Gaugings & Vacua of Maximal D=4 Supergravity

Gianluca Inverso
Università di Roma Tor Vergata

Convegno Informale di Fisica Teorica
Cortona 31/05 2012

Based on:
Dall'Agata, G.I hep-th/1112.3345 NPB859
Dall'Agata, G.I, Trigiante hep-th/1206.xxxx

Maximal D=4 Supergravity

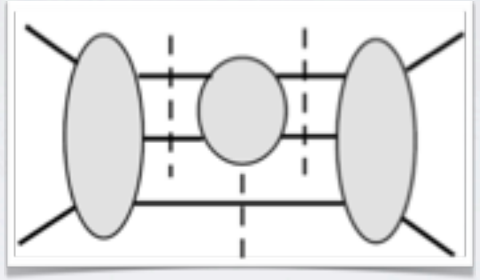
Maximal D=4 Supergravity

N=8 sugra as a unified framework for gauge interactions failed but:

Maximal D=4 Supergravity

N=8 sugra as a unified framework for gauge interactions failed but:

- Finiteness (?)

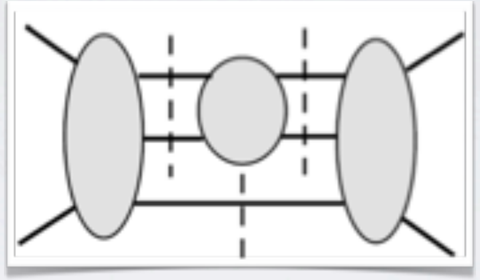


Other Mink vacua?

Maximal D=4 Supergravity

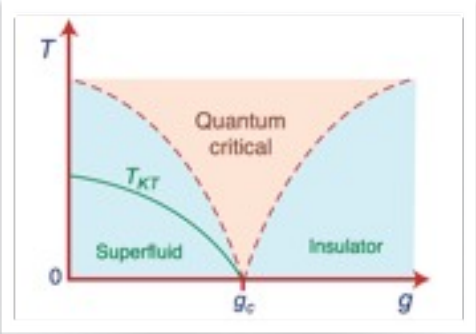
N=8 sugra as a unified framework for gauge interactions failed but:

- Finiteness (?)



Other Mink vacua?

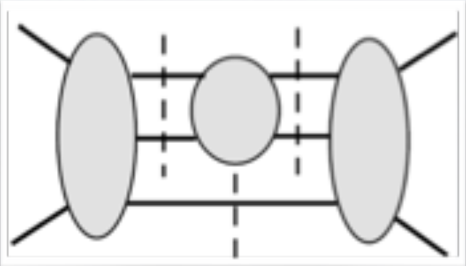
- AdS/CMT applications



Maximal D=4 Supergravity

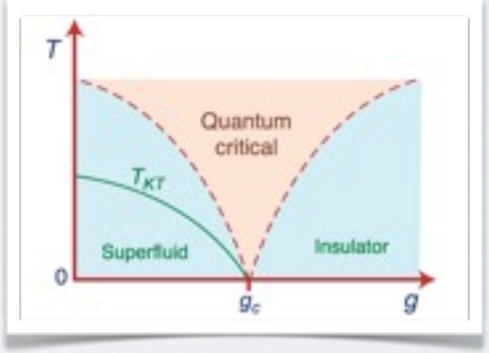
N=8 sugra as a unified framework for gauge interactions failed but:

- Finiteness (?)

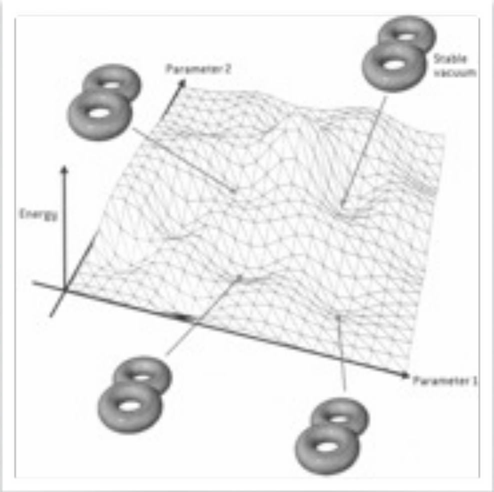


Other Mink vacua?

- AdS/CMT applications



- dS vacua in N>1
  (and eventually string theory)



SO FAR:

- gauged SO(8)

  $\simeq$ M-theory on S^7

SO FAR:

- gauged SO(8) → analytical

$\simeq$ M-theory on S^7

Symmetry	Λ_{cosm}	SUSY
SO(8)	AdS	N=8
$SO(7)_-$	AdS	N=0
$SO(7)_+$	AdS	N=0
G_2	AdS	N=1
SU(4)	AdS	N=0
SU(3) x U(1)	AdS	N=2
SO(3) x SO(3)	AdS	N=0

WARNER, HULL, NICOLAI...

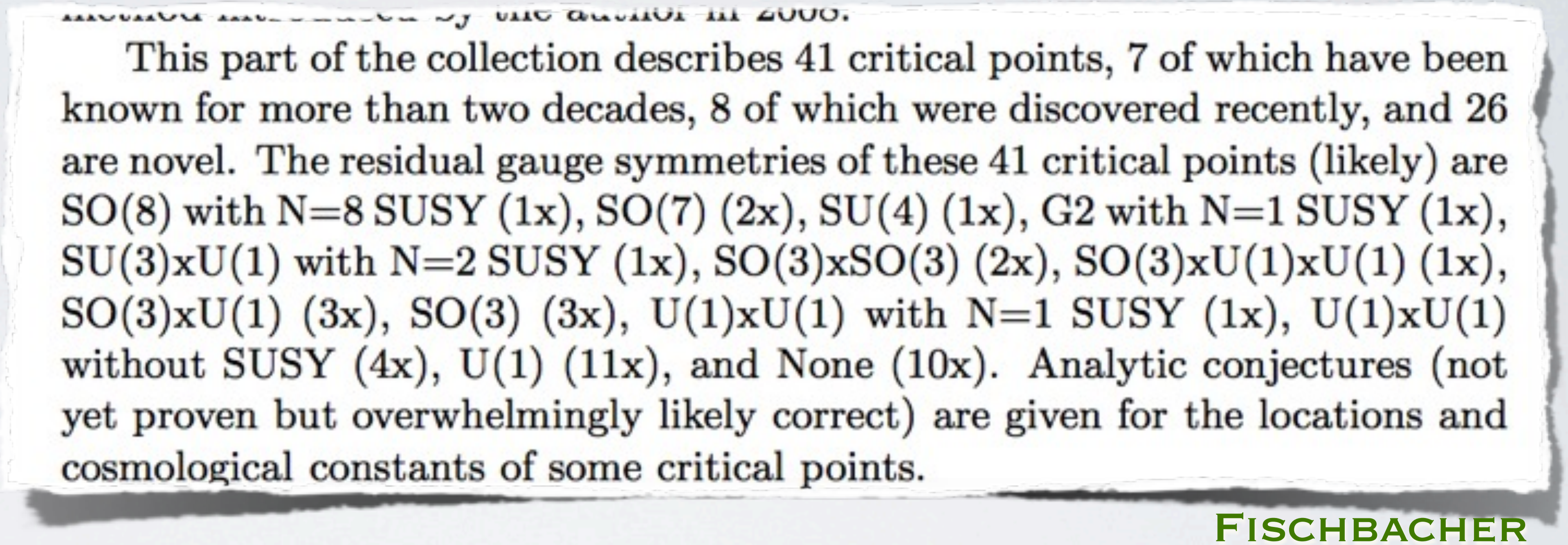
SO FAR:

- gauged SO(8)

  ≃ M-theory on S^7

analytical

numerical



This part of the collection describes 41 critical points, 7 of which have been known for more than two decades, 8 of which were discovered recently, and 26 are novel. The residual gauge symmetries of these 41 critical points (likely) are SO(8) with N=8 SUSY (1x), SO(7) (2x), SU(4) (1x), G2 with N=1 SUSY (1x), SU(3)xU(1) with N=2 SUSY (1x), SO(3)xSO(3) (2x), SO(3)xU(1)xU(1) (1x), SO(3)xU(1) (3x), SO(3) (3x), U(1)xU(1) with N=1 SUSY (1x), U(1)xU(1) without SUSY (4x), U(1) (11x), and None (10x). Analytic conjectures (not yet proven but overwhelmingly likely correct) are given for the locations and cosmological constants of some critical points.

Fischbacher

SO FAR:

- gauged SO(8) → analytical, → numerical

  $\simeq$ M-theory on S^7

- *un*stable dS in SO(4,4) & SO(5,3)

- (C)SS reduction/Twisted Tori ⟶ Mink. vacua N=6,4,2,0

  $$\mathrm{U}(1) \ltimes T^r$$

One extra N=2, CSO*(6,2), Hull

SO FAR:

- gauged SO(8) → analytical / numerical

  $\simeq$ M-theory on S^7

- *un*stable dS in SO(4,4) & SO(5,3)

- (C)SS reduction/Twisted Tori → Mink. vacua N=6,4,2,0

  Cremmer, Scherk, Schwarz

One extra N=2, CSO*(6,2), Hull

$$\mathrm{U}(1) \ltimes T^r$$

HERE:

✤Effective method to study vacua & gaugings of Maximal D=4 Supergravity

- New vacua, new class of ~~susy~~ Mink. solutions
- New gaugings

✤Inequivalent SO(8) and SO(p,q) gauged supergravities

HERE:

- ✤Effective method to study vacua & gaugings of Maximal D=4 Supergravity
  - New vacua, new class of ~~susy~~ Mink. solutions
  - New gaugings

Dall'Agata, G.I.

- ✤Inequivalent SO(8) and SO(p,q) gauged supergravities

Dall'Agata, G.I, Trigiante

Vacua of gauged maximal Supergravity

Gauged max. Sugra: Embedding Tensor

Reduce M-theory (e.g.) on T^7. ‘Fluxes’:

$\mathbf{1}_{+\mathbf{7}}$	g_7	$(\mathbf{140}+\mathbf{7})_{+\mathbf{3}}$	$\tau^i_{jk}+\delta^i_j\tau_k$	$\mathbf{28}_{-\mathbf{1}}$	$\theta_{(ij)}$
$\mathbf{1}_{-\mathbf{7}}$	$\tilde{g}_7$	$(\mathbf{140}'+\mathbf{7}')_{-3}$	$Q^{jk}_i+\delta^j_i Q^k$	$\mathbf{28}'_{+1}$	$\xi^{(ij)}$
$\mathbf{35}_{-\mathbf{5}}$	h^{ijkl}	$\mathbf{224}_{-1}$	f^i_{jkl}	$\mathbf{21}_{-\mathbf{1}}$	$\theta_{[ij]}$
$\mathbf{35}'_{+5}$	g_{ijkl}	$\mathbf{224}'_{+1}$	R^{jkl}_i	$\mathbf{21}'_{+1}$	$\xi^{[ij]}$

Gauged max. Sugra: Embedding Tensor

Reduce M-theory (e.g.) on T^7. ‘Fluxes’:

$\mathbf{1}_{+7}$	g_7	$(\mathbf{140}+\mathbf{7})_{+3}$	$\tau^i_{jk}+\delta^i_j\tau_k$	$\mathbf{28}_{-1}$	$\theta_{(ij)}$
$\mathbf{1}_{-7}$	$\tilde{g}_7$	$(\mathbf{140}'+\mathbf{7}')_{-3}$	$Q^{jk}_i+\delta^j_i Q^k$	$\mathbf{28}'_{+1}$	$\xi^{(ij)}$
$\mathbf{35}_{-5}$	h^{ijkl}	$\mathbf{224}_{-1}$	f^i_{jkl}	$\mathbf{21}_{-1}$	$\theta_{[ij]}$
$\mathbf{35}'_{+5}$	g_{ijkl}	$\mathbf{224}'_{+1}$	R^{jkl}_i	$\mathbf{21}'_{+1}$	$\xi^{[ij]}$

$$\mathcal{D}_\mu = \partial_\mu + A_\mu^M \Theta_M{}^\alpha t_\alpha$$

closure: $[X_M, X_N] = -X_{MN}{}^P X_P$

locality: $\Theta_M{}^\alpha \Theta^{M\beta} = 0$

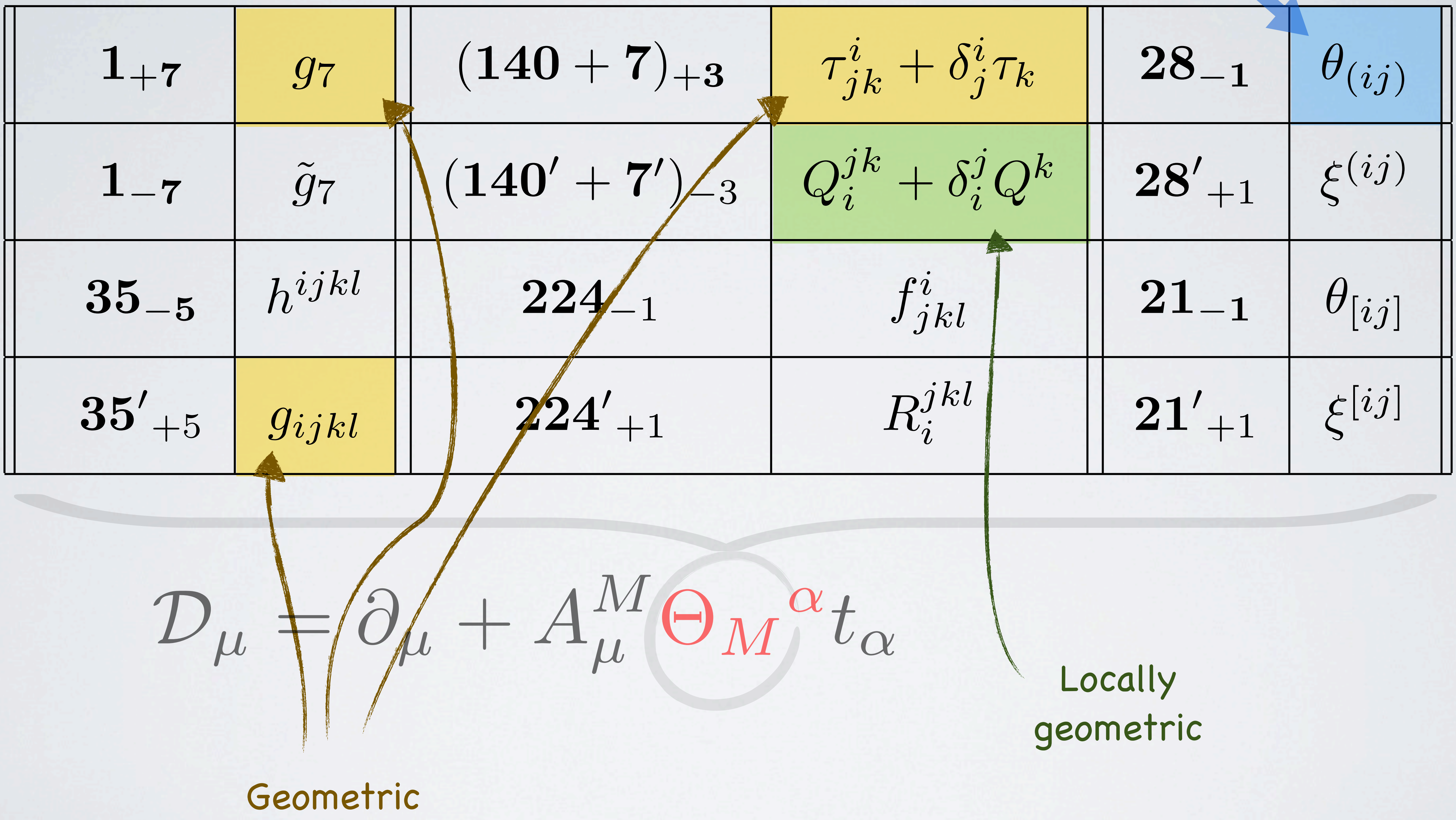
susy: $\Theta \in \mathbf{912}$ of $E_{7(7)}$

Nicolai et al.;
deWit, Samtleben, Trigiante;

Gauged max. Sugra:

11D→4D on T^7:

Geometric on S^7 !!



$\mathbf{1}_{+7}$	g_7	$(\mathbf{140}+\mathbf{7})_{+3}$	$\tau^i_{jk}+\delta^i_j\tau_k$	$\mathbf{28}_{-1}$	$\theta_{(ij)}$
$\mathbf{1}_{-7}$	$\tilde{g}_7$	$(\mathbf{140}'+\mathbf{7}')_{-3}$	$Q^{jk}_i+\delta^j_i Q^k$	$\mathbf{28}'_{+1}$	$\xi^{(ij)}$
$\mathbf{35}_{-5}$	h^{ijkl}	$\mathbf{224}_{-1}$	f^i_{jkl}	$\mathbf{21}_{-1}$	$\theta_{[ij]}$
$\mathbf{35}'_{+5}$	g_{ijkl}	$\mathbf{224}'_{+1}$	R^{jkl}_i	$\mathbf{21}'_{+1}$	$\xi^{[ij]}$

$$\mathcal{D}_\mu = \partial_\mu + A_\mu^M \Theta_M{}^\alpha t_\alpha$$

Locally geometric

Geometric

Gauged max. Sugra:

$\mathbf{1}_{+7}$	g_7	$(\mathbf{140}+\mathbf{7})_{+3}$	$\tau^i_{jk}+\delta^i_j\tau_k$	$\mathbf{28}_{-1}$	$\theta_{(ij)}$
$\mathbf{1}_{-7}$	$\tilde{g}_7$	$(\mathbf{140}'+\mathbf{7}')_{-3}$	$Q_i^{jk}+\delta_i^j Q^k$	$\mathbf{28}'_{+1}$	$\xi^{(ij)}$
$\mathbf{35}_{-5}$	h^{ijkl}	$\mathbf{224}_{-1}$	f^i_{jkl}	$\mathbf{21}_{-1}$	$\theta_{[ij]}$
$\mathbf{35}'_{+5}$	g_{ijkl}	$\mathbf{224}'_{+1}$	R_i^{jkl}	$\mathbf{21}'_{+1}$	$\xi^{[ij]}$

$$\mathcal{D}_\mu = \partial_\mu + A_\mu^M \Theta_M{}^\alpha t_\alpha$$

Focus on g_7, $\theta_{(ij)}$, and their ‘*magnetic duals*’

More than SO(8) on S^7: SO(p,q), CSO(p,q,r),….!

Find extrema of scalar potential

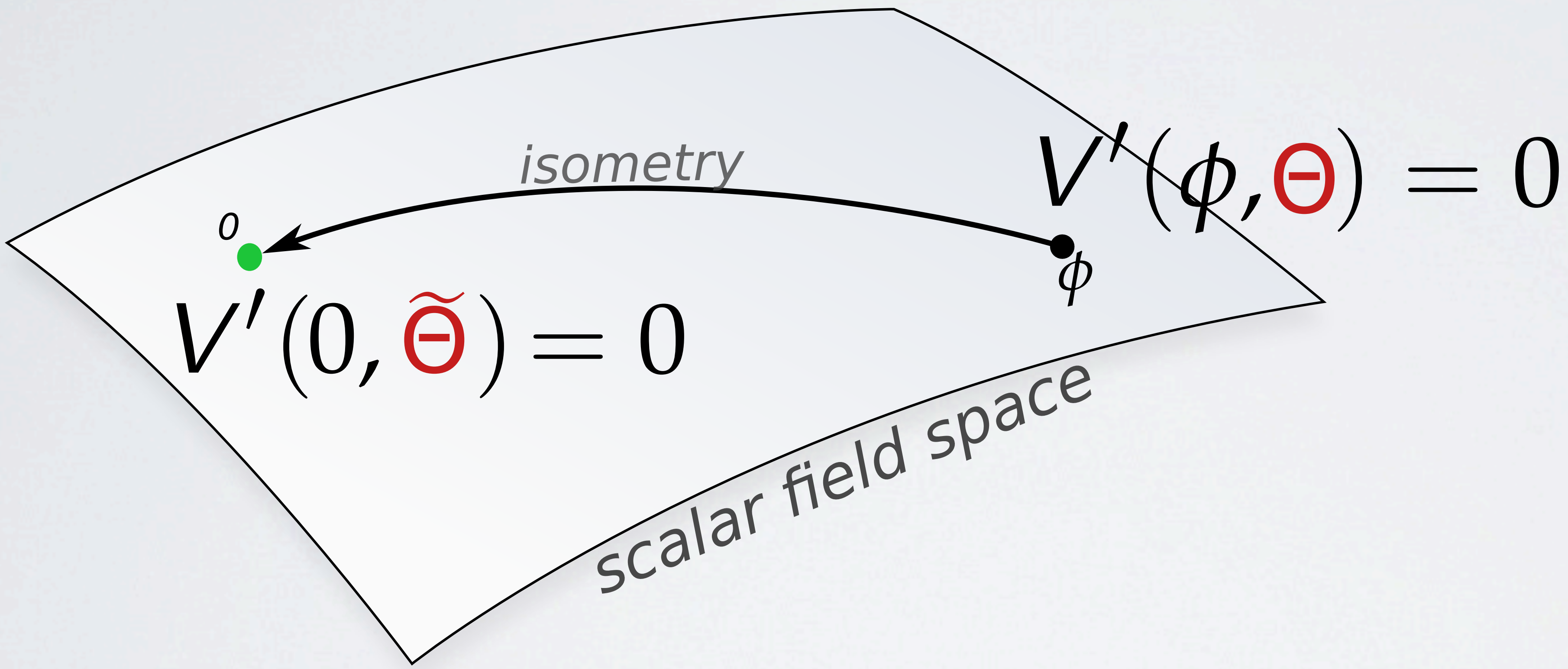
$$V(\phi) = \mathcal{M}(\phi)^{MN}\underline{\Theta_M{}^\alpha\Theta_N{}^\beta}\left(\mathcal{M}(\phi)_{\alpha\beta} + 7\eta_{\alpha\beta}\right)$$

Find extrema of scalar potential

$$V(\phi) = \mathcal{M}(\phi)^{MN} \underline{\Theta_M{}^\alpha \Theta_N{}^\beta} \left(\mathcal{M}(\phi)_{\alpha\beta} + 7\eta_{\alpha\beta}\right)$$

Scalar field space is homogeneous: $E_{7(7)}/SU(8)$

Dall'Agata, G.I.;
Dibitetto et al.



$$V'(\phi,\Theta) = 0$$

$$V'(0,\tilde{\Theta}) = 0$$

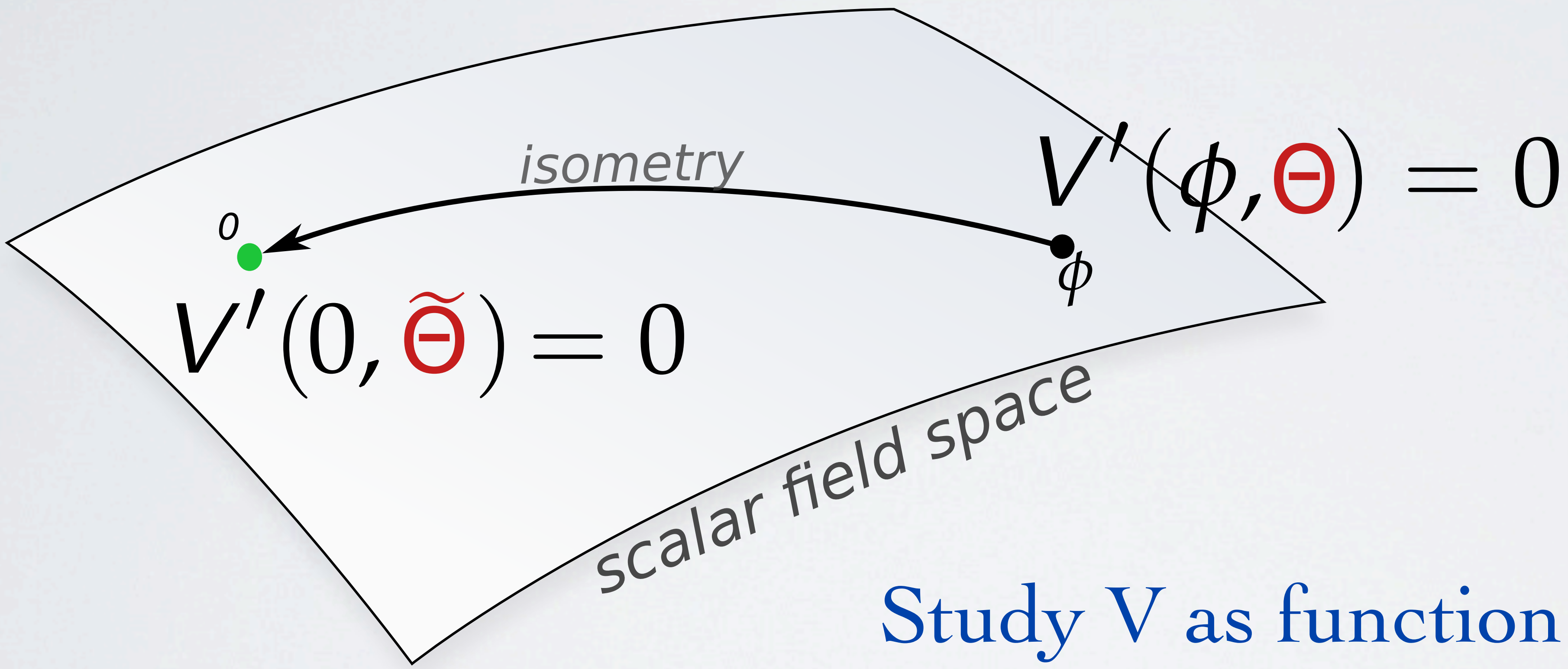
$$\Theta \xrightarrow{E_{7(7)}} \tilde{\Theta}$$

Find extrema of scalar potential

$$V(\Theta) = \delta^{MN}\underline{\Theta_M{}^{\alpha}\Theta_N{}^{\beta}}\left(\delta_{\alpha\beta} + 7\eta_{\alpha\beta}\right)$$

Scalar field space is homogeneous: $E_{7(7)}/SU(8)$

Dall'Agata, G.I.;
Dibitetto et al.

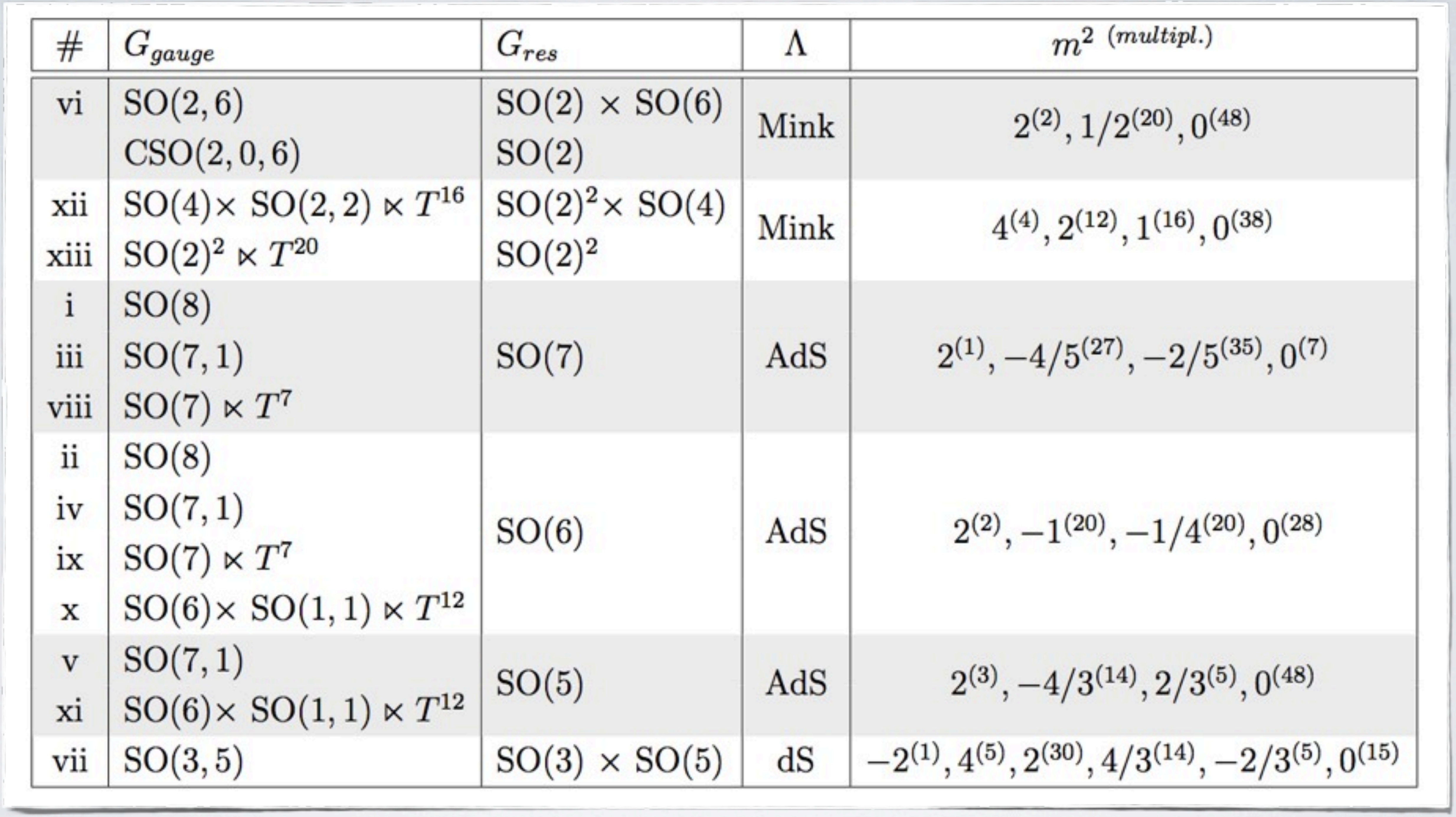


$$\Theta \xrightarrow{E_{7(7)}} \tilde{\Theta}$$

Study V as function of gauging Θ
instead of ϕ

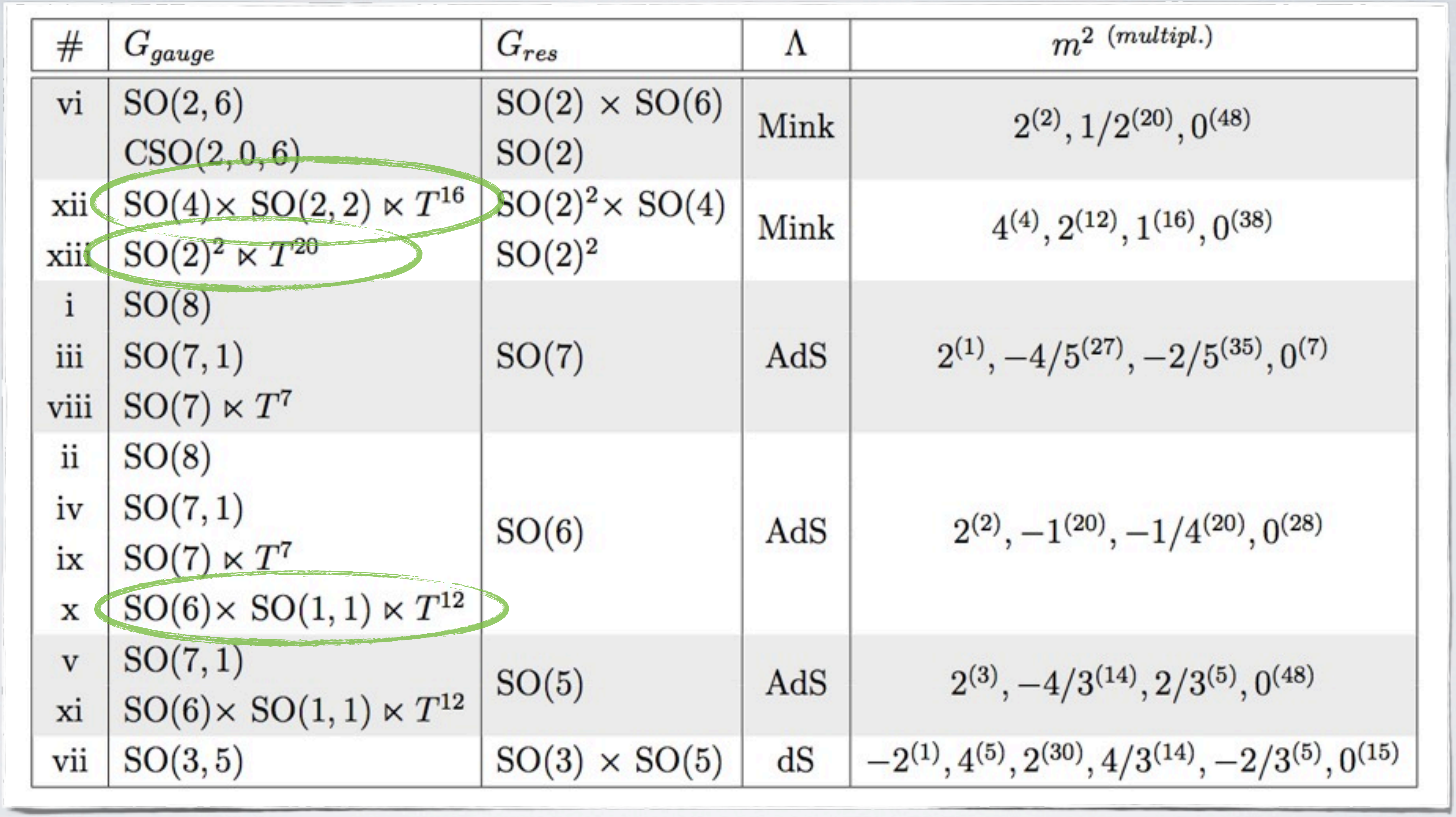
✓ Advantages: exhaustive (in principle!), algebraic, quadratic function, full mass spectra, study more theories at same time

Vacua & Full Mass Spectra!



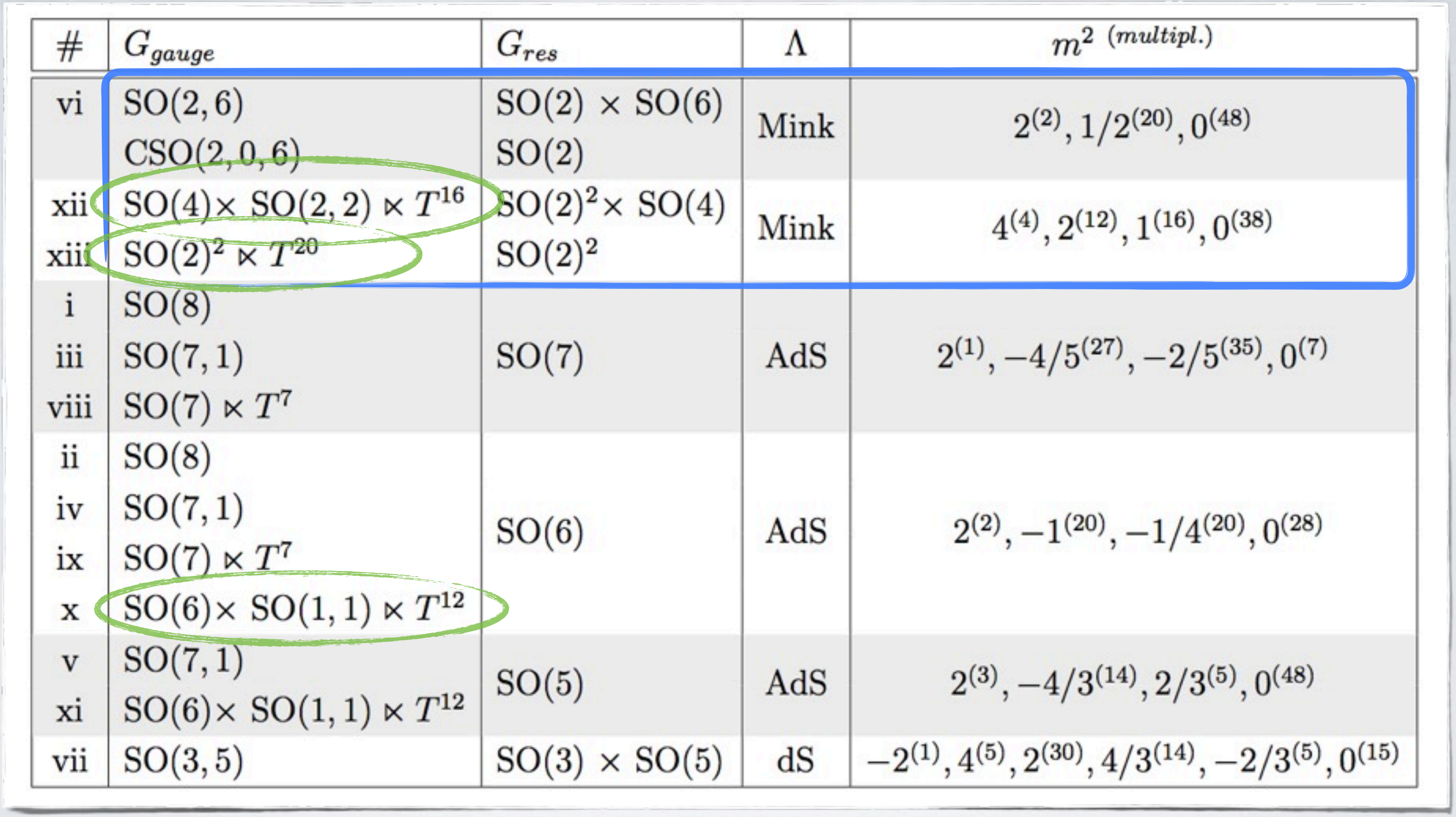
#	G_{gauge}	G_{res}	Λ	m^2 $^{(multipl.)}$
vi 	$\mathrm{SO}(2,6)$ $\mathrm{CSO}(2,0,6)$	$\mathrm{SO}(2)\times\mathrm{SO}(6)$ $\mathrm{SO}(2)$	Mink	$2^{(2)}, 1/2^{(20)}, 0^{(48)}$
xii xiii	$\mathrm{SO}(4)\times\mathrm{SO}(2,2)\ltimes T^{16}$ $\mathrm{SO}(2)^2\ltimes T^{20}$	$\mathrm{SO}(2)^2\times\mathrm{SO}(4)$ $\mathrm{SO}(2)^2$	Mink	$4^{(4)}, 2^{(12)}, 1^{(16)}, 0^{(38)}$
i iii viii	$\mathrm{SO}(8)$ $\mathrm{SO}(7,1)$ $\mathrm{SO}(7)\ltimes T^7$	$\mathrm{SO}(7)$	AdS	$2^{(1)}, -4/5^{(27)}, -2/5^{(35)}, 0^{(7)}$
ii iv ix x	$\mathrm{SO}(8)$ $\mathrm{SO}(7,1)$ $\mathrm{SO}(7)\ltimes T^7$ $\mathrm{SO}(6)\times\mathrm{SO}(1,1)\ltimes T^{12}$	$\mathrm{SO}(6)$	AdS	$2^{(2)}, -1^{(20)}, -1/4^{(20)}, 0^{(28)}$
v xi	$\mathrm{SO}(7,1)$ $\mathrm{SO}(6)\times\mathrm{SO}(1,1)\ltimes T^{12}$	$\mathrm{SO}(5)$	AdS	$2^{(3)}, -4/3^{(14)}, 2/3^{(5)}, 0^{(48)}$
vii	$\mathrm{SO}(3,5)$	$\mathrm{SO}(3)\times\mathrm{SO}(5)$	dS	$-2^{(1)}, 4^{(5)}, 2^{(30)}, 4/3^{(14)}, -2/3^{(5)}, 0^{(15)}$

Vacua & Full Mass Spectra!



#	G_{gauge}	G_{res}	Λ	m^2 (*multipl.*)
vi	$\mathrm{SO}(2,6)$	$\mathrm{SO}(2) \times \mathrm{SO}(6)$	Mink	$2^{(2)}, 1/2^{(20)}, 0^{(48)}$
	$\mathrm{CSO}(2,0,6)$	$\mathrm{SO}(2)$		
xii	$\mathrm{SO}(4) \times \mathrm{SO}(2,2) \ltimes T^{16}$	$\mathrm{SO}(2)^2 \times \mathrm{SO}(4)$	Mink	$4^{(4)}, 2^{(12)}, 1^{(16)}, 0^{(38)}$
xiii	$\mathrm{SO}(2)^2 \ltimes T^{20}$	$\mathrm{SO}(2)^2$		
i	$\mathrm{SO}(8)$	$\mathrm{SO}(7)$	AdS	$2^{(1)}, -4/5^{(27)}, -2/5^{(35)}, 0^{(7)}$
iii	$\mathrm{SO}(7,1)$			
viii	$\mathrm{SO}(7) \ltimes T^7$			
ii	$\mathrm{SO}(8)$	$\mathrm{SO}(6)$	AdS	$2^{(2)}, -1^{(20)}, -1/4^{(20)}, 0^{(28)}$
iv	$\mathrm{SO}(7,1)$			
ix	$\mathrm{SO}(7) \ltimes T^7$			
x	$\mathrm{SO}(6) \times \mathrm{SO}(1,1) \ltimes T^{12}$			
v	$\mathrm{SO}(7,1)$	$\mathrm{SO}(5)$	AdS	$2^{(3)}, -4/3^{(14)}, 2/3^{(5)}, 0^{(48)}$
xi	$\mathrm{SO}(6) \times \mathrm{SO}(1,1) \ltimes T^{12}$			
vii	$\mathrm{SO}(3,5)$	$\mathrm{SO}(3) \times \mathrm{SO}(5)$	dS	$-2^{(1)}, 4^{(5)}, 2^{(30)}, 4/3^{(14)}, -2/3^{(5)}, 0^{(15)}$

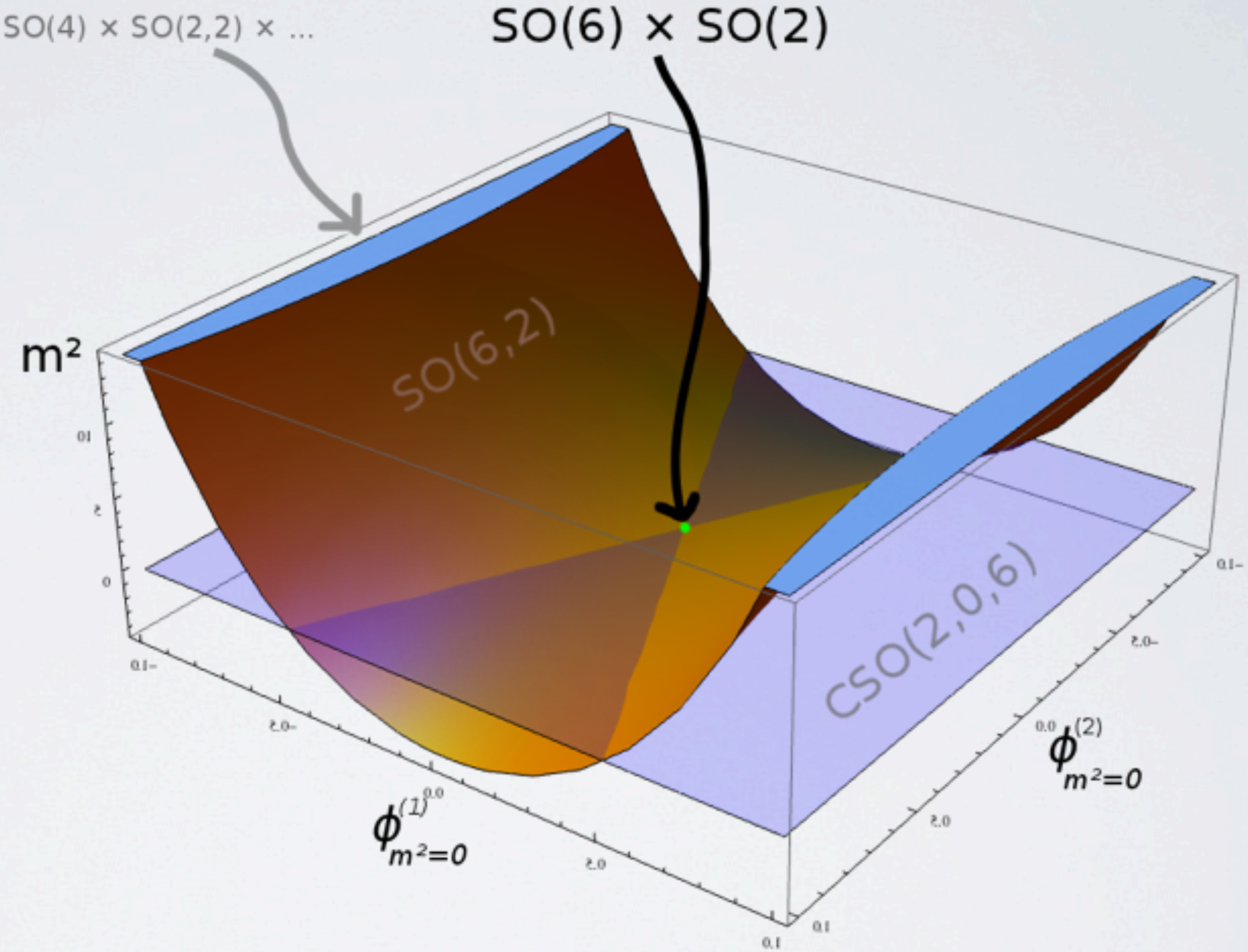
Vacua & Full Mass Spectra!



#	G_{gauge}	G_{res}	Λ	m^2 (multipl.)
vi	$SO(2,6)$	$SO(2) \times SO(6)$	Mink	$2^{(2)}, 1/2^{(20)}, 0^{(48)}$
	$CSO(2,0,6)$	$SO(2)$		
xii	$SO(4)\times SO(2,2) \ltimes T^{16}$	$SO(2)^2 \times SO(4)$	Mink	$4^{(4)}, 2^{(12)}, 1^{(16)}, 0^{(38)}$
xiii	$SO(2)^2 \ltimes T^{20}$	$SO(2)^2$		
i	$SO(8)$	$SO(7)$	AdS	$2^{(1)}, -4/5^{(27)}, -2/5^{(35)}, 0^{(7)}$
iii	$SO(7,1)$			
viii	$SO(7) \ltimes T^7$			
ii	$SO(8)$	$SO(6)$	AdS	$2^{(2)}, -1^{(20)}, -1/4^{(20)}, 0^{(28)}$
iv	$SO(7,1)$			
ix	$SO(7) \ltimes T^7$			
x	$SO(6)\times SO(1,1) \ltimes T^{12}$			
v	$SO(7,1)$	$SO(5)$	AdS	$2^{(3)}, -4/3^{(14)}, 2/3^{(5)}, 0^{(48)}$
xi	$SO(6)\times SO(1,1) \ltimes T^{12}$			
vii	$SO(3,5)$	$SO(3) \times SO(5)$	dS	$-2^{(1)}, 4^{(5)}, 2^{(30)}, 4/3^{(14)}, -2/3^{(5)}, 0^{(15)}$

CSO(2,0,6) is N=0 SS reduction. SO(6,2) has same spectrum!

Beyond quadratic order:



First ~~susy~~
Mink. vacua
not SS!

More on *classical* & *quantum* properties: Dall'Agata Zwirner 1205.4711

New SO(8) Supergravities

Supergravity with local SO(8) invariance

- 80's: first gauged Maximal Sugra

Cremmer, Julia;
deWit, Nicolai

↳ M-theory on S^7

- We have an **infinite class** of SO(8) models!

Dall'Agata, G.I, Trigiante

Supergravity with local SO(8) invariance

- 80's: first gauged Maximal Sugra

  ↳ M-theory on S^7

Cremmer, Julia;
deWit, Nicolai

- We have an **infinite class** of SO(8) models!

  ↳ Turn on 'dual flux' $\xi = c\, \theta^{-1}$ $\qquad c \in [0, 1]$

Dall'Agata, G.I, Trigiante

$$A_\mu^{\text{gauge}} \sim A_\mu + c\, A_\mu^{\text{e.m.dual}}$$

Same gauge group,
different symplectic
embedding

Are they *really* distinct theories?

- Models with same gauge group could just be dual to each other

- ANALOGY:

black hole solutions are classified by duality invariants

e.g. $$I_4(Q) = \underbrace{d_{MNPQ}}_{\text{E}_7\text{ invariant}} \, Q^M \, Q^N \, Q^P \, \underbrace{Q^Q}_{\text{BH charges}}$$ Ferrara,...

Or: tensor classifiers Trigiante et al.

- Models with same gauge group could just be dual to each other

- ANALOGY:

  black hole solutions are classified by duality invariants

e.g. $$I_4(Q) = \underset{\text{E}_7\text{ invariant}}{\underline{d_{MNPQ}}}\, Q^M\, Q^N\, Q^P\, \underset{\text{BH charges}}{Q^Q}$$

Ferrara,...

Or: tensor classifiers Trigiante et al.

- Here: classify gauge theories:

  'charges' = embedding tensor $\Theta_M{}^\alpha$

- Models with same gauge group could just be dual to each other

- ANALOGY:

black hole solutions are classified by duality invariants

e.g. $$I_4(Q) = \underbrace{d_{MNPQ}}_{\text{E}_7\text{ invariant}}\, Q^M\, Q^N\, Q^P\, \underbrace{Q^Q}_{\text{BH charges}}$$ Ferrara,...

Or: tensor classifiers Trigiante et al.

- Here: classify gauge theories:

'charges' = embedding tensor $\Theta_M{}^\alpha$

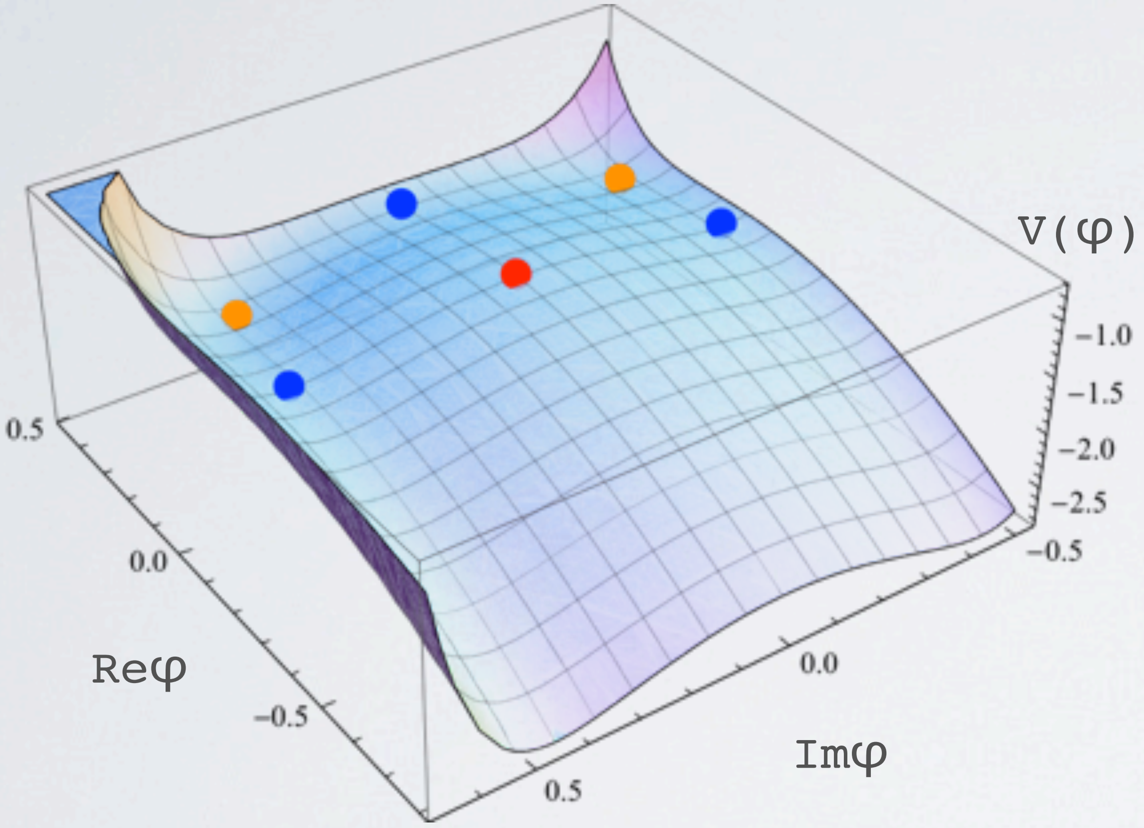
$$I_4(\Theta) = 0\,, \;\; \text{but} \quad \boxed{\underline{B_{\alpha\beta}{}^{\gamma\delta}} \equiv d^{MNPQ}\Theta_{M\alpha}\,\Theta_{N\beta}\,\Theta_P{}^\gamma\,\Theta_Q{}^\delta}$$

has duality-invariant spectrum!!

Vacuum structure changes with c!

$$\xi = c\theta^{-1}$$

$$A_\mu^{\text{gauge}} \sim A_\mu + c\, A_\mu^{\text{e.m.dual}}$$

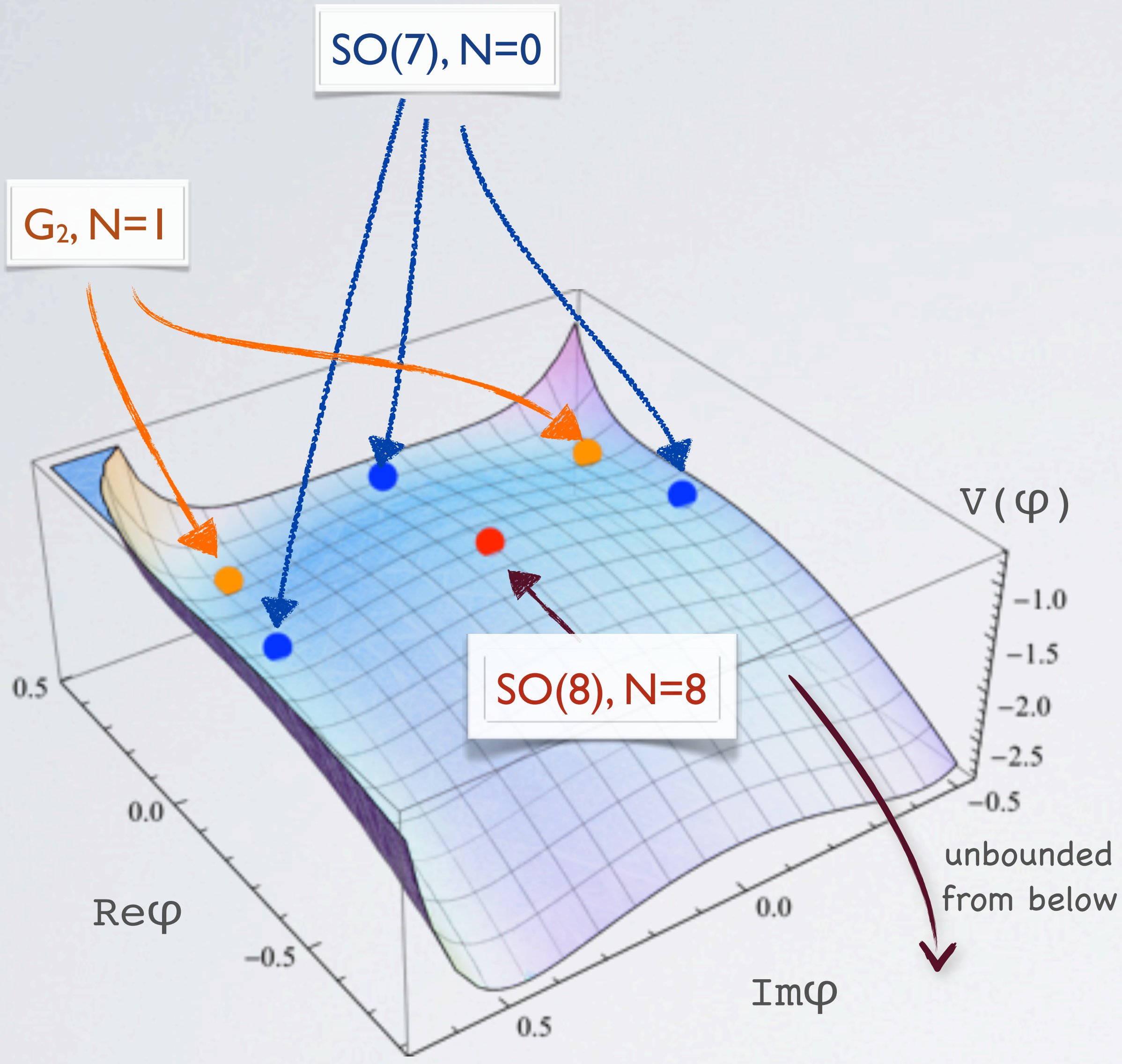


c=0

Vacuum structure changes with c!

$$\xi = c\theta^{-1}$$

$$A_\mu^{\text{gauge}} \sim A_\mu + c\, A_\mu^{\text{e.m.dual}}$$

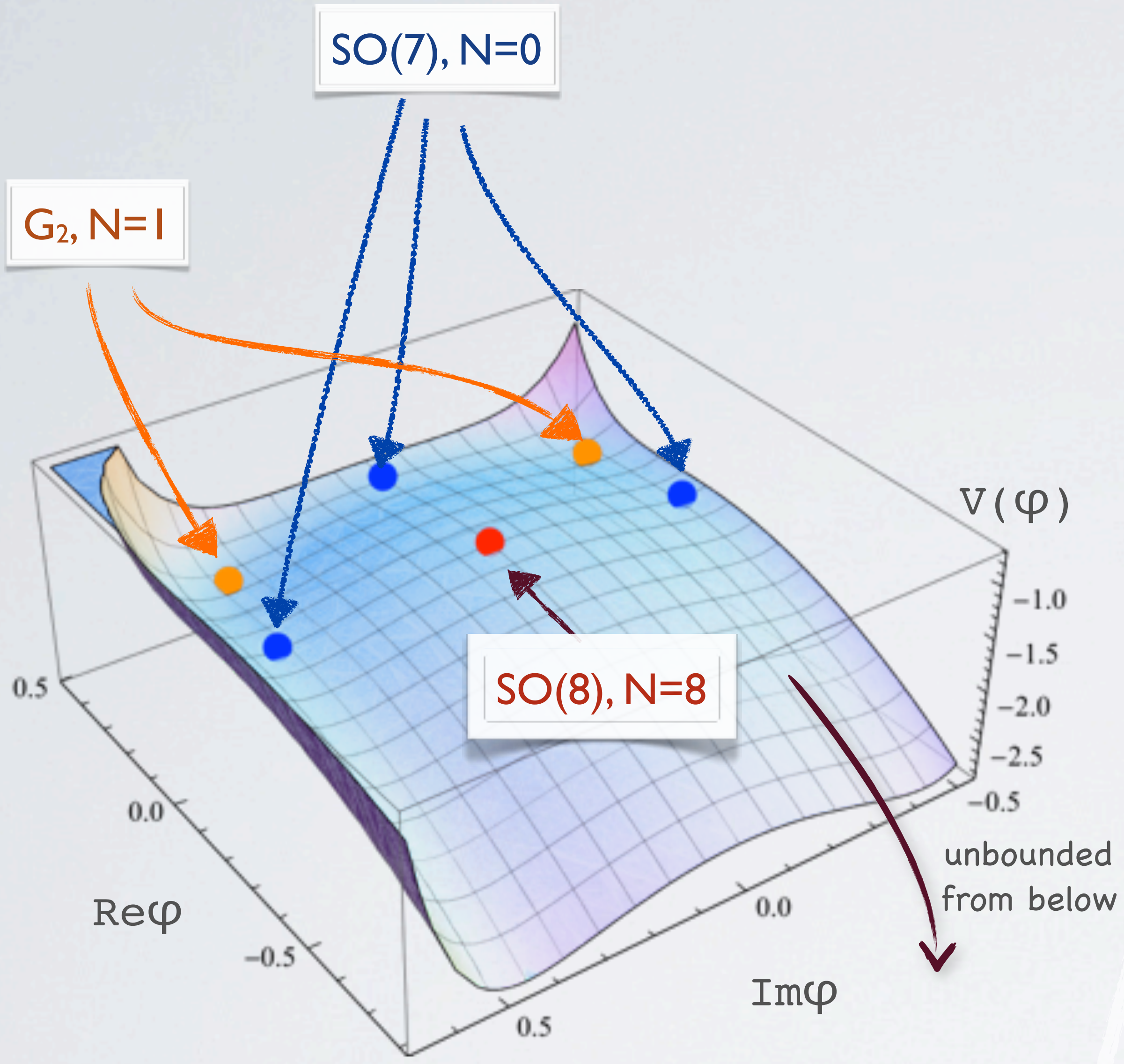


c=0

Vacuum structure changes with c!

$$\xi = c\theta^{-1}$$

$$A_\mu^{\text{gauge}} \sim A_\mu + c\, A_\mu^{\text{e.m.dual}}$$


SO(7), N=0

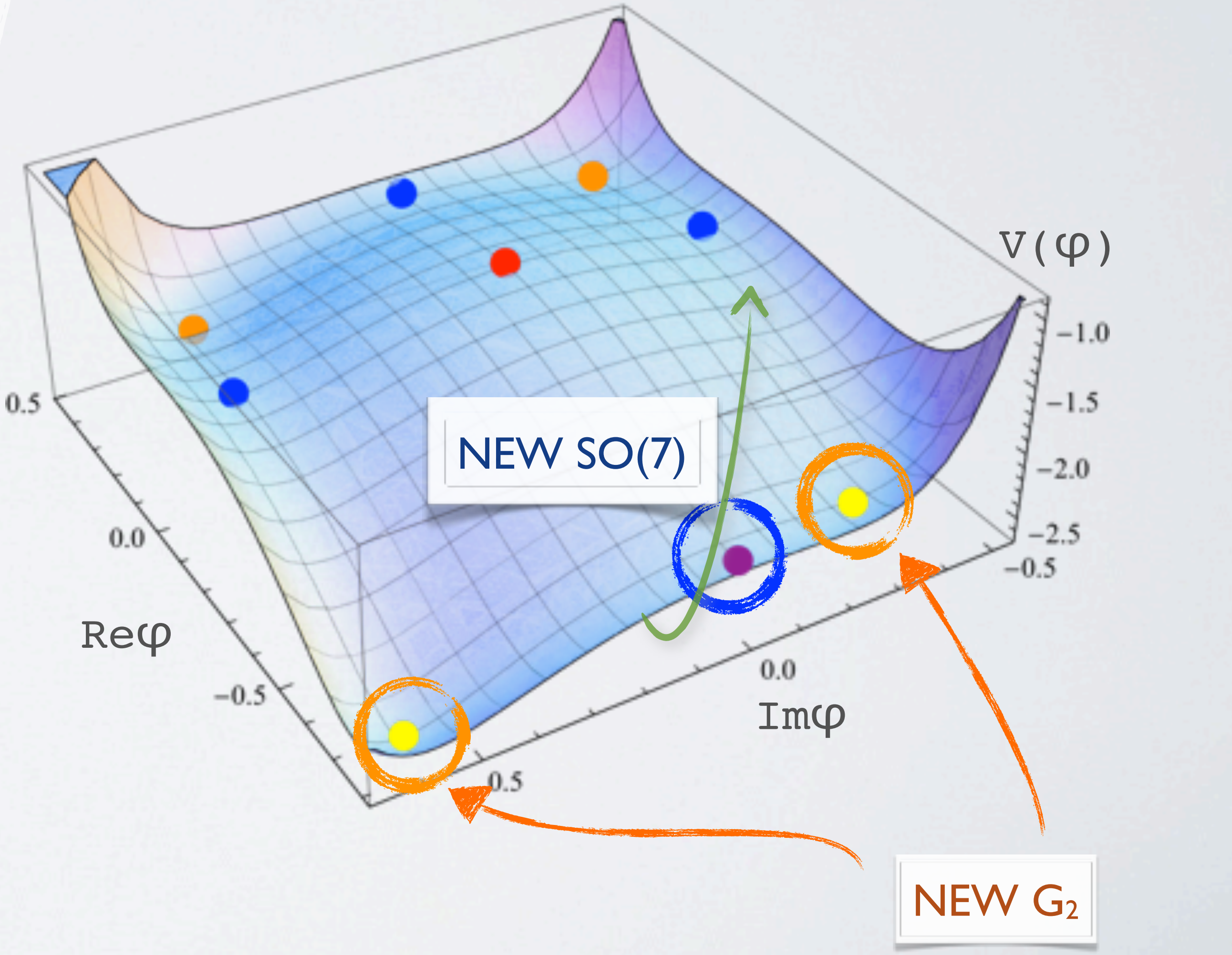
G₂, N=1

SO(8), N=8

unbounded from below

c=0

c=1/4


NEW SO(7)

NEW G₂

Summary & Outlook

- ✓ New method for finding vacua of N=8 gauged Sugra
- ✓ *New* vacua (dS, AdS, Mink.) & full mass spectrum
- ✓ relation masses - residual symmetries
  & interesting moduli space
- ✓ Infinite family of SO(8), SO(p,q) gaugings

- ❖ more vacua? dS/Mink.?
- ❖ explain relations between spectra, moduli spaces.
- ❖ interpretation of inequivalent gaugings,
  more differences?