Superconformal points in $\mathcal{N} = 2$ SQCD

Simone Giacomelli

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Introduction

The set of vacua (moduli space) of a $\mathcal{N} = 2$ gauge theory is a complex manifold and at a generic point in the moduli space the effective low energy theory is abelian. The effective action is encoded in the Seiberg-Witten curve: N. Seiberg, E. Witten '94.

$$y^2 = P(x, \Lambda^{b_1}, u_i, m_i)$$

Breaking softly $\mathcal{N} = 2$ SUSY with a mass term for the adjoint

$$\mathcal{W} = \mu \operatorname{Tr} \Phi^2 + \sqrt{2} \tilde{Q}^i \Phi Q_i + m_i \tilde{Q}^i Q_i,$$

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Softly broken $SU(N_c)$ $\mathcal{N}=2$ SQCD

- **At the classical level:** The EoMs impose the constraint

$$\Phi \propto \text{Diag}(-m_1, \dots, -m_r, c, \dots, c), \quad c = \frac{\sum_{i=1}^{r} m_i}{N_c - r}.$$

There are $N_c - r$ vacua $\forall r \leq min(N_c - 1, N_f)$.
The effective theory has r colors and N_f flavors (when all masses m_i are equal).

- **At the quantum level:** The Seiberg-Witten curve factorizes as P. Argyres, R. Plesser, N. Seiberg '96.

$$y^2 = (x+m)^{2r}(x-a)(x-b)Q^2(x), \quad r \leq N_f/2.$$

For $r \leq N_f - N_c$ there are $N_c - r$ vacua.
For $N_f - N_c < r \leq N_f/2$ there are $2N_c - N_f$ vacua.
The pattern of flavor symmetry breaking is $U(N_f) \to U(r) \times U(N_f - r)$ $\forall r$. G. Carlino, K. Konishi, H. Murayama '00.

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Duality between classical and quantum vacua

Both the pattern of flavor symmetry breaking and the counting of vacua ($N_c - r + N_c + r - N_f = 2N_c - N_f$) suggest the correspondence $r, N_f - r \to r$ between classical and quantum vacua.

In order to prove this correspondence one needs to determine the SW curve at the classical r vacua. L. Di Pietro, SG '12.

$$y^2 = P_N^2(x) - 4\Lambda^{2N-N_f}(x+m)^{N_f}.$$

This can be achieved using the relation F. Cachazo, N. Seiberg, E. Witten '03.

$$P_N(x) = x^N e^{-\sum_i \frac{U_i}{x^i}} + \Lambda^{2N-N_f}\frac{(x+m)^{N_f}}{x^N} e^{\sum_i \frac{U_i}{x^i}}, \quad U_i = \frac{\langle \mathrm{Tr}\,\Phi^i \rangle}{i}$$

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The generalized Konishi anomaly

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Chiral operators: Operators which anticommute with $\bar{Q}_{\dot{\alpha}}$ (the lowest components of chiral superfields).

Correlation functions of gauge invariant chiral operators can be computed exactly using the **generalized Konishi anomaly**!

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$$0=\langle \bar{D}^2 J_{\Phi}\rangle = \left\langle \frac{\partial \mathcal{W}}{\partial \Phi}\delta\Phi + \mathcal{A}\right\rangle, \quad \delta\Phi \propto \Phi^n.$$

Considering the transformations

$$\delta\Phi = \frac{1}{z-\Phi},\ \delta\Phi = \frac{W_\alpha W^\alpha}{z-\Phi},\ \delta Q_i = \frac{1}{z-\Phi}Q_i,$$

we can calculate the functions

$$\left\langle \mathrm{Tr}\frac{1}{z-\Phi}\right\rangle,\ \left\langle \tilde{Q}^i\frac{1}{z-\Phi}Q_i\right\rangle,\ R(z)=\frac{-1}{32\pi^2}\left\langle \mathrm{Tr}\frac{W_\alpha W^\alpha}{z-\Phi}\right\rangle.$$

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$\mathcal{N}=1$ curve and r vacua

$$R(z)=\frac{1}{2}\left(\mu(z-a)-\sqrt{\mu^2(z-a)^2-4S\mu}\right),\quad a\equiv\frac{\sqrt{2}}{N_c\mu}\langle\tilde{Q}^i Q_i\rangle.$$

This function is single-valued on the $\mathcal{N}=1$ curve:

$$\Sigma : y^2=\mu^2(z-a)^2-4\mu S.$$

The other generating functions also have poles!

$$\left\langle \tilde{Q}^i\frac{1}{z-\Phi}Q_i\right\rangle=\frac{\mu Na+N_f R(z)}{\sqrt{2}z+m}.$$

SUSY vacua are obtained locating N_f poles at $z=-\frac{m}{\sqrt{2}}$.
Classical r vacua correspond to configurations with r poles on the first sheet (i.e. $R(z)\to 0$ for $z\to\infty$)!

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Classical vs quantum r vacua

- We obtained a non perturbative notion of "classical" r vacua.
- Using the generalized Konishi anomaly we can confirm this correspondence computing the SW curve at the $\mathcal{N} = 1$ vacua.

For $m \gg \Lambda$ the gauge group is higgsed to $SU(r)$ by $\langle \Phi \rangle \sim m$. If $r > N_f/2$ it is further broken dynamically at scale Λ to $SU(N_f - r)$.

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Fixed points in SQCD

We found vacua with N_f massless flavors and $SU(r)$ gauge symmetry ($r \leq N_f/2$). The SW curve is $y^2 \approx x^{2r}$.

For $r = N_f/2$ the theory is interacting! Tuning m we find even more singular points (for N_f even): T. Eguchi, K. Hori, K. Ito, S. Yang '96.

$$y^2 \approx x^k, \quad N_f < k \leq N + \frac{N_f}{2}.$$

The study of $SO(N)$ or $Usp(2N)$ gauge theories reveals the same structure, as long as $m \neq 0$!

For $m = 0$ the global symmetry is enhanced from $U(N_f)$ to $Usp(2N_f)$ and $SO(2N_f)$ respectively. In particular, all the r vacua merge in this limit, giving an interacting fixed point.

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Scaling dimensions of chiral operators

Let us analyze the “neighbourhood” of the fixed point in parameter space:

$$y^2 = x^k + \sum_i u_i x^{k-i}, \quad \lambda \approx \frac{y}{x^{N_f/2}} dx.$$

One can determine the scaling dimensions of chiral operators imposing

$$[\lambda] = 1 \ \ (2[y] = 2 + (N_f - 2)[x]); \quad 2[y] = k[x].$$

When the theory has a nonAbelian global symmetry there is another constraint: P. Argyres, M. Douglas, N. Seiberg, E. Witten '96.

$$\prod_i (x - m_i^2) = x^{N_f} + \sum_i c_{2i} x^{N_f - i}; \quad [c_i] = 2i.$$

This condition requires $[x] = 2$.

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Fixed points in $Usp(2N)$ theory

A possible solution is to introduce two sectors, with a different scaling of x. D. Gaiotto, N. Seiberg, Y. Tachikawa '10.

The SW curve for the $Usp(2N)$ theory is

$$xy^2 = [xP_N(x) + 2\Lambda^{2N-N_f+2}\prod_i m_i]^2 - 4\Lambda^{4N-2N_f+4}\prod_i(x - m_i^2).$$

We find the maximal superconformal point setting (for $m_i = 0$)

$$xP_N(x) = x^{N+1} + 2\Lambda^{2N-N_f+2}x^{N_f/2} \Longrightarrow y^2 \approx x^{N+N_f/2}.$$

The collision of r vacua produces a singular point of the form

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Two sector proposal

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$$(x^{k+1-N_f}+\sum_{i=1}^{N} u_i x^{k+1-N_f-i}+4\Lambda^{2k+2-2N_f}); \quad \lambda=\frac{\tilde{y}}{x}dx.$$

We now introduce two scales $\epsilon_A, \epsilon_B \ll 1$.

- In one sector ($x\sim\epsilon_A$) we impose $[x]=2$, so $\tilde{y}^2\sim\epsilon_A$.
- In the other sector ($x\sim\epsilon_B$) we get $\tilde{y}^2\sim x^{k+2-N_f}$.

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We now introduce two scales $\epsilon_A, \epsilon_B \ll 1$.

- In one sector ($x \sim \epsilon_A$) we impose $[x] = 2$, so $\tilde{y}^2 \sim \epsilon_A$.
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Two sector proposal

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Identifying the two sectors

Superconformal points in $\mathcal{N} = 2$ SQCD

Simone Giacomelli

Scuola Normale Superiore, INFN Pisa

Classical and quantum vacua

Interacting fixed points

- In the B sector $(x \sim \epsilon_B)$ the curve is

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A SECTOR $\Longleftarrow SU(2) \Longrightarrow$ B SECTOR

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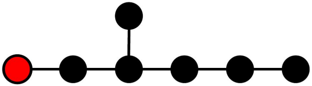
Some limiting cases

- For $N_f = 6$ the flavor symmetry of the A sector is enhanced from $SU(2) \times SO(12)$ to E_7. Y. Tachikawa '09.



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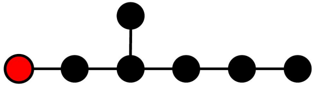

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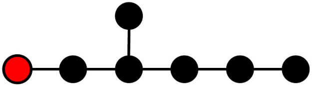

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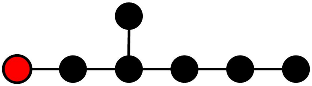

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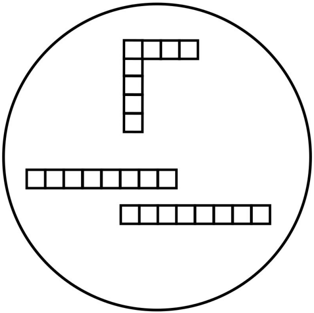
Grazie per l'attenzione!

The A sector



$$\lambda^{2N} = \sum_{k=1}^{N} \lambda^{2N-2k} \phi_{2k}(z); \quad \lambda = xdz.$$

The order of the poles at the punctures are:

$$\{1, 2, \ldots, 2; 1\}; \quad \{1, \ldots, 2N-3; N-1\}.$$