Holographic three-point functions

of giant gravitons

Agnese Bissi

Niels Bohr Institute
Niels Bohr International Academy

June 1,2012

based on hep-th 1103.4079 (JHEP 1106 (2011) 085)
with C. Kristjansen, D. Young and K. Zoubos

CORTONA 2012, Convegno Informale di Fisica Teorica

AdS/CFT correspondence

AdS/CFT correspondence

- Type II B string theory on $AdS_5 \times S^5$
- $g_s = \frac{4\pi\lambda}{N}$, $\frac{R^2}{\alpha'} = \sqrt{\lambda}$
- Single string states of energy Δ

- $\mathcal{N} = 4$ SYM with gauge group $SU(N)$
- $\lambda = g_{YM}^2 N$
- Single trace operators of conformal dimension Δ

Motivations

- Building blocks in conformal field theories are 2-point and 3-point functions of local gauge invariant operators.

$$\langle \mathcal{O}_i(x)\, \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

Spectral problem:
Find $\Delta = \Delta\,(\lambda, N)$ by diagonalizing the dilatation operator

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \mathcal{O}_k(z) \rangle = \frac{C_{ijk}}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}|y-z|^{\Delta_j+\Delta_k-\Delta_i}|z-x|^{\Delta_i+\Delta_k-\Delta_j}}$$

- In principle all the higher point correlation functions are known by using the OPE:

$$\mathcal{O}_\alpha(x_1)\mathcal{O}_\beta(x_2) \sim \sum_\gamma \frac{C_{\alpha\beta\gamma}}{|\ x_{12}\ |^{\Delta_\alpha+\Delta_\beta-\Delta_\gamma}} \mathcal{O}_\gamma(x_2)$$

Holographic prescription

- Correlation function using AdS/CFT:

$$Z_{bulk}\left[\phi\left(\vec{x},z\right)|_{z=0}\right]=\phi_0\left(\vec{x}\right)\right]=\langle e^{\int d^4x\,\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{\text{field theory}}$$

  $\phi_0\left(\vec{x}\right)$ is an arbitrary function specifying the boundary values of the bulk field ϕ.
- Taking derivatives with respect to ϕ_0 and setting it to zero we obtain the correlation functions of the operator.
- Changes in the boundary conditions of AdS correspond to changes in the Lagrangian of the field theory. Infinitesimal changes in the boundary condition correspond to the insertion of an operator.

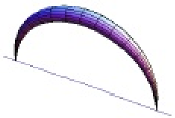
State of the art: String theory side 1

- Protected operators dual to SUGRA modes

[Freedman, Mathur, Matusis and Rastelli, 1998]

[Arutyunov and Frolov, 2000]

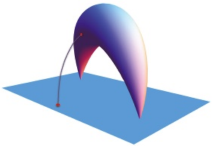
- New techniques for computing 2-point correlation functions (of non BPS operators)



[Buchbinder, 2010]
[Buchbinder and Tseytlin, 2010]
[Janik, Surowka and Wereszczynski, 2010]

State of the art: String theory side 2

- 2 operators are semiclassical and one is dual to a SUGRA mode



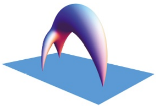
[Zarembo, 2010], [Costa, Monteiro, Santos and Zoakos,2010]
+
[Roiban and Tseytlin, 2010], [Hernandez, 2011], [Ryang, 2011], [Georgiou, 2011],[Russo and Tseytlin, 2011], [Park and Lee, 2011],[Buchbinder and Tseytlin, 2011],[Buchbinder and Tseytlin, 2011],[Bak, Chen and Wu,2011],[AB, Kristjansen, Young and Zoubos, 2011],[Arnaudov, Rashkov and Vetso, 2011],[Escobedo, Gromov, Sever and Vieira, 2011], [Hernandez, 2011], [Ahn and Bozhilov , 2011],[Caputa, de Mello Koch and Zoubos, 2012], [Hirano, Kristjansen and Young, 2012]

- Geodesic approximation for the 3 operators

[Klose and McLoughlin, 2011]

State of the art: String theory side 3

- Contributions to the 3-point function of 3 heavy operators



[Janik and Wereszczynski, 2011]
[Kazama and Komatsu, 2011]
[Buchbinder and Tseytlin, 2011]

- Correlator of BMN operators by using 3-string vertex matrix elements (also one-loop)

[Grignani and Zayakin, 2012]

- One (and two) loop correction for 3-point function of two semiclassical operators and one light BPS

[Russo and Tseytlin, 2011]
[AB, Harmark and Orselli, 2011]
[Gromov and Vieira, 2012]

Figures from Zarembo's paper

State of the art: Gauge theory side 1

- BMN operators

[Kristjansen,Plefka, Semenoff and Staudacher, 2002]
[Constable, Freedman, Headrick and Minwalla, 2002]
[Chu, Khoze and Travaglini, 2002]
[Beisert,Kristjansen,Plefka, Semenoff and Staudacher, 2002]

- Spin chain approach

[Roiban and Volovich, 2004]
[Okuyama and Tseng, 2004]
[Alday, David, Gava and Narain, 2005]

- Various operators in BMN limit (2 BPS and 1 non BPS)

[Georgiou, Gili and Russo, 2009]

- 3 BMN operators

[Grignani and Zayakin, 2012]

- 2 Schur polynomials and 1 chiral primary

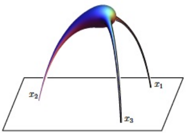
[A.B., Kristjansen, Young and Zoubos, 2011]
[Caputa, de Mello Koch and Zoubos, 2012]
[Hirano, Kristjansen and Young, 2012]

State of the art: Gauge theory side 2

- Novel spin chain approaches

[Escobedo, Gromov, Sever and Vieira, 2011]
[Foda, 2012]
[Serban, 2012]
[Kostov, 2012]
[AB, Kristjansen, Martirosyan and Orselli, in progress]

- 2 operators are light and 1 is heavy



[Gromov, Sever and Vieira, 2011]

- Three point function at one loop for scalar operators up to lenght five

[Georgiou, Gili, Grossardt and Plefka, 2012]

State of the art: Gauge theory side 3

- One loop result for three-point functions of scalar single trace operators

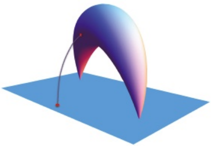
  [AB, Harmark and Orselli, 2011]
  [Gromov and Vieira, 2012]

- One loop result for BMN operators

  [Grignani and Zayakin, 2012]

Semiclassical method

$$\langle \mathcal{O}_I(x) \rangle_{\mathcal{W}} = \frac{\langle \mathcal{W}\, \mathcal{O}_I(x) \rangle}{\langle \mathcal{W} \rangle}$$



- $\mathcal{W} = \bar{\mathcal{O}}_J(x_1)\mathcal{O}_k(x_2)$: non local operator dual to classical string
- $\mathcal{O}_I(x)$: local operator dual to a sugra mode

$$\langle \mathcal{O}_I(y) \rangle_{\mathcal{W}} = \lim_{\varepsilon \to 0} \frac{\pi}{\varepsilon^{\Delta_I}} \sqrt{\frac{2}{\Delta_I - 1}} \left\langle \phi_I(y, \varepsilon)\, \frac{1}{Z_{\mathrm{str}}} \int \mathcal{D}\mathbb{X}\; \mathrm{e}^{\,-S_{\mathrm{str}}[\mathbb{X}]} \right\rangle_{\mathrm{bulk}}$$

$$S_{\mathrm{str}} = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma\, \sqrt{h} h^{\mathbf{ab}} \partial_{\mathbf{a}} \mathbb{X}^M \partial_{\mathbf{b}} \mathbb{X}^N G_{MN} + \dots$$

$$G_{MN} = g_{MN} + \gamma_{MN}$$

$\boxed{\gamma_{MN}}$ is the disturbance created by the local operator insertion

Our settings

STRING SIDE

- Giant gravitons as heavy operators: dimension $\sim \sqrt{\lambda}$
- Pointlike graviton as light operator: dimension ~ 1

GAUGE SIDE

- Schur polynomials
- Single trace chiral primary

STRING SIDE

1. Giant gravitons wrap an $\boxed{S^3 \subset S^5}$
2. Giant gravitons wrap an $\boxed{S^3 \subset AdS_5}$

GAUGE SIDE

1. Schur polynomials in an antisymmetric rep
2. Schur polynomials in a symmetric rep

Giant gravitons & light operator

- Two point function: D-brane solution continued to the Euclidean Poincaré patch
- Vary the Euclidean D-brane action in accordance with the supergravity fluctuations
- Evaluate the fluctuations on the Wick rotated giant graviton solution

Giant graviton on S^5

- Giant graviton with worldvolume $\mathbb{R}(\subset AdS_5) \times S^3(\subset S^5)$.
- The action for the D3-brane is

$$S_{D3} = -\frac{N}{2\pi^2}\int d^4\sigma\left(\sqrt{-g} - P[C_4]\right)$$

  - $g_{ab} = \partial_a X^M \partial_b X_M$, $a, b = 0, \ldots, 3 \rightarrow$ worldvolume coordinates, $X^M \rightarrow$ embedding coordinates
  - $C_{\phi\chi_1\chi_2\chi_3} = \cos^4\theta \, \mathrm{Vol}(\Omega_3)$

[Grisaru Myers Tafjord, 2000]

2-point function

- The brane ansatz as

$$\rho = 0, \quad \sigma^0 = t, \quad \phi = \phi(t), \quad \sigma^i = \chi_i$$

- The action becomes

$$S = \int dt\, L = -N \int dt \left[\cos^3\theta \sqrt{1 - \dot{\phi}^2 \sin^2\theta} - \dot{\phi} \cos^4\theta \right]$$

- The energy is minimized by

$$\cos^2\theta = \frac{k}{N}, \qquad E_{\text{min.}} = k, \qquad S_{\text{min.}} = 0,$$

where k is the conserved angular momentum.

Giant graviton on AdS$_5$

- Giant graviton with worldvolume $\mathbb{R} \times S^3 (\subset AdS_5)$
- The action is

$$S_{D3} = -\frac{N}{2\pi^2} \int d^4\sigma \left(\sqrt{-g} + P[C_4]\right)$$

where

$$C_{t\widetilde{\chi}_1\widetilde{\chi}_2\widetilde{\chi}_3} = -\sinh^4 \rho \, \mathrm{Vol}(\widetilde{\Omega}_3)$$

[Grisaru Myers Tafjord, 2000]

- The angular momentum is unbounded.

Fluctuations

- The supergravity modes are *fluctuations* dual to *chiral primary operators* with R-charge Δ in $\mathcal{N}=4$ SYM:

$$\delta g_{\mu\nu} = \left[-\frac{6\,\Delta}{5}\,g_{\mu\nu} + \frac{4}{\Delta+1}\,\nabla_{(\mu}\nabla_{\nu)}\right] s^{\Delta}(X)\,Y_{\Delta}(\Omega),$$
$$\delta g_{\alpha\beta} = 2\,\Delta\,g_{\alpha\beta}\,s^{\Delta}(X)\,Y_{\Delta}(\Omega),$$
$$\delta C_{\mu_1\mu_2\mu_3\mu_4} = -4\,\epsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5}\nabla^{\mu_5}\,s^{\Delta}(X)\,Y_{\Delta}(\Omega),$$
$$\delta C_{\alpha_1\alpha_2\alpha_3\alpha_4} = 4\epsilon_{\alpha\alpha_1\alpha_2\alpha_3\alpha_4}s^{\Delta}(X)\nabla^{\alpha}Y_{\Delta}\,(\Omega)$$

  where $Y_{\Delta}(\Omega)$ are the *spherical harmonics* on the five-sphere and $s^{\Delta}(X)$ represent a scalar field propagating on AdS_5.
- The *bulk-to-boundary propagator* for s^{Δ} is

$$\sqrt{\frac{\alpha_0}{B_\Delta}}\frac{z^{\Delta}}{((x-x_B)^2+z^2)^{\Delta}} \simeq \sqrt{\frac{\alpha_0}{B_\Delta}}\frac{z^{\Delta}}{x_B^{2\Delta}} \quad \text{where} \quad \alpha_0 = \frac{\Delta-1}{2\pi^2},\ B_{\Delta} = \frac{2^{3-\Delta}N^2\Delta(\Delta-1)}{\pi^2\,(\Delta+1)^2}$$

- Note that $z=\frac{R}{\cosh t_E}$, $\quad x_E^0 = R\tanh t_E$. (Euclidean path)

Results

- The three point function structure constant (for giant gravitons on S^5) is

$$\boxed{C^{A}_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2}}$$

- The three point function structure constant (for giant gravitons on AdS_5) is

$$\boxed{C^{S}_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{J/2} - \left(1+\frac{k}{N}\right)^{-J/2}\right)}$$

Dual objects

- Giant gravitons are dual to Schur polynomial operators:

  [Corley Jevicki Ramgoolam, 2002]

$$\chi_{R_n}(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_{R_n}(\sigma)\, Z_{i_1}^{i_{\sigma(1)}} \dots Z_{i_n}^{i_{\sigma(n)}}$$

  where R_n is an irreducible representation of $U(N)$ described in terms of a Young tableau with n boxes and $\chi_{R_n}(\sigma)$ is the character of the element σ in the representation R_n.

  - S^5 giant gravitons $\Rightarrow$ $\boxed{\text{Antisymmetric}}$ representation
  - AdS_5 giant gravitons $\Rightarrow$ $\boxed{\text{Symmetric}}$ representation
- Light operator dual to single trace chiral primary

Note:There is no limit in which the Schur polynomial reduces to a chiral primary. F.i.

$$\chi_{S_3}(Z) = \frac{1}{6}(\mathrm{Tr}Z)^3 + \frac{1}{2}\mathrm{Tr}\left(Z^2\right)\mathrm{Tr}\left(Z\right) + \frac{1}{3}\mathrm{Tr}\left(Z^3\right)$$

Three point function

$$\boxed{C_{k,k-J,J} \equiv \frac{\langle \chi_k(\bar{Z})\chi_{k-J}(Z)\mathrm{Tr}Z^J\rangle}{\sqrt{\langle \chi_k(\bar{Z})\chi_k(Z)\rangle\langle \chi_{k-J}(\bar{Z})\chi_{k-J}(Z)\rangle\langle \mathrm{Tr}\bar{Z}^J\mathrm{Tr}Z^J\rangle}}}$$

$$\langle \mathrm{Tr}Z^J\,\mathrm{Tr}\bar{Z}^J\rangle = \frac{1}{J+1}\left\{\frac{\Gamma(N+J+1)}{\Gamma(N)} - \frac{\Gamma(N+1)}{\Gamma(N-J)}\right\}$$

[Kristjansen Plefka Semenoff Staudacher, 2002]

$$\boxed{\langle \chi_k^S(\bar{Z})\chi_k^S(Z)\rangle = \prod_{j=1}^k (N-1+j)}$$

$$\boxed{\langle \chi_k^A(\bar{Z})\chi_k^A(Z)\rangle = \prod_{i=1}^k (N-i+1)}$$

Numerator

- Expand $\mathrm{Tr}Z^J$ in the basis of Schur polynomials

$$\mathrm{Tr}\, Z^J = \sum_{R_J} \chi_{R_J}(\sigma_0)\chi_{R_J}(Z)$$

$$\boxed{C^S_{k,k-J,J} = \frac{\sqrt{\prod_{p=k-J+1}^{k}(N+p-1)}}{\sqrt{JN^J(1+c(J)\,\frac{1}{N^2}+\ldots)}}}$$

$$\boxed{C^A_{k,k-J,J} = (-1)^{(J-1)}\frac{\sqrt{\prod_{p=k-J+1}^{k}(N-p+1)}}{\sqrt{JN^J(1+c(J)\,\frac{1}{N^2}+\ldots)}}}$$

Limits

- Schur polynomials $\Rightarrow$ large Young tableaux
- Chiral primary $\Rightarrow$ small operator

$$\boxed{N \to \infty, \qquad k \to \infty, \qquad \frac{k}{N} \ \text{finite}, \qquad J \ll k, \qquad J \ll \sqrt{N}}$$

$$\boxed{C^{S}_{k,k-J,J} = \frac{1}{\sqrt{J}} \left(1 + \frac{k}{N}\right)^{J/2}}$$

$$\boxed{C^{A}_{k,k-J,J} = (-1)^{(J-1)} \frac{1}{\sqrt{J}} \left(1 - \frac{k}{N}\right)^{J/2}}$$

Limits

GIANT GRAVITONS & LIGHT OPERATOR

$$C^A_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2} \xrightarrow[\boxed{\frac{k}{N}\to 0}]{} \boxed{\sqrt{J}\frac{k}{N}}$$

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{J/2} - \left(1+\frac{k}{N}\right)^{-J/2}\right) \xrightarrow[\boxed{\frac{k}{N}\to 0}]{} \boxed{\sqrt{J}\frac{k}{N}}$$

SCHUR POLYNOMIALS & CHIRAL PRIMARY

$$C^A_{k,k-J,J} = (-1)^{(J-1)}\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}$$

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(1+\frac{k}{N}\right)^{J/2}$$

Comparison

GIANT GRAVITONS & LIGHT OPERATOR

$$C^A_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2} \xrightarrow[\boxed{\frac{k}{N}\to 1}]{} \boxed{J\left(\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}\right)}$$

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{\frac{J}{2}} - \left(1+\frac{k}{N}\right)^{-\frac{J}{2}}\right) \xrightarrow[\boxed{\frac{k}{N}\to\infty}]{} \boxed{\frac{1}{\sqrt{J}}\left(1+\frac{k}{N}\right)^{J/2}}$$

SCHUR POLYNOMIALS & CHIRAL PRIMARY

$$C^A_{k,k-J,J} = \boxed{(-1)^{(J-1)}\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}}$$

$$C^S_{k,k-J,J} = \boxed{\frac{1}{\sqrt{J}}\left(1+\frac{k}{N}\right)^{J/2}}$$

Extremal vs Non-extremal correlators

- *Extremal correlators*: correlators in which the conformal dimension of one of the operators is precisely equal to the sum of the dimensions of the other two operators.
- It is recently shown in [Caputa, de Mello Koch and Zoubos, 2012] that the non extremal three point functions for 2 giant gravitons and 1 point like graviton using semiclassical probe approach and field theory are in perfect agreement.

Why?

Problem with supergravity computation for extremal correlators: in the extremal case the contributions from single and multitraces are of the same order in N, on the contrary in the non-extremal case the contributions from multitrace operators are subleading.

[D'Hoker, Freedman, Mathur, Matusis and Rastelli, 1999]

Conclusions

We compute

- 3 point function of 2 giant gravitons and a pointlike graviton, both where the giant gravitons wrap an $\boxed{S^3 \subset S^5}$ and where they wrap an $\boxed{S^3 \subset AdS_5}$
- 3 point function of 2 Schur polynomial and a chiral primary, both in the $\boxed{\text{antisymmetric}}$ and in the $\boxed{\text{symmetric}}$ representation.

Open problems

- understand how to treat extremal correlators.
- compute the same quantities for 3 giant gravitons/3 Schur polynomials.