Avatars of the generalized cusp in $\mathcal{N} = 6$ Super Chern-Simons theories

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Summary

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Physical Context

- Duality between CFT in D dimensions and String theory in (D+1$\times$ internal spaces) dimensions

Two paradigmatic examples of the conjecture

• Limit of large λ di SYM $\mathcal{N} = 4$ for $N \to \infty$ • Expansion in $\lambda^{-\frac{1}{2}}$	$\Longleftrightarrow$	• Classical Supergravity IIB on $AdS_5 \times S^5$ • Expansion in α'
• 't Hooft's limit of ABJM $\mathcal{N} = 6$ fixed $\lambda = N/k$ • Expansion in $\lambda = N/k$	$\Longleftrightarrow$	• Classical Supergravity IIA on $AdS_4 \times \mathbb{CP}_3$ • Expansion in α'

- Need to find observables calculable at strong coupling
- Example: scale dimension of a primary operator, Wilson Loops

Summary

The Wilson Loop in $\mathcal{N} = 4$ SYM (1)

$$W_f(C) = \frac{1}{\dim(f)} \mathrm{Tr}_f \left(P \exp \left(i \oint_C A_\mu dx^\mu \right) \right)$$

- one of the most important objects in QCD: order parameter for confinement

Wilson Loop in $\mathcal{N} = 4$ SYM

$$\langle W(C) \rangle = \frac{1}{N} \left\langle Tr P \exp i \oint_C ds \left(A_\mu \dot{x}^\mu(s) + \phi_I \theta^I \sqrt{\dot{x}^2(s)} \right) \right\rangle$$

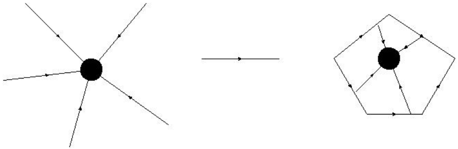
- Some of them are exactly computable supersymmetric observables
- Example: circular Wilson Loop 1/2 BPS$\Rightarrow \langle W(\text{circle}) \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$
- non trivial interpolating function $F(g^2 N)$ between weak and strong coupling: comparison with string theory
- potential: two different expansions at weak and strong coupling

The Wilson Loop in $\mathcal{N} = 4$ SYM (2)

Wilson Loops appear in two different and somewhat unrelated contexts

- BPS non trivial observables computable by localization (Pestun, Kapustin et al.)
- Scattering amplitudes/cusp physics computable by integrability
- Duality WL/Scattering Amplitudes, realization of a field theoretical idea from QCD, equivalence in the Regge limit

$$\log M_n^{MHV} = \log W_n + cost$$

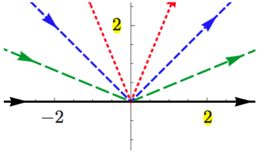


Summary

IDEA

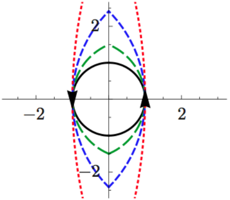
Drukker, Forini in $\mathcal{N} = 4$ SYM

- Deform smoothly the contour to interpolate between BPS and non BPS configurations (potentials)
- Deform scalar couplings to join BPS and non BPS configurations



Euclidean cusp

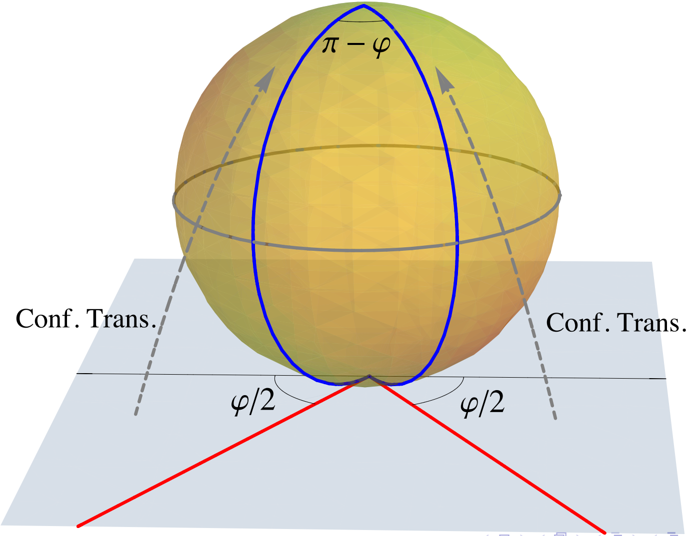
Parameters: angle of R-symmetry ϑ and a space-time angle φ



Conformal mapping: wedge

- Circle: $\varphi = 0$ BPS
- Antiparallel lines: $\varphi = \pi$ NON BPS

Conformal transformations



Results in $\mathcal{N} = 4$ SYM

- Try to interpolate the two expansions

$$< W > \equiv \exp[-TV(\lambda, \vartheta, \varphi)]$$

$$V(\lambda, \vartheta, \varphi) = \sum_{n=1}^{\infty} (\frac{\lambda}{16\pi^2})^2 V^{(n)}(\vartheta, \varphi)$$

$$V(\lambda, \vartheta, \varphi) = \frac{\sqrt{\lambda}}{4\pi} \sum_{n=0}^{\infty} (\frac{4\pi}{\sqrt{\lambda}})^n V^n_{AdS}(\vartheta, \varphi)$$

$$< W_{cusp} > \equiv \exp[-\log(\frac{L}{\epsilon})\Gamma_{cusp}(\lambda, \vartheta, \varphi)] \qquad \lim_{\varphi \to \infty} \Gamma_{cusp}(\lambda, \vartheta, i\varphi) = \frac{\varphi \gamma_{cusp}(\lambda)}{4}$$

- By the plane to cylinder map $\Gamma_{cusp}(\lambda, \vartheta, \varphi)$ is identical with the energy of a static q and $\bar{q}$ ($V(\lambda, \vartheta, \varphi)$), sitting on S^3 at an angle $\pi - \varphi$

Weak coupling BPS condition for $\vartheta = \pm\phi$ and correct potential limit 1/L

$$V^{(1)}(\vartheta, \varphi) = -2\frac{\cos\vartheta - \cos\varphi}{\sin\varphi}\varphi$$

Motivations

Calculation of cusp anomalous dimension

- relation with the wedge: calculable with localization procedure (Pestun)
- set of TBA equations to calculate the generalized cusp using integrability (Maldacena et al): propagation of a magnon moving on a long strip with two boundaries associated to the Wilson Loop

Relation between $\mathcal{N} = 4$ SYM and ABJM

- $\gamma^{SYM}_{cusp}(\lambda) = \gamma^{ABJM}_{cusp}(h(\lambda))$??? (h(λ) function that is present in the dispersion relation of a Giant Magnon)
- scattering amplitudes: $< W_4 >^{1-loop}_{SYM} = < W_4 >^{2-loop}_{ABJM}$, $< W_4 >^{1-loop}_{ABJM} = 0$

$$< W_4 >^{1-loop}_{SYM} = \frac{\lambda}{8\pi^2}[-(\frac{(-\mu^2 x^2_{13})^\epsilon}{\epsilon^2} + \frac{(-\mu^2 x^2_{24})^\epsilon}{\epsilon^2}) + \frac{1}{2}\log^2(\frac{x^2_{13}}{x^2_{24}}) + cost]$$

Summary

Short review of ABJ(M) theories

$$L_{ABJM} = L_{CS} + \widehat{L}_{CS} + L_{matter};$$

$$L_{CS} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}[Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho)];$$

$$\widehat{L}_{CS} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}[Tr(\widehat{A}_\mu \partial_\nu \widehat{A}_\rho - \frac{2i}{3}\widehat{A}_\mu \widehat{A}_\nu \widehat{A}_\rho)];$$

$$L_{matter} = Tr(D_\mu C^I D^\mu \bar{C}_I) + iTr(\bar{\psi}^I \not{D}\psi_I) + L_{int};$$

- dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$
- 't Hooft limit $\rightarrow$ string theory IIA on $AdS_4 \times \mathbb{CP}_3$
- CFT in 3-d, with N=6 Susy, Dim(A_μ)=Dim(ψ)=1, Dim(C_I)=1/2
- gauge group $U(N)_k \times U(N)_{-k}$ for ABJM theory
- gauge group $U(N)_k \times U(M)_{-k}$ for ABJ theory
- two coupling constants $\lambda = N/k$ and $\lambda' = M/k$

Wilson Loop in SCS theories

$$W_{\mathcal{R}} = Tr_{\mathcal{R}}\mathcal{P}\exp\left(i\int A_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}|M_J^I C_I \bar{C}^J \, d\tau\right)$$

Properties of this WL

- In D=3 WL is quadratic in the scalar coupling
- $\delta_{SUSY} W = 0$ implies $M_J^I = diag(1, 1, -1, -1)$
- Circle is 1/6 BPS

Problem

- the fundamental string in AdS_4 ending on a circle at the boundary is 1/2 BPS
- the fundamental string preserves SU(3) R-symmetry: 1/6 BPS WL has only SU(2)×SU(2) R-symmetry
- 1/6 BPS WL exists in both of the groups of ABJM and in $\mathcal{N} = 2$ SCS: no enhancement of SUSY from $\mathcal{N} = 2$ to $\mathcal{N} = 6$

Wilson Loop in ABJ(M) theories

- Need to coupling fermions to have 1/2 BPS WL (Drukker, Trancanelli)

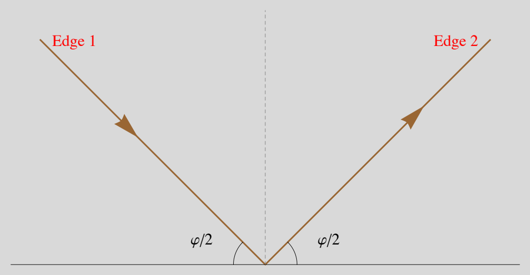
$$W_{\mathcal{R}} = Tr_{\mathcal{R}}\mathcal{P}\exp\left(-i\int L\, d\tau\right)$$

$$L = \begin{pmatrix} A_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}|M_J^I C_I \bar{C}^J & \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\eta_I^\alpha \bar{\psi}_\alpha^I \\ \sqrt{\frac{2\pi}{k}}\,|\dot{x}|\,\psi_I^\alpha \bar{\eta}_\alpha^I & \widehat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k}|\dot{x}|\widehat{M}_J^I \bar{C}^J C_I \end{pmatrix}$$

Properties of this WL

- To obtain 1/2 BPS solution we must replace $\delta_{SUSY} W = 0$ with the weaker condition $\delta_{SUSY} L(\tau) = \mathcal{D}_\tau G = \partial_\tau G + i\{L, G\}$
- Fermionic coupling are Grassman even
- For the straight line we have $M_J^I = diag(1, -1, -1, -1)$
- Susy conditions $\bar{\eta}_\alpha^I \eta_I^\beta = i(1 + \dot{x}^\mu \sigma_\mu)_\alpha^\beta$ $\quad M_J^I = \delta_J^I + i\eta_J^\beta \bar{\eta}_\beta^I$ for generic circuits

Construction of the generalized cusp(1)



Fermionic coupling

- $\eta^{\alpha}_{l1} = i n_{l1} \eta^{\alpha}_{1} = i \begin{pmatrix} \cos(\vartheta/4) \\ \sin(\vartheta/4) \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} e^{\frac{i\varphi}{4}} \\ e^{\frac{-i\varphi}{4}} \end{pmatrix}$ and $\bar{\eta}_{l1} = n_{l1} \bar{\eta}^{\alpha}_{1}$
- $\eta^{\alpha}_{l2} = i n_{l2} \eta^{\alpha}_{2} = i e^{i\delta} \begin{pmatrix} \cos(\vartheta/4) \\ -\sin(\vartheta/4) \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} e^{\frac{-i\varphi}{4}} \\ e^{\frac{i\varphi}{4}} \end{pmatrix}$ and $\bar{\eta}_{l2} = n_{l2} \bar{\eta}^{\alpha}_{2}$

Construction of the generalized cusp(2)

Coordinates

- $x^0 = 0$, $x^1 = s\cos\frac{\varphi}{2}$, $x^2 = |s|\sin\frac{\varphi}{2}$

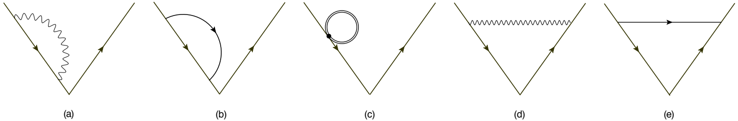
Scalar coupling

- $M_1(\theta) = \begin{pmatrix} -\cos(\vartheta/2) & -\sin(\vartheta/2) & 0 & 0 \\ -\sin(\vartheta/2) & \cos(\vartheta/2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- $M_2(\theta) = \begin{pmatrix} -\cos(\vartheta/2) & \sin(\vartheta/2) & 0 & 0 \\ \sin(\vartheta/2) & \cos(\vartheta/2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Properties

- BPS condition imposes that $\vartheta = \pm\varphi$
- gauge transformation continuous when we cross the cusp implies $\delta = 0$

Perturbative analysis: one loop results



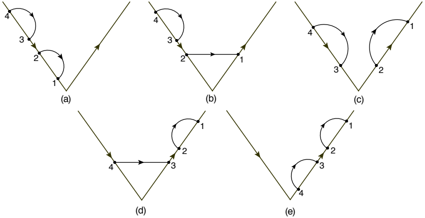
- the exchange of a single scalar is not permitted
- the exchange of a gluon is zero for any planar loop for the antisymmetry of the Levi-Civita tensor
- the only contribution comes from the exchange of a fermion

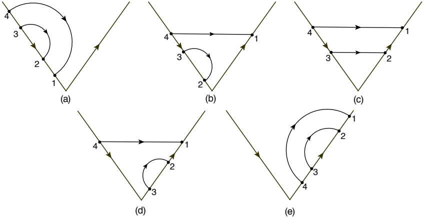
1-loop results: correct BPS condition

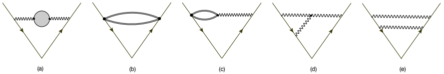
- $< W_{cusp} >^{1-loop} =$
  $$\left(\frac{2\pi}{k}\right) MN\left(\frac{\Gamma(1/2-\epsilon)}{4\pi^{3/2-\epsilon}}\right)(\mu L)^{2\epsilon}\left(\frac{1}{\epsilon}\left(\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}-2\right)-2\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}\log\left(\sec\frac{\varphi}{2}+1\right)\right)$$
- after renormalization $< W_{cusp} >^{1-loop} =$
  $$\left(\frac{2\pi}{k}\right) MN\left(\frac{\Gamma(1/2-\epsilon)}{4\pi^{3/2-\epsilon}}\right)(\mu L)^{2\epsilon}\left(\frac{1}{\epsilon}\left(\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}-1\right)-2\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}\log\left(\sec\frac{\varphi}{2}+1\right)\right)$$

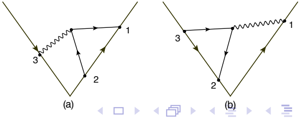
Perturbative analysis: two loop results(1)

- proliferation of diagrams
- there are diagrams which contribute to exponentiation and diagrams which contribute to the potential

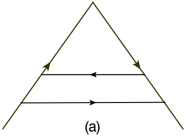




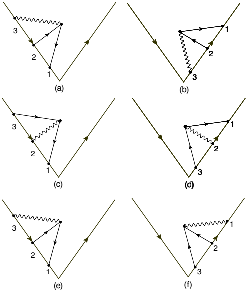


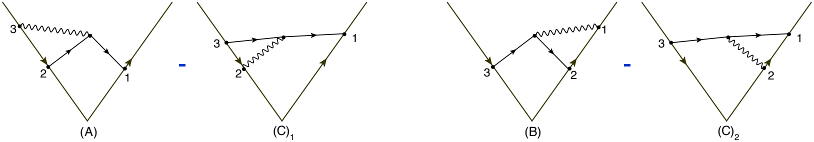


Perturbative analysis: two loop results(2)



(a)





Perturbative analysis: two loop results(3)

- we find a result compatible with this exponentiation

$$< W > \equiv \frac{1}{N+M}(N \exp[-TV_M(\lambda', \vartheta, \varphi)] + M \exp[-TV_N(\lambda, \vartheta, \varphi)])$$

- some diagrams give a contribution 4-d

$$G(x, y) \propto \frac{1}{(x-y)^2}.$$

- we find a similar result to that Drukker-Forini compatible with the Maldacena's proposal $\gamma^{SYM}_{cusp}(\lambda) = \gamma^{ABJM}_{cusp}(h(\lambda))$

$$V^{(2)}(N, M, \vartheta, \varphi) = -4NM(N+M)\frac{\cos\vartheta - \cos\varphi}{\sin\varphi}\varphi$$

- correct BPS condition $\vartheta = \pm\varphi$ also at 2-loop

Conclusions and future developments

Results

- we have constructed a new family of WL in ABJM theory
- we have calculated the generalized cusp at 1-loop and 2-loop
- we find the same BPS condition of Drukker-Forini

Future developments

- calculate the wedge with localization procedure
- verify the relation $\gamma_{cusp}^{SYM}(\lambda) = \gamma_{cusp}^{ABJM}(h(\lambda))$
- classify the Wilson Loops in ABJM theory