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# Nonequilibrium Quantum Dynamics in Many-Body Problems

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1- Introduction
2- Experiments
3- Theoretical Issues

A. Polkovnikov, K. Sengupta, A. Silva and M. Vengalattore,
Review Modern Physics Colloquium
arXiv:1007.5331

Text

# Equilibrium

**Equilibrium**: all microstates of a system consistent with the same macroscopic state are equally probable.

↓

**Statistical ensembles (1901)**

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**Mean field theory**

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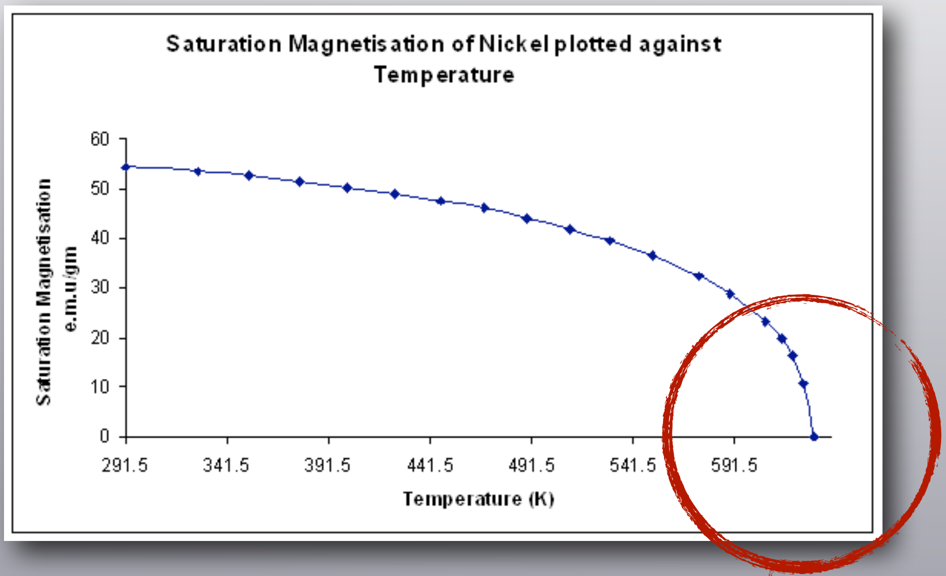
**Renormalization group (1971)**

Statistical Physics
Part 2

Course of Theoretical Physics

# Universality



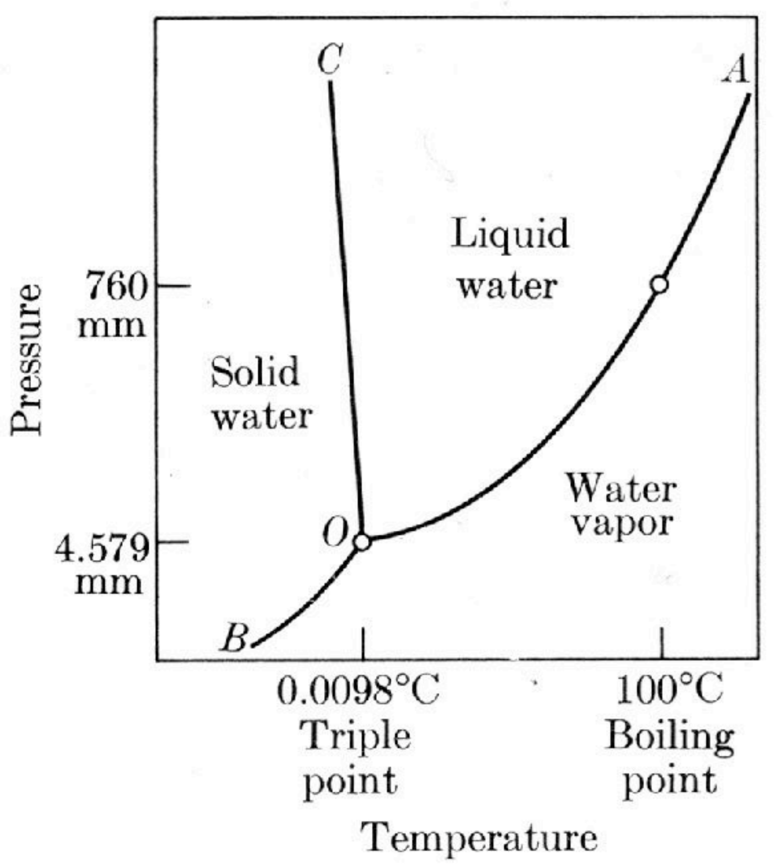
Don't need to describe all the details !!!

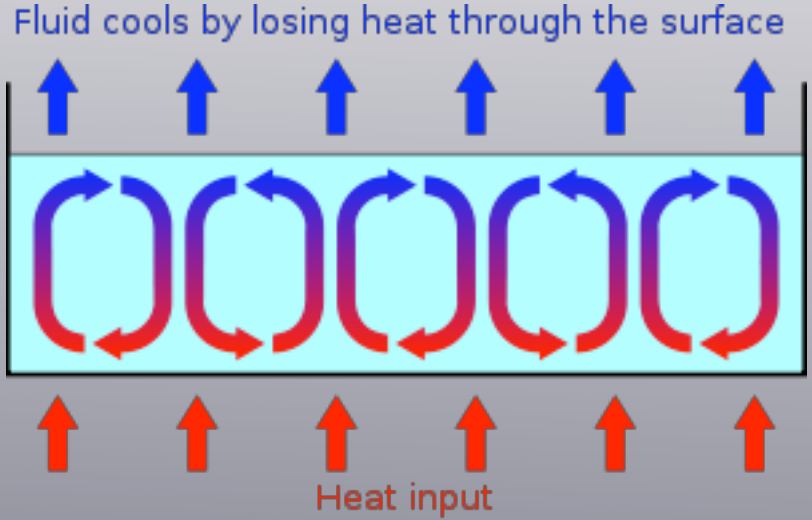
Physics at **low energies, long distances** ...

**Effective models**

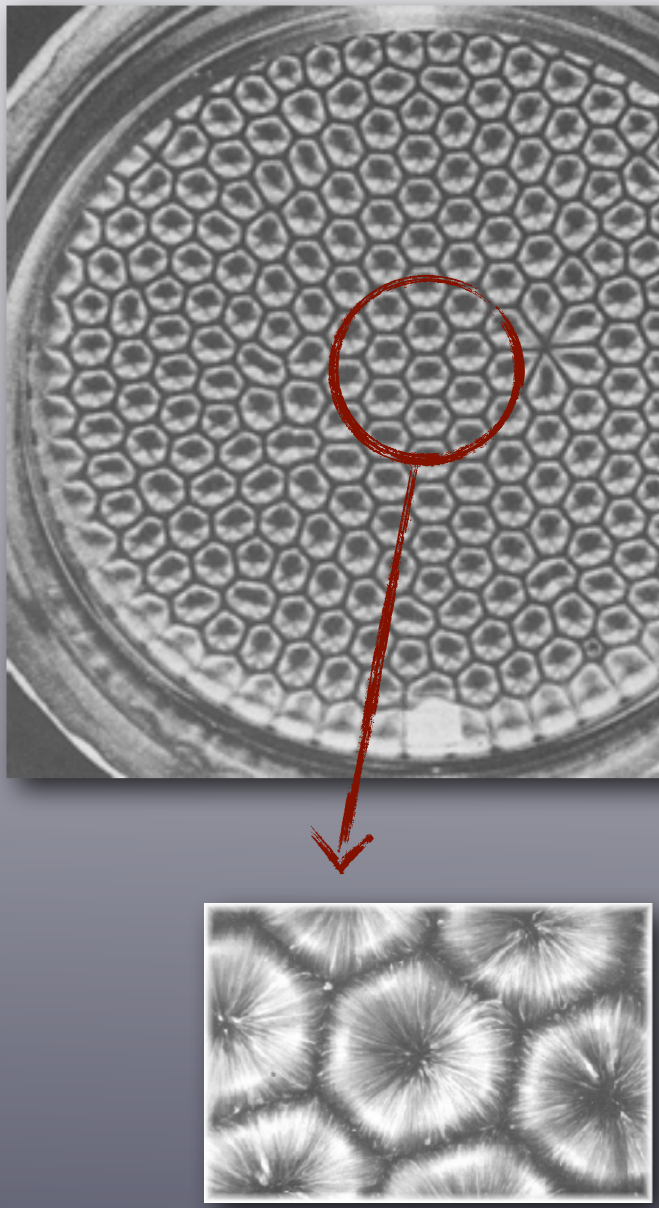
$$H = -\sum_{\langle ij\rangle} J\, S_i\, S_j$$

# Equilibrium: is that all ?





# Consequences of non-equilibrium behavior



**Important Ingredients:**

**Non-equilibrium**

+

**Non-linearity**
(**interactions**)

J. P. **Gollub** and J. S. **Langer**,
**"Pattern formation in nonequilibrium physics"**
Rev. Mod. Phys. **71**, S396 (1999)

# Questions

1) - **Fundamental** description: entropy, work, heat, fluctuations, effective ensembles ?

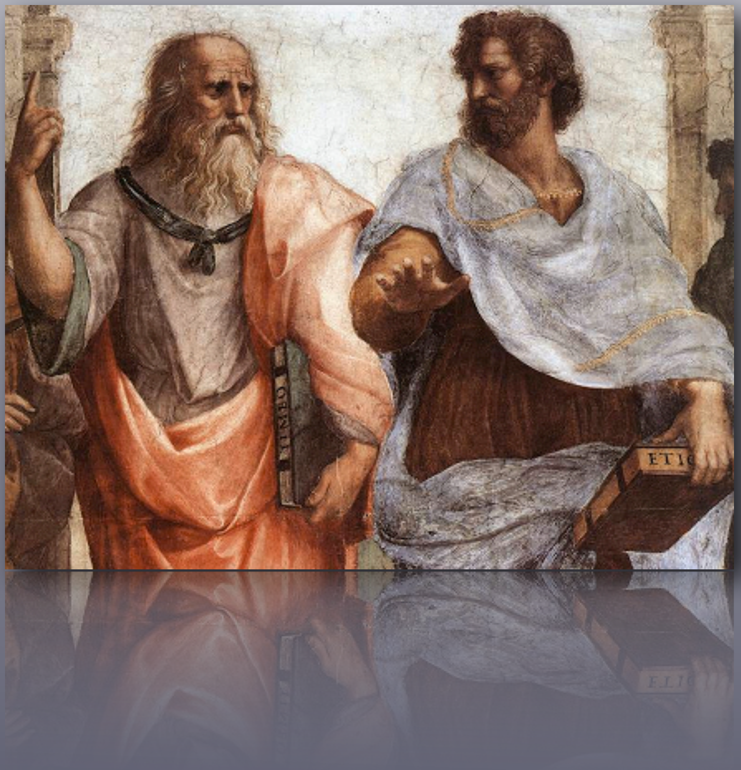
2) - **Universal** predictions ?

3) - **Generic** connections (e.g. integrability thermalization) ?

UP TO 10 YEARS AGO

IN QUANTUM SYSTEMS

ACADEMIC QUESTIONS



# Quantum Systems

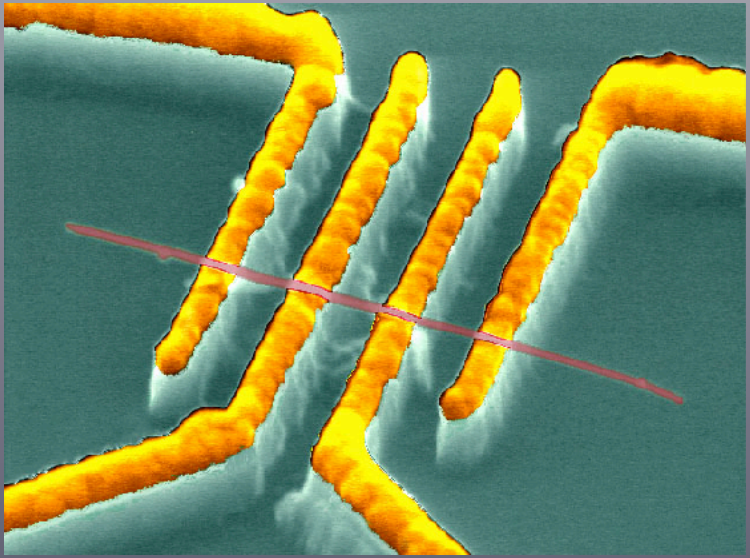
**Nonequilibrium + Nonlinearity** + **Quantum Coherence**

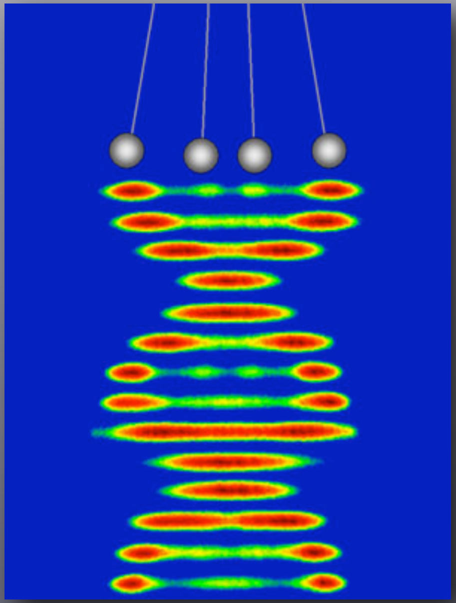
**Transport**

non-equilibrium boundary conditions

**Time dependent**

response to variation of system parameters



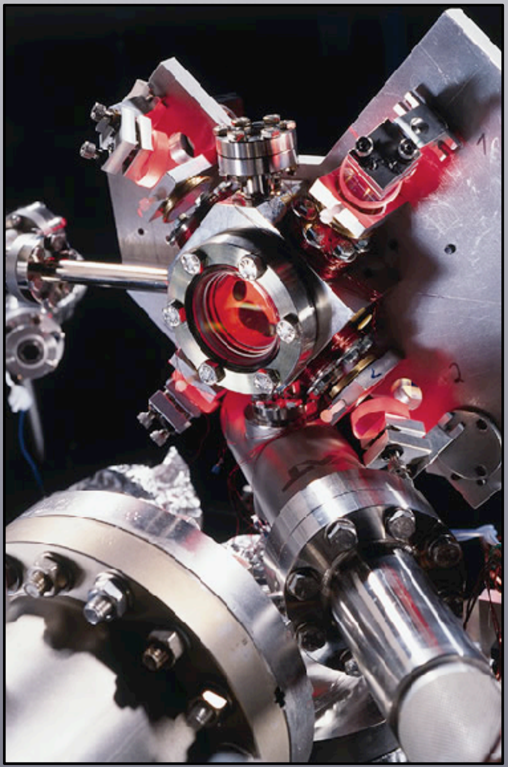


# Cold Atoms

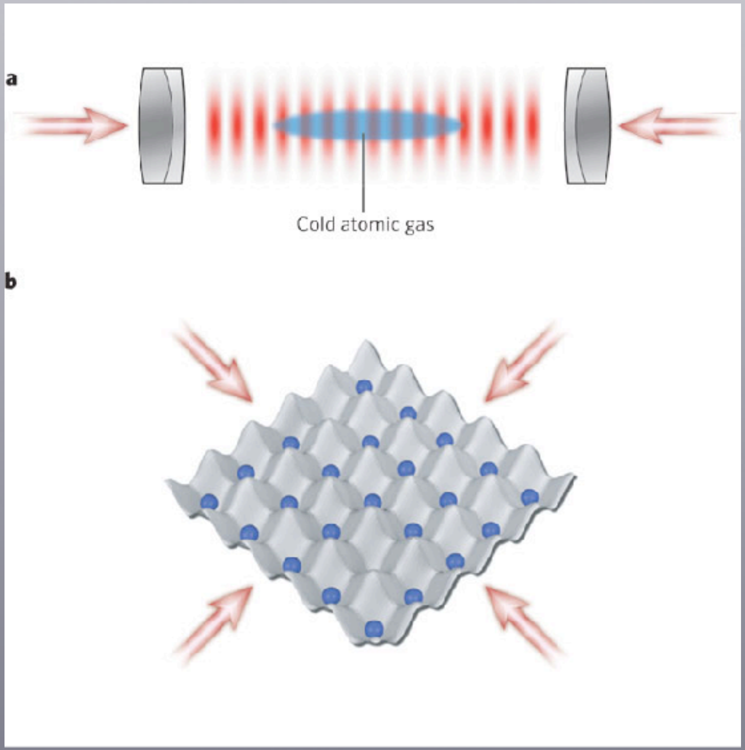
**highly isolated** = little decoherence

**highly tunable** = dimensionality, geometry, interactions.

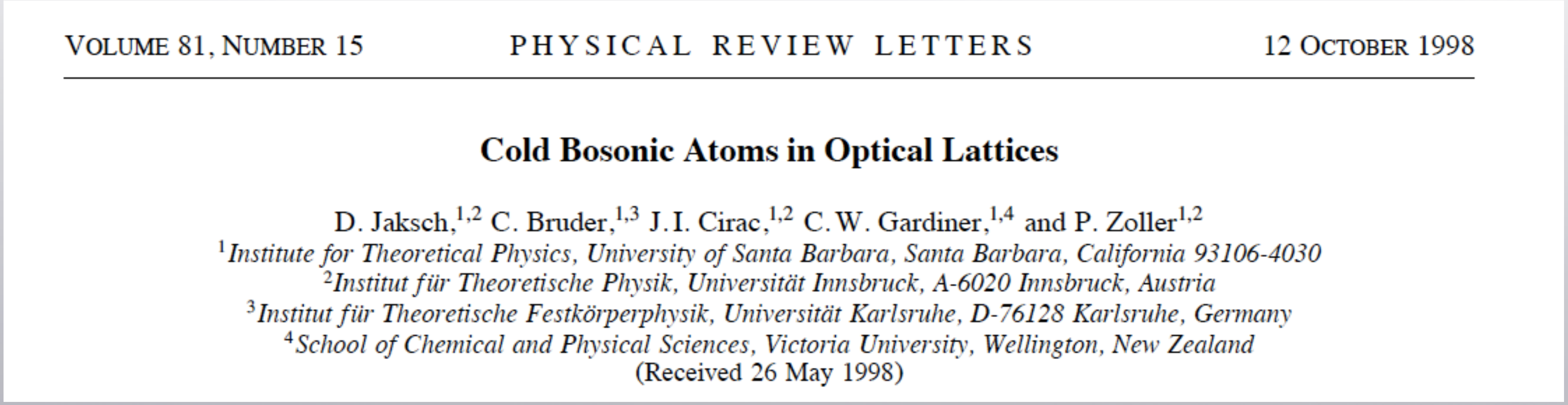
**highly versatile** = equilibrium + non-equilibrium experiments.



**"Many-body physics with ultracold gases"**
I. Bloch, J. Dalibard, and W. Zwerger
Rev. Mod. Phys. **80**, 885 (2008)



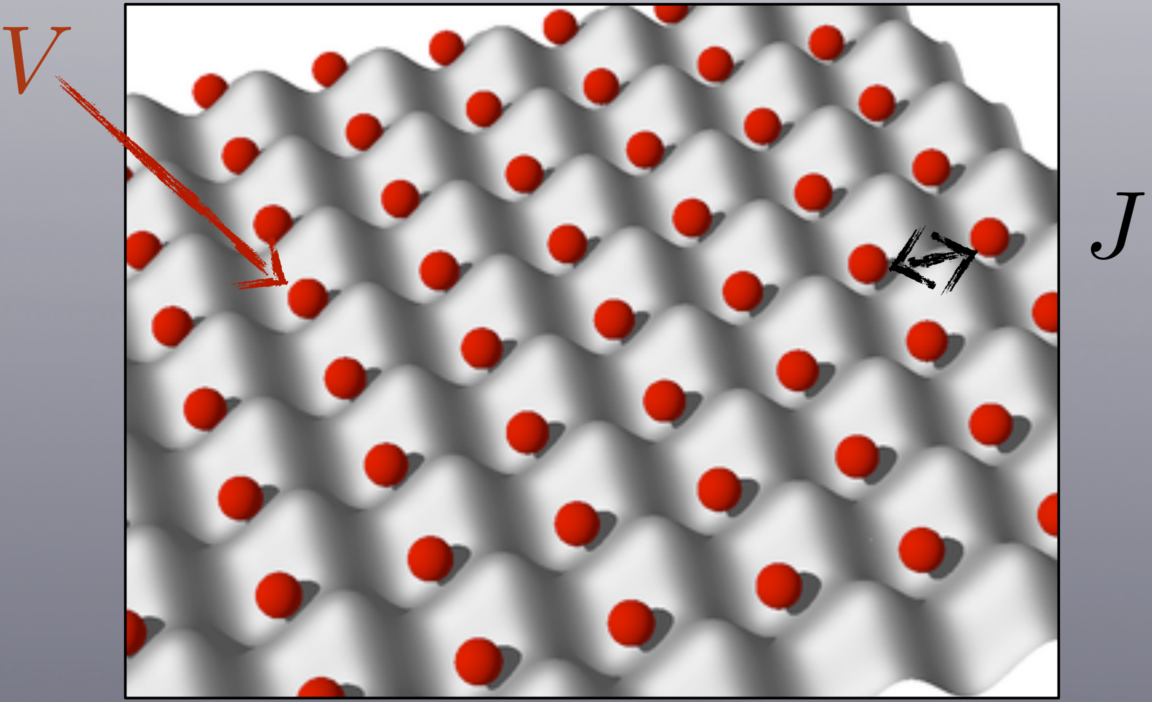
# Quantum Phase Transitions



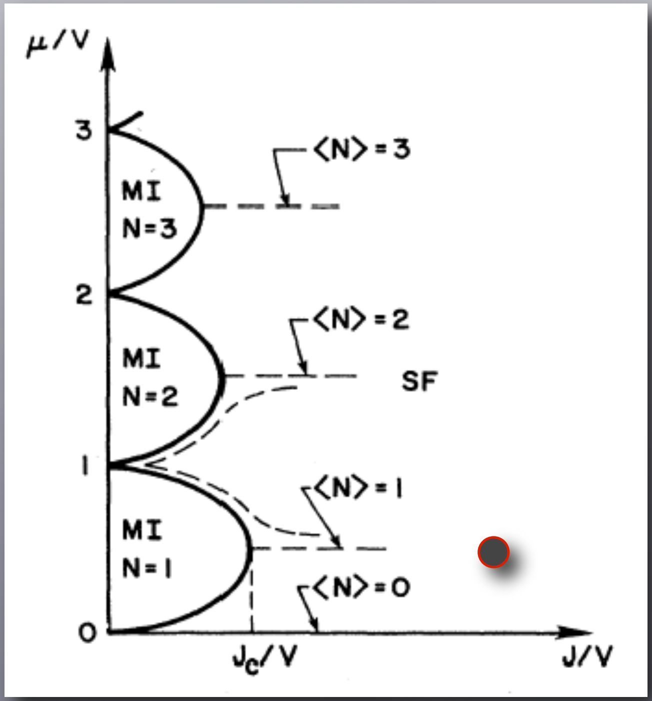
VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

## Cold Bosonic Atoms in Optical Lattices

D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]
[1] *Institute for Theoretical Physics, University of Santa Barbara, Santa Barbara, California 93106-4030*
[2] *Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[3] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
[4] *School of Chemical and Physical Sciences, Victoria University, Wellington, New Zealand*
(Received 26 May 1998)

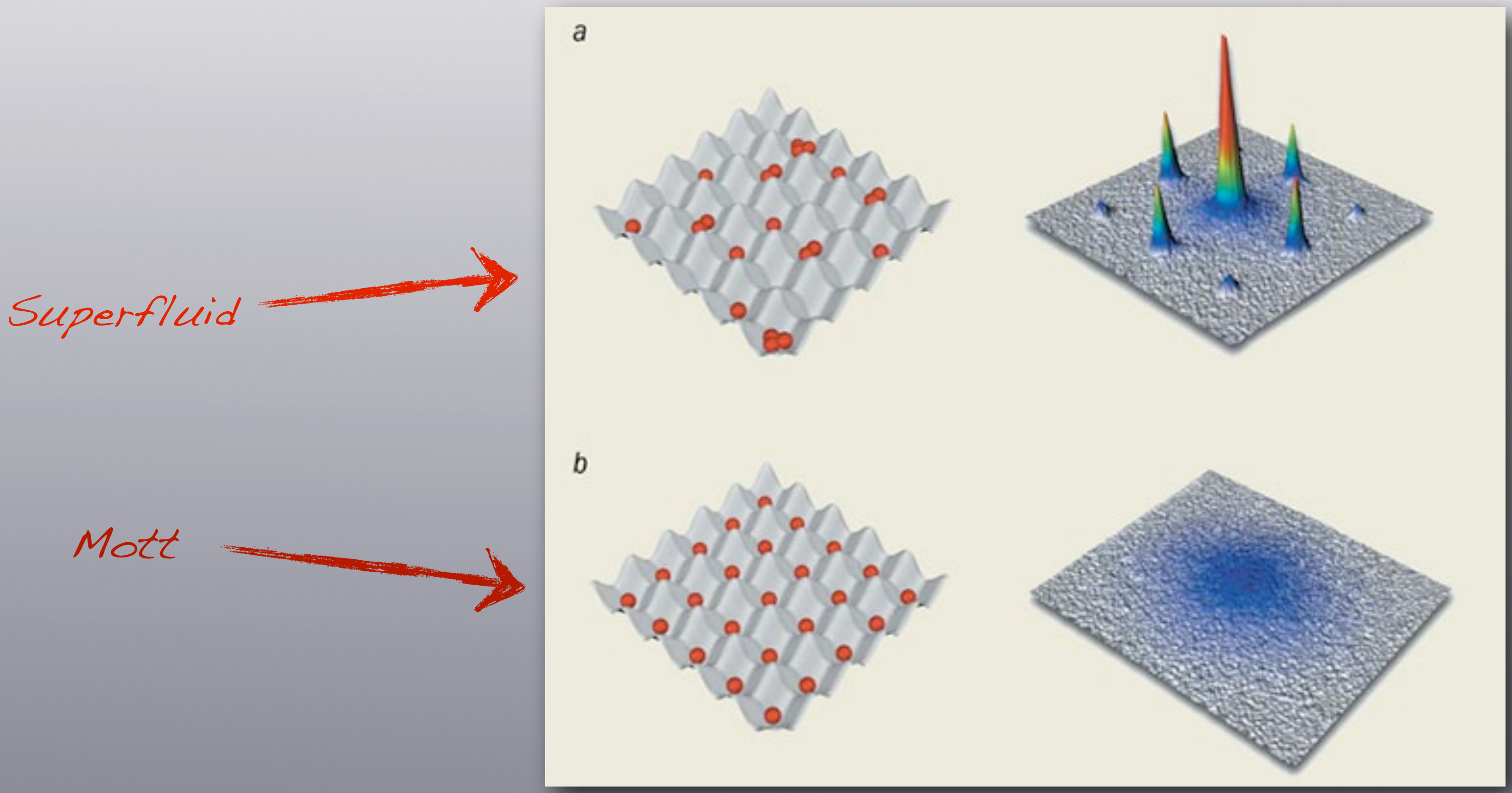


http://www.phys.uu.nl/~stoof



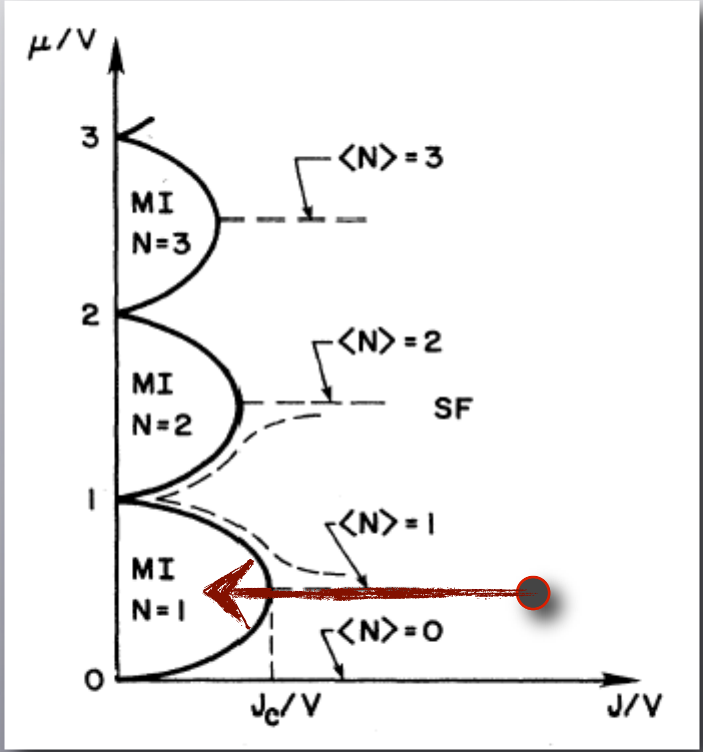
From: Fisher et al, Phys Rev B 40, 546 (1989).

# Experiments



M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, and I. Bloch, Nature 415, 39 (2002)

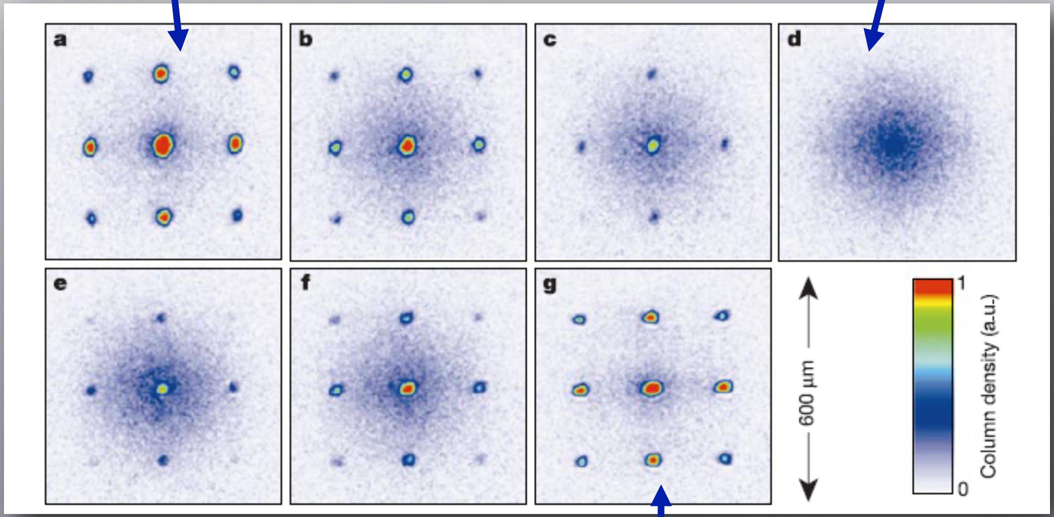
# Out of equilibrium



From: Fisher et al, Phys Rev B 40, 546 (1989).

**Superfluid**

**Mott**



**Superfluid**

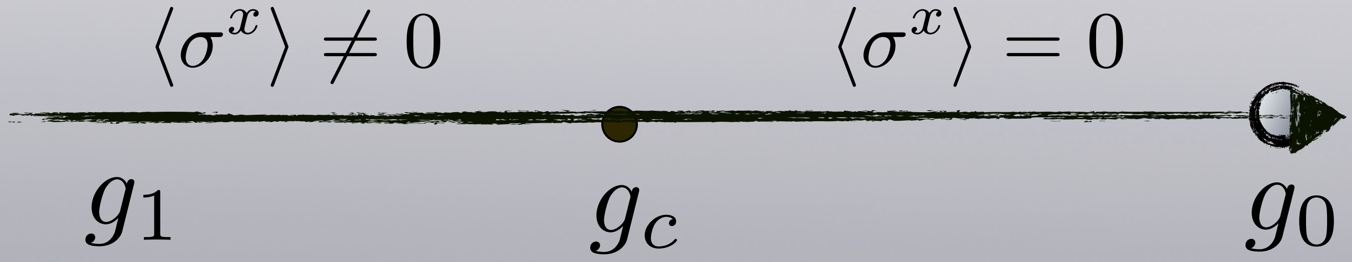
From: Greiner et al, Nature 419, 51 (2002)

High degree of coherence despite of many body interactions

# THEORY

# A paradigm: the quantum quench

**“Time Dependence of Correlation Functions Following a Quantum Quench”**
P. Calabrese and J. Cardy, Phys. Rev. Lett. **96**, 136801 (2006)

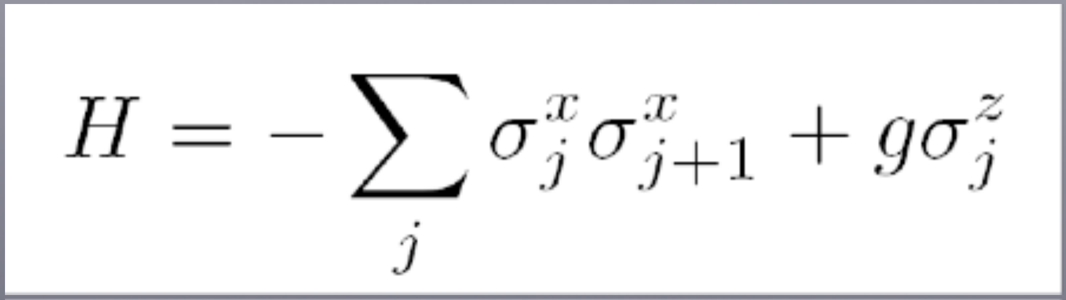


$\langle\sigma^x\rangle \neq 0$ $\qquad$ $\langle\sigma^x\rangle = 0$

$g_1$ $\qquad$ $g_c$ $\qquad$ $g_0$

$$H(g_0) \rightarrow_{\gamma} H(g_1)$$

*Example:*



$$H = -\sum_j \sigma_j^x \sigma_{j+1}^x + g\sigma_j^z$$

# Theory

1) - **Fundamental** description: entropy, work, heat, fluctuations, effective ensembles ?

**Polkovnikov ('08)**
**Silva ('08)**
**Barankov and Polkovnikov ('09)**
**Kehrein ('09-'10)**
.

2) - **Universal** predictions ?

**Igloi and Riegel ('01)**
**Altman and Auebarch ('02)**
**Sengupta, Powell, Sachdev ('04)**
**Polkovnikov ('05)**
**Zurek, Dorner and Zoller ('05)**
**Calabrese and Cardy ('06)**
**Gritsev and Polkovnikov ('07)**
**Patane', Silva, Amico, Fazio, Santoro ('08-'09)**
.

3) - **Generic** connections
(e.g. integrability thermalization) ?

**Rigol et al ('06-'08)**
**Kollath et al. ('07)**
**Cazalilla ('07)**
**Gangardt and Pustilnik ('08)**
**Barthel and Schollwock ('08)**
**Rossini,Mussardo, Santoro, Silva ('09)**
**Fioretto and Mussardo ('10)**
**Canovi, Rossini, fazio, Santoro, Silva ('10)**

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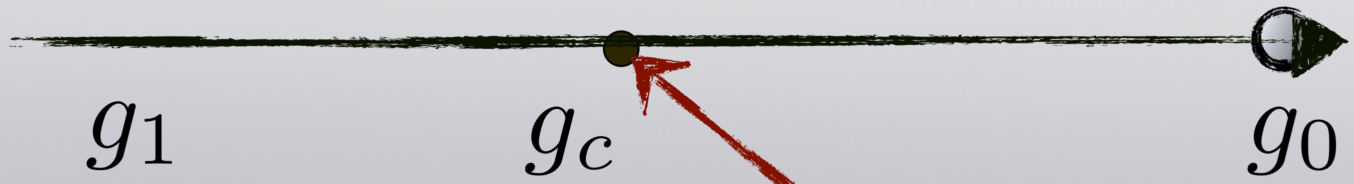
# Fundamental characterizations and universality



$$\Delta \simeq | g - g_c |^{z\nu}$$

$$\xi \simeq | g - g_c |^{-\nu}$$

No matter how slow you are

**EXCITATIONS**

**HOW MANY ??**

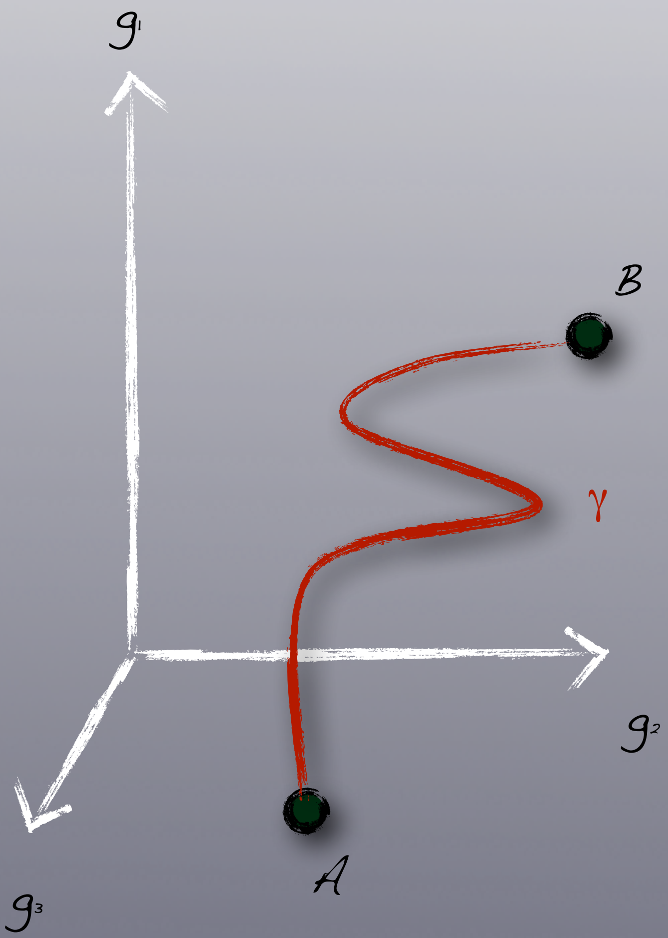
$$n_{ex} \approx | v |^{d\nu/(z\nu+1)}$$

Zurek, Nature **317**, 505 (1985)
Zurek, Dorner, Zoller, Phys. Rev. Lett. **95**, 105701 (2005)
Polkovnikov, Phys. Rev. B **72**, 161201 (2005)

# A statistical characterization

**Think thermodynamics !!!!**

A.Silva, Phys. Rev. Lett. 101, 120603 (2008)



A,B = points in parameters space

$\gamma$ = path

Thermodynamic transformation

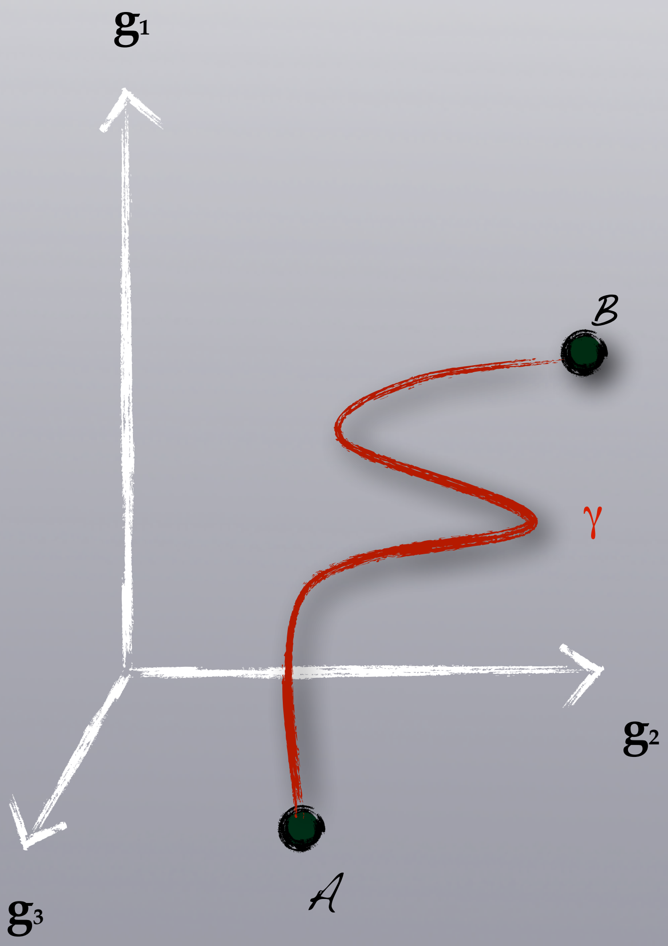
Statistical Physics
3rd Edition Part 1
Landau and Lifshitz
Course of Theoretical Physics
Volume 5
L. D. Landau and E. M. Lifshitz
Translated by J.B. Sykes and M. J. Kearsley, Oxford

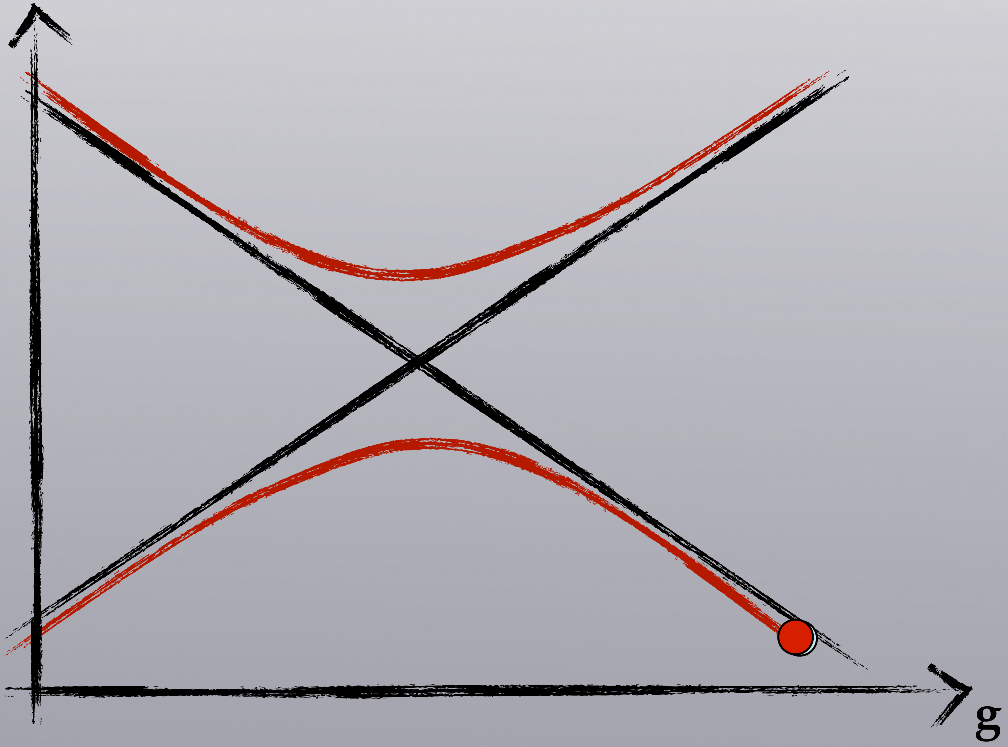
**Work**
**Entropy**
**Heat**

Closed systems

# Nonequilibrium=Statistics

**Quasistatic transformation** $\longrightarrow$ $W = \Delta F$



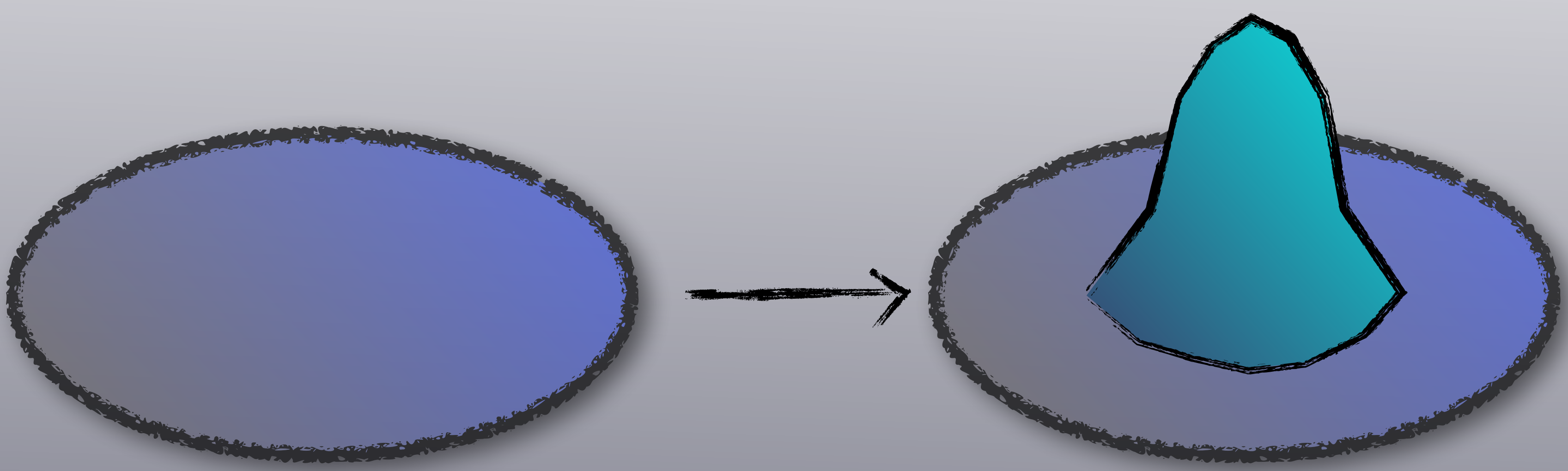


**Out of equilibrium** $P(W)$

Statistics depends on path, time dependence, etc…

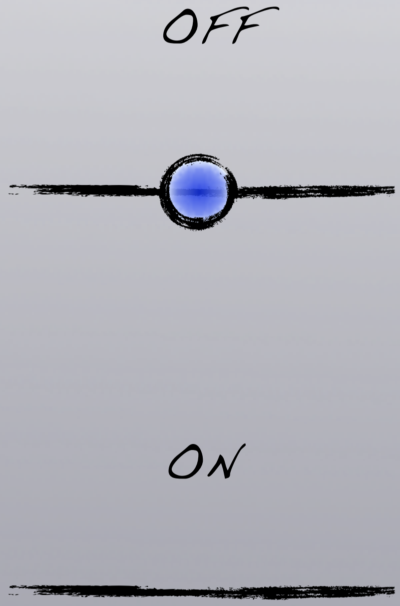
Classical systems: Jarzynski ('97), Crooks ('99)

# The simplest possible quench



# Quantum Switches

Deep level = **Quantum Switch**



Energy of photon = Work needed to turn the switch !!!!

# Absorbtion spectra

**Michele Campisi, Peter Hänggi, Peter Talkner, Rev. Mod. Phys. 83, 771-791 (2011)**

$$P(W) = \sum_n | \langle \varphi_n(g_1) \mid \varphi_0(g_0) \rangle |^2 \, \delta(W - (E_n(g_1) - E_0(g_0)))$$

= absorbtion spectrum of photons

$$G(u) = \left\langle e^{iH_0 u} e^{-iH_1 u} \right\rangle$$

Characteristic function

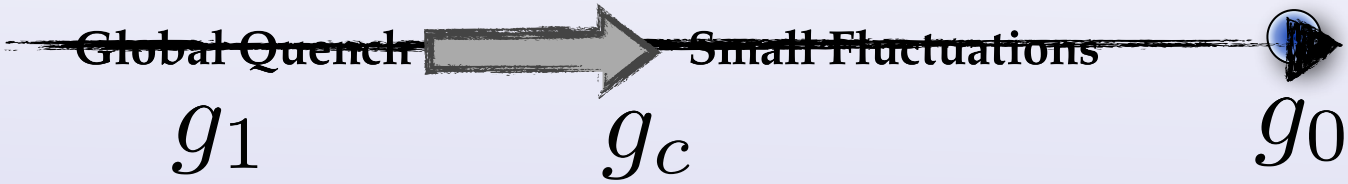
Loschmidt echo

$$G(u) = \left\langle e^{iH_0 u} U^\dagger(\Gamma(t)) e^{-iH_1 u} U(\Gamma(t)) \right\rangle$$

# Global Quenches and Universality

# Global quantum quench

$$H_0 = -\sum_i \sigma_i^x \sigma_{i+1}^x + g\sigma_i^z$$


Global Quench

Small Fluctuations

$P(w)$ **Work** per **unit volume** $w = W/V$

**Fluctuations** $1/\sqrt{V}$

$$G(t) = e^{iE(g_0)t} \langle \Psi(g_0) \mid e^{-iH(g_1)t} \mid \Psi(g_0) \rangle$$

# Loschmidt echo for global quench

A. Gambassi and A.Silva, arxiv ('11)

**System size**

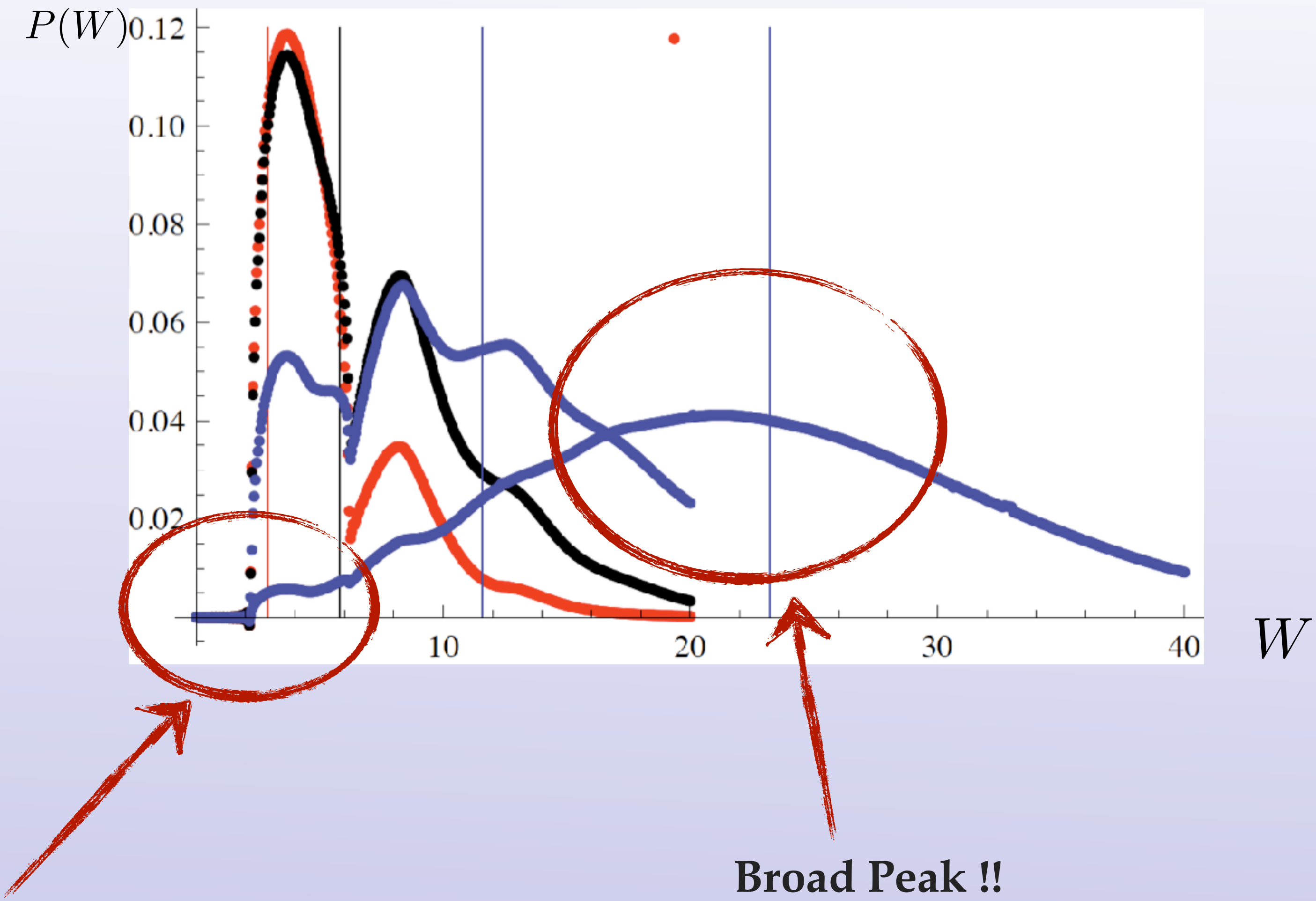
$$\mathcal{G}(t) = e^{-i\delta E t}\, e^{L \int_0^\pi \frac{dk}{2\pi} \log\left[\frac{1+|B(k)|^2 e^{-2iE_k t}}{1+|B(k)|^2}\right]}$$

**Expand and get all cumulants**

**Difference in ground state energies**

$$|\Psi_0\rangle = \frac{1}{\mathcal{N}} \exp\left[-\sum_{k>0} B(k)\gamma_k^\dagger \gamma_{-k}^\dagger\right] |0\rangle$$

# How does it look like ?



$g_i = 101$

$g_f = 2.2$

**Broad Peak !!**

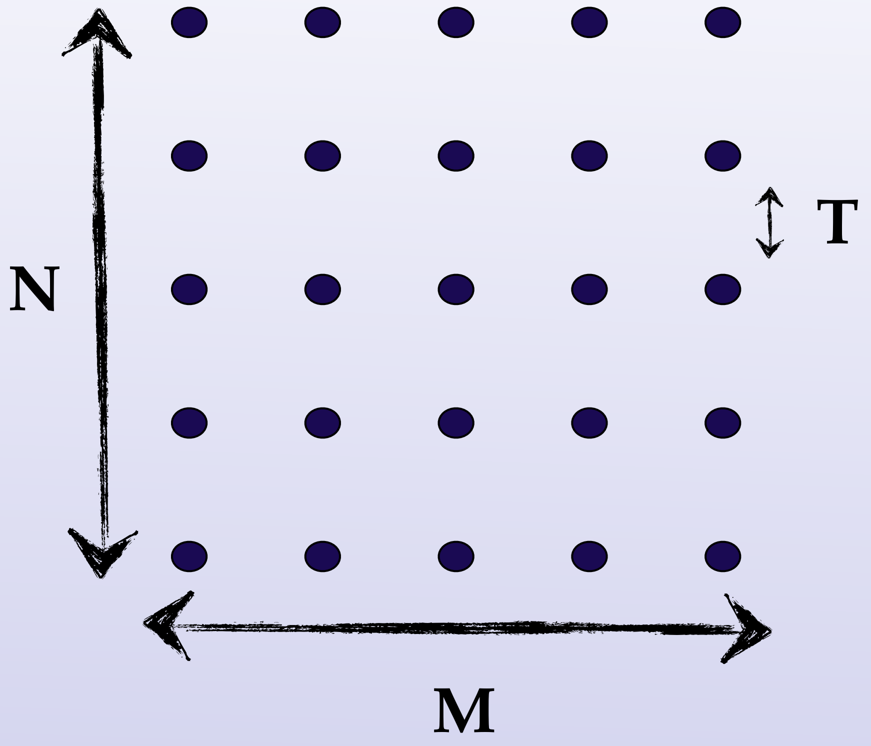
**Edge singularity !!**

$L = 50, 100, 200, 400$

# Low W part and universality

$$P(w) \simeq \mathcal{F}^2 \delta(w) + \mathcal{C}\, \Theta(w - qm)\, (w - qm)^{\alpha} + \ldots$$

# Quantum to classical correspondence ...



Periodic Boundary Conditions

$$Z = \mathrm{Tr}\left[T^N\right]$$

Other Boundary Conditions

$$Z = \langle \Psi_0 \mid T^N \mid \Psi_0 \rangle$$

QUANTUM to Classical correspondence

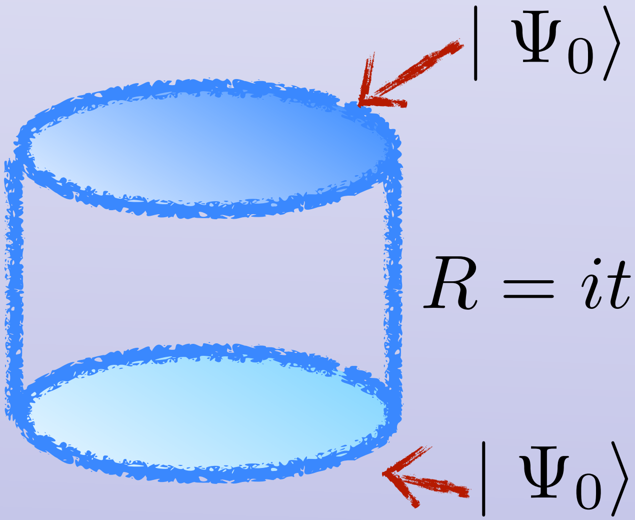
$$T = e^{-H}$$

# .... at work !!

QUANTUM ISING MODEL

$$G(t) = \langle e^{iH_0 t} e^{-iH_1 t} \rangle$$

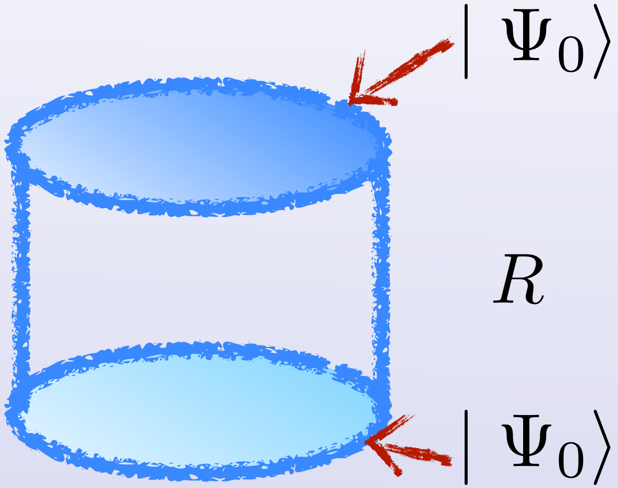
$$= e^{iE_0 t} \langle \Psi_0 \mid e^{-iH_1 t} \mid \Psi_0 \rangle$$



Partition function of a 2 dimensional Ising model with boundaries

# Statistics of the work and boundary stat. mech.

A. Silva and A. Gambassi, arxiv (2011)



$$\langle \Psi_0 | (e^{-H})^R | \Psi_0 \rangle$$

Surface

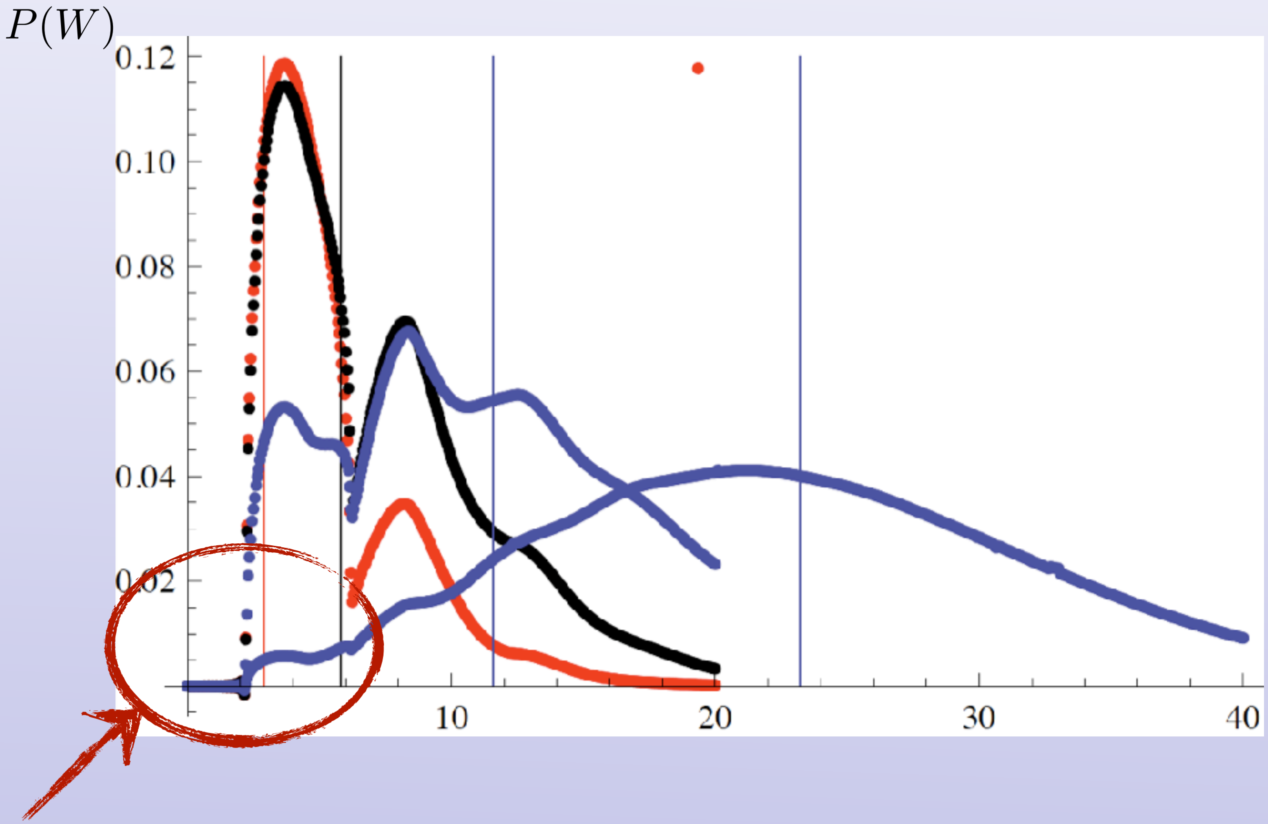
Casimir

$$G(R) = e^{-(L\ R\ f_b + L\ f_s + L\ f_c(R))}$$

Bulk

# The threshold ....

$$G(t) = e^{-L\times(it)\times f_b} \times \;....$$

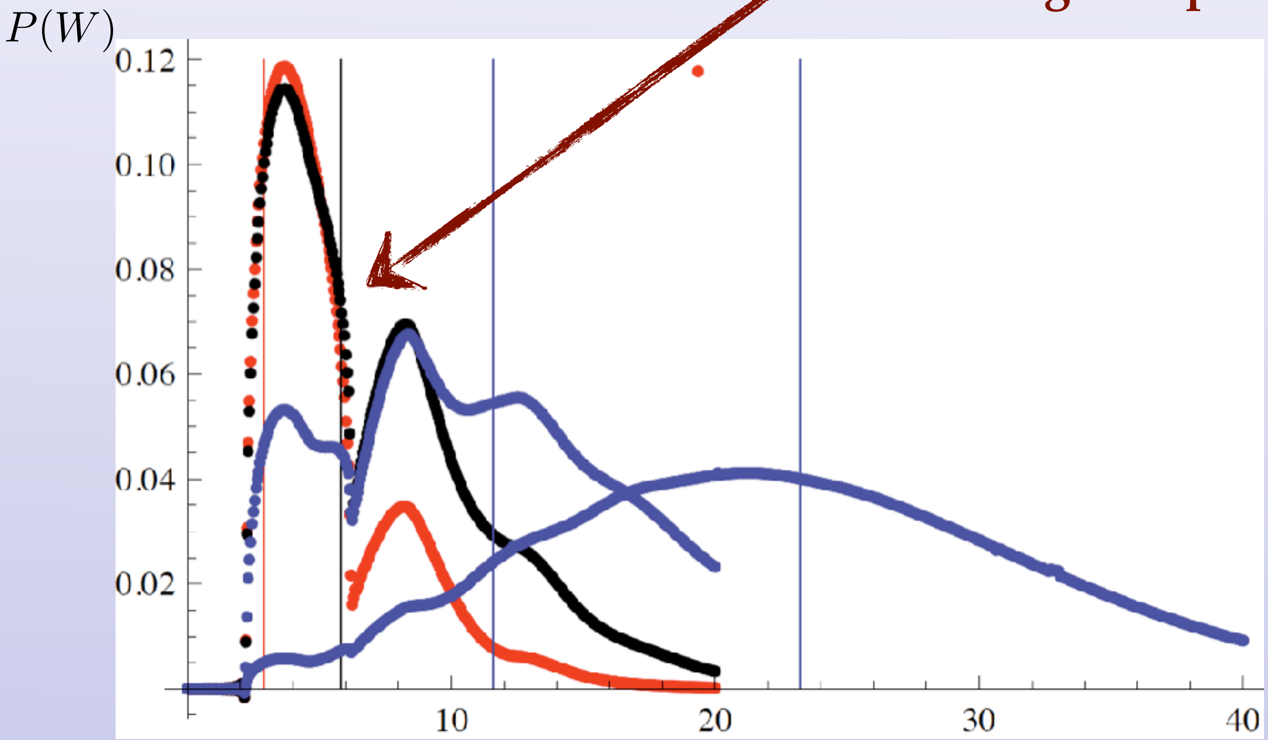


$$\delta E = L \times f_b$$

# The overall prefactor ....

$$G(t) = e^{-L\times(it)\times f_b} \times e^{-L\times f_s} \times \ldots$$

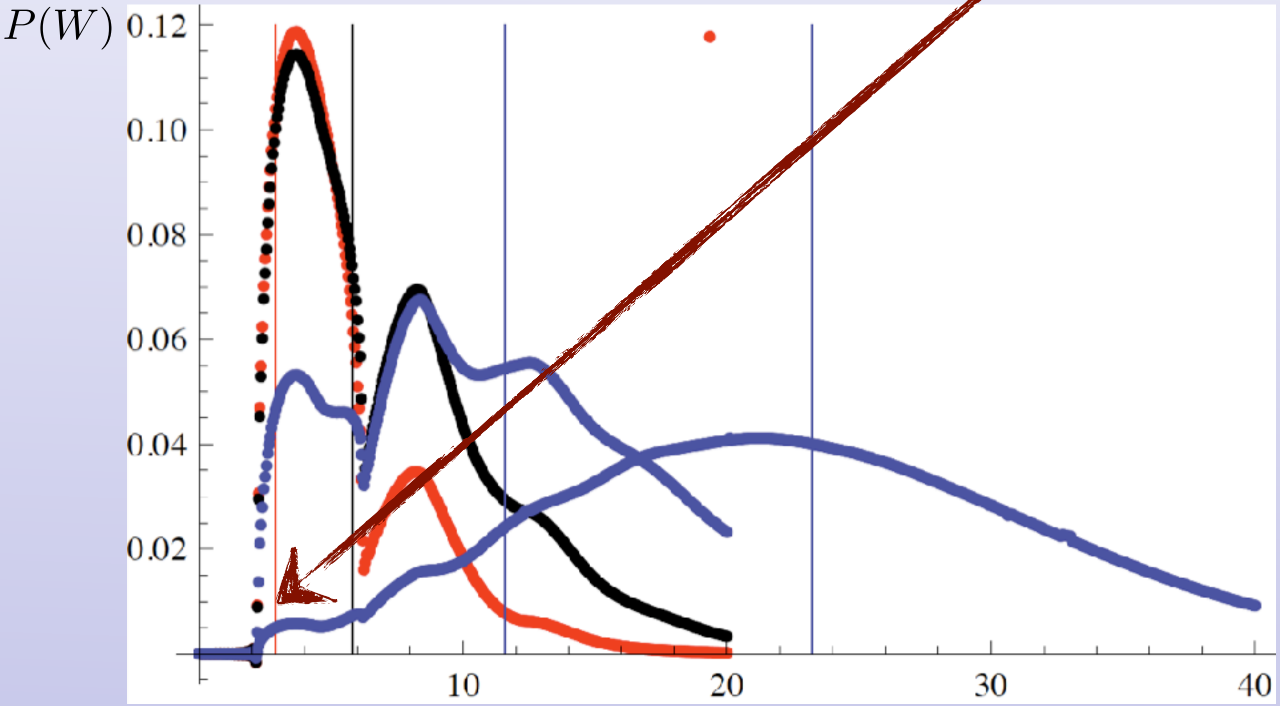
**Strenght of peaks ....**



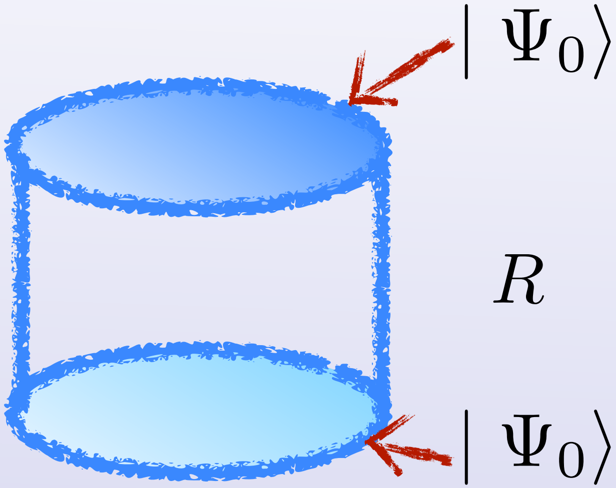
# .... and the singularity ....

$$G(t) = e^{-L\times(it)\times f_b} \times e^{-L\times f_s} \times e^{-L f_c(R)}$$

**Power law at threshold ...**



# .... and the singularity ....



$$f_c(R) = R^{-d} F(R/\xi_+)$$

$$F(x) = \frac{\mathcal{C}}{x^a}\, \mathrm{e}^{-\mathrm{qx}}$$

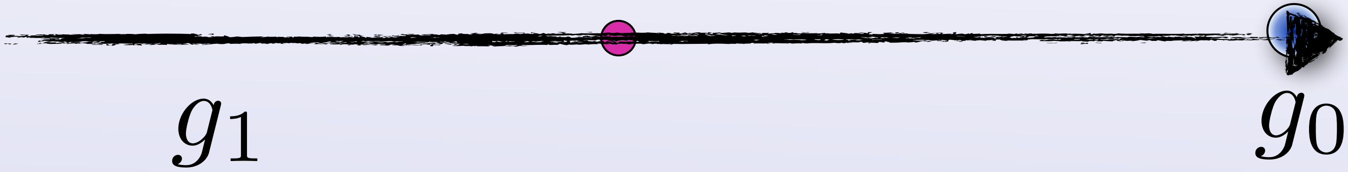
**Analytically continue ...**

$$P(w) \simeq \mathcal{F}^2 \delta(w) + \mathcal{C}\; \Theta(w - qm)\; (w - qm)^{\alpha} + \ldots$$

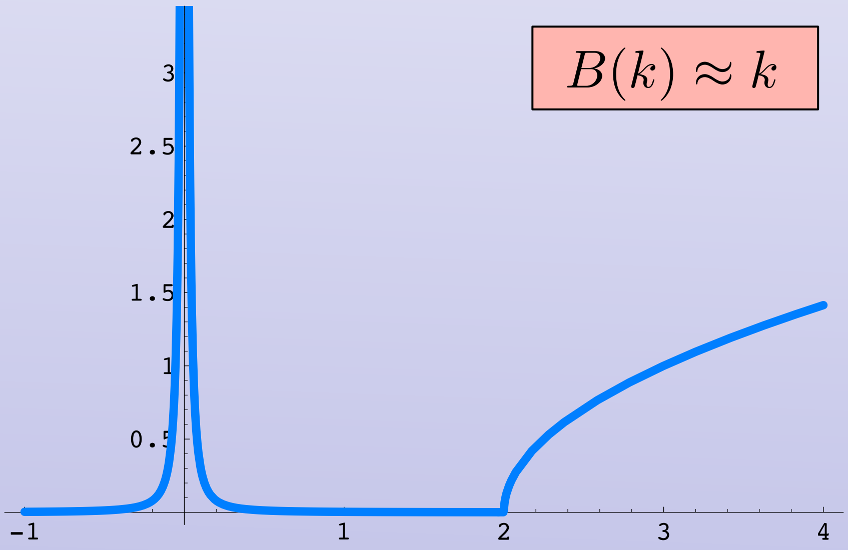
$$\alpha = d + a - 1$$

# E.g.: quantum Ising chain .....

$$H_0 = -\sum_i \sigma_i^x \sigma_{i+1}^x + g\sigma_i^z$$

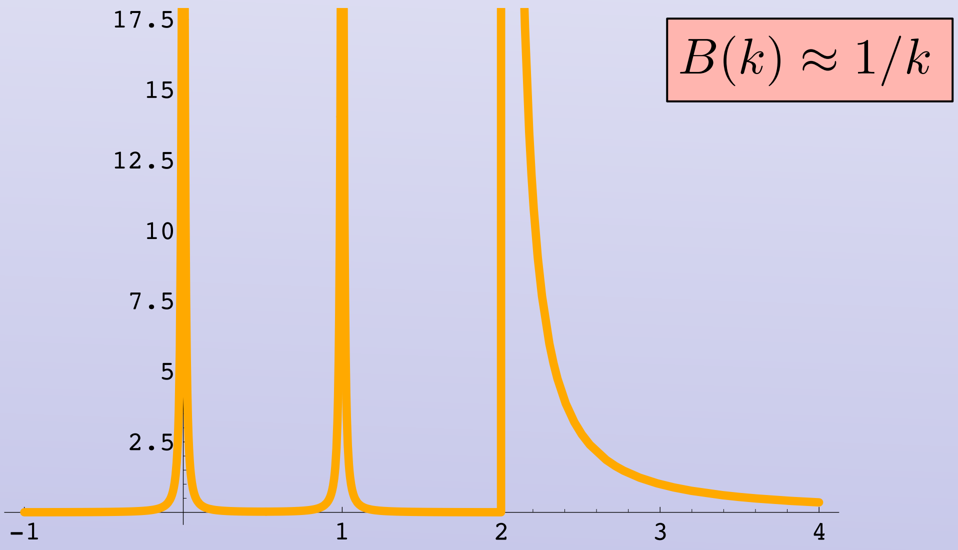


**Within same phase**



$B(k) \approx k$

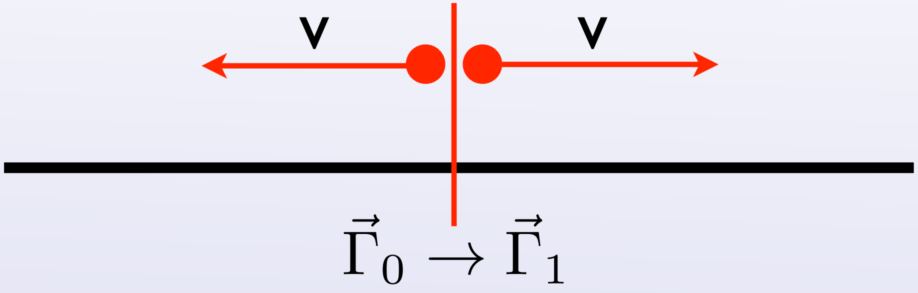
**Across critical point**



$B(k) \approx 1/k$

# Local Quenches and Generic Protocols

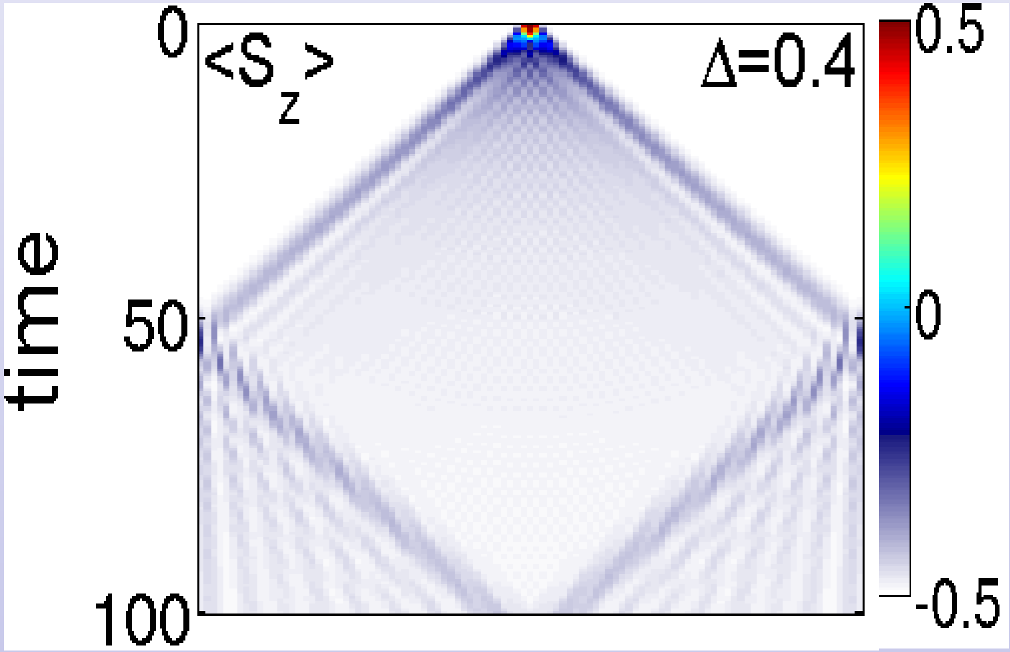
# Local Protocols



quasi-particle emission

$\Downarrow$

“light-cone” effect



Ganahl, Rabel, Essler, Evertz, 2012

Measuring entanglement entropies ...

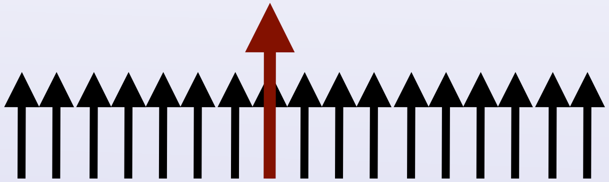
J. Cardy, PRL.106, 150404 (2011).
I. Klich and L. Levitov, PRL. 102, 100502 (2009).
D. Abanin, E. Demler, arxiv (2012)

Spectrum of Excitations ?

Dependence on Protocol ?

# Local Quenches

$$H_0 = H(g) \longrightarrow H_0 + V$$



$$V = -\delta g \sigma^z(0)$$

$$\xrightarrow[\text{Limit}]{\text{Scaling}} \quad \boxed{H_t = -\frac{i}{2}\int dx\,[\varphi\partial_x\varphi - \bar{\varphi}\partial_x\bar{\varphi}] + im(t)\bar{\varphi}\varphi_{|_{x=0}}}$$

Majorana fermion

Pietro Smacchia and A.S. ,(2012)

# Magnetization

ultraviolet cutoff

$$\langle \mathcal{M}(x,t)\rangle = -\frac{2|x|}{\pi(4x^2+\alpha^2)}\sin\left(m(t-|x|)\right)$$

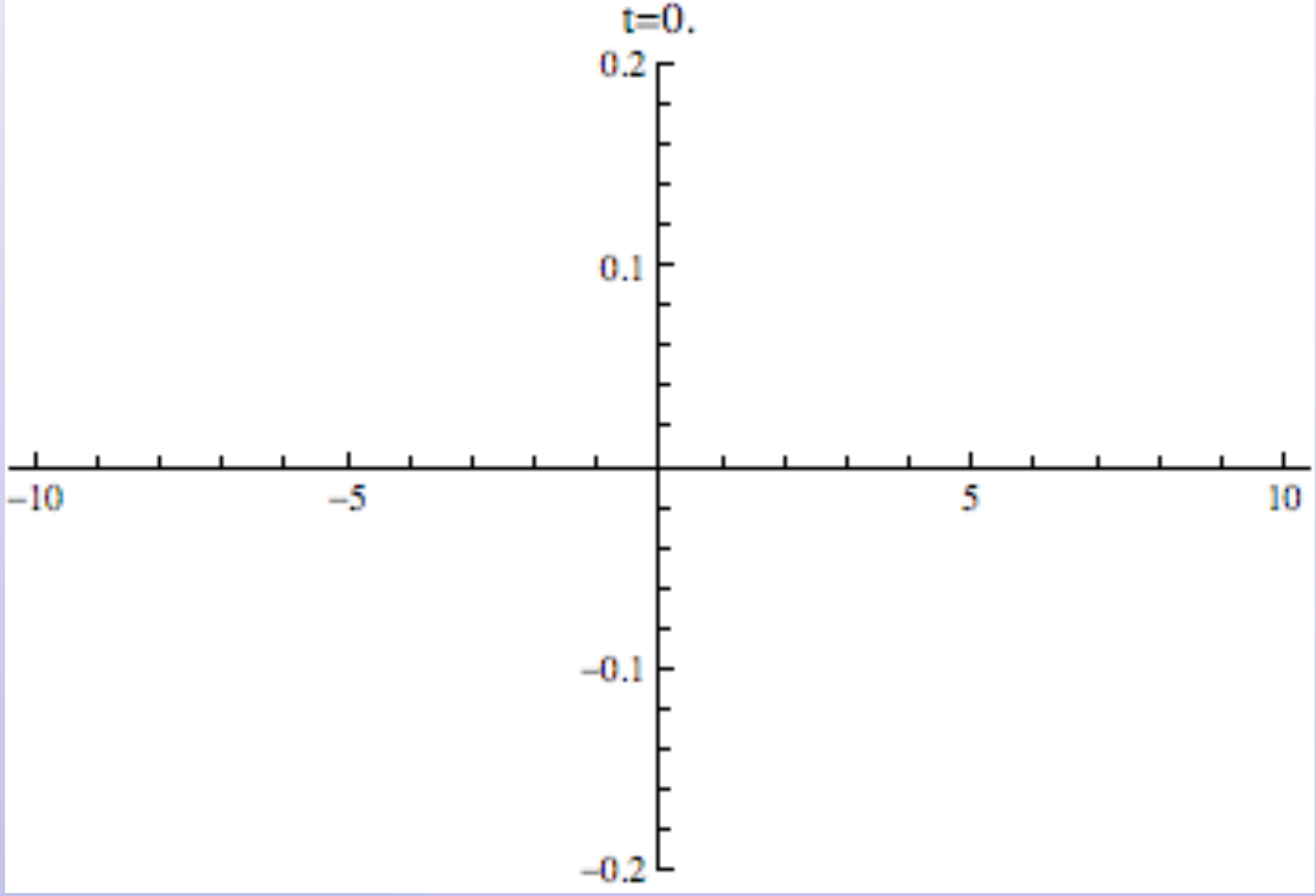
symmetric "emission" of a signal from the origin

magnetization goes down $\sim 1/x$

Ex:

$$m(t) = \theta(t)\,t$$

$$\alpha = 1$$



# Correlations

Correlation between opposite point

$$\mathcal{C}(x,t) = \frac{1}{2\pi^2(4x^2+\alpha^2)}\left(\cos\left(2m(t-|x|)\right)-1\right)$$

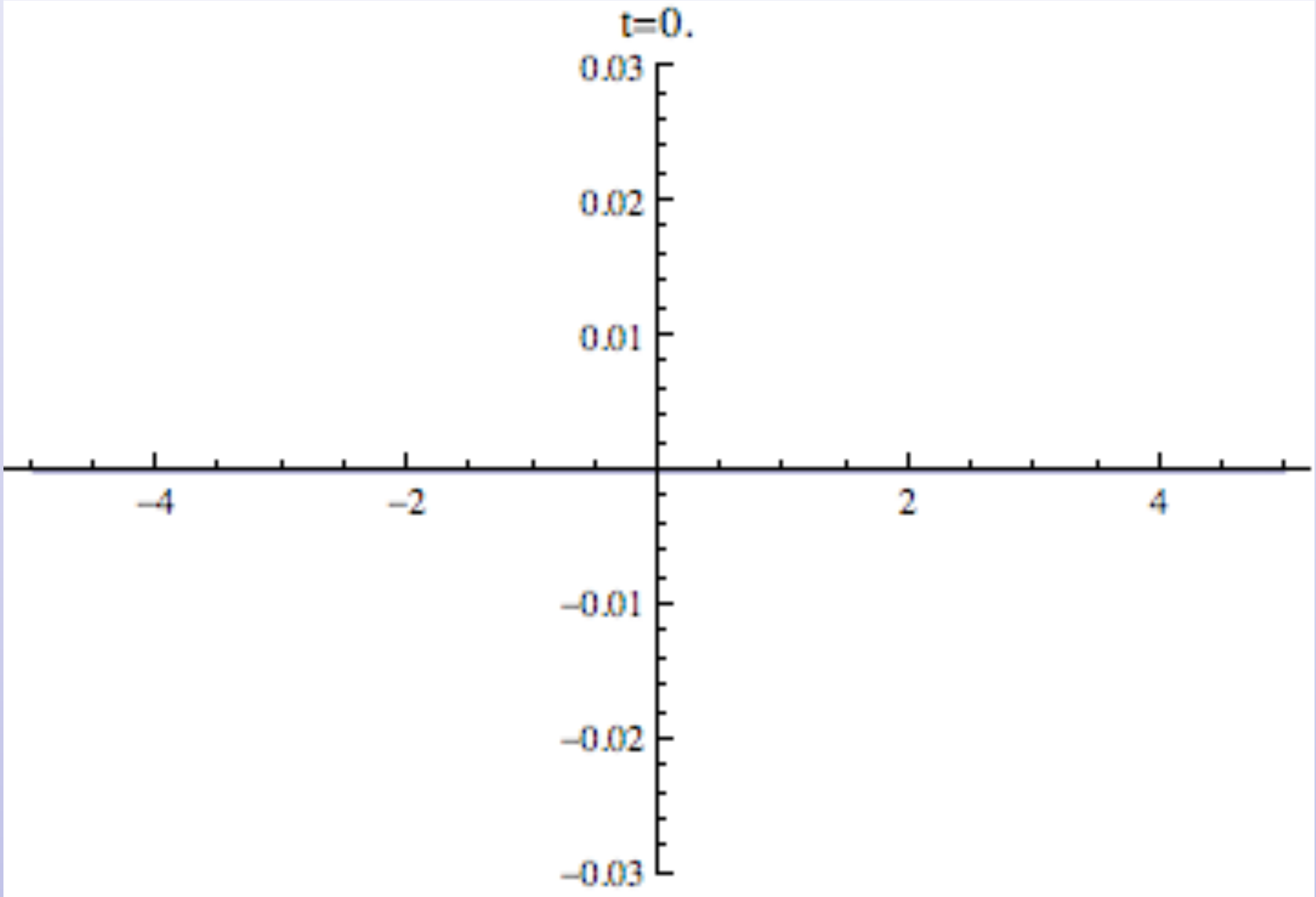
excess correlation

down $\sim 1/x^2$

Ex:

$$m(t) = \theta(t)\,t$$

$$\alpha = 1$$



# Edge Singularity

$$\mathcal{G}_i(u) = \exp\left[\frac{1}{4\pi^2}\int_{-\infty}^{T} dt \int_{-\infty}^{T} dt' \partial_t m(t) \partial_{t'} m(t') \log \frac{\alpha - i(t-t')}{\alpha - i(t-t'+u)}\right]$$

$m(T) \neq 0$

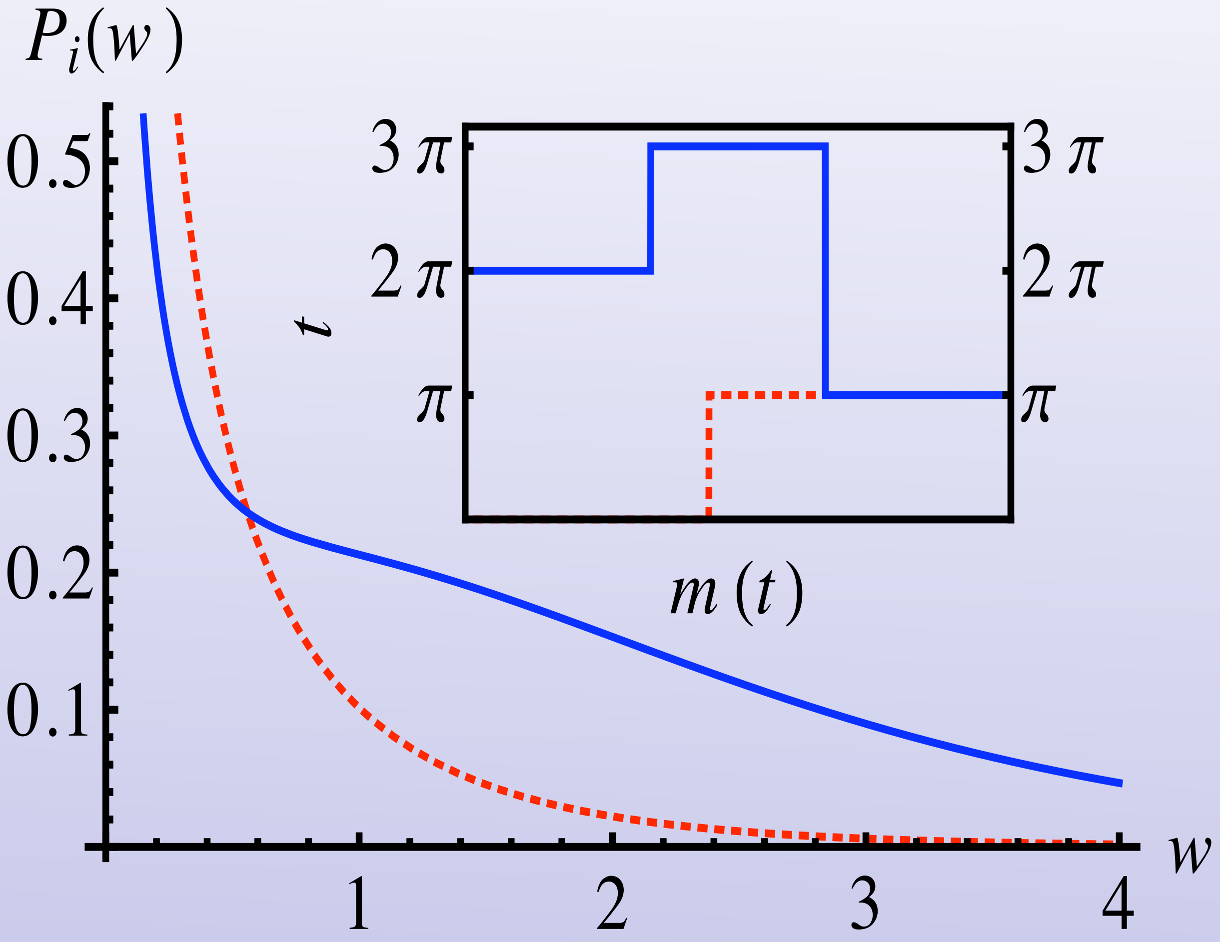
Expanding for large u:

$$\mathcal{G}_i(u) \sim (-iu)^{-\frac{m(T)^2}{4\pi^2}} \Rightarrow \boxed{P(w) \overset{w\to 0}{\sim} w^{\frac{m(T)^2}{4\pi^2}-1}}$$

Edge singularity

Exponent dependent only on the final value of m! Not on the path

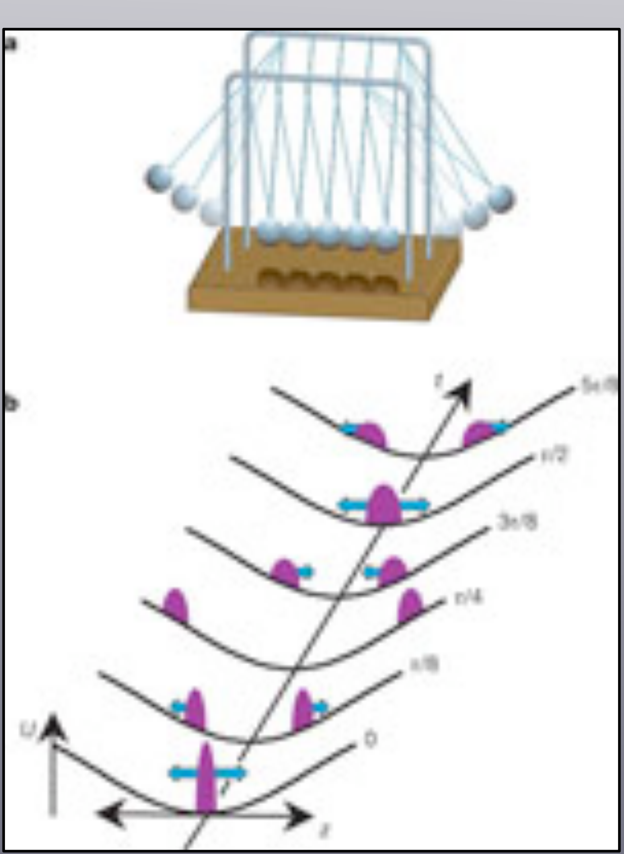
# Example



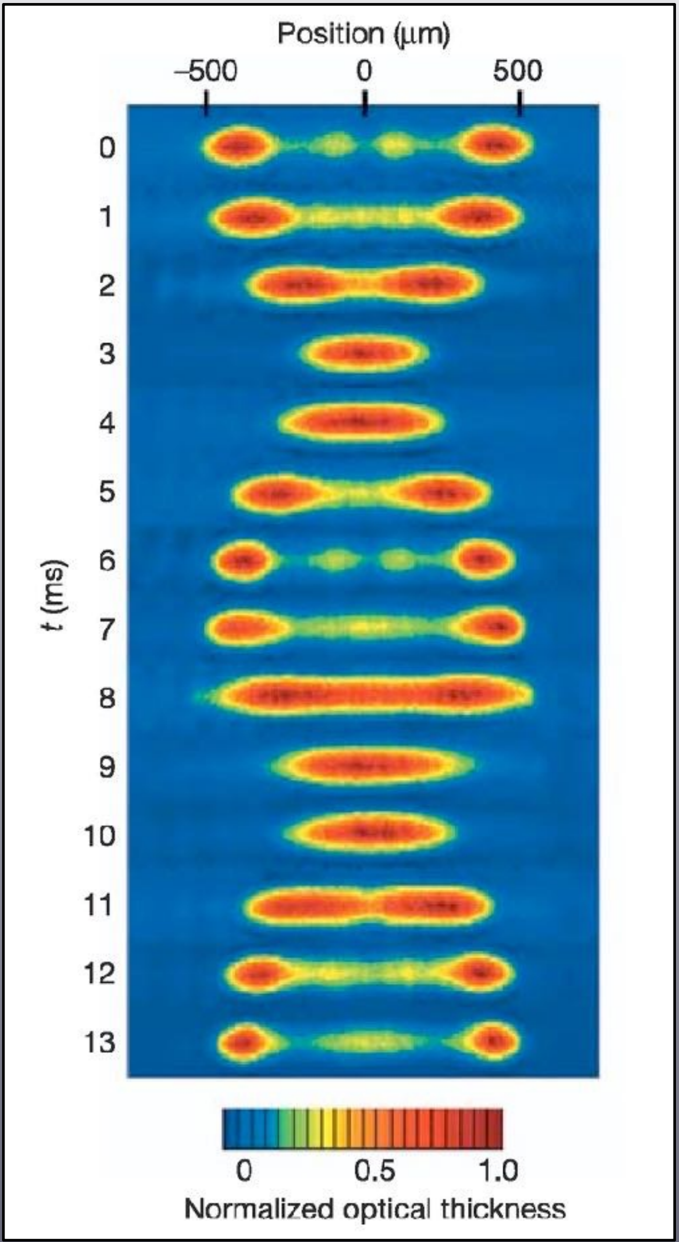
# Thermalization ....

# Generic features

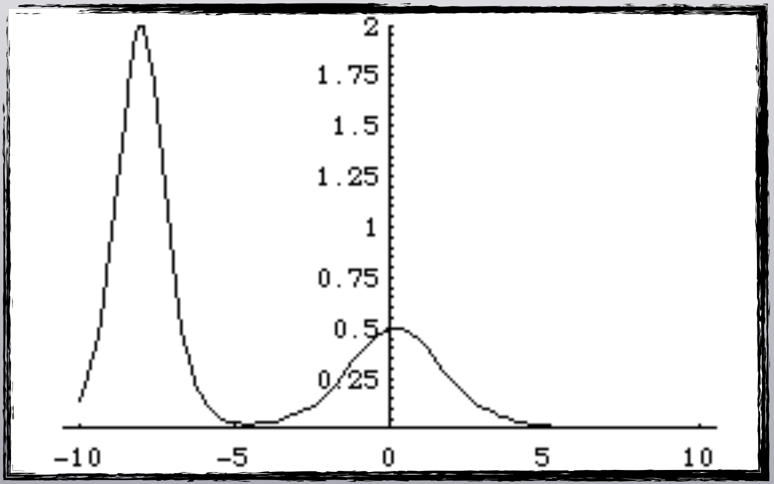
✓ Lack of thermalization in 1D condensates

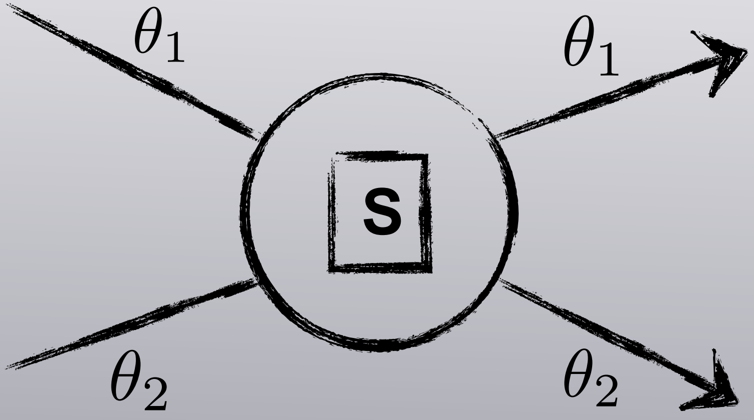


Kinoshita et al, Nature ***440**, 900 (2006)*



# Integrable Models





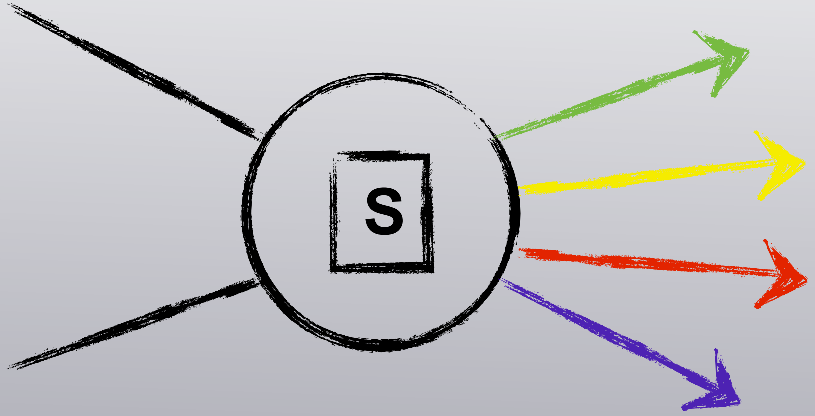
Thermalization should **not** occur:
steady states remembers the
initial conditions (as in classical physics)

**Rigol, Dunjko, Yurovsky, & Olshanii, *PRL (2007)***
Rigol, Muramatsu, & Olshanii, *PRA (2006)*
Cazalilla, *PRL (2006)*
Calabrese & Cardy, *PRL (2006), JSTAT (2007)*
**Gangardt & Pustilnik, PRA (2008)**
Eckstein *& Kollar, PRL (2008), PRA (2008)*
Iucci & Cazalilla, *arXiv (2009)*
Barther, Schollwoeck, *PRL (2008)*

$$\rho_G = \frac{e^{-\sum_\theta \beta(\theta)\, n(\theta)}}{Z}$$

# Breaking Integrability

.... ergodicity ?



Gas of N particles in a box .... eigenstates = *pseudo-random superpositions of plane waves* (Berry's conjecture) .... (Srednicki '94)

$$\int d\mathbf{p}_2 d\mathbf{p}_3 \ldots \langle | \Psi_\alpha(\mathbf{p}, \mathbf{p_2}, \ldots) | \rangle^2 = \frac{e^{-\frac{\mathbf{p}^2}{2mkT}}}{(2\pi mkT)^{3/2}}$$

**Eigenstate thermalization hypothesis**

**Deutsch, *PRA (1991)***
**Srednicki, *PRE (1994)***
Rigol, Dunjko, & Olshanii, *Nature (2008)*
Kollath, Lauchli & Altman, *PRL (2007)*
Manmana, Wessel, Noack, & Muramatsu, *PRL (2007)*
Rigol *PRL (2009), PRA (2009)*
Biroli et al. *arXiv 0907.3731*
.....

# Thermalization and Pre-Thermalization

Wetterich and Berges, 2004, Kehrein 2010, Kitagawa et al 2011

A. **Prethermalization** (some observables look thermal, qp distribution is not thermal)

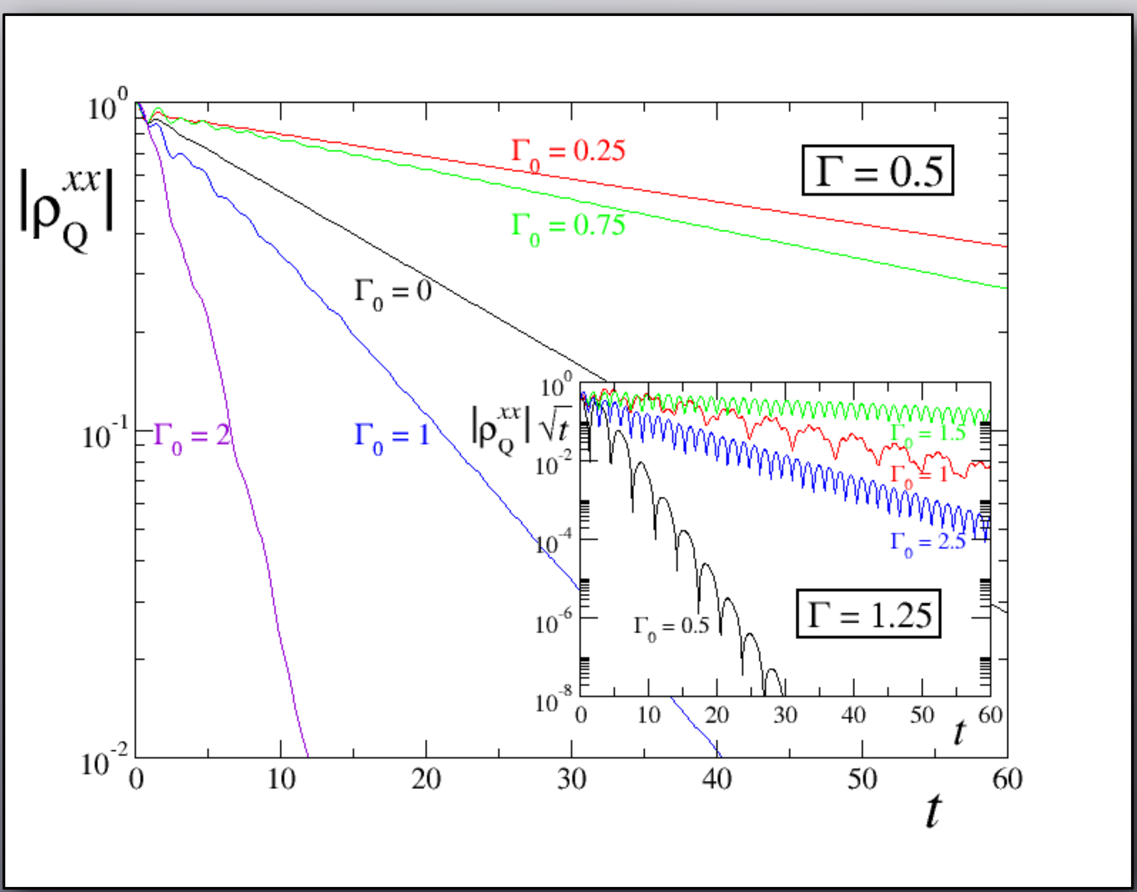
Prethermalized state of weakly non-integrable QFT = GGE.

B. **Thermalization** sets in at later times (qp distribution becomes thermal as well).

# Is that true ?

**Not really**: things are more **complex** .... and **interesting**

D. Rossini, A. Silva, G. Mussardo and G. Santoro, PRL (2009)
P. Calabrese, F. Essler, and M. Fagotti, PRL (2011)



Always exponential!

Integrable !!

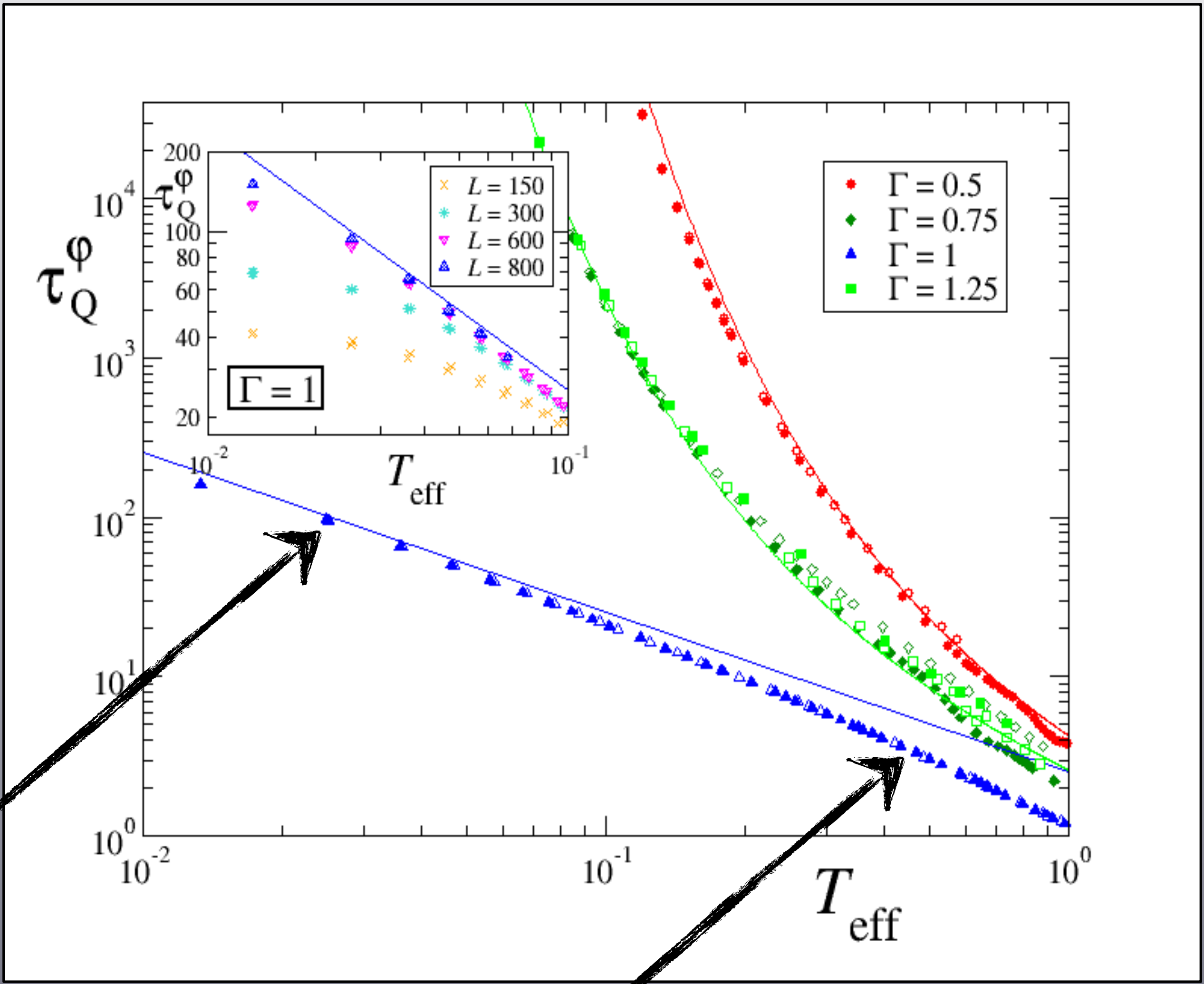
$$\rho_Q^{xx} \sim e^{-t/\tau_Q^{\varphi}}$$

**like in equilibrium with T>0...**

$$\rho_T^{xx} \sim e^{-t/\tau_T^{\varphi}}$$

Should not thermalize

# Effective temperature ...



**Effective T**

**Generalized Gibbs**

# What is going on ???

Quantum Quasiparticles are **NOT** simple objects

↓

## Thermalization and Localization

**Pal and Huse ('10), Rigol Santos ('10),**
**Canovi et al. ('10), Neuenhahn Marquardt ('10)**

[redacted]

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