Non-perturbative aspects of gauge/gravity duality

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Introduction

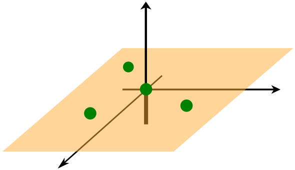
- D-branes are susceptible of dual descriptions: open/closed strings $\Rightarrow$ gauge/gravity correspondence
- Long ago, gravitational solutions dual to $\mathcal{N} = 2$ gauge theories were found at the perturbative level
- Non-perturbative effects needed to remove singularities
- Previously, explicit microscopic derivation of exact axio-dilaton profile in an orientifold model
- Here, same program for fractional branes on orbifold
- Based on forthcoming paper with M. Billó, M. Frau, A. Lerda, F. Fucito, J.F. Morales, D. Ricci Pacifici

Outline

Brane Setup

Type IIB string theory on orbifold background

$$\mathbb{R}^4 \times \mathbb{C}^2/\mathbb{Z}_2 \times \mathbb{C}$$



- Consider fractional D3 branes placed at the orbifold fixed point $X^6 = X^7 = X^8 = X^9 = 0$
- Two types $D3_0, D3_1$ corresponding to two irreps of $\mathbb{Z}_2$
- Massless states of open strings with endpoints on them give rise to field theory on 4d worldvolume

Brane Setup

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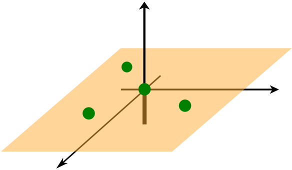

- $\mathcal{N} = 2$ $U(N_0) \times U(N_1)$ quiver theory on 4d worldvolume
  - adjoint vector multiplet

$$\Phi = \phi + \theta\lambda + \frac{1}{2}\,\theta\gamma^{\mu\nu}\theta\, F_{\mu\nu} + \cdots\,, \quad \Phi = \begin{pmatrix} \Phi_0 & 0 \\ 0 & \Phi_1 \end{pmatrix}$$

  - Two bifundamental hypers

Brane Setup

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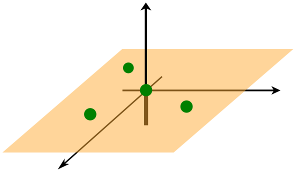
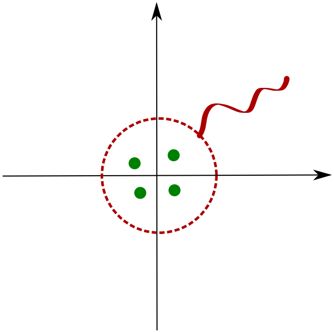


- Discard dynamics on $D3_1$ branes and set $N_0 = N_1 = N$
- Conformal $\mathcal{N} = 2$ $SU(N)$ gauge theory with $2N$ flavours

The twisted scalar

Branes are sources for closed string fields ⇒ classical solutions for sugra

- Non-trivial metric and F_5
- Constant axio-dilaton $C_0 + \mathrm{i}\mathrm{e}^{-\varphi} \to \frac{\mathrm{i}}{g_s}$



The twisted scalar

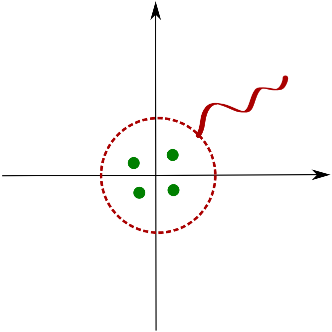
Branes are sources for closed string fields $\Rightarrow$ classical solutions for sugra

- In the twisted sector, two scalars from NS and R sector b, c complexified into a holomorphic field

$$t = c + \frac{\mathrm{i}}{g_s} b$$

- Lowest component of chiral superfield

$$T = t + \ldots + \theta^4 \frac{\partial^2}{\partial z^2} \bar{t} + \ldots$$



The classical t profile

- Classically t is the gauge coupling on fractional D3 branes

$$S_{\rm D3} \propto \int d^4x \; t \; \mathrm{tr}\, F^2 + \ldots$$

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$$S_{\mathrm{D3}} \propto \int d^4x \; t \; \mathrm{tr}\, F^2 + \ldots$$

- Brane action gives rise to source terms in e.o.m. for t

$$\frac{\delta}{\delta \bar{t}}(S_{\mathrm{bulk}} + S_{\mathrm{D3}}) = 0$$

- When all D3's are at the origin (conformal case)

$$\mathrm{i}\pi t(z) = \mathrm{i}\pi t_0$$

The classical t profile

- When branes are away from the origin, scalars get non-vanishing vevs

$$\langle \phi \rangle = \mathrm{diag}\,(a_1, \cdots, a_N), \quad \langle m \rangle = \mathrm{diag}\,(m_1, \cdots m_N)$$

- Conformal symmetry broken $\Rightarrow$ non-trivial profile for t

$$\Rightarrow \mathrm{i}\pi t(z) = \mathrm{i}\pi t_0 - 2\,\mathrm{tr}\log\frac{z - \langle \phi \rangle}{\mu} + 2\,\mathrm{tr}\log\frac{z - \langle m \rangle}{\mu}$$

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- Non-trivial source in e.o.m. for t

$$\Box t = J_{\mathrm{cl}}\delta^2(z)$$

The classical source and prepotential

- J_{cl} is encoded in effective action for massless open string fields
- Compute disk diagrams involving l adjoint scalars on the boundary and a twisted scalar, e.g. b, in the bulk

$$\sum_{l=0}^{\infty} \frac{1}{l!} \langle \underbrace{V_\phi \ldots V_\phi}_{l} V_b \rangle_{\mathrm{D3}_0} = \frac{\pi}{g_s} \sum_{l=0}^{\infty} \frac{1}{l!} \mathrm{tr}\langle \phi \rangle^l (\mathrm{i}\bar{p})^l b$$

- Taking into account c and the susy completion yields the linear part of the classical prepotential

$$F_{\mathrm{cl}} = \mathrm{i}\pi \sum_{l=1}^{\infty} \frac{(\mathrm{i}\bar{p})^l}{l!} \left(\mathrm{tr}\langle \Phi \rangle^l - \mathrm{tr}\langle M \rangle^l \right) \frac{T}{\bar{p}^2} + \cdots$$

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- Integration over fermionic superspace variables yields the classical source through

$$J_{\text{cl}} = \frac{\bar{p}^2}{\pi} \frac{\delta F_{\text{cl}}}{\delta T}\bigg|_0 = \sum_{l=1}^{\infty} \frac{\mathrm{i}}{l!} \Big(\mathrm{tr}\langle \phi \rangle^l - \mathrm{tr}\langle m \rangle^l \Big)$$

D(−1) branes as instantons

- The exact t profile gets contributions from non-perturbative effects on the source D3 branes
- Instantonic configurations of the gauge theory are realized by adding k fractional D(−1) branes at the orbifold fixed point
- Physical excitations of $-1/-1$ and $-1/3$ strings correspond to instanton moduli

(ϕ, ψ)	$U(k) \times SU(N_0)_g \times SU(N_1)_f$
$(a^\mu, M^\mu = M^{\alpha\dot{a}})$	(adj, 1, 1)
$(\bar{\chi}, \eta = \epsilon_{\dot{\alpha}\dot{a}}\lambda^{\dot{\alpha}\dot{a}})$	(adj, 1, 1)
$(\eta^c = (\tau^c)_{\dot{\alpha}\dot{a}}\lambda^{\dot{\alpha}\dot{a}}, D^c)$	(adj, 1, 1)
$(w_{\dot{\alpha}}, \mu_{\dot{a}})$	(k, $\bar{N}$, 1)+h.c.
(μ'_a, h_a)	(k, 1, $\bar{N}$)+h.c.

Non-perturbative corrections

- The non-perturbative source can be extracted from the non-perturbative prepotential

$$F_{\text{n.p.}} = \sum_k \int d\widehat{\mathcal{M}}_k \mathrm{e}^{-S_{\text{inst}}(\mathcal{M}_k, \Phi, T)}$$

- $S_{\text{inst}}(\mathcal{M}_k, \Phi, T)$ computed by D(-1) diagrams with insertions of T and moduli

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- $S_{\text{inst}}(\mathcal{M}_k, \Phi, T)$ computed by D(-1) diagrams with insertions of T and moduli
- Simplest diagrams are k D(-1) disks with only t inserted, corresponding to classical instanton action

$$S_{\text{cl}} = -\mathrm{i}\pi k t$$

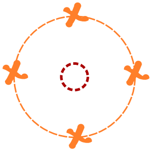
- S_{cl} weighs k-instanton contribution to $F_{\text{n.p.}}$

$$\mathrm{e}^{-S_{\text{cl}}} = q^k, \quad q = \mathrm{e}^{\mathrm{i}\pi t}$$

Moduli interactions

- Relevant contributions to S_{inst} come from diagrams with the insertion of e.g. b and l χ moduli

$$\sum_{l=0}^{\infty} \frac{1}{l!} \langle \underbrace{V_\chi \dots V_\chi}_{l} V_b \rangle_{\mathrm{D}(-1)_0}$$
$$= -\frac{\pi}{g_s} \sum_{l=0}^{\infty} \frac{1}{l!} \mathrm{tr}_k\, \chi^l (\mathrm{i}\bar{p})^l b$$



- This gives a linear non-perturbative prepotential

$$F_{\text{n.p.}} = \mathrm{i}\pi T \sum_k q^k \int d\widehat{\mathcal{M}}_k \mathrm{e}^{-S'_{\text{inst}}(\mathcal{M}_k, \Phi)} \sum_{l=1}^{\infty} \frac{1}{l!} \mathrm{tr}_k\, \chi^l (\mathrm{i}\bar{p})^l + \dots$$

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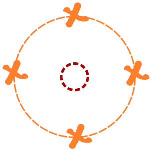


- It is expressed through chiral ring elements $\langle \mathrm{tr}\, \phi^l \rangle$ of the gauge theory

$$F_{\text{n.p.}} = \mathrm{i}\pi \sum_{l=1}^{\infty} \frac{(\mathrm{i}\bar{p})^l}{l!} \langle \mathrm{tr}\, \Phi^l \rangle_{\text{n.p.}} \frac{T}{\bar{p}^2} + \ldots$$

The exact t profile

The inclusion of instantonic corrections amounts to promote classical vevs to full quantum vevs

$$\Rightarrow J = \frac{\bar{p}^2}{\pi} \frac{\delta F}{\delta T}\bigg|_0 = \mathrm{i} \sum_{l=1}^{\infty} \frac{(\mathrm{i}\pi)^l}{l!} \Big(\langle \mathrm{tr}\, \phi^l \rangle - \mathrm{tr} \langle m \rangle^l \Big)$$

yielding the exact t profile

$$\mathrm{i}\pi t(z) = \mathrm{i}\pi t_0 - 2 \left\langle \mathrm{tr} \log \frac{z - \phi}{\mu} \right\rangle + 2\, \mathrm{tr} \log \frac{z - \langle m \rangle}{\mu}$$

The exact t profile

- Exact description from SW curve for D3 gauge theory

$$y^2 = P(z)^2 - g^2 Q(z),$$

$$P = \prod_{i=1}^{N}(z - a_i), Q = \prod_{k=1}^{N}(z - m_k)^2, g^2 = \frac{4q}{(1+q)^2}$$

- Correlator appearing in t profile computable from SW curve

$$\left\langle \mathrm{tr} \log \frac{z-\phi}{\mu} \right\rangle = \log \frac{P(z) + \sqrt{P(z)^2 - g^2 Q(z)}}{\mu^N} - \log\left(1 + \sqrt{1-g^2}\right)$$

- Exact t profile emitted from brane system

$$\pi i t(z) = \log \frac{P(z) - \sqrt{P^2(z) - g^2 Q(z)}}{P(z) + \sqrt{P^2(z) - g^2 Q(z)}}$$

t and gauge couplings

- Focus on special vacuum of gauge theory having only one scale left $\mathbf{v} = \langle \mathrm{tr}\, \phi^N \rangle$
- Corresponds to symmetric arrangement of D3's

$$a_i = a\omega^{i-1}, \quad m_i = m\omega^{i-1}, \quad \omega = \mathrm{e}^{2\mathrm{i}\pi/N}$$

- On the gravity side, evaluating t with all invariants set to zero leaves only one scale, z
- How is the twisted scalar evaluated at z related to the gauge coupling on this slice of moduli space?

t and gauge couplings

- In the massless case t is constant

$$\mathrm{i}\pi t(z)\Big|_{\mathbf{v}=0} = \log q = \mathrm{i}\pi t_0$$

- But effective gauge coupling gets corrections even in conformal case $\Rightarrow$ t can't simply be τ
- Classically, matrix of couplings takes simple form

$$2\pi\mathrm{i}\,\tau^{ij}_{\mathrm{tree}} = \begin{pmatrix} 2 & 1 & 1 & \dots \\ 1 & 2 & 1 & \dots \\ 1 & 1 & 2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \log q_0$$

- At quantum level, additional matrix structures emerge for $N > 3$

t and τ for $SU(2)$

- From explicit instanton computations, UV t_0 and IR τ couplings for $m=0$ related by

$$\mathrm{i}\pi\tau = \log q + \mathrm{i}\pi - \log 16 + \frac{1}{2} q_+ \frac{13}{64} q_0^2 + \dots$$

- Inverting it,

$$\mathrm{e}^{\mathrm{i}\pi t_0} = -16(\mathrm{e}^{\mathrm{i}\pi\tau} + 8\mathrm{e}^{2\mathrm{i}\pi\tau} + \dots) = -16\frac{\eta^8(4\tau)}{\eta^8(\tau)}$$

- Equation holds true for non-constant $t(z)$ for $m \neq 0$ after identyfing $z^2 \leftrightarrow \mathbf{v}$
- Still true for non-conformal cases reached by decoupling some flavours

t and τ for $SU(3)$

- For $SU(3)$, matrix of couplings retains its form at the exact level

$$2\pi\mathrm{i}\,\tau^{ij}_{\mathrm{SU(3)}} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \pi\mathrm{i}\,\tau \ .$$

- After identifying $z^3 \leftrightarrow \mathbf{v}$, t and τ related by

$$\mathrm{e}^{\mathrm{i}\pi t} = -27(\mathrm{e}^{\mathrm{i}\pi\tau} + 12\mathrm{e}^{2\mathrm{i}\pi\tau} + \dots) = -27\frac{\eta^{12}(3\tau)}{\eta^{12}(\tau)}$$

t and τ for $SU(4)$

- For $SU(4)$ 1-loop and instantonic corrections spoil classical matrix structure

$$2\pi\mathrm{i}\,\tau^{ij}_{\mathrm{SU}(4)} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \pi\mathrm{i}\frac{\tau_+ + \tau_-}{2} + \begin{pmatrix} 0 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & 0 \end{pmatrix} \pi\mathrm{i}\frac{\tau_+ - \tau_-}{2}$$

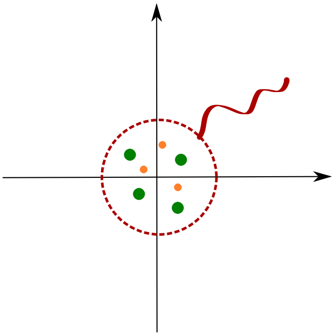
- Again t and τ_+, τ_- related through modular functions after identifying $z^4 \leftrightarrow \mathbf{v}$

$$\mathrm{e}^{\mathrm{i}\pi\, t_0} = -16\left(\mathrm{e}^{\mathrm{i}\pi\tau_+} + 8\,\mathrm{e}^{2\mathrm{i}\pi\tau_+} + \dots\right) = -16\,\frac{\eta^8(4\tau_+)}{\eta^8(\tau_+)}\ ,$$

$$\mathrm{e}^{\mathrm{i}\pi\, t_0} = -64\left(\mathrm{e}^{\mathrm{i}\pi\tau_-} + 24\,\mathrm{e}^{2\mathrm{i}\pi\tau_-} + \dots\right) = -64\,\frac{\eta^{24}(2\tau_-)}{\eta^{24}(\tau_-)}\ .$$

Conclusions

- Explicit microscopic derivation of exact t profile from D(-1) emission possible for the full quiver theory
- Non-trivial relationship between twisted scalar and effective gauge couplings



Thanks!