ON THE CUBIC INTERACTIONS OF MASSIVE AND PARTIALLY-MASSLESS HIGHER SPINS IN (A)DS

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Motivations

- String Theory (ST) $\supset \infty$ massive HS: $\varphi_{\mu_1 \ldots \mu_{s_1}\,;\,\nu_1 \ldots \nu_{s_2}\,;\,\ldots}$.
  Important for:
  - Planar duality, open-closed duality, modular invariance
  - Soft high energy behaviour of the amplitudes
  - Deeper understanding of ST: geometry, quantum DoF,. . .

  Tantalizing idea: ST $\equiv$ "broken fase" of HS gauge theory

  [Gross; Sundborg; Sezgin-Sundell; Klebanov-Polyakov; Bianchi-Morales-Samtleben; Bianchi-Riccioni;. . .]
- Many challenges to theoretical physics:
  - ∞-dimensional HS gauge symmetries (∞ HS) [Vasiliev]
  - Vasiliev's system in $AdS_4 \overset{\text{dual}}{\longleftrightarrow}$ free theory

    [Sezgin-Sundell; Klebanov-Polyakov; Giombi-Yin]

No-go & yes-go results

No-go:

- Massless HS in flat space
  - No long-range interactions [Weinberg]
  - No minimal coupling to gravity [Weinberg-Witten; Porrati; Aragone-Deser]
  - No HS generators: $G = \mathcal{P} \oplus g$ [Coleman-Mandula]
- Massive HS in flat space
  - Unphysical DoF and loss of causality [Fierz-Pauli; Velo-Zwanziger]

Yes-go:

- Massless HS
  - "Minimal-like" interactions avaible in (A)dS [Fradkin-Vasiliev]
  - Vasiliev system: ∞ symmetric HS [Vasiliev]
  - Higher-derivative cubic vertices in flat space (light-cone) [Metsaev]
- Massive HS $\in$ Metsaev $\supset$ ST

HS in flat space

1. Metric-like approach: generalization of $g_{\mu\nu}$ [Singh-Hagen; Fronsdal; Fang-Fronsdal]
2. Frame-like approach: generalization of (e^a, ω^{ab}) [Vasiliev]

1. Massive & massless spin-s: [Dirac; Fierz-Pauli]

$$
\begin{aligned}
(\Box - m^2)\,\varphi_{\mu_1\ldots\mu_s} &= 0 && \text{mass-shell} \\
\partial^{\mu_1}\,\varphi_{\mu_1\ldots\mu_s} &= 0 && \mathcal{P} \to SO(d-1) \\
\eta^{\mu_1\mu_2}\,\varphi_{\mu_1\ldots\mu_s} &= 0 && \text{irreducibility} \\
\varphi_{\mu_1\ldots\mu_s} &\sim \varphi_{\mu_1\ldots\mu_s} + \partial_{(\mu_1}\varepsilon_{\mu_2\ldots\mu_s)} && \mathcal{P} \to SO(d-2)
\end{aligned}
$$

- Massive: Lagrangians involves extra fields [Singh-Hagen]
- Massless: natural generalizations of $s = 1, 2$: [Fronsdal; Fang-Fronsdal]

$$\Box\varphi_{\mu_1\ldots\mu_s} - (\partial_{\mu_1}\partial\cdot\varphi_{\mu_2\ldots\mu_s} + \ldots) + (\partial_{\mu_1}\partial_{\mu_2}\varphi'_{\mu_3\ldots\mu_s} + \ldots) = 0$$

with constraints: $\varepsilon'_{\mu_1\ldots\mu_{s-3}} = 0\,,\ \varphi''_{\mu_1\ldots\mu_{s-4}} = 0$ (Lagrangian)

HS in flat space

- Unconstrained: extra fields [Buchbinder-Pashnev-Tsulaia; Francia-Mourad-Sagnotti]

Generating function: $\varphi(x,u) = \sum_{s=0}^{\infty} \frac{1}{s!}\, \varphi_{\mu_1 \dots \mu_s}(x)\, u^{\mu_1} \cdots u^{\mu_s}$

- Divergence

$$\partial_u \cdot \partial_x\, \varphi(x,u) = \sum_{s=0}^{\infty} \frac{1}{(s-1)!}\, \partial^{\mu_s} \varphi_{\mu_s \mu_1 \dots \mu_{s-1}}(x)\, u^{\mu_1} \cdots u^{\mu_{s-1}}$$

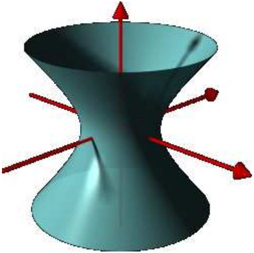
- Trace

$$\partial_u^2\, \varphi(x,u) = \sum_{s=0}^{\infty} \frac{1}{(s-2)!}\, {\varphi'}_{\mu_1 \dots \mu_{s-2}}(x)\, u^{\mu_1} \cdots u^{\mu_{s-2}}$$

- Gradient

$$u \cdot \partial_x\, \varphi(x,u) = \sum_{s=0}^{\infty} \frac{1}{s!}\, \partial_{(\mu_1} \varphi_{\mu_2 \dots \mu_{s+1})}(x)\, u^{\mu_1} \cdots u^{\mu_{s+1}}$$

Ambient-space formalism for HS in (A)dS



$\mathbb{R}^{1,d}$: $ds^2_{\text{Amb}} = \eta_{MN}\, dX^M\, dX^N \quad \eta = (-,+,\ldots,+)$

- de Sitter: $X^2 = L^2$
- Anti se Sitter: $X^2 = -L^2$

Ambient-space HS: $\Phi(X,U) = \left(\frac{R}{L}\right)^{\Delta_h} \varphi(x,u)$ [Fronsdal; Metsaev; Biswas-Siegel,. . .]

- Homogeneity: $X\cdot\partial_X \Phi = \Delta_h \Phi \qquad X = (R, x)$
- Tangentiality: $X\cdot\partial_U \Phi = 0 \qquad U = (v, u)$

Ambient-space formalism for HS in (A)dS

Action in ambient space: $S^{(2)}_{\text{Amb}} \sim \int_0^\infty R^{d+2(s-\mu-3)}\, S^{(2)}_{\text{(A)dS}}$

- $S^{(2)}_{\text{Amb}} \sim \int_0^\infty d^{d+1}X\, \Phi \cdot \partial_X^2\, \Phi + \dots$
- $S^{(2)}_{\text{(A)dS}} = \int d^dx\, \varphi \cdot (\Box_{(A)ds} - m_\mu^2)\, \varphi + \dots$

Diverging factor from the radial integral:

- Remove it introducing $\delta(R-L)$ (total-derivative terms $\neq 0$)
- Work with commuting ∂_{X^M} (great simplification wrt ∇_x)

$\delta^{(0)}\,\Phi = U\cdot\partial_X\, E$ depends on $\Delta_h = U\cdot\partial_U - 2 - \mu$ [Joung-LL-Taronna]

- Massless: $\mu = 0, \quad X\cdot\partial_U\, E = 0, \quad \delta^{(0)}\,\Phi = U\cdot\partial_X\, E$
- Massive: $\mu \notin \{0, \dots, s-1\}, \quad E = 0,$ no gauge sym.
- Partially-massless (only dS): $\mu \in \{0, \dots, s-1\}$,

$$E = (U\cdot\partial_X)^\mu\, \Omega\ ,\ X\cdot\partial_U\, \Omega = 0\ ,\ \delta^{(0)}\,\Phi = (U\cdot\partial_X)^{\mu+1}\, \Omega$$

HS cubic interactions

Noether procedure: [Berends-Burgers-Van Dam;...]

- Deformation of free system: $S = S^{(2)} + S^{(3)} + S^{(4)} + \dots$
- Deformation of gauge sym.: $\delta\varphi = \underbrace{\delta^{(0)}\,\varphi}_{u\cdot\partial_x\varepsilon} + \delta^{(1)}\,\varphi + \delta^{(2)}\,\varphi + \dots$

Gauge invariance $\delta\, S = 0 \rightarrow$ find a solution order by order

- $\delta^{(0)}\, S^{(2)} = 0$
- $\delta^{(0)}\, S^{(3)} + \underbrace{\delta^{(1)}\, S^{(2)}}_{\approx 0} = 0 \rightarrow$ Cubic $\delta^{(0)}\, S^{(3)} \approx 0$
- ...

TT part: light-cone $\subset$ TT $\subset$"Full" [Manvelyan-Mkrtchyan-Ruehl; Sagnotti-Taronna]

- $S^{(3)} = S^{(3)}_{TT_\varphi} + \text{rest}$
- $\delta^{(0)}\, S^{(3)} = [\delta^{(0)}\, S^{(3)}_{TT_\varphi}]_{TT_{\varphi,\varepsilon} + \text{rest}} + [\delta^{(0)}\, \text{rest}]_{\text{rest}} \approx 0$
- Indipendent Nother procedure $[\delta^{(0)}\, S^{(3)}_{TT}]_{TT} \approx 0$

Massless HS cubic interactions in flat-space

- Cubic action [Manvelyan-Mkrtchyan-Ruehl; Sagnotti-Taronna; Joung-Taronna]

$$S^{(3)} \sim \int C(\partial_{u_1}, \partial_{u_2}, \partial_{u_3}; \partial_{x_1}, \partial_{x_2}, \partial_{x_3})\, \varphi(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\Big|_{\substack{x_i=x\\u_i=0}}$$

$$\sim \int C(y_i\,,\, z_i)\,\varphi(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\Big|_{\substack{x_i=x\\u_i=0}}$$

Parity-preserving invariants: $y_i = \partial_{u_i}\cdot\partial_{x_{i+1}},\ z_i = \partial_{u_{i+1}}\cdot\partial_{u_{i-1}}\ [i \simeq i+3]$

Ex: $\partial^\nu\varphi_{\mu_1\mu_2}\partial^{\rho_1}\varphi_\nu{}^{\mu_1\mu_2\rho_2}\varphi_{\rho_1\rho_2} \leftrightarrow C = \partial_{u_2}\cdot\partial_{x_1}\,\partial_{u_3}\cdot\partial_{x_2}\,(\partial_{u_1}\cdot\partial_{u_2})^2\,\partial_{u_2}\cdot\partial_{u_3}$

- Gauge invariance

$$\delta_1^{(0)}S^{(3)} \sim \int \underbrace{\big[C(y,z)\,,\,u_1\cdot\partial_x\big]}_{(y_3\,\partial_{z_2}-y_2\,\partial_{z_3})C(y,z)}\ \varepsilon(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\Big|_{\substack{x_i=x\\u_i=0}}$$

$$(y_i\,\partial_{z_{i+1}} - y_{i+1}\,\partial_{z_{i+1}})\,C(y,z) = 0 \quad [i \simeq i+3]$$

General solution: $C = \mathcal{K}(y_1,y_2,y_3,g) \quad g = y_1\,z_1 + y_2\,z_2 + y_3\,z_3$

HS cubic interactions in (A)dS

- Cubic action in (A)dS [Joung-Taronna; Joung-LL-Taronna; Vasiliev]

$$S^{(3)} \sim \int \delta(\sqrt{\epsilon\, X^2} - L)\, C(\tfrac{1}{L}, Y, Z)\, \Phi(X_1, U_1)\, \Phi(X_2, U_2)\, \Phi(X_3, U_3)\big|_{\substack{X_i = X \\ U_i = 0}}$$

- Massless $[\delta(\sqrt{\epsilon\, X^2} - L)\, \partial_{X^M} = -\frac{\hat{\delta}}{L}\, \delta(\sqrt{\epsilon\, X^2} - L)\, X_M]$

$$\delta^{(0)} S^{(3)} \sim \int \delta(\sqrt{\epsilon\, X^2} - L)\, [C(\tfrac{1}{L}, Y, Z)\,,\; U_1 \cdot \partial_{X_1}]\, E_1\, \Phi_2\, \Phi_3$$

$$\Big[Y_2 \partial_{Z_3} - Y_3 \partial_{Z_2} + \tfrac{\hat{\delta}}{L}\left(Y_2 \partial_{Y_2} - Y_3 \partial_{Y_3} - \tfrac{\mu_2 - \mu_3}{2}\right) \partial_{Y_1}\Big]\, C(\hat{\delta}; Y, Z) = 0$$

- Partially-massless

$$\delta^{(0)} S^{(3)} \sim \int \delta(\sqrt{\epsilon\, X^2} - L)\, [C(\tfrac{1}{L}, Y, Z)\,,\; (U_1 \cdot \partial_{X_1})^{\mu_1 + 1}]\, \Omega_1\, \Phi_2\, \Phi_3$$

$$\prod_{n=0}^{\mu_1} \Big[Y_2 \partial_{Z_3} - Y_3 \partial_{Z_2} + \tfrac{\hat{\delta}}{L}\left(Y_2 \partial_{Y_2} - Y_3 \partial_{Y_3} - \tfrac{\mu_1 + \mu_2 - \mu_3 - 2\, n}{2}\right) \partial_{Y_1}\Big]\, C(\hat{\delta}; Y, Z) = 0$$

Massive & massless cubic interactions in (A)dS

Generating functions of all massive & massless HS couplings:

- 3 massive $\qquad C = \mathcal{K}(Y_1, Y_2, Y_3, Z_1, Z_2, Z_3)$
- 2 massive $(\mu_2 = \mu_3)$ & 1 massless $\qquad C = \mathcal{K}(\tilde{Y}_1, \tilde{Y}_2, \tilde{Y}_3, Z_1, \tilde{G})$
- 2 massive $(\mu_2 \neq \mu_3)$ & 1 massless $\qquad C = \mathcal{K}(Y_2, Y_3, Z_1, \tilde{H}_2, \tilde{H}_3)$
- 1 massive $(\mu_3 \neq 0)$ & 2 massless $\qquad C = \mathcal{K}(Y_3, \tilde{H}_1, \tilde{H}_2, \tilde{H}_3)$
- 3 massless $\qquad C = \mathcal{K}(\tilde{Y}_1, \tilde{Y}_2, \tilde{Y}_3, \tilde{G})$

$$\tilde{Y}_i = Y_i + \alpha_i\,\partial_{U_i}\cdot\partial_X$$
$$\tilde{G} = \textstyle\sum_i (Y_i + \beta_i\,\partial_{U_i}\cdot\partial_X) Z_i$$
$$\tilde{H}_i = Y_{i+1}\,(Y_{i-1} - \partial_X\cdot\partial_{U_{i-1}}) - \tfrac{1}{2}\,\partial_X\cdot(\partial_{X_i} - \partial_{X_{i+1}} - \partial_{X_{i-1}})\,Z_i$$
$$\alpha_1 = \alpha\,,\ \alpha_2 = -\tfrac{1}{\alpha+1}\,,\ \alpha_3 = -\tfrac{\alpha+1}{\alpha}$$
$$\beta_1 = \beta\,,\ \beta_2 = -\tfrac{\beta+1}{\alpha+1}\,,\ \alpha_3 = -\tfrac{\alpha-\beta}{\alpha}$$

Flat-space limit ($X^M \to X^M + L\,\hat{N}^M$, $L \to \infty$): Metsaev's results

Summary & outlook

- Consistency conditions$\rightarrow$ PDE for cubic vertices
  - Massive & massless: generating functions
  - Partially-massless: fixed spins
- Stückelberg formulation & massless limit
- First Regge trajectory of bosonic string:

$$\mathcal{K} \sim \exp[i\sqrt{2\alpha'}\,(y_1 + y_2 + y_3) + z_1 + z_2 + z_3]$$ [Taronna; Sagnotti-Taronna]

  - Taylor coefficients $\cup$ spectrum $=$ consistent couplings
  - Relation to ST properties and AdS counterpart
- Fermions and mixed-symmetry fields
- Deformations of HS gauge algebra
- AdS/CFT computations