Phenomenology in ghost free Massive Gravity

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Modification of Gravity at Large Distances

General Relativity (only one parameter!) works really good in the range

$$\text{cm} < L < 100 \text{ AU}$$

Do we really understand Gravity at large distances?

- Solar System OK
- Galactic scale (Dark Matter needed to explain rotational curves)
- Cosmological scale (Dark Energy needed to explain present acceleration)

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Simplest modification at large distances

Give mass to the Graviton

Linearized GR

Weak field limit in General Relativity:

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\ , \qquad |h_{\mu\nu}| \ll 1$
- H.E. Lagrangian expansion to the second order in $h_{\mu\nu}$
- Equation of motion: $\quad -\Box \hat{h}_{\mu\nu} = 16\pi G T_{\mu\nu}$

Diff. invariance: $10 - 4 \cdot 2 = 2$ DoF $\rightarrow$ **Graviton**

Phenomenologically in good agreement ($\sim 10^{-5}$) with Solar System experimental texts
(e.g. light deflection from the Sun)

Linearized Massive Gravity

General mass term for the graviton:

$$L_m^{(2)} = -\frac{\alpha}{4} h_{\mu\nu} h^{\mu\nu} - \frac{\beta}{4} h^2$$

No diff. invariance: 10 - 4 = 5 + 1 DoF   This is a scalar ghost

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van Dam - Veltman - Zakharov (1970)

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- Fail to reproduce light deflection from the Sun (25%)

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Boulware - Deser (1972)

Instability reappears when:

- higher order terms to second ones in $h_{\mu\nu}$ are considered
- we expand around a curved background

Ghost free Massive Gravity

de Rham - Gabadadze - Tolley (2010)

Find a nonlinear completion of PF free of ghost up to the fourth order, avoiding BD instability

- To provide the needed new DoF: introduce an extra tensor $f_{\mu\nu}$
- Nonderivative coupling with the metric: $X^\mu_\nu = g^{\mu\alpha} f_{\alpha\nu}$

$$S = \int d^4x \sqrt{g} \left[M_{pl}^2 \left(\mathcal{R} - 2m^2\, V \right) + L_{\text{matt}} \right]$$

$$V = V(Y) \qquad Y^\mu_\nu = \left(\sqrt{X} \right)^\mu_\nu$$

$$V = \text{function of the symmetric polynomials of } Y^\mu_\nu$$

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Hassan - Rosen (2011)

Generalize the proof to non perturbative level and extend it to Bigravity

$$S = \int d^4x \sqrt{g} \left[M_{pl}^2 \left(\mathcal{R} - 2m^2\, V \right) + L_{\text{matt}} \right] + \int d^4x \sqrt{\tilde{g}}\, \kappa\, M_{pl}^2\, \tilde{\mathcal{R}}$$

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Comelli - MC - Nesti - Pilo (2012)

L. Pilo's yesterday talk [arXiv:1204.1027]

Conditions for a general potential $V(X)$ to have 5 DoF $\longrightarrow$ maybe **other** ghost free potentials

Spherically Symmetric Solutions

D. Comelli, MC, F. Nesti, L. Pilo, Phys. Rev. D **85** (2012) 024044

$$ds^2 = -J(r)\,dt^2 + K(r)\,dr^2 + r^2\,d\Omega^2$$

$$d\tilde{s}^2 = -C(r)\,dt^2 + A(r)\,dr^2 + 2\,D(r)\,dt\,dr + B(r)\,d\Omega^2$$

Two branches:

- $D(r) \neq 0, \qquad B(r) = \omega^2 r^2$ with $\omega =$ const.

The graviton mass is zero!

Schwarzschild (A)dS solutions

$$J = 1 - \frac{2\,m_1}{r} + \Lambda_1\, r^2\,, \quad K = \frac{1}{J}, \quad D^2 + A\,C = c^2\omega^4$$

$$C = c^2\omega^2\left(1 - \frac{2\kappa^{-1}m_2}{r} + \Lambda_2\, r^2\right) \quad A = \frac{(c^2+1)\,\omega^2\; J - C}{J^2}$$

No vDVZ discontinuity since $JK = 1$

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Two branches:

- $D(r) = 0$ Perturbative solutions

$$J = 1 + \delta J\,,\quad K = 1 + \delta K\,,\quad C = \omega^2(1 + \delta C)$$
$$A = \omega^2(1 + \delta A)\,,\quad B = \omega^2(r^2 + \delta B)$$

$$\delta K = -\frac{m_2\, e^{-\frac{r}{\lambda_g}}\,(r + \lambda_g)}{r\,\lambda_g} + \frac{2\,m_1}{r} \qquad \delta J = \frac{2\,m_2\,e^{-\frac{r}{\lambda_g}}}{r} - \frac{2\,m_1}{r}$$

$$\delta B = m_2\,e^{-r/\lambda_g}\,\frac{(\kappa\omega^2 + 1)\,(\lambda_g^2 + r^2 + \lambda_g r)}{\kappa\,\omega^2\,r} \qquad \delta C = -\frac{2m_1}{r} - \frac{2\,m_2\,e^{-r/\lambda_g}}{\kappa\,\omega^2\,r}$$

$$\delta A = \frac{2m_1}{r} - m_2\,e^{-r/\lambda_g}\,\frac{(\lambda_g + r)\,[2\lambda_g^2\,(\kappa\,\omega^2 + 1) + \kappa\,\omega^2\,r^2]}{\kappa\,\omega^2\,\lambda_g\,r^3}$$

vDVZ discontinuity since $\delta J + \delta K \to \dfrac{m_2}{r}$ ...but it is a tricky limit!

Cosmological solutions

D. Comelli, MC, F. Nesti, L. Pilo, JHEP **03** (2012) 067

$$ds^2 = a^2(t)\left(-dt^2 + dr^2 + r^2\, d\Omega^2\right)$$
$$\tilde{ds}^2 = \omega^2(t)\left[-c^2(t)\, dt^2 + 2D(t)\, dt\, dr + dr^2 + r^2\, d\Omega^2\right]$$

From the EoM $\longrightarrow D(r) = 0$

Two branches: defining $\xi = \omega/a\,,\ H = a'/a^2\,,\ H_\omega = \omega'/\omega^2$

- $\xi(t) = \bar{\xi}$ with $\bar{\xi} =$ const.

The graviton mass is zero!

FRW + CC solutions

$$H^2 = \frac{8\pi G}{3}\,\rho_m + m^2\Lambda$$
$$H_\omega = \frac{H}{\bar{\xi}} \qquad c^2 \sim \frac{3\kappa\, H^2}{2m^2}$$

In the limit of frozen second metric
$H = H_\omega = 0$ unless to introduce spatial
curvature $k < 0$

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Perturbations

- scalar and vector sector:
  the same of GR: 1S+0V propagate
- tensor sector:
  2T propagate = 4 DoF
- $8 - (4+1) = 3$ DoF (1S+1V) are probably strongly coulped

Cosmological solutions

D. Comelli, MC, F. Nesti, L. Pilo, JHEP **03** (2012) 067

- $c = \frac{H_\omega}{H}\,\xi \qquad H^2 = \frac{8\pi G}{3}\left[\rho_m + \rho_g(\xi)\right] \qquad F(\xi) = \frac{8\pi G\,\rho_m}{3\,m^2}$

Fixed point for $\rho_m \to 0$: $\quad \xi = \bar{\xi}, \quad c = 1 \quad$ (A)de Sitter phase

At early time $G\rho_m/m^2 \gg 1$ e.g. $\left(m^2 M_{Pl}^2 \propto \Lambda \longrightarrow \frac{G\rho_m}{m^2} \sim \frac{\Omega_m}{\Omega_\Lambda}\, z^{3(w+1)} \right)$

Viable cosmology only for **small** $\xi \propto \mathcal{O}(m^2/G\,\rho_m)$

At leading order: $\quad \rho_m + \rho_g = \rho_m\,, \quad w_{eff} = w\,, \quad c = 4 + 3w$

- Radiation dominated $\left(\xi \sim 10\,(1+z)^{-4} \ll 1\right)$

$$a(t) = \frac{t}{t_U} + a_1 m^2 \frac{t^5}{t_U^3} + \cdots \qquad t_U \sim \left(\frac{1}{G\,\rho_c}\right)^{1/2}$$

- Matter dominated $\left(\xi \sim 10\,(1+z)^{-3} \ll 1\right)$

$$a(t) = \frac{t^2}{t_U^2} + a_2 m^2 \frac{t^8}{t_U^6} + \cdots$$

This branch does not exist in the limit of frozen second metric

Cosmological perturbations

D. Comelli, MC, L. Pilo, JHEP to appear [arXiv:1202.1986]

Metrics expansion:

$$g_{\mu\nu} = \bar{g}_{1\,\mu\nu} + a^2\, h_{1\,\mu\nu} \qquad \tilde{g}_{\mu\nu} = \bar{g}_{2\,\mu\nu} + \omega^2\, h_{2\,\mu\nu}$$

Decomposition in SO(3) representations: scalars + vectors + tensors

- dS phase background $\left(\rho = \text{const.} \;\; \Rightarrow \;\; \xi = \text{const.} \;\; \Rightarrow \;\; c = 1\,, \;\; H_\omega = H/\bar{\xi}\;\right)$

  All the 7 DoF propagate: 1 scalar + 1 vector + 2 tensors   No instabilities

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- Equations

$$GR(\Phi_1) + \boxed{a^2 m^2\, f(\xi)}\;\, F(\Phi_1, \Phi_2) = G\, a^2\, \delta\rho$$

$$a^2\, m^2\, f(\xi) \approx a^2\, m^2\, \xi \approx a^2\, \frac{m^4}{G\,\rho_m} \overset{m\to 0}{\to} 0$$

GR

$$GR(\Phi_2) + \boxed{\frac{m^2 a^2 f(\xi)}{\kappa\,\xi^2}}\;\, F(\Phi_1, \Phi_2) = 0$$

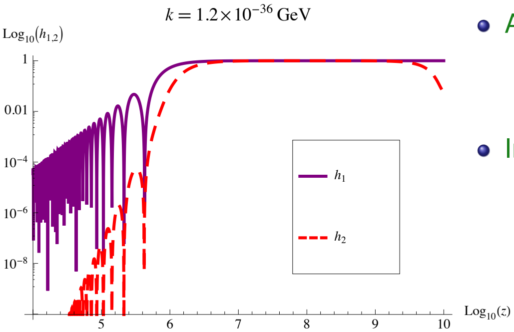
$$\frac{m^2\, a^2\, f(\xi)}{\kappa\,\xi^2} \approx \frac{m^2\, a^2}{\xi} \approx a^2\, G\,\rho_m \overset{m\to 0}{\to} \text{finite}$$

Nontrivial structure in k

Cosmological perturbations

D. Comelli, MC, L. Pilo, JHEP to appear [arXiv:1202.1986]

- Vector sector: 1 vector propagates with no instabilities
- Tensor sector: 2 tensors propagate with no instabilities
  RD era:



- Above horizon $(k\,t \ll 1)$

$$h_2 = h_1$$

- Inside horizon $(k\,t \gg 1)$

$$h_1 \sim \frac{1}{t}$$

$$h_2 \sim \frac{1}{t^5}$$

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- Scalar sector: 2 modes propagate
  RD era:
  - Above horizon $(k\,t \ll 1)$

$$\Phi_1 \sim \text{const.}$$

$$\Phi_2'' + \frac{14}{t}\,\Phi_2' + \frac{37\ \Phi_2 - 39\ \Phi_1}{t^2} = 0 \quad \Rightarrow \quad \Phi_2 \sim \frac{39}{37}\,\Phi_1 = \text{const.}$$

Both scalar perturbations are frozen

  - Inside horizon $(k\,t \gg 1)$

$$\Phi_1 \sim \frac{1}{k^2\,t^2}\;\cos(kt)$$

$$\Phi_2'' + \frac{10}{t}\Phi_2' - \frac{5\,k^2}{3}\;\Phi_2 - \frac{36}{k^2\,t^3}\Phi_1' - \frac{15}{t^2}\Phi_1 = 0 \quad \Rightarrow \quad \Phi_2 \sim \frac{1}{(k\,t)^{1/2}}e^{+(\frac{5}{3})^{1/2}\,k\,t} + \mathcal{O}(\Phi_1)$$

Tachyonic instability

The exponential growth of Φ_2 invalidate perturbation theory at time $t \sim 1/k$

Already in the RD era sub-horizon perturbations become non perturbative

($\neq$ GR where this happens in MD era due to Jeans instability)

Conclusions

- We studied the phenomenology of ghost free Massive Gravity in a Bigravity contest
- Bigravity seems to be very important in formulating Massive Gravity, and more than a tool
- Spherically symmetric solutions that deviate from the Schwarzschild one's exist but are difficult to find analytically
- Viable cosmology exists, with small correction to GR at the next to leading order, and with a late time de Sitter phase
- Unfortunately, cosmological perturbations show a tachyonic instability in the scalar sector for perturbations inside the horizon, invalidating perturbations theory already in the radiation dominated era

Thank you

GR cosmological perturbations

- Scalar sector

$$\left[-w\,\Delta + (3w+1)\mathcal{H}^2 + 2\mathcal{H}'\right]\Phi + 3(w+1)\mathcal{H}\Phi' + \Phi'' = 0$$

- Vector sector

$$\Delta V_i - 16\pi G\, a^2\,(\rho+p)\,\delta v_i = 0$$

$$\partial_{(i}V'_{j)} + 2\mathcal{H}\partial_{(i}V_{j)} = 0$$

- Tensor sector

$$h^{TT''}{}_{ij} + 2\mathcal{H}\,h^{TT'}{}_{ij} - \Delta h^{TT}{}_{ij} = 0$$

dS phase perturbations

- Scalar sector

$$\Phi'' + 2\mathcal{H}\Phi' \left[\frac{2k^4}{9a^2\mathcal{H}^2 m_\Phi^2 + k^4 - 18\mathcal{H}^4} - 1\right] + \frac{1}{3}\Phi\left[\frac{4\left(k^6 - 3k^4\mathcal{H}^2\right)}{9a^2\mathcal{H}^2 m_\Phi^2 + k^4 - 18\mathcal{H}^4} + 3a^2 m_\Phi^2 - k^2 - 6\mathcal{H}^2\right] = 0$$

- Vector sector

$$\mathcal{V}_i'' + \frac{2\,\mathcal{H}\,\mathcal{V}_i'\left(2\,k^2 + a^2\,m_\Phi^2\right)}{k^2 + a^2\,m_\Phi^2} + \mathcal{V}_i\left(k^2 + a^2\,m_\Phi^2\right) = 0$$

- Tensor sector

$$h^{TT\,\prime\prime}_{+\,ij} + 2\,\mathcal{H}\,h^{TT\,\prime}_{+\,ij} + k^2\,h^{TT}_{+\,ij} = 0$$

$$h^{TT\,\prime\prime}_{-\,ij} + 2\,\mathcal{H}\,h^{TT\,\prime}_{-\,ij} + \left(k^2 + a^2\,m_\Phi^2\right)\,h^{TT}_{-\,ij} + a^2\,m_\Phi^2\,\frac{\left(\xi^2\kappa - 1\right)}{\left(\xi^2\kappa + 1\right)}h^{TT}_{+\,ij} = 0$$

Branch 2 perturbations in RD era at leading order

- Scalar sector

$$\Phi_1'' + \frac{4}{t}\,\Phi_1' + \frac{k^2}{3}\,\Phi_1 = 0$$

$$\Phi_2'' + \frac{10x^2+42}{t(x^2+3)}\Phi_2' + \frac{-5x^6-15x^4+333x^2+999}{3t^2(x^2+3)^2}\Phi_2 - \frac{36}{t(x^2+3)}\Phi_1' - \frac{3(5x^2+39)}{t^2(x^2+3)}\Phi_1 = 0$$

$$x = k\,t$$

- Vector sector

$$\mathcal{V}'' + \frac{8\,k^2\,t^2+50}{t(k^2\,t^2+5)}\mathcal{V}' + \frac{3}{t^2}(k^2\,t^2+5)\,\mathcal{V} - \frac{48\,k^2\,t^2+320}{k^2\,t^3\,(k^2\,t^2+5)}\delta v = 0$$

- Tensor sector

$$h_1'' + \frac{2}{t}\,h_1' + k^2\,h_1 = 0$$

$$h_2'' + \frac{10}{t}h_2' + 25k^2 h_2 + \frac{15}{t^2}\,(h_1 - h_2) = 0$$