Corrections to Navier-Stokes equations from bulk fermionic bilinears

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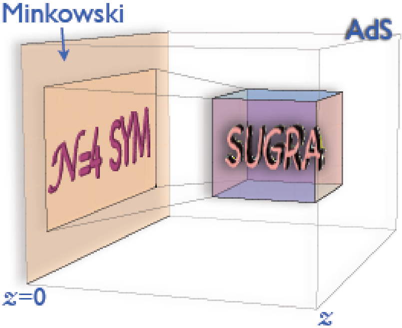
11 maggio 2012

Contents

The AdS/CFT correspondence

The AdS/CFT correspondence relates the type IIB string on $AdS_5 \times \mathcal{S}^5$ with a stack of N D3-brane to a dual to an $SU(N)$ gauge theory, $3+1$-dimensional on AdS boundary. Such a theory will be an $\mathcal{N}=4$ CFT.



Parameters

String theory

- String lenght $\ell = \sqrt{\alpha'}$
- String coupling g_s
- AdS radius R

- $\alpha' \to 0$ or $R \to \infty$
  $\Rightarrow \frac{R^4}{\alpha'^2} = 4\pi g_s N = \lambda \to \infty$
- Pure supergravity

Gauge theory

- Gauge group $SU(N)$
- 't Hooft coupling constant
  $\lambda = g_{YM}^2 N$

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$$Z_S[\phi_0(x)] = \left\langle e^{\int_{\partial AdS_{d+1}} \phi_0(x)\, \mathcal{O}(x)} \right\rangle_{\mathrm{CFT}}$$

partition function

generating functional of the correlator

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$$\Rightarrow Z_{\text{string}} \approx e^{-S_{\text{sugra}}}$$

Non-linear fluid dynamics from gravity

We start with a black brane in AdS_5

$$\begin{aligned} ds^2 &= -r^2 f(br)dt^2 + \frac{dr^2}{r^2 f(br)} + r^2\delta_{ij}dx^i dx^j \\ f(r) &= 1 - \frac{\mu}{r^4} \end{aligned}$$

Performing a boost and a dilatation we get a "*boosted black brane*"

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$$\begin{aligned} ds^2 &= \frac{dr^2}{r^2 f(br)} + r^2[-f(br)u_\mu u_\nu + P_{\mu\nu}]dx^\mu dx^\nu \\ u^t &= \left(1-\beta^2\right)^{-\frac{1}{2}} \quad u^i = \beta^i\left(1-\beta^2\right)^{-\frac{1}{2}} \end{aligned}$$

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and imposing Einstein's equation for the new metric you get exactly the linearized Navier-Stokes equation for a perfect fluid[Minwalla et al., 08]

$$\begin{aligned} \partial_i b &= \partial_0 \beta_i \\ \partial_i \beta^i &= 3\partial_0 b \end{aligned}$$

It is possible to work a more general frame, promoting Killing vectors parameters to local[L.G.C.G., P.A. Grassi, A. Mezzalira, 11].

Calculation technology

Perturbative expansion

Coefficients computation

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Coefficients computation

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Coefficients computation

- Metric "corrections"
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- Boundary stress-energy tensor
- Exact fluid-dynamics coefficients

Dictionary

(Super)Gravity

Fluid dynamics

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- $b \rightarrow$ B.H. temperature

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- Fluid temperature
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Note that both b and β_i are obtained promoting Killing vectors parameters to local.

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- $b \to$ B.H. temperature
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Fluid dynamics

- Fluid temperature
- Fluid velocity
- Navier-Stokes eqs (strongly coupled)

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- $b \to$ B.H. temperature
- $\beta \to$ Black hole boost velocity
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- Other zero modes

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Note that both b and β_i are obtained promoting Killing vectors parameters to local.

Black hole superpartners

AdS_5 is provided with 8 Killing spinors that solve the Killing spinors equation

$$d\epsilon_0 + \frac{1}{4}\omega^{ab}_{AdS}\Gamma_{ab}\epsilon_0 + \frac{1}{2}e^{a}_{AdS}\Gamma_a\epsilon_0 = 0$$

If we insert this ϵ_0 into the same equation with $\omega_{b.h.}$ and $e^a_{b.h.}$ we get gravitino's variation [Sabra et al. 98, Sabra et al. 99, Sabra et al. 05]

$$d\epsilon_0 + \frac{1}{4}\omega^{ab}_{b.h.}\Gamma_{ab}\epsilon_0 + \frac{1}{2}e^{a}_{b.h.}\Gamma_a\epsilon_0 \neq 0$$

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$$\mathrm{d}\epsilon_0 + \frac{1}{4}\omega^{ab}_{b.h.}\Gamma_{ab}\epsilon_0 + \frac{1}{2}e^a_{b.h.}\Gamma_a\epsilon_0 = \delta\psi$$

Once we decompose in components the Killing spinor equation we obtain

$$\delta\psi_r = \partial_r\epsilon + \frac{1}{2\sqrt{f^2+\frac{\mu}{r^2}}}\Gamma_r\epsilon$$

$$\delta\psi_t = \partial_t\epsilon + \frac{1}{2}\Gamma_t\left[\left(r-\frac{\mu}{r^3}\right)\Gamma_r + \sqrt{f^2+\frac{\mu}{r^2}}\right]\epsilon$$

$$\delta\psi_i = \nabla_i\epsilon + \frac{1}{2}e_i{}^b\Gamma_b\left[\frac{1}{r}\sqrt{f^2+\frac{\mu}{r^2}}\Gamma_r + 1\right]\epsilon$$

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that can be simplified using the identity for AdS spinor and expanding over $\mu = 0$

$$\delta\psi_r = -\frac{1}{4f^3}\frac{\mu}{r^2}\Gamma_r\epsilon \quad \delta\psi_t = \frac{1}{2}\Gamma_t\left(-\frac{1}{r}\Gamma_r + \frac{1}{2f}\right)\frac{\mu}{r^2}\epsilon$$

$$\delta\psi_i = \frac{r}{4f}\hat{\Gamma}_i\Gamma_r\frac{\mu}{r^2}\epsilon$$

Correction to metric

In this way we can work out

$$\delta_{\epsilon}^{2} g_{\mu\nu} = \mathrm{Re}\ \left(\bar{\epsilon}\Gamma_{(\mu}\delta\psi_{\nu)}\right)$$

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$$\delta_\epsilon^2 g_{rr} = \frac{1}{f^2}\frac{2\mu}{r^2 f^2}\lambda N \qquad \delta_\epsilon^2 g_{tt} = f^2\frac{2\mu}{r^2 f^2}\lambda N$$

$$\delta_\epsilon^2 g_{ti} = \tfrac{3\mu}{r^2}\lambda\hat{K}_i$$

and all other zero, where we have defined $\lambda = \varepsilon^\dagger\varepsilon, N = \eta^\dagger\eta, \hat{K}_i = \eta^\dagger\frac{\sigma_j e_i^j}{r}\eta$ and where ε are 2−dimensional Majorana (real) spinors and η are spinors for the sphere.

Correction to Navier-Stokes equations

Once we impose Einstein's equation we get the constraints (large r limit has been taken)

$$(\partial_i \beta_i - 3\partial_0 b) + \frac{1}{2}(w_i \partial_i \lambda + N \partial_0 \lambda) = 0$$
$$(\partial_0 \beta_i - 3\partial_i b) - \frac{1}{2}(N \partial_i \lambda + 3 w_i \partial_0 \lambda) = 0$$

where $w_i = \varepsilon_{ijk} w_{jk}$ and w_{kj} a 3−dimensional antisymmetric matrix of parameters.
Note that the compatibility condition is

$$N^2 = 3 w_i w_i$$

Future development

The next step to take is compute full metric at first order and then the stress energy tensor using Brown and York prescription[Brown, York 93].
We define the constraint that foliate the spacetime at constant r:

$$\Phi = r - c = 0$$

with $c \in \mathcal{R}$.

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We define the constraint that foliate the spacetime at constant r:

$$\Phi = r - c = 0$$

with $c \in \mathcal{R}$. The outward pointing normal vector to the boundary $\partial\mathcal{M}_{r=c}$ is defined as

$$n_M = \frac{\partial_M \Phi}{\sqrt{g^{RS}\partial_R \Phi \partial_S \Phi}}$$

Using n_M we can define the boundary metric γ:

$$\hat{\gamma}_{MN} = g_{MN} - n_M n_N$$

In order to obtain a 4−dimensional metric we have to eliminate from $\hat{\gamma}$ the column and the row corresponding to r:

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$$\gamma_{MN} = \left(\begin{array}{c|cc} \gamma_{rr} & \gamma_{rt} & \gamma_{rj} \\ \hline \gamma_{tr} & & \\ \gamma_{ir} & \multicolumn{2}{c}{\gamma_{\mu\nu}} \end{array}\right)$$

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In a similar fashion we calculate the extrinsic curvature Θ_{MN} and then $\Theta_{\mu\nu}$:

$$\Theta_{MN} = -\frac{1}{2}\left(\nabla_M n_N - \nabla_N n_M\right)$$

Finally we can define our (boundary) stress energy tensor as[Balasubramanian, Kraus 99]

$$T^{\mu\nu} = \frac{1}{8\pi G}\left(\Theta^{\mu\nu} - \Theta\gamma^{\mu\nu} - 3\gamma^{\mu\nu} - \frac{1}{2}G^{\mu\nu}\right)$$

where Θ is defined as the trace of $\Theta^{\mu\nu}$ and $G^{\mu\nu}$ is the Einstein tensor build from $\gamma^{\mu\nu}$. Note that we set $R_{AdS} = 1$.

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- Instead of taking as local functions only the bilinears, one can promote the fermions themselves to be local functions of the boundary coordinates. In that case, one needs to study the constraints coming from the Rarita-Schwinger equations in the same spirit as done in work on Minwalla et al.

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- Instead of taking as local functions only the bilinears, one can promote the fermions themselves to be local functions of the boundary coordinates. In that case, one needs to study the constraints coming from the Rarita-Schwinger equations in the same spirit as done in work on Minwalla et al.
- What is the form of the complete metric transformation? Compute it up to fourth order is the only way to obtain the full stress energy tensor. [L.G.C.G., P.A. Grassi, A. Mezzalira]