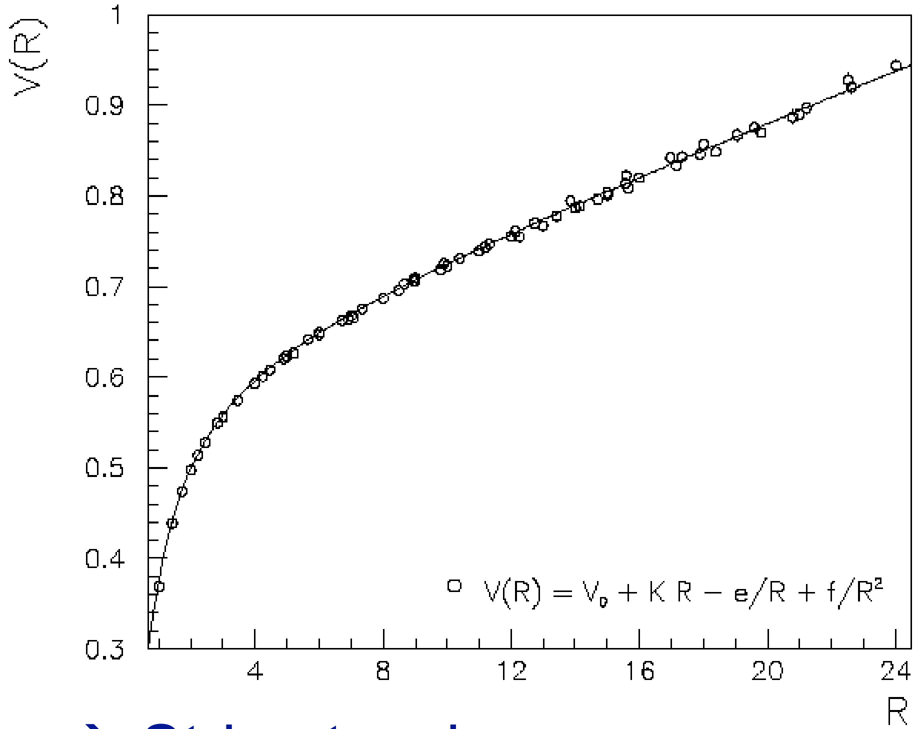
→linear potential → confinement

Interquark potential

Can measure from quarkonia spectra:



or from lattice QCD:



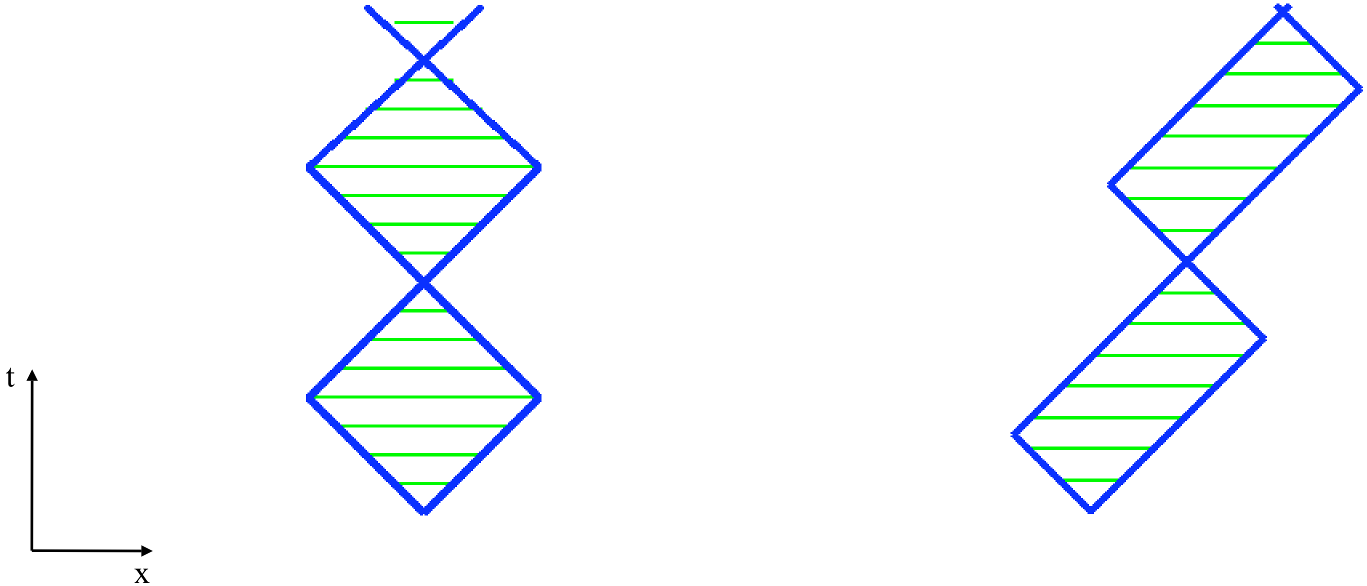
→ String tension

$$\kappa \approx 1\ \text{GeV/fm}.$$

String Model of Mesons

Light quarks connected by string.

L=0 mesons only have 'yo-yo' modes:



Obeys area law: $m^2 = 2\kappa^2$ area

The Lund String Model

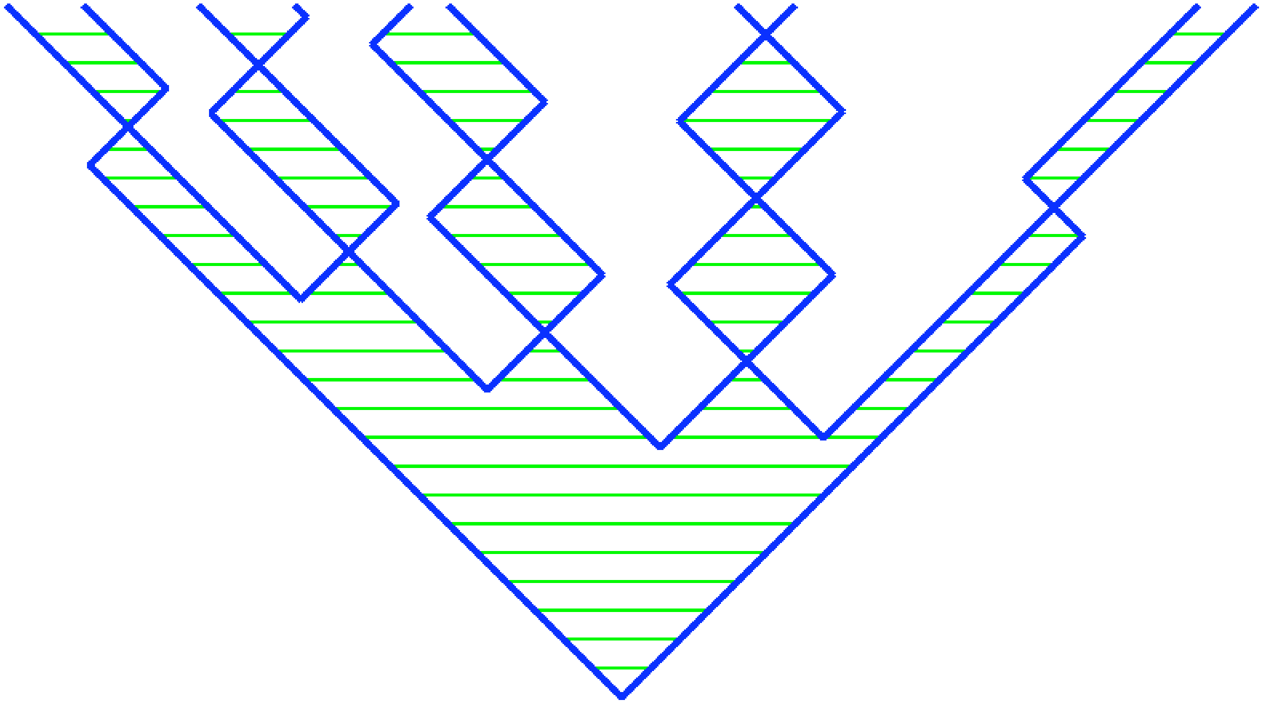
Start by ignoring gluon radiation:

e^+e^- annihilation = pointlike source of $q\bar{q}$ pairs

Intense chromomagnetic field within string → $q\bar{q}$ pairs created by tunnelling. Analogy with QED:

$$\frac{d(\text{Probability})}{dx\; dt} \propto \exp(-\pi m_q^2/\kappa)$$

Expanding string breaks into mesons long before yo-yo point.



Lund Symmetric Fragmentation Function

String picture → constraints on fragmentation function:

- Lorentz invariance
- Acausality
- Left—right symmetry

$$f(z) \propto z^{a_\alpha - a_\beta - 1}(1-z)^{a_\beta}$$

$a_{\alpha,\beta}$ adjustable parameters for quarks α and β.

Fermi motion → Gaussian transverse momentum.

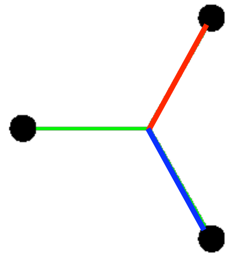
Tunnelling probability becomes

$$\exp\left[-b(m_q^2 + p_t^2)\right]$$

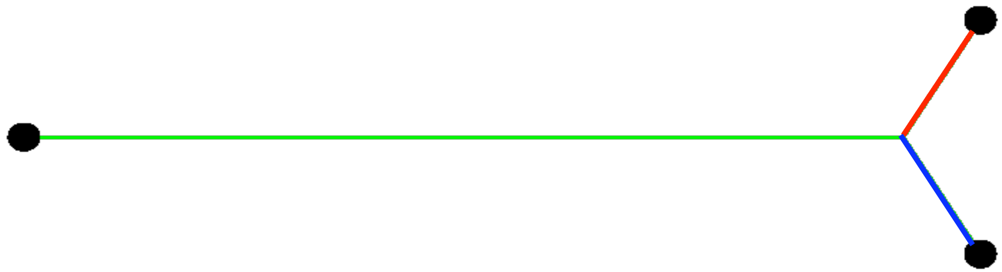
a, b and m_q^2 = main tuneable parameters of model

Baryon Production

Baryon pictured as three quarks attached to a common centre:



At large separation, can consider two quarks tightly bound: diquark



→ diquark treated like antiquark.

Two quarks can tunnel nearby in phase space: baryon—antibaryon pair

Extra adjustable parameter for each diquark!

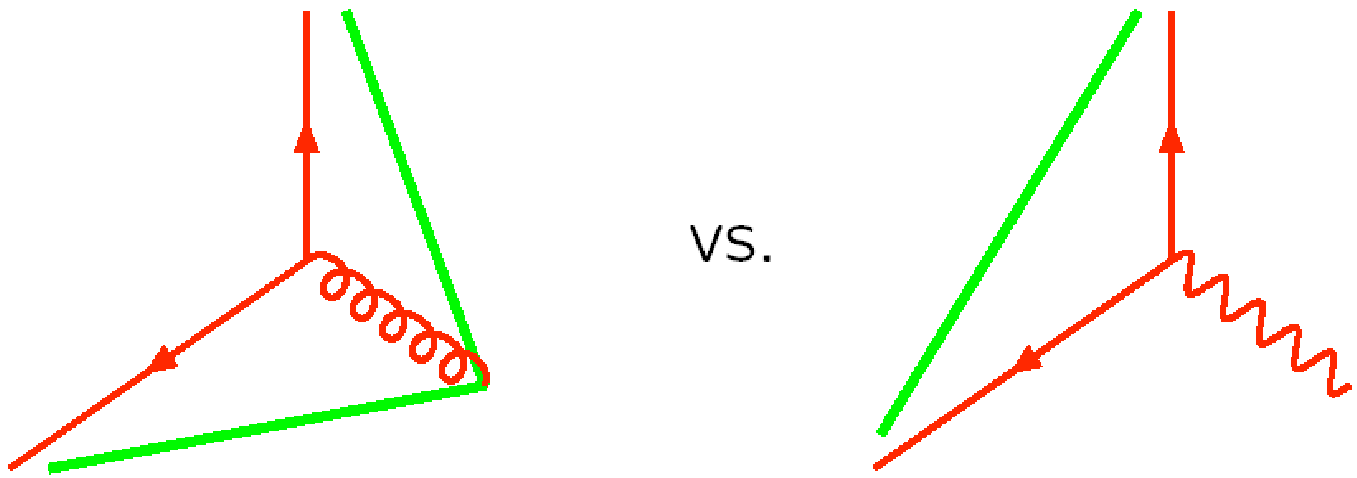
Alternative “popcorn” model:

Three-Jet Events

So far: string model = motivated, constrained independent fragmentation!

New feature: universal

Gluon = kink on string → the string effect



Infrared safe matching with parton shower: gluons with $k_\perp <$ inverse string width irrelevant.

String Model Summary

- String model strongly physically motivated.
- Very successful fit to data.
- Universal: fitted to e^+e^-, little freedom elsewhere.

- How does motivation translate to prediction?
  ~ one free parameter per hadron/effect!

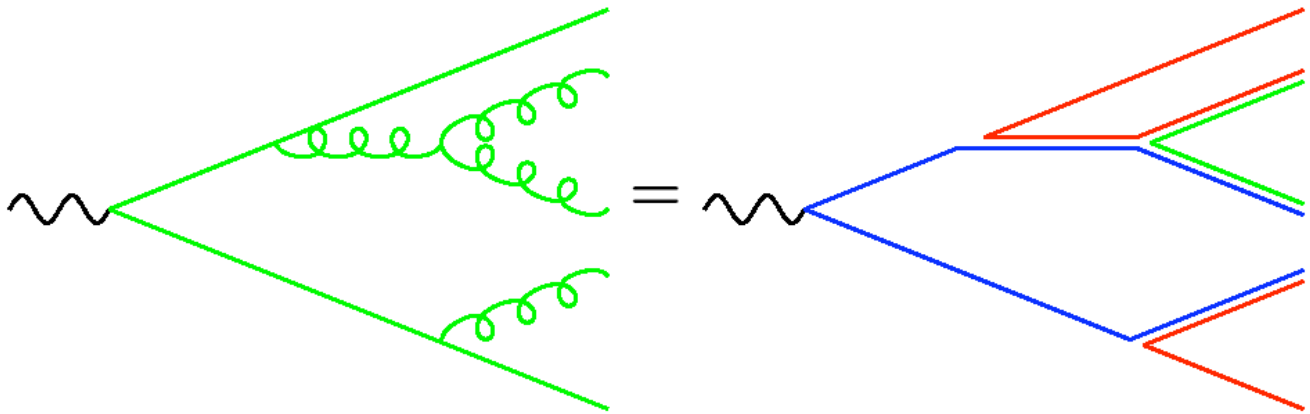
- Blankets too much perturbative information?

- Can we get by with a simpler model?

Preconfinement

Planar approximation: gluon = colour—anticolour pair.

Follow colour structure of parton shower: colour-singlet pairs end up close in phase space

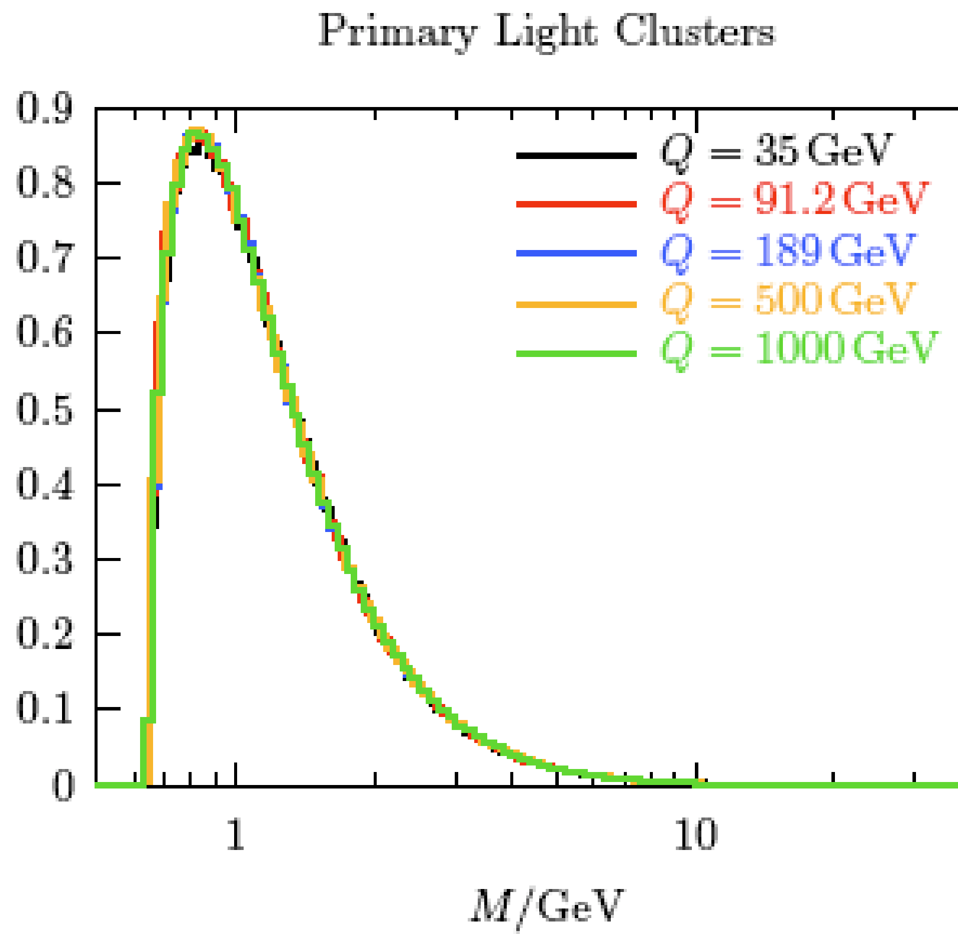


Mass spectrum of colour-singlet pairs asymptotically independent of energy, production mechanism, …

Peaked at low mass $\sim Q_0$.

Cluster mass distribution

- Independent of shower scale Q
  - depends on Q_0 and Λ



The Naïve Cluster Model

Project colour singlets onto continuum of high-mass mesonic resonances (=clusters). Decay to lighter well-known resonances and stable hadrons.

Assume spin information washed out:

decay = pure phase space.

→ heavier hadrons suppressed

→ baryon & strangeness suppression 'for free' (i.e. untuneable).

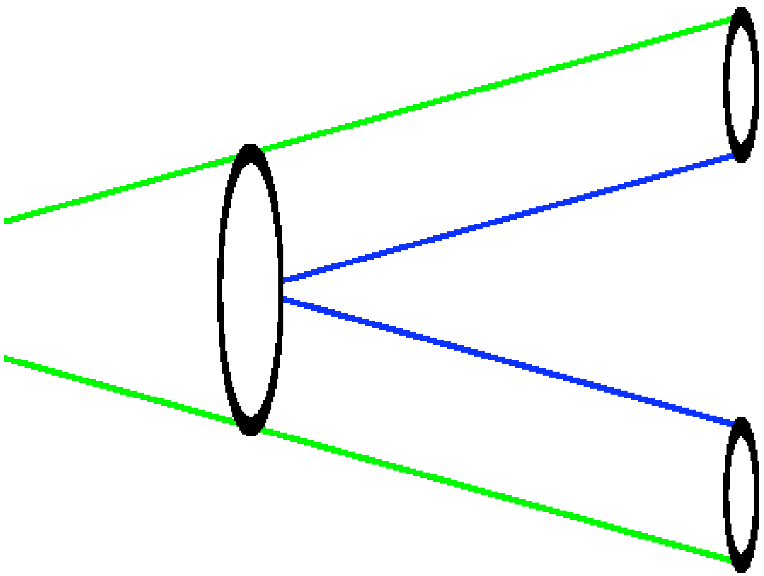
Hadron-level properties fully determined by cluster mass spectrum, i.e. by perturbative parameters.

Shower cutoff Q_0 becomes parameter of model.

The Cluster Model

Although cluster mass spectrum peaked at small m, broad tail at high m.

"Small fraction of clusters too heavy for isotropic two-body decay to be a good approximation" → Longitudinal cluster fission:



Rather string-like.

Fission threshold becomes crucial parameter.

~15% of primary clusters get split but ~50% of hadrons come from them.

The Cluster Model

"Leading hadrons are too soft"

→ 'perturbative' quarks remember their direction somewhat

$$P(\theta^2) \sim \exp(-\theta^2/2\theta_0^2)$$

Rather string-like.

Extra adjustable parameter.

Strings

"Hadrons are produced by hadronization: you must get the non-perturbative dynamics right"

Improving data has meant successively refining perturbative phase of evolution…

Clusters

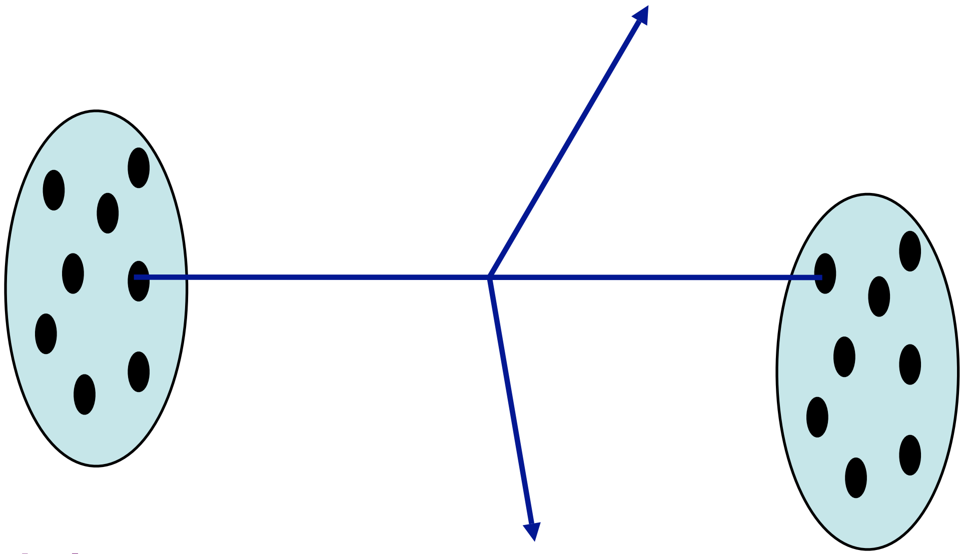
"Get the perturbative phase right and any old hadronization model will be good enough"

Improving data has meant successively making non-perturbative phase more string-like…

???

The Underlying Event

- Protons are extended objects
- After a parton has been scattered out of each, what happens to the remnants?

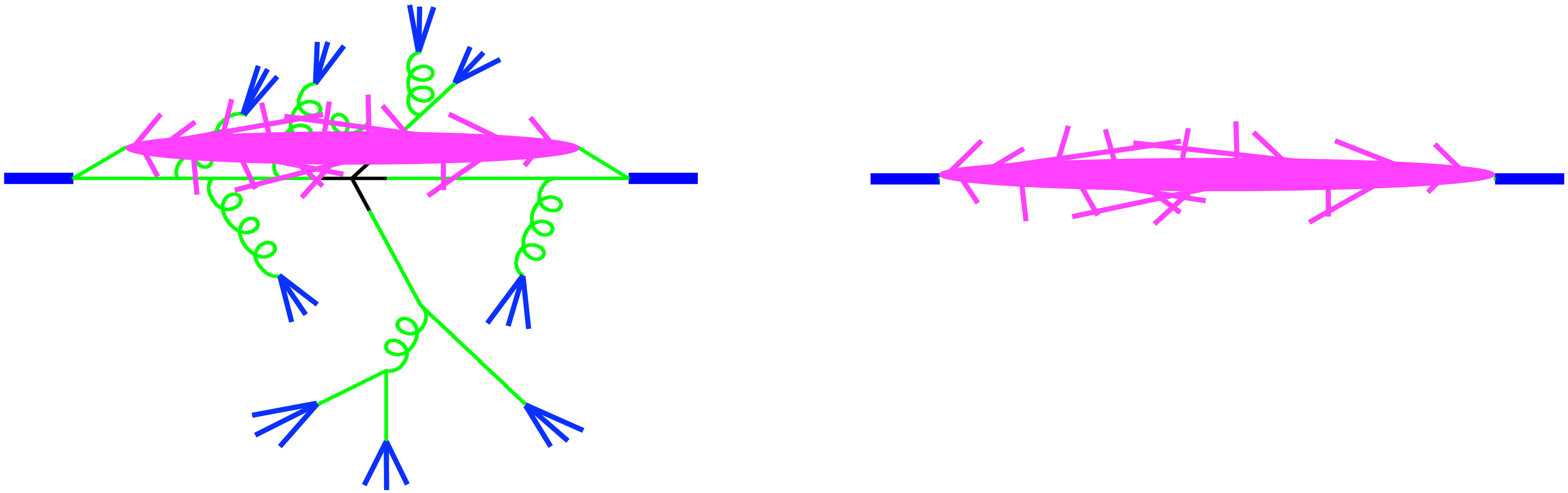


Two models:

- Non-perturbative: Soft parton—parton cross section is so large that the remnants always undergo a soft collision.
- Perturbative: 'Hard' parton—parton cross section huge at low p_t, high energy, dominates inelastic cross section and is calculable.

Soft Underlying Event Model (HERWIG)

Compare underlying event with 'minimum bias' collision
('typical' inelastic proton—proton collision)

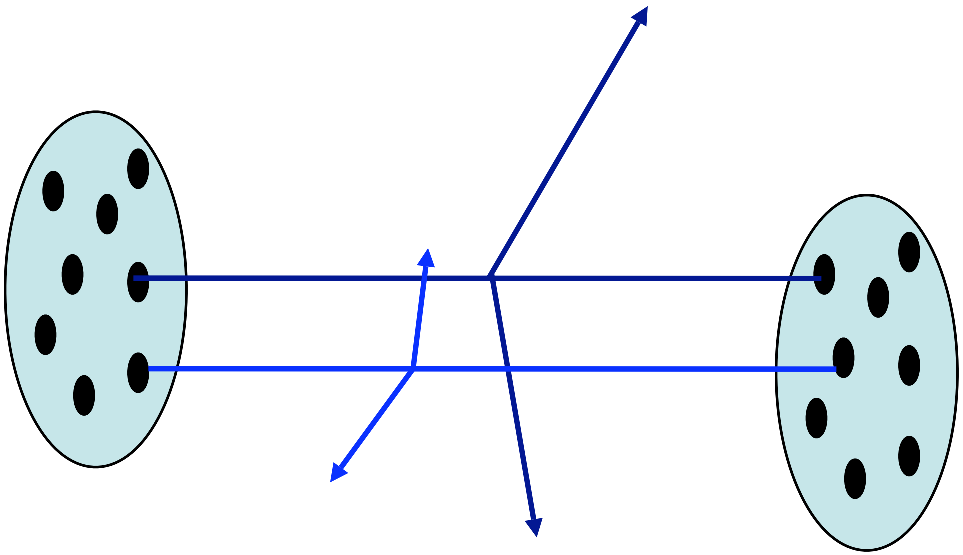


Parametrization of (UA5) data + model of energy dependence

Multiparton Interaction Model (PYTHIA/JIMMY)

For small $p_{t\ min}$ and high energy inclusive parton—parton cross section is larger than total proton—proton cross section.

→ More than one parton—parton scatter per proton—proton



Need a model of spatial distribution within proton

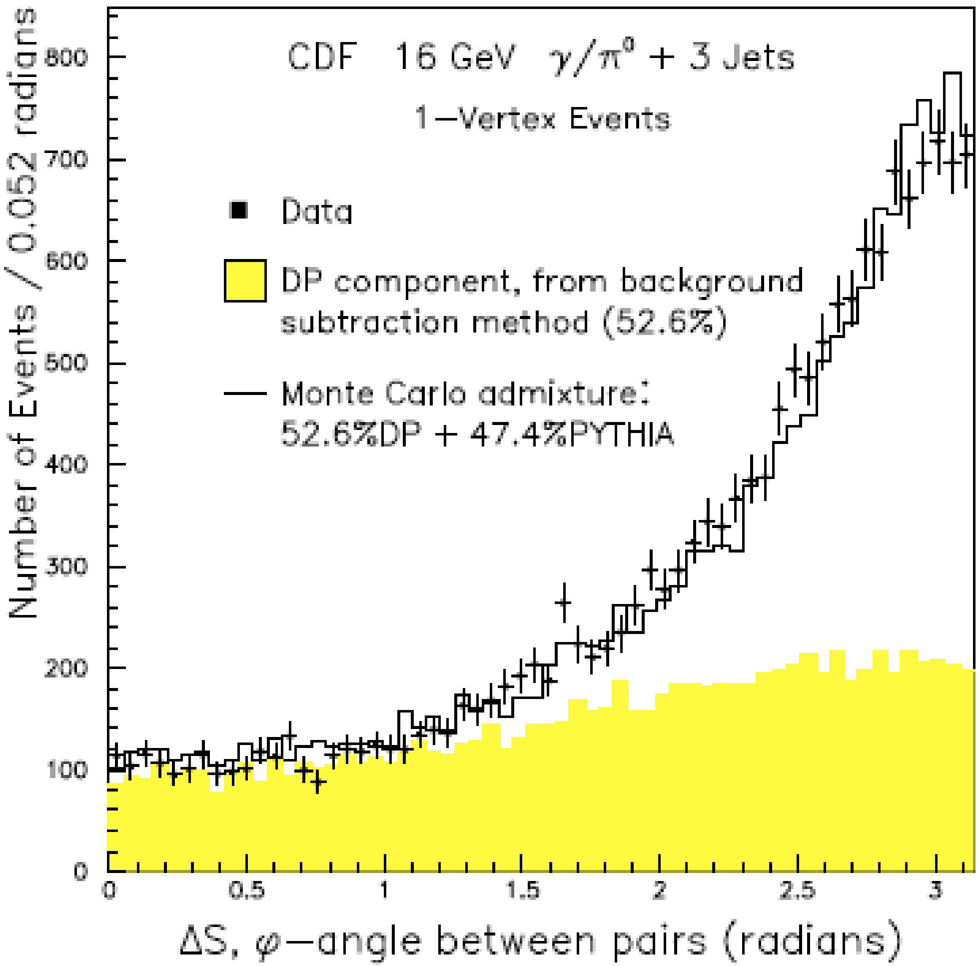
→ Perturbation theory gives you n-scatter distributions

Double Parton Scattering

- CDF Collaboration, PR D56 (1997) 3811

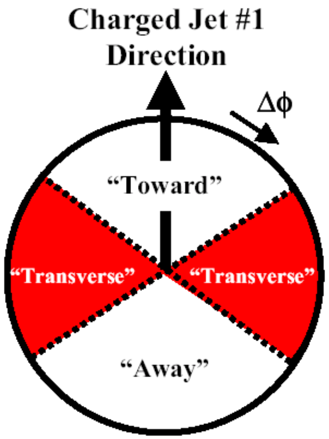
$$\sigma_{\text{DPS}} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\text{eff}}}$$

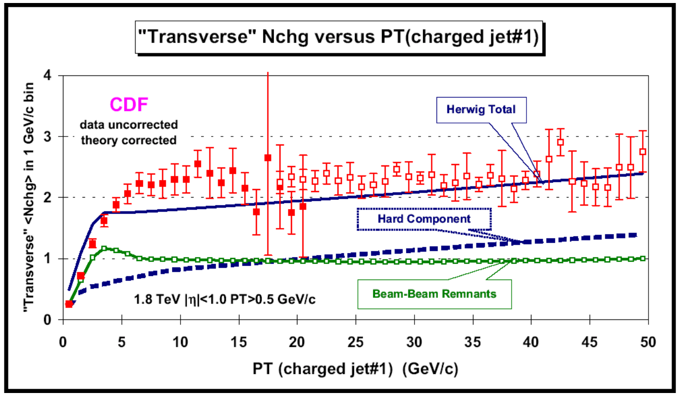
$$\sigma_{\text{eff}} = 14 \pm 1.7^{+1.7}_{-2.3}\ \text{mb}$$

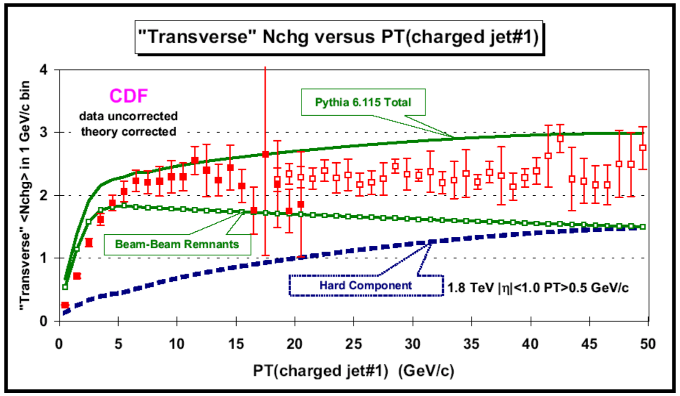


Some Warnings

- Not everyone means same thing by "underlying event"
  - Remnant—remnant interaction
  - Everything except hard process final state
- Separation into components is model dependent
  - Operational definition (R Field): "transverse" regions

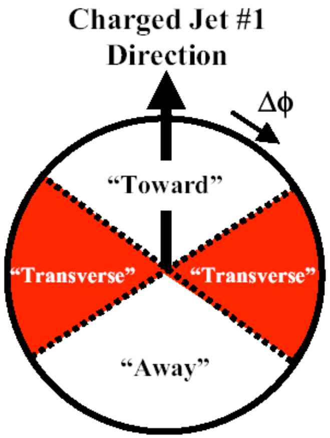


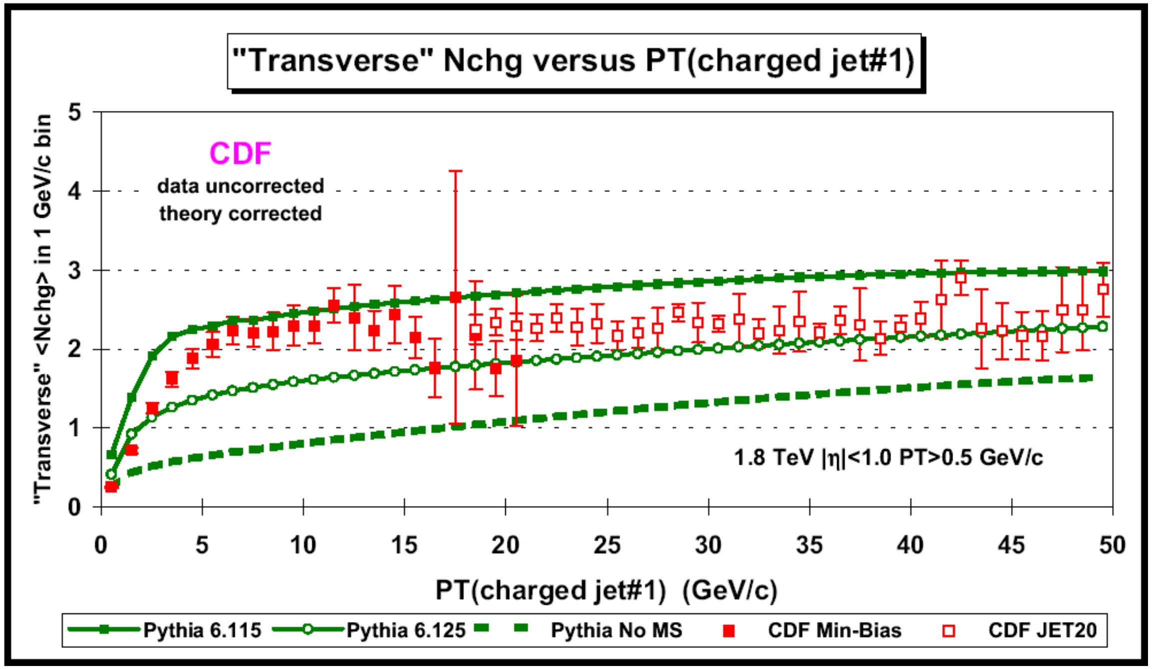




Tuning PYTHIA to the Underlying Event

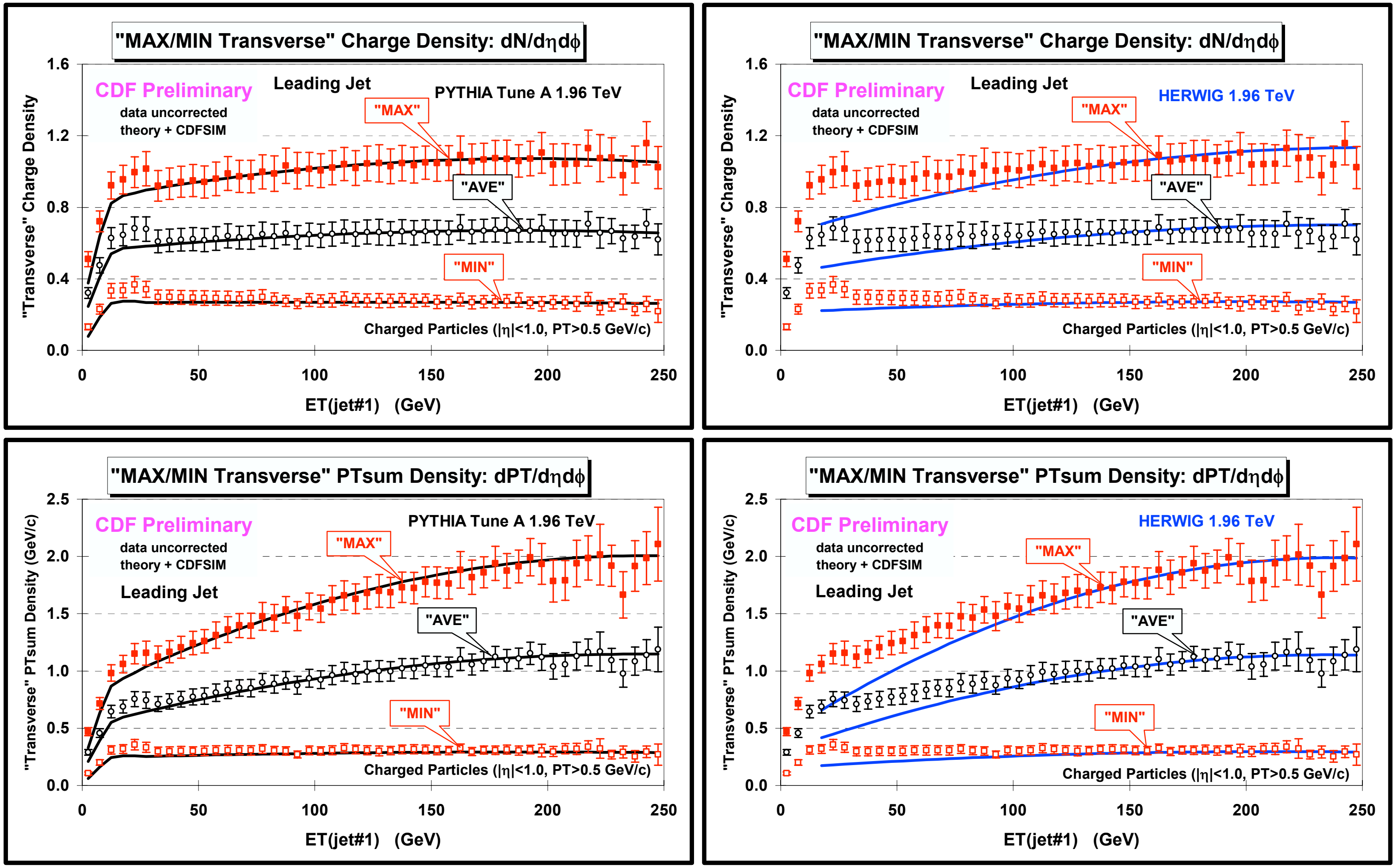
- Rick Field (CDF): keep all parameters that can be fixed by LEP or HERA at their default values. What's left?
- Underlying event. Big uncertainties at LHC...





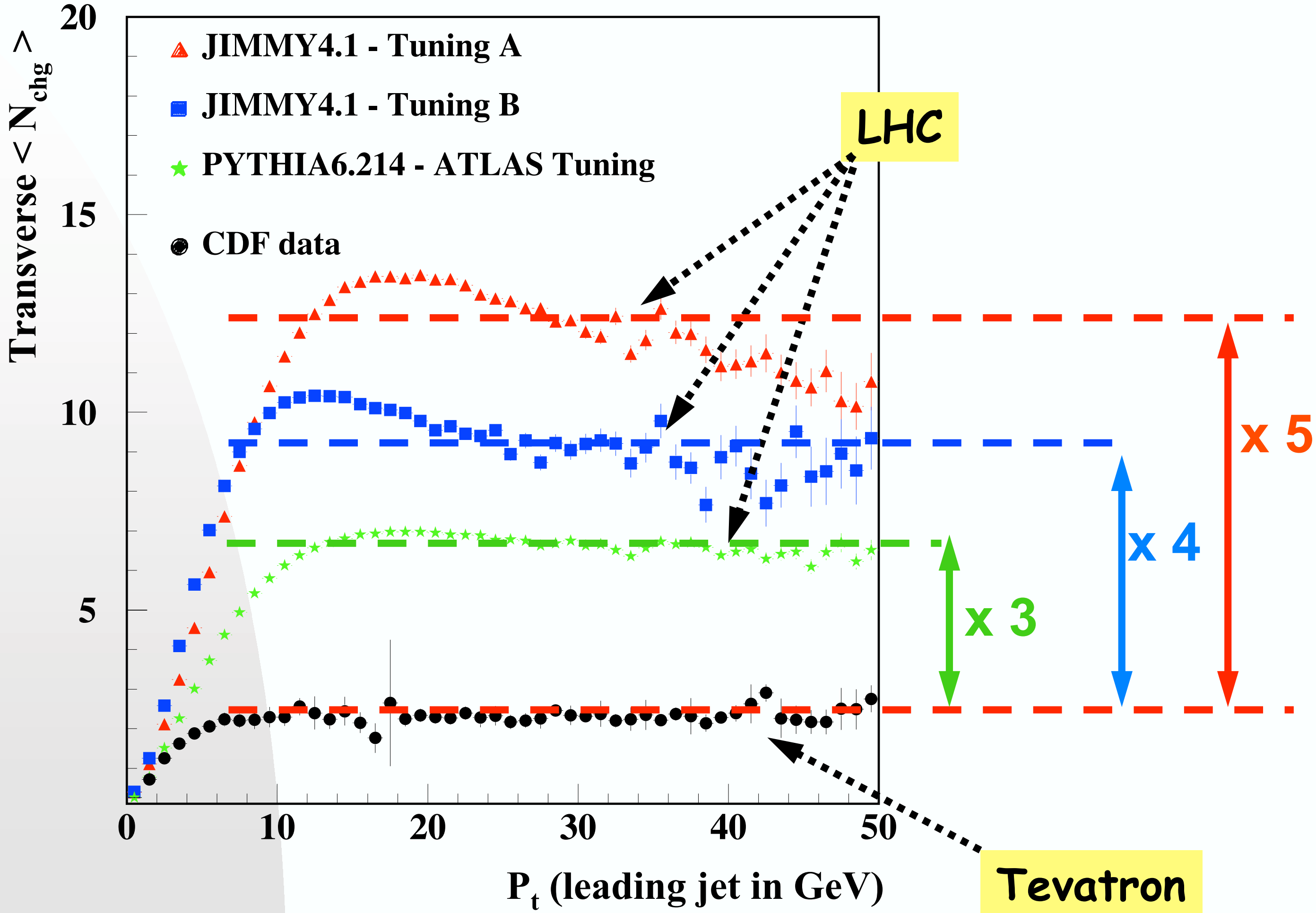
Leading Jet: "MAX & MIN Transverse" Densities

PYTHIA Tune A HERWIG



Charged particle density and PTsum density for "leading jet" events versus E_T(jet#1) for PYTHIA Tune A and HERWIG.

LHC predictions: JIMMY4.1 Tunings A and B vs. PYTHIA6.214 – ATLAS Tuning (DC2)



A. M. Moraes

Minimum-bias and the Underlying Event at the LHC

5th November 2004

Summary

- Hard Process is very well understood: firm perturbative basis
- Parton Shower is fairly well understood: perturbative basis, with various approximations
- Hadronization is less well understood: modelled, but well constrained by data. Extrapolation to LHC fairly reliable.
- Underlying event least understood: modelled and only weakly constrained by existing data. Extrapolation?

- Always ask "What physics is dominating my effect?"

Problems on Event Generator Physics

Use the event generator of your choice for the following exercises.

1. Generate hadronic Z^0 decays via $e^+e^- \to Z^0 \to q\bar{q}$ ($q = d, u, s, c, b$). Compare the charged multiplicity distribution and the distribution of $\ln(1/x_p)$ ($x_p = 2|\mathbf{p}|/\sqrt{s}$) with LEP1 data.

2. Generate $e^+e^- \to q\bar{q}$ ($q = d, u, s, c, b$) at higher energies, $\sqrt{s} = 200, 500, 1000$ GeV. (Turn off QED radiation, otherwise you will be dominated by $e^+e^- \to Z^0\gamma$.)

   (a) Compare the mean charged multiplicity with the QCD prediction

   $$\langle n_{\rm ch}\rangle \sim a\frac{\exp\sqrt{cL}}{\sqrt{L}}$$

   where a is a non-perturbative constant, $c = 72/23$ and $L = \ln(s/\Lambda^2)$. At each energy, compute the variance and hence the error in your MC result.

   (b) Compare the position ξ_p of the peak in the distribution of $\ln(1/x_p)$ with the QCD prediction

   $$\xi_p \sim \text{const} + \frac{1}{4}\ln s\ .$$

3. Generate $e^+e^- \to t\bar{t}$ at threshold and force the tops to decay leptonically (to e or μ).

   (a) Compare the charged and neutral lepton p_T distributions with those shown in the lectures.

   (b) Explain why the neutrinos tend to have higher p_T than the charged leptons. How would things change if the decay went via $t \to bH^+$ with $m_{H^+} = m_W$?

4. Same as qu.3, but for $pp \to t\bar{t}X$ at LHC energy ($\sqrt{s} = 14$ TeV). Here p_T should be defined relative to the direction of motion of the parent t or $\bar{t}$.