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Effective action of non-Abelian monopole vortex complex

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to appear...

INTRODUCTION

Well known pathologies of non-Abelian monopoles in the Coulomb phase

- Non-normalizable modes related to unbroken color group
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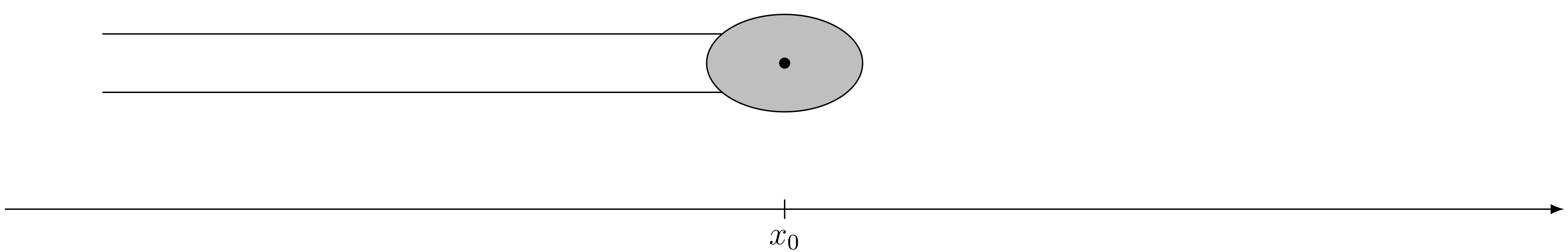
Non BPS

Abelian / Non-Abelian

M.C. , D. Dorigoni, S. B. Gudnason, K. Konishi, A. Michelini
Phys. Rev. D84 (2011) 0450024



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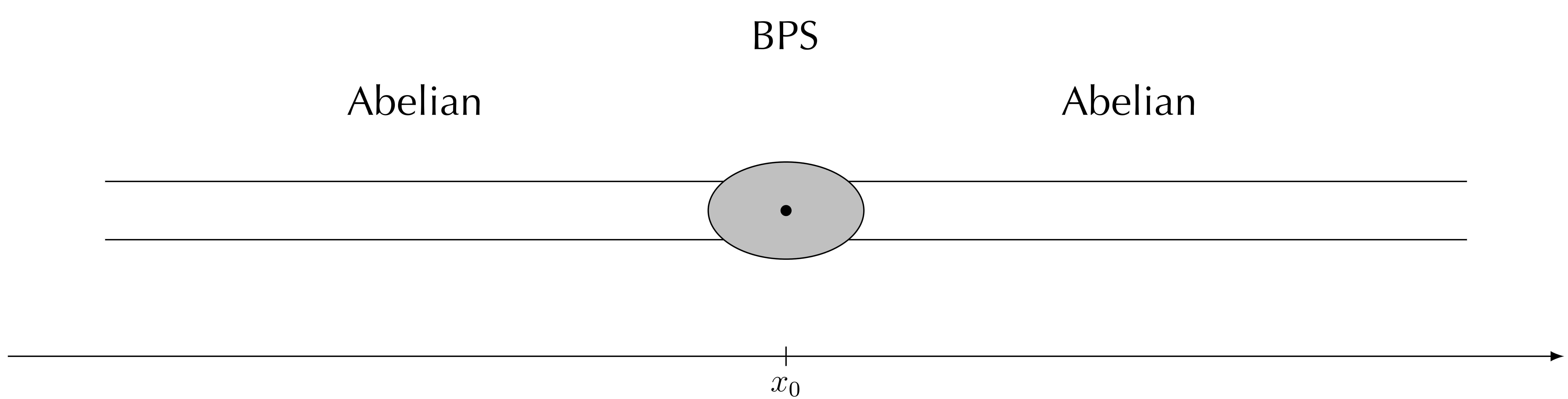

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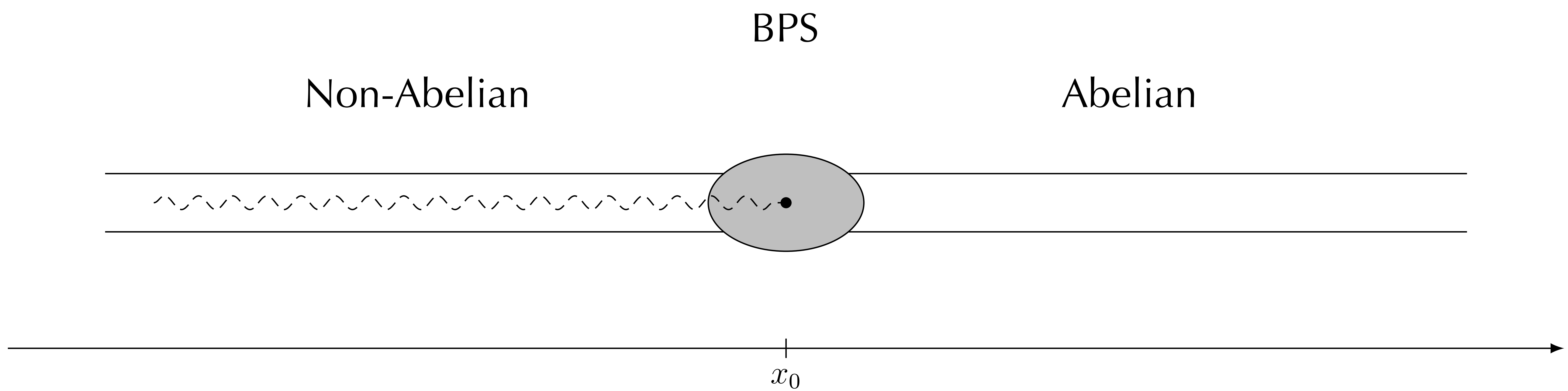

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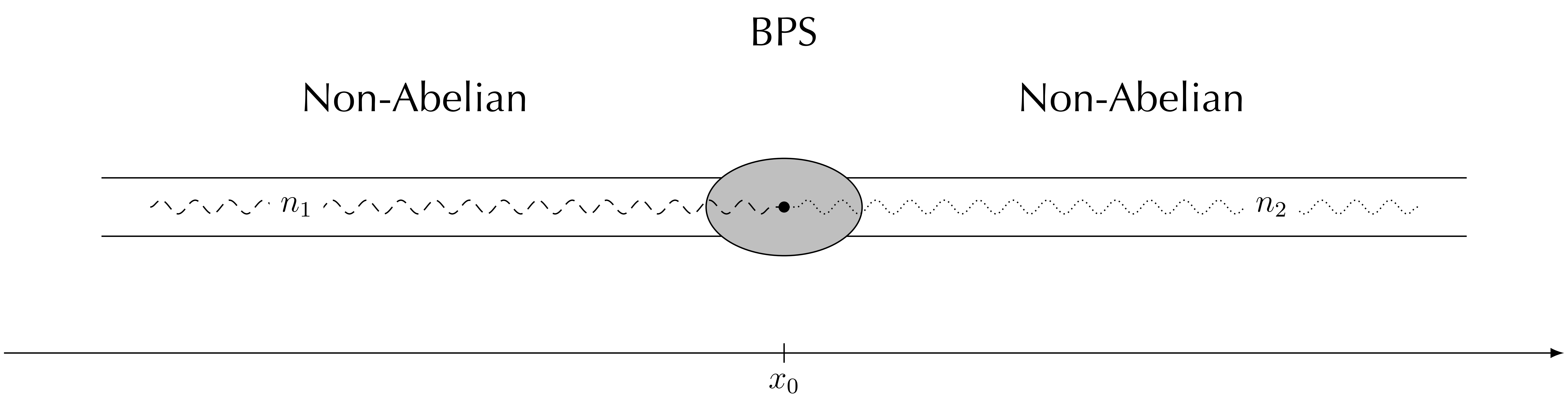

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In the monopole-vortex complex monopole modes are related to vortex moduli

The model

We consider the bosonic part of an $\mathcal{N}=2$ supersymmetric $U(N)$ gauge theory with $N_f = N$ quark flavors

$$S = \int d^4x \, \mathrm{Tr} \left\{ \frac{1}{2g^2} F_{\mu\nu}^2 + \frac{1}{g^2} |\mathcal{D}_\mu \Phi|^2 + |\mathcal{D}_\mu Q|^2 - |\Phi Q - QM|^2 - \frac{g^2}{4} (QQ^\dagger - \xi)^2 \right\}$$

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$$M = \mathrm{diag}(m_1, \cdots, m_N) \qquad \sum_{i=1}^{N} m_i = 0$$

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The vacuum is invariant under color-flavor locked symmetry

$$\Phi \overset{H_{C+F}}{\longrightarrow} H_C \,\Phi\, H_C^{-1}, \quad Q \overset{H_{C+F}}{\longrightarrow} H_C \,Q\, H_F^{-1}, \quad H_C = H_F \subset G$$

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If some masses are equal the symmetry of the vacuum is

$$H_{C+F} = S\left(U(n_1) \times U(n_2) \times \cdots \times U(n_q)\right)$$

MODULI MATRIX FORMALISM

$$
\begin{aligned}
E = \int d^3x \ \mathrm{Tr} \Bigg[\ & \frac{1}{g^2} \left(F_{12} - \mathcal{D}_3 \Phi + \frac{g^2}{2} \left(Q Q^\dagger - \xi \mathbf{1}_N \right) \right)^2 \\
& + |\mathcal{D}_1 Q + i \mathcal{D}_2 Q|^2 + |\mathcal{D}_3 Q + \Phi Q - Q M|^2 \\
& + \frac{1}{g^2} \left(F_{23} - \mathcal{D}_1 \Phi \right)^2 + \frac{1}{g^2} \left(F_{13} - \mathcal{D}_2 \Phi \right)^2 \\
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BPS equations

$$\mathcal{D}_1 Q + i\mathcal{D}_2 Q \ = \ \mathcal{D}_3 Q + \Phi Q - QM \ = \ 0$$
$$F_{23} - \mathcal{D}_1\Phi \ = \ F_{31} - \mathcal{D}_2\Phi \ = \ 0$$
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BPS equations can be formally solved by

$$A_{\bar{z}} = i S^{-1} \partial_{\bar{z}} S \,, \quad A_3 - i \Phi = i S^{-1} \partial_3 S \,, \quad Q = \sqrt{\xi} \, S^{-1} H_0(z) e^{M x_3} \qquad z = x_1 + i \, x_2$$

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Moduli matrix formalism

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The last BPS equation yields the *master equation*

$$
\boxed{\frac{1}{g^2 \xi} \left[4 \partial_z \left(\Omega \partial_{\bar z} \Omega^{-1} \right) + \partial_3 \left(\Omega \partial_3 \Omega^{-1} \right) \right] = \Omega_0 \Omega^{-1} - \xi \mathbf{1}_N}
\qquad \Omega = S S^\dagger, \quad \Omega_0 = H_0 e^{2M\, x_3} H_0^\dagger
$$

U(3) color group: one non-Abelian vortex

The moduli matrix and the *S* matrix can be obtained from the embedded ANO solution

$$S' = \begin{pmatrix} e^{-\frac{1}{2}\psi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad H_0' = \begin{pmatrix} z & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$\Omega = SS^\dagger$ satisfies the master equation when all quarks are massless: $M = 0$

$$\frac{1}{g^2\xi}\left[4\partial_z\left(\Omega\partial_{\bar{z}}\Omega^{-1}\right) + \partial_3\left(\Omega\partial_3\Omega^{-1}\right)\right] = \Omega_0\Omega^{-1} - \xi\mathbf{1}_N \qquad \Omega_0 = H_0H_0^\dagger$$

The ansatz for the quark field is: $Q = \sqrt{\xi}\, S^{-1}H_0(z)$

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When $M \neq 0$, the ansatz is

$$Q = S^{-1}(z, \bar{z}, b, \bar{b}, x_3)\, H_0(z, b, \bar{b})\, P(x_3) \qquad P(x_3) = e^{Mx_3} \qquad M = \frac{1}{3}\begin{pmatrix} -2m & & \\ & m & \\ & & m \end{pmatrix}$$

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We can rewrite the last two factors as follows

$$H_0\, P(x_3) = P(x_3) \widetilde{H}_0 \quad \Rightarrow \quad \begin{cases} b_1 \to \tilde{b}_1 = b_1\, e^{-m\, x_3} \\ b_2 \to \tilde{b}_2 = b_2\, e^{-m\, x_3} \end{cases}$$

Moduli parameters depend on the mass and vary along the x_3 axis

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$$\Omega(x_3, z, \bar{z}, \tilde{b}, \bar{\tilde{b}}) = e^{M\, x_3}\left(S\, S^{\dagger}\right)(x_3, z, \bar{z}, \tilde{b}, \bar{\tilde{b}})e^{M\, x_3}$$

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This Ω matrix satisfies the master equation up to order $\mathrm{O}\left(\dfrac{m^2}{g^2\xi}\right)$

We need to consider always $m \ll \sqrt{\xi}$

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The asymptotic form of the moduli matrix

$$H_0 = \begin{pmatrix} z & 0 & 0 \\ b_1 e^{-m x_3} & 1 & 0 \\ b_2 e^{-m x_3} & 0 & 1 \end{pmatrix}$$

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At negative infinity:

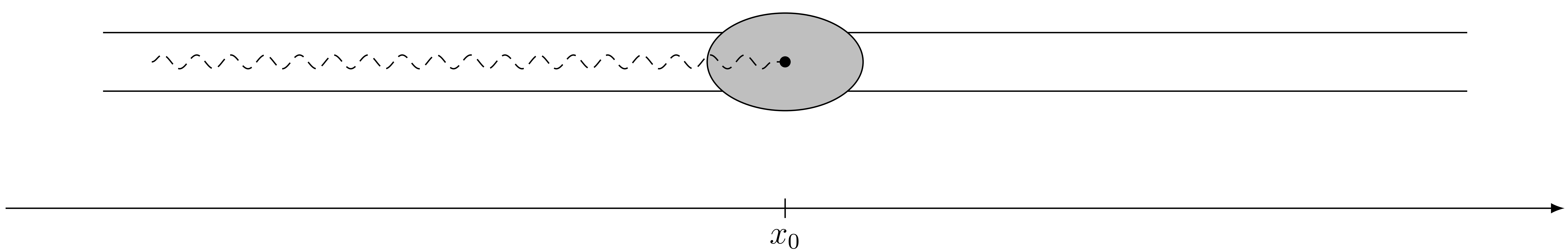
$$H_0^- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{b_1}{b_2} \\ 0 & 0 & z \end{pmatrix}$$

non-Abelian vortex

At positive infinity:

$$H_0^+ = \begin{pmatrix} z & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Abelian vortex



U(3) COLOR GROUP: ONE NON-ABELIAN VORTEX

The asymptotic form of the moduli matrix

$$H_0 = \begin{pmatrix} z & 0 & 0 \\ b_1 e^{-mx_3} & 1 & 0 \\ b_2 e^{-mx_3} & 0 & 1 \end{pmatrix}$$

At negative infinity:

$$H_0^- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{b_1}{b_2} \\ 0 & 0 & z \end{pmatrix}$$

non-Abelian vortex

$$M = \frac{1}{3}\begin{pmatrix} -2m & & \\ & m & \\ & & m \end{pmatrix}$$

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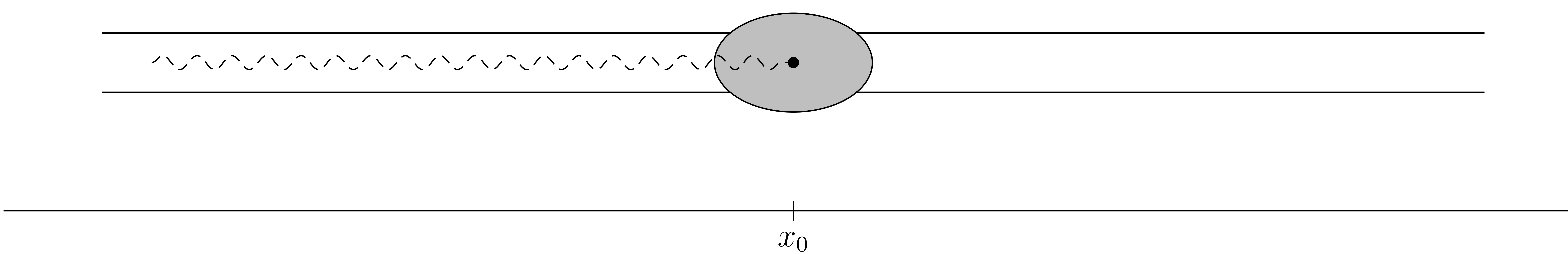

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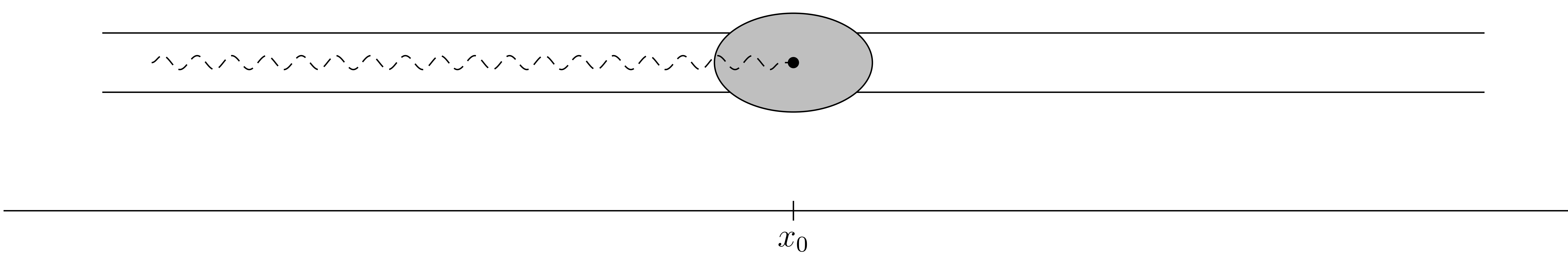

U(3) color group: one non-Abelian vortex

When moduli parameters are allowed to depend on x_3 and t

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \longrightarrow \begin{pmatrix} b_1(x_3, t) \\ b_2(x_3, t) \end{pmatrix}$$

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We find the correct ansatz for the gauge fields A_0, A_3 by solving consistently the second order equations of motion obtained from the bulk action

$$\frac{2}{g^2}\mathcal{D}_i F_{i\alpha} = \frac{i}{g^2}\left[\left[\Phi,(\mathcal{D}_\alpha\Phi)^\dagger\right] - \left[(\mathcal{D}_\alpha\Phi),\Phi^\dagger\right]\right] - i\left((\mathcal{D}_\alpha Q)Q^\dagger - Q(\mathcal{D}_\alpha Q)^\dagger\right) \qquad \begin{array}{c}\alpha = 0,3 \\ i = 1,2,3\end{array}$$

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Up to order $\mathrm{O}\left(\dfrac{m^2}{g^2\xi}\right)$ we find

$$A_{\tilde{0}} = i\left(\delta_0 S^\dagger S^{\dagger\,-1} - S^{-1}\delta_{\bar{0}} S\right)$$

$$A_{\tilde{3}} = \frac{i}{2}\left(\partial_{\dot{3}} S^\dagger S^{\dagger\,-1} - S^{-1}\partial_{\dot{3}} S\right) + i\left(\delta_3 S^\dagger S^{\dagger\,-1} - S^{-1}\delta_{\bar{3}} S\right) + O\left(\frac{m^2}{g^2\xi}\right)$$

$$\delta_\alpha = \frac{\partial b(x_3,t)}{\partial x_\alpha}\frac{\partial}{\partial b(x_3,t)}$$

$$\delta_{\bar{\alpha}} = \frac{\partial \bar{b}(x_3,t)}{\partial x_\alpha}\frac{\partial}{\partial \bar{b}(x_3,t)}$$

U(3) COLOR GROUP: ONE NON-ABELIAN VORTEX

By using the master equation we can recast the action in the form

$$S = \int \left\{ \mathrm{Tr}\left[\frac{4}{g^2} \left(\partial_z \left(\Omega^{-1} \delta_{\bar{\alpha}} \Omega \right) \delta_\alpha \left(\Omega^{-1} \partial_{\bar{z}} \Omega \right) - \delta_\alpha \left(\Omega^{-1} \delta_{\bar{\alpha}} \Omega \right) \partial_z \left(\Omega^{-1} \partial_{\bar{z}} \Omega \right) \right) \right] + \xi \delta_\alpha \delta_{\bar{\alpha}} \log\det\Omega \right\}$$

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Making the moduli dependence explicit, we obtain

$$S = C \int \mathrm{d}x \mathrm{d}t \frac{\left(e^{2m\,x} + |\vec{b}|^2\right) \left(\partial_\alpha \vec{b}^\dagger \partial_\alpha \vec{b} \right) - |\vec{b}^\dagger \partial_\alpha \vec{b}|^2}{\left(e^{2m\,x} + |\vec{b}|^2 \right)^2}$$

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The constant C is obtained by the integral

$$C = \frac{4}{g^2} \int \mathrm{d}z \mathrm{d}\bar{z} \left[\partial_z \partial_{\bar{z}} \psi \left(1 - z\bar{z}e^{-\psi} \right) + e^{-\psi} \left(1 - z\partial_z \psi \right) \left(1 - \bar{z} \partial_{\bar{z}} \psi \right) \right] = \frac{4\pi}{g^2}$$

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By redefining the moduli parameters as

$$b_1 = e^{m\,x_0} n_1 \qquad b_1^\dagger = e^{m\,x_0} n_1^\dagger$$

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$$\vec{n}^\dagger \cdot \vec{n} = 1$$

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$$S = \frac{4\pi}{g^2} \int dt dx \Big[\left(\frac{m}{2} \dot{x}_0^2 + \frac{1}{2m} |\vec{n}^\dagger \cdot \partial_\alpha \vec{n}|^2 \right) m\, \mathrm{sech}^2(m(x - x_0)) + |D_\alpha \vec{n}|^2\, e^{-2m(x-x_0)} \mathrm{sech}(m(x - x_0)) + \lambda \left(\vec{n}^\dagger \cdot \vec{n} - 1 \right) \Big]$$

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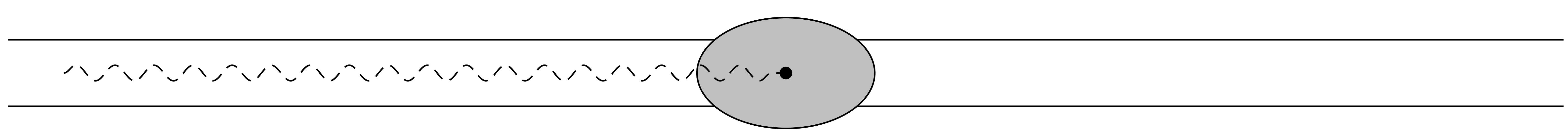

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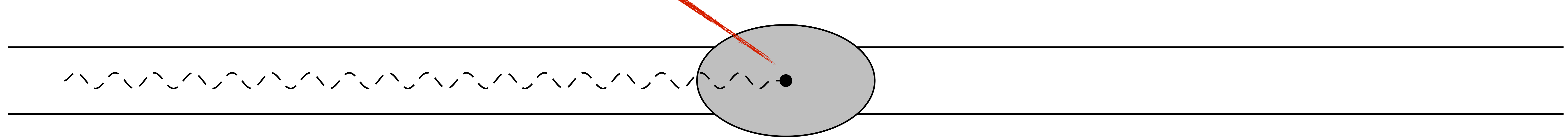

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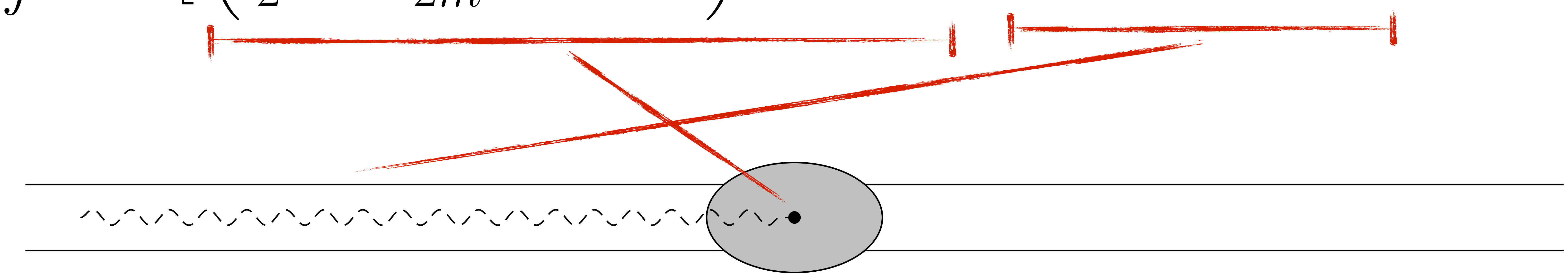

U(3) color group: one non-Abelian vortex

Simpler approach:

Take the $\mathbb{C}P^2$ vortex worldsheet action

$$S = \frac{4\pi}{g^2}\int dxdt\,\left(\partial_\alpha\vec{\Phi}^\dagger\cdot\partial^\alpha\vec{\Phi} - \left|\Phi^\dagger\cdot\partial_\alpha\Phi\right|^2\right)$$

$$\Phi^T = (\Phi^1, \Phi^2, \Phi^3)$$

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and substitute the kink solution

$$\Phi = \frac{1}{\sqrt{e^{4m(x-x_0)} + e^{-2m(x-x_0)}}}\left(n_0 e^{2m(x-x_0(t))}, \frac{n_1 e^{-m(x-x_0(t))}}{\sqrt{|n_1|^2+|n_2|^2}}, \frac{n_2 e^{-m(x-x_0(t))}}{\sqrt{|n_1|^2+|n_2|^2}}\right)$$

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In the limit $m \longrightarrow \infty$ we obtain again

$$S = \frac{4\pi}{g^2}\int dtdx\left[\left(\frac{m}{2}\dot{x}_0^2 + \frac{1}{2m}|\vec{n}^\dagger\cdot\partial_\alpha\vec{n}|^2\right)\delta(x-x_0) + |D_\alpha\vec{n}|^2\,\theta(x-x_0) + \lambda\left(\vec{n}^\dagger\cdot\vec{n}-1\right)\right]$$

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This method proves useful when dealing with more complicated setups.

U(4) color group: fully non-Abelian setting

The mass matrix is chosen as

$$M = \frac{1}{2}\begin{pmatrix} m & & & \\ & m & & \\ & & -m & \\ & & & -m \end{pmatrix}$$

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The mass matrix is chosen as

$$M = \frac{1}{2}\begin{pmatrix} m & & & \\ & m & & \\ & & -m & \\ & & & -m \end{pmatrix}$$

The most general moduli matrix is

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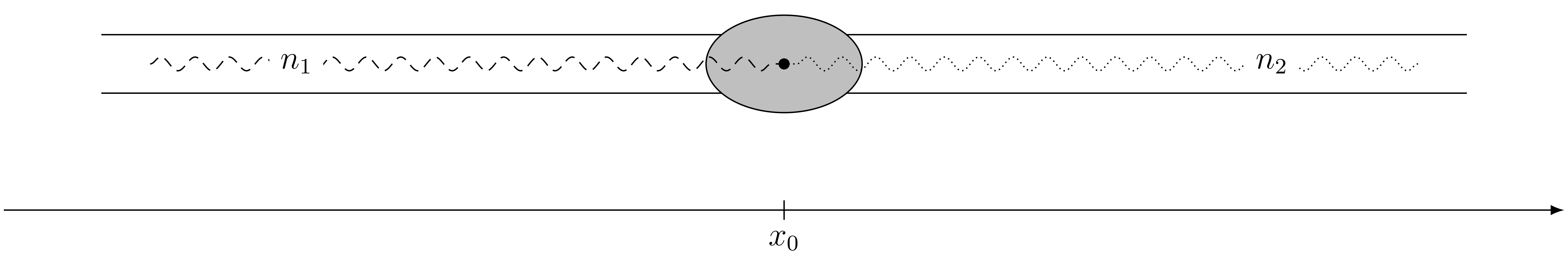

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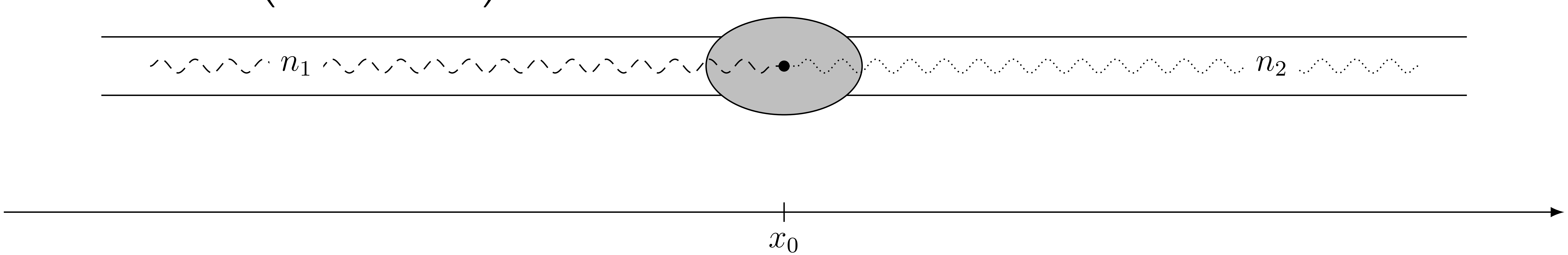

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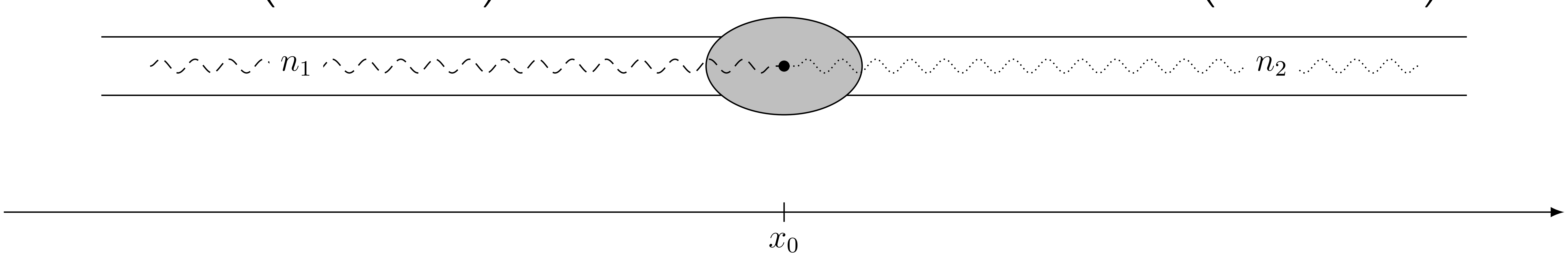

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Unbroken $SU(2)$

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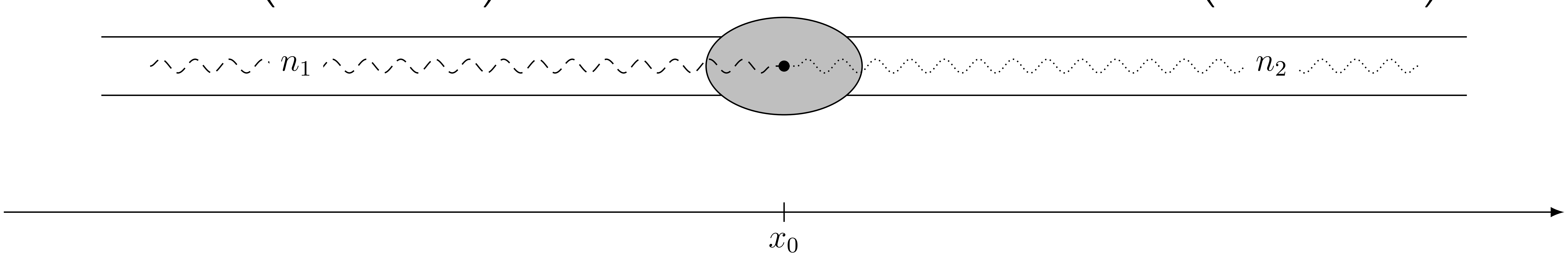

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$$b_1 \longrightarrow b_1$$

$$b_2 \longrightarrow b_2\, e^{mx}$$

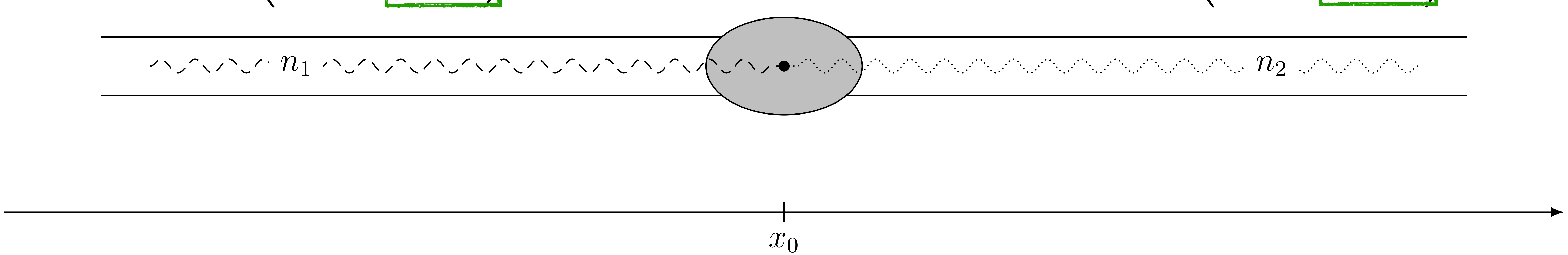
$$b_3 \longrightarrow b_3\, e^{mx}$$

At negative infinity:

$$H_0^- = \begin{pmatrix} z & & & \\ b_1 & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

At positive infinity:

$$H_0^+ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \frac{b_2}{b_3} \\ & & & z \end{pmatrix}$$



U(4) color group: fully non-Abelian setting

We take the vortex worldsheet action: $\mathbb{C}P^3$ NLSM

$$S = \frac{4\pi}{g^2}\int dxdt\,\left(\partial_\alpha\vec{\Phi}^\dagger\cdot\partial^\alpha\vec{\Phi} - \left|\Phi^\dagger\cdot\partial_\alpha\Phi\right|^2\right)$$

$$\vec{\Phi}^T = (\Phi^1,\Phi^2,\Phi^3,\Phi^4)$$

$$\vec{\Phi}^\dagger\cdot\vec{\Phi} = 1$$

and substitute the kink solution $\vec{\Phi} = \dfrac{1}{\sqrt{2\cosh(m(x-x_0))}}\left(\dfrac{\vec{n}_1}{|\vec{n}_1|}e^{-m(x-x_0)},\dfrac{\vec{n}_2}{|\vec{n}_2|}e^{m(x-x_0)}\right)$

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2-components vectors

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In the limit $m \longrightarrow \infty$ we obtain the effective action for the complex

$$S = \frac{4\pi}{g^2}\int dxdt\,\left[-\frac{m}{4}\,\delta(x-x_0) + \frac{m}{4}\,\delta(x-x_0)\,\dot{x_0}^2 + \frac{1}{4m}\delta(x-x_0)\left|\vec{n}_1^\dagger\cdot\partial_\alpha\vec{n}_1 - \vec{n}_2^\dagger\cdot\partial_\alpha\vec{n}_2\right|^2\right.$$
$$\left. + \theta(x_0-x)\left|\left(\partial_\alpha - \vec{n}_1^\dagger\cdot\partial_\alpha\vec{n}_1\right)\vec{n}_1\right|^2 + \theta(x-x_0)\left|\left(\partial_\alpha - \vec{n}_2^\dagger\cdot\partial_\alpha\vec{n}_2\right)\vec{n}_2\right|^2\right]$$

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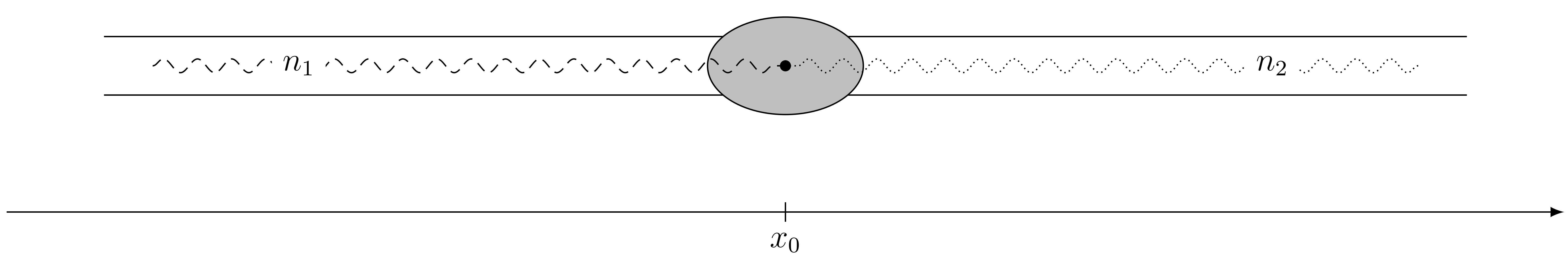

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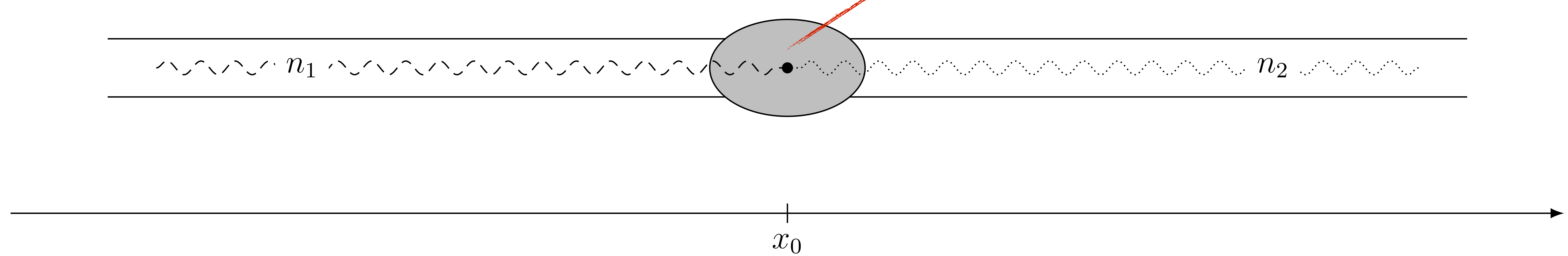

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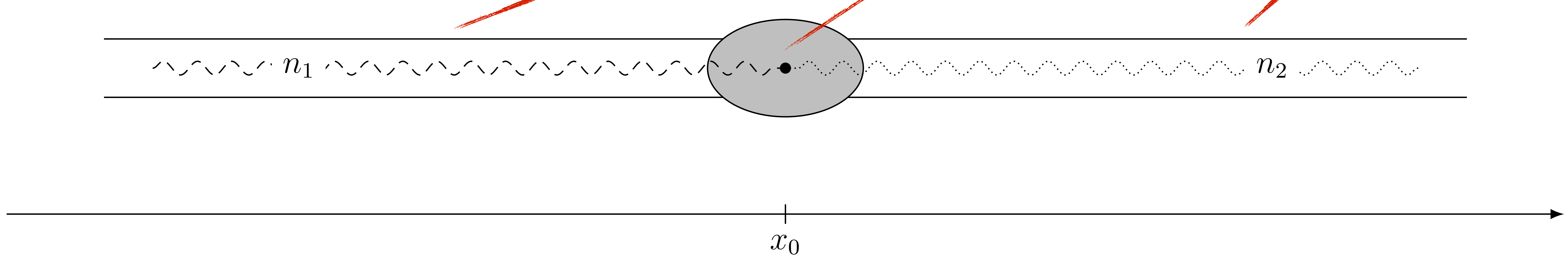

General U(N) color group

The mass matrix can be taken as

$$M = \frac{1}{2}\begin{array}{c}\left(\begin{array}{ccc|ccc} m & & & & & \\ & \ddots & & & & \\ & & m & & & \\ \hline & & & -m & & \\ & & & & \ddots & \\ & & & & & -m \end{array}\right) \\ \underbrace{\hphantom{m \quad \ddots \quad m}}_{n}\underbrace{\hphantom{-m \quad \ddots \quad -m}}_{n}\end{array}\begin{array}{l}\left.\vphantom{\begin{array}{c} m \\ \ddots \\ m \end{array}}\right\} n \\ \left.\vphantom{\begin{array}{c} -m \\ \ddots \\ -m \end{array}}\right\} n \\ \vphantom{\underbrace{m}_{n}}\end{array}$$

$$n = \frac{N}{2}$$

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n components vectors

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Conclusions

We found the effective action for orientational moduli in a junction of two Abelian or non-Abelian vortices with a non-Abelian monopole

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Future study of the orientational moduli dynamics: sigma models with boundaries

Thank You!