# **3D Topological States of Matter and Spin-Charge Separation** *Cortona, Maggio 2012*

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P. Sodano, C.A. Trugenberger and MCD, arXiv:1104.2485 to appear in NJP; C.A. Trugenberger and MCD, arXiv:1105.5375, Phys. Rev. B 84 (2011) 094520; arXiv:1112.3281

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- Conclusions

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- Low energy effective theories are topological field theories: background independent, ground state degeneracy; quasiparticles have fractional statistics. Margin 2012 - p. 3/1

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- Which is the topological field theory that describes this new phase of matter? Topological BF action Cortona, Maggio 2012 - p. 4/18

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gauge invariance:

 $a_\mu \to a_\mu + \partial_\mu \xi \ , b_{\mu\nu} \to b_{\mu\nu} + \partial_\mu \eta_\nu - \partial_\nu \eta_\mu$  .

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$$S_{TM} = S_{BF} + \int d^4 x \left[ \left( \frac{1}{4e^2 \lambda} f_{ij}^2 + \frac{1}{4e^2 \eta} f_{0i}^2 \right) + \left( \frac{1}{2g^2 \eta} g_0^2 + \frac{1}{2g^2 \lambda} g_i^2 \right) \right]$$

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- The model describes strong topological insulator (Moore)

# **3D description of fractionalization**

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- Charge fields must be described by a single scalar degree of freedom; in 3D described by antisymmetric gauge potential  $b_{\mu\nu}$  with gauge invariance under the transformations:  $b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_{\mu}\eta_{\nu} \partial_{\nu}\eta_{\mu}$

### **Phases of BF model in (3+1) dimensions**

- ■  $U(1) \times U(1)$  gauge symmetry ⇒ the dual field strengths contain singularities (Polyakov):
  - chargeons (electric topological defects): world-line described by a (singular) current  $J_{\mu}$  and coupling  $ika_{\mu}J_{\mu}$ ;
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- Chargeons and spinons are represented as topological quasi-particle excitations of charge and spin gauge fields that arise due to the compactness of the corresponding gauge groups mediating the emergent gauge.

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- T = 0 quantum phase structure ( $\alpha = ml ≥ O(1)$ , l = lattice spacing):
  - $\epsilon \gg \frac{k^2 e^2}{\alpha^2}$ ,  $\mu \ll \frac{\pi^2}{e^2}$ ; chargeons condensation phase  $\rightarrow$  top. superconductor
  - intermediate regime; no condensation  $\rightarrow$  top. insulator
  - $\epsilon \ll \frac{k^2 e^2}{\alpha^2}$ ,  $\mu \gg \frac{\pi^2}{e^2}$ ; spinons condensation phase  $\rightarrow$  top. confinement

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- These new modes are the long wavelength fluctuations of the continuous distribution of topological defects that det promoted to a continuous two-form antisymmetric 2012-10.11/18

Spinons are diluted; chargeons get promoted to a continuous two-form antisymmetric field  $B_{\mu\nu}$ :

$$S_{\text{eff}}^{TS} = \int d^4x \; i\pi k \; B_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta} + \frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$
$$+ \frac{1}{12\Lambda^2} H_{\mu\nu\alpha} H_{\mu\nu\alpha}$$

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- Mass arises as a consequence of quantum mechanical condensation of topological excitations: mechanism of topological superconductivity (Allen, Bowick, Lahiri)

• Spinons get promoted to a continuous two-form:  $S_{\text{eff}}^{TC} = \int d^4x \left[ \frac{i\theta}{32\pi^2} \left( B_{\mu\nu} - F_{\mu\nu} \right) \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta} \right. \\ \left. + \frac{1}{4} \left( B_{\mu\nu} - F_{\mu\nu} \right) \left( B_{\mu\nu} - F_{\mu\nu} \right) + \frac{1}{12\Lambda^2} H_{\mu\nu\alpha} H_{\mu\nu\alpha} \right],$ 

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- reabsorbed  $F_{\mu\nu}$ :  $B_{\mu\nu} \rightarrow B_{\mu\nu} + F_{\mu\nu}$

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- The mass term for the Kalb-Ramond fields gives:  $W(C) = \exp(-TA(S)), T \propto \Lambda^2$
- Area-law  $\Rightarrow$  linear potential between charges: topological matter with a compact BF term can realize U(1) confinement via the Stückelberg mechanism.

• 3 marginal term can still be added:  $b_{\mu\nu}b_{\mu\nu}$ ,  $b_{\mu\nu}f_{\mu\nu}$  and  $b_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}b_{\alpha\beta}$  to  $S_{TM}$ 

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- these terms, taken one by one, break the gauge invariance; however:

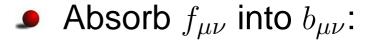
$$S = \int d^4x \left[ \frac{ik^2}{2\theta} \left( b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \epsilon_{\mu\nu\alpha\beta} \left( b_{\alpha\beta} + \frac{\theta}{4\pi k} f_{\alpha\beta} \right) + \frac{4\pi^2 k^2}{e^2 \theta^2} \left( b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \left( b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \right],$$

is gauge invariant if

$$b_{\mu\nu} \to b_{\mu\nu} + \partial_{\mu}\eta_{\nu} - \partial_{\nu}\eta_{\mu}, a_{\mu} \to a_{\mu} - \frac{4\pi k}{\theta}\eta_{\mu}.$$

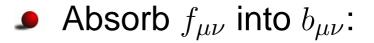
• Absorb  $f_{\mu\nu}$  into  $b_{\mu\nu}$ :

$$S = \int d^4x \left[ \frac{i}{32\theta} b_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} b_{\alpha\beta} + \frac{\pi^2}{4e^2\theta^2} b_{\mu\nu} b_{\mu\nu} + \frac{i}{16\pi} b_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta} + i b_{\mu\nu} \Phi_{\mu\nu} \right]$$



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- Image the factor  $\theta/\pi$ , that takes over the role of charge unit.
- The original gauge symmetry appears as broken; the scalar longitudinal polarization "eats up" two transverse polarizations:  $a_{\mu}$  and  $b_{\mu\nu}$  merge into a single massive tensor, 3 degrees of freedom.

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• The closed boundaries of the open surfaces represent the world-lines of composite chargeon-spinon fluctuations: charge  $\theta/\pi$ , current  $J_{\mu} = (1/2\pi)\partial_{\nu}\Phi_{\mu\nu}$ . • Compute the induced action for  $\Phi_{\mu\nu}$ :  $S_{QP}$ 

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- $\nu$  represents the (signed) self-intersection number of the world-surface.
- If  $\theta/\pi = 1$  this topological term is just another representation of the spin factor of a point particle with spin 1/2 (Pawelczyk): in this confinement phase chargeons and spinons recombine into a single particle with charge 1 and spin 1/2.

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- 3 possible phases: topological superconductor, topological insulator and charge confinement
- No SSB, photon acquires a topological mass through the BF mechanism or it becomes a massive antisymmetric tensor via the Stückelberg mechanism