*Convegno Informale di Fisica Teorica Cortona 2012*

# Pair Creation of Particles and Strings

1. History.
2. One-loop QED .
3. Multiloop QED.
4. One-loop open string theory.

# 1. History

F. Sauter 1931: Dirac's theory predicts spontaneous pair creation from vacuum, induced by an external field (first for weak static homogeneous fields).

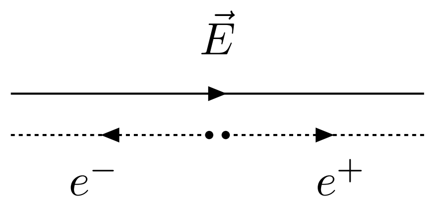
J. Schwinger 1951:

$$
\begin{aligned}
\mathrm{Im}\mathcal{L}^{(1)}_{\rm spin}(E) &= \frac{m^4}{8\pi^3}\beta^2 \sum_{k=1}^{\infty} \frac{1}{k^2} \exp\left[-\frac{\pi k}{\beta}\right] \\
\mathrm{Im}\mathcal{L}^{(1)}_{\rm scal}(E) &= -\frac{m^4}{16\pi^3}\beta^2 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \exp\left[-\frac{\pi k}{\beta}\right]
\end{aligned}
$$

$(\beta = \frac{eE}{m^2})$

- Rate of pair creation per unit volume $= w \approx -2\mathrm{Im}\mathcal{L}$
- $k$th term $\leftrightarrow$ coherent production of $k$ pairs by the field

Im$\mathcal{L}(E)$ nonperturbative in the field $\leftrightarrow$ tunneling picture: A virtual pair turns real by separating out along the field and drawing a sufficient energy to make up for its rest mass energies.




Pair creation rate exponentially small for $E \ll E_{\text{cr}} = 10^{16} V/cm$.

Lasers are now coming close:

- Polaris (Jena): $\frac{E^2}{E_{\text{cr}}^2} \sim 10^{-7}$
- ELI (Extreme Light Infrastructure): $\frac{E^2}{E_{\text{cr}}^2} \sim 10^{-3}$
- XFEL (DESY): $\frac{E^2}{E_{\text{cr}}^2} \sim 10^{-2}$

The pair creation threshold could be substantially lower for laser field configurations, for example:

- Counterpropagating laser beams of linear polarization (M. Ruf, G. R. Mocken, C. Müller, K. Z. Hatsagortsyan, C. H. Keitel, PRL 102, 080402, 2009).
- Superimposing a plane-wave X-ray beam with a strongly focused optical laser pulse (G.D. Dunne, H. Gies, R. Schützhold, PRD 80:111301, 2009).
- .......... (many more)

However, the calculation of the pair creation rate for generic electric fields requires approximative methods.
Some classical results on pair creation by more general electric fields (mostly using WKB):

- Keldysh 1965
- Brezin & Itzykson 1970
- Narozhnyi & Nikishov 1970
- Popov 1972
- Popov & Marinov 1972

# The worldline instanton method

I.K. Affleck, O. Alvarez, N.S. Manton, NPB 197, 509 (1982)

## Worldline representation of the QED effective action

- R.P. Feynman, PR 80, (1950) 440 (Scalar QED)
- R.P. Feynman, PR 84 (1951) 108 (Spinor QED)

Scalar QED:

*(Quenched) effective action* $\Gamma(A)$

$$\Gamma(A) \;=\; \int d^4x\, \mathcal{L}(A) = \int_0^\infty \frac{dT}{T}\, \mathrm{e}^{-m^2T} \int_{x(T)=x(0)} \mathcal{D}x(\tau)\, e^{-S[x(\tau)]}$$

$T \;=\;$ *proper time of the loop scalar*

$S[x(\tau)] \;=\;$ *worldline action*

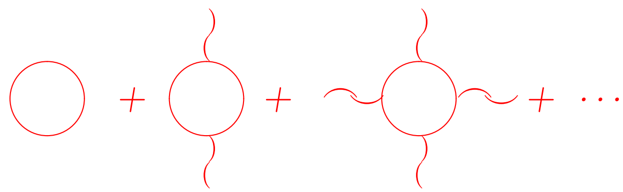
$$S \;=\; S_0 + S_{\rm ext} + S_{\rm int}$$

$$S_0 \;=\; \int_0^T d\tau \frac{\dot{x}^2}{4}$$

(free propagation)

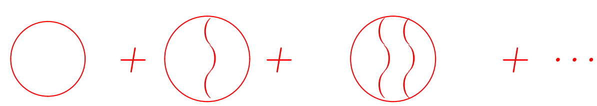
$$S_{\mathrm{ext}} \;=\; ie \int_0^T \dot{x}^\mu A_\mu(x(\tau))$$

(external photons)



$$S_{\mathrm{int}} \;=\; -\frac{e^2}{8\pi^2} \int_0^T d\tau_1 \int_0^T d\tau_2 \frac{\dot{x}(\tau_1) \cdot \dot{x}(\tau_2)}{(x(\tau_1) - x(\tau_2))^2}$$

(internal photons)



*Start at one-loop:*

$$\Gamma_{\rm scalar}[A] \;=\; \int_0^\infty \frac{dT}{T}\, {\rm e}^{-m^2T} \int \mathcal{D}x\, {\rm e}^{-\int_0^T d\tau\left(\frac{\dot{x}^2}{4}+ieA\cdot\dot{x}\right)}$$

*Rescaling* $\tau = Tu$*, this becomes*

$$\Gamma[A] \;=\; \int_0^\infty \frac{dT}{T}\, {\rm e}^{-m^2T} \int \mathcal{D}x\, {\rm e}^{-\left(\frac{1}{T}\int_0^1 du\,\dot{x}^2+ie\int_0^1 du A\cdot\dot{x}\right)}$$

*The* $T$ *integral has a stationary point at*

$$T_c \;=\; \frac{\sqrt{\int du\,\dot{x}^2}}{m}$$

$$\rightarrow \qquad {\rm Im}\Gamma \;=\; \frac{1}{m}\sqrt{\frac{2\pi}{T_c}}\,{\rm Im}\int \mathcal{D}x\, {\rm e}^{-\left(m\sqrt{\int du\,\dot{x}^2}+ie\int_0^1 du A\cdot\dot{x}\right)}$$

*The new worldline action,*

$$S = m\sqrt{\int du\,\dot{x}^2} + ie\int_0^1 du A\cdot\dot{x}$$

*is stationary if*

$$m\frac{\ddot{x}_\mu}{\sqrt{\int du\,\dot{x}^2}} = ieF_{\mu\nu}\dot{x}_\nu$$

*Contract with* $\dot{x}^\mu \Rightarrow \dot{x}^2 = \mathrm{const.} \equiv a^2 \Rightarrow m\ddot{x}_\mu = ieaF_{\mu\nu}\dot{x}_\nu$

$\Rightarrow$ *the extremal action trajectory* $x^{\mathrm{cl}}(u)$ *is a periodic solution of the Lorentz force equation*
*(*$\equiv$ *worldline instanton )*

*semiclassical approximation (= weak field approximation)*

$$\mathrm{Im}\mathcal{L}(E) \overset{\mathrm{E}\to 0}{\sim} \mathrm{e}^{-S[x^{\mathrm{cl}}]}$$

*Constant field case:*

$\vec{E} = (0, 0, E) = const.$

*Periodicity condition* $\Rightarrow a = \frac{m}{eE} 2n\pi, \qquad n \in \mathbf{Z}^+$

$$\begin{aligned} x^{\mathrm{cl}}(u) &= \frac{m}{eE} (x_1, x_2, \cos(2n\pi u), \sin(2n\pi u)) \\ S[x^{\mathrm{cl}}] &= n\pi \frac{m^2}{eE} \end{aligned}$$

*$nth$ worldline instanton* $\quad \Rightarrow \quad$ *$nth$ Schwinger exponential*

**Generalization to arbitrary electric fields**

*G.V. Dunne, C.S., PRD 72, 105004 (2005)*

*Example: Single bump space dependent electric field*

$$E(x_3) = E\,\mathrm{sech}^2(kx_3)$$

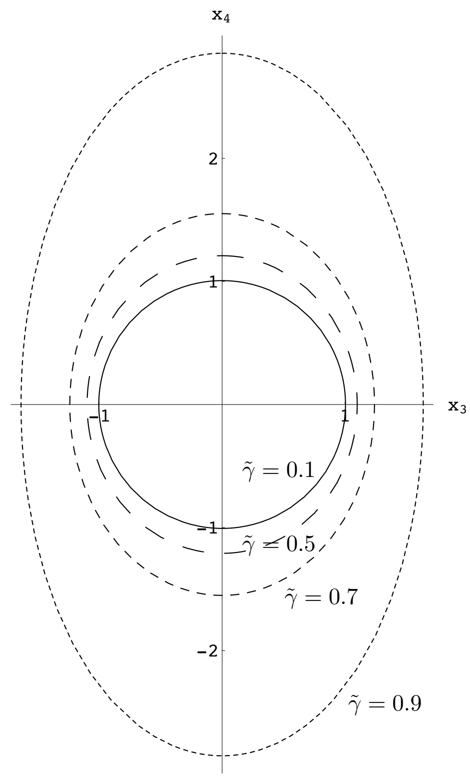
$$\begin{aligned} x_3(u) &= \frac{m}{eE}\frac{1}{\tilde{\gamma}}\,\mathrm{arcsinh}\Big(\frac{\tilde{\gamma}}{\sqrt{1-\tilde{\gamma}^2}}\;\sin(2n\pi u)\Big) \\ x_4(u) &= \frac{m}{eE}\frac{1}{\tilde{\gamma}\sqrt{1-\tilde{\gamma}^2}}\,\mathrm{arcsin}(\tilde{\gamma}\cos(2n\pi u)) \end{aligned}$$

$$\tilde{\gamma} \equiv \frac{mk}{eE} \qquad\qquad \text{(inhomogeneity parameter)}$$

*Stationary action:*

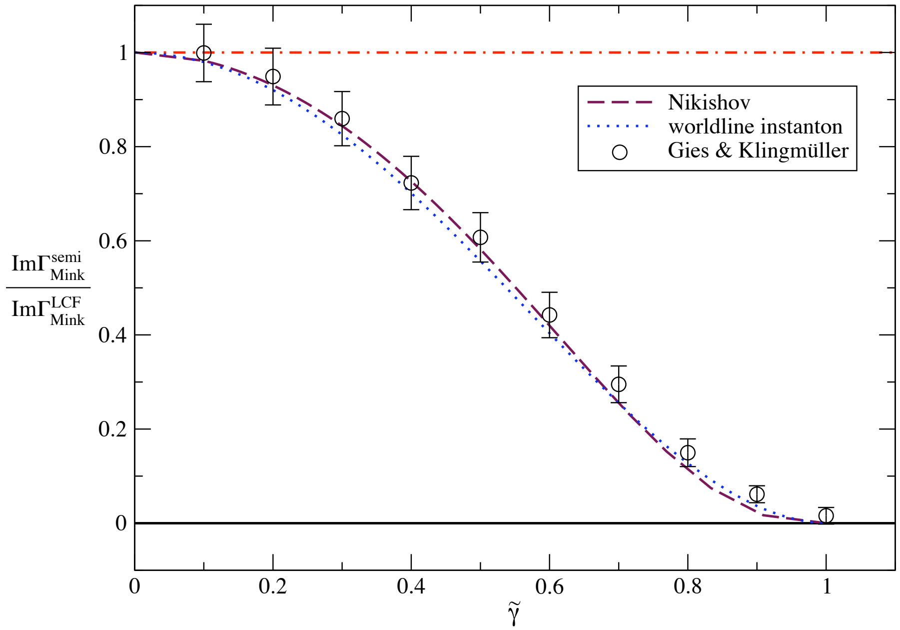
$$S_0 \;=\; n\,\frac{m^2\pi}{eE}\left(\frac{2}{1+\sqrt{1-\tilde{\gamma}^2}}\right)$$

$S_0$ *increases with* $\tilde{\gamma} \to$ *decrease of pair creation rate*




*The instantons cease to exist for $\tilde{\gamma} > 1$ (the total energy that can be extracted from the field is then less than $2m$).*

*Comparison of the pair creation rate with exact results:*




*No pair creation for $\tilde{\gamma} > 1$!*

# 3. Multiloop QED

*I.K. Affleck, O. Alvarez, N.S. Manton, NPB 197, 509 (1982)*

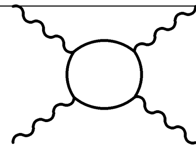
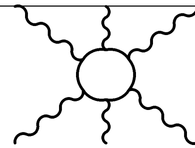
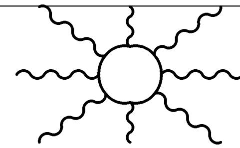
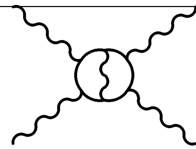
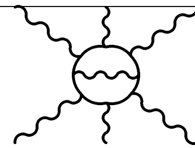
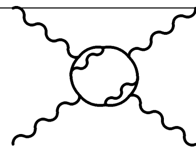
*The worldline instanton remains a stationary trajectory even with photon insertions. Evaluating the photon insertion term $S_i$ on this trajectory gives an all-loop formula:*

$$\mathcal{L}_{\rm scal}^{\rm all-loop}(E) = \sum_{l=1}^{\infty} {\rm Im}\mathcal{L}_{\rm scal}^{(l)}(E) \overset{\beta\to 0}{\sim} \frac{m^4\beta^2}{8\pi^3}\exp[-\frac{\pi}{\beta}+\alpha\pi] = \mathcal{L}_{\rm scal}^{(1)}\, {\rm e}^{\alpha\pi}$$

$(\beta = eE/m^2)$.

*The mass is the physically renormalized one!*

*This corresponds to an infinite set of Feynman diagrams:*

| | Number of external legs | | | |
|---|---|---|---|---|
| Number of loops | 4 | 6 | 8 | $\cdots$ |
| 1 |  |  |  | $\cdots$ |
| 2 |  |  | $\cdots$ | $\cdots$ |
| 3 |  | $\cdots$ | $\ddots$ | $\vdots$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\ddots$ | $\vdots$ |

*All mass renormalization counterdiagrams are also included.*

*S.L. Lebedev and V.I. Ritus 1984:*

- *Independent derivation from Coulomb corrections to the tunneling picture.*
- *Explicit two-loop check,*

$$\sum_{l=1}^{2} \mathrm{Im}\mathcal{L}^{(l)}_{\mathrm{scal}}(E) \overset{\beta\to 0}{\sim} \frac{m^4\beta^2}{8\pi^3}\exp(-\frac{\pi}{\beta})(1+\alpha\pi)$$

Program: From the AAM formula one can, using Borel dispersion relations, obtain information on the $l$ - loop $N$ - photon amplitudes:

- High-order behaviour qualitatively different for physical and generic mass renormalization.
- For large $N$ it is dominated by the quenched (one fermion loop) contribution.
- The perturbation series may converge in the quenched approximation.

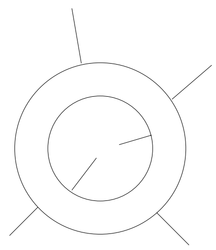
Gerald V. Dunne and C.S., JHEP 0206:042 (2002);
J. Phys.: Conf. Ser. 37 (2006) 59.
L.C. Martin, C.S., V.M. Villanueva, NPB 668, 335 (2003).
I. Huet, D.G.C. McKeon, C.S., JHEP 12 (2010) 036.
I. Huet, M. Rausch de Traubenberg, C.S., arXiv:1112.1049 [hep-th].

# 4. One-loop open string theory

*Pair creation of open strings by a constant electric field:*

*C.P. Burgess, Nucl. Phys. B 294, 427 (1987).*
*C. Bachas and M. Porrati, Phys. Lett. B 296, 77 (1992).*

*At one-loop, have to replace the particle loop by a string annulus:*



*Open strings interact only at the boundary, have to insert*

$$\int_0^T d\tau \, ieA \cdot \dot{x}$$

*on both boundaries.*

*Bachas and Porrati:*

$$\mathrm{Im}\mathcal{L}_{\mathrm{string}}(E) = \frac{1}{4(2\pi)^{D-1}}\sum_{\mathrm{states\,S}}\frac{\beta_1+\beta_2}{\pi\epsilon}\sum_{k=1}^{\infty}(-)^{k+1}\Bigl(\frac{|\epsilon|}{k}\Bigr)^{D/2}\exp\Bigl(-\frac{\pi k}{|\epsilon|}(M_S^2+\epsilon^2)\Bigr)$$

*Here the first sum is over the physical states of the bosonic string, with $M_S$ the mass of the state. The second sum is a Schwinger-type sum.*

$$\beta_{1,2} \;=\; \pi q_{1,2}E$$

*where $q_{1,2}$ are the $U(1)$ charges at the string endpoints, and*

$$\epsilon \;=\; \frac{1}{\pi}(\mathrm{arctanh}\beta_1 + \mathrm{arctanh}\beta_2)$$

*This formula reproduces in the weak – field limit the Schwinger formula for arbitrary integer spin $J$.*
*For stronger fields it diverges at a critical field strength*

$$E_{\mathrm{cr}} \;=\; \frac{1}{\pi\,|\max q_i|}$$

*Heuristically, a field of this strength would break the string apart.*

*A. Torrielli and C. S., JPA 43:402003 (2010): Use worldsheet instantons:*

$$\Gamma[F] \;=\; \frac{1}{2} \sum_{\text{oriented states}} \int_0^\infty \frac{dT}{T} (4\pi^2 T)^{-\frac{D}{2}} \mathrm{e}^{-\pi T M_S^2} \int \mathcal{D}x \, \mathrm{e}^{-S_E[x,F]}$$

*Path integral over the embeddings of the annulus at fixed $T$ into $D$ - dimensional flat space.*

*Worldsheet action:*

$$S_E = \frac{1}{4\pi\alpha'} \int d\sigma d\tau \, \partial_a x^\mu \partial^a x_\mu - i\frac{q_1}{2} \int d\tau \, x^\mu \partial_\tau x^\nu F_{\mu\nu}|_{\sigma=0} - i\frac{q_2}{2} \int d\tau \, x^\mu \partial_\tau x^\nu F_{\mu\nu}|_{\sigma=\frac{1}{2}}$$

*Here $\alpha'$ is the Regge slope, which will be set equal to $\frac{1}{2}$ in the following. The worldsheet is parameterized as a rectangle $\sigma \in [0, \frac{1}{2}]$ and $\tau \in [0, T]$ where $\tau = T$ is identified with $\tau = 0$.*

*Rescale $\tau = Tu$ and do the $T$ - integral by the method of steepest descent. Stationary point:*

$$T_0 \;=\; \sqrt{\frac{I_u}{I_\sigma + 2\pi^2 M_S^2}}$$

$$\begin{aligned} I_\sigma &:= \int_0^1 du \int_0^{\frac{1}{2}} d\sigma\, \partial_\sigma x^\mu \partial_\sigma x_\mu \\ I_u &:= \int_0^1 du \int_0^{\frac{1}{2}} d\sigma\, \partial_u x^\mu \partial_u x_\mu \end{aligned}$$

*New worldsheet action:*

$$\begin{aligned} S_{\rm eff} \;=\; & \frac{1}{\pi}\sqrt{I_u}\sqrt{I_\sigma + 2\pi^2 M_S^2} - i\frac{q_1}{2}\int d\tau\, x^\mu \partial_\tau x^\nu F_{\mu\nu}|_{\sigma=0} \\ & - i\frac{q_2}{2}\int d\tau\, x^\mu \partial_\tau x^\nu F_{\mu\nu}|_{\sigma=\frac{1}{2}} \end{aligned}$$

*$k$th worldsheet instanton:*

$$
\begin{aligned}
x_k^{D-1} &= \frac{2\pi M_S}{|a|}\cos(2\pi k u)\cosh(b - a\sigma) \\
x_k^{D} &= \frac{2\pi M_S}{|a|}\sin(2\pi k u)\cosh(b - a\sigma)
\end{aligned}
$$

*(with the remaining coordinates constants).*

$$
\begin{aligned}
b &= \mathrm{arctanh}\beta_1 \\
a &= 2(\mathrm{arctanh}\beta_1 + \mathrm{arctanh}\beta_2)
\end{aligned}
$$

*Worldsheet action:*

$$
S[x_k^\mu] = 2\pi^2 M_S^2 \frac{k}{a}
$$

*This correctly reproduces the exponents of the Bachas-Porrati formula in the large $M_S$ limit.*

*Next:*

- *On to more general electric field backgrounds (for strings, so far only the constant field case has been feasible).*
- *The boundaries of the worldsheet instanton are worldline instantons. Will this be true for the general case?*