Holographic Metals

Valentina Giangreco Marotta Puletti

Chalmers Institute of Technology

Cortona 2012, May 30, 2012

based on arXiv:1110.4601 [hep-th]
(and also on arXiv:1011.6261 [hep-th] + work in progress)

in collaborations with Sean Nowling, Lárus Thorlacius and Tobias Zingg

Overview

- Holography + metals: apply AdS/CFT to Condensed Matter physics (AdS/CMT)

- What do I mean by AdS/CFT here?

  gravity in (asymptotical) AdS_{d+1} (BULK) $\Leftrightarrow$ QFT in $\mathbf{R}^{d-1,1}$ (BOUNDARY)

  weakly/strongly coupled duality

  radial coordinate z $\Leftrightarrow$ energy scale

- How do I want to apply AdS/CFT to CMT?

  - as an *effective* theory,
  - in the large N limit and when the QFT is strongly coupled:

  $$\frac{L^2}{\kappa^2} \sim N^{\#} >> 1$$

  where L =AdS curvature radius and $\kappa^2 = 8\pi G_N =$ Newton's constant

- Why?

  - strongly coupled QFT and "traditional" perturbative methods fail here

  - AdS/CFT can give a geometrical picture of these systems

Outline

- Introductions and Motivations
- Friedel oscillations
- Introductions to electron star and AdS hard wall geometry
- Electron star vs AdS hard wall: ...bad and good!
- Conclusions +Future

Intro and Motivations: The "broad" picture

- Goal?

- Holographic description of (2+1) strongly correlated charged fermions at finite density (μ) and at very low temperature (or zero T) ($T \ll \mu$)

- breakdown of Landau-Fermi theory: Non-Fermi Liquid (NFL)

- Why?

- FL can be tuned to a Quantum Critical Point (QCP)

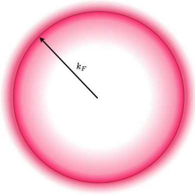
- and develop "strange" metallic behaviour (NFL : $r \sim T$)

- Non-Fermi Liquid:

  - central role of FERMI SURFACE (FS) (but no quasi-particles)

  - anisotropic scaling properties: $\omega \to \lambda\,\omega$, $k \to \lambda^s k$,

    with $s =$ critical dynamical exponent



Intro and Motivations: The "broad" picture

- Goal?

- Holographic description of (2+1) strongly correlated charged fermions at finite density (μ) and at very low temperature (or zero T) ($T \ll \mu$)
- breakdown of Landau-Fermi theory: Non-Fermi Liquid (NFL)

- Why?

- FL can be tuned to a Quantum Critical Point (QCP)
- and develop "strange" metallic behaviour (NFL : $r \sim T$)
- Non-Fermi Liquid:
  - central role of FERMI SURFACE (FS) (but no quasi-particles)

$$G^{-1}(\omega = 0, k = k_F) = 0 \qquad G: \text{ fermionic Green's function}$$

  - anisotropic scaling properties: $\omega \to \lambda\,\omega$, $k \to \lambda^s k$,

    with s = critical dynamical exponent

Intro and Motivations: Our work

- **Goal:**
  - Which are the "good" ingredients in bulk? Test holographic models on the market!
  - Fermi surface is our key but how is it encoded in a holographic geometry?
  - When do *bulk* Fermi features (FS) induce *boundary* Fermi features (FS)?
- **How to study?**
  - probe fermion approximation: spectral function shows poles (= Fermi Surfaces) in holographic models [Liu,McGreevy,Vegh '09],[Hartnoll, Hofman, Tavanfar '11],[Cubrovic, Liu, Schalm, Sun, Zaanen '11];
  - the notion of Fermi surface must be consistently encoded in all the observables!
  - use the "internal" d.o.f. to study the low-energy dynamics ($\neq$ probe approx): Friedel oscillations as diagnostic!
- **Strategy:**
  - search for Friedel oscillations in the response function for two holographic models (electron star and hard wall AdS)

Friedel oscillations

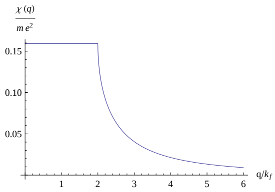
- What?

- oscillations in configuration space present in static response functions, like current current correlation functions, at very low temperature (also $T = 0$);
- due to the presence of a sharp Fermi surface: this is why we use them as a diagnostic!
- present in FL and NFL

- Example: Non relativistic degenerate fermions in (2+1) dimensions:

  ρ = the density current

$$\langle \delta\rho(k) \rangle = \chi(k)\delta A_t(k)\,, \qquad \chi(k) \sim \langle \rho(-k)\rho(k) \rangle$$



$2k_F$ singularity!

Friedel oscillations

▶ What?

- oscillations in configuration space present in static response functions, like current current correlation functions, at very low temperature (also $T = 0$);
- due to the presence of a sharp Fermi surface: this is why we use them as a diagnostic!
- present in FL and NFL

▶ Example: Non relativistic degenerate fermions in (2+1) dimensions:

ρ = the density current

$$\langle \delta\rho(k) \rangle = \chi(k)\delta A_t(k)\,, \qquad \chi(k) \sim \langle \rho(-k)\rho(k) \rangle$$

$$\begin{aligned}
\chi(k) \equiv \langle \rho(-k)\rho(k) \rangle_R = -\Pi_R(k) \quad &= \quad -\frac{e^2}{2\pi^2}\mathcal{P}\int dp\, d\theta\, \theta(k_f - p)\frac{p}{E_{p+k} - E_p} \\
&= \quad -\frac{e^2 m}{2\pi}\left(1 - \sqrt{1 - \left(\frac{2k_f}{k}\right)^2}\theta(k - 2k_f)\right)\,.
\end{aligned}$$

Friedel oscillations

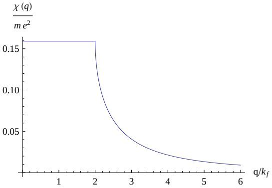
- **What?**

- oscillations in configuration space present in static response functions, like current current correlation functions, at very low temperature (also $T = 0$);
- due to the presence of a sharp Fermi surface: this is why we use them as a diagnostic!
- present in FL and NFL

- **Example**: Non relativistic degenerate fermions in (2+1) dimensions:

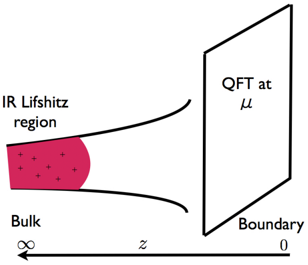
ρ = the density current

$$\langle \delta\rho(k) \rangle = \chi(k)\delta A_t(k)\,, \qquad \chi(k) \sim \langle \rho(-k)\rho(k) \rangle$$



$2k_F$ singularity!

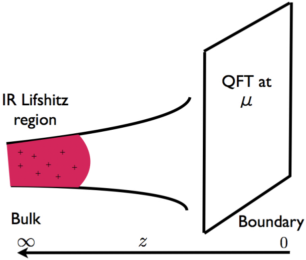
Intro to Electron Star (ES) [Hartnoll et al. '09] [Arsiwala et al. '10] [Hartnoll&Tavanfar'10]



- Ingredients in the bulk:

- Maxwell gauge field: $A_t = \frac{eL}{\kappa} h(z)$ $\quad \lim_{z\to 0} A_t = \mu_{|\partial}$
- high density of fermions (Thomas-Fermi approx):
  - $T = 0$ perfect fluid of free charged fermions in the **bulk**
  - fermions are in a **local** Lorentz frame (LL) at each value of z: $\mu_{\text{loc}}(z) = \frac{A_t}{\sqrt{-g_{tt}}}$
  - fluid thermodynamic variables: $p(z)$, $\rho(z)$ and $\sigma(z)$ with $\mu_{\text{loc}}(z)$
- asymptotically AdS metric: $ds^2 = \frac{L^2}{z^2}(-f(z)dt^2 + g(z)dz^2 + dx^2 + dy^2)$

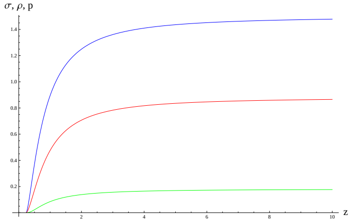
Intro to Electron Star (ES) [Hartnoll et al. '09] [Arsiwala et al. '10] [Hartnoll&Tavanfar'10]



- The action

$$
\begin{aligned}
S &= S_{HE} + S_M + S_{fluid} = \\
&= \frac{1}{2\kappa^2}\int d^4x\sqrt{-G}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}\int d^4x\sqrt{-G}\,F_{\mu\nu}F^{\mu\nu} + \int d^4x\sqrt{-G}\,p
\end{aligned}
$$

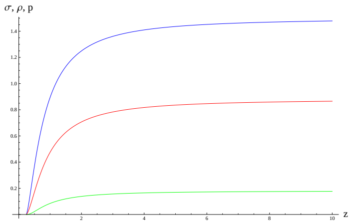
Intro to Electron Star (ES) [Hartnoll et al. '09] [Arsiwala et al. '10] [Hartnoll&Tavanfar'10]



- <u>The action</u>

$$
\begin{aligned}
S &= S_{HE} + S_M + S_{fluid} = \\
&= \frac{1}{2\kappa^2}\int d^4x\sqrt{-G}\left(R+\frac{6}{L^2}\right) - \frac{1}{4e^2}\int d^4x\sqrt{-G}\,F_{\mu\nu}F^{\mu\nu} + \int d^4x\sqrt{-G}\,p
\end{aligned}
$$

Intro to Electron Star (ES) [Hartnoll et al. '09] [Arsiwala et al. '10] [Hartnoll&Tavanfar'10]



- Properties of ES: Why this model?

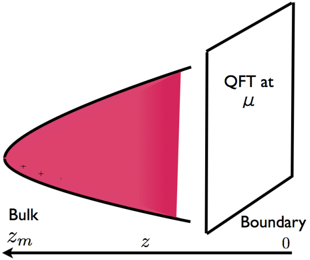
- back-reaction of the metric w.r.t. fluid in a controlled approximation

- In the interior IR emergent Lifshitz geom $\Rightarrow$ in the boundary emergent critical exponent s!

$$f = z^{-2s+2}\,, g = g_\infty\,, h = h_\infty \quad \Rightarrow \quad (z,x,y) \to \lambda(z,x,y)\,, t \to \lambda^s t$$

- promising geometry to characterise metallic quantum criticality!

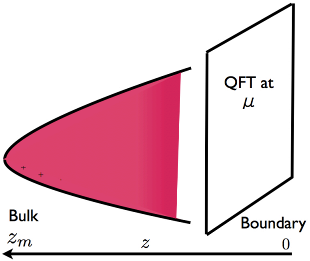
Intro to AdS Hard Wall (AdS-HW) [Sachdev'11]



▶ Ingredients in the bulk:

- the gauge field $A_t = \frac{eL}{\kappa} h(z)$
- the metric is frozen: AdS_4 truncated at $z = z_m$ (hard wall boundary cond at z_m)
  $ds^2 = \frac{L^2}{z^2}(-dt^2 + dz^2 + dx^2 + dy^2)$
- fermions of charge q and mass m: Ψ = bulk single-particle wave function

Intro to AdS Hard Wall (AdS-HW) [Sachdev'11]

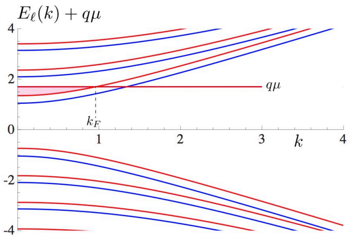


- The action:

$$
\begin{aligned}
S &= S_M + S_D \\
&= -\frac{1}{4e^2}\int d^4x\,\sqrt{-G}\,F_{\mu\nu}\,F^{\mu\nu} + \int d^4x\,\sqrt{-G}\,i\left(\bar{\Psi}\Gamma^{\mu}D_{\mu}\Psi + m\bar{\Psi}\Psi\right)
\end{aligned}
$$

Intro to AdS Hard Wall (AdS-HW) [Sachdev'11]

- Bulk Fermion Spectrum



Picture from [Sachdev'11]

- Properties

- it is a confining geometry: confinement scale sets the spacing
- different role for the fermions: here are really treated QM
- discrete number of **bulk** Fermi surfaces

Go ahead!

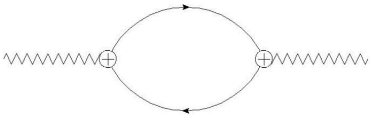
▶ Summary so far

- Friedel oscillations as a diagnostic in order to detect signals of Fermi Surface in ES and AdS-HW.
- Compute static current-current correlation functions, which are bosonic observables
- but we want to detect a fermionic structure... 1-loop diagrams!

Go ahead!

▶ Summary so far

- Friedel oscillations as a diagnostic in order to detect signals of Fermi Surface in ES and AdS-HW.
- Compute static current-current correlation functions, which are bosonic observables
- but we want to detect a fermionic structure... 1-loop diagrams!



Go ahead!

▶ Summary so far

- Friedel oscillations as a diagnostic in order to detect signals of Fermi Surface in ES and AdS-HW.
- Compute static current-current correlation functions, which are bosonic observables
- but we want to detect a fermionic structure... 1-loop diagrams!

▶ How do we proceed?

- Insert a "disturbance" in the system
  - AdS-HW: δA_t a perturbation in the gauge sector
  - ES: $\delta A_x, \delta g_{tx}, \delta g_{yx}, \delta u_x$, shear modes
- Compute the correlators:
  - AdS HW: $\langle \rho(-k)\rho(k) \rangle$ with $1/N$ corrections (density-density correlator)
  - ES: $\langle J_x(-k) J_x(k) \rangle$ with $1/N$ corrections (induced magnetic effect correlator)

Friedel oscillations in AdS Hard Wall

- Set-up

- introduce an effective term in the action

$$S_{Pol} = \frac{e^2}{2} \int dz dz' d\vec{x}\, A_\mu(z, -\vec{k}) \Pi^{\mu\nu}(z, z', \vec{k}) A_\nu(z', \vec{k}) \,.$$

- with the 1-loop vacuum polarization tensor

$$\Pi^{\mu\nu}(z, z', \nu, \vec{k}) = -\int \frac{d\omega d^2 p}{(2\pi)^3} \mathrm{Tr}\, \left[M^\mu G(\omega, p, z, z') M^\nu G(\omega + \nu, k + p, z', z) \right] \,,$$

where $M^\mu = -q\Gamma^0\Gamma^\mu$. We want to compute it for $\nu = 0$.

- G is the fermionic Green's function (in Lemhann representation):

$$G(\omega, k, z, z') = \sum_{\ell \neq 0} \left(\frac{1}{\omega - E_\ell(k) + i\eta\, \mathrm{sign}(E_\ell(k))} \right) \chi_{\ell,k}(z) \chi^\dagger_{\ell,k}(z') \,,$$

Friedel oscillations in AdS Hard Wall

- Set-up

- introduce an effective term in the action

$$S_{Pol} = \frac{e^2}{2}\int dz dz' d\vec{x}\, A_\mu(z,-\vec{k})\Pi^{\mu\nu}(z,z',\vec{k})A_\nu(z',\vec{k})\,.$$

- with the 1-loop vacuum polarization tensor

$$\Pi^{\mu\nu}(z,z',\nu,\vec{k}) = -\int \frac{d\omega d^2p}{(2\pi)^3}\mathrm{Tr}\,\left[M^\mu G(\omega,p,z,z')M^\nu G(\omega+\nu,k+p,z',z)\right]\,,$$

where $M^\mu = -q\Gamma^0\Gamma^\mu$. We want to compute it for $\nu = 0$.

- G is the fermionic Green's function (in Lemhann representation):

$$G(\omega,k,z,z') = \sum_{\ell\neq 0}\left(\frac{1}{\omega - E_\ell(k) + i\eta\,\mathrm{sign}(E_\ell(k))}\right)\chi_{\ell,k}(z)\chi^\dagger_{\ell,k}(z')\,,$$

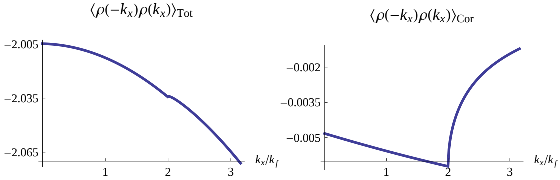
- Concrete computations

- The gap discretizes the spectrum: we can isolate one single **bulk** Fermi surface
- approximate the dispersion relation $E_\ell(k)$ with a non-relativistic fermion
- use for the Π the expression for the 2+1 non-relativistic fermions

$$\Pi^{\mu\nu}_{rel}(z,z',\vec{p}) \sim \#\mathrm{Tr}\left(\Gamma^\mu\chi\chi^\dagger_{|k}\Gamma^\nu\chi\chi^\dagger_{|k+p}\right)_{|z,z'}\times\mathcal{P}\int\frac{d^2k}{(2\pi)^2}\left(\frac{\theta(|k+p|-k_f)\theta(k_f-|k|)}{E_1(k+p)-E_1(k)}\right)$$

Friedel oscillations in AdS Hard Wall

- Results: we do have a boundary FS! :)



- Key-points

- factorization of the radius w.r.t. the boundary coordinates ⇒ factorization of the radius and the boundary momentum!
- discreteness of the bulk FS (due to the gapped spectrum)
- delocalization of bulk FS: the wave functions fill all the way the space-time, they know all the geometry!

(Absence of!) Friedel oscillations in Electron Star

- Set-up

- the effective action

$$S_{Pol} = \frac{e^2}{2} \int dz dz' d^2k \sqrt{|g(z)||g(z')|}\delta A_\mu(z,-k)\Pi^{\mu\nu}_{CG,EC}(z,z',k)\delta A_\nu(z',k)$$

where $\Pi^{\mu\nu}_{CG,EC}$ is the coarse grained polarization tensor in flat space.

- The fermionic loop: Recall the fermions live in a local Lorentz frame!

$$\Pi^{\mu\nu}_{CG,flat}(z,z',k_{L.L.}) = \delta(z-z')\Pi^{\mu\nu}(0,k_{L.L.})$$

- We need to project in the LL all the quantities entering in Π: $k_{L.L.} = \frac{k}{\sqrt{g_{yy}}} = k\,z$

- Use for Π the expression for (3+1) QED.

$$\text{Singular part:} \quad \Pi^{\mu\nu}(0,k_{L.L.}) \sim N^{\mu\nu}(k_{L.L.}) \ln\left(\frac{k_{L.L.}-2k_f}{k_{L.L.}+2k_f}\right) + \dots,$$

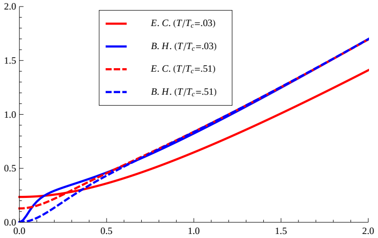
the crucial point are the quantities entering here:

$$k_{L.L.} = z\,k\,, \quad k_F(z) = \sqrt{\mu_{\rm loc}(z)^2 - m^2} = \sqrt{\frac{(z\,A_t(z))^2}{f(z)} - m^2}$$

$\Rightarrow$ the Fermi momentum depends on z: there is a different **bulk** FS for each point in the radial direction!

(Absence of!) Friedel oscillations in Electron Star

▶ Results: we do NOT have a boundary FS! :(



▶ Key-points

- continuum of bulk FS, each at each point in z
- each bulk FS is different: set by different value of the local μ_{loc}
- the bulk FS are very localized:... this is the ultimate meaning of the Thomas-Fermi approx!

▶ Lesson:

each different FS at a different radius will not act coherently $\Rightarrow$ summing from the deep interior (IR) to the boundary (UV) they are just smeared out!

cf. [Kulaxizi, Parnachev '08] for $D4 - D8 - \bar{D}8$

Summary and Future

- Summary:
  - Review of the electron star and AdS hard wall geometry;
  - Brief intro to Friedel oscillations;
  - Strategy to compute current-current correlators including 1-loop effect (effective action with Π);
  - Results: static current-current correlations show boundary FS in hard wall geometries but NOT in the electron star geometry
  - Lesson: In order that bulk FS induces boundary FS in all correlation functions, it is necessary for the bulk FS to be non-local and discrete

- Future:
  - Better "stringy" embedding for AdS Hard wall geometry? Yes: AdS soliton! It works the same!
  - Gravitational back-reaction in AdS Hard wall?
  - What about the RG flow?
  - Non-Fermi liquid?

- Thanks!

Bonus Track I:

- Classical: Bulk-to Boundary propagator in AdS hard wall

- The classical eom for the gauge field in AdS hard wall geom:

$$\left(\partial_z^2 - k_x^2\right) A_0(k_x, z) = 0\,.$$

- the classical bulk to boundary Green's function (*i.e.* the solution which goes to one at the boundary):

$$G_0^{B\partial}(k_x, z) = \cosh(k_x z) - \tanh(k_x z_m)\sinh(k_x z) \Rightarrow \langle \rho(-k_x)\rho(k_x)\rangle_0 = -k_x \tanh(k_x z_m)$$

- One-loop:

- the one-loop corrected eom for the gauge field:

$$\left(\partial_z^2 - k_x^2\right) A_0(k_x, z) \quad = \quad -e^2 \int dz' \left[\Pi_{rel}^{00}(z, z', k_x) A_0(k_x, z') + \Pi_{rel}^{0x}(z, z', k_x) A_x(k, z')\right]$$

- with a high confinement scale and a small Fermi surface volume, the diffeo-integral equation can be solved perturbatively in $\lambda \sim \frac{1}{m_\star^2}$ for the bulk to boundary Green's functions

$$G^{B\partial}(k_x, z) = G_0^{B\partial}(k_x, z) + \lambda G_1^{B\partial}(k_x, z) + \ldots. \qquad (t - \text{component})$$

$$\begin{aligned} 0 &= \left(\partial_z^2 - k_x^2\right) G_0^{B\partial}(k_x, z)\,, \\ 0 &= \left(\partial_z^2 - k_x^2\right) G_1^{B\partial}(k_x, z) + \frac{e^2}{\lambda}\int dz' \Pi_{rel}^{00}(z, z', k_x) G_0^{B\partial}(k_x, z') \end{aligned}$$

Bonus Track II: Polarization tensor in AdS hard wall

- What we are computing:

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}|_{vac} + \Pi^{\mu\nu}_{rel} \; .$$

$$\begin{aligned}\Pi^{\mu\nu}_{rel}(z,z',\vec{k}) = & - \int \frac{d\omega\, d^2p}{(2\pi)^3} \mathrm{Tr}\left[M^\mu G(\omega,\vec{p},z,z') M^\nu G(\omega,\vec{k}+\vec{p},z',z)\right] \\ & - \mathrm{Tr}\left[M^\mu G^0(\omega,\vec{p},z,z') M^\nu G^0(\omega,\vec{k}+\vec{p},z',z)\right] \; .\end{aligned}$$

$$G(\omega,k,z,z') = \sum_{\ell\neq 0}\left(\frac{1}{\omega - E_\ell(k) + i\,\eta\,\mathrm{sign}(E_\ell(k))}\right)\chi_{\ell,k}(z)\chi^\dagger_{\ell,k}(z') \; ,$$

- Isolating one-single Fermi surface:

$$\Pi^{\mu\nu}_{rel}(z,z',\vec{p}) = \Pi^{\mu\nu}_{analytic} + \mathrm{Tr}\; M^\mu\; \chi_{1,k}(z)\chi^\dagger_{1,k}(z')\, M^\nu\; \chi_{1,k+p}(z')\chi^\dagger_{1,k+p}(z) \times$$

$$\int \frac{d^2k}{(2\pi)^2}\left(\frac{\theta(|k+p|-k_f)\theta(k_f-|k|)}{E_1(k+p)-E_1(k)+i\eta \mathrm{s}(E_1(k))} - \frac{\theta(k_f-|k+p|)\theta(|k|-k_f)}{E_1(k+p)-E_1(k)-i\eta \mathrm{s}(E_1(k))}\right)$$

- Non-relativistic approximation: $E_1(k) \sim a_1 + b_1 k^2$

$$\begin{aligned}&\Pi^{\mu\nu}_{rel}(z,z',\vec{p}) = \\ &= \frac{1}{pb_1}\int \frac{d^2k}{(2\pi)^2}\Big[\frac{\theta(E_1(k+p))\theta(-E_1(k))}{p+2\cos(\theta)k+i\eta\mathrm{s}(E_1(k))} \times \mathrm{Tr}\left(M^\mu\; \chi_{1,k}(z)\chi^\dagger_{1,k}(z') M^\nu\; \chi_{1,k+p}(z')\chi^\dagger_{1,k+p}(z)\right) \\ &- \frac{\theta(E_1(k-p))\theta(-E_1(k))}{p-2\cos(\theta)k-i\eta\mathrm{s}(E_1(k))}\mathrm{Tr}\left(M^\mu\; \chi_{1,k-p}(z)\chi^\dagger_{1,k-p}(z') M^\nu\; \chi_{1,k}(z')\chi^\dagger_{1,k}(z)\right)\Big]\end{aligned}$$

Bonus Track II: Polarization tensor in AdS hard wall

- The static polarization is purely real:

$$\Pi^{\mu\nu}_{rel}(z,z',\vec{p}) = \frac{1}{4b_1\pi^2 p}\mathcal{P}\int dk d\theta\ \frac{k\,\theta(k_f-k)}{p/2+k\cos(\theta)}\mathrm{Tr}\left(M^\mu\ \chi_{1,k}(z)\chi^\dagger_{1,k}(z')M^\nu\ \chi_{1,k+p}(z')\chi^\dagger_{1,k+p}(z)\right)$$

- further approx: k_f is small which means that the magnitude of the loop momentum is never large and that the wave functions are slowly varying functions of the momenta:

$$\mathrm{Tr}\left(M^0\ \chi_{1,k}(z)\chi^\dagger_{1,k}(z')M^0\ \chi_{1,k+p}(z')\chi^\dagger_{1,k+p}(z)\right) \sim \mathrm{Tr}\left(M^0\ \chi_{1,0}(z)\chi^\dagger_{1,0}(z')M^0\ \chi_{1,p}(z')\chi^\dagger_{1,p}(z)\right)\ .$$

This approximation loses some of the angular information in the wave-functions, but is sufficient to display the essential features.

- The result:

$$\Pi^{\mu\nu}_{rel}(z,z',\vec{p}) = \frac{1}{4b_1\pi^2 p}\mathrm{Tr}\left(M^\mu\ \chi_{1,0}(z)\chi^\dagger_{1,0}(z')M^\nu\ \chi_{1,p}(z')\chi^\dagger_{1,p}(z)\right)\mathcal{P}\int dk d\theta\ \frac{k\,\theta(k_f-k)}{p/2+k\cos(\theta)}$$

- Introducing the effective expansion parameter, $\lambda \equiv \frac{1}{4\pi b_1}$,

$$\Pi^{\mu\nu}_{rel}(z,z',\vec{p}) = -\lambda\mathrm{Tr}\left(M^\mu\ \chi_{1,0}(z)\chi^\dagger_{1,0}(z')M^\nu\ \chi_{1,p}(z')\chi^\dagger_{1,p}(z)\right)\left(1-\sqrt{1-\left(\frac{2k_f}{p}\right)^2}\theta(p-2k_f)\right)$$

This is the form of the polarization used in the numerics.