The Composite Higgs option

Theory and Phenomenology

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SNFT 2007 - Parma



Motivation: the Hierarchy Problem

FACT: EW precision data from LEP strongly suggest the presence of a light Higgs boson, mH~100-300 GeV





what is the nature of the Higgs boson ? is it a fundamental (= elementary) scalar field ?

naively, if the Higgs boson is a fundamental scalar, the natural value of its mass is of the order of the largest scale in the theory (UV instability) :





$$\delta m_H^2 = \frac{3}{64\pi^2} (3g_2^2 + g_1^2)\Lambda^2$$

$$\delta m_H^2 = -\frac{3}{4\pi^2} \frac{m_t^2}{v^2} \Lambda^2$$

possibility #1:

all the couplings of the theory remain weak up to the Planck scale and the Higgs is an elementary scalar (perturbative case)

then there must be a symmetry protection (and new particles) which ensures a light Higgs

example:





possibility #2:

(a subsector of) the theory becomes strongly interacting at a scale Λ and the Higgs is a composite bound state (strongly-interacting case)

for virtual momenta larger than the compositeness scale the Higgs couplings switch off (form factors)

= finite



the other resonances of the strongly-interacting sector cannot be too light in order not to spoil the success of EW precision tests:

 $m \sim \Lambda \gtrsim a$ few TeV



can be the composite Higgs be naturally lighter than the other resonances ?

yes, if it is a (pseudo) Goldstone boson

an example from QCD: the Pion

strongly interacting sector = QCD 0



QCD has other resonances, with m~1 GeV 0 ex: the ρ , m_{ρ} = 770 MeV



 \bigcirc the pion is lighter (m_{π} = 135 MeV): it is the Goldstone boson of chiral symmetry breaking consider QCD with 2 flavors in the chiral limit $(m_q=0)$: it has an SU(2)_LxSU(2)_R global symmetry under rotation of L and R quarks separately:

 $\begin{pmatrix} u_L \\ d_L \end{pmatrix} \longrightarrow U_L \cdot \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \longrightarrow U_R \cdot \begin{pmatrix} u_R \\ d_R \end{pmatrix}$

the QCD condensate breaks the chiral symmetry down to the diagonal (vectorial) subgroup:

 $\langle \bar{q}_R q_L \rangle \longrightarrow U_L \langle \bar{q}_R q_L \rangle U_R^{\dagger} \qquad \text{SU}(2)_L \times \text{SU}(2)_R \longrightarrow \text{SU}(2)_V$

the spontaneous breaking implies 3 Goldstones, transforming like a triplet under SU(2)_v: the pions, π

Goldstones = $[SU(2)_L \times SU(2)_R]/[SU(2)_V] = 3$

how a potential (hence a mass) for the pion is radiatively generated



let us turn on the photon:

1. the photon is a field external to the QCD sector (= "elementary") which gauges the $U(1)_V$ subgroup of the QCD flavor symmetry

 $Q = T_L^3 + T_R^3$

2. gauging only a subgroup of $SU(2)_L \times SU(2)_R$ is equivalent to an explicit breaking

EXPLICIT BREAKING

the (charged) pion will acquire a potential through the electromagnetic loop corrections

Π

Form factor

π

let us derive the pion potential

we treat the pion as a constant classical background field, adopting a non-linear sigma model description

 $\Sigma = \exp(i\tau^a \pi^a / f_\pi) \qquad \Sigma \to U_L \Sigma U_R^{\dagger}$



we use a trick: let us gauge the entire SU(2)_LxSU(2)_R flavor symmetry – not only the subgroup U(1)_V – by turning on the external sources L_{μ} , R_{μ}

then, the most general $[SU(2)_L \times SU(2)_R]$ -invariant effective action, at the quadratic order in the external L, R sources, is:

 $\mathcal{L}_{eff} = \frac{1}{2} \left(P_T \right)_{\mu\nu} \left(\Pi_L(p^2) \operatorname{Tr} \{ L_\mu L_\nu \} + \left(\Pi_R(p^2) \operatorname{Tr} \{ R_\mu R_\nu \} - \left(\Pi_{LR}(p^2) \operatorname{Tr} \{ \Sigma^{\dagger} L_\mu \Sigma R_\nu \} \right) \right) \right)$

 Π_L, Π_R, Π_{LR} are form factors, originating from the exchange of the pion and heavier resonances

$$\left(P_T\right)_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

in order to derive a few properties of the form factors, it is useful to

1. rewrite the action in terms of vectorial and axial sources: 1

$$V_{\mu} = \frac{1}{\sqrt{2}} \left(L_{\mu} + R_{\mu} \right) \qquad A_{\mu} = \frac{1}{\sqrt{2}} \left(R_{\mu} - L_{\mu} \right)$$

2. work in the vacuum $\langle \Sigma
angle = 1$

hence:

$$\mathcal{L}_{eff} = \frac{1}{2} \left(P_T \right)_{\mu\nu} \left[\Pi_{VV}(p^2) \operatorname{Tr}\{V_{\mu}V_{\nu}\} + \Pi_{AA}(p^2) \operatorname{Tr}\{A_{\mu}A_{\nu}\} - \Pi_{VA}(p^2) \operatorname{Tr}\{V_{\mu}A_{\nu} + A_{\mu}V_{\nu}\} \right]$$

$$\Pi_{VV} = \frac{\Pi_L + \Pi_R - \Pi_{LR}}{2} \qquad \Pi_{AA} = \frac{\Pi_L + \Pi_R + \Pi_{LR}}{2} \qquad \Pi_{VA} = \frac{\Pi_R - \Pi_L}{2}$$

now, we know that in the two-point Axial Green function $\langle A_{\mu}A_{\nu}\rangle$, and only there, there is a pole due to the exchange of the pion:

 $\langle A_{\mu}A_{\nu}\rangle = \left(p^2\eta_{\mu\nu} - p_{\mu}p_{\nu}\right)\left[f_{\pi}^2\frac{1}{p^2} + \dots\right]$

hence:

 $\Pi_{VV}(0) = 0$ $\Pi_{AA}(0) = f_{\pi}^{2}$ $\Pi_{VA}(0) = 0$

⊗----⊗

π

$$\Pi_{LR}(0) = f_{\pi}^{2}$$
$$\Pi_{L}(0) = \Pi_{R}(0) = \frac{1}{2}\Pi_{LR}(0)$$

 $\overline{\langle |A_{\mu}(x)|\pi\rangle} = -i p_{\mu} f_{\pi} e^{-ipx}$

in general, Π_{LR} is sensitive to the difference between axial and vector currents, i.e. it is an order parameter of the spontaneous breaking of the chiral symmetry:

 $\Pi_{LR}(p^2) = \Pi_{AA}(p^2) - \Pi_{VV}(p^2)$

we thus expect that it goes to zero at energies much larger than the QCD scale (= the scale at which chiral symmetry breaking occurs)

 $|\Pi_{LR}(Q^2) \to 0$ for $-p^2 = Q^2 \gg \Lambda_{QCD}^2$

using the Operator Product Expansion (OPE) of two currents, one can prove indeed that:

$$\Pi_{LR}(Q^2) \propto \frac{1}{Q^4}$$

A quick sketch of the OPE argument :

[see Shifman, Vainshtein, Zakharov NPB 147 (1979) 385 for more details]

Given any two operators $O_1(x_1), O_2(x_2)$, their time-ordered product can be expressed as a sum of local operators times c-number coefficients that depend on the separation $(x_1 - x_2)$:

$$T\{O_1(x)O_2(0)\} = \sum_n C_{12}^{(n)}(x)O_n(0)$$

• This is an equality between operators, i.e. it implies the equality of any Green function made with them.

• The sum extends over all operators with the same global symmetries of the product ${\cal O}_1 {\cal O}_2$

• In the case of the product of two conserved currents, the OPE reads:

$$i \int d^4x \ e^{iq \cdot x} T\{J^{\mu}(x)J^{\nu}(0)\} = \left(q^2 \eta^{\mu\nu} - q^{\mu}q^{\nu}\right) \sum_n C^{(n)}(x)O_n(0)$$

By dimensional analysis, the larger is the dimension of the operator O_n , the more suppressed will be its coefficient at large $Q^2 = -q^2$:

$$C^{(n)}(Q^2) \sim \frac{1}{Q^{[O_n]}}$$

Hence, to deduce the large- Q^2 behavior we can concentrate on the lowest-dimension operators.

In the case of QCD, the $SU(3)_c$ gauge-invariant (scalar) operators of dimension 6 or less that contribute to the VEV of the product of two axial or vectorial currents are:

dimension operator 1 $\left(\right)$ $O_M = \bar{\Psi} M \Psi$ 4 $O_G = G^a_{\mu\nu} G^{a\,\mu\nu}$ 4 $O_{\sigma} = \bar{\Psi} \sigma_{\mu\nu} t^a \tilde{M} \Psi \ G^{a \, \mu\nu}$ 6 $O_{\Gamma} = \left(\bar{\Psi} \Gamma_1 \Psi \right) \left(\bar{\Psi} \Gamma_2 \Psi \right)$ 6 $O_f = f^{abc} G^a_{\mu\nu} G^b_{\nu\gamma} G^c_{\gamma\mu}$ 6

- The operators O_M, O_σ break explicitly the chiral symmetry and thus must be proportional to the quark masses. They vanish in the chiral limit
- O_{Γ} is the only chiral-invariant operator whose VEV breaks spontaneously the chiral symmetry, distinguishing between axial and vectorial currents

Hence:
$$\Pi_{LR}(Q^2) = Q^2 C_{O_{\Gamma}}(Q^2) \langle O_{\Gamma} \rangle + \dots$$
$$= Q^2 \left[\frac{\delta}{Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right) \right]$$

The coefficient δ has been computed in perturbation theory (in the large-N limit):

$$\delta = 8\pi^2 \left(\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right) \left(\langle \bar{\Psi}\Psi \rangle\right)^2$$

[Shifman, Vainshtein, Zakharov]

At this point, let us go back to the chiral Lagrangian:

 $\mathcal{L}_{eff} = \frac{1}{2} \left(P_T \right)_{\mu\nu} \left[\Pi_L(p^2) \text{Tr}\{L_{\mu}L_{\nu}\} + \Pi_R(p^2) \text{Tr}\{R_{\mu}R_{\nu}\} - \Pi_{LR}(p^2) \text{Tr}\{\Sigma^{\dagger}L_{\mu}\Sigma R_{\nu}\} \right]$

Let us turn on only the photon (setting to zero the other source fields):

$$L_{\mu} = R_{\mu} = Q v_{\mu}$$

 $Q = T^3 = \frac{1}{2} \begin{pmatrix} 1 & 0\\ 0 & -1 \end{pmatrix}$

and use:

$$\Sigma = \exp\left(i\frac{\pi^a\tau^a}{f_\pi}\right) = \mathbf{1}\cos\left(\pi/f_\pi\right) + i\hat{\pi}^a\tau^a\sin\left(\pi/f_\pi\right)$$

$$\hat{\pi}^a \equiv \frac{\pi^a}{\pi} \qquad \pi \equiv \sqrt{\sum_a (\pi^a)^2}$$

one has:

$$\mathcal{L}_{eff} = \frac{1}{2} \left(P_T \right)_{\mu\nu} v_{\mu} v_{\nu} \left[\left(\Pi_L(p^2) + \Pi_R(p^2) \right) \operatorname{Tr} \{Q^2\} - \Pi_{LR}(p^2) \operatorname{Tr} \{\Sigma^{\dagger} Q \Sigma Q\} \right]$$

after a little bit of algebra one finds:

$$\operatorname{Tr}\{\Sigma^{\dagger}Q\Sigma Q\} = \operatorname{Tr}\{Q^{2}\} - \frac{(\pi^{1})^{2} + (\pi^{2})^{2}}{\pi^{2}}\sin^{2}\frac{\pi}{f_{\pi}}$$

as expected the photon couples only to the charged pion. The neutral pion remains an exact Goldstone with no potential. We can thus set

$$\pi_0 = 0$$
 $\pi^2 = (\pi_1)^2 + (\pi_2)^2$

it follows:

$$\mathcal{L}_{eff} = \frac{1}{2} \left(P_T \right)_{\mu\nu} v_{\mu} v_{\nu} \left[\Pi_{VV}(p^2) + \Pi_{LR}(p^2) \sin^2 \frac{\pi}{f_{\pi}} \right]$$

From the above Lagrangian it follows the Coleman-Weinberg effective potential:

$$V(\pi) = \frac{3}{16\pi^2} \int_0^\infty dQ^2 Q^2 \log\left(1 + \frac{1}{2} \frac{\Pi_{LR}(Q^2)}{\Pi_{VV}(Q^2)} \sin^2\frac{\pi}{f_\pi}\right)$$

Given that
$$\Pi_{VV}(Q^2) = \frac{Q^2}{e^2} + \dots$$
 $\Pi_{LR}(Q^2) \propto \frac{1}{Q^4}$

the integral can be reasonably well approximated by expanding the logarithm:

$$V(\pi) \simeq \frac{3\alpha_{em}}{8\pi^2} \sin^2 \frac{\pi}{f_\pi} \int_0^\infty dQ^2 \ \Pi_{LR}(Q^2)$$

Let us derive an estimate for the integral $\int dQ^2 \Pi_{LR}(Q^2)$

in the limit of large number of QCD colors (1/N expansion), the product of two vector and axial currents can be written in terms of an infinite sum over narrow resonances:

$$(P_T)_{\mu\nu}\Pi_{VV}(q^2) = \langle V_{\mu}V_{\nu}\rangle = (p^2\eta_{\mu\nu} - p_{\mu}p_{\nu})\sum_{n}\frac{F_{\rho_n}^2}{p^2 - m_{\rho_n}^2}$$

 $(P_T)_{\mu\nu}\Pi_{AA}(q^2) = \langle A_{\mu}A_{\nu} \rangle = (p^2\eta_{\mu\nu} - p_{\mu}p_{\nu}) \left| \frac{f_{\pi}^2}{p^2} + \sum_{\nu} \frac{F_{a_n}^2}{p^2 - m_{a_n}^2} \right|$

under this assumption, the information on the large-momentum behavior of $\Pi_{LR}(q^2)$ from the OPE

can be translated into two sum rules (Weinberg's Sum Rules):

the OPE $\lim_{Q^2 \to \infty} Q^2 \Pi_{LR}(Q^2) = 0$ $\sum [F^2 - F^2] - f^2$

$$\sum_{n} [r_{\rho_{n}} - r_{a_{n}}] = J_{\pi}$$
$$\sum_{n} [m_{\rho_{n}}^{2} F_{\rho_{n}}^{2} - m_{a_{n}}^{2} F_{a_{n}}^{2}] = 0$$

A further simplification comes by assuming that the sum is dominated by the first vectorial and axial resonances (Vector Meson Dominance):

 $F_{\rho}^{2} - F_{a}^{2} = f_{\pi}^{2}$ $m_{\rho}^{2}F_{\rho}^{2} - m_{a}^{2}F_{a}^{2} = 0$

 $\lim_{Q^2 \to \infty} \Pi_{LR}(Q^2) = 0$

Solving for F_{ρ} , F_{a} and substituting into $\Pi_{LR}(q^{2})$ one obtains:

$$\Pi_{LR}(Q^2) \simeq f_{\pi}^2 \left| 1 + \frac{m_{\rho}^2}{m_a^2 - m_{\rho}^2} \frac{Q^2}{Q^2 + m_a^2} - \frac{m_a^2}{m_a^2 - m_{\rho}^2} \frac{Q^2}{Q^2 + m_{\rho}^2} \right|$$

hence:

$$\int_{0}^{\infty} dQ^2 \ \Pi_{LR}(Q^2) = f_{\pi}^2 \ \frac{m_a^2 m_{\rho}^2}{m_a^2 - m_{\rho}^2} \ \log\left(\frac{m_a^2}{m_{\rho}^2}\right)$$

Since experimentally $m_a > m_\rho$, the integral is positive and the potential is minimized for:

$$\sin\frac{\langle \pi \rangle}{f_{\pi}} = 0$$

In other words, the radiative corrections align the vacuum in the $U(1)_Q$ -preserving direction, and the photon remains massless.

The charged pion acquires a mass, while the neutral one stays massless (still an exact Goldstone boson):

$$m_{\pi^+}^2 - m_{\pi^0}^2 = \frac{3\alpha_{em}}{4\pi} \frac{m_a^2 m_\rho^2}{m_a^2 - m_\rho^2} \log\left(\frac{m_a^2}{m_\rho^2}\right)$$

[First derived by: Das et al. PRL 18 (1967) 759]

The above formula is still valid after the explicit breaking of chiral symmetry due to the quark masses is turned on (i.e. the difference in mass is dominated by the e.m. correction). Numerically:

 $\Delta m_{\pi} |_{\mathrm{TH}} = 5.2 \,\mathrm{MeV} \qquad \Delta m_{\pi} |_{\mathrm{EXP}} = 4.6 \,\mathrm{MeV}$

a simple Composite Higgs Model

On the example of QCD, let us consider the possibility that a new strongly-interacting sector exists (i.e. with new "quarks" and new "gluons") which has a flavor symmetry G broken spontaneously down to G' by the strong dynamics





we require two conditions:

1. the SM gauge group must be $G \to G' \subset G_{SM}$ embeddable in the unbroken subgroup G':

2. G/G' must contain (at least) one $SU(2)_{L}$ doublet (the Higgs)



notice:

- in absence of the external gauging of G_{SM}, the Higgs is an exact Goldstone boson (hence massless)
- The orientation of G_{SM} compared to G' in the vacuum depends on the Higgs potential arising at 1 loop



An explicit (minimal) example:

 $G = SO(5) \times U(1)_{\times}$ $G' = SO(4) \times U(1)_{\times}$

 $SO(4) \sim SU(2)_L \times SU(2)_R$

 $Y=T_L^3+X$

Goldstones = $[SO(5)/SO(4)] = 10 - 6 \neq 4$

4 real scalars: a 4 of SO(4) = a doublet of $SU(2)_{L}$

The SO(4)~SU(2)_LxSU(2)_R isomorphism can be made explicit by associating to any SO(4) vector $v^{\hat{a}}$ a 2x2 matrix $V \equiv v^{\hat{a}}\sigma^{\hat{a}}$:

$$v^{\hat{a}} \to S^{\hat{a}\hat{b}}v^{\hat{b}}$$
 $V \equiv v^{\hat{a}}\sigma^{\hat{a}}$ $\sigma^{\hat{a}} \equiv (\vec{\sigma}, -i\mathbf{1})$
 $S \in SO(4)$ $V \to LVR^{\dagger}$ $L, R \in SU_{L,R}$

S preserves the norm of the vector, $\left|v\right|$

L, R preserve the determinant of V, where: det $V = -|v|^2$

Notice: for each matrix $S \in SO(4)$ there are two $SU(2)_L \times SU(2)_R$ transformations that act in the same way: (L, R) and (-L, -R). Hence, the exact equivalence at the level of group elements is:

$$SO(4) = \frac{SU(2)_L \times SU(2)_R}{\mathbf{Z}_2}$$

To derive the Higgs potential, let us repeat all the steps we made in the QCD example:

1. The Goldstones can be described in terms of a sigma field (transforming like a 5 of SO(5))

 $\Sigma = \Sigma_0 e^{\Pi/F_{\pi}}$

 $\Sigma_0 = (0, 0, 0, 0, 1)$ $\Pi = -i T^{\hat{a}} h^{\hat{a}} \sqrt{2}, \quad \hat{a} = 1, 2, 3, 4$ $T^{\hat{a}} \in \text{Alg}\{\text{SO}(5)/\text{SO}(4)\}$

$$\Sigma = \frac{1}{h} \sin \frac{h}{F_{\pi}} \left(h_1, h_2, h_3, h_4, h \cot \frac{h}{F_{\pi}} \right) \qquad h \equiv \sqrt{(h^{\hat{a}})^2}$$

erators:

$$T_{ij}^{a_L,a_R} = -\frac{i}{2} \left[\frac{1}{2} \epsilon^{abc} \left(\delta_i^b \delta_j^c - \delta_j^b \delta_i^c \right) \pm \left(\delta_i^a \delta_j^4 - \delta_j^a \delta_i^4 \right) \right]$$

$$T_{ij}^{\hat{a}} = -\frac{i}{\sqrt{2}} \left(\delta_i^{\hat{a}} \delta_j^5 - \delta_j^{\hat{a}} \delta_i^5 \right)$$

SO(5) generators:

2. Gauge all $SO(5) \times U(1)_X$ generators by means of external sources (think of them as classical sources)

The most general [SO(5) \times U(1) $_{\times}$]-invariant effective Lagrangian at the quadratic order in the external sources is:

$$\mathcal{L} = \frac{1}{2} (P_T)_{\mu\nu} \left[\Pi_0^X(p^2) X_\mu X_\nu + \Pi_0(p^2) \operatorname{Tr} (A_\mu A_\nu) + \Pi_1(p^2) \Sigma A_\mu A_\nu \Sigma^T \right]$$

The form factors Π_0^X, Π_0, Π_1 encode the dynamics of the strong sector and are generated by the exchange of massive resonances plus the fluctuations of the Goldstones around the vacuum Σ

(one can add bare kinetic and gauge-fixing terms for the external fields as well)

3. Extract information on the form factors by expanding around the SO(4)-preserving vacuum Σ_0 :

$$\mathcal{L} = \frac{1}{2} (P_T)_{\mu\nu} \left[\Pi_0^X(p^2) X_\mu X_\nu + \Pi_0(p^2) A_\mu A_\nu + \left(\Pi_0 + \frac{1}{2} \Pi_1 \right) A_\mu^{\hat{a}} A_\nu^{\hat{a}} \right]$$

SO(4) sources

SO(5)/SO(4) sources

hence:

$$\Pi_a = \Pi_0$$
$$\Pi_{\hat{a}} = \left(\Pi_0 + \frac{1}{2}\Pi_1\right)$$

$$(P_T)_{\mu\nu}\Pi_a(p^2) = \langle J^a_{\mu}J^a_{\nu}\rangle = \left(p^2\eta_{\mu\nu} - p_{\mu}p_{\nu}\right)\sum_n \frac{F^2_{\rho_n}}{p^2 - m^2_{\rho_n}}$$
$$(P_T)_{\mu\nu}\Pi_{\hat{a}}(p^2) = \langle J^{\hat{a}}_{\mu}J^{\hat{a}}_{\nu}\rangle = \left(p^2\eta_{\mu\nu} - p_{\mu}p_{\nu}\right)\left[\frac{1}{p^2}\frac{F^2_{\pi}}{2} + \sum_n \frac{F^2_{a_n}}{p^2 - m^2_{a_n}}\right]$$

we then deduce:

 $\checkmark \quad \Pi_1(0) = F_\pi^2 \qquad \qquad \Pi_0(0) = \Pi_0^X(0) = 0$

 $\Pi_1(q^2)$ is sensitive to the difference between currents associated with broken and unbroken generators, i.e. it is an order parameter of the spontaneous breaking of the SO(5) symmetry.

We thus expect that it goes to zero at energies much larger than the compositeness scale:

for $Q^2 \gg (4\pi F_\pi)^2$

$$\Pi_1(Q^2) \propto \frac{1}{(Q^2)^n}$$
4. Turn on only the SU(2)_LxU(1)_Y elementary gauge fields (in the Σ background). The effective Lagrangian reads:

$$\mathcal{L} = \frac{1}{2} (P_T)_{\mu\nu} \left[\left(\Pi_0^X + \Pi_0 + \frac{\Pi_1}{4} \sin^2 \frac{h}{F_\pi} \right) B_\mu B_\nu + \left(\Pi_0 + \frac{\Pi_1}{4} \sin^2 \frac{h}{F_\pi} \right) A_\mu^{a_L} A_\nu^{a_L} + 2 \sin^2 \frac{h}{F_\pi} \Pi_1(p^2) \hat{H}^{\dagger} T^{a_L} Y \hat{H} A_\mu^{a_L} B_\nu \right]$$

A one-loop exchange of gauge fields then leads to the following Coleman-Weinberg effective potential for the Higgs (neglecting for simplicity the contribution from $U(1)_Y$):

$$\hat{H} \equiv \frac{1}{h} \begin{bmatrix} h_1 - ih_2 \\ h_3 - ih_4 \end{bmatrix}$$

$$V(h) = \frac{9}{2} \int \frac{d^4 Q}{(2\pi)^4} \log\left(1 + \frac{1}{4} \frac{\Pi_1(Q^2)}{\Pi_0(Q^2)} \sin^2\frac{h}{F_\pi}\right)$$

At this point, without specifying the strong dynamics one can <u>not of course proceed further</u> and evaluate the integral.

However, it is reasonable to assume that $\Pi_1(q^2)$ will go to zero fast enough at energies much higher than the compositeness scale, so that the integral will converge:

$$\Pi_1(Q^2) \propto \frac{1}{(Q^2)^n}$$

with $n \ge 2$

 $V(h) = \frac{9}{128\pi^2} \sin^2\left(\frac{h}{F_{\pi}}\right) \int_0^\infty dQ^2 \ Q^2 \ \frac{\Pi_1(Q^2)}{\Pi_0(Q^2)} + \dots$

Then, as in the case of QCD, one can derive two Weinberg's sum rules and compute the integral in terms of the resonances' masses and decay constants assuming a large N and Vector Meson Dominance.

Also, if the integral is positive, as in QCD, then the potential is minimized for $\langle h \rangle = 0$ and the Electroweak symmetry is unbroken.

an example of strong dynamics

A composite Higgs from an warped extra dimension

Consider a 5-dimensional theory with one extra spatial coordinate, \mathcal{Y} , compactified on an interval ($0 \le y \le L$):

Randall and Sundrum have shown that a metric of the form (5D Anti-deSitter spacetime, AdS5)

$$ds^2 = e^{-2ky} dx^{\mu} dx^{\nu} \eta_{\mu\nu} - dy^2$$

is a solution of Einstein's equations of motion if one carefully chooses the cosmological constants in the bulk and on the branes.

brane	Bulk	brane
$\int \overset{x^{\mu}}{\searrow} y$		
y = 0		y = L

an example of strong dynamics

A composite Higgs from an warped extra dimension

UV

The "warp" factor

$$ds^2 = e^{-2ky} dx^{\mu} dx^{\nu} \eta_{\mu\nu} - dy^2$$

implies a rescaling (redshift) of all the dimensionful quantities by moving along the extra dimension.

In other words:

translation in y = Weyl rescaling in 4D



an example of strong dynamics

A composite Higgs from an warped extra dimension

From that it follows a possible solution of the Hierarchy Problem by the geography of wave functions in the 5D bulk:

 $k \sim M_{Pl}$, $k e^{-2kL} \sim \text{TeV}$





the Higgs structure along the extra dimension appears like a form factor for an observer on the UV brane The 5D theory is a calculable model of the strong dynamics

One can easily build an SO(5)/SO(4) model and compute the form factors $\Pi_{0,1}$ analytically. One finds:

 $\Pi_1(Q^2)$ dies off exponentially $\Pi_1(Q^2) \propto e^{-QM_{IR}}$ at large momenta: $M_{IR} = k e^{-kL} \sim \text{TeV}$

The integral $\int_0^{\infty} dQ^2 Q^2 \Pi_1(Q^2)/\Pi_0(Q^2)$ is positive, so that the gauge contribution to the Higgs potential does not trigger the Electroweak symmetry breaking

The EW symmetry is broken by the top quark contribution, which is larger than the gauge one

elementary VS composite

- how to distinguish an elementary from a composite Higgs -

Let us assume that the EW symmetry is broken: $\sin(\langle h \rangle / F_{\pi}) \neq 0$ We can derive the couplings of the Higgs to the gauge fields from the Lagrangian

$$\mathcal{L} = \frac{1}{2} (P_T)_{\mu\nu} \left[\left(\Pi_0^X + \Pi_0 + \frac{\Pi_1}{4} \sin^2 \frac{h}{F_\pi} \right) B_\mu B_\nu + \left(\Pi_0 + \frac{\Pi_1}{4} \sin^2 \frac{h}{F_\pi} \right) A_\mu^{a_L} A_\nu^{a_L} + 2 \sin^2 \frac{h}{F_\pi} \Pi_1(p^2) \hat{H}^{\dagger} T^{a_L} Y \hat{H} A_\mu^{a_L} B_\nu \right]$$

by expanding the form factors at order O(p²) and setting $\hat{H}=inom{0}{1}$:

 $\mathcal{L} = (P_T)_{\mu\nu} \left[\frac{1}{2} \left(\frac{F_{\pi}^2}{4} \sin^2 \frac{\langle h \rangle}{F_{\pi}} \right) \left(B_{\mu} B_{\nu} + W_{\mu}^3 W_{\nu}^3 - 2W_{\mu}^3 B_{\nu} \right) + \left(\frac{F_{\pi}^2}{4} \sin^2 \frac{\langle h \rangle}{F_{\pi}} \right) W_{\mu}^+ W_{\nu}^- \right. \\ \left. + \frac{1}{2} p^2 \left[\Pi_0'(0) W_{\mu}^a W_{\nu}^a + \left(\Pi_0'(0) + \Pi_0^{X'}(0) \right) B_{\mu} B_{\nu} \right] + \dots \right]$

it follows:

the gap between the EW scale and the scale
$$F_{\pi}$$
 at which the Higgs is formed as a bound state is a dynamically-fixed fundamental parameter

$$v = F_{\pi} \sin \frac{\langle h \rangle}{F_{\pi}}$$

$$\epsilon \equiv \frac{v}{F_{\pi}} = \sin \frac{\langle h \rangle}{F_{\pi}}$$

the limit $\epsilon \to 0$ ($F_{\pi} \to \infty$) is a decoupling limit in which one recovers the SM at low energy

$$m_{\rho} \sim \frac{4\pi F_{\pi}}{\sqrt{N}}$$

 $m_{
ho}$

h \overline{m}_h

 $\overline{m_W}$

All the corrections to the precision observables due to the exchange of the heavy resonances will be suppressed by powers of ϵ :

The LEP precision tests will put a upper bound on ϵ :

 $|\Delta\epsilon_{1,3,b}\propto\epsilon^2|$

 $\epsilon^2 \leq 0.3 - 0.1$

By expanding around the Higgs vev one can easily obtain the couplings of the composite Higgs to the gauge fields:

$$\mathcal{L} = (P_T)_{\mu\nu} \left[\frac{1}{2} \left(\frac{F_{\pi}^2}{4} \sin^2 \frac{h}{F_{\pi}} \right) \left(B_{\mu} B_{\nu} + W_{\mu}^3 W_{\nu}^3 - 2W_{\mu}^3 B_{\nu} \right) + \left(\frac{F_{\pi}^2}{4} \sin^2 \frac{h}{F_{\pi}} \right) W_{\mu}^+ W_{\nu}^- \right. \\ \left. + \frac{1}{2} p^2 \left[\Pi_0'(0) W_{\mu}^a W_{\nu}^a + \left(\Pi_0'(0) + \Pi_0^{X'}(0) \right) B_{\mu} B_{\nu} \right] + \dots \right]$$



 $g_{VVh} = g_{VVh}^{SM} \sqrt{1 - \epsilon^2}$ $g_{VVhh} = g_{VVhh}^{SM} \left(1 - 2\epsilon^2\right)$

Prediction:

 $O(\epsilon^2)$ shifts in the couplings of the Higgs

hence:

A precise measurement of its couplings will tell if the Higgs is composite or elementary, and determine the scale F_{π}



The Higgs compositeness also reflects into an only partial unitarization of the WW scattering:

In absence of the physical Higgs boson the scattering of longitudinal W's violates unitarity at energies higher than $\Lambda_0\sim 1.2~{\rm TeV}$



An elementary (SM-like) Higgs fully unitarizes the scattering [the SM is complete, i.e. renormalizable up to arbitrary high energies] A composite Higgs had modified couplings and only partly unitarizes the WW scattering.

Unitarity is lost at a scale: $\Lambda = \frac{\Lambda_0}{\epsilon}$

$$A\left(W_{L}^{+}W_{L}^{-} \to W_{L}^{+}W_{L}^{-}\right) = \frac{g_{2}^{2}}{4M_{W}^{2}}\left[s - \frac{s^{2}\left(1 - \epsilon^{2}\right)}{s - m_{h}^{2}} + t - \frac{t\left(1 - \epsilon^{2}\right)}{t - m_{h}^{2}}\right]$$



Full unitarity is recovered thanks to the exchange of the heavy vectorial resonances

hence:

An excess of events in the WW scattering compared to the SM prediction will be the signal Higgs compositeness

experimentally difficult at the LHC:

with $L=200~{
m fb}^{-1}$ the LHC should be sensitive up to $~\epsilon^2\simeq 0.5-0.7$

[Giudice et al. JHEP 0706:045, 2007]



• EM correction to the pion mass + Weinberg's sum rules

→ Das et al. PRL 18 (1967) 759

de Rafael hep-ph/9802448

• 1/N expansion

➡ E.Witten, NPB 160 (1979) 57



• Composite Higgs

- → D.B. Kaplan, H. Georgi PLB 136 (1984) 183
- → Dungan, Georgi, Kaplan NPB 254 (1985) 299

Composite Higgs from a warped extra-dimension

K.Agashe et al. NPB 719 (2005) 165

• Randall-Sundrum (warped) models

- → L. Randall, R. Sundrum PRL 83 (1999) 83
- → L.Randall, R. Sundrum PRL 83 (1999) 4690



• Composite vs Elementary Higgs: couplings and WW scattering

Giudice et al. JHEP 0706:045, 2007

• WW scattering (& the role of the Higgs in it)

→ B.W. Lee, C. Quigg, H.B. Thacker, PRD 16 (1977) 1519

M. Chanowitz Ann. Rev. Nucl. Part. Sci. 38 (1988) 323

M.Veltman "Reflections on the Higgs system" CERN Lectures 1996-1997 (available on the SLAC archive)

THE COMPOSITE HIGGS OPTION

Theory and Phenomenology

Roberto Contino (CERN)

SNFT 2007 - Parma

PART 2.

PHENOMENOLOGY

AN EFFECTIVE THEORY FOR THE LIGHTEST RESONANCES

Let us consider the case in which the elementary fields are linearly coupled to the strong sector:



 $\mathcal{L}_{int} = A_{\mu}J^{\mu} + \bar{\Psi}O + h.c.$

This is for example the case for the class of 5D composite Higgs models from a warped extra dimension.

As we will see, linear couplings are also motivated by the absence of large Flavor-Changing Neutral Currents.

As in QCD, the composite operators will have the right quantum numbers to excite a tower of bound states

$$\langle 0|J_{\mu}|\rho\rangle = \epsilon_{\mu}^{r} f_{\rho} m_{\rho}$$

vectorial resonance

 $\langle 0|O|\chi\rangle = \Delta$

fermionic resonance

Hence, linear couplings imply mass mixings to the composites:



mass mixing

$$\mathcal{L}_{mix} = \sum_{n} \Delta_n \, \bar{\Psi} \chi_n + h.c.$$

To build a low-energy effective theory:

Keep only the first resonance of each tower



$$\mathcal{L}_{mix} = \Delta \,\overline{\Psi} \chi + h.c.$$

Motivation:



At the LHC we will most probably discover only the first resonance of each tower, if any (!)



Keeping only one resonance greatly simplifies the mathematics



Even this rough truncation of the spectrum fully captures the low-energy phenomenology

Linear Mixings for vector fields:



same as
$$\rho$$
-photon
mixing in QCD $\longrightarrow \mathcal{L}_{mix} = \frac{M_*^2}{2} \left(\frac{g_{el}}{g_*} A_{\mu} - \rho_{\mu}\right)^2$

$$=\frac{M_*^2}{2}\rho_{\mu}^2 - M_*^2 \frac{g_{el}}{g_*} A_{\mu}\rho_{\mu} + \frac{M_*^2}{2} \left(\frac{g_{el}}{g_*}\right)^2 A_{\mu}^2$$

Cutoff scale of the effective theory

The truncated theory is not just an approximation to the complete one; like all effective theories, it is valid up to a cutoff energy:





•Elementary sector:

{SM - Higgs} inter-elementary coupling: g_{el} ~ 1

•Composite sector:

{ ρ, χ + Higgs} [\supset excited massive copy of the SM] inter-composite coupling: $4\pi \gg g_* \gg 1$

•Mixing:

only mass mixings allowed

•Higgs:

H couples only to ρ and χ

A SIMPLE TWO-SITES SO(5)/SO(4) MODEL

$$\begin{split} &\mathrm{SO}(5)\times\mathrm{U}(1)_X\to\mathrm{SO}(4)\times\mathrm{U}(1)_X\\ &\mathrm{SO}(4)\sim\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R \end{split}$$

For simplicity: we focus on just the 3rd-generation up quarks



composite sector

Spectrum of Composite Fermions

$$\chi = \begin{bmatrix} \begin{pmatrix} Q' \\ Q \end{pmatrix} \\ \tilde{T} \end{bmatrix} \qquad \qquad Q = \begin{bmatrix} T \\ B \end{bmatrix} \qquad \qquad Q' = \begin{bmatrix} T_{5/3} \\ T_{2/3} \end{bmatrix}$$
$$Y[Q] = 1/6 \qquad \qquad Y[Q'] = 7/6$$

 $[{f 5}=({f 2},{f 2})\oplus({f 1},{f 1})]$

Spectrum of Composite Vectors

 $\rho_{\mu} = \{ W_{\mu}^{*L}, B_{\mu}^{*}, \tilde{B}_{\mu}, \tilde{W}_{\mu}^{\pm}, \tilde{A}_{\mu} \} \qquad [\mathbf{10} = (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{2}, \mathbf{2})]$ under SU(2)_L×U(1)_Y: $\mathbf{3}_{0}$ $\mathbf{1}_{0}$ $\mathbf{1}_{0}$ $\mathbf{1}_{\pm 1}$ $\mathbf{2}_{\frac{1}{2}}$ The Lagrangian: $\mathcal{L} = \mathcal{L}_{elementary} + \mathcal{L}_{composite} + \mathcal{L}_{mixing}$

$$\mathcal{L}_{elementary} = -\frac{1}{4} (W^{a_L}_{\mu\nu})^2 - \frac{1}{4} (B_{\mu\nu})^2 + \bar{q}_L i \not\!\!\!D q_L + \bar{t}_R i \not\!\!\!D t_R$$

$$\mathcal{L}_{composite} = -\frac{1}{4} (\rho_{\mu\nu}^{A})^{2} + \frac{M_{*}^{2}}{2} (\rho_{\mu}^{A})^{2} + \frac{F_{\pi}^{2}}{2} (D_{\mu}\Sigma) (D^{\mu}\Sigma)^{T} + \bar{\chi} (i\not\!\!D - m)\chi - m_{\Sigma} \,\bar{\chi}_{i} \Sigma_{i} \Sigma_{j} \chi_{j}$$

$$\mathcal{L}_{mixing} = -M_*^2 \frac{g_{2,el}}{g_*} W_{\mu}^{a_L} W_{\mu}^{*a_L} + \frac{M_*^2}{2} \left(\frac{g_{2,el}}{g_*} W_{\mu}^{a_L}\right)^2 -M_*^2 \frac{g_{1,el}}{g_*} B_{\mu} B_{\mu}^* + \frac{M_*^2}{2} \left(\frac{g_{1,el}}{g_*} B_{\mu}\right)^2 + \Delta_q \bar{q}_L Q_R + \Delta_{t_R} \bar{t}_R \tilde{T}_L + h.c.$$

The mass mixings in \mathcal{L}_{mixing} break explicitly the SO(5) symmetry, though in a soft way (soft breaking terms). The Higgs potential is still calculable:



there are enough propagators to have a finite result

The exchange of the composite states is in fact a way to model the form factors:



$$\cdots + \cdots = \cdots$$

$$\Pi_1(p^2) = \frac{F_\pi^2 M_*^4}{(p^2 - M_*^2)^2}$$
$$\Pi_0(p^2) = \frac{M_*^2}{g_*^2} \frac{p^2}{p^2 - M_*^2}$$

$$\frac{-i}{\frac{p^2}{g_{el}^2} - \Pi_0(p^2)} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

DIAGONALIZATION:

elementary/composite \rightarrow light/heavy

$$\begin{pmatrix} A_{\mu} \\ \rho_{\mu} \end{pmatrix} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} A_{\mu} \\ \rho_{\mu} \end{pmatrix}$$

$$\tan \theta = \frac{g_{el}}{g_*}, \qquad g_{SM} = \frac{g_{el}g_*}{\sqrt{g_{el}^2 + g_*^2}} \simeq g_{el}$$

$$\begin{pmatrix} q_L \\ Q_L \end{pmatrix} \to \begin{pmatrix} \cos \varphi_L & -\sin \varphi_L \\ \sin \varphi_L & \cos \varphi_L \end{pmatrix} \begin{pmatrix} q_L \\ Q_L \end{pmatrix} \qquad \qquad \tan \varphi_L = \frac{\Delta_{q_L}}{m}$$

$$\begin{pmatrix} t_R \\ \tilde{T}_R \end{pmatrix} \to \begin{pmatrix} \cos \varphi_{t_R} & -\sin \varphi_{t_R} \\ \sin \varphi_{t_R} & \cos \varphi_{t_R} \end{pmatrix} \begin{pmatrix} t_R \\ \tilde{T}_R \end{pmatrix} \qquad \qquad \tan \varphi_{t_R} = \frac{\Delta_{t_R}}{\tilde{m}}$$

$$|\mathrm{SM}\rangle = \cos\varphi |\Psi\rangle + \sin\varphi |\chi\rangle$$
$$|\mathrm{heavy}\rangle = -\sin\varphi |\Psi\rangle + \cos\varphi |\chi\rangle$$

 \mathfrak{B} θ, φ parametrize the <u>degree of partial compositeness</u>

In the mass-eigenstate basis, the finiteness of the Higgs potential comes from a cancellation among divergent diagrams:

 W^L_μ, B_μ $\tilde{B}_{\mu}, \tilde{W}^{\pm}_{\mu}, \tilde{A}_{\mu}$ $, B^*_{\mu}$ finite $\frac{g_{SM}^2}{\sin^2\theta}$ $g_{SM}^2 \, \frac{\cos^2 \theta}{\sin^2 \theta}$ g_{SM}^2
CONSEQUENCES OF PARTIAL COMPOSITENESS

$$|\mathrm{SM}\rangle = \cos\varphi |\Psi\rangle + \sin\varphi |\chi\rangle$$



the larger φ the more "composite" will be a SM particle



the Higgs is a full composite (= solution to the Hierarchy Problem)



heavier SM particles = more composites light SM particles = almost elementary

$$y_t = Y_* \sin \varphi_L \sin \varphi_{t_R}$$

induced Yukawa coupling

Flavor-Changing Neutral Currents : a sort of GIM mechanism

small enough for light fermions



 $\left(\bar{\Psi}\Psi\right)^2 \left(\frac{\sin^4\varphi}{M_{\star}^2}\right)$

FCNC: generated (only) after rotating to the SM mass eigenbasis

 $\mathcal{L}_{FCNC} = A_{ijmn} \left(\bar{d}_{L,i} \gamma^{\mu} d_{L,j} \right) \left(\bar{d}_{R,m} \gamma_{\mu} d_{R,n} \right)$

$$A_{ijmn} = \left(-\frac{g_*^2}{M_*^2}\right) (D_L^{\dagger})_{ki} (D_L)_{kj} \left(\sin^2 \varphi_{L,k} \, \sin^2 \varphi_{R,l}\right) (D_R^{\dagger})_{lm} (D_R)_{ln}$$

DISCOVERING THE NEW RESONANCES AT THE LHC

VECTORIAL RESONANCES

Single production of the heavy vectors can proceeds either through their composite or its elementary component: $|\mathrm{SM}\rangle = \cos \varphi |\Psi\rangle + \sin \varphi |\chi\rangle$

 $|\text{heavy}\rangle = -\sin\varphi |\Psi\rangle + \cos\varphi |\chi\rangle$

 $\mathcal{A}\left[\mathrm{SM}_{1} + \mathrm{SM}_{2} \to \mathrm{heavy}\right] \propto g_{*} \sin \varphi_{1} \sin \varphi_{2} \cos \varphi_{\mathrm{heavy}} - \left(g_{el} \cos \varphi_{1} \cos \varphi_{2} \sin \varphi_{\mathrm{heavy}}\right)$

if SM_{1,2} are light fermions g_* more than compensated by $\varphi_1 \varphi_2$ suppression

> might be cheaper to proceed via the elementary component of the heavy state

Example:

Z^{*} SINGLE PRODUCTION

• Drell-Yan production through the elementary component of the Z*:



• vector-boson fusion production through the composite component of the Z*:

despite the larger coupling, vector-boson fusion is subdominant compared to Drell-Yan because of:

- low luminosity of longitudinal W's, Z's in the proton
- 3 body final state vs 1 body

Drell-Yan production cross sections of SU(2)_L heavy vectors



[From Burdman, Perelstein, Pierce hep-ph/0212228]

Once produced the heavy resonances will decay mostly to the SM particles with the largest mixing angle: H, W_{long}, Z_{long}, top, bottom



<u>Example</u>: the case of W^{*3}

$$\Gamma(W^{*\,3} \to q\bar{q}) = 3\,\Gamma(W^{*\,3} \to l\bar{l}) \simeq \frac{g_2^2 \tan^2 \theta_2}{32\pi} M_*$$

$$\Gamma(W^{*\,3} \to t\bar{t}) = \Gamma(W^{*\,3} \to b\bar{b}) = \left(\sin^2 \varphi_{t_L} \cot \theta_2 - \cos^2 \varphi_{t_L} \tan \theta_2\right)^2 \frac{g_2^2}{32\pi} M_*$$

$$\Gamma(W^{*\,3} \to ZH) = \Gamma(W^{*\,3} \to W^+W^-) = \frac{g_2^2 \cot^2 \theta_2}{192\pi} M_*$$

for
$$\begin{cases} M_* = 3 \text{ TeV} \\ \tan \theta_2 = 1/6 \\ \sin \varphi_{t_L} = 0.4 \end{cases}$$

 $\Gamma_{tot} = 170 \text{ GeV}$ $BR(ee, \mu\mu) = 0.1\%$ BR(tt, bb) = 5% **Characteria Content** (when kinematically allowed):



\bigstar Production and decay of the heavy gluon G^{*}



• Also possible: $G^* \to t\bar{T}, T\bar{T}, b\bar{B}, B\bar{B}$

HEAVY PARTNERS OF THE TOP



PAIR PRODUCTION

When the fermionic resonances are not too heavy, the easiest way to produce them is via pair production:



DECAYS

 $\int_{\frac{W_L^+/Z_L,h}{t_R}}$ $T_{5/3}/T_{2/3}$ $\lambda_{Q'} = Y_* \sin \varphi_{t_R}$

 $\int_{S}^{S} \frac{Z_L, h / W_L^-}{t_R}$ T/B

 $\lambda_Q = Y_* \sin \varphi_{t_R} \cos \varphi_L$



 $\lambda_{\tilde{T}} = Y_* \sin \varphi_L \cos \varphi_{t_R}$

SINGLE PRODUCTION

Single production becomes important for large masses of the heavy fermions. It proceeds via the same couplings that appear in the decay processes:





\tilde{T} production cross section at the LCH



[From Azuelos et al. hep-ph/0402037]

Discovering the exotic $T_{5/3}$



LITERATURE

• Linear Couplings & Two-Site Effective Lagrangian

R.C., T. Kramer, M. Son, R. Sundrum JHEP 05 (2007) 074

Production of heavy vector resonances

K.Agashe et al. hep-ph/0612015 (heavy gluon)

Azuelos et al. hep-ph/0402037 (W^* , Z^*)

Production of heavy fermionic resonances

Aguilar-Saavedra hep-ph/0603199 (T, pair production) Azuelos et al. hep-ph/0402037 (T, single production) W. Skiba, D. Tucker-Smith hep-ph/0701247 (B, pair production)