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On the Pure Spinor Superstring in $\mathrm{AdS}_4 \times \mathbb{CP}^3$

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  - central charge
  - one-loop effective action

Pure Spinor superstring in semi-symmetric superspaces

Semi-symmetric superspace: supercoset manifold G/H_0 with

- $\mathrm{Bos}[G]$ symmetries of the space
- Ω automorphism involutive on $\mathrm{Bos}[\mathcal{G}]$ $\quad\Rightarrow$
- $\mathbb{Z}_4$-graded Lie superalgebra

$$\mathcal{G} = \bigoplus_{k=0}^{3} \mathcal{H}_k \quad \text{with} \quad \mathcal{H}_k = \{H_k \in \mathcal{G} : \Omega(H_k) = i^k H_k\}_{k=0,1,2,3}$$

in particular

$$(\mathcal{H}_0; \mathcal{H}_2) \text{ bosonic} \qquad\qquad (\mathcal{H}_1, \mathcal{H}_3) \text{ fermionic}$$

$\mathcal{H}_0$ closed subalgebra $\Rightarrow$ generates the subgroup H_0.

Canonical form in $\mathcal{G}$:

$$J \equiv g^{-1}dg = \sum_{i=0}^{3} J_i \qquad J_i \in \mathcal{H}_i \,, g \in G$$

Invariance of J under *global* left multiplication

$$g \to g'g \qquad g' \in G$$

Transformation of J under *local* right multiplication

$$g \to gh \qquad h \in H_0$$

$$J_0 \to h^{-1}J_0 h + h^{-1}dh \qquad J_i \to h^{-1}J_i h \qquad i = 1, 2, 3$$

$\Rightarrow$ J_i matter fields

$\Rightarrow$ J_0 gauge connection

Superstring action (Berkovits, 2000; Berkovits, Chandia, 2001)

$$S_{PS} = \frac{\sqrt{\lambda'}}{2\pi} \int d^2z \, \mathrm{STr} \Big[\frac{1}{2} J_2 \overline{J}_2 + \frac{3}{4} J_3 \overline{J}_1 + \frac{1}{4} J_1 \overline{J}_3 + \\ - w_3 \overline{\nabla} \lambda_1 - w_1 \nabla \lambda_3 - \{w_3, \lambda_1\}\{w_1, \lambda_3\} \Big]$$

where $\nabla \lambda \equiv \partial \lambda + [J_0, \lambda]$ covariant derivative and

$\lambda_{1,3} \in \mathcal{H}_{1,3}$ ghosts $\qquad$ $w_{3,1} \in \mathcal{H}_{3,1}$ conjugate momenta

BRST transformation:

$$Q(g) = g(\lambda_1 + \lambda_3) \quad \Rightarrow \quad Q(J) = \partial(\lambda_1 + \lambda_3) + [J, \lambda_1 + \lambda_3]$$

$$Q(\lambda_1) = 0 \qquad Q(\lambda_3) = 0 \qquad Q(w_3) = J_3 \qquad Q(w_1) = \overline{J}_1$$

Invariance of S_{PS} and nilpotency of Q are guaranteed by the constraints

$$\{\lambda_1, \lambda_1\} = 0 \qquad \{\lambda_3, \lambda_3\} = 0$$

that correspond to the pure spinor constraint in flat space.

The constraints also allow an additional local invariance

$$\delta\lambda_1 = 0 \qquad \delta\lambda_3 = 0 \qquad \delta w_3 = [\lambda_1, \Omega_2] \qquad \delta w_1 = [\lambda_3, \Omega_2]$$

with $\Omega_2 \in \mathcal{H}_2$.

$\mathrm{OSP}(4|6)$ **supercoset**

Supermatrix $(4+6)\times(4+6)$

$$A = \begin{pmatrix} X & \theta \\ \eta & Y \end{pmatrix}$$

with X, Y Grassmann even and θ, η Grassmann odd.

Definition: $A \in osp(4|6)$ superalgebra if

$$A^{st}K + KA = 0$$

with K 4-symplectic/6-euclidean metric $\Rightarrow$

$$\mathrm{Bos}[\mathrm{OSP}(4|6)] \cong \mathrm{Sp}(4)\times\mathrm{SO}(6) \cong \mathrm{SO}(3,2)\times\mathrm{SU}(4)$$

Ω involutive automorphism provides $\mathcal{H}_0 = so(3,1)\oplus u(3) \Rightarrow$

$$\mathrm{Bos}\left[\frac{\mathrm{OSP}(4|6)}{\mathrm{SO}(3,1)\times\mathrm{U}(3)}\right] \cong \frac{\mathrm{SO}(3,2)}{\mathrm{SO}(3,1)}\times\frac{\mathrm{SU}(4)}{\mathrm{U}(3)} \cong \mathrm{AdS}_4\times\mathbb{CP}^3$$

In addition $[\Omega(A)]^* = \Omega(A^*) \Rightarrow$ one-to-one correspondence

$$\mathcal{H}_3{}^* \equiv \mathcal{H}_1$$

$so(3,1) \oplus u(3)$ basis for $osp(4|6)$

$$\mathcal{H}_0 : \quad \begin{array}{ll} M^{mn} & \in so(3,1) \\ V_a{}^b & \in u(3) \end{array} \qquad \mathcal{H}_2 : \quad \begin{array}{ll} P^m & \in so(3,2) \setminus so(3,1) \\ V_a\,, V^a & \in su(4) \setminus u(3) \end{array}$$

$$\mathcal{H}_1 : \quad \mathcal{O}_{\alpha a}\,, \mathcal{O}^{\dot{\alpha} a} \qquad \mathcal{H}_3 : \quad \mathcal{O}_\alpha{}^a\,, \mathcal{O}^{\dot{\alpha}}{}_a$$

Ghost components

$$\lambda_1 = \lambda^{\alpha a} \mathcal{O}_{\alpha a} + \lambda_{\dot{\alpha} a} \mathcal{O}^{\dot{\alpha} a} \qquad \lambda_3 = \lambda^\alpha{}_a \mathcal{O}_\alpha{}^a + \lambda_{\dot{\alpha}}{}^a \mathcal{O}^{\dot{\alpha}}{}_a$$

$\alpha/\dot{\alpha}$ transforms under $(\frac{1}{2}, 0)/(0, \frac{1}{2})$ representation of $\mathrm{SO}(3,1)$

a super/sub-script transforms under $\mathbf{3}/\mathbf{3}^*$ representation of $\mathrm{U}(3)$

Solution of the constraints

By means of the $\mathrm{OSP}(4|6)$ superalgebra the ghost constraints becomes

$$\epsilon_{abc}\lambda^{\alpha a}\varepsilon_{\alpha\beta}\lambda^{\beta b} = 0 \qquad \epsilon^{abc}\lambda^{\alpha}{}_{a}\varepsilon_{\alpha\beta}\lambda^{\beta}{}_{b} = 0$$

$$\epsilon^{abc}\lambda_{\dot\alpha a}\varepsilon^{\dot\alpha\dot\beta}\lambda^{\dot\beta b} = 0 \qquad \epsilon_{abc}\lambda_{\dot\alpha}{}^{a}\varepsilon^{\dot\alpha\dot\beta}\lambda_{\dot\beta}{}^{b} = 0$$

$$\lambda^{\alpha a}(\sigma^m)_{\alpha}{}^{\dot\beta}\lambda_{\dot\beta a} = 0 \qquad \lambda_{\dot\alpha}{}^{a}(\overline{\sigma}^m)^{\dot\alpha}{}_{\beta}\lambda^{\beta}{}_{a} = 0$$

Solution for λ_1 (Fré, Grassi, 2008)

$$\lambda^{\alpha a} = \theta^{\alpha} u^{a} \qquad \lambda_{\dot\alpha a} = \psi_{\dot\alpha} v_a$$

with

$$u^a v_a = 0$$

scaling $\quad u^a \to c u^a \qquad \theta^\alpha \to \frac{1}{c}\,\theta^\alpha \quad$ and the analogue for $v, \psi \Rightarrow$

$$|u|^2 \equiv u^a u^*_a = 1 \qquad |v|^2 \equiv v^{a*} v_a = 1$$

Matrix

$$U = \begin{pmatrix} u^a & \epsilon^{abc} v_b u_c^* & v^{a*} \end{pmatrix} \in \mathrm{SU}(3)$$

It is still possible $u^a \to e^{i\phi_u} u^a$ and $v_a \to e^{i\phi_v} v_a \Rightarrow$

$$(u, v) \quad \leftrightarrow \quad \frac{\mathrm{SU}(3)}{\mathrm{U}(1)_u \times \mathrm{U}(1)_v}$$

Projection of the covariant canonical form $U^{-1}\nabla U$ on the coset $su(3)\diagdown[u(1) \oplus u(1)]$

$$j \equiv \begin{pmatrix} 0 & -{j_1}^* & -{j_2}^* \\ j_1 & 0 & -{j_3}^* \\ j_2 & j_3 & 0 \end{pmatrix}$$

$$j_1 = \epsilon_{abc} v^{a*} u^b \nabla u^c \qquad j_2 = v_a \nabla u^a \qquad j_3 = \epsilon^{abc} u_a^* v_b \nabla v_c$$

with

$$\nabla u^a = \partial u^a + i{J^a}_b u^b \qquad \nabla v_a = \partial v_a - i v_b {J^b}_a$$

Solution for λ_3 (recalling $\mathcal{H}_3{}^* \equiv \mathcal{H}_1$)

$$\lambda^\alpha{}_a = \bar\psi^\alpha v_a \qquad \lambda_{\dot\alpha}{}^a = \bar\theta_{\dot\alpha} u^a$$

with

$$\bar\psi^\alpha = \psi^*_{\dot\alpha}(\bar\sigma^2)^{\dot\alpha\alpha} \qquad \bar\theta_{\dot\alpha} = \theta^{\alpha *}(\sigma^2)_{\alpha\dot\alpha}$$

Parametrization of $w_{3,1}$: using the residual local invariance

$$w^\alpha{}_a = \omega^\alpha u^*_a \qquad w_{\dot\alpha}{}^a = \rho_{\dot\alpha} v^{a*}$$

$$w^{\alpha a} = \bar\rho^\alpha v^{a*} \qquad w_{\dot\alpha a} = \bar\omega_{\dot\alpha} u^*_a$$

with

$$\bar\rho^\alpha = \rho^*_{\dot\alpha}(\bar\sigma^2)^{\dot\alpha\alpha} \qquad \bar\omega_{\dot\alpha} = \omega^{\alpha *}(\sigma^2)_{\alpha\dot\alpha}$$

The (anti)ghost $(\omega)\theta$ are a *bc* system with action $\varepsilon_{\alpha\beta}\omega^\alpha\overline{\nabla}\theta^\beta$ where

$$\nabla\theta^\alpha = \partial\theta^\alpha - \frac{1}{2}\theta^\beta (J_{mn}\sigma^{mn})_\beta{}^\alpha$$

Action with unconstrained ghosts

$$S = \frac{R^2}{2\pi}\int d^2z\,\Big[\frac{1}{2}\eta_{mn}J^m\overline{J}^n - \frac{1}{2}J_a\overline{J}^a - \frac{1}{2}J^a\overline{J}_a +$$
$$-\frac{i}{4}\varepsilon_{\alpha\beta}\left(3J^{\alpha}{}_a\overline{J}^{\beta a} + J^{\alpha a}\overline{J}^{\beta}{}_a\right) - \frac{i}{4}\varepsilon^{\dot\alpha\dot\beta}\left(3J_{\dot\alpha}{}^a\overline{J}_{\dot\beta a} + J_{\dot\alpha a}\overline{J}_{\dot\beta}{}^a\right) +$$
$$-i\left(\varepsilon_{\alpha\beta}\omega^\alpha\overline{\nabla}\theta^\beta + \varepsilon^{\dot\alpha\dot\beta}\rho_{\dot\alpha}\overline{\nabla}\psi_{\dot\beta} + \varepsilon_{\alpha\beta}\bar\rho^\alpha\nabla\bar\psi^\beta + \varepsilon^{\dot\alpha\dot\beta}\bar\omega_{\dot\alpha}\nabla\bar\theta_{\dot\beta}\right) +$$
$$+\frac{1}{8}\left(\omega^\alpha(\sigma^{mn})_{\alpha\beta}\theta^\beta\;\bar\rho^\gamma(\sigma_{mn})_{\gamma\delta}\bar\psi^\delta + \rho_{\dot\alpha}(\bar\sigma^{\,mn})^{\dot\alpha\dot\beta}\psi_{\dot\beta}\;\bar\omega_{\dot\gamma}(\bar\sigma_{mn})^{\dot\gamma\dot\delta}\bar\theta_{\dot\delta}\right) +$$
$$+\frac{1}{2}\left(\varepsilon_{\alpha\beta}\omega^\alpha\theta^\beta\varepsilon^{\dot\alpha\dot\beta}\bar\omega_{\dot\alpha}\bar\theta_{\dot\beta} + \varepsilon^{\dot\alpha\dot\beta}\rho_{\dot\alpha}\psi_{\dot\beta}\varepsilon_{\alpha\beta}\bar\rho^\alpha\bar\psi^\beta\right)\Big] + S_\chi$$

with

$$S_\chi = \frac{R^2}{2\pi}\int d^2z\,\mathrm{Tr}(\bar{j}^\dagger j) = \frac{R^2}{2\pi}\int d^2z\,[\sum_{k=1}^{3}\bar{j}_k{}^* j_k + \text{c.c.}]$$

Trivially

- $S_{\theta,\psi} \to \mathrm{SO}(3,1) \times \mathrm{U}(3)$-invariant
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$\mathrm{U}(3)$ transformation:

$$u^a \;\to\; u^b M_b{}^a \qquad\qquad v_a \;\to\; M_a^{*\,b} v_b$$

(analogous ones for v^{a*} and u_a^*)

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(analogous ones for v^{a*} and u^*_a)

- j_2 invariant by definition
- $j_1 \;\to\; (\det M) j_1 \quad \Rightarrow \quad {\bar{j}_1}^* j_1 \;\to\; |\det M|^2 {\bar{j}_1}^* j_1 = {\bar{j}_1}^* j_1$

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- $j_3 \;\to\; (\det M^*) j_3 \quad$ etc.

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$\Rightarrow \ S_\chi \ \mathrm{U}(3)$-invariant

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- $S_{\chi} \rightarrow \mathrm{SO}(3,1)$-invariant

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- $j_1 \;\rightarrow\; (\det M) j_1 \quad \Rightarrow \quad \bar{j}_1{}^* j_1 \;\rightarrow\; |\det M|^2 \bar{j}_1{}^* j_1 = \bar{j}_1{}^* j_1$
- $j_3 \;\rightarrow\; (\det M^*) j_3$ etc.

$\Rightarrow$ S_{χ} $\mathrm{U}(3)$-invariant

$\Rightarrow$ S manifestly $\mathrm{SO}(3,1) \times \mathrm{U}(3)$-covariant

Background field expansion

$$g = \widetilde{g} e^{X/R}$$

with

$\widetilde{g} \in \mathrm{OSP}(4|6) \;\Rightarrow\; \widetilde{J}$ background configuration

$X = \sum_{i=1}^{3} X_i \in osp(4|6) \setminus [so(3,1) \oplus u(3)]$ quantum fluctuation

$$\Rightarrow \quad J = \widetilde{J} + \frac{1}{R}(dX + [\widetilde{J}, X]) + \frac{1}{2R^2}[dX + [\widetilde{J}, X], X] + O\left(\frac{1}{R^3}\right)$$

S_{matter} gives

- the kinetic term $S_{XX} \sim \bar{\partial} X \partial X$
- the interaction terms $S_{\widetilde{J}XX}$, $S_{\widetilde{J}\widetilde{J}XX}$

S_{ghost} gives

- the kinetic term $S_{\omega\theta} \sim \omega\bar{\partial}\theta, \ldots$
- the interaction terms $S_{\widetilde{J}_0\omega\theta}$, $S_{XX\omega\theta}$, $S_{\widetilde{J}_0 XX\omega\theta}$

For S_χ the expansion is

$$U = \widetilde{U} e^{x/R}$$

with

$\widetilde{U} \in \mathrm{SU}(3) \Rightarrow \tilde{j}$ background configuration

$x \in su(3) \setminus [u(1) \oplus u(1)]$ quantum fluctuation

and gives the kinetic term $S_{xx} \sim \bar{\partial} x \partial x$

From S_{XX}, $S_{\omega\theta}$, S_{xx} we obtain the fundamental OPE to calculate

- the central charge
- the one-loop divergent diagrams

Central charge

$$c = c^{(0)} + \frac{1}{R^2} c^{(2)} + \frac{1}{R^4} c^{(4)} + \cdots$$

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$$\begin{aligned} T_{matter} &= -R^2 \mathrm{STr}\left[\frac{1}{2}J_2 J_2 + J_1 J_3\right] \\ &\to -\mathrm{STr}\left[\frac{1}{2}\partial X_2 \partial X_2 + \partial X_1 \partial X_3\right] + O\left(\frac{X^4}{R^2}\right) \end{aligned}$$

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$$c^{(0)}_{bos.\ matter} = 4(\mathrm{AdS}) + 6(\mathbb{CP}) = 10 \quad c^{(0)}_{ferm.\ matter} = -24$$

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$$c^{(0)}_{bos.\ matter} = 4(\mathrm{AdS}) + 6(\mathbb{C}\mathrm{P}) = 10 \quad c^{(0)}_{ferm.\ matter} = -24$$
$$c^{(2)}_{bos.\ matter} = c^{(2)}_{ferm.\ matter} = 0$$

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$$c^{(0)}_{bos.\ matter} = 4(\mathrm{AdS}) + 6(\mathbb{C}\mathrm{P}) = 10 \quad c^{(0)}_{ferm.\ matter} = -24$$
$$c^{(2)}_{bos.\ matter} = c^{(2)}_{ferm.\ matter} = 0$$

$$T_{\theta\psi} = iR^2\left(\varepsilon_{\alpha\beta}\omega^{\alpha}\nabla\theta^{\beta} + \varepsilon^{\dot{\alpha}\dot{\beta}}\rho_{\dot{\alpha}}\nabla\psi_{\dot{\beta}}\right)$$

Central charge

$$c = c^{(0)} + \frac{1}{R^2} c^{(2)} + \frac{1}{R^4} c^{(4)} + \cdots$$

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$$c^{(0)}_{\theta\psi} = 4 + 4 = 8 \quad c^{(2)}_{\theta\psi} = 0$$

$$T_\chi = -R^2 \mathrm{Tr}\left(j^\dagger j\right) \quad \rightarrow \quad -2\sum_{k=1}^{3} \partial x_k^* \partial x_k + O\left(\frac{x^4}{R^2}\right)$$

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$$c_\chi^{(0)} = 6 \qquad c_\chi^{(2)} = 0$$

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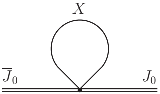
$$c_\chi^{(0)} = 6 \qquad c_\chi^{(2)} = 0$$

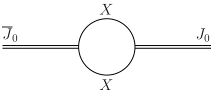
$c = c_{bos.\,matter} + c_{ferm.\,matter} + c_{\theta\psi} + c_\chi = 10 - 24 + 8 + 6 = 0$

up to the $1/R^2$ *order*

One-loop effective action

$\overline{J}_0 J_0$ function





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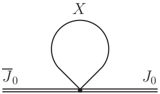

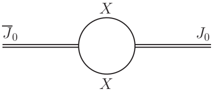

- $\mathrm{SO}(3,1)$ sector $\overline{J}_{mn} J_{kl}$ and $\mathrm{U}(3)$ sector $\overline{J}^a{}_b J^c{}_d$

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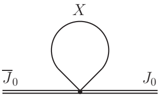

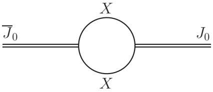

- $\mathrm{SO}(3,1)$ sector $\overline{J}_{mn} J_{kl}$ and $\mathrm{U}(3)$ sector $\overline{J}^{a}{}_{b} J^{c}{}_{d}$

  divergent parts of diagrams with bosonic and fermionic propagators sum to zero separately

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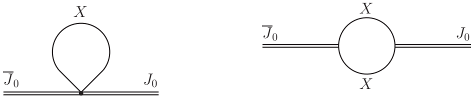

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- mixed sector $\overline{J}_{mn}J^a{}_b$

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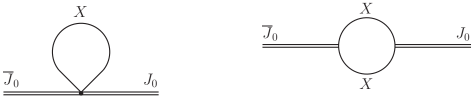

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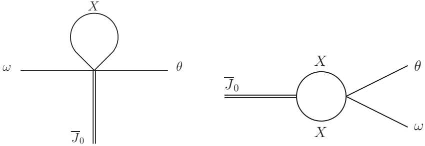
  divergent parts of diagrams with bosonic and fermionic propagators sum to zero separately

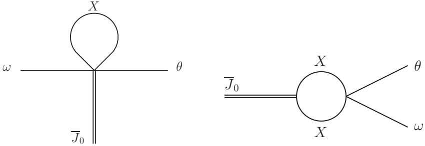
- mixed sector $\overline{J}_{mn} J^a{}_b$

  first and second order diagram are separately finite

$\overline{J}_0\,\omega\theta$ function

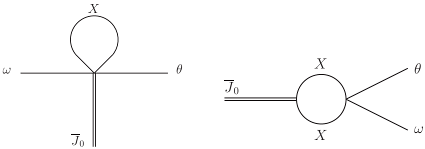


$\overline{J}_0\,\omega\theta$ function



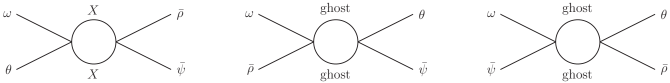
the divergent parts of the diagrams sum to zero

$\overline{J}_0\,\omega\theta$ function

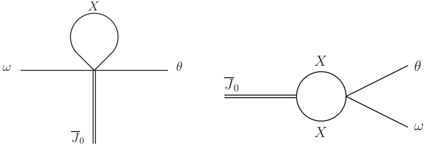


the divergent parts of the diagrams sum to zero

$\omega\theta\,\bar{\rho}\bar{\psi}$ function

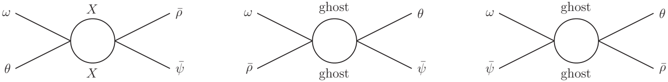


$\overline{J}_0\,\omega\theta$ function



the divergent parts of the diagrams sum to zero

$\omega\theta\,\bar{\rho}\bar{\psi}$ function



the divergent parts of the matter propagator diagram and ghost propagator diagrams sum to zero

Conclusions

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