3D Topological States of Matter and Spin-Charge Separation

Cortona, Maggio 2012

Maria Cristina Diamantini

Universitá di Perugia

P. Sodano, C.A. Trugenberger and MCD, arXiv:1104.2485 to appear in NJP; C.A. Trugenberger and MCD, arXiv:1105.5375, Phys. Rev. B 84 (2011) 094520; arXiv:1112.3281

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- Conclusions

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- Low energy effective theories are topological field theories: background independent, ground state degeneracy; quasiparticles have fractional statistics

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- Which is the topological field theory that describes this new phase of matter? Topological BF action

Wen's idea

- Excitations over top. ground states described by conserved matter currents : 2D $j_\mu \propto \epsilon_{\mu\nu\alpha}\partial_\nu b_\alpha$ = charge matter current, b_μ $U(1)$ gauge field.

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- gauge invariance:
  $a_\mu \rightarrow a_\mu + \partial_\mu \xi\ ,\ b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_\mu \eta_\nu - \partial_\nu \eta_\mu$.

- $S_{TM} = S_{BF} + \int d^4x \left[\left(\frac{1}{4e^2\lambda} {f_{ij}}^2 + \frac{1}{4e^2\eta} {f_{0i}}^2 \right) + \left(\frac{1}{2g^2\eta} {g_0}^2 + \frac{1}{2g^2\lambda} {g_i}^2 \right) \right]$

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- PT invariant; ground state degeneracy; provides a generalization of fractional statistics to 3D (Semenoff et al); supports edges excitations (Balachandran et al; Moore; Hansson)

- What are λ and η? Coupling to an external e.m. field $iej_\mu A_\mu$, induced action: $S(A) = \int d^4x \frac{1}{2}\left[\frac{1}{\lambda}\mathbf{E}^2 + \frac{1}{\eta}\mathbf{B}^2\right]$

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- The model describes strong topological insulator (Moore)

3D description of fractionalization

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- Massive vectors from SSB carry three degrees of freedom
- BF mechanism describe a vector particle in 3D with a gauge invariant mass: two helicity degrees of freedom, ideal candidates to describe spin fields.
- Charge fields must be described by a single scalar degree of freedom; in 3D described by antisymmetric gauge potential $b_{\mu\nu}$ with gauge invariance under the transformations: $b_{\mu\nu} \to b_{\mu\nu} + \partial_\mu \eta_\nu - \partial_\nu \eta_\mu$

Phases of BF model in (3+1) dimensions

- $U(1) \times U(1)$ gauge symmetry $\Rightarrow$ the dual field strengths contain singularities (Polyakov):
  - chargeons (electric topological defects): world-line described by a (singular) current J_μ and coupling $ika_\mu J_\mu$;
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- $T = 0$ quantum phase structure ($\alpha = ml \geq O(1)$, l = lattice spacing):
  - $\epsilon \gg \frac{k^2 e^2}{\alpha^2}$, $\mu \ll \frac{\pi^2}{e^2}$; chargeons condensation phase $\rightarrow$ top. superconductor
  - intermediate regime; no condensation $\rightarrow$ top. insulator
  - $\epsilon \ll \frac{k^2 e^2}{\alpha^2}$, $\mu \gg \frac{\pi^2}{e^2}$; spinons condensation phase $\rightarrow$ top. confinement

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- Julia-Toulouse mechanism (Quevedo and Trugenberger): the condensation of topological defects in solid state media generates new hydrodynamical modes for the low-energy effective theory
- These new modes are the long wavelength fluctuations of the continuous distribution of topological defects that get promoted to a continuous two-form antisymmetric

Chargeons condensation phase

- Spinons are diluted; chargeons get promoted to a continuous two-form antisymmetric field $B_{\mu\nu}$:

$$S_{\mathrm{eff}}^{TS} = \int d^4x \; i\pi k \; B_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta} + \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

$$+\frac{1}{12\Lambda^2}H_{\mu\nu\alpha}H_{\mu\nu\alpha}$$

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- Mass arises as a consequence of quantum mechanical condensation of topological excitations: mechanism of topological superconductivity (Allen, Bowick, Lahiri)

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- Spinons get promoted to a continuous two-form:
$$S_{\text{eff}}^{TC} = \int d^4x \left[\frac{i\theta}{32\pi^2} \left(B_{\mu\nu} - F_{\mu\nu}\right) \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta} + \frac{1}{4} \left(B_{\mu\nu} - F_{\mu\nu}\right) \left(B_{\mu\nu} - F_{\mu\nu}\right) + \frac{1}{12\Lambda^2} H_{\mu\nu\alpha} H_{\mu\nu\alpha} \right],$$

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- Wilson loop: $W(C) =< \exp i \int_C dx^\mu A_\mu > \to$ Stüc. mech. $\to W(S) =< \exp i \int_C dx^\mu \wedge dx^\nu B_{\mu\nu} >$

Spinons condensation phase

- Spinons get promoted to a continuous two-form:
$S_{\rm eff}^{TC} = \int d^4x \left[\frac{i\theta}{32\pi^2}\left(B_{\mu\nu} - F_{\mu\nu}\right)\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}\right.$
$\left. + \frac{1}{4}\left(B_{\mu\nu} - F_{\mu\nu}\right)\left(B_{\mu\nu} - F_{\mu\nu}\right) + \frac{1}{12\Lambda^2}H_{\mu\nu\alpha}H_{\mu\nu\alpha}\right]$,
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- Area-law $\Rightarrow$ linear potential between charges: topological matter with a compact BF term can realize $U(1)$ confinement via the Stückelberg mechanism.

- 3 marginal term can still be added: $b_{\mu\nu}b_{\mu\nu}$, $b_{\mu\nu}f_{\mu\nu}$ and $b_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}b_{\alpha\beta}$ to S_{TM}

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- these terms, taken one by one, break the gauge invariance; however:

$$S = \int d^4x \left[\frac{ik^2}{2\theta} \left(b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \epsilon_{\mu\nu\alpha\beta} \left(b_{\alpha\beta} + \frac{\theta}{4\pi k} f_{\alpha\beta} \right) + \right.$$

$$\left. + \frac{4\pi^2 k^2}{e^2\theta^2} \left(b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \left(b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \right],$$

is gauge invariant if

$$b_{\mu\nu} \to b_{\mu\nu} + \partial_\mu \eta_\nu - \partial_\nu \eta_\mu, a_\mu \to a_\mu - \frac{4\pi k}{\theta} \eta_\mu.$$

- Absorb $f_{\mu\nu}$ into $b_{\mu\nu}$:

$$S = \int d^4x \left[\frac{i}{32\theta} b_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}b_{\alpha\beta} + \frac{\pi^2}{4e^2\theta^2} b_{\mu\nu}b_{\mu\nu} + \right.$$

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- The original gauge symmetry appears as broken; the scalar longitudinal polarization "eats up" two transverse polarizations: a_μ and $b_{\mu\nu}$ merge into a single massive tensor, 3 degrees of freedom.

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- The closed boundaries of the open surfaces represent the world-lines of composite chargeon-spinon fluctuations: charge θ/π , current $J_\mu = (1/2\pi)\partial_\nu \Phi_{\mu\nu}$.

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- ν represents the (signed) self-intersection number of the world-surface.
- If $\theta/\pi = 1$ this topological term is just another representation of the spin factor of a point particle with spin 1/2 (Pawelczyk): in this confinement phase chargeons and spinons recombine into a single particle with charge 1 and spin 1/2.

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- 3 possible phases: topological superconductor, topological insulator and charge confinement
- No SSB, photon acquires a topological mass through the BF mechanism or it becomes a massive antisymmetric tensor via the Stückelberg mechanism