The four-dimensional BF theory with a planar boundary

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Outlines

1. The classical theory and the gauge-fixing
2. The introduction of the boundary
3. The physics on the boundary

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Topological field theories and their physical implication

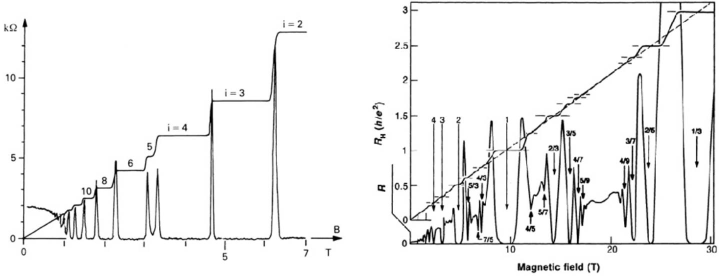

Figure: Resistance measurements for IQHE and FQHE

$$S_{CS} = \frac{k}{2\pi}\int d^3x\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho$$

X. G. Wen, *Adv. Phys 44*, 1995 (QHE)

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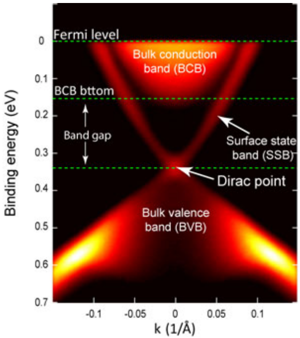

Figure: Band structure of Bi_2Se_3 (ARPES)

- Bernevig and Zhang, *Phys. Rev. Lett 96*, 2006 (TI)
- Cho and Moore, *Annals Phys. 326*, 2011 (TI)
- Diamantini et al, *Nucl. Phys B*, 1995 (Superconductivity)

The classical theory

The classical action

$$S = \int_M d^4x \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} B_{\rho\sigma}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$[A_\mu] = 1, \ \ [B_{\mu\nu}] = 2$$

Birmigham et al., *Phys. Rept., 209*, 1991

Symmetries

$$\begin{cases} \delta^{(1)} A_\mu = -\partial_\mu \theta \\ \delta^{(1)} B_{\mu\nu} = 0, \end{cases} \qquad \begin{cases} \delta^{(2)} A_\mu = 0 \\ \delta^{(2)} B_{\mu\nu} = -(\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu). \end{cases}$$

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The classical theory

We need to fix the gauge

1. Axial gauge:

$$A_3 = B_{i3} = 0, \qquad i = 0, 1, 2$$

2. We fix the gauge by adding to S the term

$$S_{gf} = \int_M d^4x (bA_3 + d^i B_{i3})$$

3. The residual gauge invariance and the Ward identities:

$$\partial_i J^i_{A^i} + \partial_3 J^3_{A^3} + \partial_3 b = 0$$
$$\partial_j J^{ij}_{B^{ij}} + \partial_3 J^{i3}_{B^{i3}} + \frac{1}{2}\partial_3 d^i = 0.$$

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The boundary $x_3 = 0$

The Symanzik's method

1. **Locality:**

$$\mathcal{L} \sim \delta^{(n)}(x_3) f(x_i, x_3)$$

2. **Separability:**

$$\Delta_{AB}(x, x') = \theta(x_3)\theta(x'_3)\Delta^{+}_{AB} + \theta(-x_3)\theta(-x'_3)\Delta^{-}_{AB}$$
$$\Delta_{AB}(x, x') = 0 \;\; if \;\; x_3 x'_3 < 0$$

K. Symanzik, *Nucl. Phys B 190*, 1981

S. Emery and O. Piguet *Helv. Phys. Acta, 64*, 1991

The boundary Lagrangian

$$\mathcal{L}_b = \delta(x_3)\left[\frac{a_1}{2}\epsilon^{ijk}\partial_i A_j A_k + a_2 A_i \tilde{B}^i + a_3 \frac{m}{2} A_i A^i\right]$$

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The boundary conditions

1. The equations of motion with a boundary term for A^i and $\tilde{B}^i$:

$$\partial_3 \tilde{B}^i = -\delta(x^3)[a_1 \epsilon^{ijk}(\partial_j A_k)^+ + a_2 \tilde{B}^{i+} + a_3 m A^{i+}]$$
$$\epsilon^{ijk}\partial_3 A_k = a_2 \delta(x^3)\epsilon^{ijk} A_k^+$$

2. The algebraic system for the fields on the boundary:

$$(1 + a_2)\tilde{B}^{i+} = -a_1 \epsilon^{ijk}\partial_j A_k^+ - a_3 m A^{i+}$$
$$(1 - a_2) A_i^+ = 0$$

3. The only acceptable boundary condition is:

$$\tilde{B}^{i+} = -\frac{a_3 m}{2} A^{i+}$$

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The Ward identities and the boundary algebra

The integrated broken Ward identities:

$$\int_{-\infty}^{\infty} dx_3 \partial_i J^i_{A^i} = \partial_i \tilde{B}^{i+}$$

$$\int_{-\infty}^{\infty} dx_3 \epsilon^{ijk} \partial_j J_{\tilde{B}^k} = -\epsilon^{ijk} \partial_j A^+_k$$

The boundary algebra

$$\partial_l \delta^{(3)}(X - X') = \partial_i \Delta_{A_l \tilde{B}^i}(X', X)$$

$$[\tilde{B}^{0+}(X), A_\beta(X')]_{t=t'} = \partial_\beta \delta^{(2)}(X' - X)$$
$$[\tilde{B}^{0+}(X), \tilde{B}^+_0(X')]_{t=t'} = 0$$
$$[A^+_\beta(X), A^{\gamma+}(X')]_{t=t'} = 0$$

A. P. Balachandran et al., *Mod.Phys.Lett. A8*, 1993

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The dimensional reduction

The new fields and their symmetries

1.
$$\begin{cases} \partial_i \tilde{B}^{i+} = 0 \\ \epsilon^{ijk}\partial_j A_k^+ = 0 \end{cases} \Rightarrow \begin{cases} \tilde{B}^{i+} = \epsilon^{ijk}\partial_j \zeta_k \\ A_i^+ = \partial_i \Lambda \end{cases}$$

2.
$$\begin{cases} \delta\Lambda = const \\ \delta\zeta = \partial\theta \end{cases}$$

The boundary condition and the duality relation

$$\epsilon^{ijk}\partial_j\zeta_k = -\frac{a_3 m}{2}\partial^i\Lambda \Rightarrow \begin{cases} \Lambda \to \frac{\Lambda}{\sqrt{m}} \\ \zeta_i \to \sqrt{m}\zeta_i \end{cases} \Rightarrow \epsilon^{ijk}\partial_j\zeta_k = \partial^i\Lambda$$

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The dimensional reduction

The canonical commutation relation for the new fields

$$[\Lambda(X), \epsilon^{\beta\gamma}\partial'_\beta\zeta_\gamma(X')]_{t=t'} = \delta^{(2)}(X - X')$$

$$[\epsilon^{\gamma\beta}\zeta_\beta(X), \partial'_\delta\Lambda(X')]_{t=t'} = \delta^\gamma_\delta\delta^{(2)}(X - X')$$

The conjugate variables and the boundary Lagrangian

1. $x_0 \sim \Lambda \qquad p_0 \sim \epsilon^{\beta\gamma}\partial_\beta\zeta_\gamma$

   $x_{1,2} \sim \epsilon^{\gamma\beta}\zeta_\beta \qquad p_{1,2} \sim \partial_\delta\Lambda$

2. $$\mathcal{L} = \sum_i p_i\dot{x}_i - H =$$

   $$\epsilon^{\alpha\beta}\partial_\alpha\zeta_\beta\partial_t\Lambda + \partial_\alpha\Lambda\epsilon^{\alpha\beta}\partial_t\zeta_\beta - \frac{1}{2}(\epsilon^{\alpha\beta}\partial_\alpha\zeta_\beta)^2 - \frac{1}{2}(\partial_\alpha\Lambda)^2$$

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The physics on the boundary

The duality relation and massless fermionic fields

$$(2D)\ \ \partial_i\Lambda + \epsilon_{ij}\partial^j\zeta = 0 \qquad \Rightarrow \qquad (3D)\ \ \epsilon^{ijk}\partial_j\zeta_k = \partial^i\Lambda$$

H. Aratyn, *Phys.Rev.*, 1983

The boundary Lagrangian

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Conclusions

1. The physics on the boundary is expressed in terms of a gauge field and a scalar field
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3. There are three pairs of canonical variables on the boundary
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