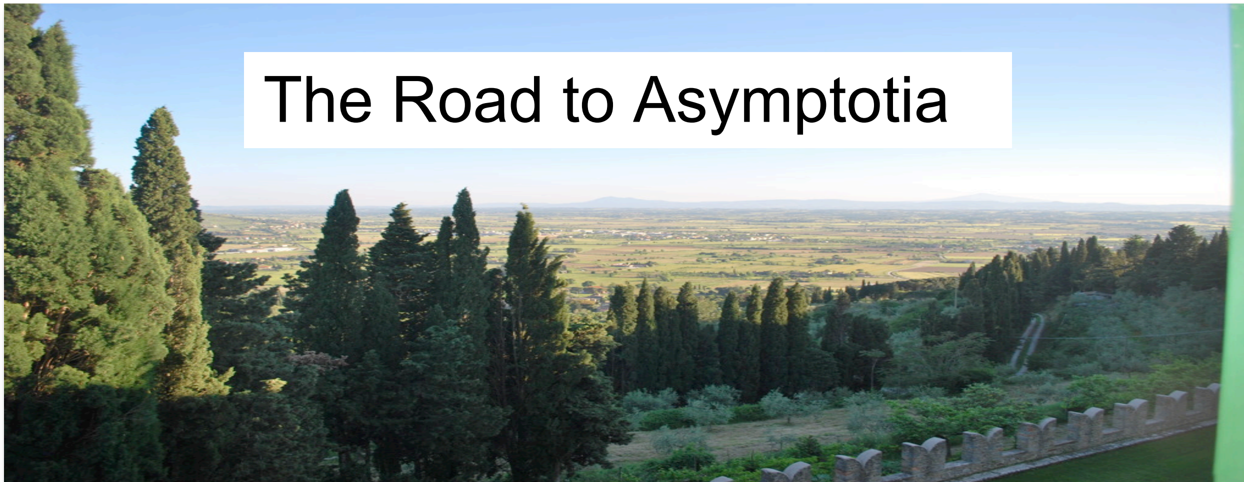
The Road to Asymptotia



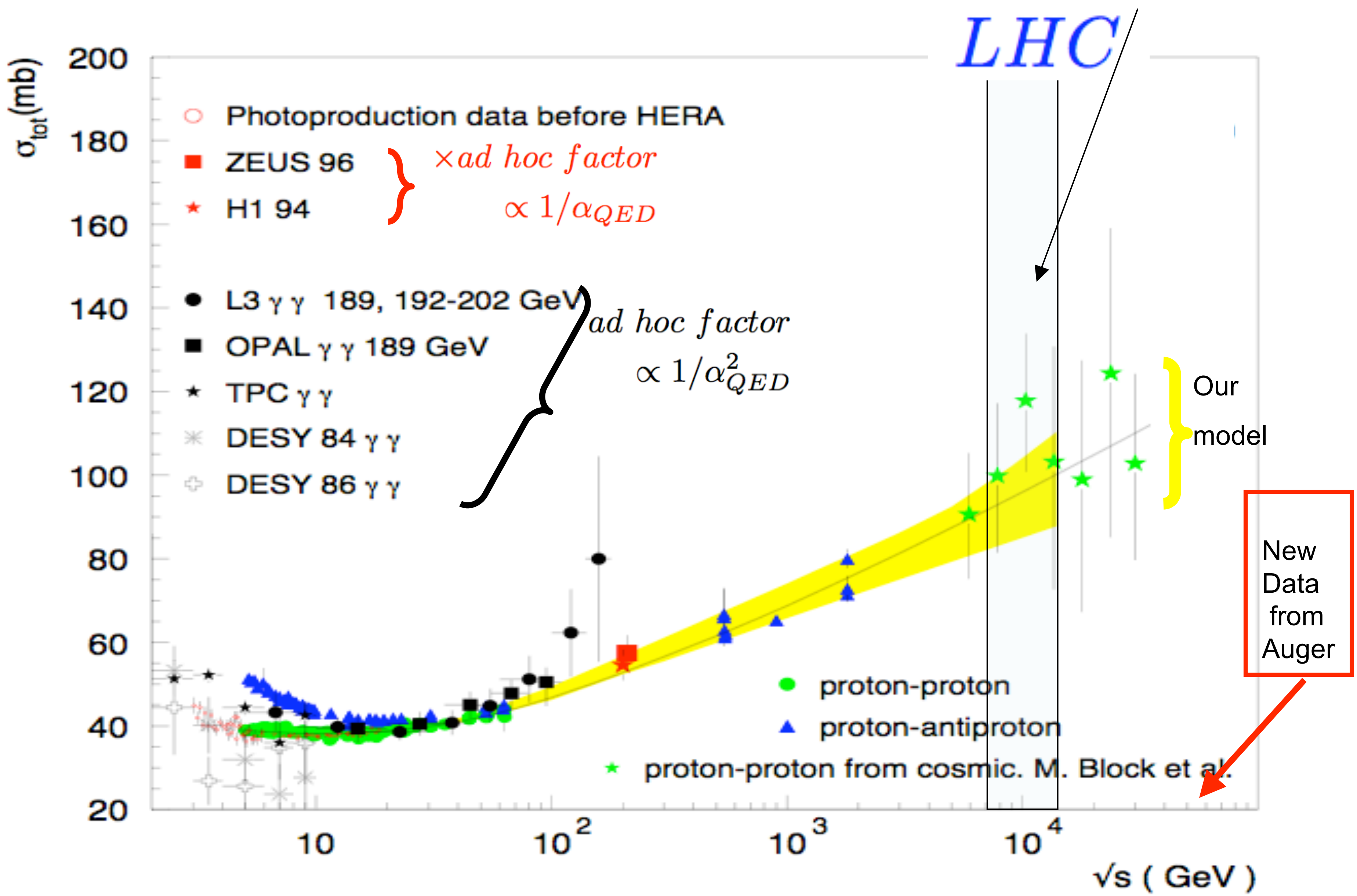
With the total cross-section: Saturation of the Froissart bound and other checks

Giulia Pancheri-INFN Frascati
with

With A. Grau, S. Pacetti and Y.N. Srivastava

Cortona 2012

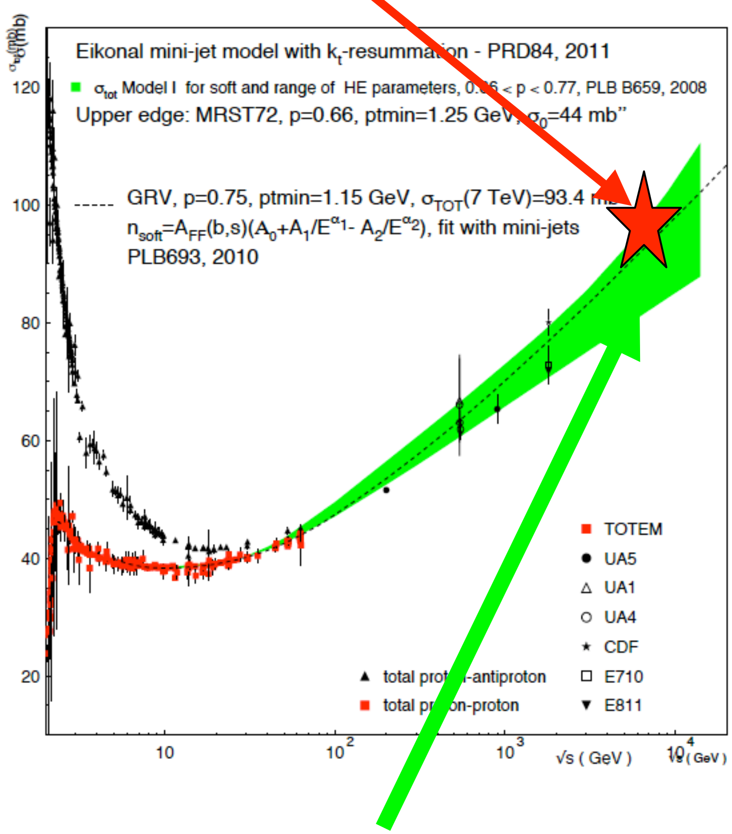
Total cross section data before 2011



TOTEM measurements in 2011

$\sigma_{total} = 98.3 \pm 3mb$

$\frac{d\sigma_{el}}{dt}$



Achilli, GP, et al, PRD84(2011)

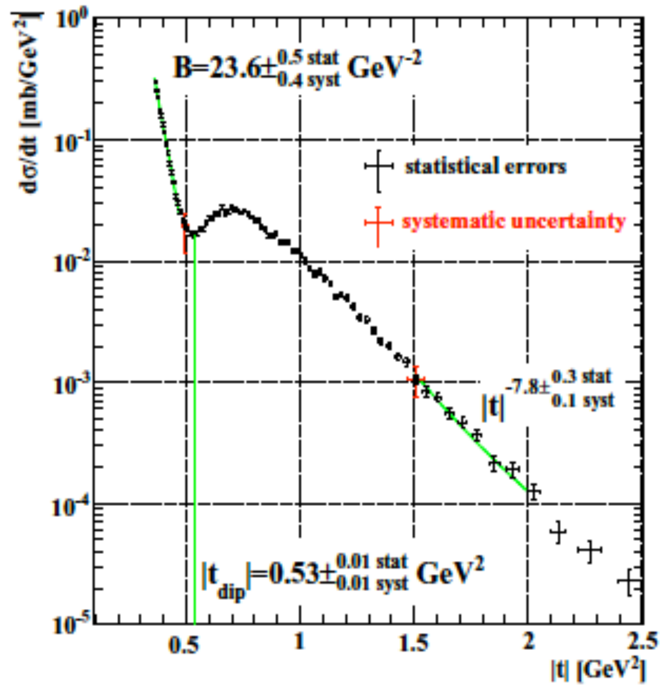
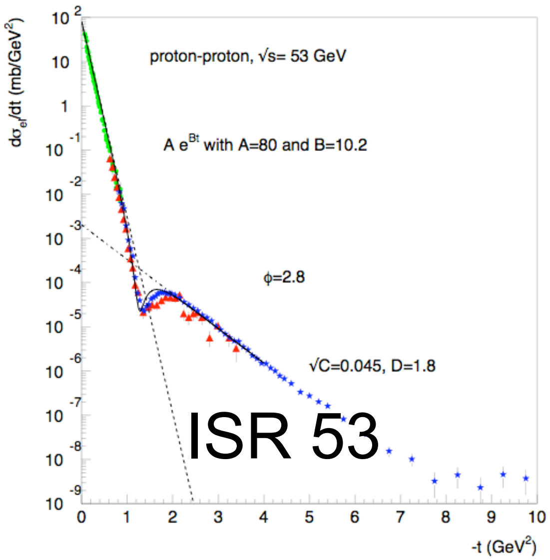


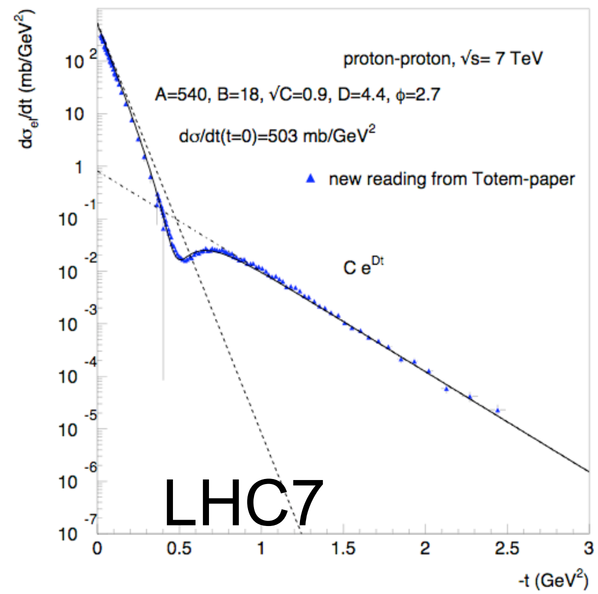
Fig. 3: The measured differential cross-section $d\sigma/dt$. The superimposed fits and their parameter values are discussed in the text.

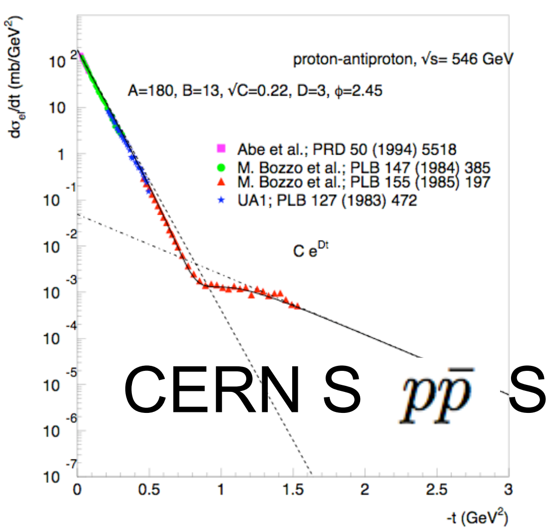
G. Antchev et al.Eur.Phys.Lett. 2011

The return of the dip: pp vs $p\bar{p}$



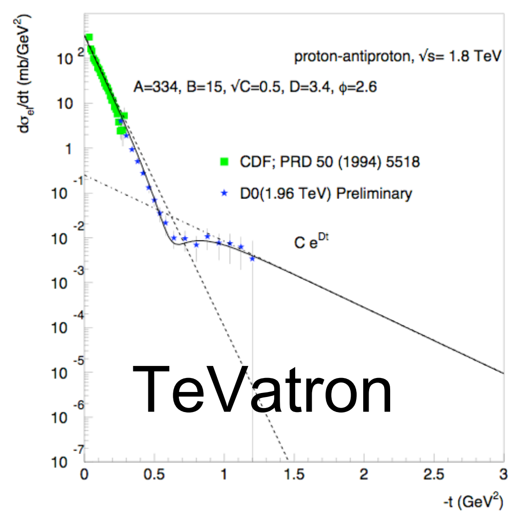
A simpler system





$p\bar{p}$

Complicated
By resonances



How can one define asymptotia?

- Saturation of the Froissart bound ?

$$\sigma_{total} \lesssim \boxed{\frac{\pi}{m_\pi^2}} [\log \frac{s}{s_0}]^2$$

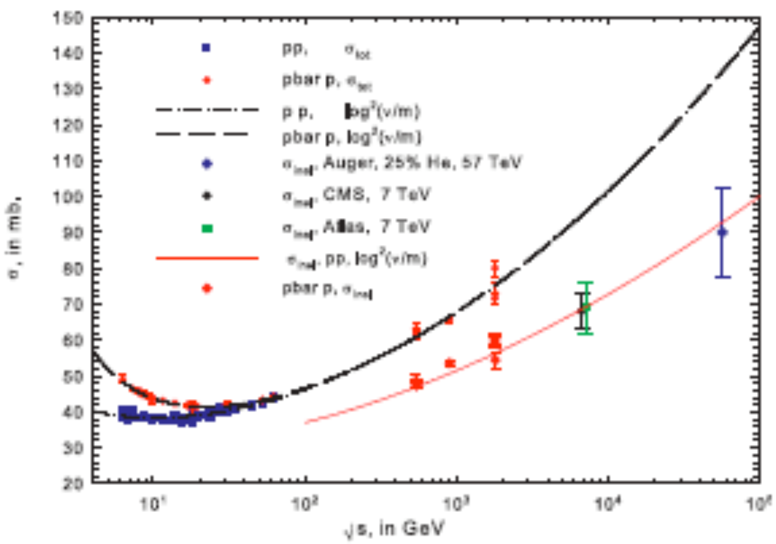
- With or without the constant (Froissart-Martin-Lukazsku)?
- What is s_0 ?anyway ?
- Black disk limit?

$$\mathcal{R}_{el} = \frac{\sigma_{el}}{\sigma_{total}} \Longrightarrow 1/2$$

Has asymptotia been reached?

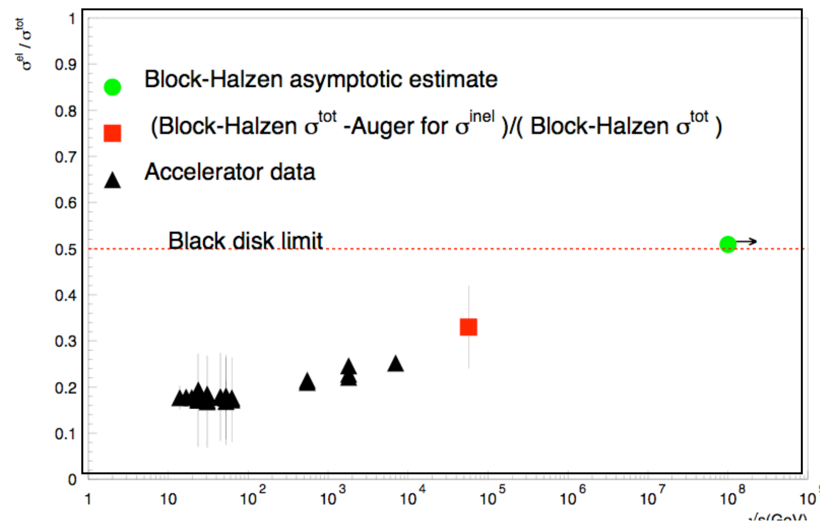
(with dire consequences for hidden extra dimensions according to Srivastava et al., arXiv:1104.2553, Block and Halzen ArXiv:1201.0960))

Experimental Confirmation that the Proton is Asymptotically a Black Disk, Martin M. Block, Francis Halzen, Phys.Rev.Lett. 107 (2011) 212002



$$\sigma_{total} \sim \log^2(s/s_0)$$

Checks for asymptotia al LHC, A. Grau. G. P., S. Pacetti and Y.N. Srivastava, May 2012, Submitted to PLB



$$\mathcal{R}_{el} \sim 0.25(\sqrt{s} = 1.8 - 7\ TeV)$$
$$0.3(\sqrt{s} = 57\ TeV)$$

Interesting?

[many people are turned off by claims of victory anytime a new measurement appears, ISR, CERN $Sp\bar{p}S$, TeVatron and now LHC7]

- ✷ Is **asymptotia** reached? i.e. is the Froissart bound (FB) for sigma total saturated? Why would this be interesting?

1. Because **saturation** of FB could exclude power-like behaviour as from hidden extra dimensions [Block Halzen 2012 , Srivastava et al, 2011]
2. Or data could hint to **new baryonic** interactions at 10-100 TeV and thus solve problems with cosmic rays composition based on current σ_{total} extrapolations [Piran, april 2012]
3. Because there is a connection between **Froissart** bound and **confinement** which the total cross-section can investigate

- ✷ Why the **dip in pp elastic differential cross-section**?

The total cross-section: confinement and deconfinement at work

$$\sigma_{total} = \sigma_{elastic} + \sigma_{inelastic}$$

A confined system: quarks and gluons remain inside the original hadrons even at high energy

deconfined

Central production: quarks and gluons scatter away and then hadronize

Fully deconfined

Single and double diffractive Production: quarks and gluons remain "close" to original hadrons and then hadronize

Different, not necessarily conflicting descriptions

Regge-Pomeron

with 1-2-3….pomerons + Regge trajectories

$$\mathcal{A}_{P/R}(s,t) = i\beta(t)\left(\frac{s}{s_0}\right)^{\alpha_{P/R}(t)-1}$$

$$\sigma_{total} = X s^{-\eta} + Y s^{\epsilon}$$

$$\epsilon = \alpha_P(0) - 1 > 1$$

☹ And we are not yet in asymptotia if this is true

Eikonal formulation

$$F(s,t) = i\int d^2\mathbf{b}e^{i\mathbf{q}\cdot\mathbf{b}}[1 - e^{i\chi(b,s)}]$$

Asymptotic Black disk limit

$$\begin{aligned} \Re e\chi &\simeq 0 \\ \Im m\ \chi &= \theta(R(s) - b) \\ R(s) &\to \log s/s_0 \end{aligned}$$

$$\begin{aligned} \sigma_{total} &\sim \log^2(s/s_0) \\ \mathcal{R}_{el} &= 1/2 \end{aligned}$$

Both descriptions have a point but

- With QCD at hand, one should look for a microspic description connected to the most interesting QCD question, now, infrared gluons and confinement
- We have developed a model (1996-2012 …) to connect IR gluons to the asymptotic behaviour of the total cross-section
- Interesting results for sigma_{tot}, sigma_{el}, sig_{inel}
- Still under progress

A. Grau, R.M. Godbole, GP, Y.N.Srivastava

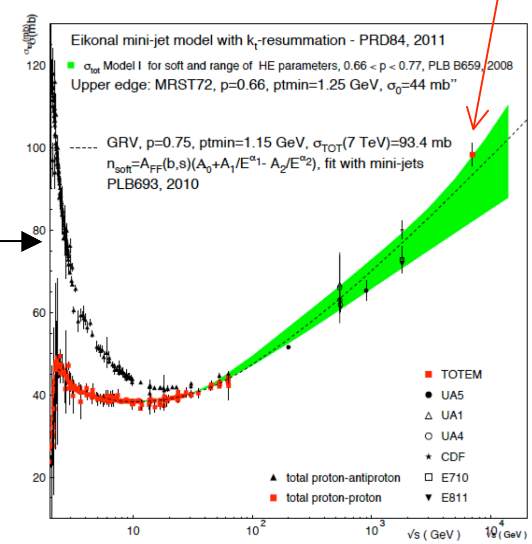
Our QCD model: a formalism to study confinement in total cross-section

Totem 2011

We have developed a model

green band in

PLB2008



which connects σ_{total}

to the study of ultra soft gluon coupling

where one can expect confinement effects to arise

In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering (mini-jets) and restores the Froissart bound

s^{ε}

$$\sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}]$$

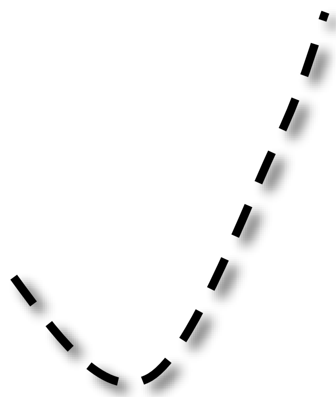
$$\sigma_{tot}(s) \rightarrow [\varepsilon \ln(s)]^{(1/p)} \qquad \frac{1}{2} < p < 1$$

Grau, Godbole,GP,Srivastava,PLB 2009

Issues in a QCD mini-jet description

What generates the rise? Low-x parton collisions

Cline,Halzen &Luthe 1973
Gaisser, Halzen,Stanev 1985
G.P., Y.N. Srivastava 1986
Durand,Pi 1987
Sjostrand, van Zijl 1987
…

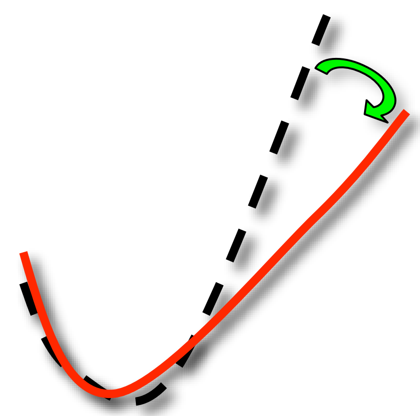


$s^{\epsilon} \quad \epsilon \sim 0.3$

What tames the rise into to a Froissart-like behavior?

A cut off obtained by [embedding into the eikonal] the acollinearity induces by IR kt-emission

[our model, G.P. et al. **Phys.Lett.B382, 1996**]]



Our model: eikonal+minijets+soft gluon resummation in the IR

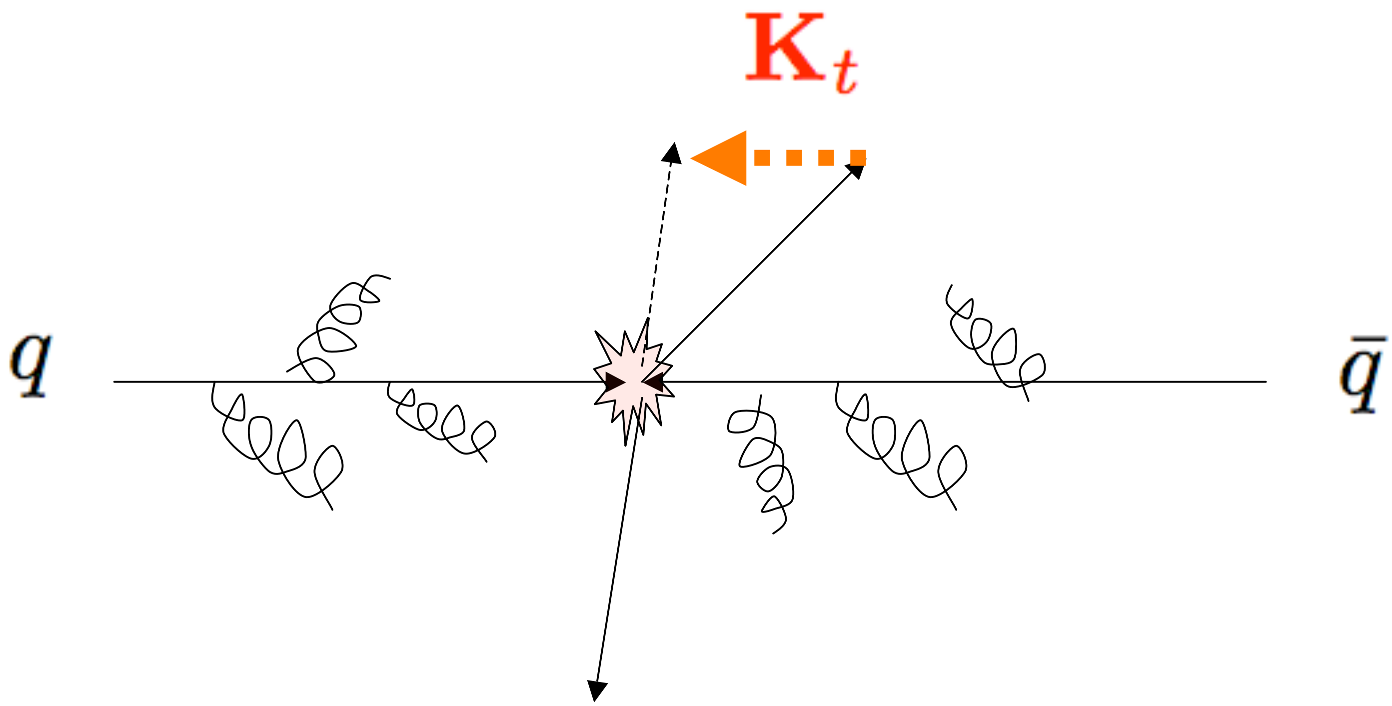
- Start with eikonal representation $\sigma_{tot}(s) = 2\int (d^2b)[1 - e^{-\bar{n}(b,s)/2}]$ $\Re e\chi \approx 0$
- Low and high energy component $\bar{n}(b,s) = \bar{n}_{low}(b,s) + \bar{n}_{high}(b,s)$
- **Low** energy component is parametrized with **No rising** term
- **High** energy (rising) component is from **PQCD** $\bar{n}_{high} = A(b,s)\sigma_{jet}(s)$

  Minijets to get the rise $p_t^{parton-out} \geq p_{tmin} \simeq 1\ GeV$

- To **tame the rise** $A(b,s)$ is obtained from $K_t - resummation$ with integration down into the infrared with an ansatz for infrared behaviour

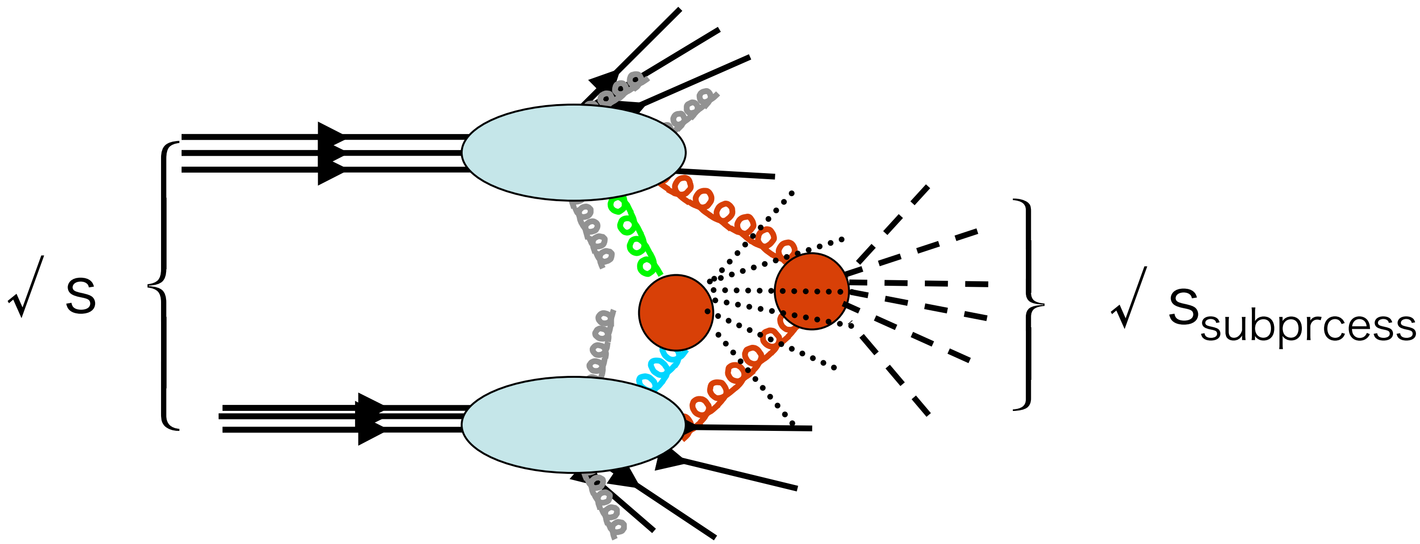
$$\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}$$

Soft gluon emission introduces acollinearity



Acollinearity reduces the collision cross-section as partons do not scatter head-on any more, i.e. the gluon cloud is too thick for partons to see each other : gluon saturation

Cartoon view of the model for σ_{total}



- QCD minijets with LOPDFs from CERNLIB to drive the rise
- Soft Gluon k_t-resummation (ISR) in the infrared main original ingredient of our model
- Multiple scattering (in Eikonal representation to implement unitarity)

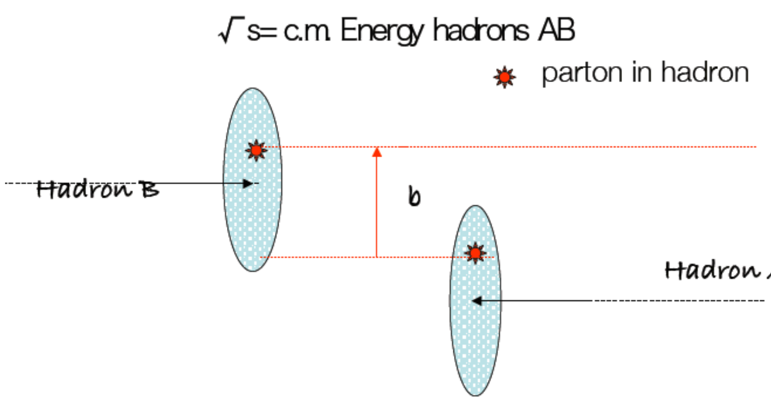
We model the impact parameter distribution as the Fourier-transform of ISR soft k_t distribution and thus obtain a cut-off at large distances : Froissart bound?

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \; e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\; e^{-h(b,q_{max})}}$$

$$h(b,E) = \frac{16}{3\pi}\int_0^{qmax} \frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln(\frac{2q_{max}}{k_t})[1-J_0(bk_t)]$$

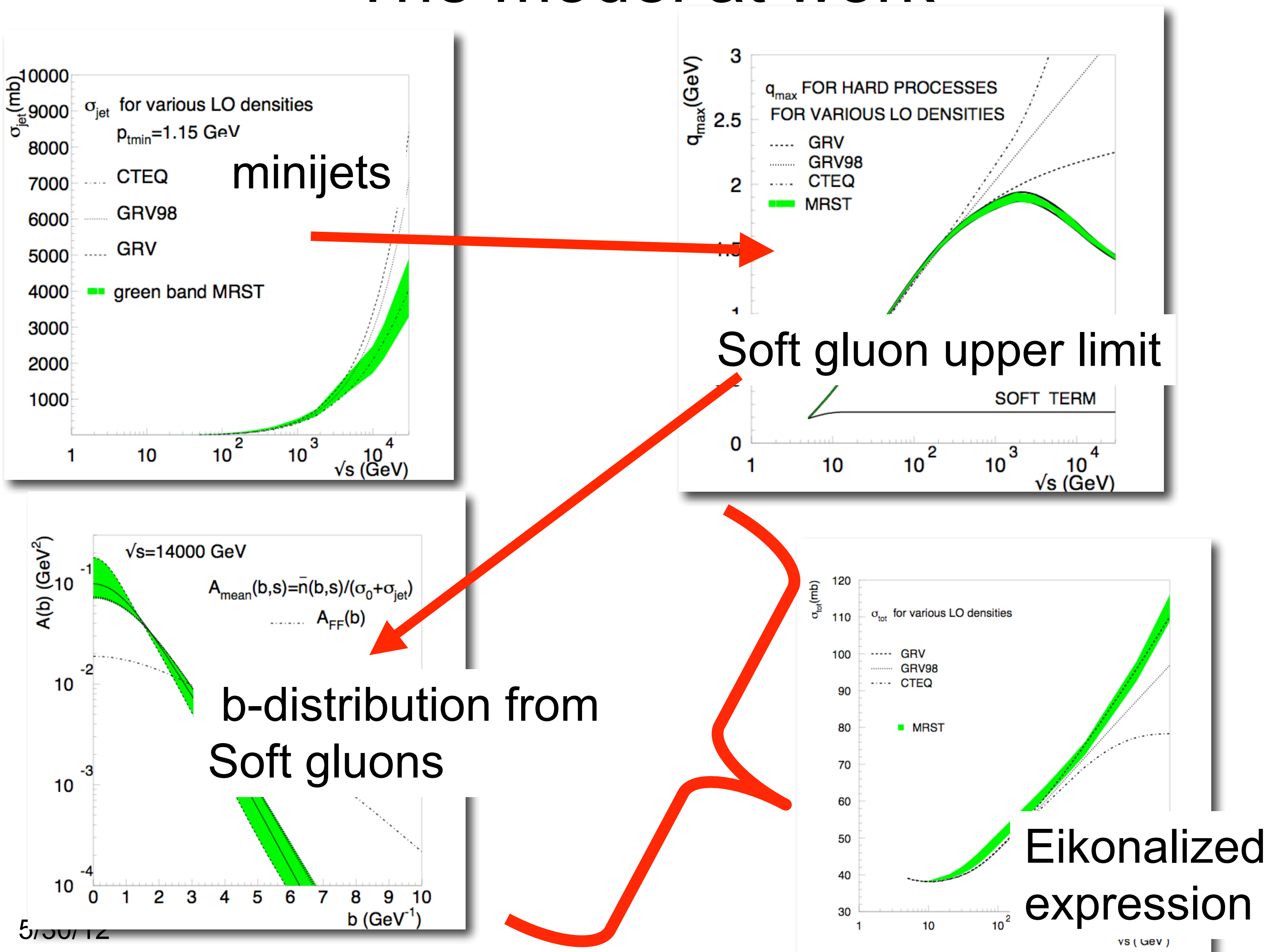
$$\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}$$

$$A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}$$



q_{tmax} ? Fixed by single gluon emission kinematics

The model at work



the large-s limit

$$\sigma_{total} \to 2\pi \int db^2 [1 - e^{-C(s)e^{-(bq)^{2p}}}]$$

$$C(s) = (s/s_0)^{\varepsilon} \sigma_1$$

$$A(b,s) \propto e^{-(bq)^{2p}}$$

Mini-jets

Ultra-soft gluons effects

$$\sigma_T \approx \frac{2\pi}{\bar{\Lambda}^2} [\varepsilon \ln \frac{s}{s_0}]^{1/p}$$

$\sim \ln^2 s \quad p = 1/2$

$\sim \ln s \quad p = 1$

Application to LHC7 data: ATLAS, CMS, TOTEM GP et al, PRD2011

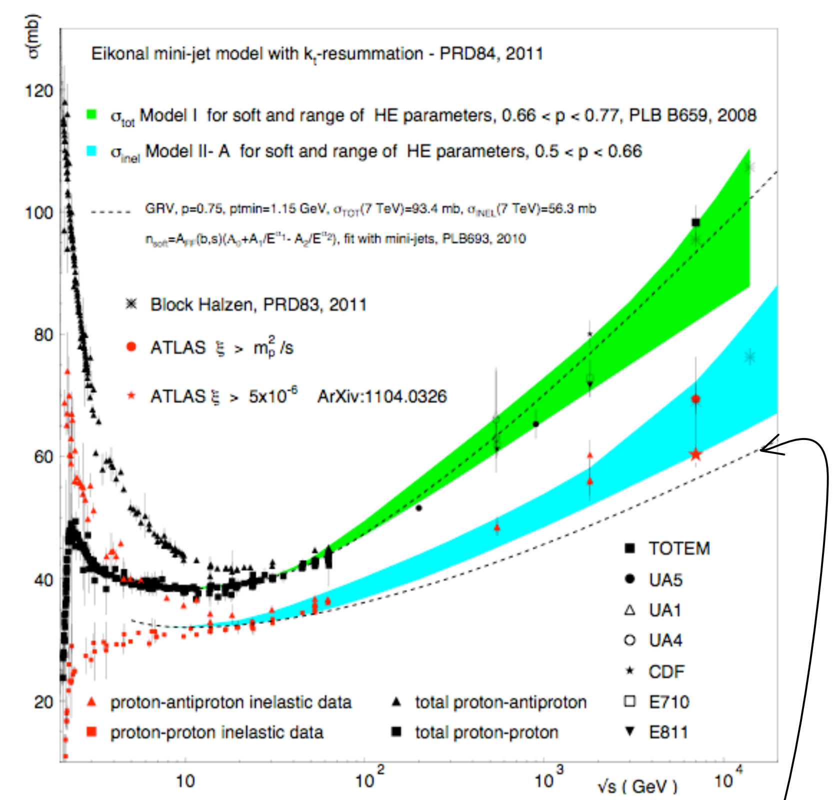
When data from ATLAS and CMS appeared, a problem with the eikonal formulation became evident:

Use independent collisions in b-space to obtain total inelastic collisions

$$P(n,\bar{n}) = \frac{(\bar{n})^n e^{-\bar{n}}}{n!}$$

$$\begin{aligned}\sigma_{inel}(s) &= \sum_{n=1} \int d^2\mathbf{b}\; P(\{n,\bar{n}\}) \\ &= \int d^2\mathbf{b}[1 - e^{-\bar{n}(b,s)}] \\ &\equiv \sigma_{tot}(s) - \sigma_{el}(s)\end{aligned}$$

$$with\ \Im m\chi(b,s) = \bar{n}(b,s)$$



- Inelastic with 2-component eikonal is only non-correlated events
- Diffraction must have been put in the elastic!

For a different approach : Lipari Lusignoli PRD2009

The eikonal 2-component formulation has problems

- Ok for the sigma total but

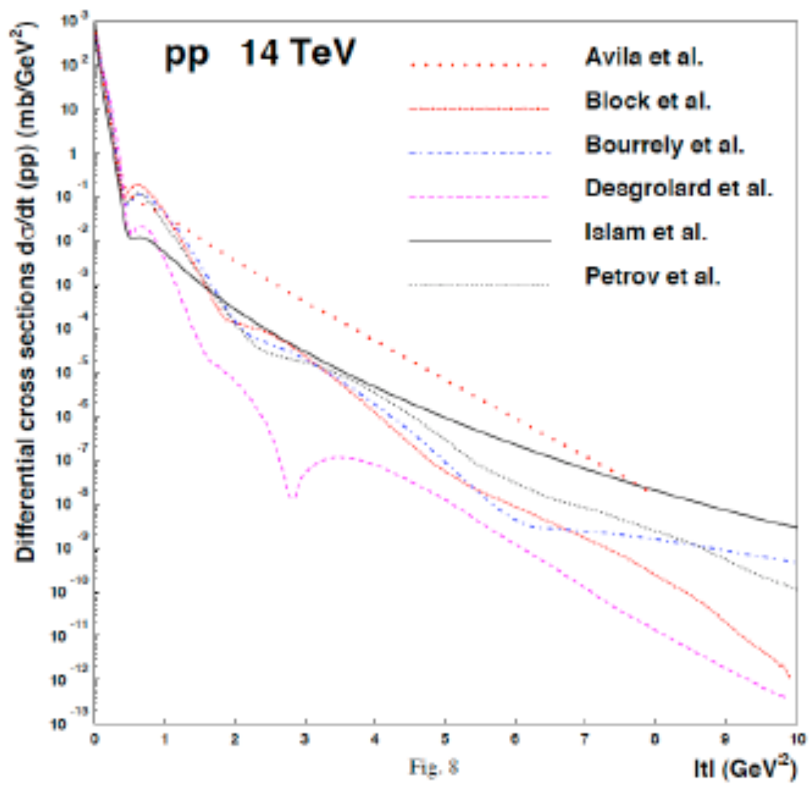
Sigma elastic and sigma inelastic get mixed up: diffraction, single and double, goes into the elastic [GP et al PRD84]

- Need for a different formalism [e.g. Lipari&Lusignoli 2009]

- And anyway further understanding

- Turn to the elastic differential to see what happens

Many predictions before 2011



TOTEM : the forward peak

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt}|_{t=0}\ e^{B_{exp} t}$$

- The slope actually changes as one measures away from t=0 to the dip region
- ~ 20 GeV^{-2} at small 0.02<-t<0.33
- ~23 GeV^{-2} at -t before the dip

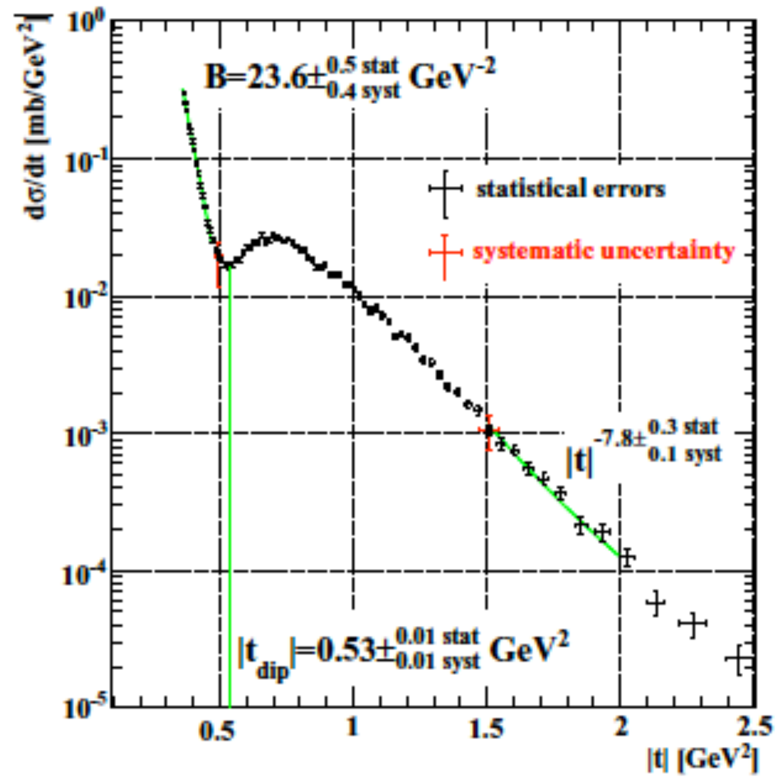
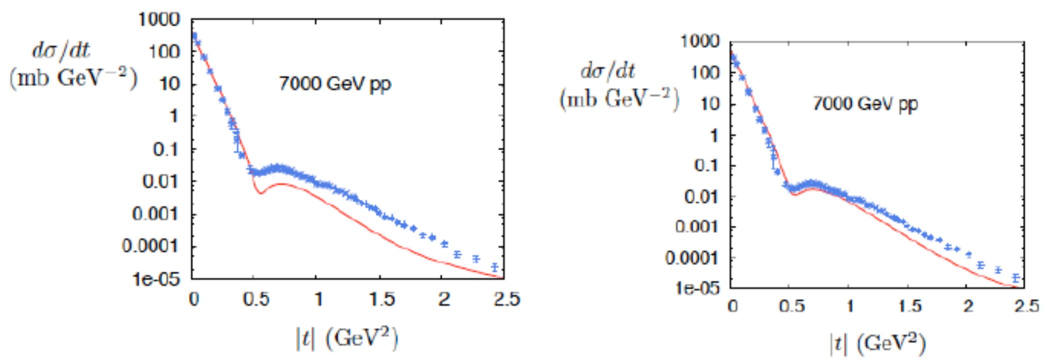


Fig. 3: The measured differential cross-section $d\sigma/dt$. The superimposed fits and their parameter values are discussed in the text.

How do models fare with the TOTEM data for elastic differential x-section?

Donnachie and Landshoff 2011

without and with hard Pomeron



- Many other attempts, with modification of previous parametrizations have now appeared

- Menon et al., Block and Halzen, ….

Turn to something old and simple toy-like: two exponential and a phase from Barger and Phillips in 1973

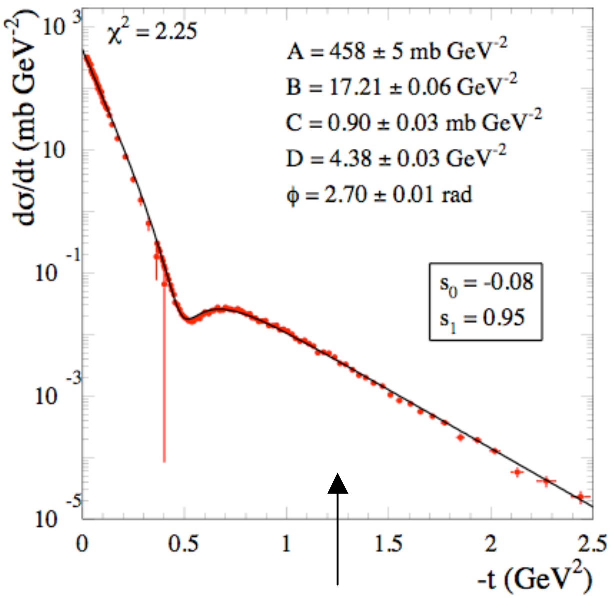
$$\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{\frac{1}{2}B(s)t} + \sqrt{C(s)}e^{i\phi(s)}e^{\frac{1}{2}D(s)t}]$$

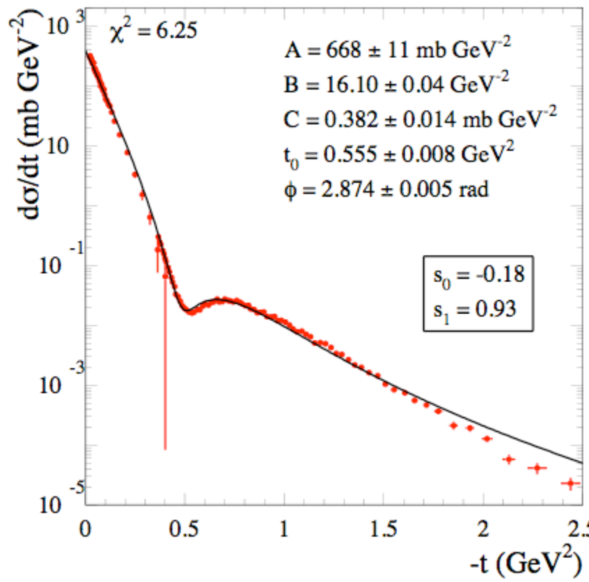
$$\frac{d\sigma}{dt} = A(s)e^{B(s)t} + C(s)e^{D(s)t} + 2\sqrt{A(s)}\sqrt{C(s)}e^{\frac{(B(s)+D(s))t}{2}}\cos\phi$$

five s-dependent real parameters, A B C D ϕ

How does it work with LHC TOTEM data?

How to describe both the diffraction peak and the tail of TOTEM data : models for the tail



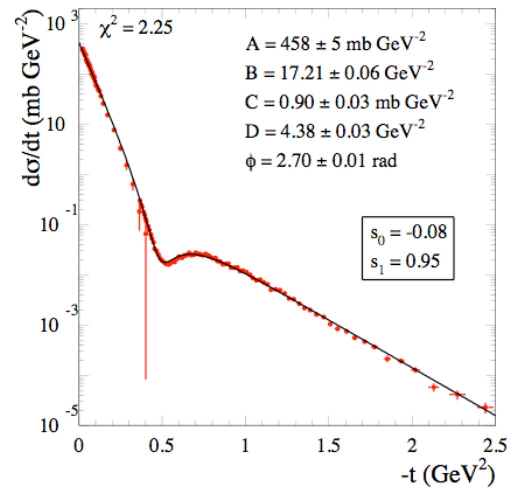


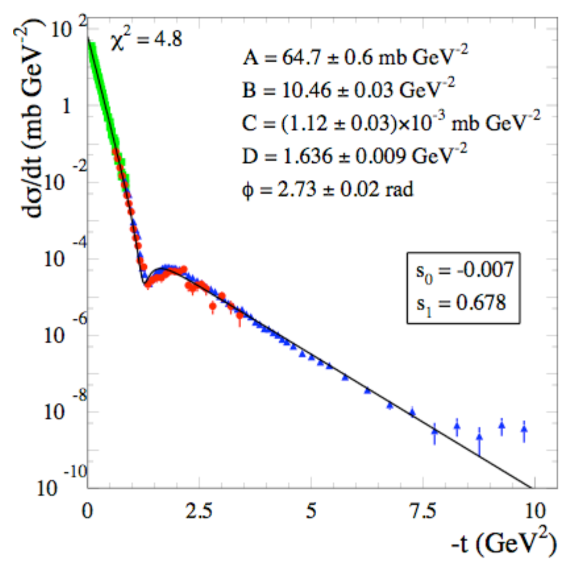
- Model 1 : two exponentials
- TOTEM $|t|^{-n}$ with $n = 7.8 \pm 0.3^{stat} \pm 0.1^{syst}$
- Donnachie and Landshoff (1996) $|t|^8$
- Model 2 : $\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{Bt/2} + \frac{\sqrt{C(s)}}{(-t+t_0)^4}e^{i\phi}]$

The two exponential gives the best fit

Two exponentials and a phase vs ISR and LHC7 data

- A model not so much …model dependent : two exponentials and a phase (Barger and Phillips 1973)
- Good description of TOTEM data and reasonable for ISR (both pp)

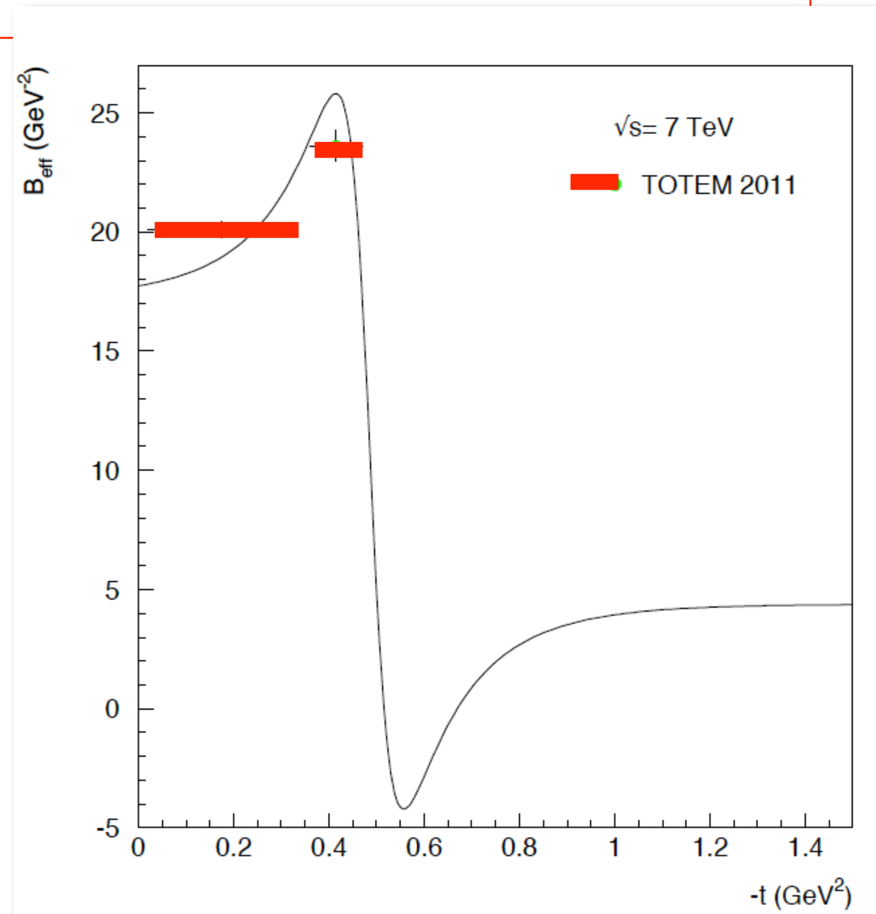




With A. Grau,S. Pacetti, Y.N. Srivastava
Submitted to PLB, May 2012

? How about the slope in the two exponential model

$$B_{eff}(s,t) \equiv \frac{d \ln \frac{d\sigma}{dt}}{dt}$$



$$B_{eff}(s,t) = \frac{ABe^{Bt} + CDe^{Dt} + \sqrt{A}\sqrt{C}(B+D)e^{(B+D)t/2}\cos\phi}{Ae^{Bt} + Ce^{Dt} + \sqrt{A}\sqrt{C}e^{(B+D)t/2}\cos\phi}$$

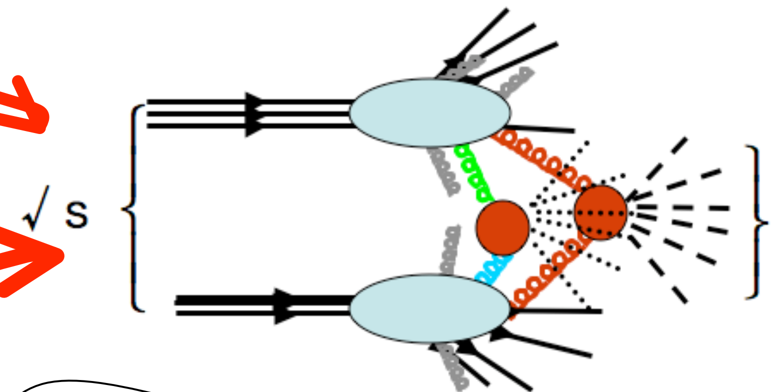
Eikonal (vs Regge-Pomeron): how to reconcile minijets with exponential shrinking?

- What is wrong with the minijets + IR resummation + eikonal picture through which the elastic amplitude is built in this model (ours)?

My guess (work in progress): a global condition on the amplitude that at t=0 no gluons, soft, IR, or otherwise escape needs to be enforced ~

~form factor as a further resummation effect forcing all the single subprocess distributions to $\sqrt{-t}$ to an overall momentum K~

reabsorption and compensation of the change of momentum



•Each subprocess gives a minijet x-section s^{ε}

•Soft IR gluons for one Such subprocess give $\exp(b\Lambda)^{p}$

•Eikonal resums multiple collisions

Conclusion

- A model with minijets and soft gluon resummation is able to describe the total cross-section from 5 GeV to cosmic rays energies
- A model with two exponential and a phase is well suited to describe the dip structure at LHC as well as the forward diffraction peak and shoud be used to parametrize future data at 8 TeV or beyond
- The connections between these two models is still under study

How to check asymptotia?

$$\mathcal{F}(s,t) = i\int_o^\infty (bdb) J_o(b\sqrt{-t})[1 - e^{2i\delta_R(b,s)} e^{-2\delta_I(b,s)}]$$

$$\sigma_{total}(s) = 4\pi \Im m \mathcal{F}(s,0)$$

- Two asymptotic sum rules in impact parameter space [EPJC 2005]

$$(\frac{1}{2})\int_{-\infty}^0 (dt) \Im m \mathcal{F}(s,t) \;\rightarrow\; 1; \;\; as\ s \rightarrow \;\infty. \qquad s_1$$

$$\int_{-\infty}^0 (dt) \Re e \mathcal{F}(s,t) \;\rightarrow\; 0; \;\; as\ s \rightarrow \;\infty \qquad s_0$$

BP model allows easy check of the sum rules

- With parameters from fit

$$s_1 = \sqrt{\frac{A}{1+\hat{\rho}^2}\frac{1}{\sqrt{\pi}B} + \frac{\sqrt{C}}{\sqrt{\pi}D}\cos\phi} = 0.94$$ at LHC7

- At ISR 53 GeV $s_1 = 0.75$

To satisfy both sum rules, add a real part to the first term

$s \leftrightarrow \ u$ Use our minijet model with soft gluon resummation with 0.66<p<0.77 PLB08

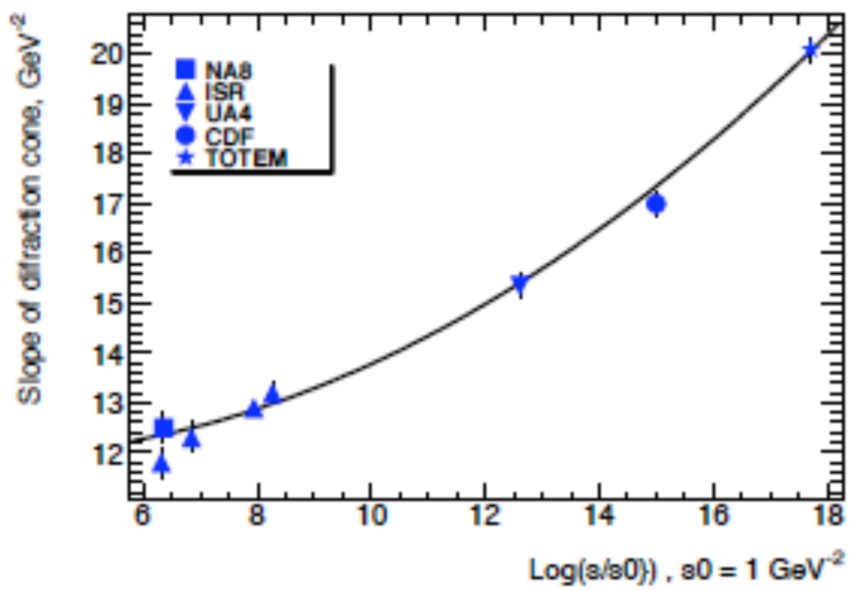
$$\mathcal{A}(s,0) \rightarrow \ i\left[ln(s/s_o e^{-i\pi/2})\right]^{1/p}$$

$$= i\big(\big(ln(s/s_o) - i\pi/2\big)\big)^{1/p}$$

$$\frac{\Re e\mathcal{A}(s,0)}{\Im m\mathcal{A}(s,0)} \rightarrow \ \frac{\pi}{2pln(s/s_o)} = 0.134 \div 0.115$$

$$s_0 \sim 0.05 \ LHC7$$

Slope from data



Ryskin 2012 : log^{2}s behaviour?

Dip or no dip?

- Before and after the dip the two processes pp and $p\bar{p}$ should be described by the same physics
- At the dip the basic amplitude is almost zero (5 orders of magnitude lower in the cross-section) so the *leftovers* from Regge exchange, present only in $p\bar{p}$, fill the dip

pp and $\bar{p}p$

R.M.Godbole, A. Grau, G.P. Y.N. Srivastava, +A. Achilli, +A.Corsetti + O. Shekhovtsova

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- Phys. Lett. 2010
- Eur.Phys.J.C63:69-85,2009. e-Print: arXiv:0812.1065 [hep-ph]
- Phys.Lett.B659:137-143,2008. e-Print: arXiv:0708.3626 [hep-ph]
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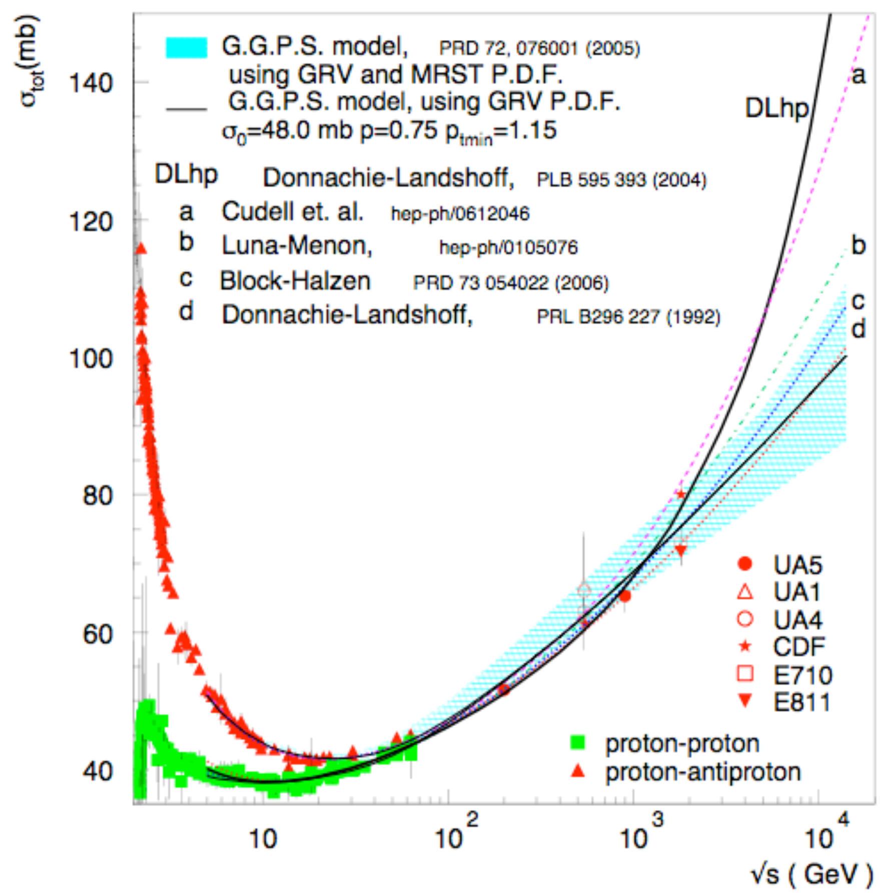
- Phys.Lett.B382:282-288,1996. e-Print: hep-ph/9605314



Some details

Mini-jets

$$\sigma_{\text{jet}}^{AB}(s; p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}.$$

DGLAP evolved

Which value of p_{tmin}?
Which densities?

Parametrize data choosing PDF and p_{tmin} to catch the early rise of σ_{total}

Mini-jets drive the rise of σ_{total}

$$\sigma_{\text{jet}}^{AB}(s, p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}$$

$$p_{tmin} \sim 1 \div 2 \; GeV$$

DGLAP evoluted PDF

Parton-parton x-sections: $parton_i + parton_j \rightarrow parton_k(p_t) + parton_l(-p_t)$

Building sigma$_{total}$

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\Im m\chi(b,s)} cos\Re e\chi(b,s)]$$

$$\Re e\chi(b,s) \simeq 0$$

$$\bar{n}(b,s) = 2\Im m\chi(b,s) \simeq A(b)\sigma(s)$$

Two component simplest model

$$\bar{n}(b,s) = \bar{n}_{soft}(b,s) + \bar{n}_{hard}(b,s)$$

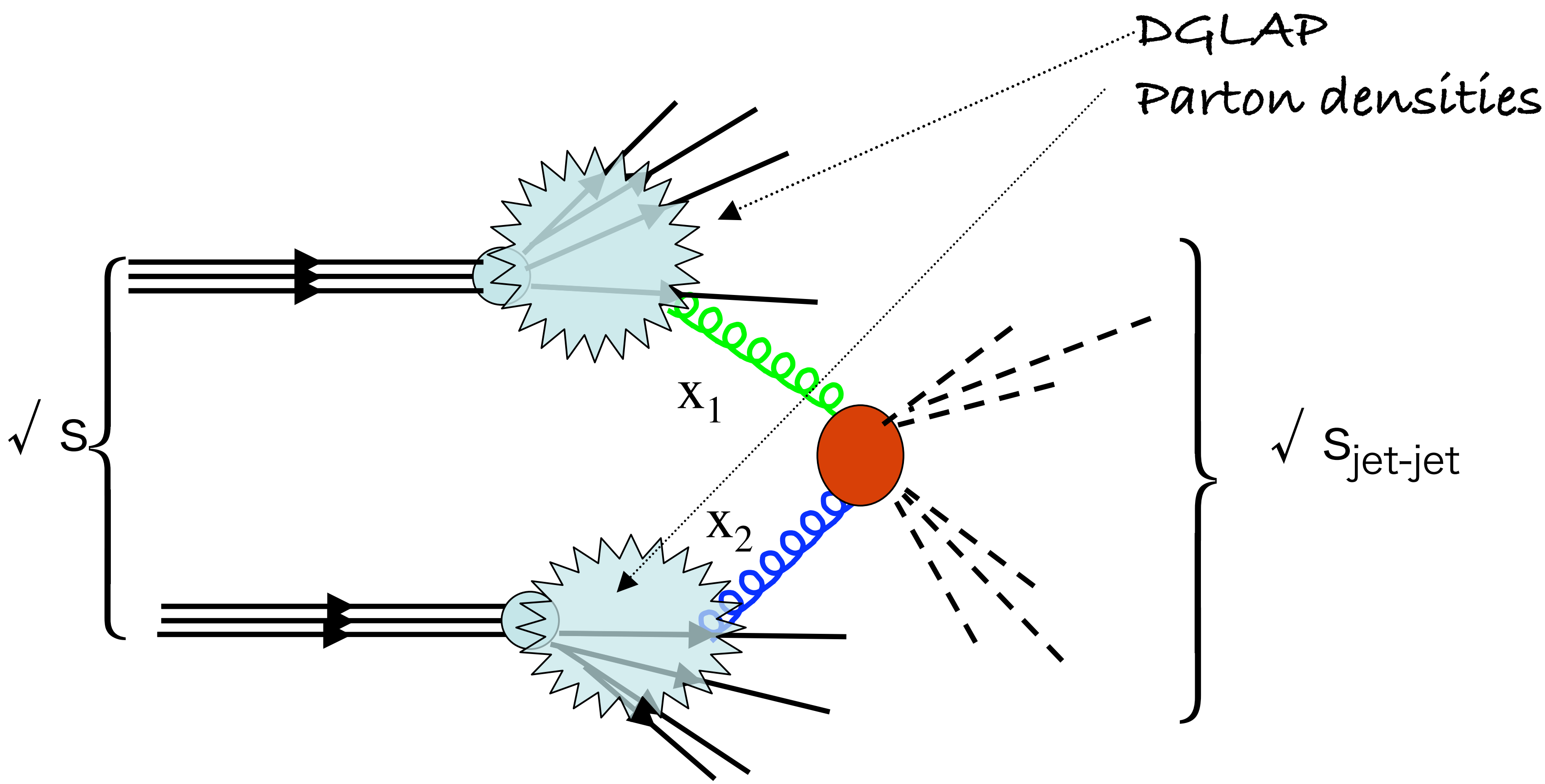
$$\bar{n}_{soft/hard}(b,s) = A_{soft/hard}(b,s)\sigma_{soft/hard}(s)$$

Overlap function

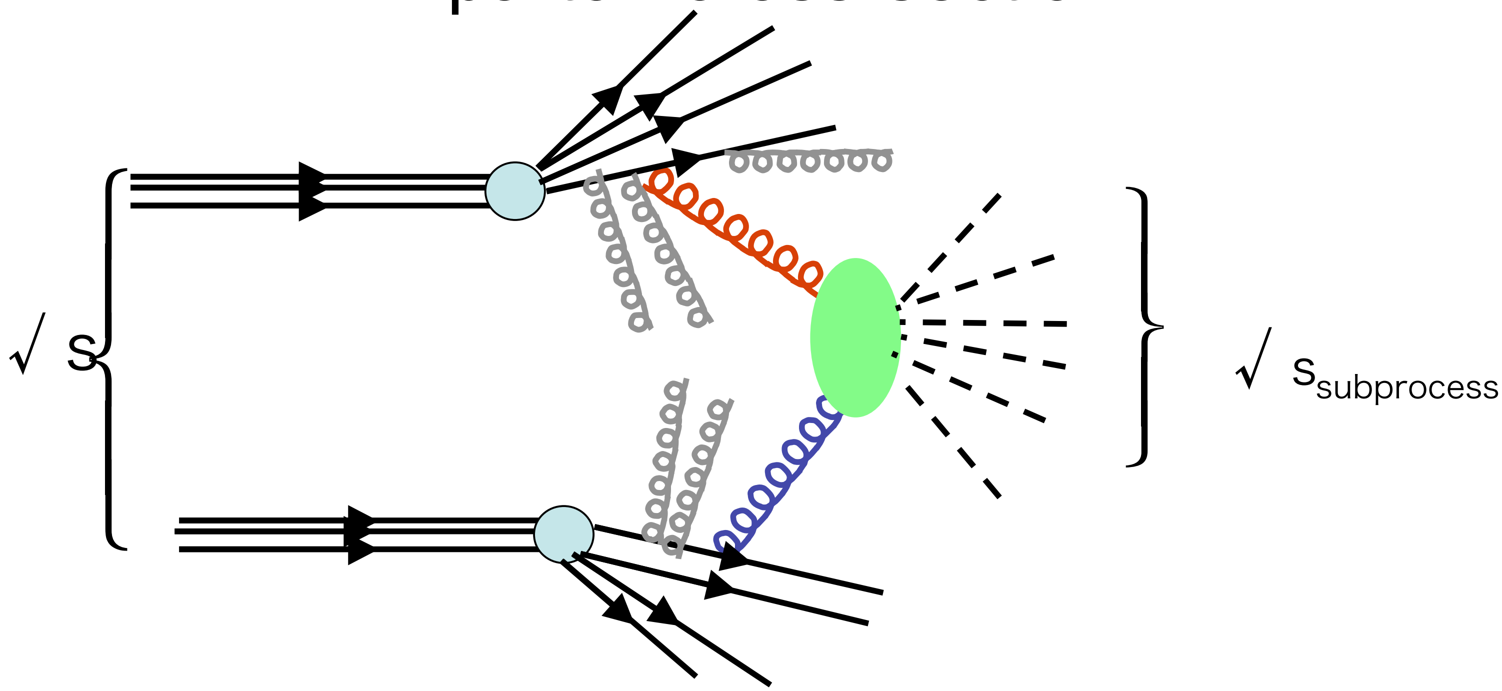
What makes the cross-section rise?

Mini-jets are responsible for the rise of the total cross-section

Cline, Halzen, Luthe 1972- Gaisser, Halzen 1985- G.P., Srivastava 1985



One component missing in the mini-jet picture is soft gluon emission from the initial state to break the collinearity and reduce the parton-parton cross-section



Eikonal models: b-distribution can quench the rise

$$n_{hard-minijets}(b) \approx A(b,s)\sigma_{jet}(s,p_{tmin})$$

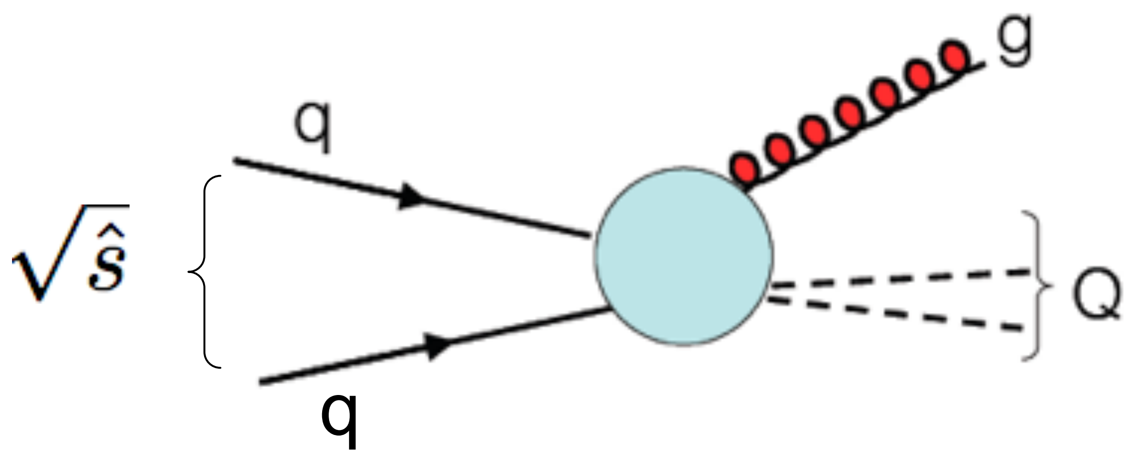
How to choose it:
Form factors?

Choice of densities for mini-jet x-section

Because we use resummation to access large distance behaviour

- LO PDFs are used, to avoid double counting the most important contribution (small kt) to observables like σ_{tot}
- LO: GRV, MRST, CTEQ
- For illustration purposes: GRV
- Bands are also presented with GRV and MRST
- We are working to include other densities

The single soft gluon Integration limit can be obtained from kinematics



$$q_{max} = \frac{\sqrt{\hat{s}}}{2}(1 - \frac{Q^2}{\hat{s}})$$

σ_{total} and the large-s limit

$$2\Im m\chi = n_{soft} + n_{hard-minijets} \qquad Re\chi \approx 0$$

$$\sigma_{total} = 2\int d^2\vec{b}[1 - e^{-n_{soft}-n_{hard-minijets}}]$$

$$n_{hard-minijets}(b) \approx A(b,s)\sigma_{jet}(s, p_{tmin}) \qquad >> n_{soft}$$

$$\sigma_{total} \to 2\pi \int db^2[1 - e^{-C(s)e^{-(bq)^{2p}}}]$$

$$A(b,s) \propto e^{-(bq)^{2p}}$$

$$C(s) = (s/s_0)^{\varepsilon}\sigma_1$$

Mini-jets

Ultra-soft gluons effects

At very large energy:
from power law to log behaviour

$$\sigma_T(s) \approx \frac{2\pi}{p}\frac{1}{\bar{\Lambda}^2}\int_0^\infty du u^{1/p-1}[1-e^{-C(s)e^{-u}}]$$

$u = (\bar{\Lambda}b)^{2p}$ $\qquad$ $I(u,s) = 1 - e^{-C(s)e^{-u}}$ has the limits

$I(u,s) \to 1$ at $u = 0$

$I(u,s) \to 0$ as $u = \infty$

$$\sigma_T \approx \frac{2\pi}{\bar{\Lambda}^2}[\varepsilon \ln \frac{s}{s_0}]^{1/p}$$

$\sim \ln^2 s \qquad p = 1/2$

$\sim \ln s \qquad p = 1$

A general scheme for various processes

- Start with PDF for the chosen process
  - Proton-proton, pion-proton, pion-pion, photons (nuclear matter, heavy ions)
  - Calculate mini-jet basic cross-section, quark-antiquark, gluon-gluon (dominant), quark-gluon
  - Calculate qmax (s) for soft emission

- Fix p (singularity) for one process, say proton-proton
- Calculate A(b,qmax(s))
- Parametrize $\bar{n}_{soft}(b,s)$
- Eikonalize and integrate