# The four-dimensional BF theory with a planar boundary 

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four-dimensional
BF theory with a
planar boundary
A. Amoretti

## Outlines

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The physics on
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Figure: Resistance measurements for IQHE and FQHE

$$
S_{C S}=\frac{k}{2 \pi} \int d^{3} x \epsilon^{\mu \nu \rho} A_{\mu} \partial_{\nu} A_{\rho}
$$

回 X. G. Wen, Adv. Phys 44, 1995 (QHE)

Topological field theories and their physical implication

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Figure: Band structure of $\mathrm{Bi}_{2} \mathrm{Se}_{3}$ (ARPES)

Bernevig and Zhang, Phys. Rev. Lett 96, 2006 (TI)
葍 Cho and Moore, Annals Phys. 326, 2011 (TI)
Diamantini et al, Nucl. Phys B, 1995 (Superconductivity)

## The classical theory

The classical action
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$$
\begin{aligned}
& S=\int_{M} d^{4} x \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu} B_{\rho \sigma} \\
& F_{\mu \nu} \equiv \partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu} \\
& {\left[A_{\mu}\right]=1, \quad\left[B_{\mu \nu}\right]=2}
\end{aligned}
$$

Birmigham et al., Phys. Rept., 209, 1991

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## Symmetries

$$
\left\{\begin{array} { l } 
{ \delta ^ { ( 1 ) } A _ { \mu } = - \partial _ { \mu } \theta } \\
{ \delta ^ { ( 1 ) } B _ { \mu \nu } = 0 , }
\end{array} \quad \left\{\begin{array}{l}
\delta^{(2)} A_{\mu}=0 \\
\delta^{(2)} B_{\mu \nu}=-\left(\partial_{\mu} \varphi_{\nu}-\partial_{\nu} \varphi_{\mu}\right)
\end{array}\right.\right.
$$

## The classical theory

We need to fix the gauge

1. Axial gauge:

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## The classical theory

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1. Axial gauge:

$$
A_{3}=B_{i 3}=0, \quad i=0,1,2
$$

2. We fix the gauge by adding to $S$ the term

$$
S_{g f}=\int_{M} d^{4} x\left(b A_{3}+d^{i} B_{i 3}\right)
$$

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3. The residual gauge invariance and the Ward identities:

$$
\begin{aligned}
& \partial_{i} J_{A^{i}}^{i}+\partial_{3} J_{A^{3}}^{3}+\partial_{3} b=0 \\
& \partial_{j} J_{B^{i j}}^{i j}+\partial_{3} J_{B^{i 3}}^{i 3}+\frac{1}{2} \partial_{3} d^{i}=0 .
\end{aligned}
$$

## The boundary $x_{3}=0$

The Symanzik's method
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1. Locality:

$$
\mathcal{L} \sim \delta^{(n)}\left(x_{3}\right) f\left(x_{i}, x_{3}\right)
$$

Separability:


R K. Symanzik, Nucl. Phys B 190, 1981
囯 S. Emery and O. Piguet Helv. Phys. Acta, 64, 1991

## The boundary $x_{3}=0$

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1. Locality:

$$
\mathcal{L} \sim \delta^{(n)}\left(x_{3}\right) f\left(x_{i}, x_{3}\right)
$$

2. Separability:

$$
\begin{gathered}
\Delta_{A B}\left(x, x^{\prime}\right)=\theta\left(x_{3}\right) \theta\left(x_{3}^{\prime}\right) \Delta_{A B}^{+}+\theta\left(-x_{3}\right) \theta\left(-x_{3}^{\prime}\right) \Delta_{A B}^{-} \\
\Delta_{A B}\left(x, x^{\prime}\right)=0 \text { if } x_{3} x_{3}^{\prime}<0
\end{gathered}
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The boundary Lagrangian

$$
\mathcal{L}_{b}=\delta\left(x_{3}\right)\left[\frac{a_{1}}{2} \epsilon^{i j k} \partial_{i} A_{j} A_{k}+a_{2} A_{i} \tilde{B}^{i}+a_{3} \frac{m}{2} A_{i} A^{i}\right]
$$

## The boundary $x_{3}=0$

The boundary conditions
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1. The equations of motion with a boundary term for $A^{i}$ and $\tilde{B}^{i}$ :

$$
\begin{aligned}
& \partial_{3} \tilde{B}^{i}=-\delta\left(x^{3}\right)\left[a_{1} \epsilon^{i j k}\left(\partial_{j} A_{k}\right)^{+}+a_{2} \tilde{B}^{i+}+a_{3} m A^{i+}\right] \\
& \epsilon^{i j k} \partial_{3} A_{k}=a_{2} \delta\left(x^{3}\right) \epsilon^{i j k} A_{k}^{+}
\end{aligned}
$$

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## The boundary $x_{3}=0$

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& \epsilon^{i j k} \partial_{3} A_{k}=a_{2} \delta\left(x^{3}\right) \epsilon^{i j k} A_{k}^{+}
\end{aligned}
$$

2. The algebraic system for the fields on the boundary:

$$
\begin{aligned}
& \left(1+a_{2}\right) \tilde{B}^{i+}=-a_{1} \epsilon^{i j k} \partial_{j} A_{k}^{+}-a_{3} m A^{i+} \\
& \left(1-a_{2}\right) A_{i}^{+}=0
\end{aligned}
$$

The only acceptable boundary condition is:

## The boundary $x_{3}=0$

## The boundary conditions

1. The equations of motion with a boundary term for $A^{i}$ and $\tilde{B}^{i}$ :

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\begin{aligned}
& \partial_{3} \tilde{B}^{i}=-\delta\left(x^{3}\right)\left[a_{1} \epsilon^{i j k}\left(\partial_{j} A_{k}\right)^{+}+a_{2} \tilde{B}^{i+}+a_{3} m A^{i+}\right] \\
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& \left(1-a_{2}\right) A_{i}^{+}=0
\end{aligned}
$$

3. The only acceptable boundary condition is:

$$
\tilde{B}^{i+}=-\frac{a_{3} m}{2} A^{i+}
$$

## The Ward identities and the boundary algebra

 The integrated broken Ward identities:The
four-dimensional
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$$
\begin{aligned}
& \int_{-\infty}^{\infty} d x_{3} \partial_{i} J_{A^{i}}^{i}=\partial_{i} \tilde{B}^{i+} \\
& \int_{-\infty}^{\infty} d x_{3} \epsilon^{i j k} \partial_{j} J_{\tilde{B}^{k}}=-\epsilon^{i j k} \partial_{j} A_{k}^{+}
\end{aligned}
$$

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\end{aligned}
$$

The boundary algebra

$$
\partial_{l} \delta^{(3)}\left(X-X^{\prime}\right)=\partial_{i} \Delta_{A_{l} \tilde{B}^{i}}\left(X^{\prime}, X\right)
$$

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The Ward identities and the boundary algebra
The integrated broken Ward identities:

$$
\begin{aligned}
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The boundary algebra

$$
\partial_{I} \delta^{(3)}\left(X-X^{\prime}\right)=\partial_{i} \Delta_{A_{i} \tilde{B}^{i}}\left(X^{\prime}, X\right)
$$

$$
\begin{aligned}
& {\left[\tilde{B}^{0+}(X), A_{\beta}\left(X^{\prime}\right)\right]_{t=t^{\prime}}=\partial_{\beta} \delta^{(2)}\left(X^{\prime}-X\right)} \\
& {\left[\tilde{B}^{0+}(X), \tilde{B}_{0}^{+}\left(X^{\prime}\right)\right]_{t=t^{\prime}}=0} \\
& {\left[A_{\beta}^{+}(X), A^{\gamma+}\left(X^{\prime}\right)\right]_{t=t^{\prime}}=0}
\end{aligned}
$$

E- A. P. Balachandran et al., Mod.Phys.Lett. A8, 1993

## The dimensional reduction

The new fields and their symmetries
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1.

$$
\left\{\begin{array}{l}
\partial_{i} \tilde{B}^{i+}=0 \\
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$$

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## The dimensional reduction

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1.

$$
\left\{\begin{array} { l } 
{ \partial _ { i } \tilde { B } ^ { i + } = 0 } \\
{ \epsilon ^ { i j k } \partial _ { j } A _ { k } ^ { + } = 0 }
\end{array} \Rightarrow \left\{\begin{array}{l}
\tilde{B}^{i+}=\epsilon^{i j k} \partial_{j} \zeta_{k} \\
A_{i}^{+}=\partial_{i} \Lambda
\end{array}\right.\right.
$$

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$$

2. 

$$
\left\{\begin{array}{l}
\delta \Lambda=\text { const } \\
\delta \zeta_{i}=\partial_{i} \theta
\end{array}\right.
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## The dimensional reduction

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2.

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\left\{\begin{array}{l}
\delta \Lambda=\text { const } \\
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\end{array}\right.
$$

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$$
\epsilon^{i j k} \partial_{j} \zeta_{k}=-\frac{a_{3} m}{2} \partial^{i} \Lambda
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$\epsilon^{i j k} \partial_{j} \zeta_{k}=-\frac{a_{3} m}{2} \partial^{i} \Lambda \Rightarrow\left\{\begin{array}{l}\Lambda \rightarrow \frac{\Lambda}{\sqrt{m}} \\ \zeta_{i} \rightarrow \sqrt{m} \zeta_{i}\end{array} \Rightarrow \epsilon^{i j k} \partial_{j} \zeta_{k}=\partial^{i} \Lambda\right.$

## The dimensional reduction

The canonical commutation relation for the new fields

$$
\begin{aligned}
& {\left[\Lambda(X), \epsilon^{\beta \gamma} \partial_{\beta}^{\prime} \zeta_{\gamma}\left(X^{\prime}\right)\right]_{t=t^{\prime}}=\delta^{(2)}\left(X-X^{\prime}\right)} \\
& {\left[\epsilon^{\gamma \beta} \zeta_{\beta}(X), \partial_{\delta}^{\prime} \Lambda\left(X^{\prime}\right)\right]_{t=t^{\prime}}=\delta_{\delta}^{\gamma} \delta^{(2)}\left(X-X^{\prime}\right)}
\end{aligned}
$$

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& {\left[\epsilon^{\gamma \beta} \zeta_{\beta}(X), \partial_{\delta}^{\prime} \Lambda\left(X^{\prime}\right)\right]_{t=t^{\prime}}=\delta_{\delta}^{\gamma} \delta^{(2)}\left(X-X^{\prime}\right)}
\end{aligned}
$$

The conjugate variables and the boundary Lagrangian

$$
\begin{aligned}
\text { 1. } & x_{0} \sim \Lambda & & p_{0} \sim \epsilon^{\beta \gamma} \partial_{\beta} \zeta_{\gamma} \\
& x_{1,2} \sim \epsilon^{\gamma \beta} \zeta_{\beta} & & p_{1,2} \sim \partial_{\delta} \Lambda
\end{aligned}
$$

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$$
\begin{aligned}
& {\left[\Lambda(X), \epsilon^{\beta \gamma} \partial_{\beta}^{\prime} \zeta_{\gamma}\left(X^{\prime}\right)\right]_{t=t^{\prime}}=\delta^{(2)}\left(X-X^{\prime}\right)} \\
& {\left[\epsilon^{\gamma \beta} \zeta_{\beta}(X), \partial_{\delta}^{\prime} \Lambda\left(X^{\prime}\right)\right]_{t=t^{\prime}}=\delta_{\delta}^{\gamma} \delta^{(2)}\left(X-X^{\prime}\right)}
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The conjugate variables and the boundary Lagrangian

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\text { 1. } \begin{array}{rlr}
x_{0} \sim \Lambda & & p_{0} \sim \epsilon^{\beta \gamma} \partial_{\beta} \zeta_{\gamma} \\
& x_{1,2} \sim \epsilon^{\gamma \beta} \zeta_{\beta} & \\
p_{1,2} \sim \partial_{\delta} \Lambda
\end{array}
$$

2. 

$$
\begin{aligned}
& \mathcal{L}=\sum_{i} p_{i} \dot{x}_{i}-H= \\
& \epsilon^{\alpha \beta} \partial_{\alpha} \zeta_{\beta} \partial_{t} \Lambda+\partial_{\alpha} \Lambda \epsilon^{\alpha \beta} \partial_{t} \zeta_{\beta}-\frac{1}{2}\left(\epsilon^{\alpha \beta} \partial_{\alpha} \zeta_{\beta}\right)^{2}-\frac{1}{2}\left(\partial_{\alpha} \Lambda\right)^{2}
\end{aligned}
$$

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$$
\text { (2D) } \partial_{i} \Lambda+\epsilon_{i j} \partial^{j} \zeta=0 \quad \Rightarrow \quad(3 D) \epsilon^{i j k} \partial_{j} \zeta_{k}=\partial^{i} \Lambda
$$

圁 H. Aratyn, Phys.Rev., 1983
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$\square$

## The physics on the boundary

The duality relation and massless fermionic fields

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$$

围 H. Aratyn, Phys.Rev., 1983
The boundary Lagrangian

$$
\mathcal{L}=\epsilon^{\alpha \beta} \partial_{\alpha} \zeta_{\beta} \partial_{t} \Lambda+\partial_{\alpha} \Lambda \epsilon^{\alpha \beta} \partial_{t} \zeta_{\beta}-\frac{1}{2}\left(\epsilon^{\alpha \beta} \partial_{\alpha} \zeta_{\beta}\right)^{2}-\frac{1}{2}\left(\partial_{\alpha} \Lambda\right)^{2}
$$

R. Cho and J. E. Moore, Annals Phys. 326, 2011

## Conclusions

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1. The physics on the boundary is expressed in terms of a gauge field and a scalar field

The only acceptable boundary condition implies massless fermionic fields
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1. The physics on the boundary is expressed in terms of a gauge field and a scalar field
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R A. P. Balachandran et al., Mod.Phys.Lett. A8, 1993
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5. Future developments:

- Non-abelian extension
- The five-dimensional BF model with a boundary

