EXTENDED GRAVITY THEORIES FROM SPACETIME NONCOMMUTATIVITY

L. CASTELLANI

U. PIEMONTE ORIENTALE

CORTONA  2012

1. NC FIELD THEORIES , $\ast$ PRODUCT

2. NC GRAVITY (VIELBEIN)
   $\ast$-DIFF. GEOMETRY : $\wedge_\ast$, d, $\int$, ℓ_A

3. COUPLINGS :
   - FERMIONS
   - GAUGE FIELDS
   - SCALARS

4. GEOMETRICAL SEIBERG-WITTEN MAP

5. EXTENDED GRAVITY ACTIONS

6. DYNAMICAL NONCOMMUTATIVITY

P. Aschieri, L.C.,
"Noncommutative D=4 gravity coupled to fermions",
JHEP 0906(2009)086

"Noncommutative supergravity in D=3 and D=4",
JHEP 0906(2009)087

"Noncommutative gravity coupled to fermions: second order expansion via the Seiberg-Witten map", 1111.4822 hep-th

" Noncommutative gauge fields coupled to noncommutative gravity", 1205.1911 hep-th

MOTIVATIONS FOR NC SPACETIME

- TO ANALYSE SMALL SCALE STRUCTURE OF SPACETIME : HIGH ENERGY CONCENTRATION IN SMALL VOLUMES → CURVATURE → NO INFORMATION UNDER THE PLANCK LENGTH

- MINIMAL LENGTH → CURE OF UV DIV. IN QFT ?

- $\star$ PRODUCTS (ENCODING NONCOMMUTATIVITY) → DEFORMED FIELD THEORIES → SW MAP → EXTENDED (HIGHER DER) FIELD TH.S → EXTENDED GRAVITY THEORIES

- Euristically:

General relativity

Quantum mechanics

Existence
of a minimal length

$$L_P = \sqrt{\frac{hG}{2\pi c^3}} \approx 10^{-33}\, cm$$

L_P: Planck scale

- To "measure" geodetics in a gravitational field:
  use freely falling particles with mass ***m***.

- How precisely can we measure geometry?

- Uncertainty in position of the particle:

$$\boxed{\Delta q \approx \frac{\hbar}{mc}}$$

corresponding to $\Delta p \sim mc$ (pair creation threshold)

- To decrease Δq, increase ***m*** $\rightarrow$ then also the curvature of spacetime increases until …

$$\boxed{\text{curvature radius } \approx \Delta q}$$

- order of magnitude for Δq ?

$$- R = \frac{8\pi G}{c^4} T \rightarrow 1/\Delta q^2 \approx \frac{8\pi G}{c^4} T^0{}_0 \approx \frac{8\pi G}{c^2} \frac{m}{\Delta q^3}$$

and substituting $m \sim h / (c\, \Delta q)$

$$\Delta q \approx (hG / c^3)^{1/2}$$

$\approx$ Planck length

It is therefore impossible to observe phenomena (or spacetime structure) under the Planck scale L_P

- This indetermination emerges automatically if the coordinates are noncommutative :

$$x\,y - y\,x = \alpha\,(L_P)^2$$

or in general:

$$\boxed{[x^i, x^j] = i\,\vartheta^{ij}}$$

with ϑ = antisymmetric tensor (constant)

~ 1930 : MINIMAL LENGTH CUTOFF IN QFT
(BREAKS LORENTZ INVARIANCE : CUTOFF DEPENDS ON REF. SYSTEM)

FLINT, MARCH, MÖGLICH, GOUDSMIT, ...

$\sim 10^{-15}$ m

~ 1936 : BRONSTEIN : GRAVITY DOES NOT ALLOW ARBITRARILY LARGE MASS CONCENTRATION IN SMALL REGION OF SPACETIME ($\Rightarrow$ BLACK HOLE)

DIFFERENT FROM E.M.

1938 : HEISENBERG (ÜBER DIE IN DER THEORIE DER ELEMENTARTEILCHEN AUFTRETENDE UNIVERSELLE LÄNGE)

- UNIVERSAL LENGTH $r_0 \sim 100\ \mathrm{fm}$

(FROM CONSIDERATIONS OF VALIDITY LIMITS OF FERMI THEORY : "EXPLODES" AT SCALES $\sim r_0$, NOT RENORMALIZABLE)

- CONCLUDES THAT GRAVITY IS IRRELEVANT SINCE $\ell_p \ll r_0$

(I.E. MINIMAL LENGTH NOT DUE TO GRAVITY)

1947: SNYDER EXPLOITS AN IDEA OF HEISENBERG

$$[X_\mu, X_\nu] = \frac{J_{\mu\nu}}{m_{PL}}$$

LORENTZ COV.

~1964: MEAD. THOUGHT EXPERIMENTS TO SHOW THAT GRAVITY PREVENTS MEASURING DISTANCES UNDER l_p

~1990: DEFORMED COMM. RELATIONS

- QUANTUM GROUPS
- QUANTUM COSETS

K-POINCARÉ, q-MINKOWSKI, ...

- FROM STRING THEORY :
  - CANNOT "RESOLVE" ARBITRARILY SMALL STRUCTURES WITH FINITE SIZE OBJECTS → GENERALIZED UNCERTAINTY PRINCIPLE
  - LOW EN. LIMIT OF OPEN STRINGS IN A BACKGROUND B-FIELD
- TODAY : NC AS A "GUIDE" TO

EXTENDED GRAVITY THEORIES

NC FIELD THEORIES, * PRODUCT

- FIELD THEORIES ON NC SPACES BECOME ESPECIALLY TRACTABLE WHEN NONCOMMUTATIVITY IS ENCODED IN A TWISTED *-PRODUCT BETWEEN ORDINARY FIELDS (NC, ASSOCIATIVE)

- EXAMPLE: MOYAL-GROENEWOLD *PRODUCT:

$$f(x) * g(x) \equiv \exp\left[\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\right] f(x)g(y)\Big|_{y\to x}$$

$$= fg + \frac{i}{2}\theta^{\mu\nu}\partial_\mu f\,\partial_\nu g + \frac{1}{2!}\left(\frac{i}{2}\right)^2\theta^{\mu\nu}\theta^{\rho\sigma}(\partial_\mu\partial_\rho f)(\partial_\nu\partial_\sigma g)$$
$$+ \cdots$$

- GENERALIZATION $\rightarrow$ ABELIAN TWIST

$$\partial_\mu \longrightarrow X_A^\mu(x)\,\partial_\mu \quad \text{WITH} \quad [X_A, X_B] = 0$$

- EXTENSION TO p-FORMS : $\wedge_\star$-PRODUCT

$$X_A \longrightarrow \ell_{X_A} \quad \text{(LIE DERIVATIVE)} :$$

$$\tau \wedge_\star \tau' \equiv \tau \wedge \tau' + \frac{i}{2}\theta^{AB}\,\ell_{X_A}\tau \wedge \ell_{X_B}\tau' +$$

$$+ \frac{1}{2!}\left(\frac{i}{2}\right)^2 \theta^{AB}\theta^{CD}\left(\ell_{X_A}\ell_{X_C}\tau\right) \wedge \left(\ell_{X_B}\ell_{X_D}\tau'\right) + \cdots$$

- NC THEORIES ARE OBTAINED BY REPLACING PROD. BETWEEN FIELDS WITH $*$ PROD IN CLASSICAL ACTIONS

$\longrightarrow$ NONLOCAL ACTIONS, HIGHER DER

$\longrightarrow$ INVARIANT UNDER NC $*$ SYMMETRIES

- EXAMPLE : NC YANG-MILLS IN FLAT SPACE

* GAUGE THEORY

INGREDIENTS :
$$A_\mu(x) = A_\mu^I(x) T_I$$
$$\varepsilon(x) = \varepsilon^I(x) T_I$$
$$[T_I, T_J] = C_{IJ}^{\ K} T_K \qquad \text{G-LIE A.}$$

FIELD STRENGTH :

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - A_\mu * A_\nu - A_\nu * A_\mu$$

GAUGE TRANSF :
$$\delta_\varepsilon A_\mu = \partial_\mu \varepsilon - A_\mu * \varepsilon + \varepsilon * A_\mu$$
$$\Rightarrow \quad \delta_\varepsilon F_{\mu\nu} = - F_{\mu\nu} * \varepsilon + \varepsilon * F_{\mu\nu}$$

25/05/12

- INVARIANT ACTION : $\int \mathrm{Tr}\left(F_{\mu\nu} * F^{\mu\nu}\right) d^4x$

(USE CYCLICITY OF Tr AND OF $\int$)

- NOTE : $A_\mu * A_\nu - A_\nu * A_\mu =$

$$\left(A^I_\mu * A^J_\nu + A^J_\nu * A^I_\mu\right)[T_I, T_J] +$$

$$\left(A^I_\mu * A^J_\nu - A^J_\nu * A^I_\mu\right)\{T_I, T_J\}$$

$\Rightarrow$ $F^I_{\mu\nu}$, A^I_μ BELONG TO THE $U(G)$

$\Rightarrow$ PROLIFERATION OF DEG. OF FREEDOM !

1) EXPLOIT PROPERTIES OF REPR OF G

FOR EX FOR $G = SU(2)$, IN THE 2-DIM REPR. (PAULI MATRICES):

$$[\tau_I, \tau_J] = \epsilon_{IJK} \tau_K$$
$$\{\tau_I, \tau_J\} = i \delta_{IJ} \mathbb{1}$$

- ONLY 1 ADDITIONAL GENERATOR ($\mathbb{1}$)
- $SU(2) \longrightarrow U(2)$

2) SEIBERG-WITTEN MAP $\hat{A} = \hat{A}(A)$, $\hat{\varepsilon} = \hat{\varepsilon}(A, \varepsilon)$

$\rightarrow$ ALL NC FIELDS IN TERMS OF CLASSICAL FIELDS

NONCOMMUTATIVE (VIELBEIN) GRAVITY

- CLASSICAL ACTION

$$S = \int R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} = -4 \int R \sqrt{-g}\, d^4x$$

$$R^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b$$

$V^a = V^a_\mu dx^\mu$
$R^{ab} = R^{ab}{}_{\mu\nu} dx^\mu \wedge dx^\nu$

- INDEX-FREE:

$$S = \int Tr\,(i\gamma_5 R \wedge V \wedge V)$$

$$V \equiv V^a \gamma_a \,,\quad \Omega \equiv \frac{1}{4} \omega^{ab} \gamma_{ab} \,,\quad R = d\Omega - \Omega \wedge \Omega$$

THEN $R = \frac{1}{4}\left(d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b\right)\gamma_{ab} = \frac{1}{4} R^{ab}\gamma_{ab}$

AND $\int \mathrm{Tr}\left(i\gamma_5 R \wedge V \wedge V\right) =$

$$\frac{1}{4}\int R^{ab} \wedge V^c \wedge V^d \underbrace{\mathrm{Tr}\left(i\gamma_5\gamma_{ab}\gamma_c\gamma_d\right)}_{4\epsilon_{abcd}}$$

IS THE USUAL EINSTEIN-HILBERT ACTION

- INVARIANCES : G.C.T. (INTEGRAL OF 4-FORM)
  LOCAL LORENTZ

LOCAL LORENTZ INVARIANCE IN INDEX-FREE NOTATION :

$$\delta_\varepsilon V = -V\varepsilon + \varepsilon V \ , \quad \delta_\varepsilon \Omega = d\varepsilon - \Omega\varepsilon + \varepsilon\Omega$$

WITH $\varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab}$

$$\Rightarrow \quad \delta_\varepsilon R = -R\varepsilon + \varepsilon R$$

$$\Rightarrow \quad \delta_\varepsilon \int Tr\left(i\gamma_5 R \wedge V \wedge V\right) = 0$$

BY CYCLICITY OF Tr AND $[\gamma_5, \varepsilon] = 0$

* GRAVITY :

$$S = \int \mathrm{Tr}\left(i\gamma_5 R \wedge_* V \wedge_* V\right)$$

WITH $R \equiv d\Omega - \Omega \wedge_* \Omega$

NOTE : $\Omega \wedge_* \Omega$ CONTAINS $[\gamma^{ab}, \gamma^{cd}]$

AND $\{\gamma^{ab}, \gamma^{cd}\} \rightarrow 1\!\!1, \gamma_5$

$$\Rightarrow \left| \begin{aligned} \Omega &= \tfrac{1}{4}\omega^{ab}\gamma_{ab} + i\omega 1\!\!1 + \tilde{\omega}\gamma_5 \\ V &= V^a\gamma_a + \tilde{V}^a\gamma_a\gamma_5 \\ R &= \tfrac{1}{4}R^{ab}\gamma_{ab} + iR 1\!\!1 + \tilde{R}\gamma_5 \end{aligned} \right.$$

- INVARIANCES :
  - GCT (S IS AN INTEGRAL OF 4-FORM)
  - $\star$- GAUGE INVARIANCE UNDER

$$\delta_\varepsilon V = -V \star \varepsilon + \varepsilon \star V$$

$$\delta_\varepsilon \Omega = d\varepsilon - \Omega \star \varepsilon + \varepsilon \star \Omega$$

$$\Rightarrow \delta_\varepsilon R = -R \star \varepsilon + \varepsilon \star R$$

$$\delta_\varepsilon S = \delta_\varepsilon \int Tr\left(i\gamma_5 R \wedge_\star V \wedge_\star V\right) = 0$$

(CYCLICITY OF Tr AND OF $\int$, AND $[\gamma_5, \varepsilon] = 0$)

$$\varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab} + i\varepsilon 1 + \tilde{\varepsilon}\gamma_5$$

- INDEX-FREE FORMALISM (→ CHAMSEDDINE 2003)
- $\star$ GRAVITY WITH COMPLEX VIELBEIN :
  CHAMSEDDINE 2003
- $\star$ GRAVITY WITH REAL VIELBEIN
  (C.C. CONDITIONS ON FIELDS, COMPATIBLE WITH $\star$-GAUGE TRANSF. P. ASCHIERI, L.C.)
  2009
- θ^2 - EXPANSION OF FIELDS AND ACTION VIA SW MAP P. ASCHIERI, L.C. 2011
- GAUGE-INV θ^2- EXPANSION P. ASCHIERI, L.C.
  M. DIMITRIJEVIC, V. RADOVANOVIC

GEOMETRICAL SW MAP FOR ABELIAN TWISTS

- RELATES NC GAUGE FIELD $\hat{\Omega}$ TO ORDINARY (CLASSICAL) Ω, AND $\hat{\varepsilon}$ TO ε AND Ω SO AS TO SATISFY

$$\hat{\Omega}(\Omega) + \hat{\delta}_{\hat{\varepsilon}}\hat{\Omega}(\Omega) = \hat{\Omega}(\Omega + \delta_\varepsilon \Omega)$$

WHERE

$$\delta_\varepsilon \Omega = d\varepsilon - \Omega\varepsilon + \varepsilon\Omega$$

$$\hat{\delta}_{\hat{\varepsilon}}\hat{\Omega} = d\hat{\varepsilon} - \hat{\Omega} * \hat{\varepsilon} + \hat{\varepsilon} * \hat{\Omega}$$

- CAN BE SOLVED ORDER BY ORDER IN θ

$$\hat{\Omega} = \Omega + \Omega^1(\Omega) + \Omega^2(\Omega) + \cdots$$

$$\hat{\varepsilon} = \varepsilon + \varepsilon^1(\varepsilon, \Omega) + \varepsilon^2(\varepsilon, \Omega) + \cdots$$

WITH $$\Omega^{n+1} = \frac{i}{4(n+1)} \theta^{AB} \{\hat{\Omega}_A, l_B \hat{\Omega} + \hat{R}_B\}_*^n$$

$$\varepsilon^{n+1} = \frac{i}{4(n+1)} \theta^{AB} \{\hat{\Omega}_A, l_B \hat{\varepsilon}\}_*^n$$

$$R^{n+1} = \frac{i}{4(n+1)} \theta^{AB} \left(\{\hat{\Omega}_A, (l_B + L_B)\hat{R}\}_*^n - [\hat{R}_A, \hat{R}_B]_*^n\right)$$

RECURSIVE REL.S: GENERALIZE ULKER (2008)

FOR EXAMPLE :

$$V^{1a} = 0$$

$$\tilde{V}^{1a} = \frac{1}{4}\theta^{AB}\chi^{\rho}_{A}\omega^{bc}_{\rho}\epsilon_{abcd}\left(\ell_B V^d - \frac{1}{2}\chi^{\sigma}_{B}\omega^{de}_{\sigma}V^e\right)$$

$$\omega^{1ab} = 0$$

$$\omega^1 = -\frac{1}{16}\theta^{AB}\chi^{\rho}_{A}\omega^{ab}_{\rho}\left(\ell_B\omega^{ab} + i_B R^{ab}\right)$$

$$\tilde{\omega}^1 = -\frac{1}{16}\theta^{AB}\chi^{\rho}_{A}\omega^{ab}_{\rho}\left(\ell_B\omega^{cd} + i_B R^{cd}\right)\epsilon_{abcd}$$

- APPLYING THE SW MAP TO THE FIELDS IN THE NC GRAVITY ACTION YIELDS A HIGHER DERIV. ACTION INVOLVING ONLY V^a, ω^{ab} (AND THE BACKGR. X_A FIELDS DEFINING THE $\star$-PRODUCT)

$$S = S^0 + S^1 + S^2 + \ldots$$

WITH $S^0 =$ CLASSICAL EINSTEIN-HILBERT ACTION

$$S^1 = 0$$

$$S^2 \neq 0$$

NOTE: THE EXPANDED ACTION, AFTER SW MAP, IS GAUGE INVARIANT UNDER USUAL GAUGE TRANSF. ORDER BY ORDER IN θ

$$S = S^0 + S^1 + S^2 + \dots$$

- BECAUSE USUAL GAUGE TRANSF. INDUCE $\star$-GAUGE TRANSF. ON THE NC FIELDS UNDER WHICH S IS INVARIANT
- BECAUSE USUAL GAUGE TRANSF DO NOT CONTAIN θ

$$
\begin{aligned}
S^2 = \vartheta^{AB}\vartheta^{CD} \int & \tfrac{1}{2} R^{rs}_{CD}\, i_B i_A \left(R^{ab} \wedge R^{db}\right) \wedge V^e \wedge V^f \epsilon_{rsef} \\
& + i_B i_A \left(L_C R^{ab} \wedge L_D R^{ad}\right) \wedge V^e \wedge V^f \epsilon_{bdef} \\
& - 2 i_B i_A \left(R^{ab}_C \wedge R^{cd}_D \wedge R^{ef}\right) \wedge V^e \wedge V^f \epsilon_{abcd} \\
& - i_B i_A \left(R^{ab}_C \wedge R^{ab}_D \wedge R^{ef}\right) \wedge V^g \wedge V^h \epsilon_{efgh} \\
& - R^{fg} \wedge L_A V^f \wedge R^{cd}_{CB} \wedge L_D V^e \epsilon_{cdeg} \\
& - i_B i_A \left(R^{ef} \wedge R^{ab}_C \wedge R^{ab}_D\right) \wedge V^g \wedge V^h \epsilon_{efgh} \\
& - \tfrac{1}{4} i_B i_A \left(R^{ab} \wedge R^{cd}\right) L_C V^e \wedge L_D V^e \epsilon_{abcd} \\
& - \tfrac{1}{4} R^{ab}_{CD} R^{cd} \wedge L_A V^e \wedge L_B V^e \epsilon_{abcd} \\
& + \tfrac{1}{4} R^{ab}_C \wedge R^{cd}_D \wedge L_A V^e \wedge L_B V^e \epsilon_{abcd} \\
& - \tfrac{1}{4} R^{ab} \wedge \left(L_C L_A V^c \wedge L_D L_B V^d\right) \epsilon_{abcd} \\
& + \tfrac{1}{8} R^{gf} \wedge R^{cd}_{CA} \wedge L_D V^e \wedge L_B V^f \epsilon_{cdeg}
\end{aligned}
$$

- S^2 IS MANIFESTLY INVARIANT UNDER LOCAL LORENZ ROTATIONS

→ FROM $L_B \equiv i_B D + D i_B$

$$L_B V = D V_B + i_B T = D(X_B^{\mu} V_{\mu}) + 2 X_B^{\mu} T_{\mu\nu} dx^{\nu}$$

$$T \equiv dV - \Omega \wedge V - V \wedge \Omega \qquad \text{(TORSION)}$$

$$V_B \equiv i_B V = X_B^{\mu} V_{\mu}$$

→ AT THIS STAGE THE X_A^{μ} VECTOR FIELDS HAVE NO DYNAMICS

COUPLINGS OF NC $\star$-GRAVITY

FERMIONS

- CLASSICAL ACTION

$$S = \int R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} +$$

$$+ \underbrace{i [\bar{\psi} \gamma^a D\psi - \overline{D\psi} \gamma^a \psi] \wedge V^b \wedge V^c \wedge V^d \epsilon_{abcd}}_{i \bar{\psi} \not{D} \psi \sqrt{-g}\, d^4x}$$

$$D\psi \equiv d\psi - \Omega\psi$$

- $\delta_\varepsilon \psi = \varepsilon \psi \qquad \left(\varepsilon = \frac{1}{4} \varepsilon^{ab} \gamma_{ab}\right)$

* FERMIONS (SPIN 1/2)

$$S = \int Tr\left[i\gamma_5 \left(R \wedge_* V \wedge_* V - (D\psi * \bar{\psi} - \psi * D\bar{\psi}) \wedge_* V \wedge_* V \wedge_* V \right) \right]$$

$$D\psi \equiv d\psi - \Omega * \psi$$

- *-GAUGE INVARIANT UNDER SAME *-TRANSF AND

$$\delta_\varepsilon \psi \doteq \varepsilon * \psi$$

* FERMIONS (SPIN 3/2)

→ NC SUPERGRAVITY

GAUGE FIELDS

NO HODGE DUAL !

- CLASSICAL (GEOMETRIC) ACTION

$$S = \int (R^{ab} + \Lambda V^a \wedge V^b) \wedge V^c \wedge V^d \epsilon_{abcd}$$
$$+ \mathrm{Tr}\left(f^{ab} F - \frac{1}{12} f^{rs} f_{rs} V^a \wedge V^b \right) \wedge V^c \wedge V^d \epsilon_{abcd}$$

- $F = dA - A \wedge A \,, \qquad A = A^I T_I$

$\equiv F_{ab} V^a \wedge V^b \qquad F = F^I T_I$

- $f_{rs} = f^I_{rs} T_I$: AUXILIARY 0-FORM

• FIELD EQS :

$$\frac{\delta}{\delta f_{ab}} \rightarrow f_{ab} = F_{ab}$$

$$\frac{\delta}{\delta A^I} \rightarrow D_a^{G\otimes SO(3,1)} F^{ab} = 0$$

$$\frac{\delta}{\delta V^a} \rightarrow R^{ac}{}_{bc} - \frac{1}{2}\delta^a_b R = -\left(F^I_{ac} F^I_{bc} - \frac{1}{4}\delta^a_b F^2\right) + 3\Lambda \delta^a_b$$

$\frac{\delta}{\delta \omega^{ab}} \rightarrow$ GIVES ω^{ab} IN TERMS OF V^a

$$(dV^a - \omega^a{}_b V^b = 0)$$

- SAME STRATEGY

1) $\Lambda \longrightarrow \Lambda_*$ IN THE ACTION

2) SW MAP

- RESULT :

$$S^1 = \frac{1}{2}\theta^{AB} \int \Big(F^I_{AB} f^J_{rs} F^L \wedge V^c \wedge V^d - \frac{1}{2} f^I_{ab} F^J_A \wedge F^L_B \wedge V^c \wedge V^d - \frac{1}{6} F^I_{AB} f^J_{ab} f^L_{ab} \Big) \mathrm{Tr}\left(i \{T_I, T_J\} T_L\right)$$

SCALARS

- CLASSICAL ACTION

$$S = \int \Big(R^{ab} \wedge V^c \wedge V^d + \frac{1}{3} \varphi^{Ia} d\Phi^I \wedge V^b \wedge V^c \wedge V^d - \frac{1}{4!} \big(\varphi^{In} \varphi^I_n + W(\Phi) \big) V^a \wedge V^b \wedge V^c \wedge V^d \Big) \epsilon_{abcd}$$

Φ^I : SCALAR (0-FORMS)

φ^{Ia} : AUXILIARY FIELDS

FIELD EQ.S $\rightarrow$ $\varphi^I_a = \partial_a \Phi^I$

- $\wedge \to \wedge_\star$ IN THE ACTION
- SW - MAP

$$\Rightarrow \quad S = \underbrace{S^0}_{} + \underbrace{S^1}_{=0} + S^2 + \cdots$$

P. ASCHIERI, L.C.

$\to$ EXTENDED GRAVITY COUPLED TO SCALARS (AND TO THE BACKGROUND FIELDS X^μ_A)

- RELATE X^μ_A TO SCALAR FIELDS

DYNAMICAL NONCOMMUTATIVITY

- RELATING X_A TO SCALARS :

$$X_A^\mu = [\partial_\mu \phi^A]^{-1}$$

OR MORE GENERALLY

$$X_A^\mu = [\partial_\mu Z(\phi)^A]^{-1}$$

$$\Rightarrow [X_A, X_B] = 0$$

P. A., L.C., M. DIMITRIJEVIC (2008) ← FLAT SPACE
P. A., L.C. 2012 ← GRAVITY TO APPEAR

CONCLUSIONS

- USING NONCOMMUTATIVITY AS A BUILDING PRINCIPLE FOR EXTENDED GRAVITY THEORIES

- DYNAMICAL X^{μ}_{A} (I.E. DYNAMICAL TWIST) IS REALIZED VIA SCALAR FIELDS