

Monte Carlo tools for the LHC

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"Quarks and mesons? No border control? Mach 15 jet
Who the hell ARE these guys?"

from "Angels and Demons" by Dan Brown



Plan

- ◆ Basics 30'
- ◆ Available Tools 60'
- ◆ On air examples 30'

Exercises:

- ◆ A few exercises will be given on the way
- ◆ Mostly borrowed from M. Seymour's Cern lessons...
- ◆ ... solutions can be found at
madgraph.hep.uiuc.edu/MC101.nb



MC101

Exercises:

MC 101

+

This is a set of solved exercises (borrowed from M. Seymour) for practising the basic notions of Monte Carlo.

- Write the simplest integration function based on the definition of average and error
- Apply an analytic transformation : importance sampling
- Von Neumann's rejection method : plain vanilla
- Von Neumann's rejection method : improved
- Dimensionality of the phase space of 1- \rightarrow n particles
- Useful functions
- Install Vegas
- Top decay
- $q_1 q_2 \rightarrow t\bar{t}$ production
- Representation of the grid in VEGAS. Each red square has the same area

send questions/corrections to maltoni@fis.uniroma3.it

Basics: references


- ◆ M. Seymour, Cern lectures,
<http://seymour.home.cern.ch/seymour/slides/CERNlectures.html>
- ◆ S. Weinzierl, Introduction to MC methods,
hep-ph/0006269

Basics:

from integration to event generation

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very complex functions

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

 $Dim[\Phi(n)] \sim 3n$

General and flexible method is needed

Integrals as averages



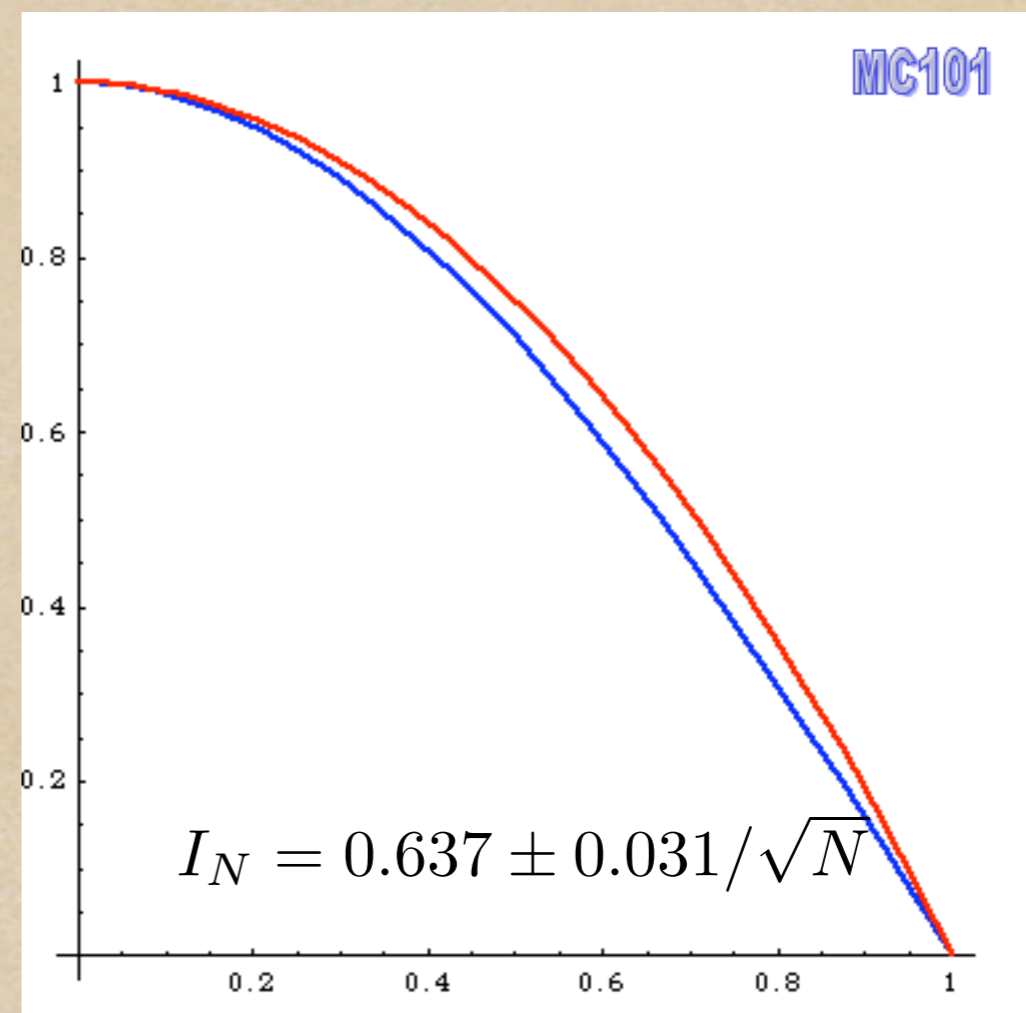
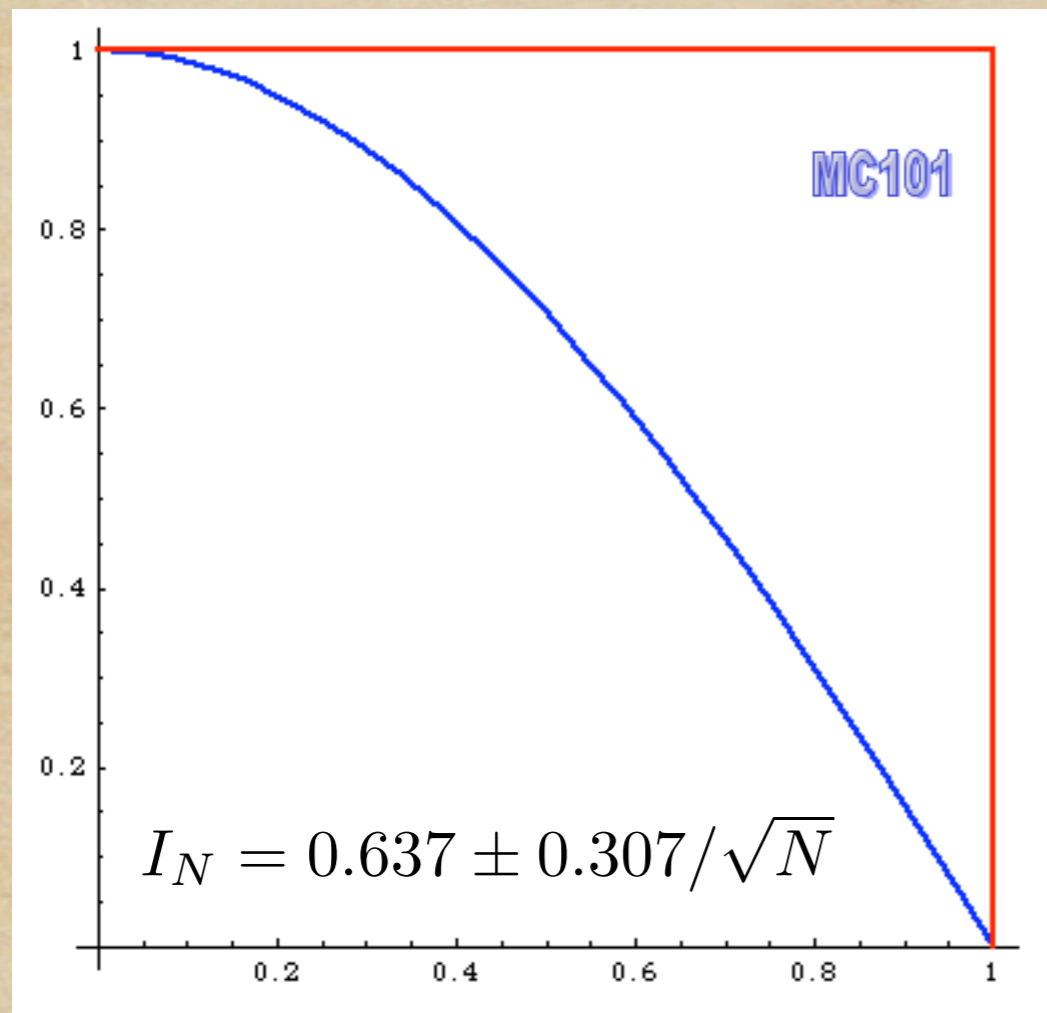
$$I = \int_{x_1}^{x_2} f(x) dx \quad \Rightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x_i)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x_i)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

- ☞ Convergence is slow but it can be estimated easily
- ☞ Error does not depend on # of dimensions!
 - ☞ Improvement by minimizing V_N
- ☞ Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$

Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I = \int_0^1 dx (1 - x^2) \frac{\cos \frac{\pi}{2} x}{1 - x^2}$$

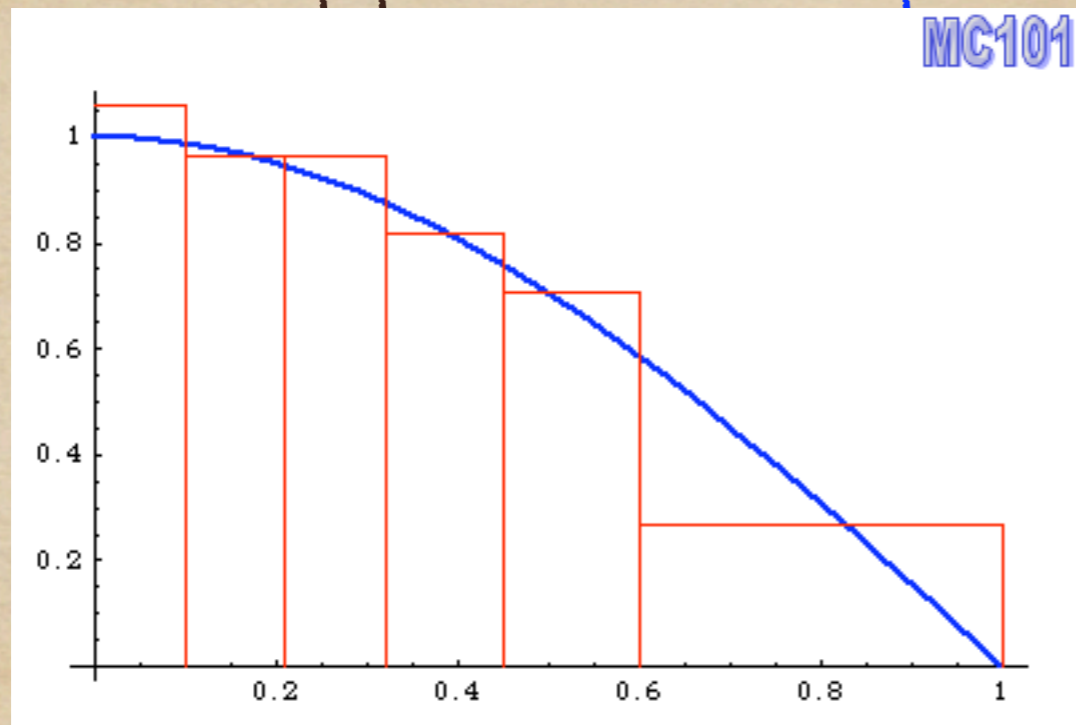
$$= \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2} \rightarrow \approx 1$$

Importance Sampling

but... you need to know too much about $f(x)$!

idea: learn during the run and build a step-function

approximation $p(x)$ of $f(x)$ \Rightarrow VEGAS



many bins where $f(x)$
is large

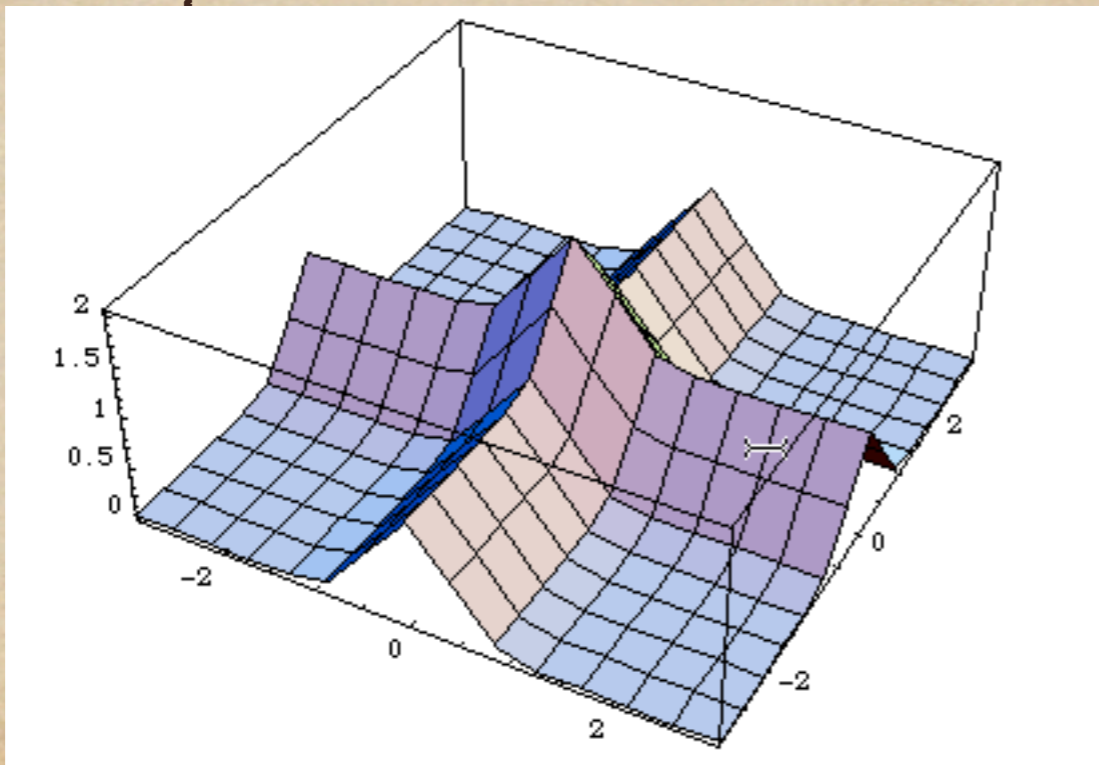
$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

Importance Sampling

can be generalized to n dimensions:

$$p(\vec{x}) = p(x) \cdot p(y) \cdot p(z) \dots$$

but the peaks of $f(\vec{x})$ need to be “aligned” to the axis!



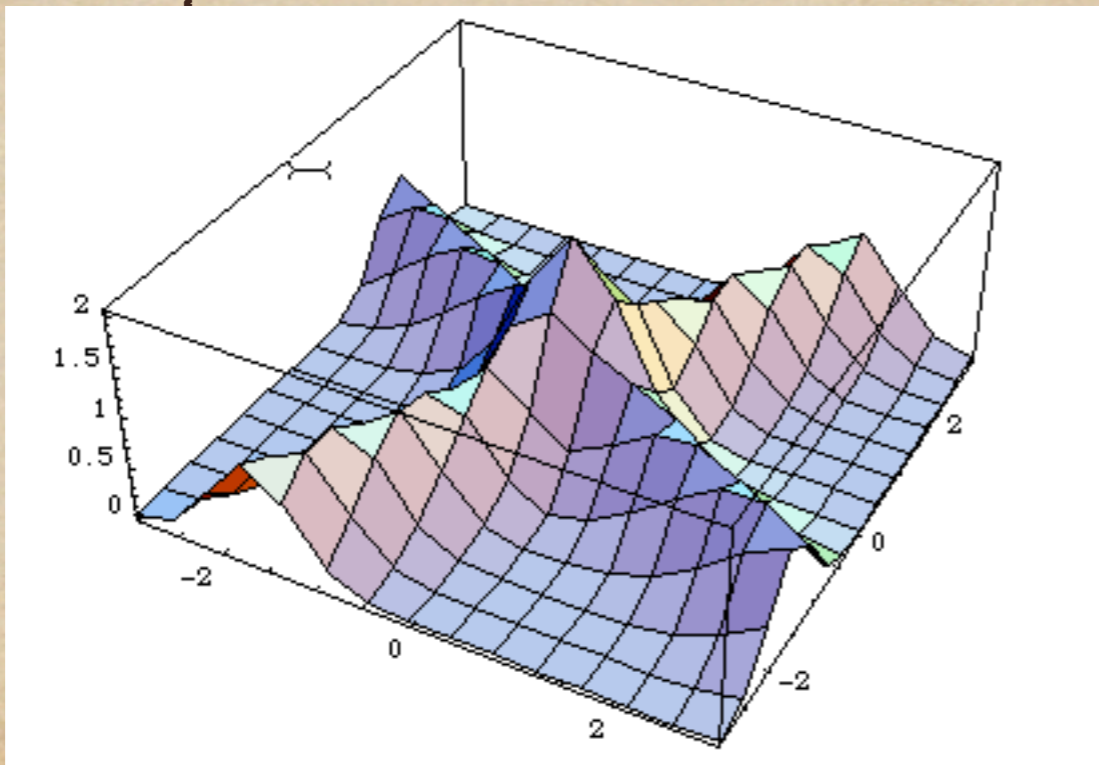
This is ok...

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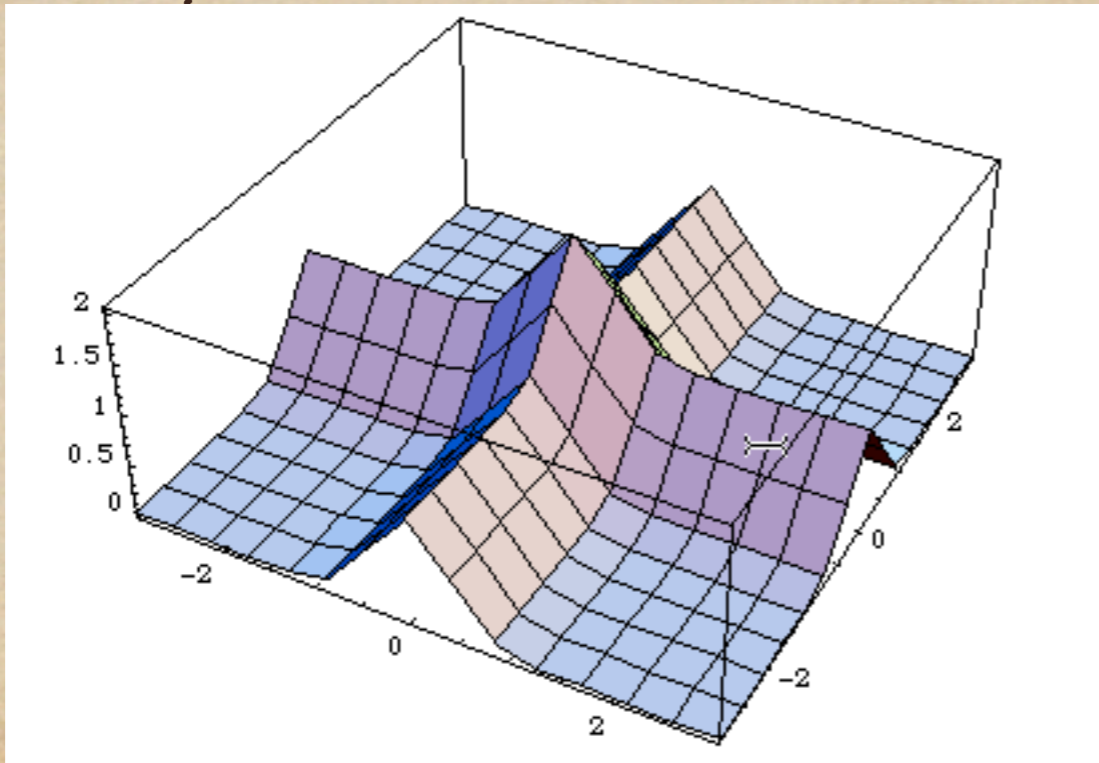
This is not ok...

Importance Sampling

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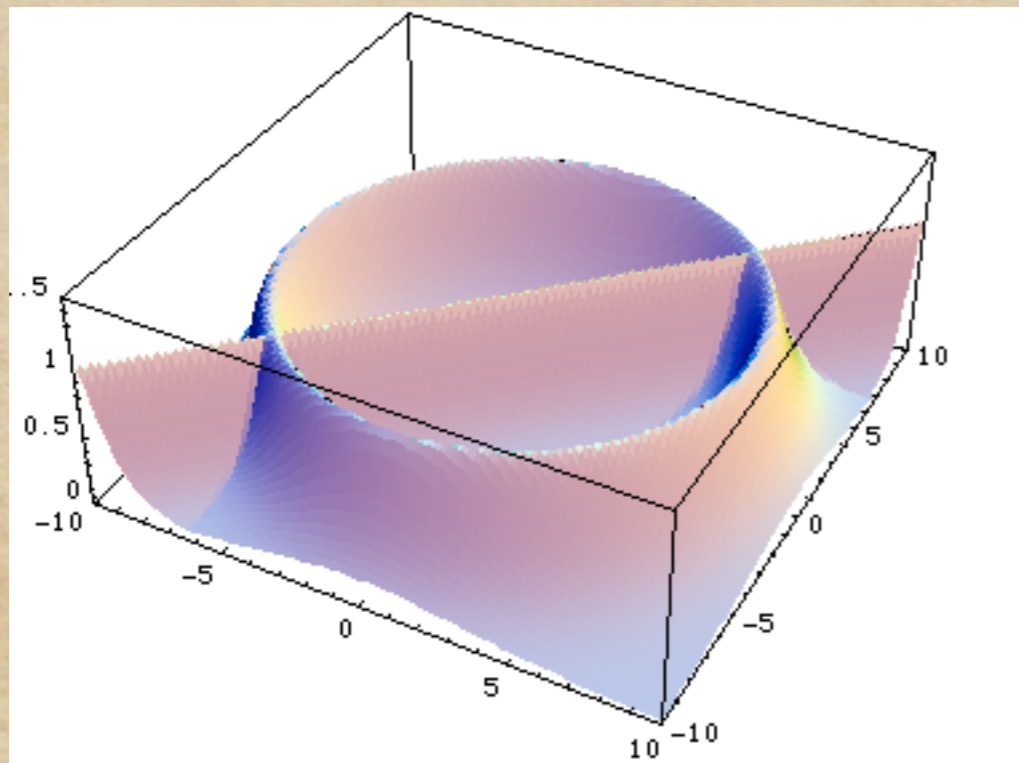
$$p(\vec{x}) = p(x) \cdot p(y) \cdot p(z) \dots$$

but the peaks of $f(\vec{x})$ need to be “aligned” to the axis!



but it is sufficient to make
a change of variables!

Multi-channel



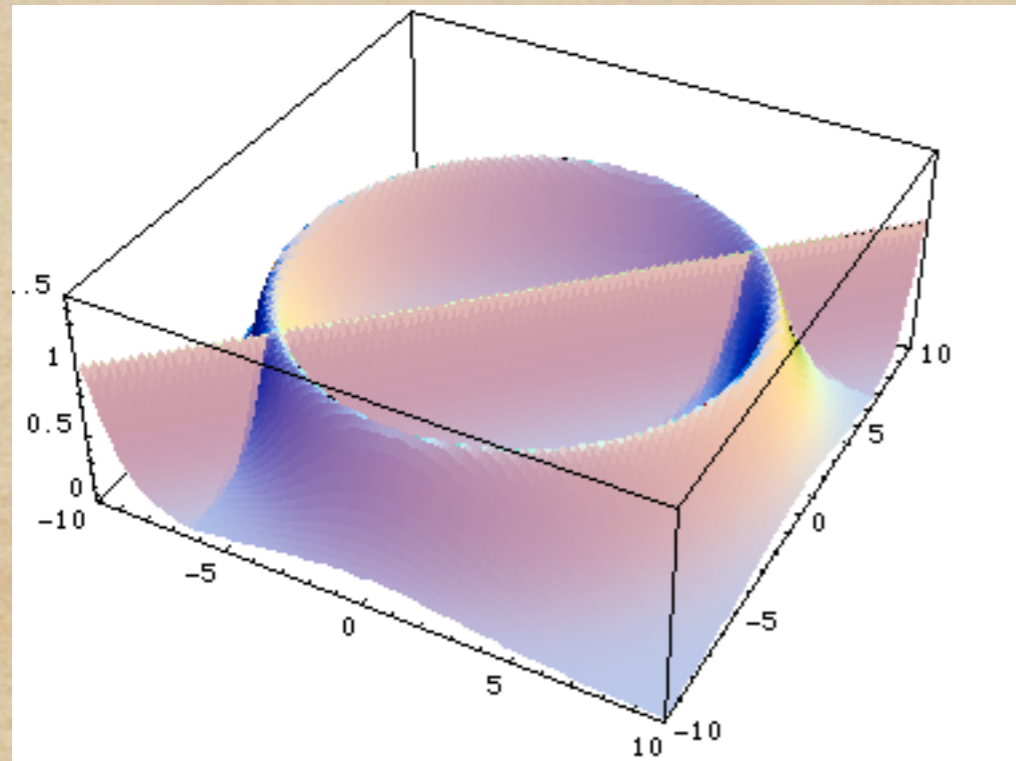
In this case there is no unique transformation:
Vegas is bound to fail!

Solution: use different transformations= channels

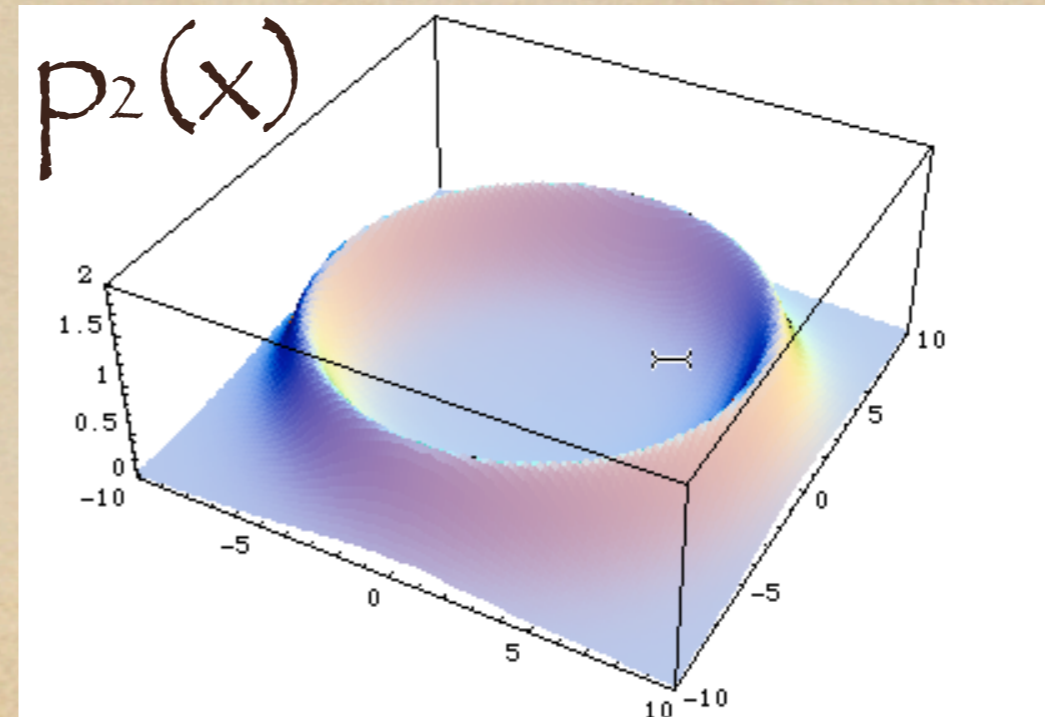
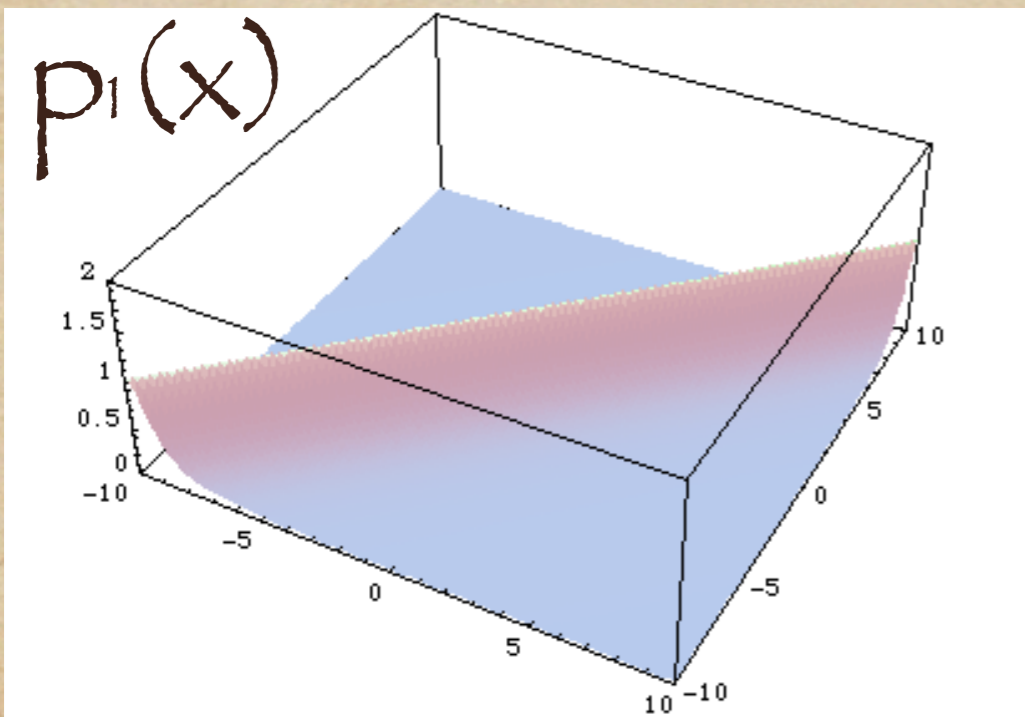
$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

with each $p_i(x)$ taking care of one “peak” at the time

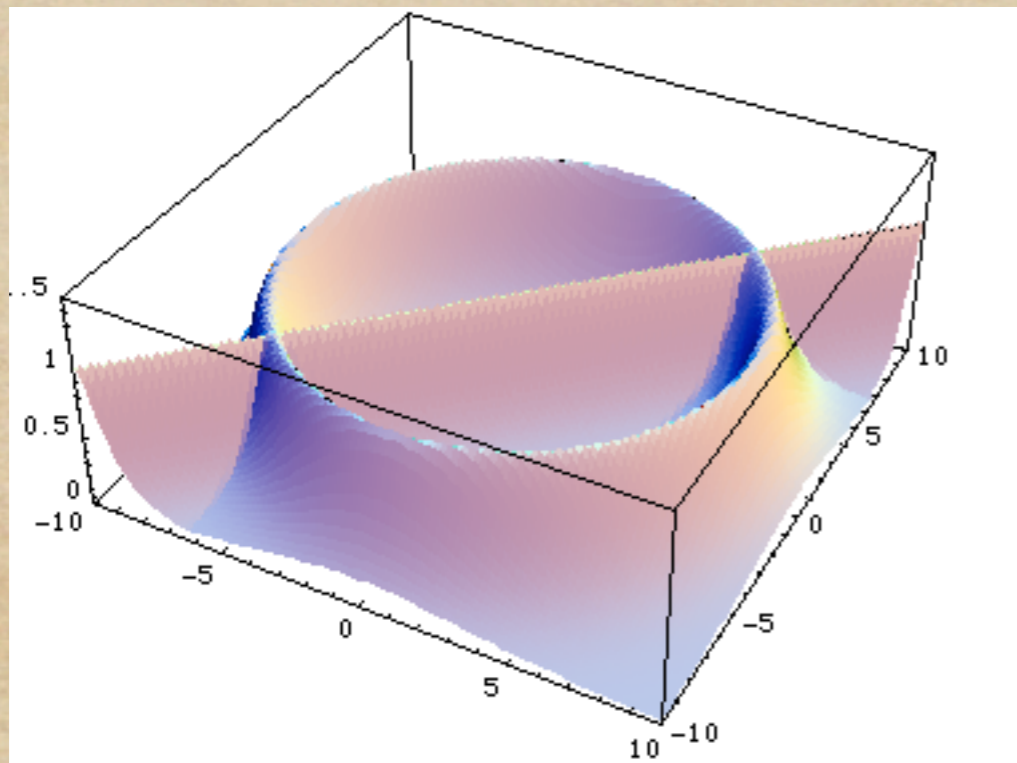
Multi-channel



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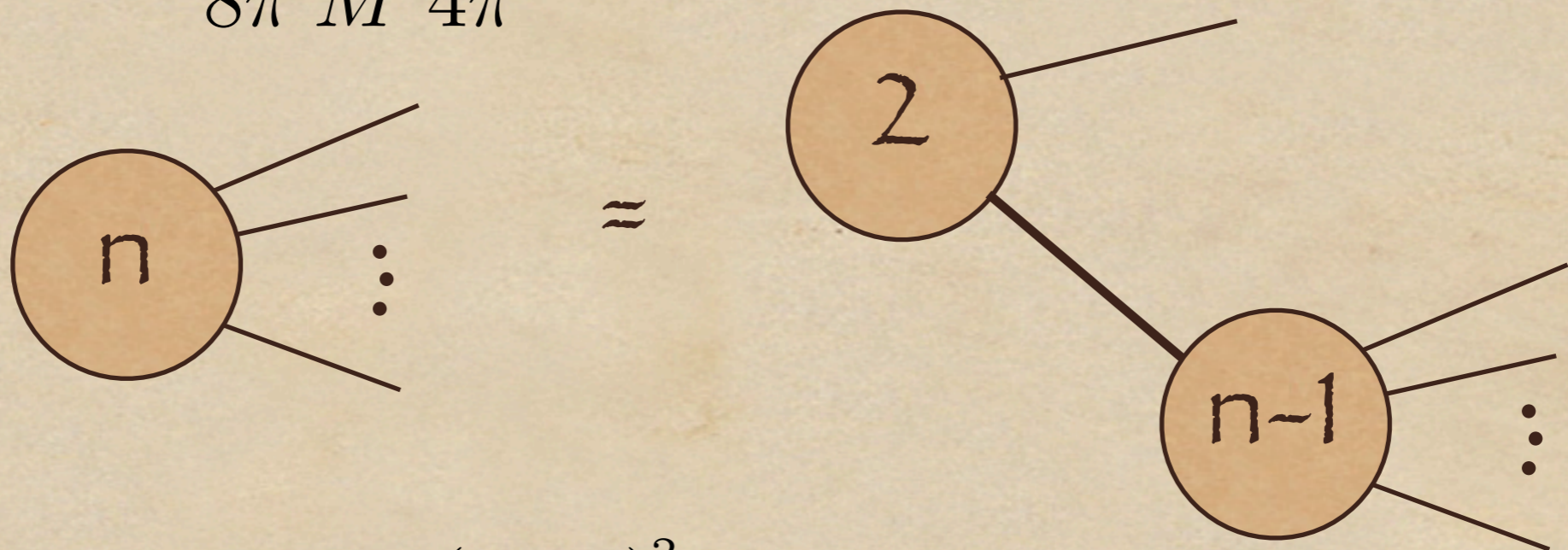
$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

Exercise: show that only V_N depends on the α_i

Phase Space

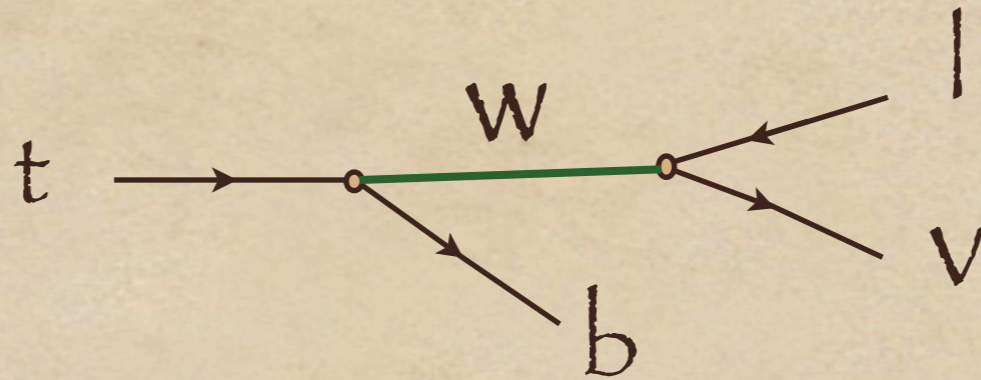
$$d\Phi_n = \left[\prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right] (2\pi)^4 \delta^{(4)} \left(p_0 - \sum_{i=1}^n p_i \right)$$

$$d\Phi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$



$$d\Phi_n(M) = \frac{1}{2\pi} \int_0^{(M-\mu)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$

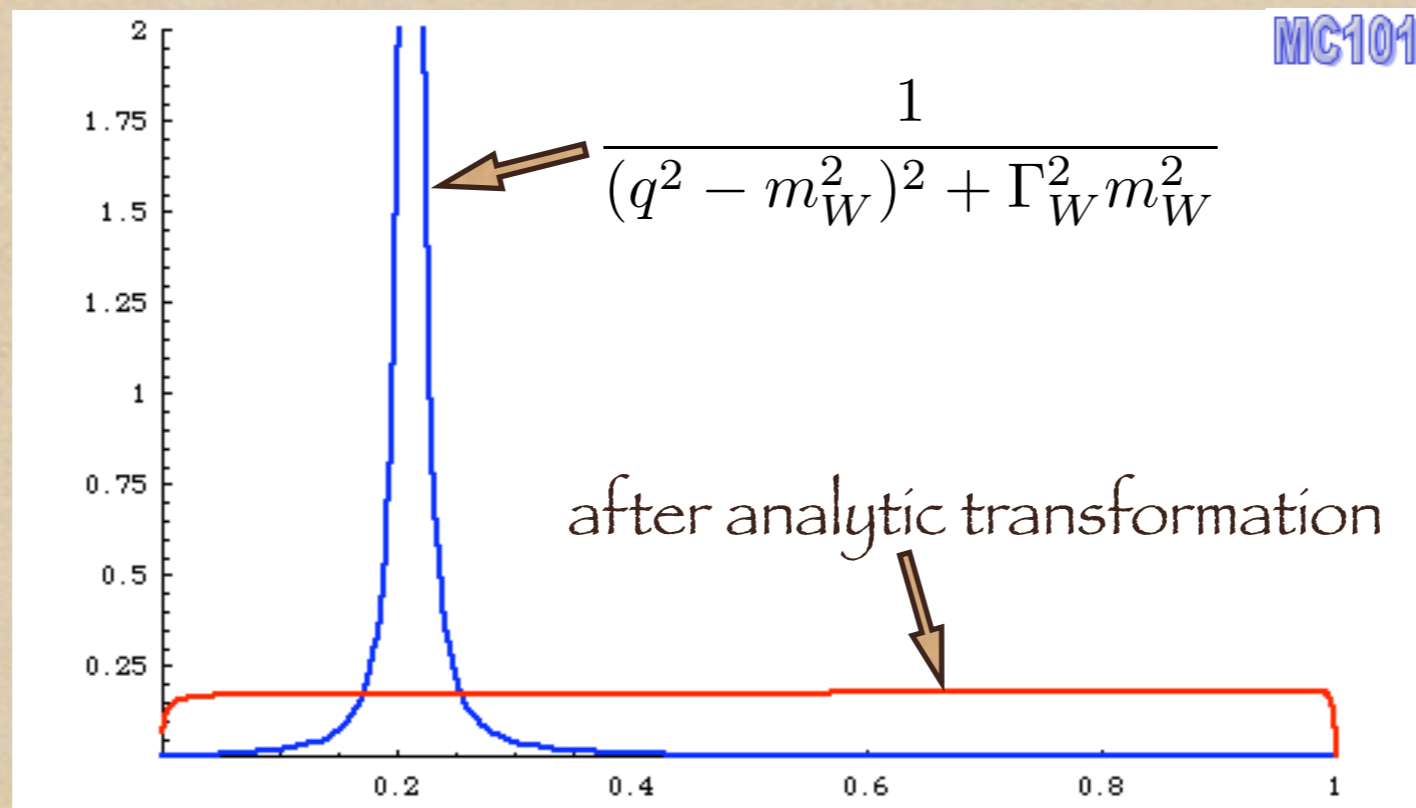
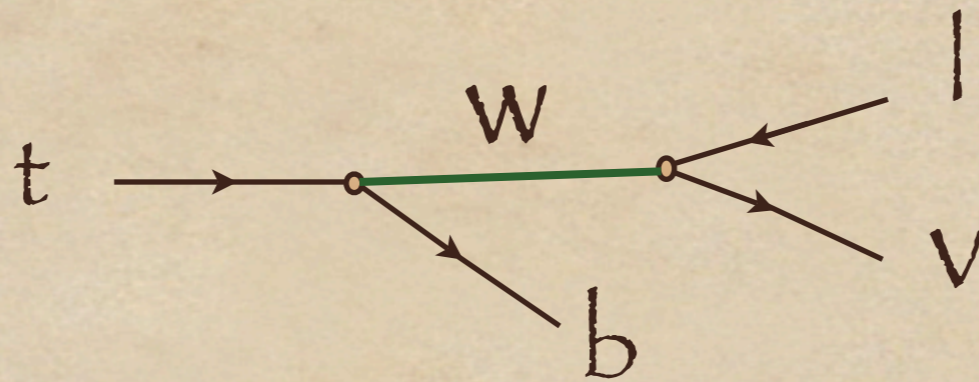
Exercise: top decay



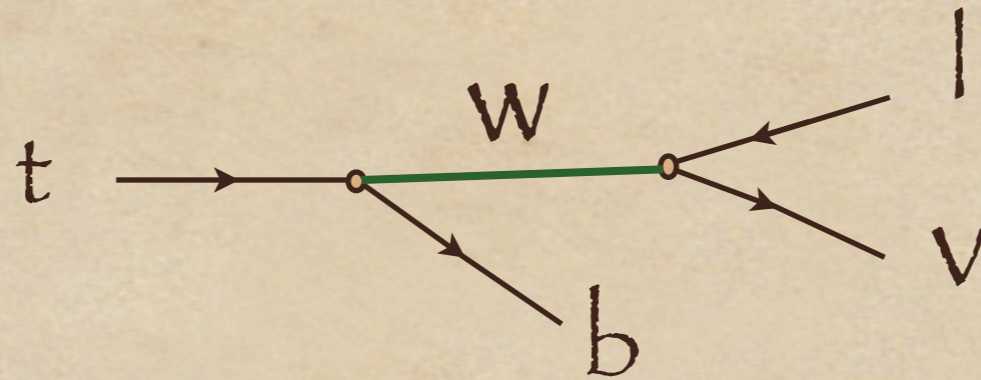
- ◆ Easy but non-trivial
- ◆ Breit-Wigner peak to be “flattened”

$$\frac{1}{(q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2}$$

Exercise: top decay



Exercise: top decay

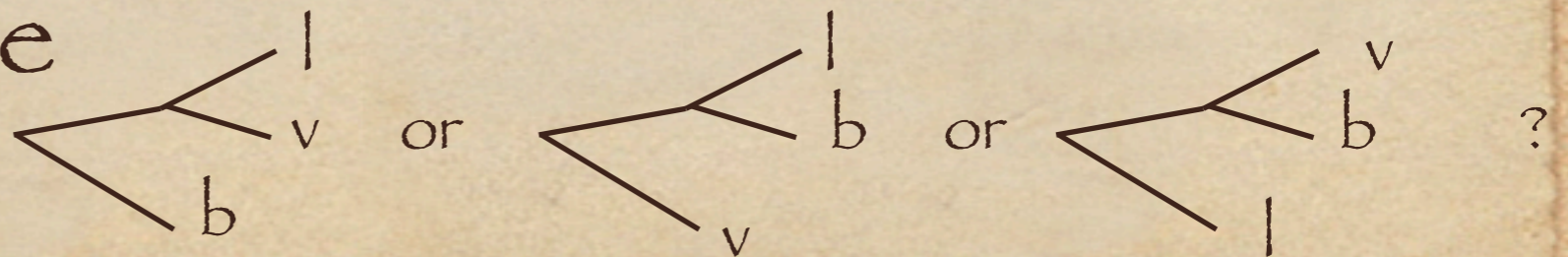


- ◆ Easy but non-trivial

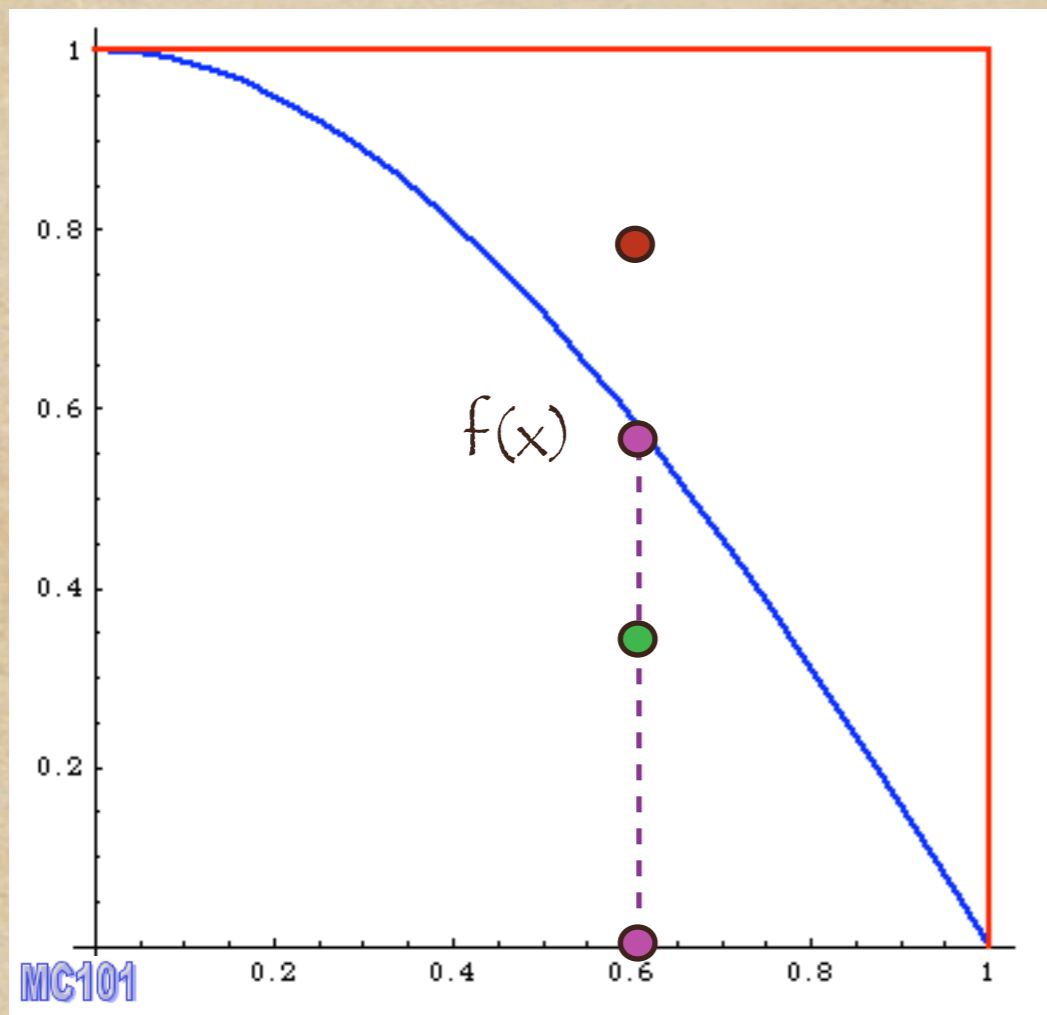
- ◆ Breit-Wigner peak to be “flattened”:

$$\frac{1}{(q^2 - m_W^2)^2 + \Gamma_W^2 m_W^2}$$

- ◆ Choose the right “channel” for the phase space



Event generation

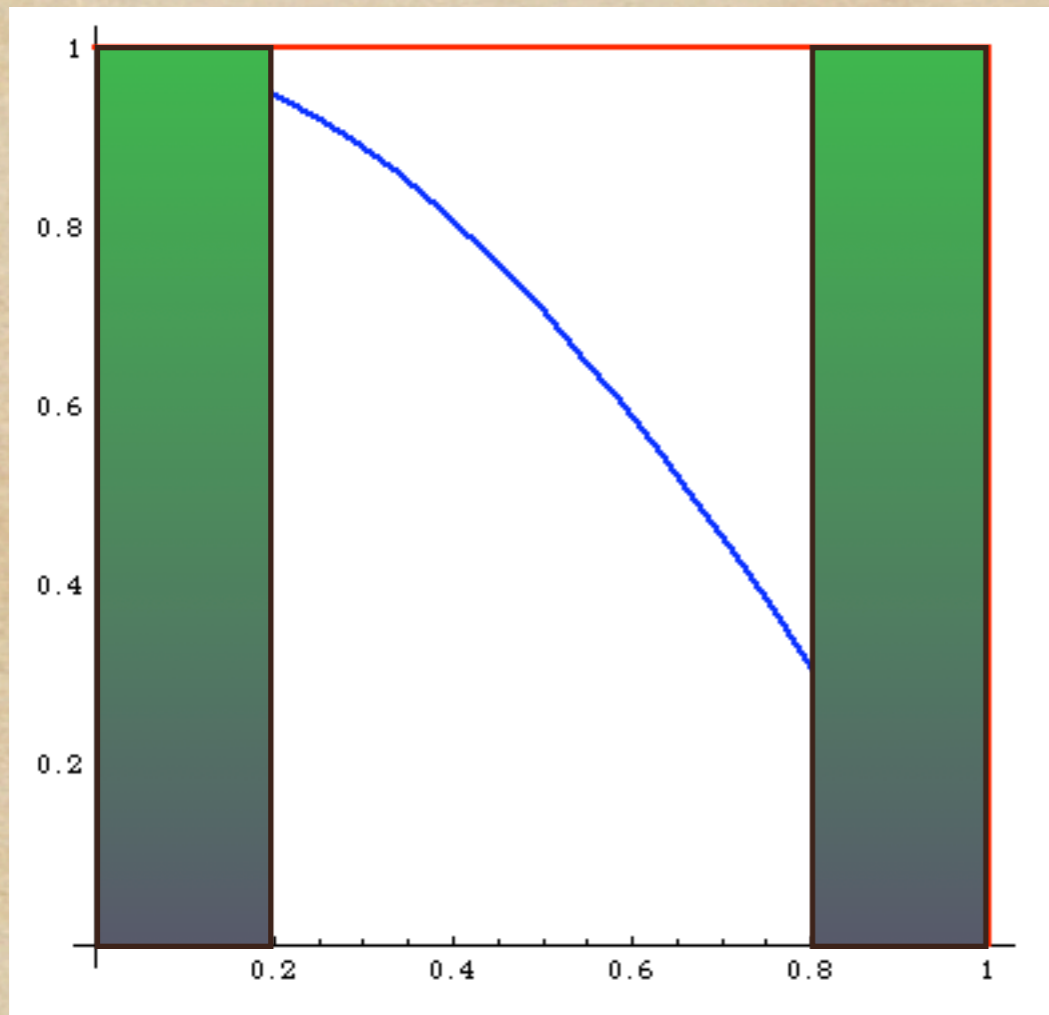


Alternative way

1. pick x
2. calculate $f(x)$
3. pick $0 < y < f_{\max}$
4. Compare:
if $f(x) > y$ accept event,
else reject it.

$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

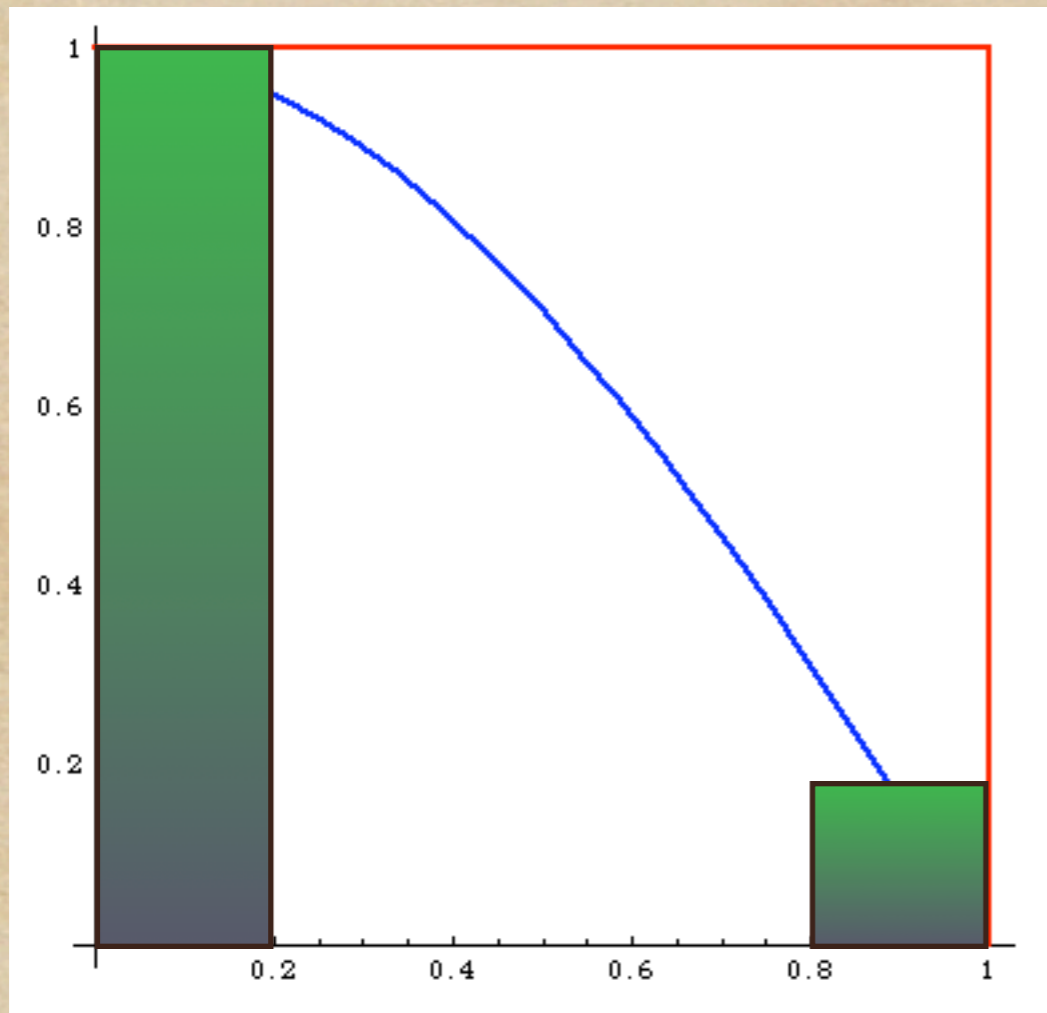


What's the difference?

before:

same # of events in areas
of phase space with very
different probabilities:
events must have
different weights

Event generation

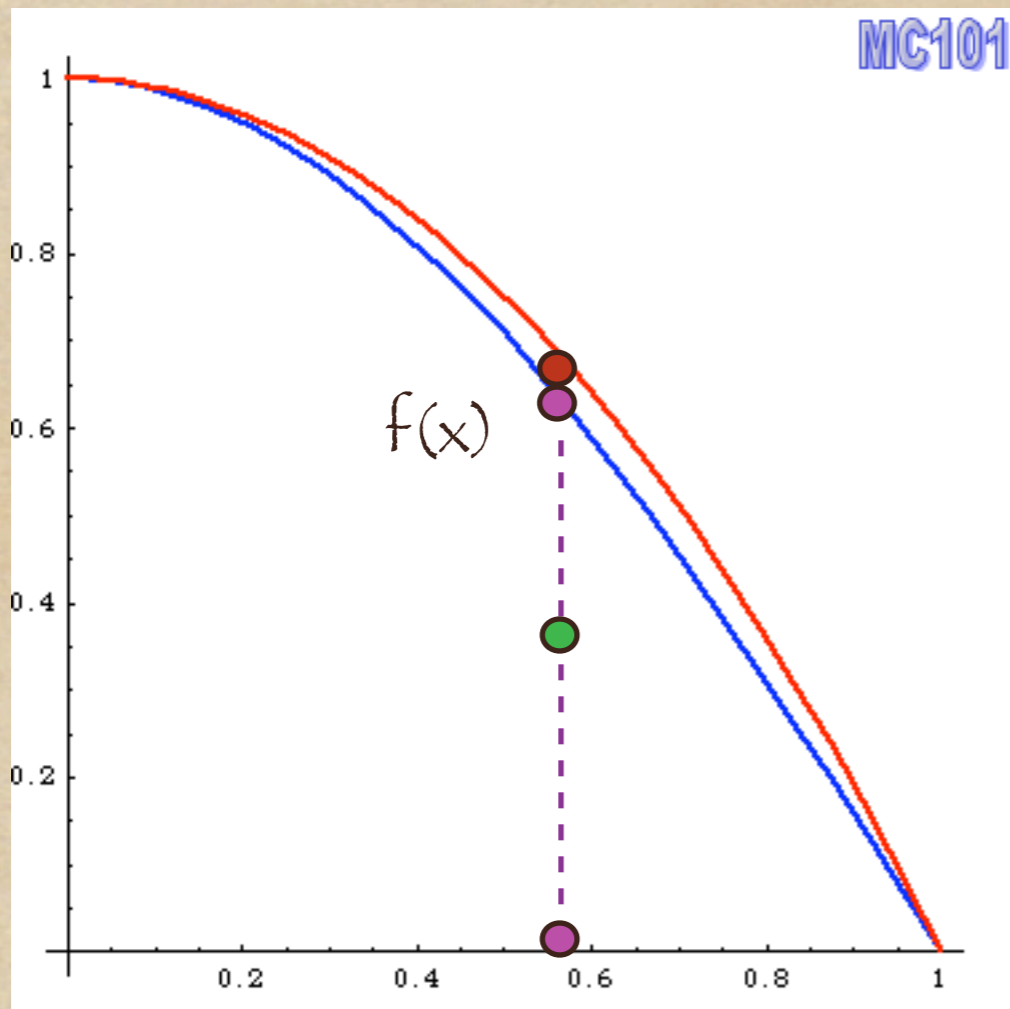


What's the difference?
after:

events is proportional
to the probability of
areas of phase space:
events have all the same
weight ("unweighted")

Events distributed as in Nature

Event generation



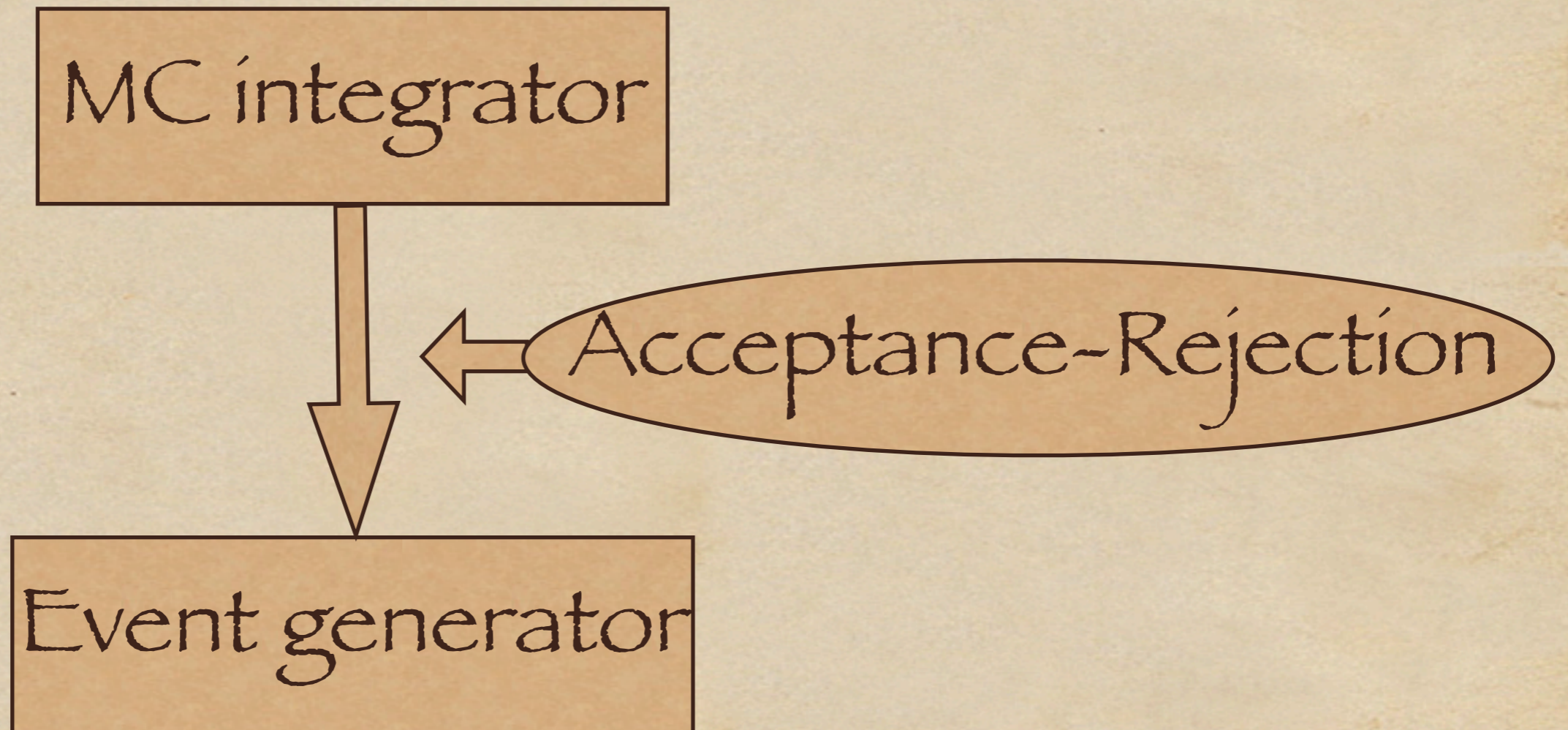
Improved

1. pick x distributed as $p(x)$
2. calculate $f(x)$ and $p(x)$
3. pick $0 < y < 1$
4. Compare:
if $f(x) > y p(x)$ accept event,
else reject it.

much better efficiency!!!

Basics:

from integration to event generation



☞ This is possible only if $f(x) < \infty$ AND has definite sign!

Basics:

from integration to event generation

to take home

1. Integrate is hard
2. Integration + unweighting = Event generation
3. EFFICIENT event generator =
need to know how to integrate the x-section
VERY well

Basics: Final Project

1. Consider $q\bar{q} \rightarrow t\bar{t}$
2. Build a MC for it
3. Include your MC for top decays
4. Make plots of the angular correlation between the charged leptons.
5. Calculate (or find) the amplitude for the full process $q\bar{q} \rightarrow t\bar{t} \rightarrow b\bar{b} e^+ e^- \nu\bar{\nu}$ and compare with the results of point 4.

Available Tools: references

- ◆ Les Houches Guide Book to MC generators for Hadron Collider Physics, hep-ph/0403045
- ◆ Links and descriptions of the codes at <http://www.ippp.dur.ac.uk/HEPCODE/>
- ◆ Recent talks by Frixione, Mrenna, Schumann, Webber @ KEK 04 and Piccinini @ IFAE 04.
- ◆ Several talks by M.L.Mangano (@ FNAL, CERN, MC4LHC, IFAE, KEK)

Available Tools

- ◆ Significant progress in the last few years, R&D still going on!
- ◆ Many different codes available
- ◆ Result: (not only) users are confused!

Available Tools

There is no

PERFECT-FOR-ALL-PURPOSES

MONTECARLO!

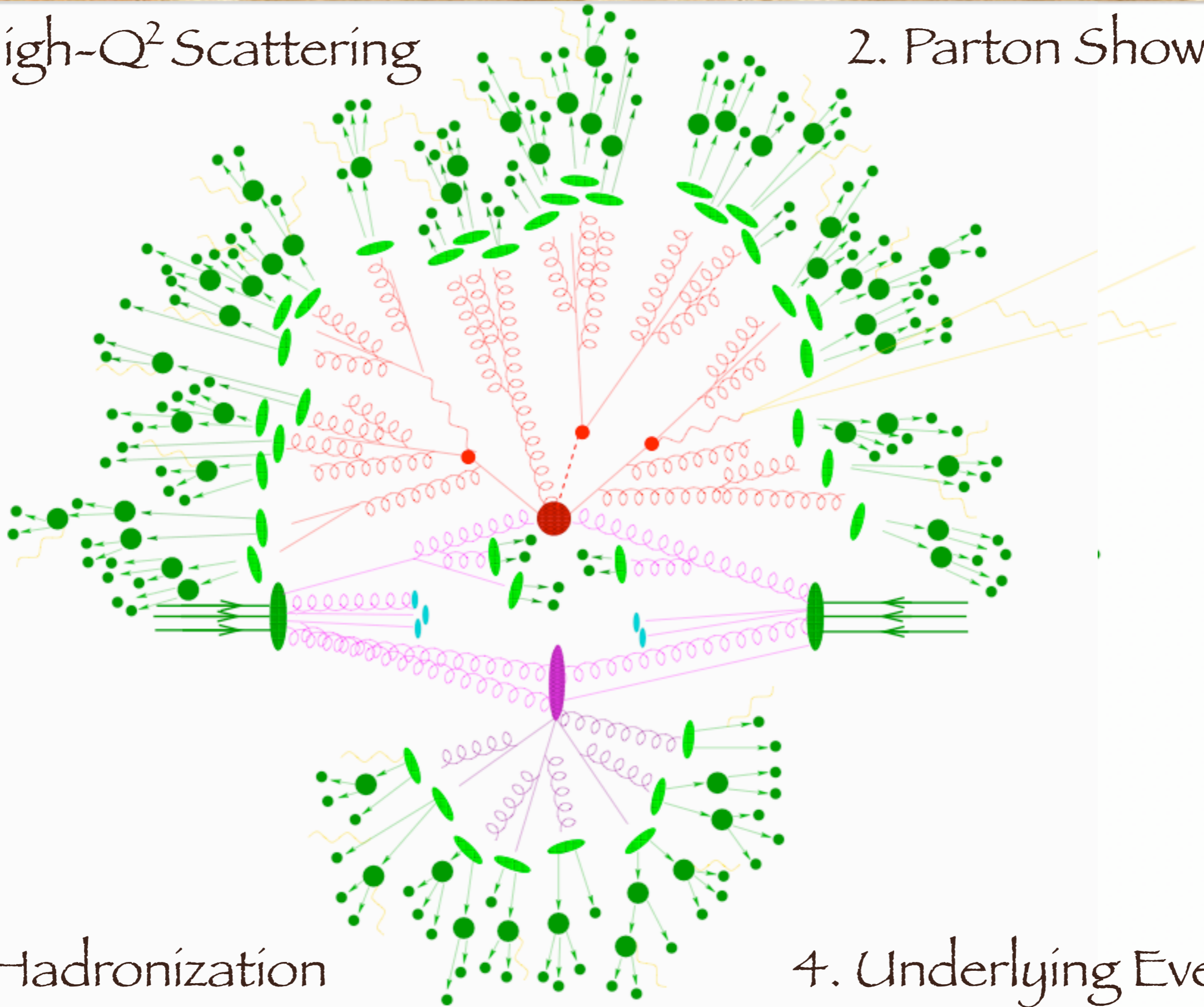
but

You'll certainly find one suitable to
your needs!

You must know what you need!!

1. High- Q^2 Scattering

2. Parton Shower



3. Hadronization

4. Underlying Event

Main classes of MC's

- ◆ MC integrators
- ◆ Parton Shower MC event generators
- ◆ Matrix-element based MC event generators
- ◆ MC@NLO

MC's integrators

- ◆ Now used only for at least NLO calculations or analytically resummed results
- ◆ Provide essential information on the normalization of the cross section
- ◆ Produce distributions of any quantity of interest but not events (due to negative weights)

The "cleanest" tools → theorists' 1st ♥

MC's integrators

- ◆ Inclusive approach (NO EVENTS)
- ◆ Predictions are at parton level only. No showering, hadronization or detector effects.
- ◆ Jets contain at most two partons

MC's integrators

- ◆ They need a lot of manual work \Rightarrow progress is slow with only few codes available for “simple processes”
- ◆ In some cases special treatment of particular areas of phase space gives an “improved” prediction (e.g. ResBos)

An experimenter's wishlist

■ Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Theoretical status

- Much smaller jet multiplicities, some categories untouched

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 2j$	$WW + \leq 0j$	$WWW + \leq 3j$	$t\bar{t} + \leq 0j$
$W + b\bar{b} + \leq 0j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 0j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 2j$	$ZZ + \leq 0j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 0j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 0j$
$Z + c\bar{c} + \leq 0j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 0j$
$\gamma + \leq 1j$	$\gamma\gamma + \leq 1j$		$b\bar{b} + \leq 0j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 0j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 0j$		
	$Z\gamma + \leq 0j$		

+ several other results

MC's integrators

A useful list at the HEPDATA web site

<http://www.ippp.dur.ac.uk/HEPCODE/>

Here some examples:

- ◆ NLOJET++ : for jets and photons
- ◆ DIPHOX family : photons w/ fragmentation
- ◆ ResBos family : resummed results
- ◆ MCFM : many processes $V, VV, VQQ...$

When should they ...

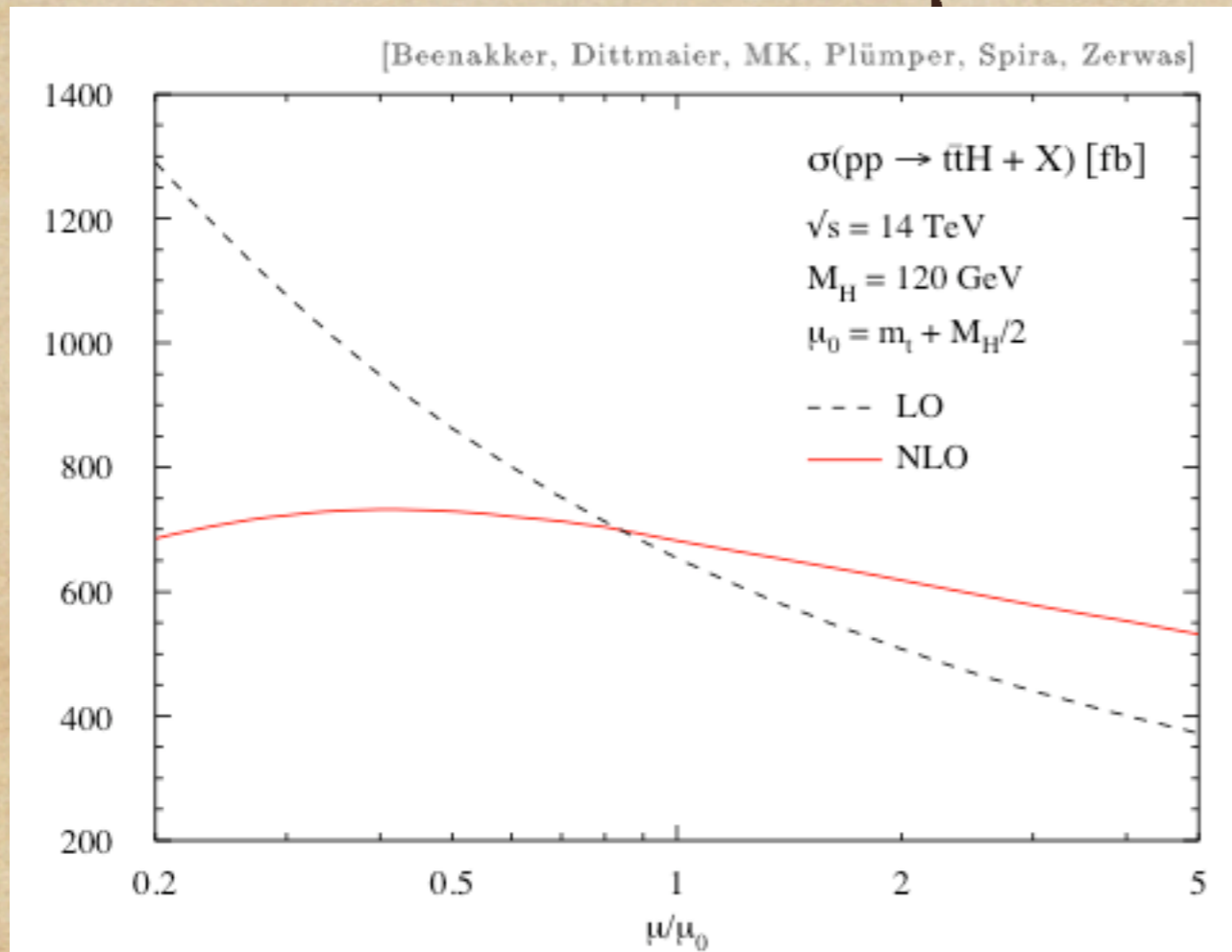
...be used?

1. When the most precise knowledge of the cross section is needed
2. The measurement is inclusive enough for hadronization effects not to be important
3. To study the “theoretical” errors of a measurement

...not be used?

1. For evaluation of the tails of the distributions
2. As “blind” k-factor estimators for LO distributions

Example: $t\bar{t}H$ @ NLO



1. Discovery channel for a light Higgs
2. Very difficult calculation: $2 \rightarrow 3$ process
3. NLO result improves our prediction of the cross section dramatically

4. p_T distribution of the Higgs are very similar at LO and NLO

5. In absence of a MC@NLO (very hard), ME+PS seems a very reasonable option

