D-brane bound state geometries from string amplitudes

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Based on

- S.G., F. Morales, R. Russo: `arXiv:0912.2270`
- W. Black, R. Russo, D. Turton: `arXiv:1007.2856`
- S.G., R. Russo, D. Turton: `arXiv:1108.6331`
- S.G., R. Russo: `arXiv:1201.2585`

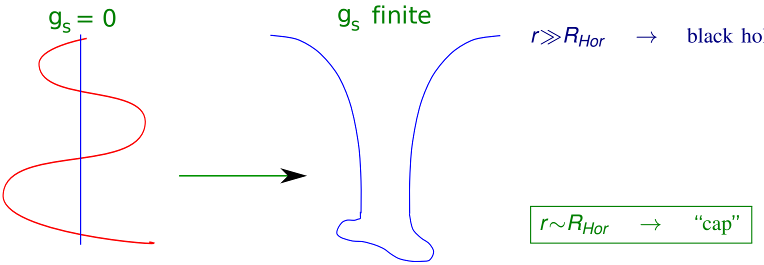
The goal:

- derive the space-time configuration sourced (at $g_s \neq 0$) by a BPS bound state of D-branes
- work in the regime in which the space-time geometry can be described by classical supergravity:
  D-brane bound state $\rightarrow$ coherent state

The motivation:

- in string theory black holes are constructed by wrapping along the compact dimensions different types of D-branes
- (BPS) black hole entropy is exactly reproduced by counting D-brane bound states (Strominger-Vafa)
- what is the space-time description of black hole microstates?

The fuzzball picture (Mathur et al.)



D-brane microstate   microstate geometry

- same charges and mass as the black hole ($r \gg R_{Hor}$)
- the region inside the horizon is replaced by a "smooth cap" which carries the information on the particular microstate
- for generic microstates the size of the cap is $\sim R_{Hor}$
- string effects are relevant at the horizon scale!

Outline

IIB on $\mathbb{R}^{1,4} \times S^1 \times T^4$

1. 1 charge (1/2 BPS):
   - D1 or D5 geometries
   - no macroscopic entropy ($S \approx 0$)
2. 2 charges (1/4 BPS):
   - D1-P or D5-P bound states
   - D1-D5 bound states
   - macroscopic entropy ($S = 2\sqrt{2}\pi\sqrt{n_1 n_5}$), but vanishing horizon area in supergravity
3. 3 charges (1/8 BPS):
   - D1-D5-P bound states
   - macroscopic entropy ($S = 2\pi\sqrt{n_1 n_5 n_p}$) and non-vanishing horizon area

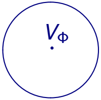
Disk amplitudes as sources

- When $g_s \neq 0$ D-branes act as sources for closed string fields $\Phi = g_{\mu\nu},\ \phi,\ B_{\mu\nu},\ C^{(p)}\ \Rightarrow$

$$\Gamma = \Gamma_{\text{bulk}} + \Gamma_{\text{brane}}$$

- The space-times fields Φ are determined by the equation

$$\frac{\delta\Gamma_{\text{bulk}}}{\delta\Phi} = -\frac{\delta\Gamma_{\text{brane}}}{\delta\Phi} \equiv \langle V_\Phi \rangle_{\text{disk}} \equiv$$



$(*)$

- V_Φ is the vertex operator for the closed field Φ; for example

$$V_{\mathcal{G}}^{(-1,-1)} = \mathcal{G}_{\mu\nu}\, c\, \psi^\mu\, e^{-\varphi}\, \bar{c}\, \bar{\psi}^\nu\, e^{-\bar{\varphi}}\, e^{i\,k\cdot x} + \text{ghosts}$$

where $\mathcal{G}_{\mu\nu} = g_{\mu\nu},\ \phi,\ B_{\mu\nu}$

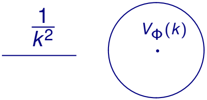
D-brane geometry (Di Vecchia, Frau, Lerda, Pesando, Russo, Sciuto)

- The geometry sourced by a D1 brane

$$ds^2 = H^{-3/4}(r)(-dt^2 + dy^2) + H^{1/4}(r)(dx_i^2 + dz_a^2)$$

$$H(r) = 1 + \frac{Q_1}{r^2}\,, \quad Q_1 = \frac{(2\pi)^4 g_s\,\alpha'^3}{V_4}\, n_1$$

  is the solution of $(*)$ with $\langle V_\Phi \rangle_{\rm disk}$ computed using D1 boundary conditions ($\bar\partial X^\mu = \partial X^\mu$ if $\mu = t, y$, $\bar\partial X^\mu = -\partial X^\mu$ if $\mu = x_i, z_a$)
- Expand around flat space $\Phi = \mathbb{I} + \delta\Phi$
- At linear order in $\delta\Phi$, $(*)$ becomes $\nabla^2 \delta\Phi = \langle V_\Phi \rangle_{\rm disk} \;\Rightarrow$

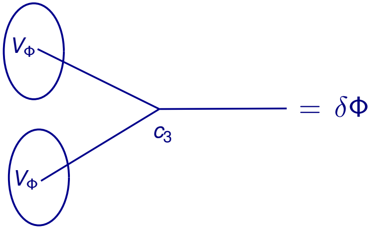
$\delta\Phi(k) =$ 

$\Leftrightarrow \quad \delta\Phi \sim \frac{Q_1}{r^2}$

- In this case $\langle V_\Phi(k) \rangle_{\rm disk}$ is k-independent; it only encodes information on the global charge Q_1

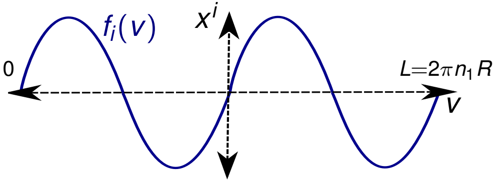
In summary

- Disk amplitudes directly give the first non-trivial terms in the asymptotic expansion of the geometry around flat space
- The full non-linear solution can be derived by recursively solving the supergravity e.o.m.'s
  $(*)$: $\nabla^2 \delta\Phi + c_3\, \delta\Phi^2 + c_4\, \delta\Phi^3 + \ldots = \langle V_\Phi \rangle_{\rm disk}$
- The higher order terms in this expansion correspond to world-sheet amplitudes with more than one boundary



D1-P geometries

- BPS bound states of D1 and P charges are given (semi-classically) by a D1-brane carrying a left-moving wave in the transverse directions $\Rightarrow$ $f_i(v)\,, \quad v = t + y$



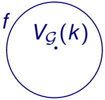
- The world-sheet boundary conditions depend on $f_i(v)$

$$\partial X^\mu = R^\mu_\nu\, \bar\partial X^\nu + \text{fermions}\,, \quad R^\mu_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 4|\dot f(V)|^2 & 1 & -4\dot f_i(V) & 0 \\ 2\dot f_i(V) & 0 & -\mathbb{I} & 0 \\ 0 & 0 & 0 & -\mathbb{I} \end{pmatrix}$$

$$\mu,\nu = (v = t + y, u = t - y, x_i, z_a)$$

- The disk amplitude is now a non-trivial function of k_i, for example

$$\langle V_{\mathcal{G}}(k)\rangle_{\text{disk},f} \equiv$$



$$\sim \frac{n_1}{L}\int_0^L dv\, e^{-i\,k_i f^i(v)}\, \mathcal{G}^{\mu\nu}\, R_{\nu\mu}(v)$$

- The geometry sourced by this is

$$ds^2 = H^{-3/4}dv(-du + K dv + 2A_i dx_i) + H^{1/4}(dx_i^2 + dz_a^2)$$

where

$$H = 1 + \frac{Q_1}{L}\int_0^L dv\,\frac{1}{|x_i - f_i|^2}\,, \quad K = \frac{Q_1}{L}\int_0^L dv\,\frac{|\dot{f}_i|^2}{|x_i - f_i|^2}\,,$$

$$A_i = -\frac{Q_1}{L}\int_0^L dv\,\frac{\dot{f}_i}{|x_i - f_i|^2}$$

- The f-dependent harmonic functions H, K, A_i are encoded in $\langle V_{\mathcal{G}}(k)\rangle_{\text{disk},f}$

Comments on D1-P geometries

- There is a continuous family of geometries, parametrized by $f_i(v)$, carrying the same global D1 and P charges and preserving the same supersymmetries (1/4 BPS)
- After quantization, these geometries account for the 2-charge entropy
- All bound states carry non-trivial oscillations in the transverse (x_i) directions and therefore break rotational symmetry (unlike the naive black hole geometry)
- For the same reason, all bound states carry local D1 and P charges along the transverse directions $\Rightarrow$ dipole charges
- All these properties are captured by disk amplitudes
- The geometries are singular at $x_i = f_i(v)$ (D1-brane source)

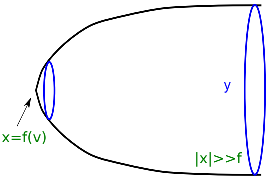
D1-D5 geometries

The metric after a U-duality

$$ds_6^2 = (H(1+K))^{-1/2}[-(dt-A)^2 + (dy+B)^2] + (H(1+K))^{1/2} dx_i^2$$

where $H, K, A \equiv A_i dx_i$ are the same as before and $dA = - *_4 \, dB$

- the term $dy + B$ describes the fiber of a KK-monopole: the y cycle vanishes smoothly at $x_i = f_i(v)$

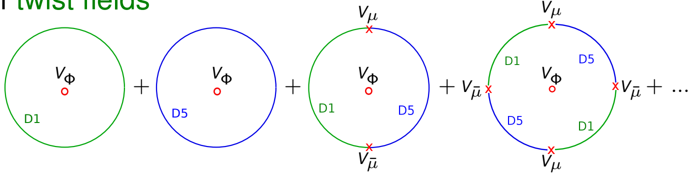


- there is a local KK-monopole charge (dipole charge), that is U-dual to the local D1 charge along the transverse directions

The D1-D5 twist fields

- In a D1-D5 bound state open strings stretched between D1 and D5 have a non-trivial vev
- On the world-sheet this vev can be described perturbatively by the insertion of twist fields

$\langle V_\Phi \rangle_{disk} =$

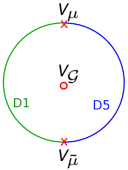


- The twist fields are $V_\mu = \mu^A e^{-\frac{\varphi}{2}} S_A \Delta\,,\ V_{\bar\mu} = \bar\mu^A e^{-\frac{\varphi}{2}} S_A \bar\Delta$ with S_A spin fields of $SO(1,5)$ acting on t, y, x_i
- The bilinears $\bar\mu^A \mu^B$ can have non-trivial vev

$$\bar\mu^A \mu^B = v_I \,(C\Gamma^I)^{[A,B]} + \frac{1}{3!}\, v_{IJK}\,(C\Gamma^{IJK})^{(A,B)}$$

parametrized by v_I, v_{IJK}

- Disk amplitudes with different numbers of twist field insertions contribute to different orders in the $1/r$ expansion: amplitudes with $2n$ twist fields go like $1/r^{(2+n)}$

- For example: the diagram 

$\sim \; \mathcal{G}^{IJ}\, R_J^M\, v_{IMK}\, \frac{x^K}{|x|^4}$ contributes to A_i, B_i

- The vev's v_{IJK} are identified with the moments of the profile f_i, for example: $v_{tij} \sim \frac{1}{L}\int_0^L dv\, \dot{f}_i\, f_j$

- Resumming amplitudes with arbitrary numbers of twist fields should reproduce the full dependence of H, K, A_i, B_i on f_i

- As before, terms non-linear in H, K, A_i, B_i come from amplitudes with more boundaries

D1-D5-P geometries

How to bind P to D1-D5?

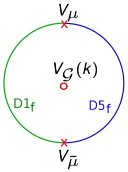
1. P can be carried by D1-D5 strings: $v_{IJK} \to v_{IJK}(v)$
2. P can be carried by D1-D1, D5-D5 strings:
   - D1-D5 vev v_{IJK}
   - left-moving wave profiles $f_{D1}(v), f_{D5}(v)$

1. is the entropically favored configuration
2. is computationally easier (if $f_{D1}(v) = f_{D5}(v) \equiv f(v)$) : take the D1-D5 result and substitute

$$R^{\mu}_{\nu} \to R^{\mu}_{\nu}(f)$$

In the following I will restrict to 2

- For example:



$$\sim \int_0^L dv\, e^{-i\,k^i f_i(v)}\, \mathcal{G}^{IJ}\, R_J^M(f)\, v_{IMK}\, k^K$$

- The dependence on $f_i(v)$ is exact, but only first order in v_{IJK}

Qualitative features of D1-D5-P geometries

- We have derived the geometry sourced by a generic 3 charge bound state up to order $1/r^4$
- All the 10D II supergravity fields are non-trivial, including $B_{\mu\nu}$, $C^{(0)}$, $C^{(4)}$, that vanish in the naive 3-charge black hole
- There are new terms that depend on all three charges, and vanish in every two charge limit:
  - a term of order $1/r^3$ (dipole term) in $B_{\mu\nu}$
  - a term of order $1/r^4$ in g_{ij} (metric along spatial $\mathbb{R}^4$), that makes g_{ij} non conformally flat but conformally hyperkahler
- What about the full non-linear solution?

Conclusions and Outlook

- The computation of disk amplitudes, together with the supergravity equations of motion, provide in principle a systematic way to derive the geometry sourced by 3 charge bound states
- We know:
  - the large r expansion of the geometry, up to order $1/r^4$: the geometry differs from the naive black hole geometry by dipole terms
  - a general exact supergravity ansatz that contains all the fields excited in the large r solution: it turns out to be equivalent to a 5D $\mathcal{N} = 2$ supergravity solution with three vector multiplets
- We would like to know:
  - an exact solution of the supergravity ansatz that reduces to the string amplitude result for large r
  - is the exact solution smooth and horizonless?

Further applications

- These techniques apply to more general bound states, as far as a world-sheet description for the bound state constituents is known (D-branes or fundamental strings):
  - four charge black hole in 4D (work in progress with F. Morales and R. Russo)
  - non-BPS bound states (even if the amplitude computation is probably harder)
- Disk amplitudes could also be used to study dynamical processes involving black holes:
  - absorption or emission from a black hole microstate