Supersymmetric Fluid Dynamics

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June 1, 2012

based on hep-th/1105.4706 , hep-th/1107.2780 and a forthcoming paper

Motivations

- Perturbative procedure for Fluid-Gravity Correspondence

  Battacharyya, Hubeney, Minwalla, Rangamani – hep-th/0712.2456

  - Gravitational derivation of Navier-Stokes Equations
  - Explicit construction of Fluid Energy Momentum Tensor

- First attempt of a supersymmetric extension of Fluid-Gravity Correspondence

  L.G.C. Gentile, P.A. Grassi, AM – hep-th/1105.4706

  - Corrections to Navier-Stokes Equations

- Supersymmetric Fluid Lagrangians

  T.S. Nyawelo, J.W. van Holten, S. Groot Nibbelink – hep-th/0307283
  R. Jackiw, V.P. Nair, S.Y. Pi, A.P. Polychronakos – hep-ph/0407101

Outlook

Aim: analysis of Supersymmetric Fluid Dynamics

- From Fluid/Gravity Correspondence
  - Finite Black Hole Superpartner
  - Boundary $T_{\mu\nu}$ from Brown-York prescription
- From Lagrangian
  - Supersymmetrization of suitable bosonic models

Preliminary Analysis: Supersymmetric Perfect Fluid

L.G.C. Gentile, P.A. Grassi, AM – in preparation

Contents

- Susy extension of Fluid–Gravity correspondence
- Supersymmetric Lagrangian Models
- Conclusions

Fluid-Gravity Correspondence

- *AdS*/*CFT* correspondence

Type IIB string theory on asymptotically $AdS_5 \times S^5$

$\mathcal{N} = 4$ SYM on AdS_5 boundary

- Effective Description: length scales >> mean free path length

Einstein Equations with negative cosmological constant

Relativistic Fluid Dynamics

- Static Solution

Black Hole or Black Branes in *AdS*. . .

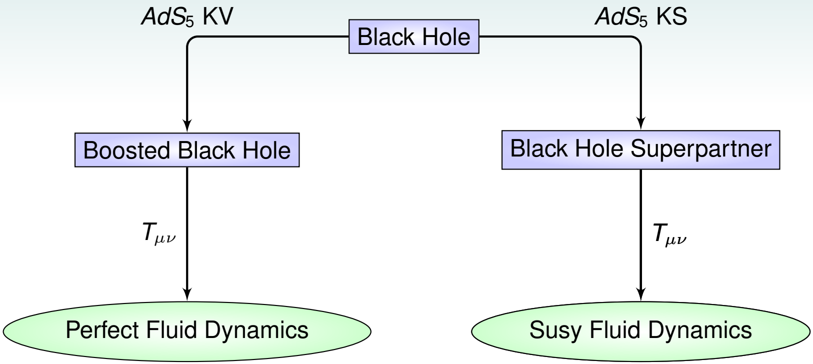
static perfect fluid configuration

- Perturbation

Evolving Black Brane

dissipative fluid flow

In Brief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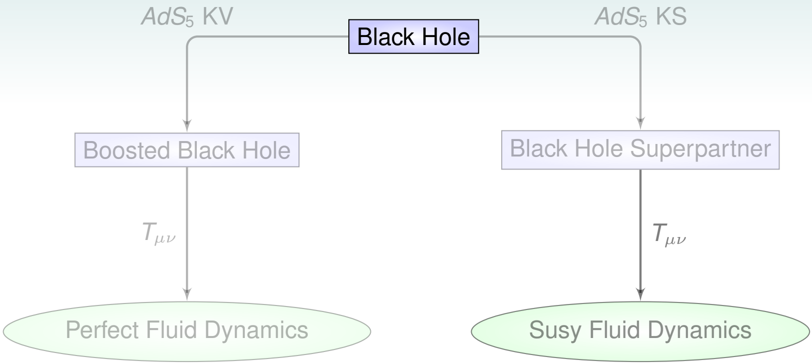
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BH from SUGRA

- BH solution in AdS_5: need of $D = 5, \mathcal{N} = 2$ gauged Sugra

A. Ceresole, G. Dall'Agata – hep-th/0004111
A. Van Proeyen *et al.* – hep-th/0403045

$$\left\{ e_\mu^a \, , \; A_\mu^I \, , \; \psi_\mu \, , \; \zeta^A \, , \; q^X \, , \; \lambda^a \ldots \right\}$$

- For our purpose $\longrightarrow$ consistent truncation:

$$\left\{ e_\mu^a \, , \; A_\mu^I \, , \; \psi_\mu \right\} + \text{constant scalars}$$

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BH: Basic Setup

Background choice

- Black Hole solution, asymptotic AdS_5 in Poincaré patch

$$ds^2_{BH} = -r^2 f(r)\,dt^2 + \frac{1}{r^2 f(r)} dr^2 + r^2 d\vec{x}^2 \,, \qquad f(r) = 1 + \frac{\mu}{r^4}$$

- Other fields: $\psi_\mu = 0$, $A^I_\mu = 0 \implies A^I_\mu \longrightarrow A_\mu$

Energy Momentum Tensor

- Brown-York procedure $\longrightarrow T_{\mu\nu}$ on the boundary of AdS_5

$$T_{\mu\nu} \propto \mu\,\mathrm{diag}\,(3\,,\,1\,,\,1\,,\,1)$$

- Energy Momentum Tensor for Perfect Conformal Fluid in Rest Frame

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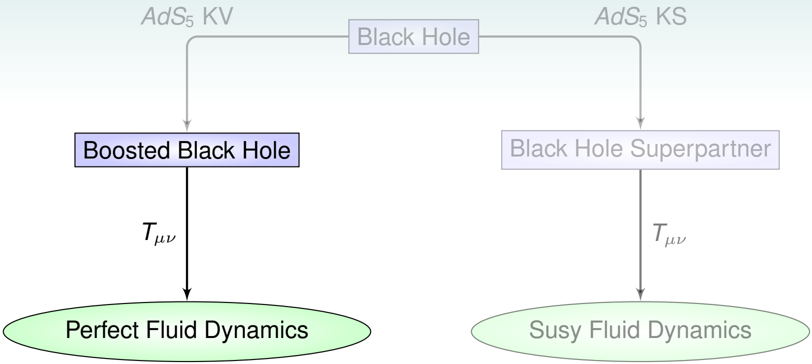

BBH from AdS_5 Isometry

- AdS_5 isometry group: $SO(2,4) \longrightarrow 15$ parameters
- BH breaks 8 isometries:
  - b: r–dilatation
  - $\{\beta_i\}$: boost
  - $\{c\,,\,\gamma_i\}$: conformal transformation
- Infinitesimal metric deformation $\longrightarrow \delta g_{BH} = \mathcal{L}_{KV} g_{BH} \neq 0$
- Boosted Black Hole: finite transformation of g_{BH} associated to $\{b\,,\,\beta_i\}$

$$ds^2 = \frac{1}{r^2 f(br)} dr^2 + r^2 \left[-f(r)\, u_\mu u_\nu + P_{\mu\nu} \right] dx^\mu dx^\nu$$

$$f(br) = 1 + \frac{\mu}{(br)^4}\,, \qquad P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}\,, \qquad u^\mu = \left\{ -\frac{1}{\sqrt{1+\beta^2}}\,,\, \frac{\beta_i}{\sqrt{1+\beta^2}} \right\}$$

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Constant $\{b\,,\ \beta_i\}$

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$$T_{\mu\nu} \propto \frac{\mu}{b^4}\left(4u_\mu u_\nu + \eta_{\mu\nu}\right)$$

Local $\{b\,,\ \beta_i\}$

- Minwalla's perturbative procedure
  - Navier–Stokes equations
  - $T_{\mu\nu}$ for dissipative fluid

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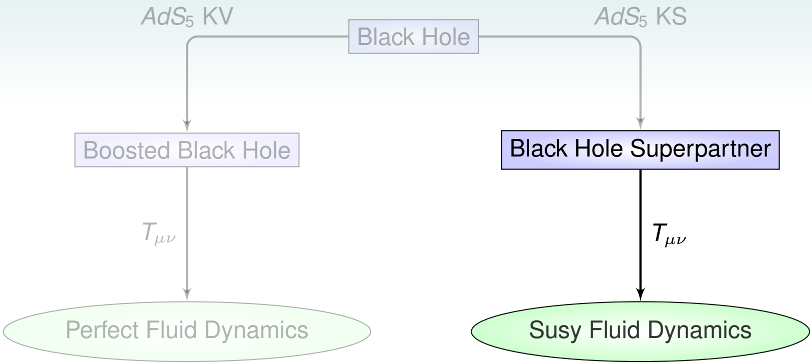
BHS: Black Hole Superpartner



BHS from Bulk Fermions

- Supersymmetry transformation with fermionic parameter ϵ:

$$\delta_\epsilon \psi_M = \mathcal{D}_M(\hat{\omega})\,\epsilon + \frac{i}{4\sqrt{6}} e_M^A \hat{F}^{BC} (\Gamma_{ABC} - 4\eta_{AB}\Gamma_C)\,\epsilon\ ,$$

$$\delta_\epsilon g_{MN} = -\frac{1}{2}\mathrm{Re}\left(i\bar{\epsilon}\Gamma_{(M}\psi_{N)}\right)\ , \qquad \delta_\epsilon A_M = -\frac{4}{\sqrt{6}}\mathrm{Re}\left(\bar{\epsilon}\psi_M\right)$$

- Presence of bulk fermions $\longrightarrow$ metric deformation (BH Superpartners)

P.C. Aichelburg, F. Embacher – Phys.Rev. D34 (1986) 3006
B.A. Burrington, J.T. Liu, W.A. Sabra – hep-th/0412155

- AdS_5 without BH: we require no deformations $\longrightarrow$ Killing Spinor ϵ

$$\delta_\epsilon \psi_M = \mathcal{D}_M(\hat{\omega})\,|_{AdS}\ \epsilon = 0$$

- Turning on BH:

$$\delta_\epsilon \psi_M = \mathcal{D}_M(\hat{\omega})\,|_{BH}\ \epsilon \neq 0 \longrightarrow \delta_\epsilon^2 g_{MN} \neq 0\ ,\ \delta_\epsilon^2 A_M \neq 0$$

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Killing Spinor

- Computation of the AdS_5 Killing Spinor ϵ:

$$\left(\partial_M + \frac{1}{4}\,\hat{\omega}_M^{AB}|_{AdS}\,\Gamma_{AB} + \frac{1}{2}\,e_M^A|_{AdS}\,\Gamma_a\right)\epsilon = 0$$

- Space-Time Splitting: $\{t, r\} \otimes \{x^i\}$, $\Gamma = \sigma \otimes \hat{\sigma}$, $\epsilon = \varepsilon \otimes \eta$

$\varepsilon \rightarrow$ 2 $\mathbb{R}$ bosonic components $\eta \rightarrow$ 2 $\mathbb{C}$ fermionic components

- Solution

$$\epsilon = \frac{1}{\sqrt{r}}\,\sigma_0\varepsilon_0 \otimes \hat{\sigma}_0\eta_1 - \sqrt{r}x^\mu\,\sigma_3\varepsilon_0 \otimes \hat{\sigma}_\mu\eta_1 - \sqrt{r}\sigma_3\varepsilon_0 \otimes \hat{\sigma}_0\eta_0$$

- ε_0 satisfy $(1-\sigma_1)\,\varepsilon_0 = 0 \rightarrow$ only 1 $\mathbb{R}$ bosonic component
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BH Superpartner

- Turning on BH: $\delta_\epsilon \psi_M \neq 0 \longrightarrow \delta^2_\epsilon g_{MN} \neq 0$, $\delta^2_\epsilon A_M \neq 0$
- $\delta^2_\epsilon g_{MN}$, $\delta^2_\epsilon A_M$:
  - depend on fermionic bilinears $\eta_i^\dagger \sigma_\mu \eta_j$
  - are infinitesimal corrections
- Minwalla's procedure: local bilinears
  - EoM's for $\eta_i^\dagger \sigma_\mu \eta_j$
  - Metric corrections in bilinear derivative expansion

But, to compute $T_{\mu\nu}$ $\longrightarrow$ we need Finite Transformation!

- First step: constant bilinears $\longrightarrow$ analogue to $T_{\mu\nu}$ for perfect fluid

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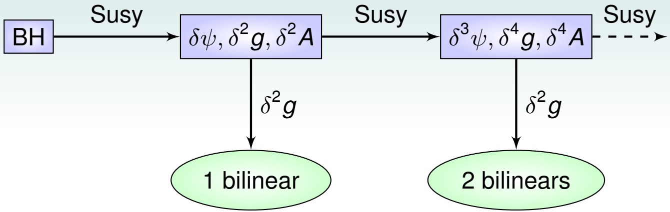
Perturbative Procedure



- Fermionic bilinears $\longrightarrow$ series truncates!
- Development of algorithms to compute, order by order

$$\left\{ \psi_M \ , \ e_M^A \ , \ A_M \ , \ \hat{\omega}_M^{AB} \right\}$$

- Implementation of algorithms in `Mathematica` code

Result Analysis

$\eta_1 \neq 0\ ,\ \eta_0 \neq 0$:

- δg_{MN} is t–dependent
- $\delta A_M \neq 0$
- Computation is cumbersome$\rightarrow$ work in progress

$\eta_1 \neq 0\ ,\ \eta_0 = 0$:

- δg_{MN} is t–dependent
- $\delta A_M \neq 0$
- Finite metric is really complicated

$\eta_1 = 0\ ,\ \eta_0 \neq 0$:

- δg_{MN} is t–independent
- $\delta A_M = 0$
- Finite metric: depend on $M = \eta_0^\dagger \eta_0$ and $V_i = \eta_0^\dagger \sigma_i \eta_0$

Simplest case: $\eta_1 = 0 \, , \; \eta_0 \neq 0$

- Finite Metric at Large r

$$
\begin{aligned}
ds^2 = & - r^2 f(r)\, dt^2 + \frac{1}{r^2 f(r)} dr^2 + r^2 d\vec{x}^2 + \\
& - \frac{\mu}{r^3} M \, dt dr - \frac{3\mu}{32} M^2 \, dt^2 + \frac{3\mu}{32} M^2 \, d\vec{x}^2 - \frac{3\mu}{16 r^4} M^2 \, dr^2
\end{aligned}
$$

- Energy Momentum Tensor

$$T_{\mu\nu} = -\frac{\mu}{2} \mathrm{diag}(3 \, , \; 1 \, , \; 1 \, , \; 1)$$

No contributions from this BH Superpartner

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Road to Supersymmetric Fluid Dynamics

Interpretation of modifyed $T_{\mu\nu}$? $\longrightarrow$ we need a Supersymmetric Fluid Dynamics

P.A. Grassi, AM, L. Sommovigo – hep-th/1107.2780
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- Idea: supersymmetrize an appropriate bosonic Lagrangian

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- Simplest bosonic model using Clebsh Parametrization:

$$\mathcal{L} = \sqrt{-g}\left(j^{\mu}\left(\partial_{\mu}a + \alpha\partial_{\mu}\beta\right) + f\left(j^{2}\right)\right)$$

- EoM for $a \longrightarrow \partial_{\mu}j^{\mu} = 0$
- Other EoM's $\oplus$ manipulations $\longrightarrow$ Euler equations (NS with no dissipation)

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Supersymmetrization

- $j_\mu \to$ linear superfield J satisfying $\bar{D}DJ = 0$
- $\partial_\mu a + \alpha \partial_\mu \beta \to$ real superfield A in Wess-Zumino gauge
- Introduction of $\mathcal{J}^\mu \propto \left(\bar{D}\gamma_5\gamma_\mu D\right) J$

$$S = \int \mathrm{d}^4x \int \mathrm{d}^4\theta \left(-J\,A + F\left(\mathcal{J}_\mu \mathcal{J}^\mu\right) J^2\right)$$

- Component expansion $\longrightarrow$ first comparison with Fluid-Gravity results

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Results and Open Issues

Fluid-Gravity Correspondence:

- Construction of finite Black Hole Superpartner in different cases
- Computation of boundary Energy Momentum Tensors

Models for Supersymmetric Fluid Dynamics:

- Construction of suitable supersymmetric lagrangians
- Analysis of different sectors

In preparation:

- Deeper results analysis
- Dissipative contribution
- Link between the above results

Fluid Dynamics from BBH: Minwalla's Procedure

- Local parameters $\longrightarrow$ g_{BBH} is not a Einstein solution
- Perturbative expansion for metric and parameters

$$G = G^0(\beta_i, b) + \varepsilon G^1(\beta_i, b) + O\left(\varepsilon^2\right)$$

$$\beta_i = \beta_i^{(0)} + \varepsilon \beta_i^{(1)} + O\left(\varepsilon^2\right) \ , \qquad b = b^{(0)} + \varepsilon b^{(1)} + O\left(\varepsilon^2\right)$$

- Imposing in ε the Einstein equations (example: first order):
  - EoM's for parameters $\longrightarrow$ Navier-Stokes equations
  - correction to metric: G^1
- From G^1 $\longrightarrow$ finite metric $\longrightarrow$ $T_{\mu\nu}$

$$T_{\mu\nu} = \frac{1}{b^4}\left(4u_\mu u_\nu + \eta_{\mu\nu}\right) - 2\frac{1}{b^3}\sigma_{\mu\nu}$$