Thermal string probes in AdS and finite temperature Wilson loops

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Motivations and Overview

Subject of the talk

Holographic description of Wilson loops for finite temperature gauge theory

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- New method to describe F-string (and D-brane) probes in thermal backgrounds
  - includes the thermal excitations of the probe
  - demands the probe to be in thermal equilibrium with the background

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- Standard method uses extremal fundamental string probes even though the background is at finite temperature
- New method to describe F-string (and D-brane) probes in thermal backgrounds
  - includes the thermal excitations of the probe
  - demands the probe to be in thermal equilibrium with the background
- Method of this talk can be used to find new finite temperature effects in AdS/CFT

References

Talk based on:

- G. Grignani, T. Harmark, A. Marini, N. Obers, M. Orselli
  "Thermal string probes in AdS and finite temperature Wilson loops"
  `arXiv:1201.4862[hep-th]`

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See also:

- G. Grignani, T. Harmark, A. Marini, N. Obers, M. Orselli
  "Heating up the BIon"
  JHEP **1106** (2011), `arXiv:1012.1494 [hep-th]`

- G. Grignani, T. Harmark, A. Marini, N. Obers, M. Orselli
  "Thermodynamics of the hot BIon"
  Nucl.Phys. **B851** (2011), `arXiv:1101.1297 [hep-th]`

Wilson loops

Wilson loop (WL) operator → phase factor associated with the trajectory of a heavy quark in the fundamental representation of the gauge group moving along a closed path $\mathcal{C}$

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- For a $SU(N)$ pure YM theory

$$W(\mathcal{C}) = \frac{1}{N} \mathrm{Tr}\, \mathcal{P} \exp \left[i \oint_{\mathcal{C}} A_\mu \dot{x}^\mu ds \right]$$

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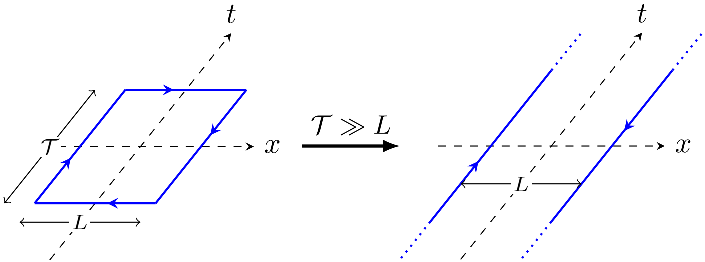
- Generalization for $\mathcal{N}=4$ SYM which includes also six scalar fields ϕ_I

$$W(\mathcal{C}) = \frac{1}{N}\mathrm{Tr}\,\mathcal{P}\exp\left[\oint_{\mathcal{C}} \left(iA_\mu \dot{x}^\mu + |\dot{x}|\theta^I\phi^I\right) ds\right]$$

$\theta^I =$ unit vector in $\mathbb{R}^6$ → position on S^5

Rectangular Wilson loop



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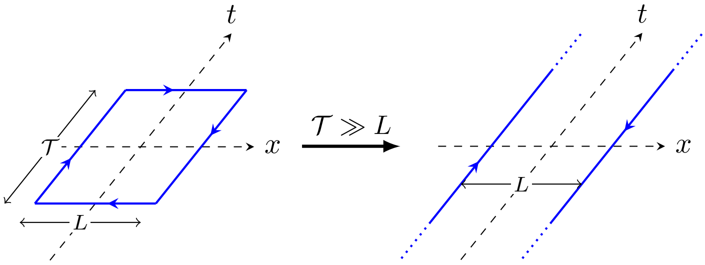


Rectangular WL with $\mathcal{T} \gg L$

- This configuration corresponds to a static quark-antiquark ($q\bar{q}$) pair separated by a distance L
- The potential energy $V_{q\bar{q}}(L)$ of the pair is related to the expectation value of the WL

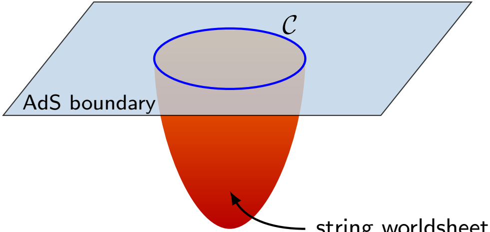
$$\langle W(\mathcal{C}) \rangle \approx \exp[-\mathcal{T} V_{q\bar{q}}(L)]$$

Wilson loops at strong coupling

Consider the case of infinitely heavy (non-dynamical) quarks

- In the AdS/CFT correspondence a Wilson loop is dual to a fundamental string probe with its worldsheet extending into the bulk of AdS space and ending at the location of the loop on the boundary of AdS

[Rey, Yee (1998); Maldacena (1998)]



Wilson loops at strong coupling

For $N \gg 1$, $\lambda \gg 1 \quad \longrightarrow \quad \langle W(\mathcal{C})\rangle = e^{-S(\mathcal{C})}$

$S(\mathcal{C})$ = classical string action $\longrightarrow$ extremize the Nambu-Goto action for the string worldsheet

$$S_{\rm NG} = \frac{1}{2\pi l_s^2}\int d\tau d\sigma\sqrt{\gamma} \qquad \gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$$

$\gamma = \det\gamma_{ab}$, γ_{ab} is the induced metric on the string world-sheet and $g_{\mu\nu}$ the spacetime metric

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- However the classical NG action provides an effective description of the F-string probe only at zero temperature

NG string as a probe of thermal backgrounds

Equations of motion for any probe brane → Carter equation

extrinsic curvature

worldsheet EM tensor

$$K_{ab}{}^{\rho}\,T^{ab} = J \cdot F^{\rho}$$

possible external forces

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- For a string probe → NG EOM ≡ Carter eq with T^{ab} the same EM tensor as in the zero-temperature (extremal) case

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NG is not accurate for the description of F-string probes in thermal backgrounds

- Thermal equilibrium ⟶ the string probe gains the same temperature as the background
- String DOF's are "heated up" by the temperature of the background ⟶ this changes the EM tensor

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New method

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EM tensor → $T_{00} = A r_0^6 (7 + 6 \sinh^2 \alpha), \quad T_{11} = -A r_0^6 (1 + 6 \sinh^2 \alpha)$

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Temperature ⟶ $\dfrac{3}{2\pi r_0 \cosh\alpha}$, Charge ⟶ $kT_{\mathrm{F1}} = 6Ar_0^6 \cosh\alpha \sinh\alpha$

with $A = \Omega_7/(16\pi G)$ and $T_{\mathrm{F1}} = 1/(2\pi l_s^2)$

Setup

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$$k \text{ strings with } 1 \ll k \ll N \text{ and } g_s^2 k \gg 1$$

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- SUGRA F-string requires having many coincident strings:

  k strings with $1 \ll k \ll N$ and $g_s^2 k \gg 1$

  $\longrightarrow$ WL in the k-symmetric product of the fundamental representation

- The expectation value of the WL gives the potential for a static Q-$\bar{Q}$ pair

  $\longrightarrow$ Q ($\bar{Q}$) corresponds to the symmetric rep. of k (anti-)quarks

Background metric

- The dual background is the AdS black hole in the Poincaré patch times a five-sphere

$$ds^2 = \frac{R^2}{z^2}(-f dt^2 + dx^2 + dy_1^2 + dy_2^2 + f^{-1} dz^2) + R^2 d\Omega_5^2$$

$$f(z) = 1 - \frac{z^4}{z_0^4}$$

  - R is the AdS radius
  - the boundary of AdS is at $z = 0$
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- $T = \frac{1}{\pi z_0}$ ⟶ temperature of the black hole as measured by an asymptotic observer

Embedding of the F-string

Static F-string probe between Q and $\bar{Q}$

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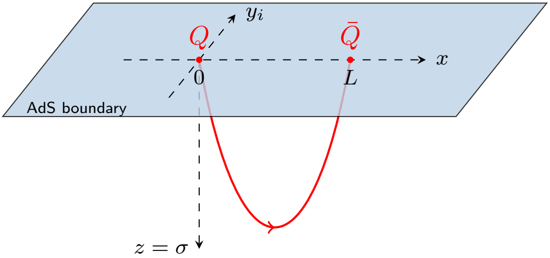
- Ansatz for the embedding: $t = \tau$, $z = \sigma$, $x = x(\sigma)$, $y_1 = y_2 = 0$



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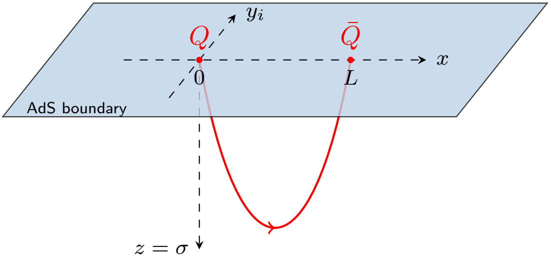

- The induced metric γ_{ab} for this embedding is

$$\gamma_{ab} d\sigma^a d\sigma^b = \frac{R^2}{\sigma^2} \left[-f d\tau^2 + \left(f^{-1} + x'^2 \right) d\sigma^2 \right]$$

Local temperature and thermal equilibrium

- Redshift factor R_0

$$R_0(\sigma) = \sqrt{-\gamma_{00}} = \frac{R}{\sigma}\sqrt{f(\sigma)}$$

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[Tolman (1930); Tolman, Ehrenfest (1930)]

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[Tolman (1930); Tolman, Ehrenfest (1930)]

$$T_{\rm local} = \frac{T}{R_0}$$

- Thermal equilibrium between the F-string probe and the background

$$\frac{T}{R_0} = \frac{3}{2\pi r_0 \cosh\alpha}$$

Local temperature

SUGRA F-string temperature

Action principle and EOM

For a stationary blackfold $\longrightarrow$ action = free energy
Free energy for a SUGRA F-string probe in a general background with redshift factor R_0

$$\mathcal{F} = A \int dV_{(1)} R_0 r_0^6 (1 + 6 \sinh^2 \alpha)$$

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With the setup defined before the free energy for the thermal F-string probe

$$\mathcal{F} = A \left(\frac{3}{2\pi T} \right)^6 \int d\sigma \sqrt{1 + f(\sigma) x'(\sigma)^2} G(\sigma) \,, \quad G(\sigma) \equiv \frac{R^8}{\sigma^8} f(\sigma)^3 \frac{1 + 6 \sinh^2 \alpha(\sigma)}{\cosh^6 \alpha(\sigma)}$$

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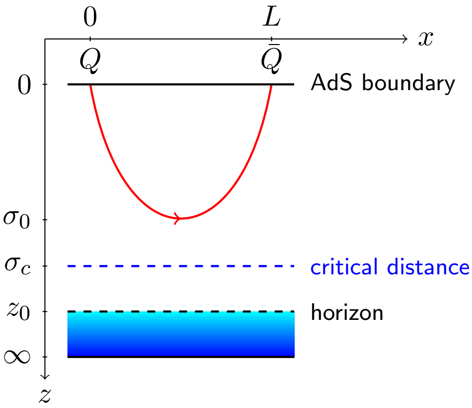
Varying with respect to $x(\sigma)$ this gives the EOM

$$\left(\frac{f(\sigma)x'(\sigma)}{\sqrt{1 + f(\sigma)x'(\sigma)^2}} G(\sigma)\right)' = 0$$

Solution

Boundary conditions: $x(0) = 0$ and $\lim_{\sigma \to \sigma_0} x'(\sigma) = \infty$

general solution $\longrightarrow$ $$x'(\sigma) = \left(\frac{f(\sigma)^2 G(\sigma)^2}{f(\sigma_0) G(\sigma_0)^2} - f(\sigma) \right)^{-\frac{1}{2}}$$



Introduce the dimensionless coordinate $\hat{\sigma} = \pi T \sigma$

$$LT = \frac{2}{\pi} \int_0^{\hat{\sigma}_0} d\hat{\sigma} \left(\frac{f(\hat{\sigma})^2 H(\hat{\sigma})^2}{f(\hat{\sigma}_0) H(\hat{\sigma}_0)^2} - f(\hat{\sigma}) \right)^{-\frac{1}{2}}$$

with $f(\hat{\sigma}) = 1 - \hat{\sigma}^4$, $\hat{\sigma}_0 = \pi T \sigma_0$,

$$H(\hat{\sigma}) = \frac{f(\hat{\sigma})^3}{\hat{\sigma}^8} \frac{1 + 6 \sinh^2 \alpha(\hat{\sigma})}{\cosh^6 \alpha(\hat{\sigma})}, \quad \kappa \equiv \frac{2^5 k T_{\mathrm{F1}}}{3^7 A R^6} = \frac{f(\hat{\sigma})^3}{\hat{\sigma}^6} \frac{\sinh \alpha(\hat{\sigma})}{\cosh^5 \alpha(\hat{\sigma})}$$

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- LT depends only on the dimensionless quantities κ and $\hat{\sigma}_0$
- The equation for κ enforces the charge conservation and can be used to find $\alpha(\hat{\sigma})$
- In terms of the gauge theory variables k, λ and N, κ can be written

$$\kappa = \frac{2^7}{3^6} \frac{k\sqrt{\lambda}}{N^2}$$

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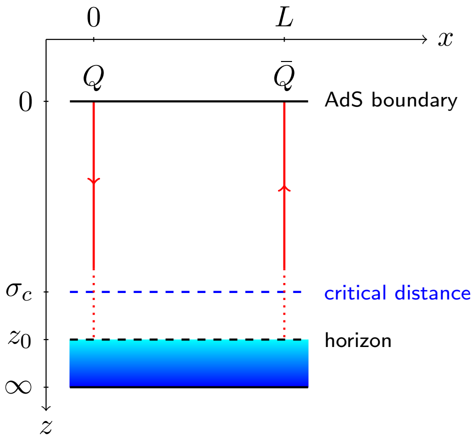

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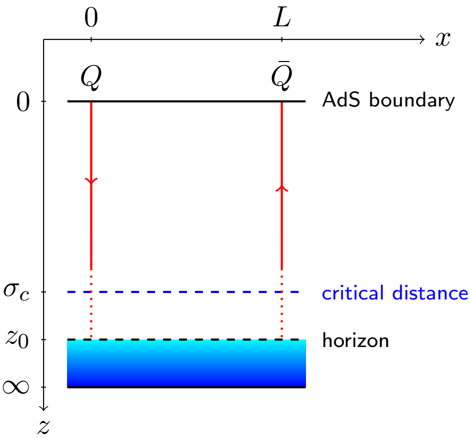

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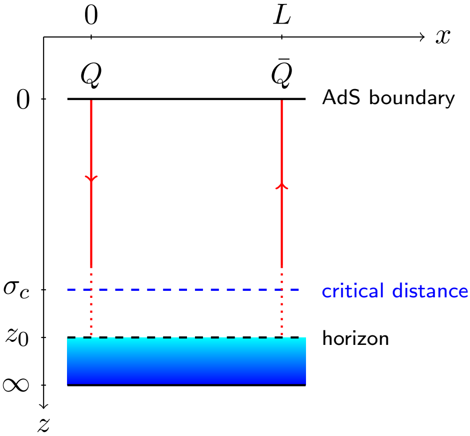

- Disconnected configuration → two straight strings stretching from the charges Q and $\bar{Q}$ towards the horizon
- In the gauge theory side this corresponds to two Polyakov loops, *i.e.* Wilson lines along the thermal circle
- Pair of free charges Q and $\bar{Q}$ → screened pair

Critical distance

From the equation for κ and the fact that $\sinh\alpha/\cosh^5\alpha \leq 2^4/5^{5/2}$ for a given κ the equation can only be satisfied provided

$$\hat{\sigma} \leq \hat{\sigma}_c \quad \text{with} \;\; \hat{\sigma}_c^2 = \sqrt{1 + \frac{5^{5/3}}{2^{14/3}}\kappa^{\frac{2}{3}}} - \frac{5^{5/6}}{2^{7/3}}\kappa^{\frac{1}{3}}$$

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  - The SUGRA F-string has a maximal temperature for a given k
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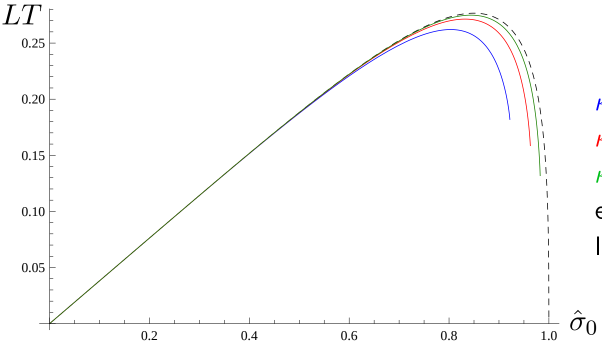
- Physical interpretation: F-string probe is in thermal equilibrium with the background
  - The SUGRA F-string has a maximal temperature for a given k
  - Tolman law $\longrightarrow$ the local temperature goes to infinity as one approaches the black hole
- This is a qualitatively new effect which means that the probe description breaks down beyond the critical distance $\hat{\sigma}_c$

LT as a function of $\hat{\sigma}_0$

Plot of LT as a function of $\hat{\sigma}_0$ for various values of κ:



$\kappa = 0.01$ (blue line)
$\kappa = 0.001$ (red line)
$\kappa = 0.0001$ (green line)
extremal probe (black dashed line)

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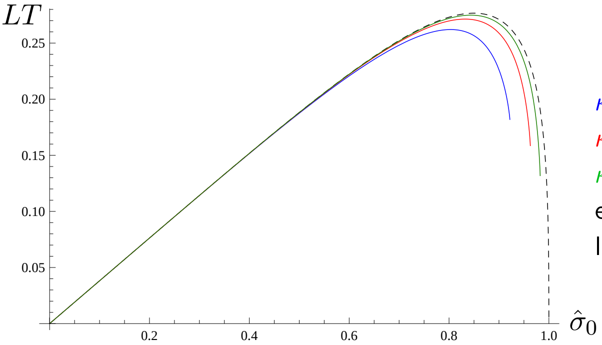
Plot of LT as a function of $\hat{\sigma}_0$ for various values of κ:



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extremal probe (black dashed line)

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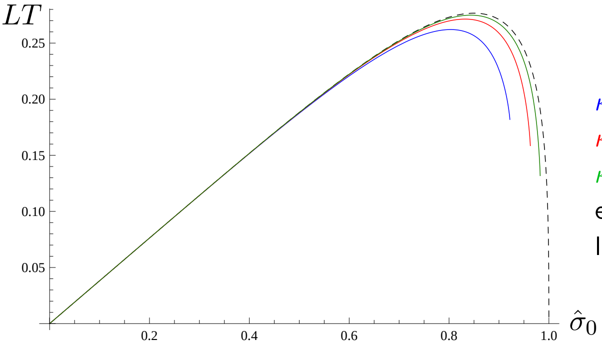

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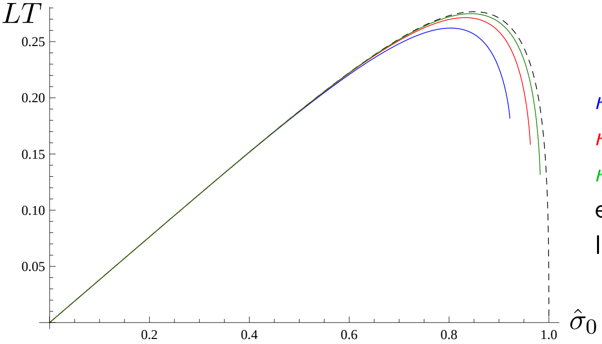

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- LT always shows a maximum $(LT)_{\max}$
- For $LT > (LT)_{\max} \longrightarrow$ no connected solutions

The extremal F-string probe corresponds to $\kappa = 0$

$$LT|_{\kappa=0} = \frac{2\sqrt{2\pi}}{\Gamma\left(\frac{1}{4}\right)^2}\hat{\sigma}_0\sqrt{1-\hat{\sigma}_0^4}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\hat{\sigma}_0^4\right)$$

→ matches with the result found using the NG string probe

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- Terms with κ mark an important departure from the results obtained using the extremal F-string

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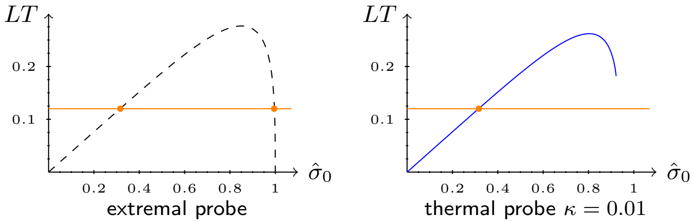
  - extremal probe ⟶ two solutions available for given $LT < (LT)_{\max}$
  - thermal F-string probe ⟶ only one solution for $LT < LT|_{\hat{\sigma}_c}$



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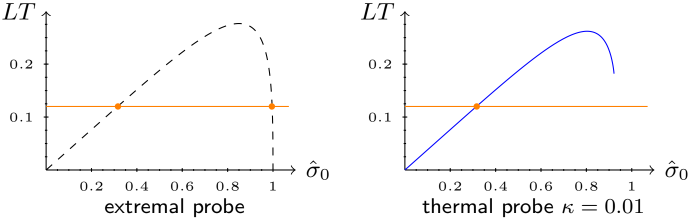

- Quantitative differences
  - in the dependence of LT on $\hat{\sigma}_0$
  - $(LT)_{\max}$ receives a $O(\sqrt{\kappa})$ correction for small κ ⟶ $1/N$ effect missed by the less accurate extremal F-string probe

Free energy

The free energy of the string extended between Q and $\bar{Q}$

$$\mathcal{F} = \sqrt{\lambda} k T \int_0^{\hat{\sigma}_0} \frac{d\hat{\sigma}}{\hat{\sigma}^2} (1-X) \sqrt{1 + f x'^2}, \quad X \equiv 1 - \tanh\alpha - \frac{1}{6\cosh\alpha \sinh\alpha}$$

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Regularization → subtract the free energy of two Polyakov loops (has the same UV divergences)

To do this in a controlled way, introduce an infrared cutoff at $z=\sigma_{\rm cut}$ near the event horizon with $\sigma_{\rm cut} \leq \sigma_c$

$$\mathcal{F}_{\rm sub} = \sqrt{\lambda} kT \int_0^{\hat{\sigma}_{\rm cut}} \frac{d\hat{\sigma}}{\hat{\sigma}^2}(1-X)$$

Regularized free energy

The difference $\Delta\mathcal{F} = \mathcal{F} - \mathcal{F}_{\text{sub}}$ is the regularized free energy and it can be written as

$$\Delta\mathcal{F} = \mathcal{F}_{\text{loop}} - 2\mathcal{F}_{\text{charge}}$$

with

$$\mathcal{F}_{\text{loop}}(T, L, k, \lambda) = \sqrt{\lambda} kT \left(-\frac{1}{\hat{\sigma}_0} + \int_0^{\hat{\sigma}_0} d\hat{\sigma} \frac{(1-X)\sqrt{1+fx'^2} - 1}{\hat{\sigma}^2} \right)$$

$$\mathcal{F}_{\text{charge}}(T, k, \lambda, \sigma_{\text{cut}}) = -\frac{1}{2}\sqrt{\lambda} kT \left(\frac{1}{\hat{\sigma}_{\text{cut}}} + \int_0^{\hat{\sigma}_{\text{cut}}} d\hat{\sigma} \frac{X}{\hat{\sigma}^2} \right) \underset{\kappa\to 0}{\simeq} -\frac{1}{2}\sqrt{\lambda} kT$$

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- $\mathcal{F}_{\text{loop}}$ $\longrightarrow$ regularized free energy of the rectangular Wilson loop
- $\mathcal{F}_{\text{charge}}$ $\longrightarrow$ regularized free energy for each of the Polyakov loops

Q-$\bar{Q}$ potential

Regularized free energy of the rectangular Wilson loop for small LT

$$\mathcal{F}_{\text{loop}} = -\frac{\sqrt{\lambda}k}{L}\left(\frac{4\pi^2}{\Gamma(\frac{1}{4})^4} + \frac{\Gamma(\frac{1}{4})^4}{96}\sqrt{\kappa}(LT)^3 + \frac{3\Gamma(\frac{1}{4})^4}{160}(LT)^4 + \cdots\right)$$

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- For $\kappa = 0 \longrightarrow$ same results found by using an extremal F-string probe in the AdS black hole background
  [Brandhuber et al. (1998); Rey et al. (1998)]
- Higher order term $\sim \sqrt{\kappa}(LT)^3 \longrightarrow$ "new term"
  - dominant correction to the Coulomb term provided

$$LT \ll \frac{\sqrt{k}\lambda^{1/4}}{N}$$

  - can only be seen by including the thermal excitations of the F-string probe

Debye screening

- $\Delta\mathcal{F} < 0 \longrightarrow$ Wilson loop is thermodynamically preferred
- $\Delta\mathcal{F} > 0 \longrightarrow$ Polyakov loop is thermodynamically preferred
- $\Delta\mathcal{F} = 0 \longrightarrow$ phase transition $\longrightarrow$ Debye screening of the Q-$\bar{Q}$ pair

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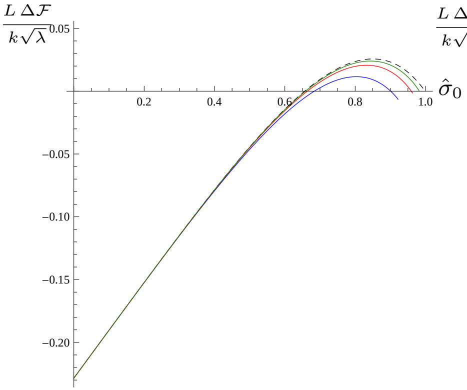

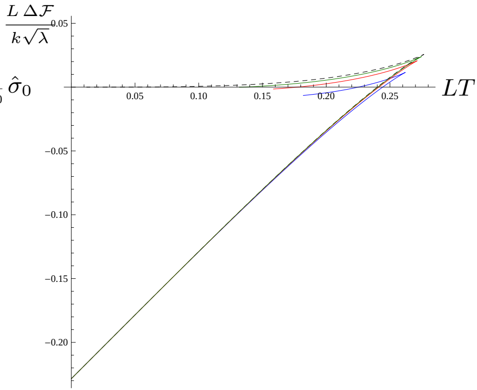

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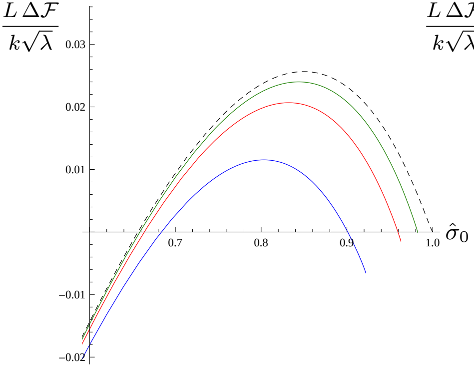

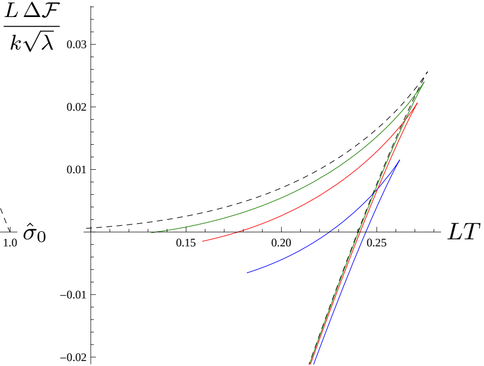

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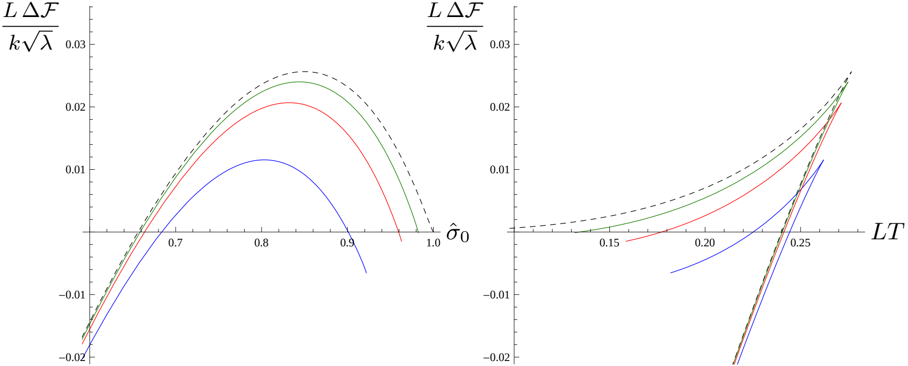

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- $\hat{\sigma}_0|_{\Delta\mathcal{F}=0}$ and $(LT)|_{\Delta\mathcal{F}=0}$ $\longrightarrow$ onset for charge screening
- $(LT)_{\max}$ never reached since the charges are screened before $\longrightarrow$ $(LT)|_{\Delta\mathcal{F}=0} < (LT)_{\max}$

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    $\longrightarrow$ increasing κ the onset moves to higher values of LT and $\hat{\sigma}_0$

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- Qualitative difference
  - Using the thermal F-string probe the pair is less easily screened compared to the extremal F-string probe
    $\longrightarrow$ increasing κ the onset moves to higher values of LT and $\hat{\sigma}_0$

- Quantitative differences
  - For a finite value of LT in the small κ limit $\Delta\mathcal{F}$ receives $\sqrt{\kappa}$ corrections to the extremal probe results
  - Onset of charge screening for small κ

$$(LT)|_{\Delta\mathcal{F}=0} \simeq 0.240038 + 0.0379706\sqrt{\kappa}$$

  - For the gauge theory $\longrightarrow$ $\sqrt{k}\lambda^{1/4}/N$ correction to
    - potential between the charges
    - critical value $(LT)|_{\Delta\mathcal{F}=0}$ where the charges become screened

Summary

- New method to describe thermal probes in finite temperature backgrounds

  $\longrightarrow$ it keeps into account the fact that for extended probes the internal degrees of freedom should be in thermal equilibrium with the background

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  → it keeps into account the fact that for extended probes the internal degrees of freedom should be in thermal equilibrium with the background

- Application of this method to the study of Wilson loops in finite temperature $\mathcal{N} = 4$ SYM using thermal F-string probes in the AdS black hole background

  → there are both qualitative and quantitative differences with respect to the "standard approach" based on the Nambu-Goto action

Outlook

- Examine holographic aspects of quark-gluon plasma physics
  - *e.g.* energy loss of a heavy quark moving through the plasma

    [Herzog, Karch, Kovtun, Kozcaz, Yaffe (2006); Gubser (2006)]

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- Revisit the thermal generalization of the Wilson loop in higher representations, in the regime where it involves a "blown-up" version:
  - D3-brane (symmetric representation)
  - D5-brane (antisymmetric representation)

$\longrightarrow$ interesting in view of the discrepancies between gauge theory and gravity results found for the symmetric representation

[Hartnoll, Kumar (2006); Grignani, Karczmarek, Semenoff (2010)]

Part II

Extra slides

Probe approximation

The probe approximation means that we should be able to piece the string probe together out of small pieces of SUGRA F-strings in hot flat ten-dimensional space-time

- local length scale of the string probe (= string "thickness") $\ll$ length scale of each of the pieces of F-string in hot flat space

We consider the branch of the SUGRA F-string connected to the extremal F-string $\longrightarrow$ the thickness of the F-string is the charge radius

$$r_c = r_0(\cosh\alpha \sinh\alpha)^{1/6} \;\Rightarrow\; r_c \propto \kappa^{1/6} R$$

Length scales in the metric of the AdS BH $\longrightarrow$ R and $z_0 \propto 1/T$ $\longrightarrow$ $r_c \ll R$ and $r_c \ll 1/T$ $\longrightarrow$

$$\boxed{\kappa \ll 1\,, \qquad RT \ll \kappa^{-\frac{1}{6}}}$$

- Given that $\kappa \ll 1$, the condition $RT \ll \kappa^{-1/6}$ is easily fulfilled as it just gives a rather weak upper bound on how high the asymptotic temperature T can be

- $\kappa \ll 1 \longrightarrow$ the critical distance $\hat{\sigma}_c$ is very close to the horizon: $1 - \hat{\sigma}_c \propto \kappa^{1/3}$ for small κ

- The regime of validity of the SUGRA F-string is $1 \ll k \ll N$ and $g_s^2 k \gg 1$; instead, the probe approximation requires $\kappa \ll 1$. These two conditions are consistent provided that $g_s \ll N^3 \longrightarrow$ trivially satisfied since we assume weak string coupling $g_s \ll 1$

In the near horizon region, $z_0 - \sigma \ll z_0$, a relevant quantity to consider is the local temperature $T_{\rm local} = T/R_0$

$T_{\rm local}$ must vary over sufficiently large length scales $\longrightarrow$ the probe is locally a SUGRA F-string in hot flat space of temperature $T_{\rm local}$ $\longrightarrow$

$$A \equiv \frac{r_c T'_{\rm local}}{T\sqrt{1 + x'(\sigma)^2}} \ll 1$$

$i.e.$ the variation of $T_{\rm local} \ll$ the length scale of the F-string probe

For the straight string solution in the $\kappa \to 0$ limit $\longrightarrow$ $z_0 - \sigma \gg \kappa^{1/9} z_0$

For the connected string configuration, assuming that $x' \gg 1$, the condition $A \ll 1$ is equivalent to $z_0 - \sigma \gg \kappa^{1/6} z_0$

In both cases the probe approximation breaks down before we reach the critical distance $\sigma = \sigma_c$

- We should also consider the extrinsic curvature of the solution

- Since we already took into account the variation of the background, the easiest way to analyze the extrinsic curvature is to neglect the derivatives of the metric; doing this we should require

$$z_0 - \sigma \gg \kappa^{1/3} z_0$$

- However, this is already guaranteed by the stronger condition $z_0 - \sigma \gg \kappa^{1/9} z_0$ for the straight string solution or $z_0 - \sigma \gg \kappa^{1/6} z_0$ for the connected string solution which we found above