Phase Space for Massive Yang-Mills Theory

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Preamble: The high energy behavior of massive nonabelian gauge theories (e.g. in the Electroweak Model) is compared in the two cases of Higgs Mechanism (HM) and of Non Linear Realization (NLR) of the gauge group (Stückelberg mass). In most extreme cases this problem can be translated into the study of the zero-mass limit. In this way the question becomes a fundamental issue since a massless vector meson has two degrees of freedom while a massive one has three.

The two scenarios (HM and NLR) have strikingly different behavior: in the HM there is a metamorphosis of the longitudinal state into the Goldstone scalars, while in the NLR a phase transition line separates the massive theory from the pure massless case (i.e. no Goldstone modes and no longitudinal states).

There are phemenological consequences of this differences, in particular in the case of HM all the Higgs fields become physical modes. This signature might be detected at high energy at LHC and in a linear collider.

From a theoretical point of view, the aim of this work is to show that the problem of unitarity at high energy in nonabelian gauge theory with no Higgs boson can open new perspectives in quantum field theory.

Cortona, 1.6.2012

0. Plan of the Talk

1. $M^2 \to 0$, $\Longrightarrow$ change in the number of degree of freedom: two helicity states versus three spin-one states.

2. In HM metamorphosis of longitudinal modes into once-Goldstone-modes (via Equivalence Theorem) .

3. In NLR (i.e. Stückelberg mass) decoupling of both longitudinal modes and Goldstone-modes. Phase transition?

4. A lattice gauge model.

5. The Transition Line. Confined and deconfined phase space regions.

6. Spectrum. Weak coupling limit.

1. Higgs Mechanism versus Stückelberg Mass

We are going to compare the usual HM generated mass term in a $SU(2)$ Yang-Mills

$$S_{YM} + \Lambda^{D-4} \int d^D x \; [(\partial_\mu - igA_\mu)\Phi]^\dagger (\partial_\mu - igA_\mu)\Phi$$

with the Stückelberg mass

$$S_{YM} + \Lambda^{D-4} M^2 \int d^D x \; Tr\{[gA_\mu - i\Omega\partial_\mu \Omega^\dagger]^2\}.$$

Notations: in HM we have

$$\Phi = \begin{pmatrix} i\phi_1 + \phi_2 \\ \phi_0 - i\phi_3 \end{pmatrix}$$

and ϕ_0 is the field of the Higgs boson.

While in the NLR

$$\Omega = \phi_0 + i\phi_a\tau_a = \begin{pmatrix} \phi_0 + i\phi_3 & i\phi_1 + \phi_2 \\ i\phi_1 - \phi_2 & \phi_0 - i\phi_3 \end{pmatrix}.$$

The theory is expected to be fundamental (not an effective one), where the number of parameters is fixed. Two questions are discussed here.

A) The suggested subtraction procedure is based on dimensional regularization . What about other regularization procedures?

B) Although perturbative unitarity is valid, the behavior of some cross sections at high energy, evaluated at fixed order, is untenable (e.g. the celebrated case of $W_L W_L$ elastic scattering).

2. The Unitarity Conundrum

In their seminal paper Lee, Quigg and Thacker [Phys. Rev. D 16, 1519 (1977)] correctly remark that, at very high energies, the vanishing of the most divergent terms proceeds through the cancellation of various contributions, which includes the one of the Higgs boson. [1]

Many have shortcutted this correct statement by concluding that *by removing the Higgs boson, unitarity is violated* **. To our opinion this conclusion has to be carefully reconsidered.**

[1] See the comment after their eq. (2.3)

Lets us recall briefly the argument for the case of $SU(2)$, with Higgs Mechanism. For longitudinally polarized vector bosons

$$\epsilon_L = \frac{1}{M}\left(|\vec{p}|, \hat{p}E\right)$$

the sum of the gauge tree graphs for $W_L^+ W_L^-$ elastic amplitude in the center of mass behaves like

$$M_{gauge} = g^2 \frac{s}{8M_W^2}(\cos\theta + 1) + \mathcal{O}(s^0).$$

The Higgs $s, t-$channels contribution cancels this bad behavior and

$$M_{gauge} + M_H = g^2 [\frac{3 + \cos^2\theta}{4\cos^2\theta_W (\cos\theta - 1)} - \frac{M_H^2}{2M_W^2} + \mathcal{O}(s^{-1})].$$

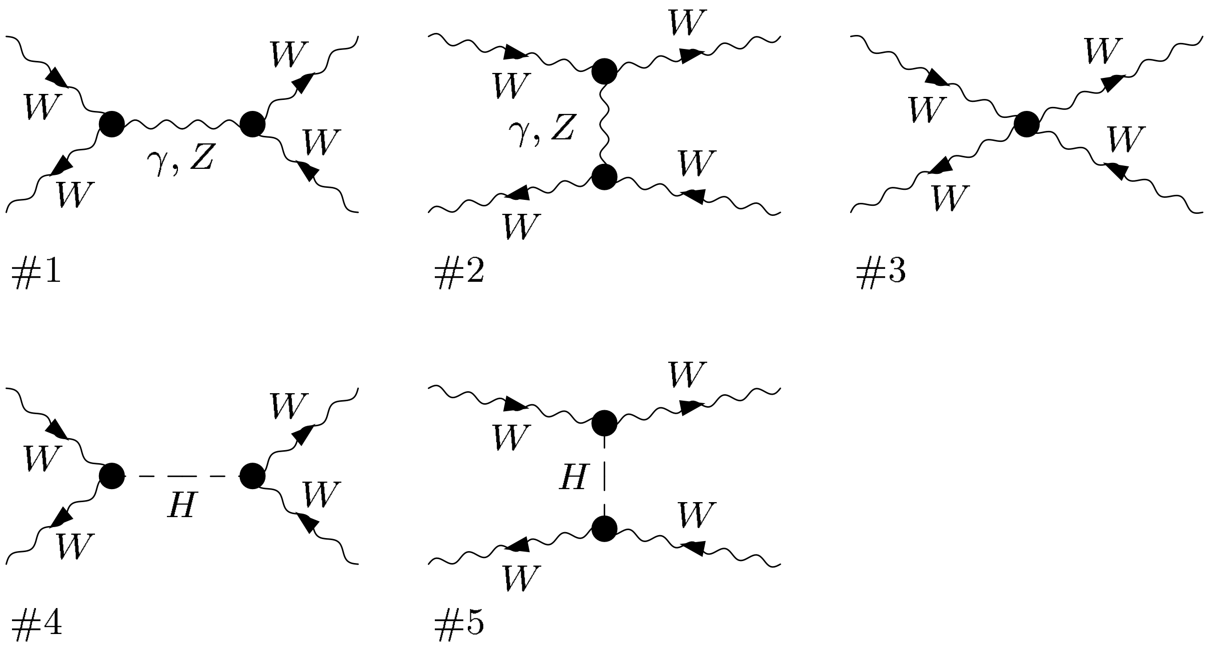

Figure 1:

Born diagrams for W^+W^- elastic scattering.

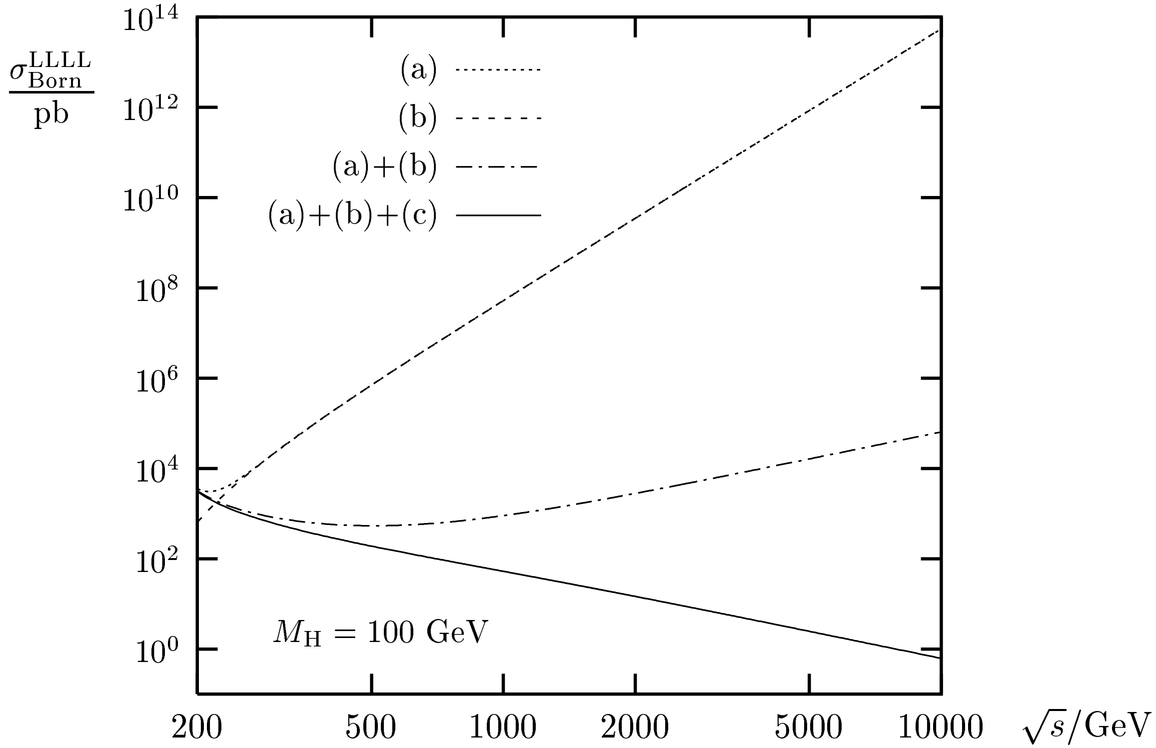

Figure 2:

Cross section with $\theta_{\mathrm{cut}} = 10^0$. (a) 3-prongs gauge, (b) 4-prongs gauge (c) Higgs diagrams. From Denner & Hahn (97).

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Very High Energy with Stückelberg: Problems

The behavior of an nonabelian gauge theory in the limit of small M is very singular. The self-coupling vertex has a coupling $\simeq M^2$ but the ϕ - propagator has a factor $\simeq M^{-2}$. Thus a graph with a vertex with many ϕ - prongs has a singular behavior. If the infrared ($M \sim 0$) behavior is dominated by the nonlinear sigma model features, then the forward scattering amplitude at high energy (neglecting infrared divergences!) behaves like

$$T^{(n)}_{\phi\phi}(s) \sim (\frac{s}{M^2})^{(n+1)},$$

where n is the number of loops.

Although perturbative unitarity is preserved, the series seems bad.

Some troublesome Questions

In extreme processes we can translate the problem of high energy to the more fundamental one of the limit $M \to 0$.

- **For $M = 0$ only two polarizations are physical while for $M \neq 0$ they are three : problems with the matching among unphysical vector meson modes, Goldstone bosons and Faddeev-Popov ghosts in order to provide Physical Unitarity .**
- **Do longitudinal polarizations decouple from physical states?**
- **Or else?**

3. Scenario at very High Energy (Higgs): a prologue

At very high energy the longitudinal polarization of a vector boson

$$\epsilon_L = \frac{1}{M}\left(|\vec{p}|, \hat{p}E\right)$$

becomes indistinguishable from a spin zero particle described by a scalar boson $\partial_\mu \phi$

$$\epsilon_{J=0} = \frac{1}{M}\left(E, \vec{p}\right) \quad .$$

Thus the experimental setup provides a cut-off energy E_c. Only for $E < E_c$ one can distinguish the two polarization states. E_c depends on the precision of the momentum and energy measurements.

In the limit $v \to 0$ the symmetry is restored and the longitudinal polarization modes transform into the (former, $v \neq 0$) Goldstone bosons. The metamorphosis is abrupt. At the same time the vector mesons carry two physical- and two unphysical modes.

In the limit the Higgs field doublet is a physical mode, therefore the nonabelian gauge theory is not asymptotically free. A small value of v provides a very good infrared regulator for the otherwise ill-defined massless theory, since all the requirements are met: physical unitarity, BRST, locality, etc. (arXiv:1106.5537)

Equivalence Theorem

Some help from the Equivalence Theorem. Use ($M_G^2 =$ mass^2 of the Goldstone boson $= \xi^{-1}M^2$ at the tree level)

$$\epsilon_L = \frac{1}{M}\left(|\vec{p}|, \hat{p}E\right) \quad = \frac{p_\mu}{M} + \frac{1}{M}\left(-\frac{M_G^2}{|\vec{p}|+E_G}, \hat{p}\frac{M^2}{|\vec{p}|+E}\right) = \frac{p_\mu}{M} + \mathcal{O}(M).$$

Then for very large energy processes ($s, t >> M^2$) we can consider the limit $M \to 0$. EQ theorem says that for $M \to 0$

$$\epsilon_L^{\mu_1} \cdots \epsilon_L^{\mu_k} W_{\widehat{A^{\mu_1}(p_1)}\cdots\widehat{A^{\mu_k}(p_k)}***}\Big|_{p^2=M^2} \simeq [iR]^k W_{\widehat{\phi(p_1)}\cdots\widehat{\phi(p_k)}***}|_{p^2=M_G^2},$$

$$R \equiv i\frac{p^\nu \Gamma_{\phi A^\nu}}{M\Gamma_{\phi\phi}}|_{p^2=M_\alpha^2} = \xi\frac{M_G^2}{M^2}$$

Summary of the limit $M = 0$ in the Higgs case

- **The longitudinal modes transforms into the Goldstone bosons**
- **The Goldstone bosons become physical, like the Higgs singlet.**
- **The vector gauge field (massless) describe two transverse and physical modes and two unphysical.**
- **The theory is not asymptotically free, even if the mass of the gauge field is zero.**

Scenario at very High Energy (Stückelberg): Asymptotic Freedom?

The Equivalence Theorem is based on the Slavnov-Taylor identities, thus it works also in the nonlinear case. However a metamorphosis of the longitudinal modes is not allowed, because the Goldstone bosons remain unphysical, if not decoupled form physical states (due to BRST).

We dare an educated guess on the limit $M = 0$: it is not allowed by the presence of a phase transition line. The line is supposed to separate the particle phase from the confinement (asymptotic freedom). This guess is pertinent only for extreme processes, where kinematically the $M = 0$ limit reproduces the large energy regime.

4. Lattice Simulation

The guess is supported by the lattice simulation of the *massive* **Yang-Mills theory. The lattice action is (arXiv:1112.2982)**

$$S_E = -\frac{\beta}{2}\,\mathfrak{Re}\sum_{\square} Tr(U_{\square}) - \frac{\beta}{2}m^2\mathfrak{Re}\sum_{x\mu} Tr\{\Omega(x)^{\dagger}U(x,\mu)\Omega(x+\mu)\},$$

where $\beta = \frac{4}{g^2}$ **and** $m^2 \equiv M^2a^2$**. Thanks to the limit (classical)**

$$\begin{aligned} &- \lim_{a=0}\frac{\beta}{2}M^2a^2\mathfrak{Re}\sum_{x\mu} Tr\{\Omega(x)^{\dagger}U(x,\mu)\Omega(x+\mu) - 1\} \\ &= \frac{M^2}{g^2}\int d^4x Tr\ \{(A_\mu - i\Omega\partial_\mu\Omega^{\dagger})^2\} \\ &= \frac{M^2}{g^2}\int d^4x Tr\ \{[(i\partial_\mu + A_\mu)\Omega]^{\dagger}(i\partial_\mu + A_\mu)\Omega\}. \end{aligned} \tag{1}$$

5. The Phase Diagramm (β, m^2)

The simulation is on the partition function

$$Z[\beta, m^2, N, D] = \sum_{\{U,\Omega\}} e^{-S_E}.$$

Moreover we introduce the order parameter

$$\mathfrak{C} = \frac{1}{DN\beta}\frac{\partial}{\partial m^2}\ln Z = \frac{1}{2ND}\langle \mathfrak{Re} \sum_{x\mu} Tr\{\Omega^\dagger(x)U(x,\mu)\Omega(x+\mu)\}\rangle. \quad (2)$$

A transition line is found where the order parameter and energy have inflection points. The end point is around $\beta = 2.2$ and $m^2 = 0.381$. (E.Fradkin & S.H. Shenker, J. Greensite & S. Olejnik, W. Caudy & J. Greensite, I. Campos, C. Bonati, G.Cossu, M. D'Elia & A. Di Giacomo and others)

6. Confined and Deconfined Regions

In the two regions (above and below the line) the two-point functions of the *gauge invariant* **fields**

$$C(x,\mu) \equiv \Omega^\dagger(x) U(x,\mu) \Omega(x+\mu) = C_0(x,\mu) + i\tau_a C_a(x,\mu) \tag{3}$$

have different behavior. Above they show the presence of an energy gap, which disappears by crossing the line. In the continuum we have

$$C(x,\mu) = \Omega^\dagger(x)(1 - iaA_\mu(x))(\Omega(x) + a\partial_\mu \Omega) + \mathcal{O}(a^2). \tag{4}$$

Thus for C_1, C_2, C_3 **one gets**

$$i\tau_a C_a(x,\mu) = -ia\Omega^\dagger(A_\mu(x) - i\Omega\partial_\mu\Omega^\dagger)\Omega + \mathcal{O}(a^2). \tag{5}$$

While for C_0

$$C_0(x,\mu) = 1 - \frac{a^2}{4} Tr\ \{(A_\mu - i\Omega\partial_\mu\Omega^\dagger)^2\} + \mathcal{O}(a^4). \tag{6}$$

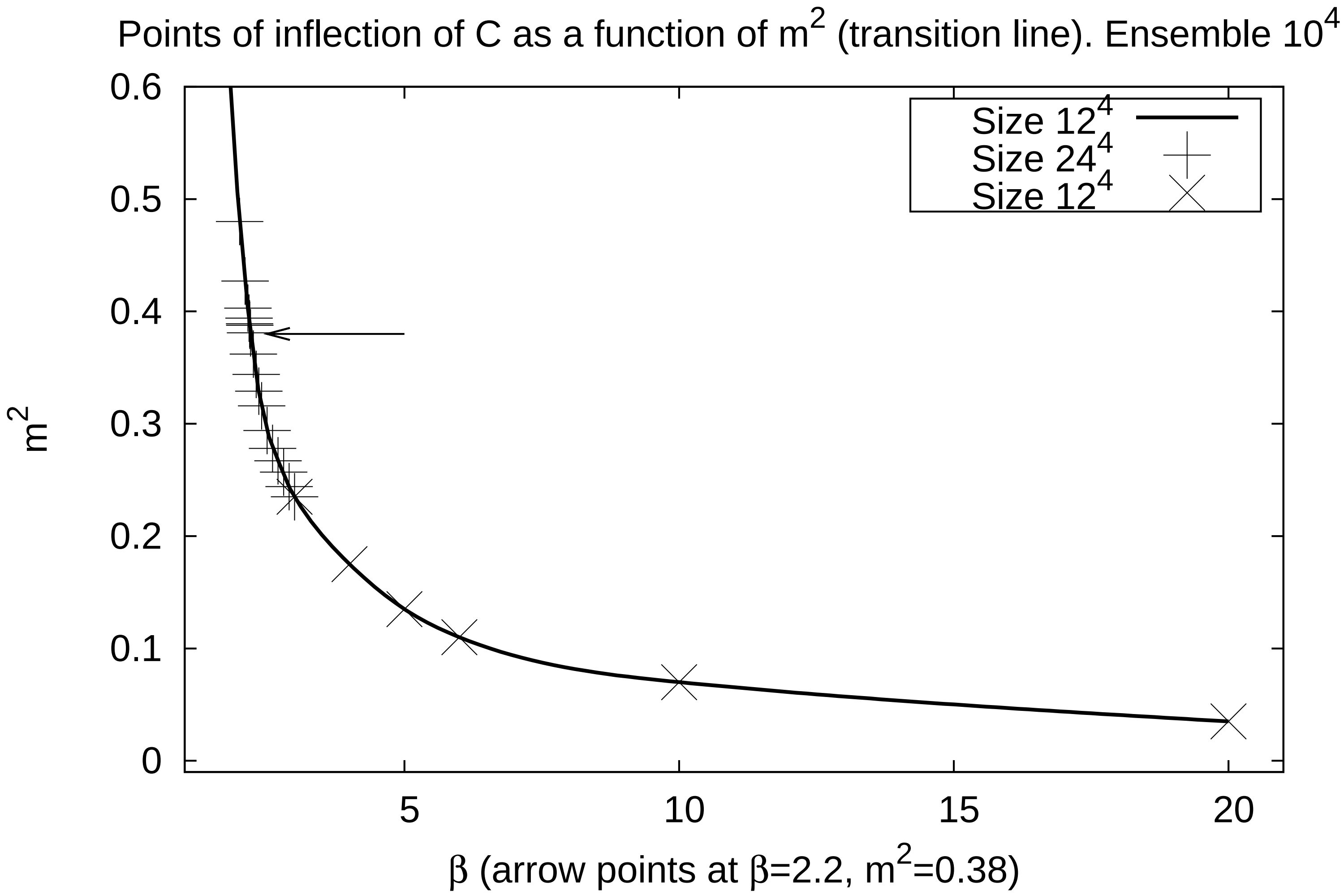

Figure 3: Phase diagram

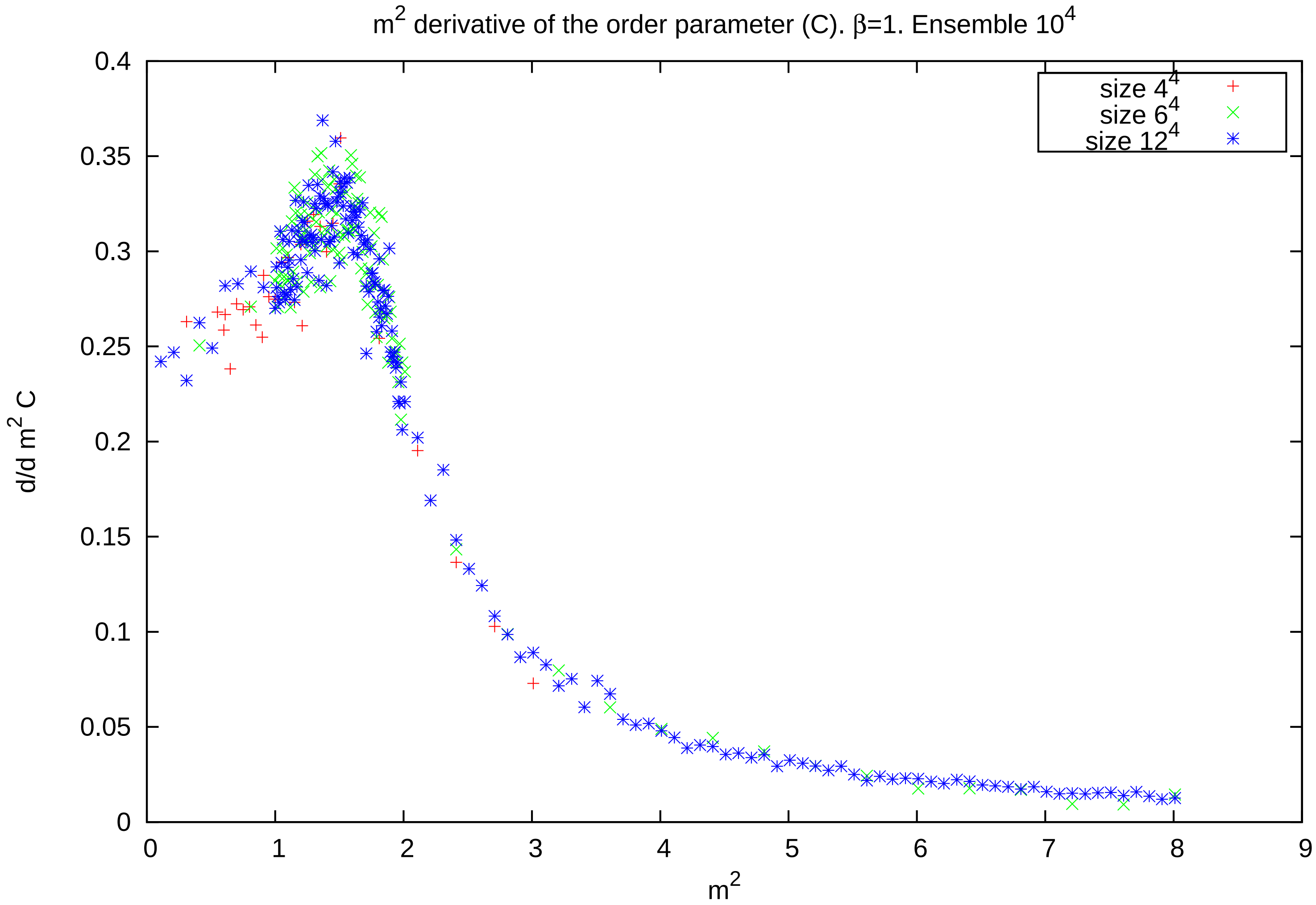

Figure 4: m^2 derivative of the order parameter. $\beta = 1.0$

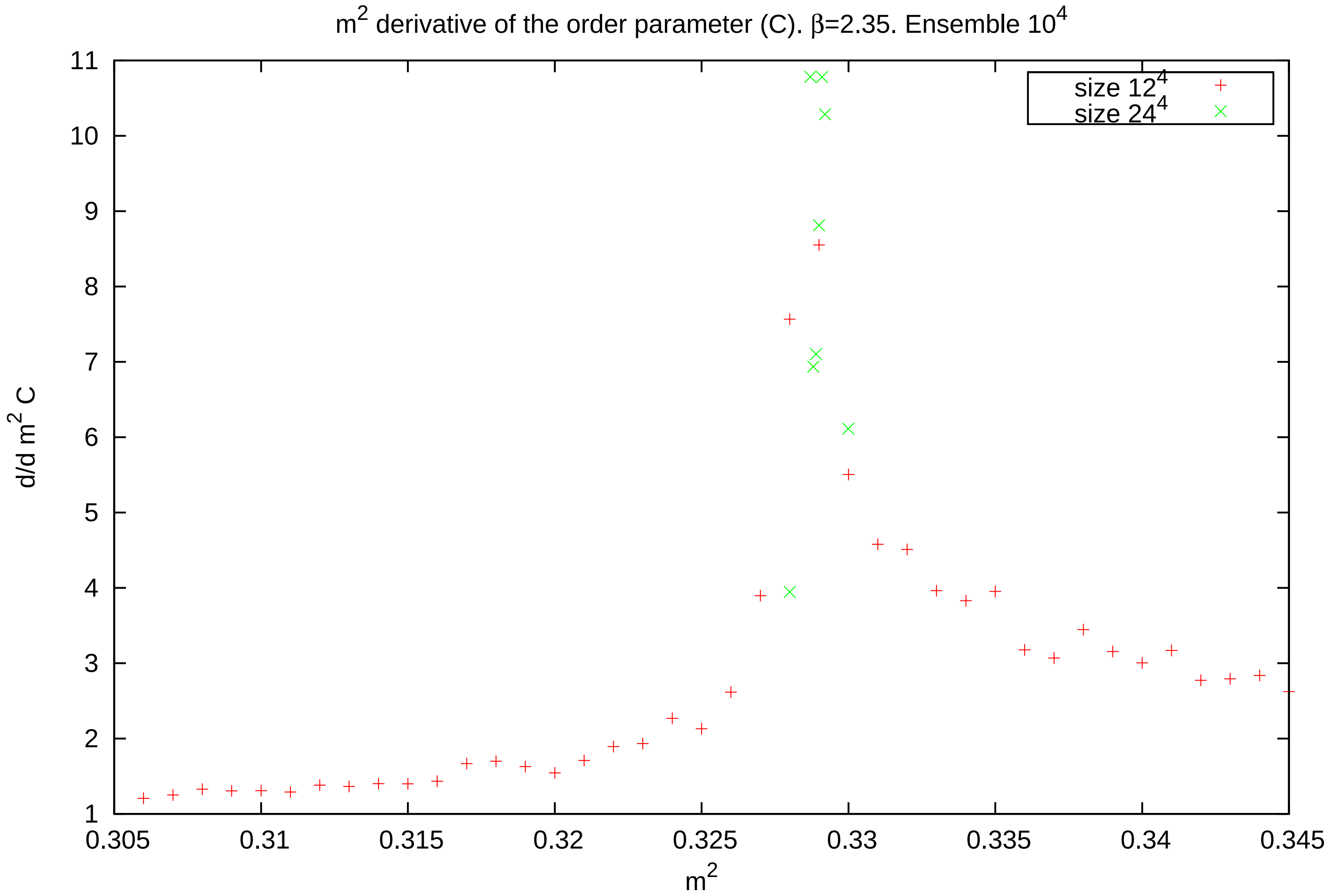

Figure 5: m^2 derivative of the order parameter. $\beta = 2.35$

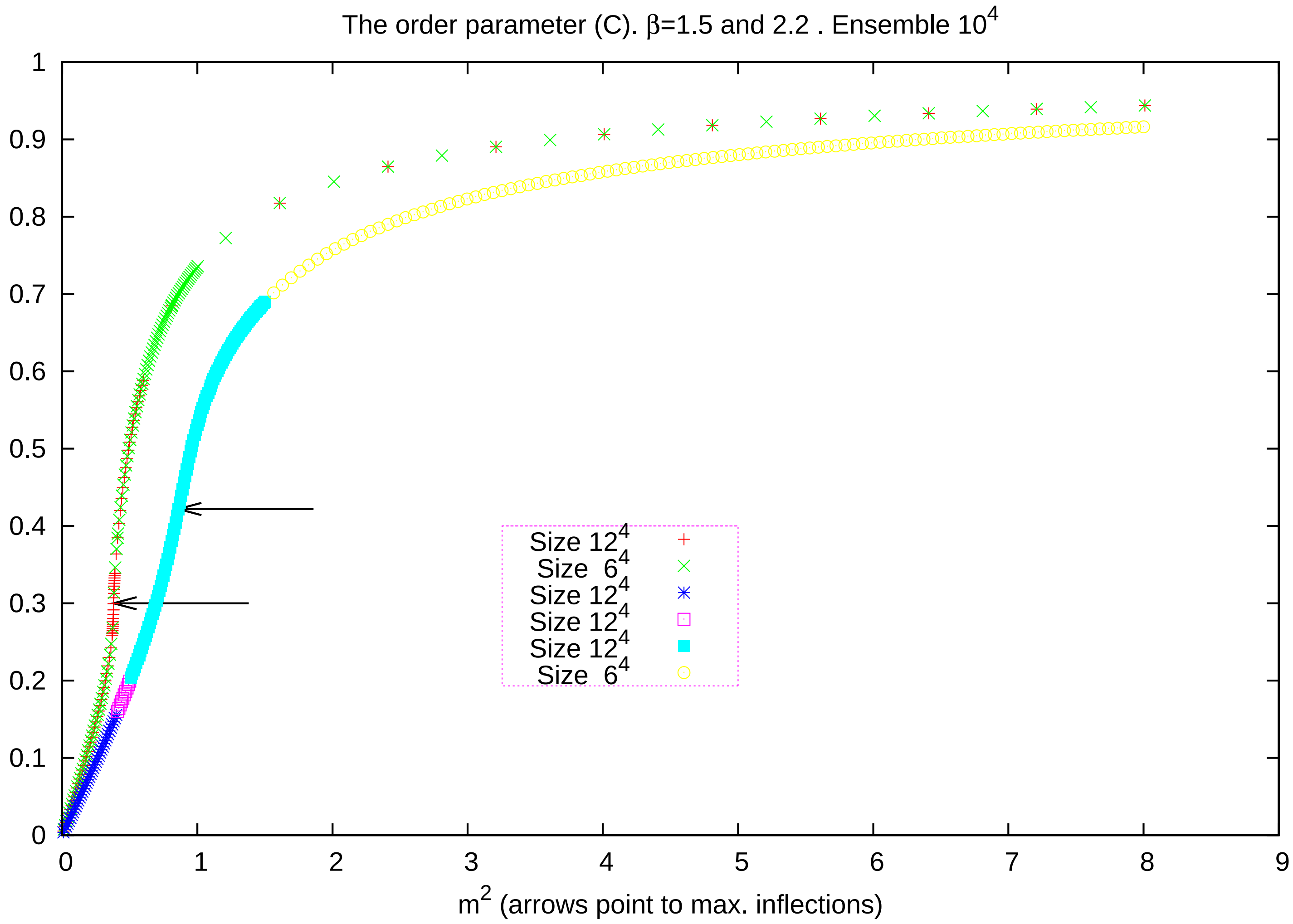

Figure 6: Order parameter at $\beta = 1.5$ and $\beta = 2.2$.

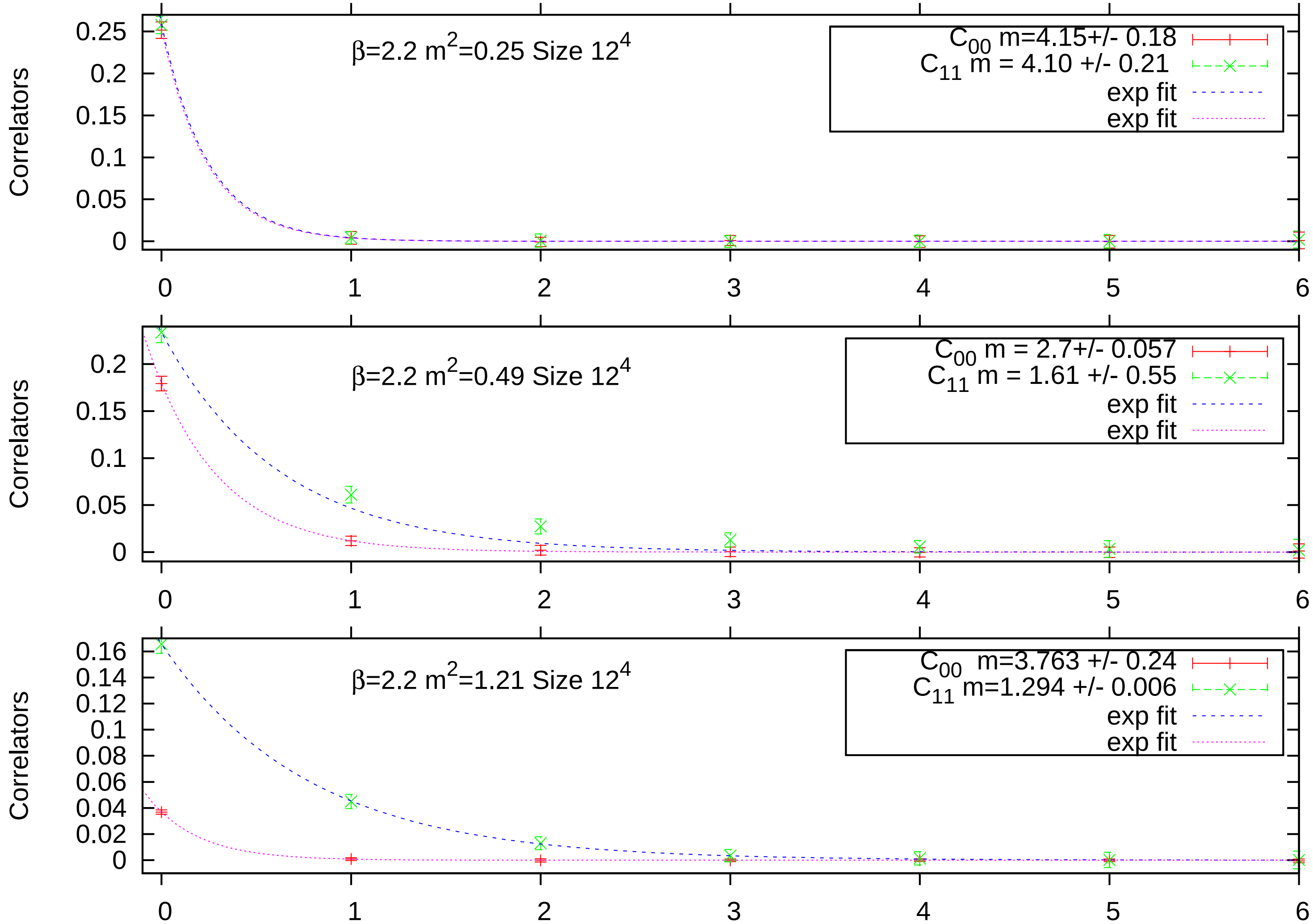

Figure 7: Time correlators. The polarizations are the same

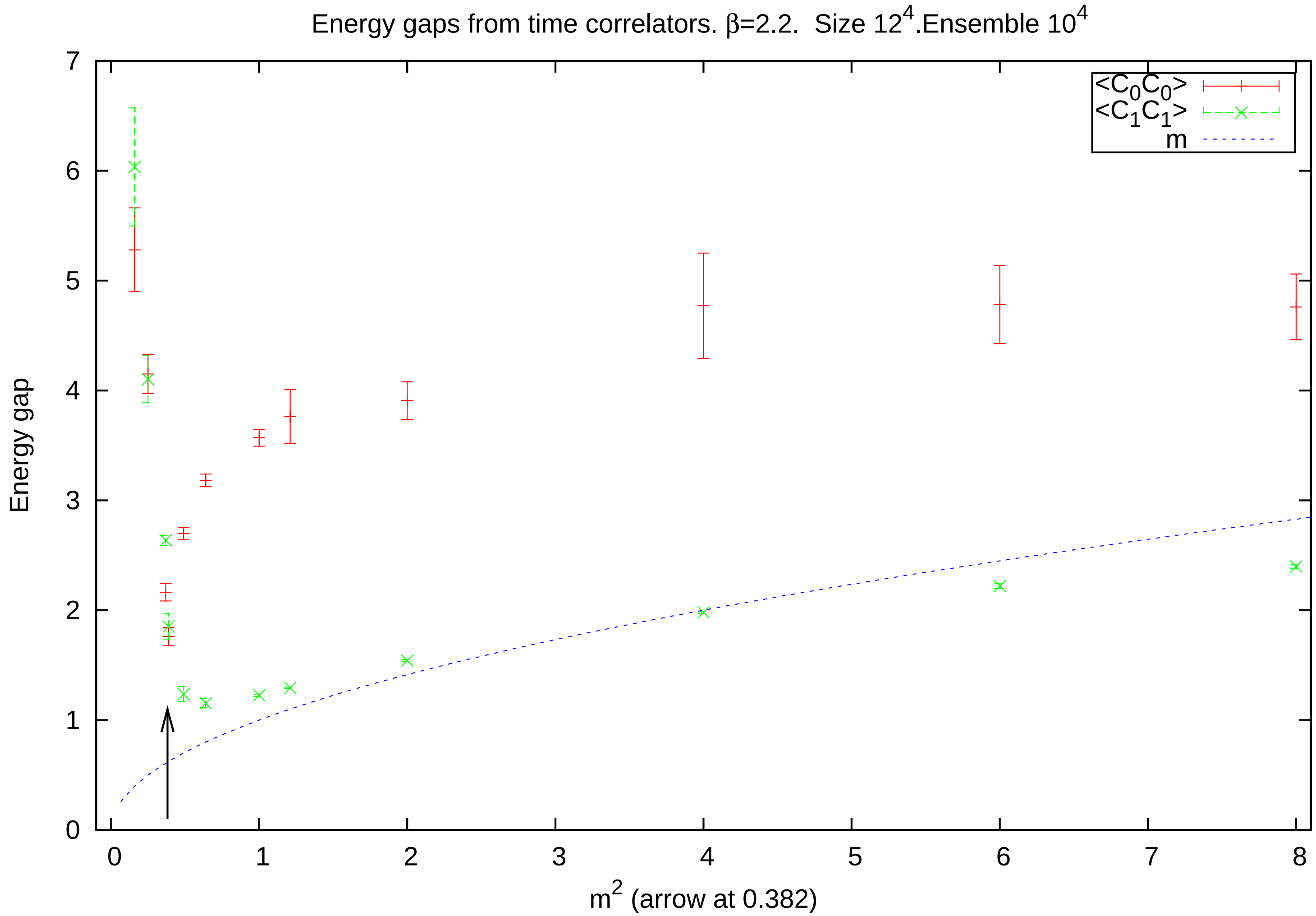

Figure 8: Energy gaps for $\beta = 2.2$ (fit with $e^{-m_0 t}$)

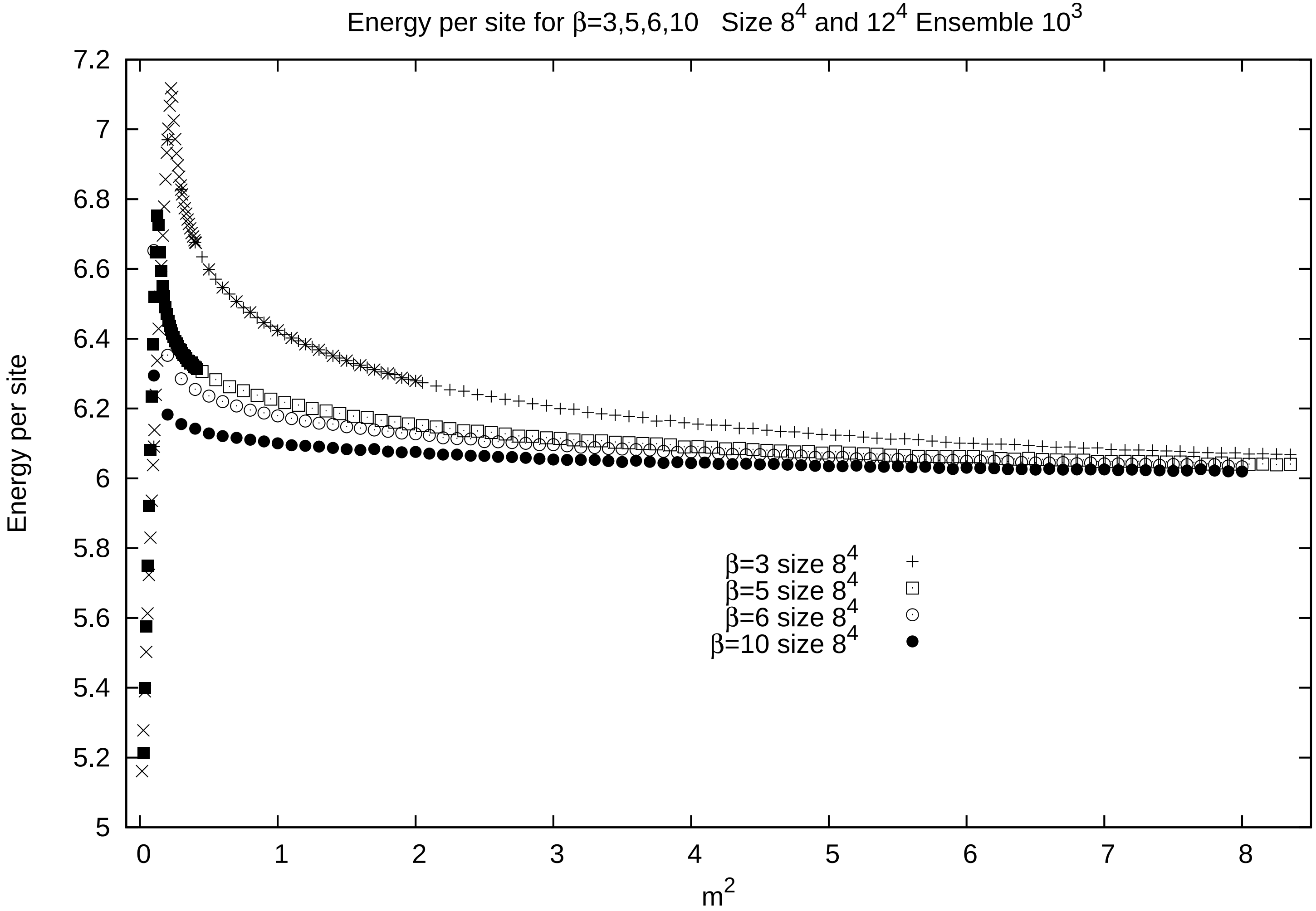

Figure 9: Energy per site at $\beta = 3, 5, 6, 10$ as function of m^2.

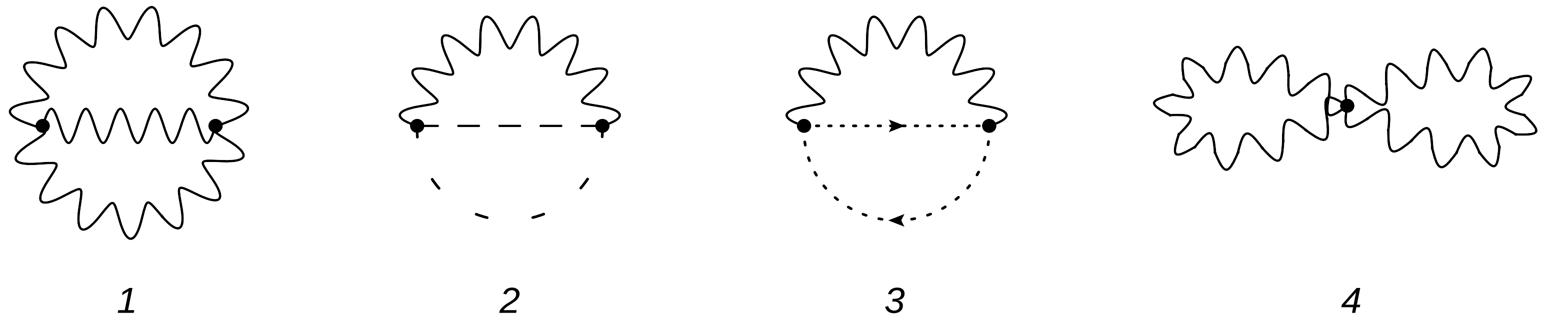

Figure 10: The Goldstone boson lines are dashed. The Faddeev-Popov propagators are dotted.

Weak Coupling Limit

The Physics of the lattice model has to be worked out: properties of the Phase Transition, weak coupling limit, value of the parameters β and m^2. For instance in the Weak Coupling Limit ($\beta \to \infty$) one can evaluate the "Energy" of the vacuum both in the continuum and on the lattice. Some of the lattice results are depicted in Fig. 9, which shows how the independence from β is established for large values of m^2. The subtraction procedure yields for the vacuum graphs in Fig. 10 (Davidychev & Tausk (95)) ($\Delta = \frac{e^{\gamma-1}M^2}{4\pi\Lambda^2}$)

$$\mathcal{E}^{(1)} = -\frac{3}{2}g^2\frac{M^4}{(4\pi)^4}\{ -\frac{69}{2}\frac{1}{(D-4)^2} - \frac{1}{(D-4)}(-\frac{91}{4}+\frac{69}{2}\ln(\Delta)\ ) \\ -\frac{163}{8}+\frac{91}{4}\ln(\Delta) - \frac{69}{4}\ln^2(\Delta) - \frac{23}{16}\pi^2 + \frac{99}{2\sqrt{3}}Cl_2(\frac{\pi}{3})\}$$

and the counterterm contribution

$$\mathcal{E}^{(2)} = -\frac{3}{2}\frac{g^2M^4}{(4\pi)^4}\{\frac{69}{(D-4)^2} + \frac{1}{D-4}(23+\frac{69}{2}\ln\Delta\ ) + \frac{23}{2}\ln\Delta$$
$$+\frac{69}{8}[\ln^2\Delta + 1 + \frac{\pi^2}{6}]\}.$$

The final result for the energy of the vacuum is

$$\mathcal{E} = \mathcal{E}^{(1)} + \mathcal{E}^{(2)} = -\frac{3}{2}g^2\frac{M^4}{(4\pi)^4}\{\frac{69}{2}\frac{1}{(D-4)^2} + \frac{183}{4}\frac{1}{(D-4)}$$
$$-\frac{69}{8}\ln^2(\Delta) + \frac{137}{4}\ln\Delta - \frac{94}{8} + \frac{99}{2\sqrt{3}}Cl_2(\frac{\pi}{3})\}.$$

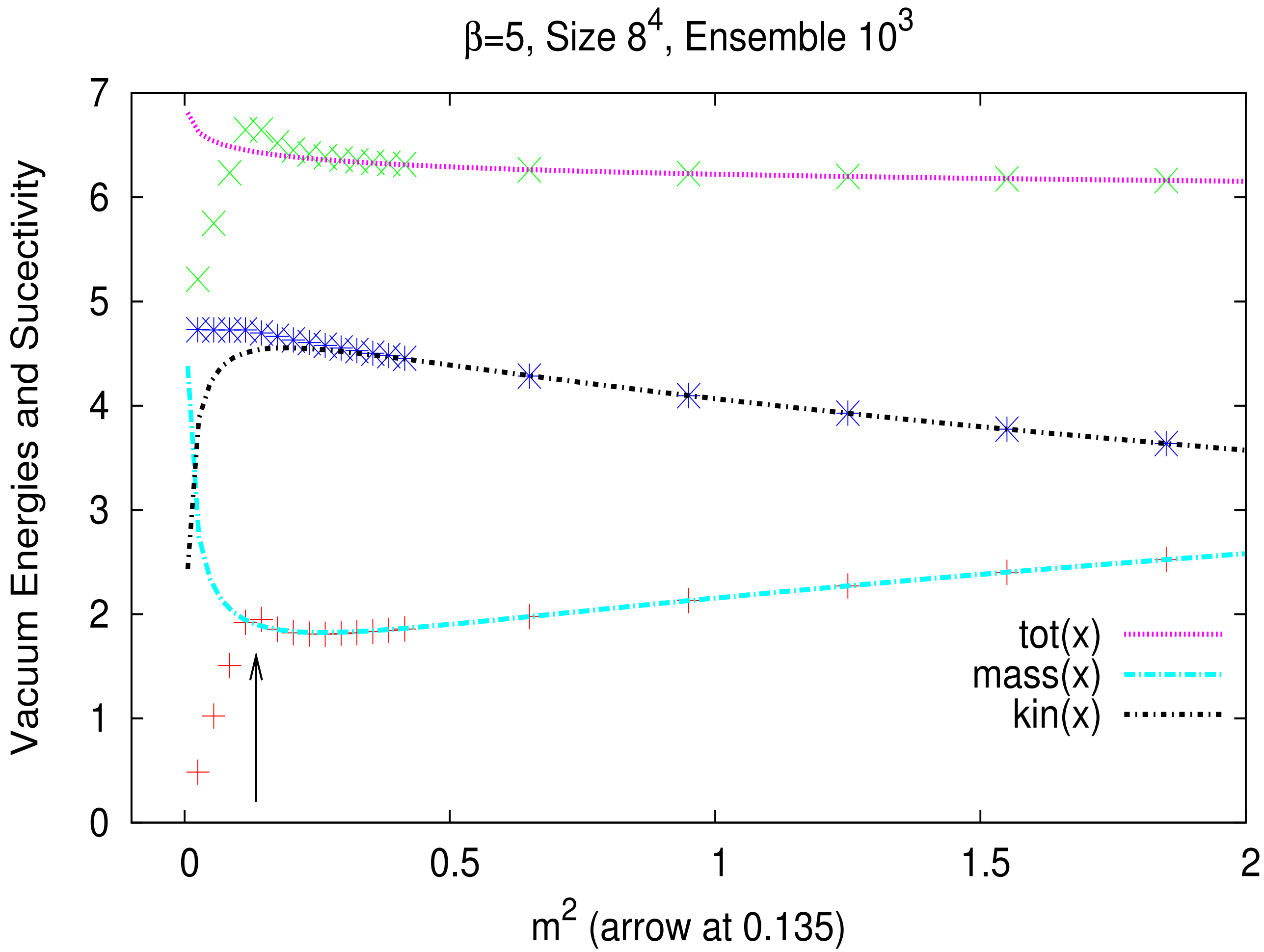

Figure 11: The line "tot" is the total energy, while "mass" is the m^2 part and "kin" is the Wilson action.

6. Conclusions

- **In the Higgs Mechanism the $M = 0$ limit is consistent via a metamorphosis of the longitudinal modes into the Goldstone bosons. A massless gauge theory coupled to scalars.**

- **In the nonlinear case we envisage a phase transition where longitudinal modes and Goldstone bosons are decoupled (asymptotic freedom).**

- **Lattice simulations support the conjecture that $m^2 \neq 0$ and $m^2 = 0$ are different phases (for high β).**

- **In lattice gauge theory the evaluation of amplitudes near the transition line is at reach.**

THANKS FOR THE ATTENTION!