Gregory-Laflamme instability and conformal symmetry

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✧ How does **black hole physics** change as the **dimension D increases**?

~ **Vacuum** general relativity $(R_{\mu\nu} = 0)$

~ **Spacetime dimension** is the **only parameter** of vacuum GR!

✧ In what sense **black holes are like fluids**?

~ Early analogies (e.g. the **membrane paradigm**) **Damour** '70, **Price & Thorne** '80 ...

~ An example: The similarities between **black hole physics** and **lumps of fluids**

* Both tend to **extremize surface area**
* Their surface is **elastic**
* **Evaporation** when microphysics included (Hawking radiation)
* The evaporation rate is larger for small drops

✧ Fluid dynamics:

~ **Effective theory** for fluctuations away from thermodynamic equilibrium

$$\omega \to 0, \quad \lambda \to \infty$$

~ **Long wavelength expansion:** $\lambda \gg \lambda_{\rm mfp}, \quad \lambda \gg \lambda_{\rm thermal}$

~ The theory has a **scale** that can for example distinguish between **small/large** liquid droplets

✧ Classical General Relativity:

~ The classical vacuum theory is **scale-invariant**

~ Length scale set by black hole size $r_s \sim GM$

~ "Hydrodynamic modes" $\lambda \gg r_s$ **don't fit!**

We need an additional scale for fluid behavior

~ Scale from cosmological constant : $\lambda \gg \ell_{\rm cosmo}$

Large black holes in AdS -> AdS/Fluid correspondence Bhattacharya *et al.* 2007

~ **Widely separated scales** arise naturally for higher D black holes in some regimes

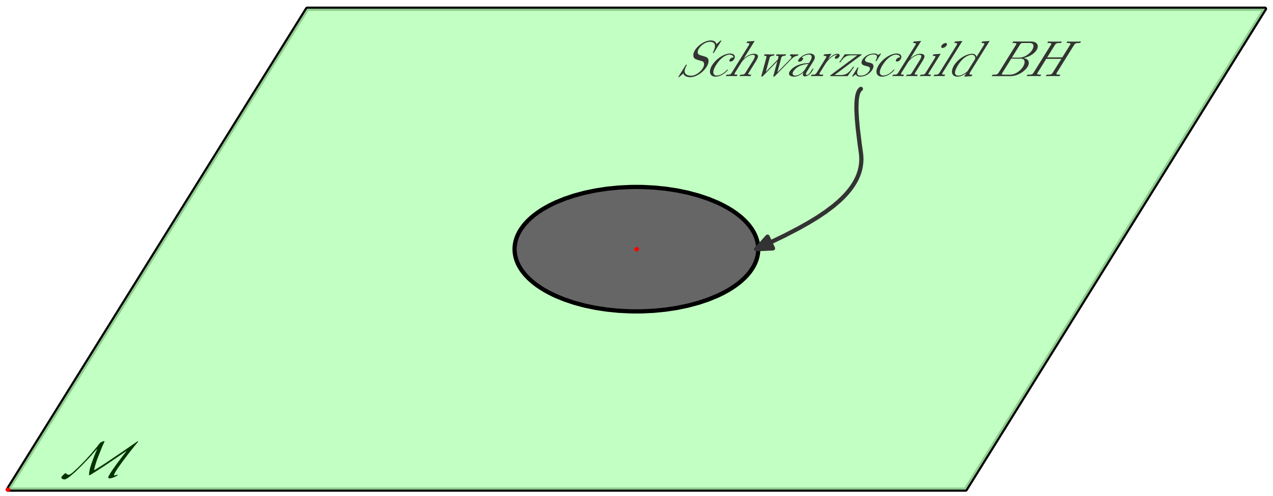
THE BLACK STRING

~ where we observe the emergence of fluid behavior for the first time ~

Extended horizons $(D > 4)$

Start with a Schwarzschild black hole

(Ricci flat)



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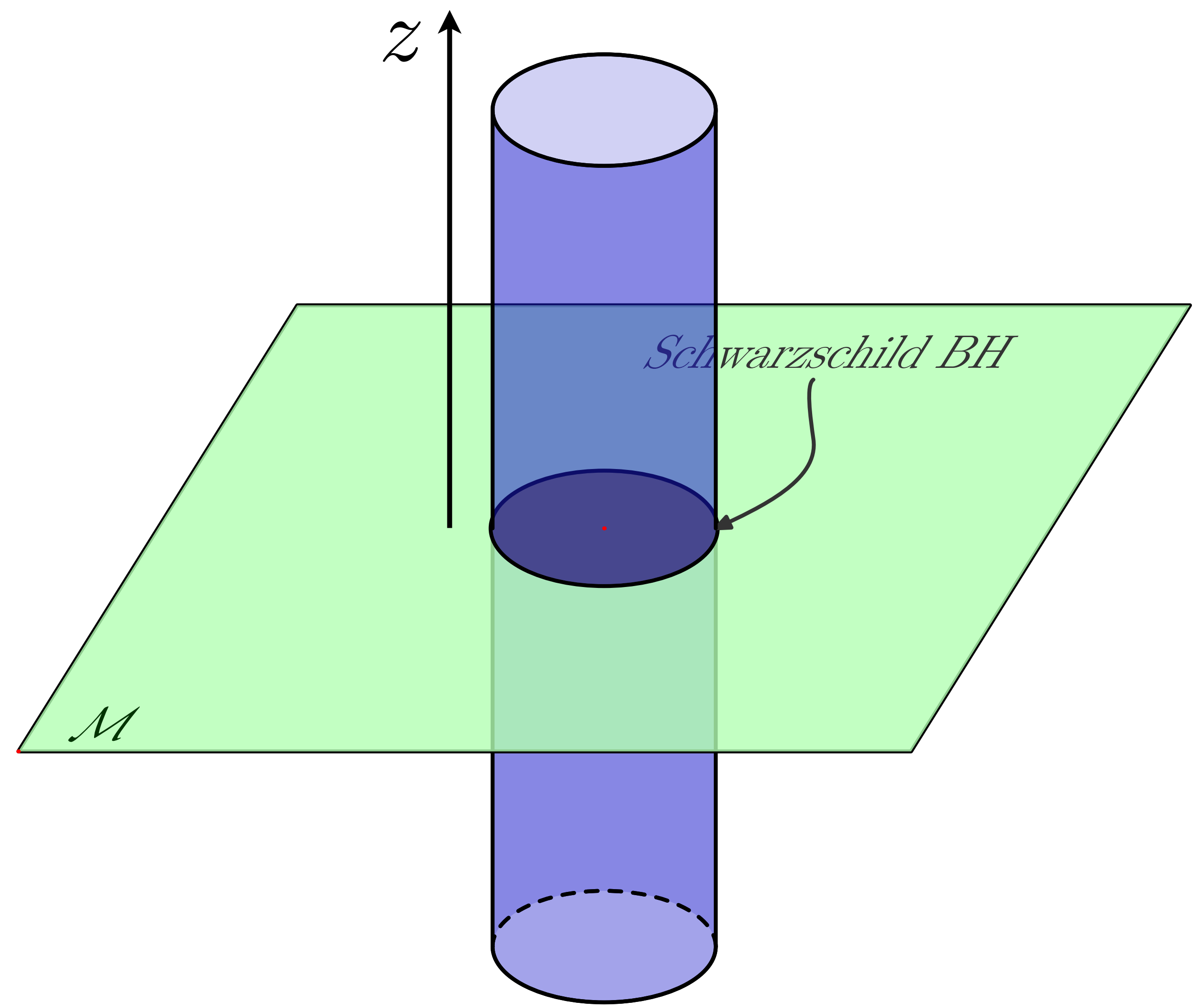


Take the direct product with a flat extra dimension

(still Ricci flat)

$$\text{Schwarzschild} \times \mathbb{R}$$

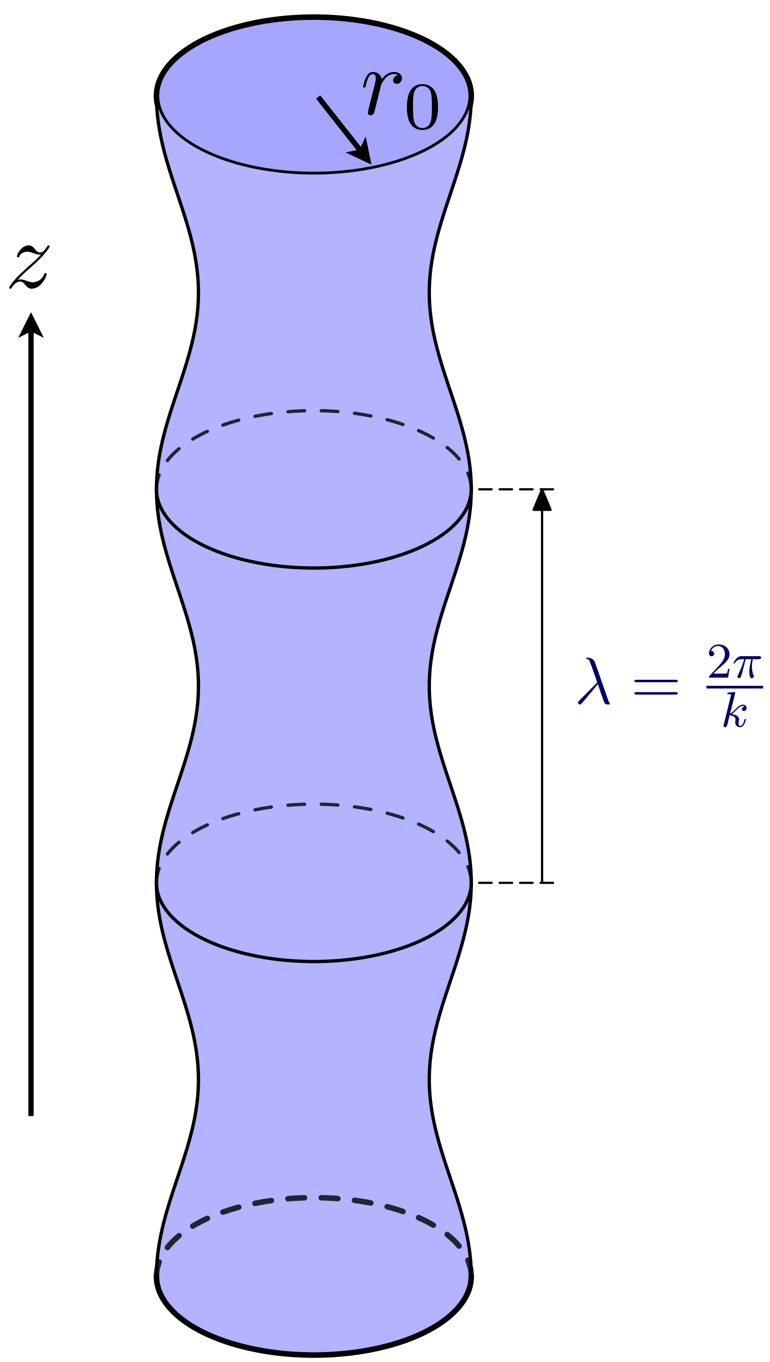
$$ds^2 = ds^2_{\text{Schw}} + dz^2$$

BLACK STRING

[black membranes and black branes with an analogous construction]

Gregory-Laflamme instability

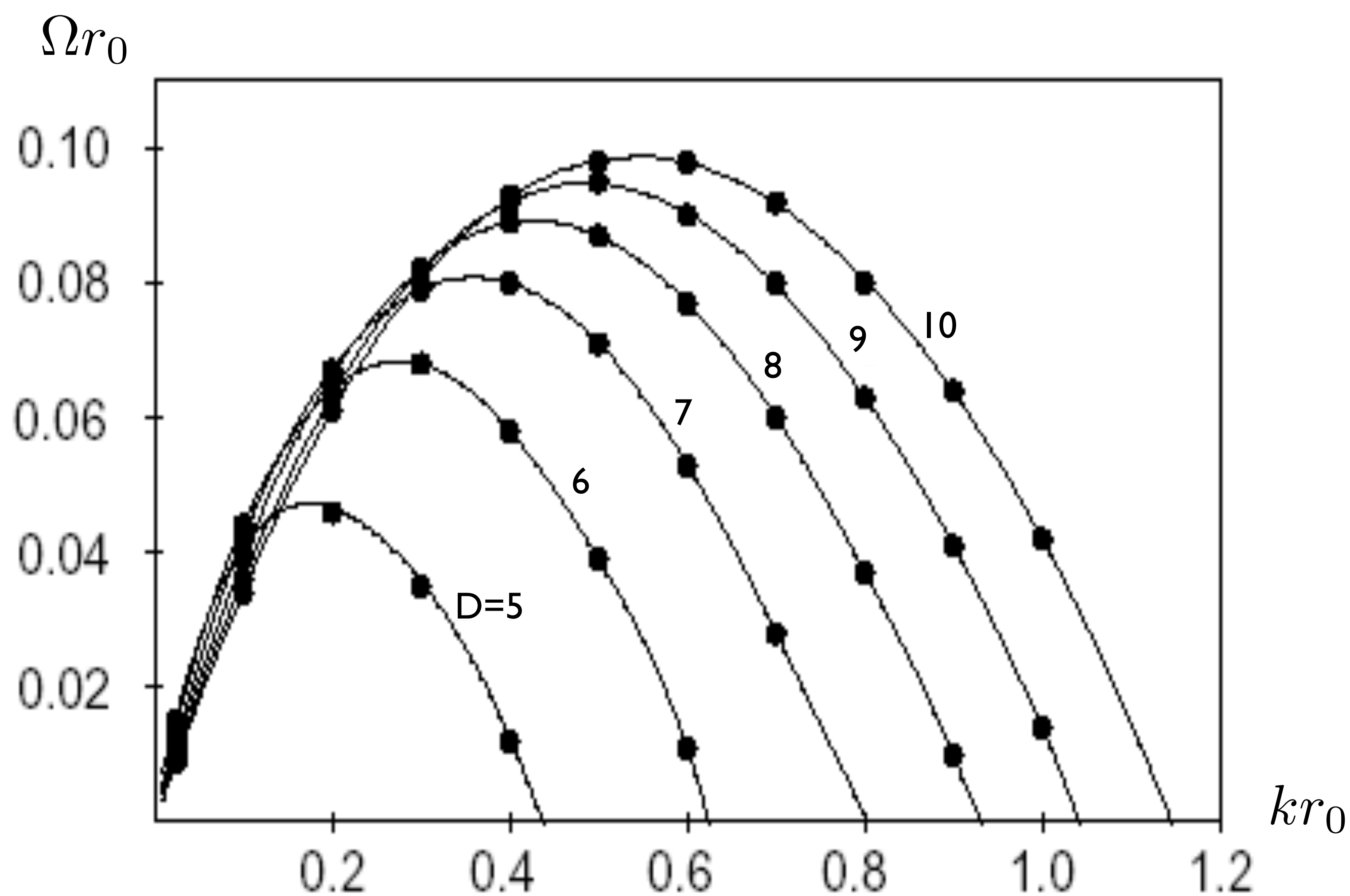
Gregory & Laflamme '94



Linearized perturbations: $g_{\mu\nu} + \epsilon h_{\mu\nu}$

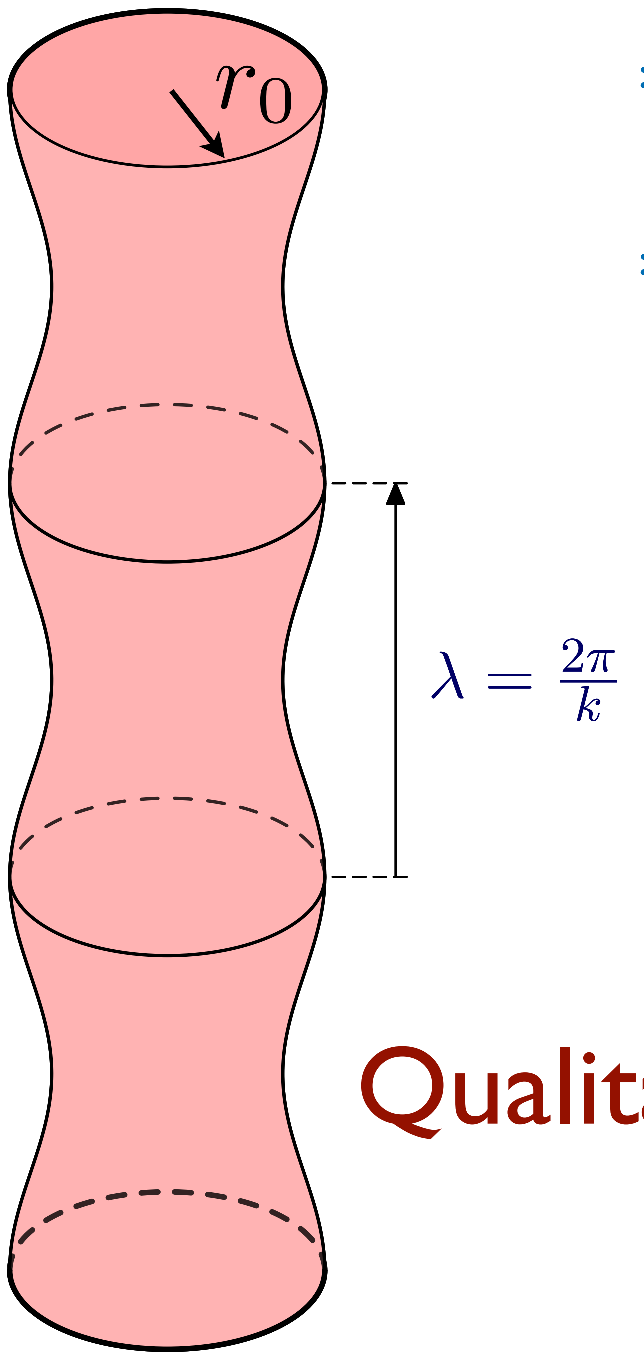
$$\delta r_0 \sim e^{\Omega t + ikz}$$

Instability for $\lambda \gtrsim r_0$



Black holes as fluid droplets

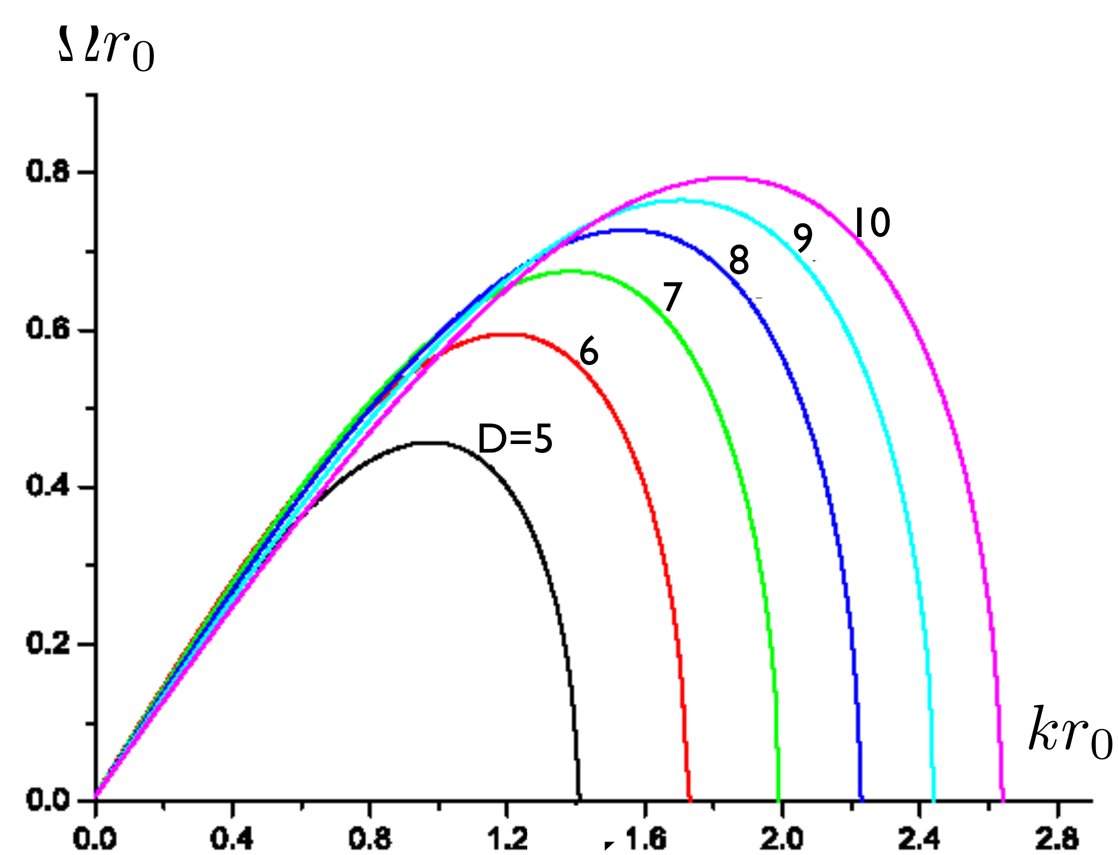
~ Column of fluid held by surface tension ~



* If long enough, **capillary breakup** is energetically advantageous
* Perturbations of wavelength longer than circumference **grow** exponentially

Rayleigh-Plateau instability

Qualitatively, RP=GL!



Cardoso & Dias '06

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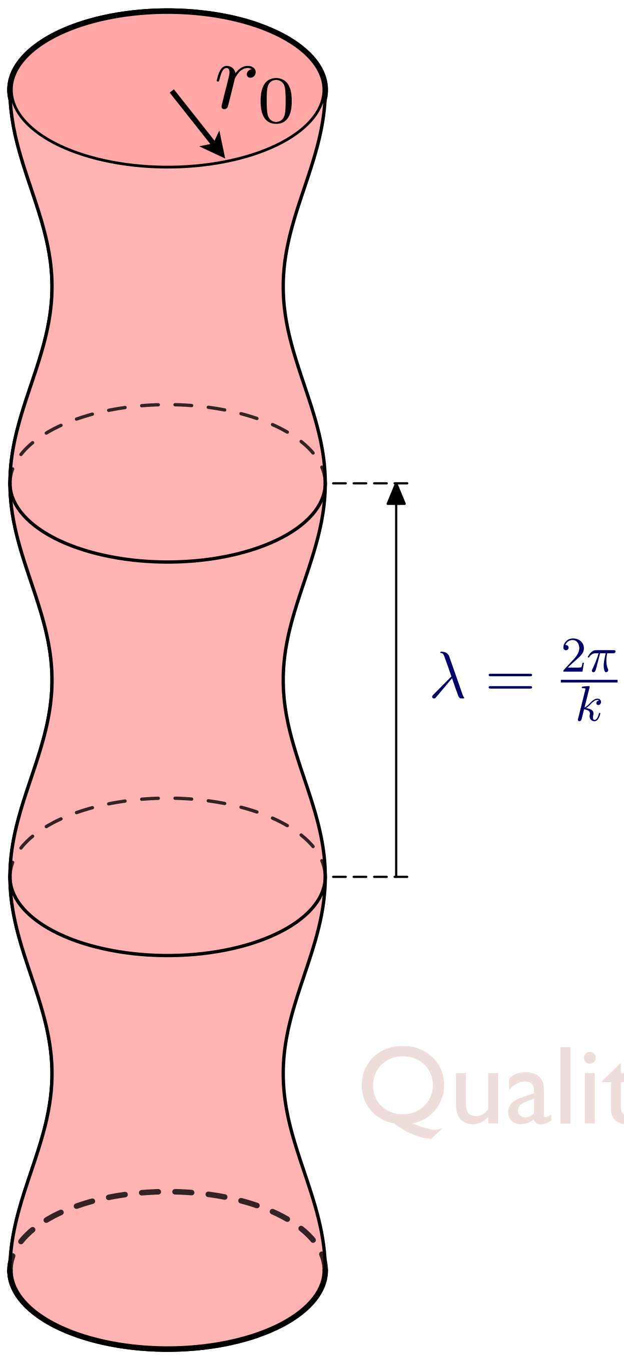

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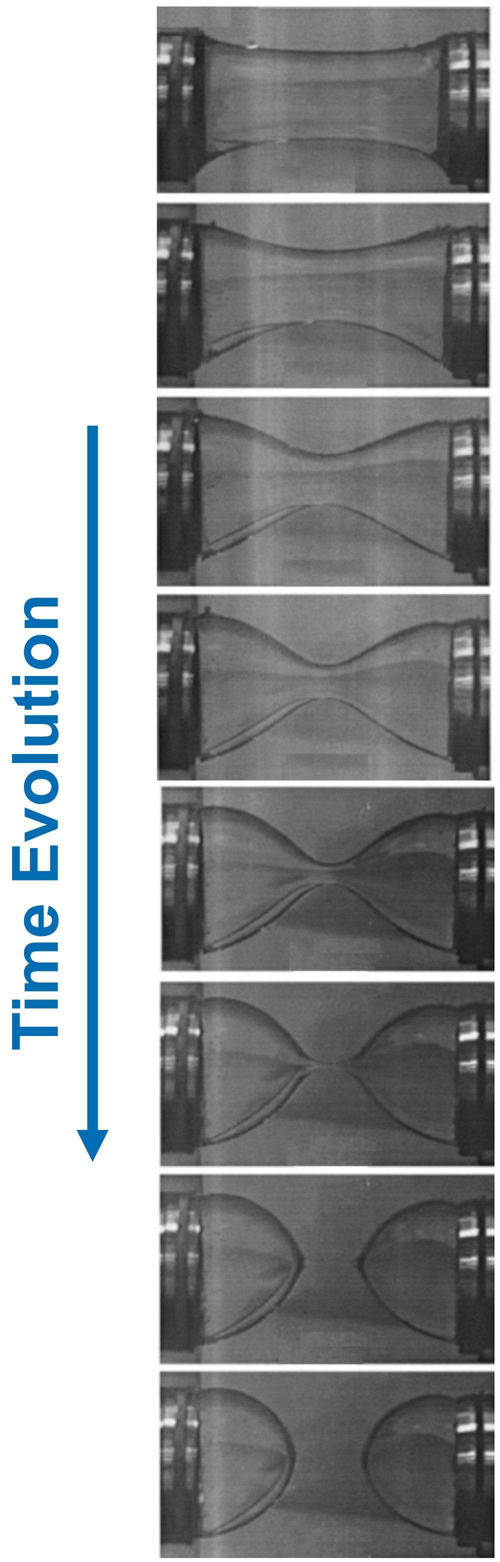
Rayle… instability

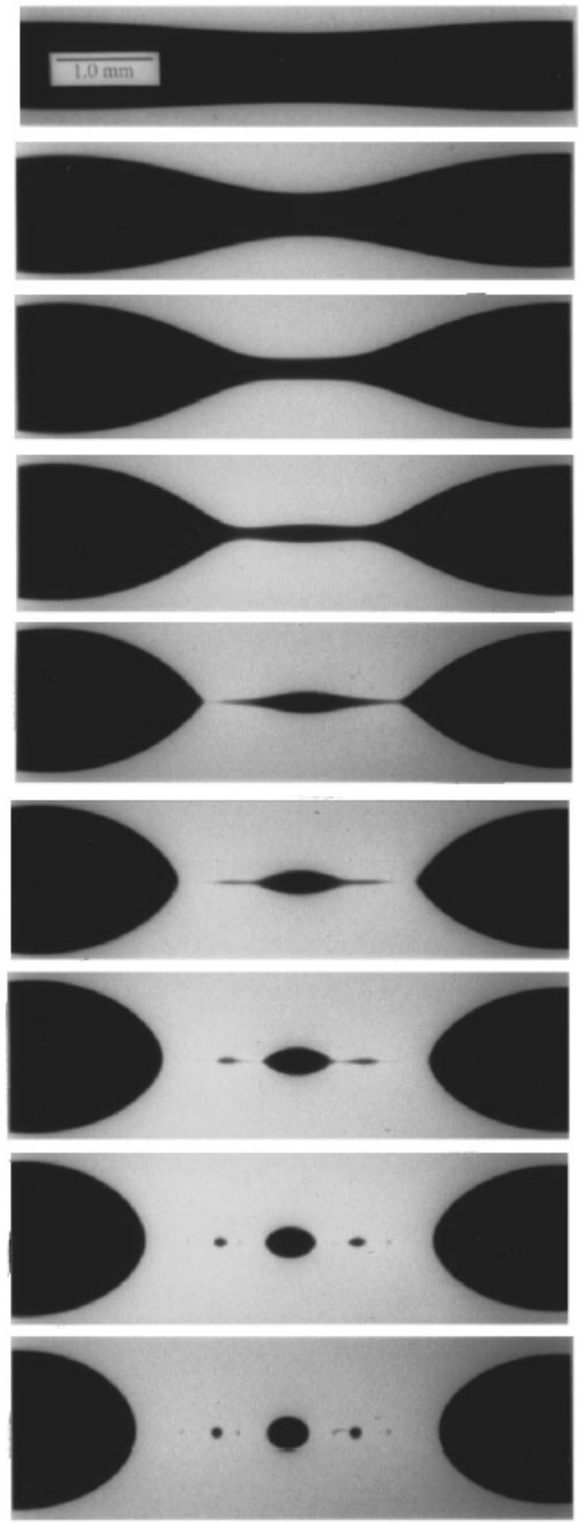
Shi, Brenner & Nagel 1994

Qualitatively, RP:

Cardoso & Dias '06

Rayleigh-Plateau Time Evolution

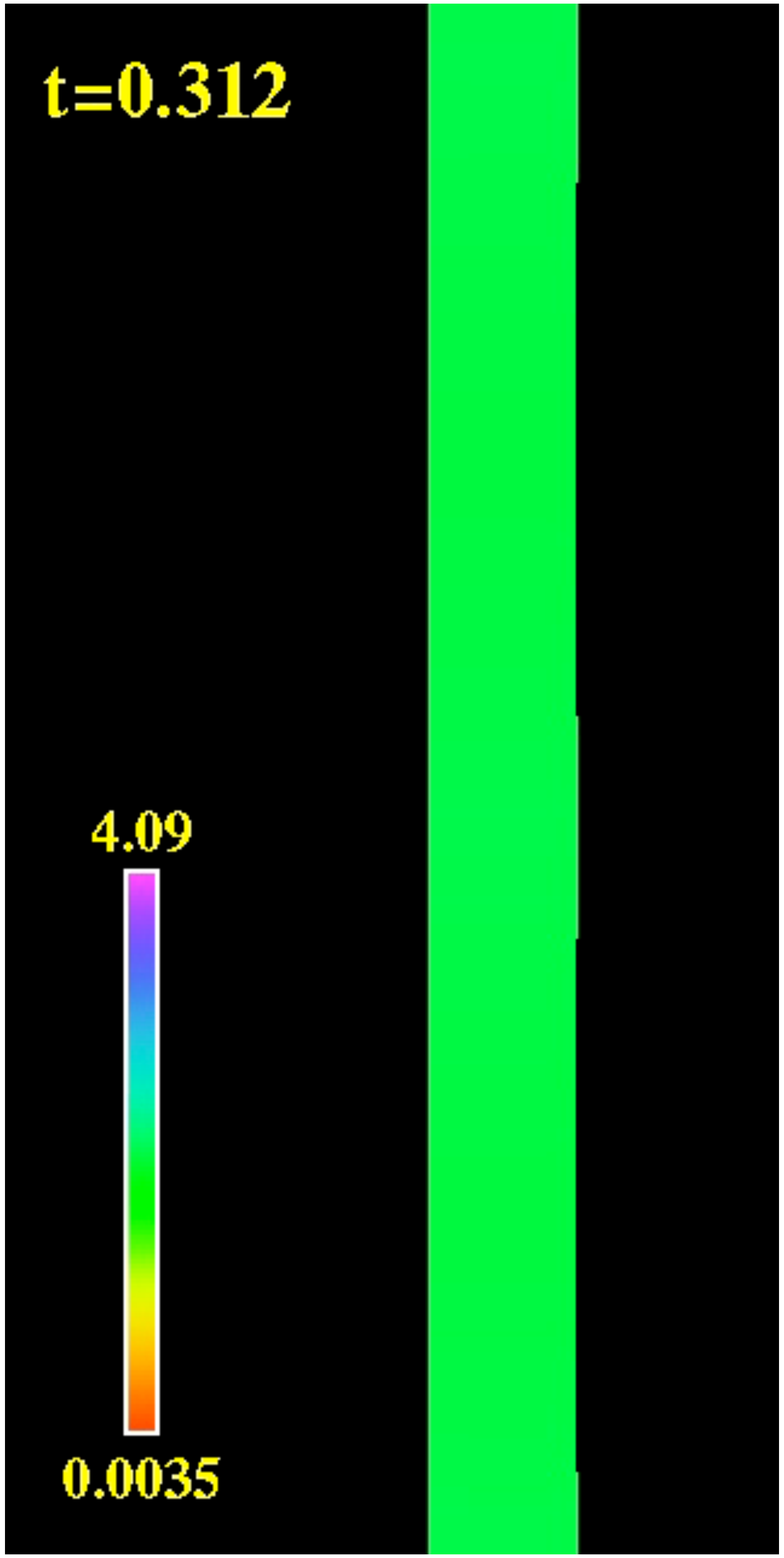




Tjahjadi, Stone & Ottino 1992

Spiegelberg Gaudet & McKinley 1994

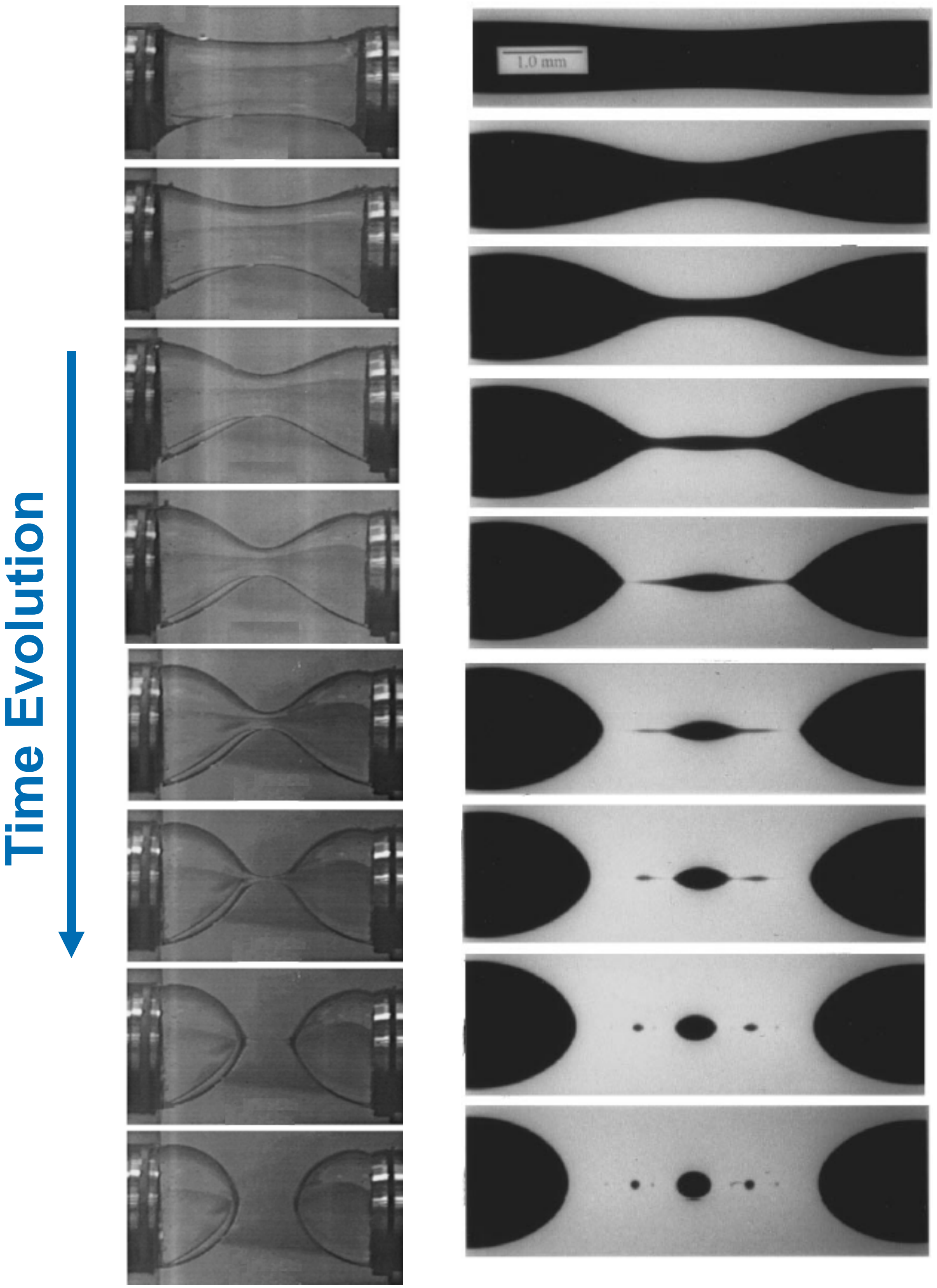
Gregory-Laflamme Time Evolution



Lehner & Pretorius 2010

Rayleigh-Plateau Time Evolution

Gregory-Laflamme Time Evolution



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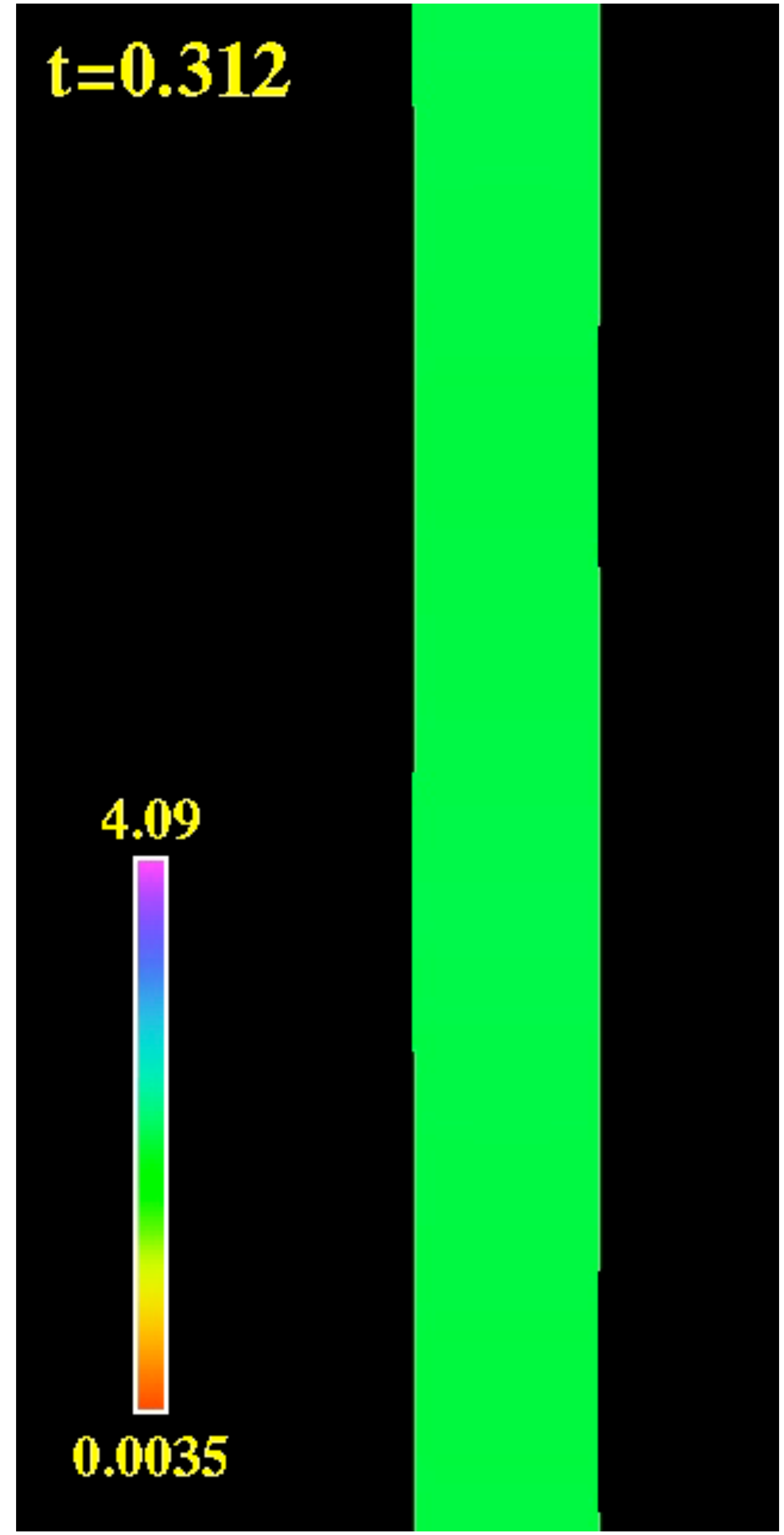


Lehner & Pretorius 2010

* Gravitational duals to confining theories: Aharony, Minwalla & Wiseman '05
BHs dual to **plasma balls**, with surface tension Lahiri & Minwalla '07

Large BHs in AdS: fluid lumps of deconfined plasma

Gregory Laflamme/Rayleigh-Plateau analogy
becomes a precise **duality** MC, Dias, Emparan & Klemm '08
Miyamoto & Maeda '08

* **No intrinsic scale** of vacuum gravity:
no way to tell **small/large** black holes

However, the black string is "**thin**"

* **D>4** black holes: emergence of **widely separated scales**
BHs **effectively** described by a **fluid** that lives on
a **dynamical worldvolume**

On black hole phases and instabilities

~ where we learn that black holes rotating extremely fast resemble black strings and black branes ~

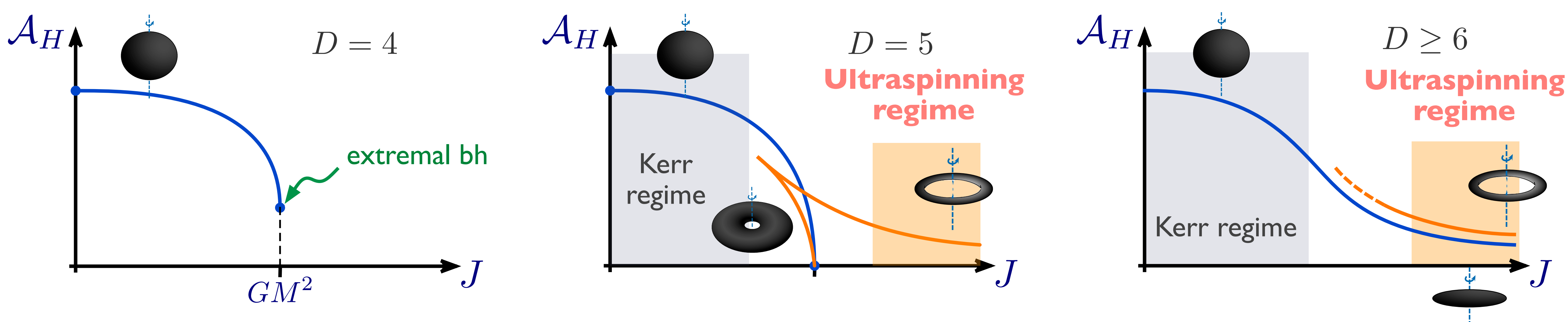
The Origin of Scales

★ **Newtonian** vs. **Rotational** potential competition:

$$V_{\text{Newton}} \sim \frac{1}{r^{D-3}} \qquad V_{\text{centrif}} \sim \frac{1}{r^2}$$

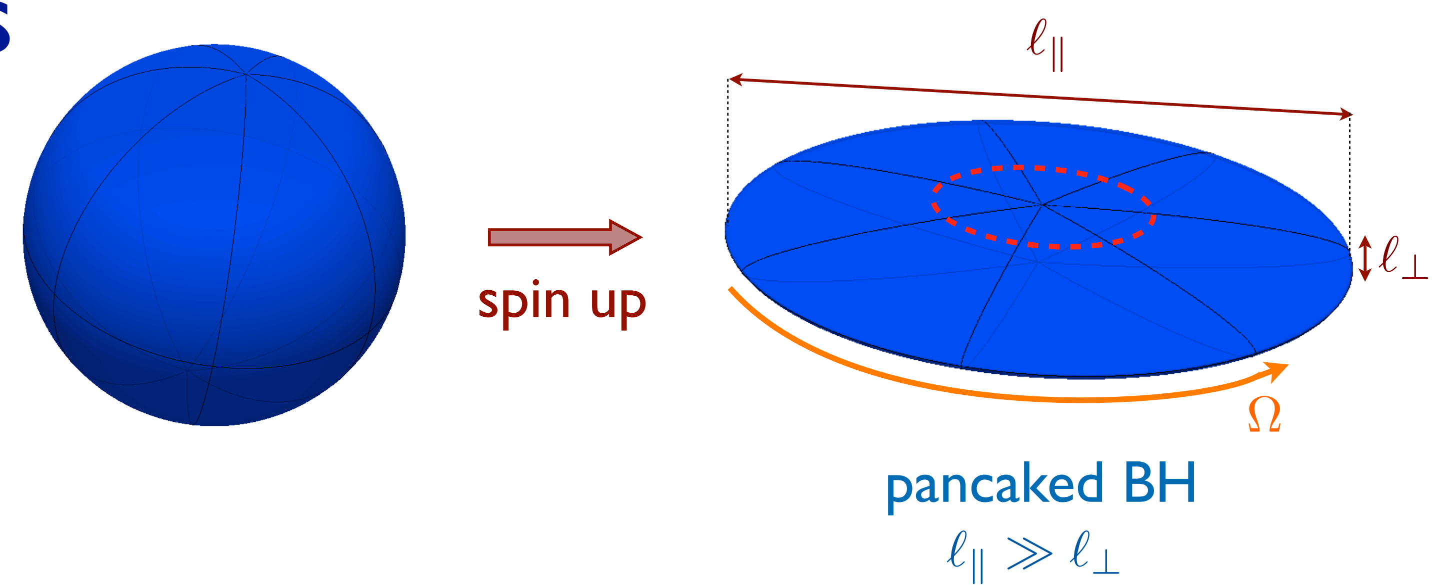
When $D \geq 6$ the **centrifugal potential dominates**

➡ rotating objects tend to **pancake**



★ **Ultraspinning black holes**

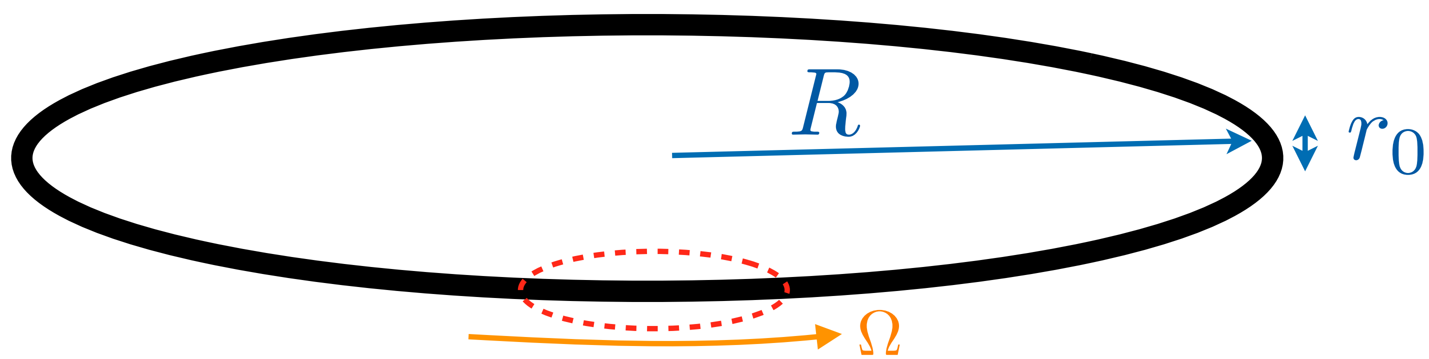
locally resemble a **boosted black brane**



★ Black rings: **large** and **thin** when ultraspinning

locally resemble a boosted black string

$$R \gg r_0$$

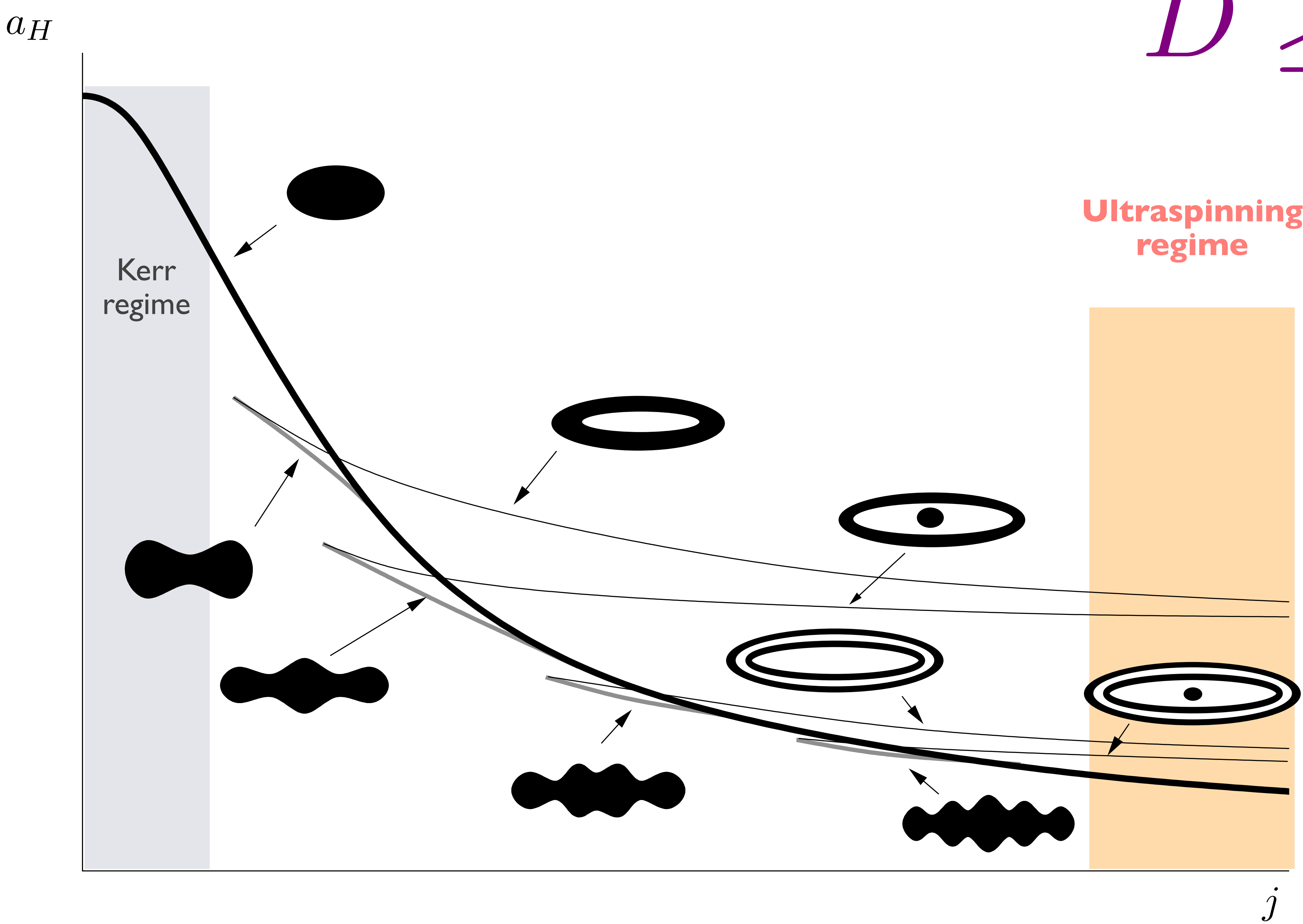


* Length scales $\quad \ell_M \sim (GM)^{\frac{1}{D-3}} \qquad \ell_J \sim \frac{J}{M}$

* Ultraspinning regime when $\quad \ell_J \gg \ell_M$

Towards full phase diagram

$D \geq 6$

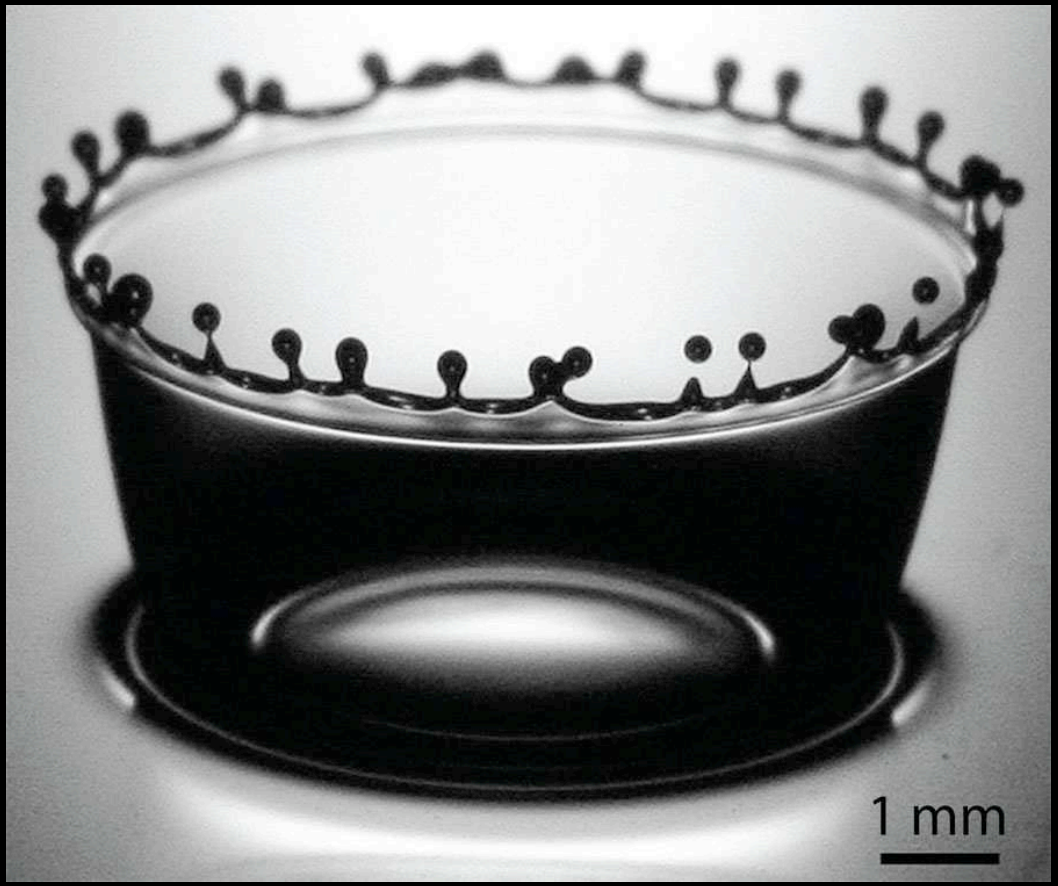


Emparan Harmark Niarchos Obers & Rodriguez 2007

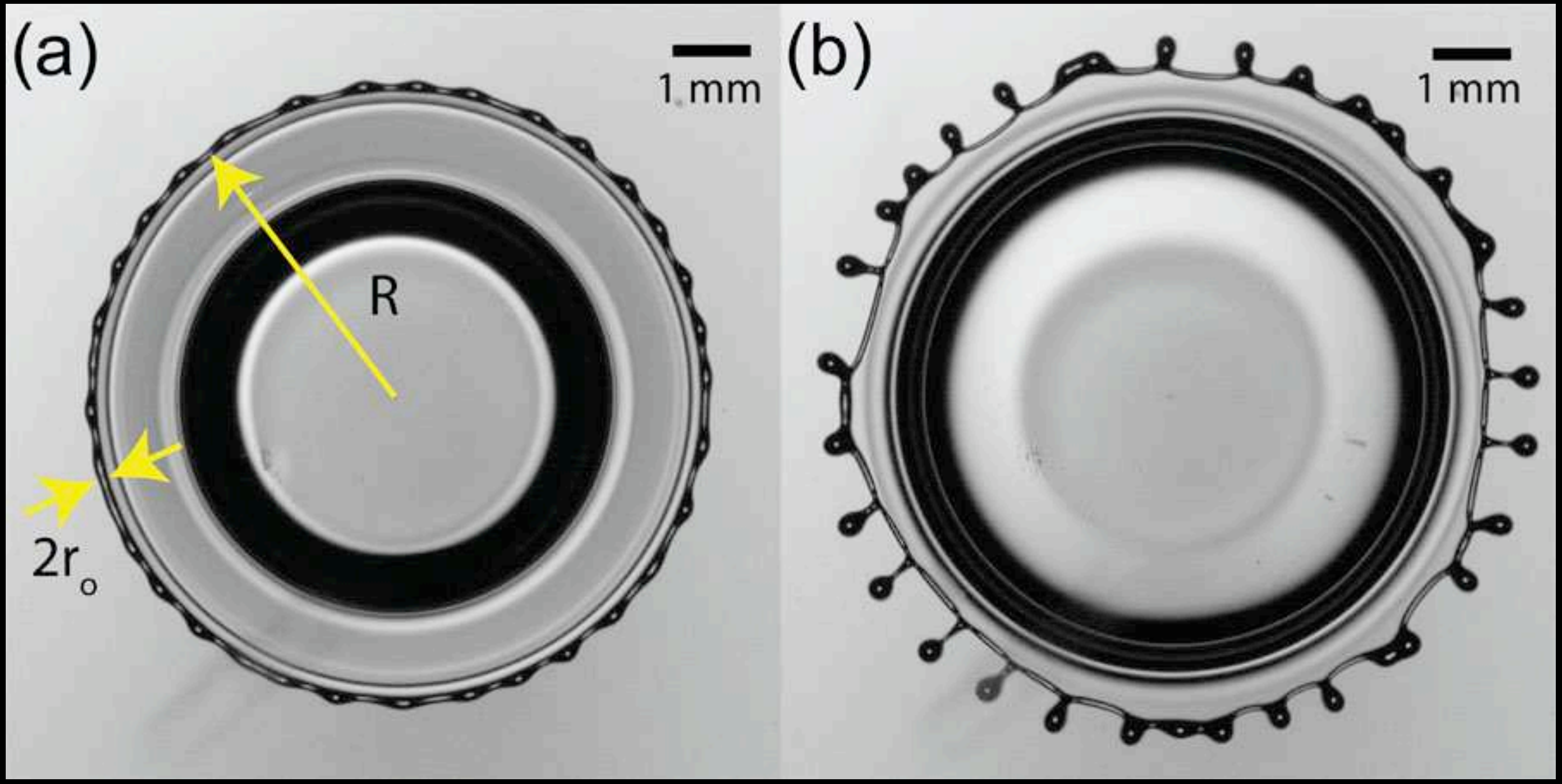
Emparan Figueras 2010

Kleihaus, Kunz & Radu 2012

Crown splash ~ thin black ring instability ?



Deegan, Brunet & Eggers '08



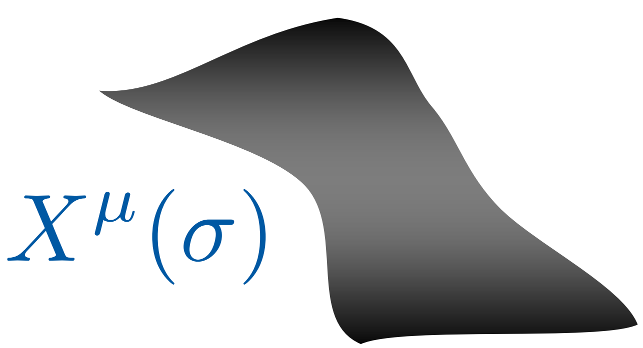
BLACKFOLDS

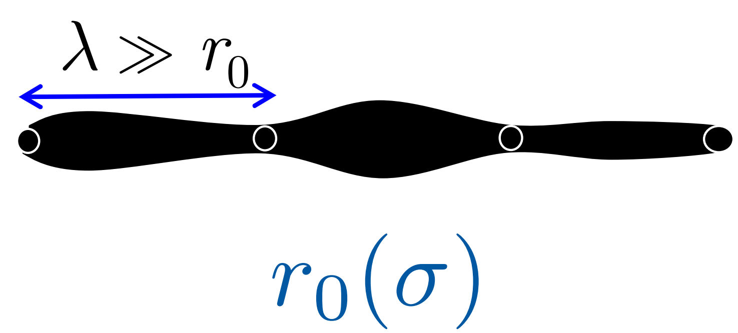
~ where the separation of scales is used to describe effectively black holes as fluids living in a dynamical worldvolume ~

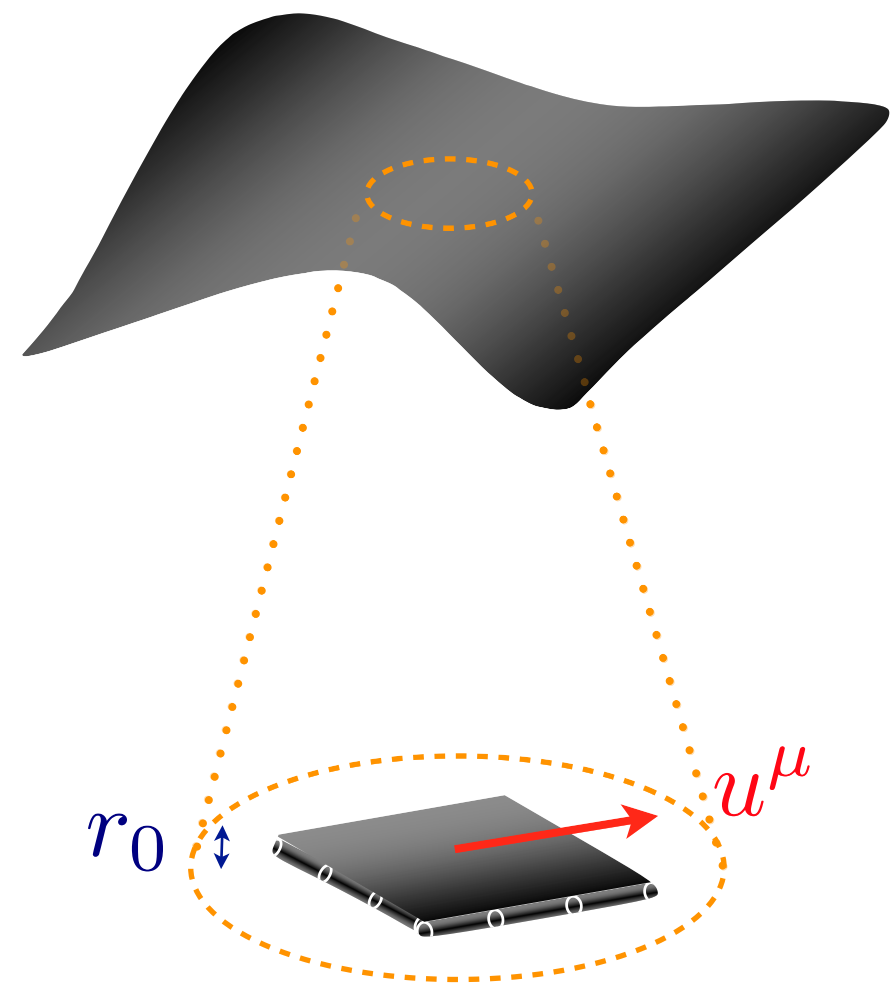
BLACKFOLDS

Emparan, Harmark, Niarchos & Obers '09

★ Long wavelength dynamics captured by an effective worldvolume theory







Collective variables:

(can vary on scales $R \gg r_0$)

$X^\mu(\sigma^a)$	embedding functions
$r_0(\sigma^a)$	thickness of the horizon
$u^a(\sigma^a)$	local boost field

BLACKFOLDS

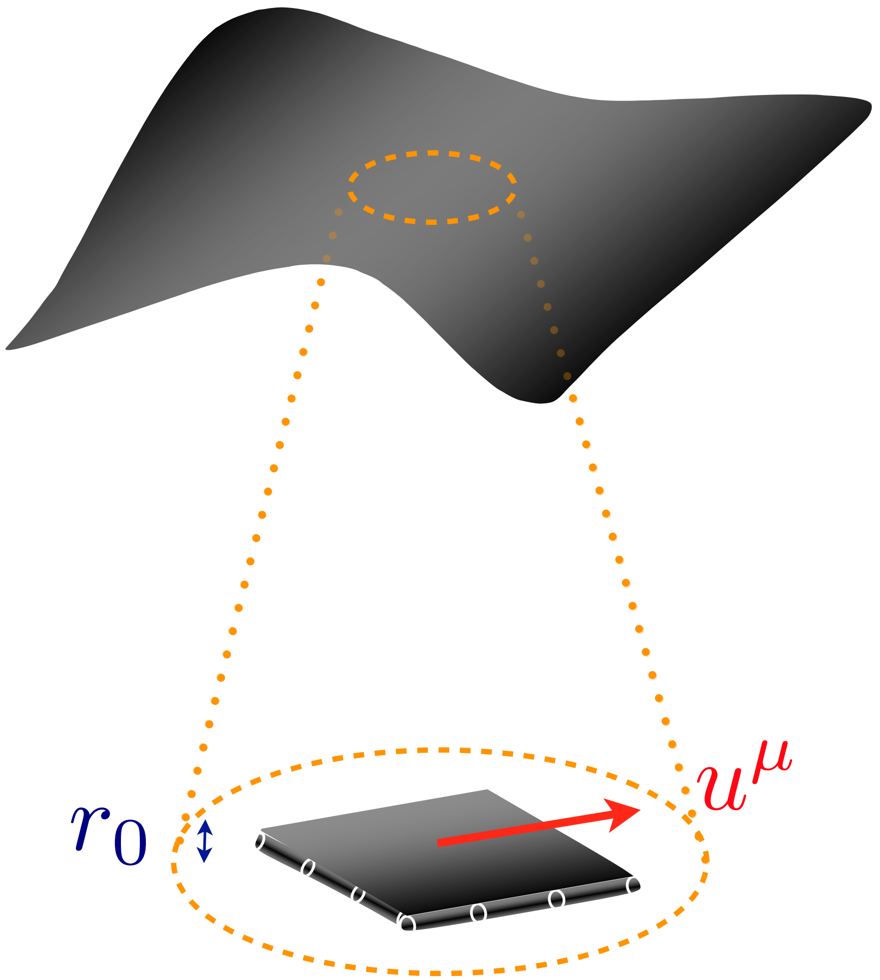
Emparan, Harmark, Niarchos & Obers '09

Underlying conservative dynamics:

$$\nabla_\mu T^{\mu\nu} = 0$$

Quasi-local stress energy tensor of Brown and York in weak field region $r_0 \ll r \ll R$

$$16\pi G\, T_{\mu\nu} = K_{\mu\nu} - h_{\mu\nu} K - \left(\hat{K}_{\mu\nu} - h_{\mu\nu} \hat{K} \right)$$



⇨ Perfect fluid stress tensor

$$\varepsilon = \frac{\Omega_{(n+1)}}{16\pi G} (n+1)\, r_0^n$$

$$P = -\frac{\Omega_{(n+1)}}{16\pi G} r_0^n$$

$$T_{\mu\nu} = (\varepsilon + P) u_\mu u_\nu + P h_{\mu\nu}$$

$n = D - 3 - p$ (at leading order...)

(for the black string: $n = D - 4$)

BLACKFOLDS

Emparan, Harmark, Niarchos & Obers '09

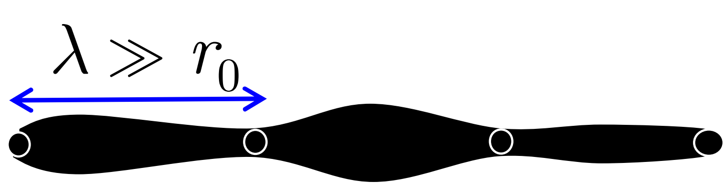
Underlying conservative dynamics:

$$\nabla_\mu T^{\mu\nu} = 0$$

Intrinsic dynamics (fluid excitations)

$$D_a T^{ab} = 0$$ [fluid]

fluid equations (with conserved particle/string numbers)

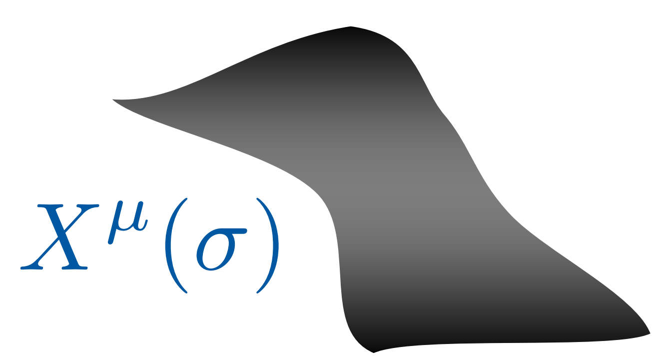


$$r_0(\sigma)$$

Extrinsic dynamics (elastic deformations)

$$T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0$$ [Carter's equation]

balance of forces on the black brane worldvolume



BLACKFOLDS

Emparan, Harmark, Niarchos & Obers '09

* Blackfold eqns can be derived from Einstein eqns
* The horizon of the black brane remains regular

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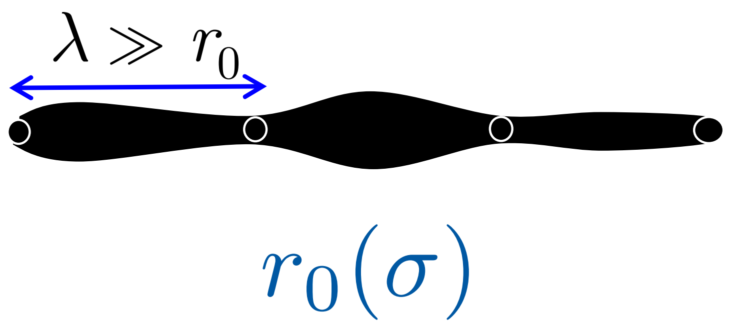

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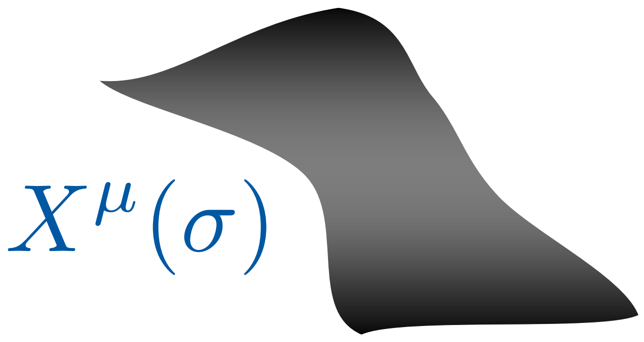
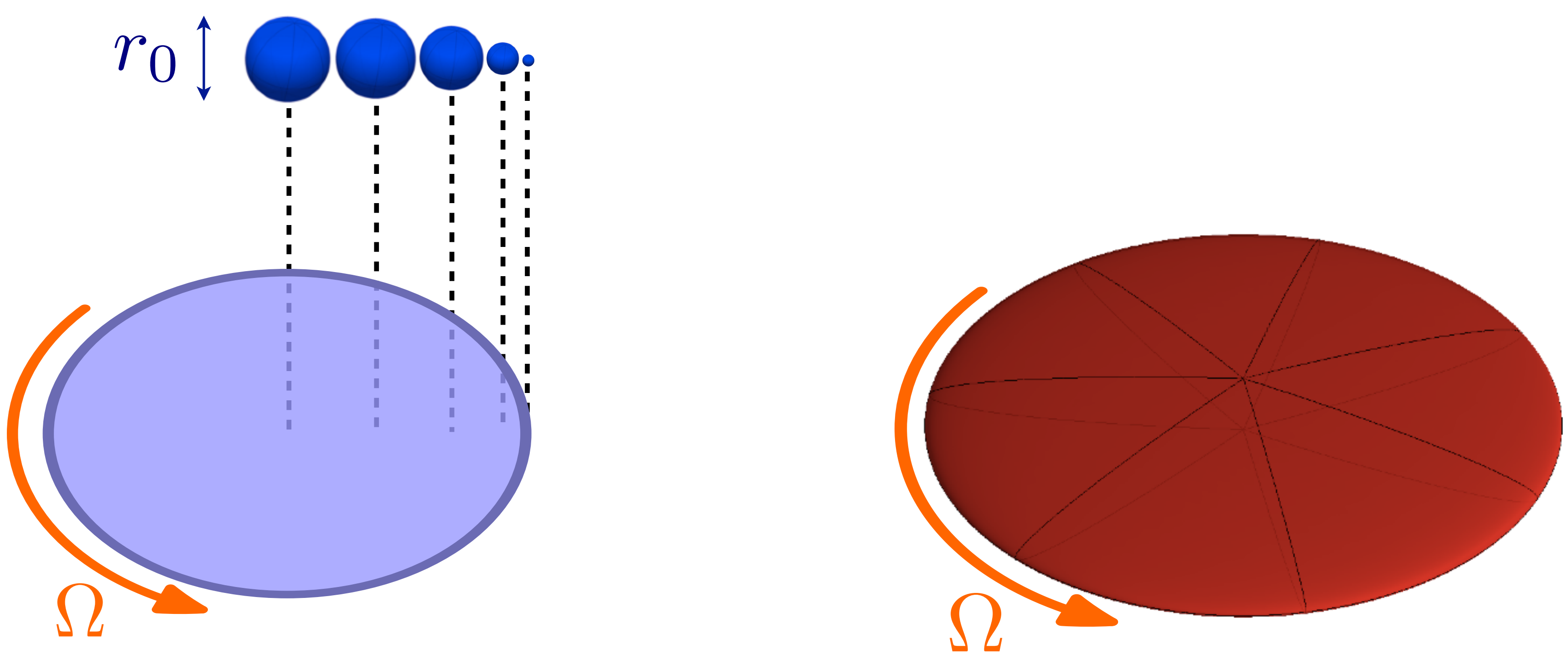
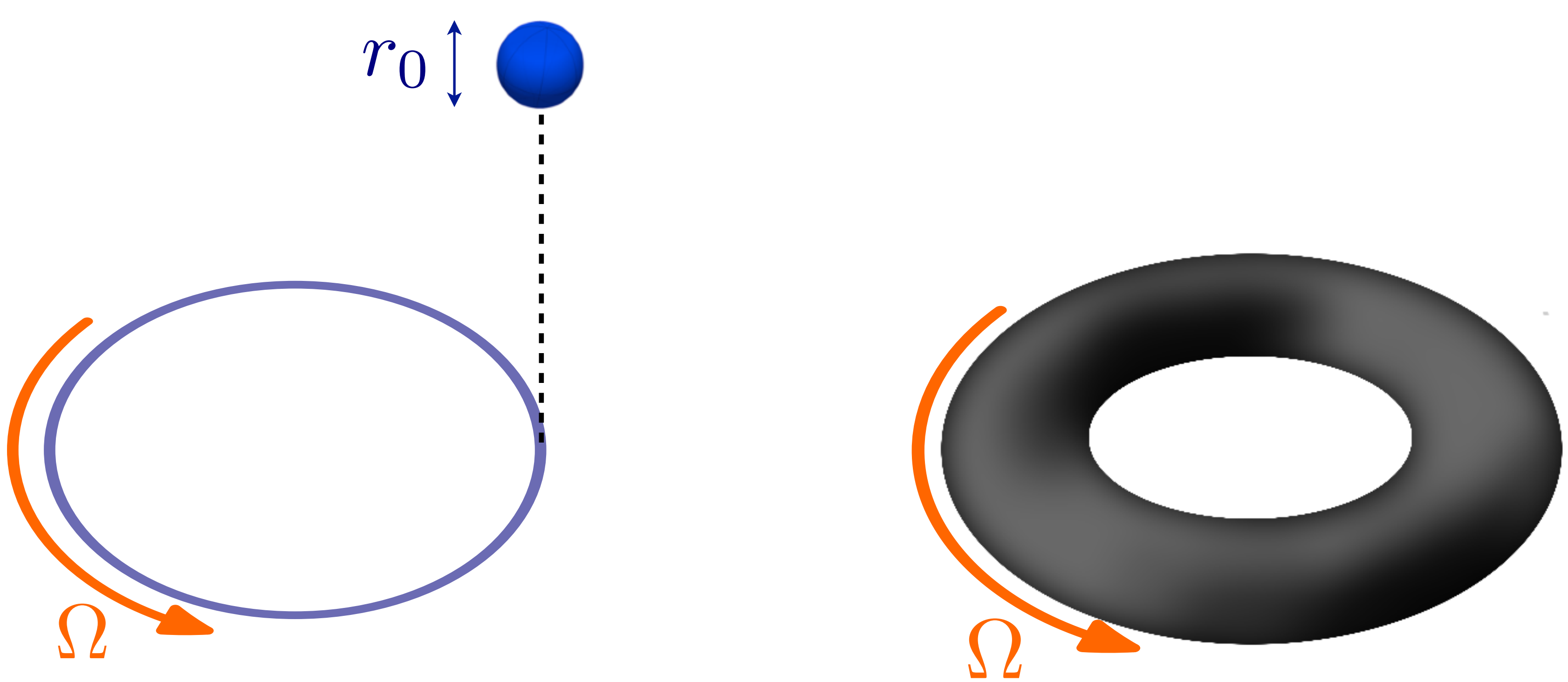


* Straightforward to study properties of ultraspinning MP & BR



Pancaked Black Hole
S^{n+3} horizon



Black Ring
$S^1 \times S^{n+1}$ horizon

Sound waves on a black string/brane

Intrinsic fluctuations δr_0 ➡ pressure/density fluctuations

*** sound waves ***

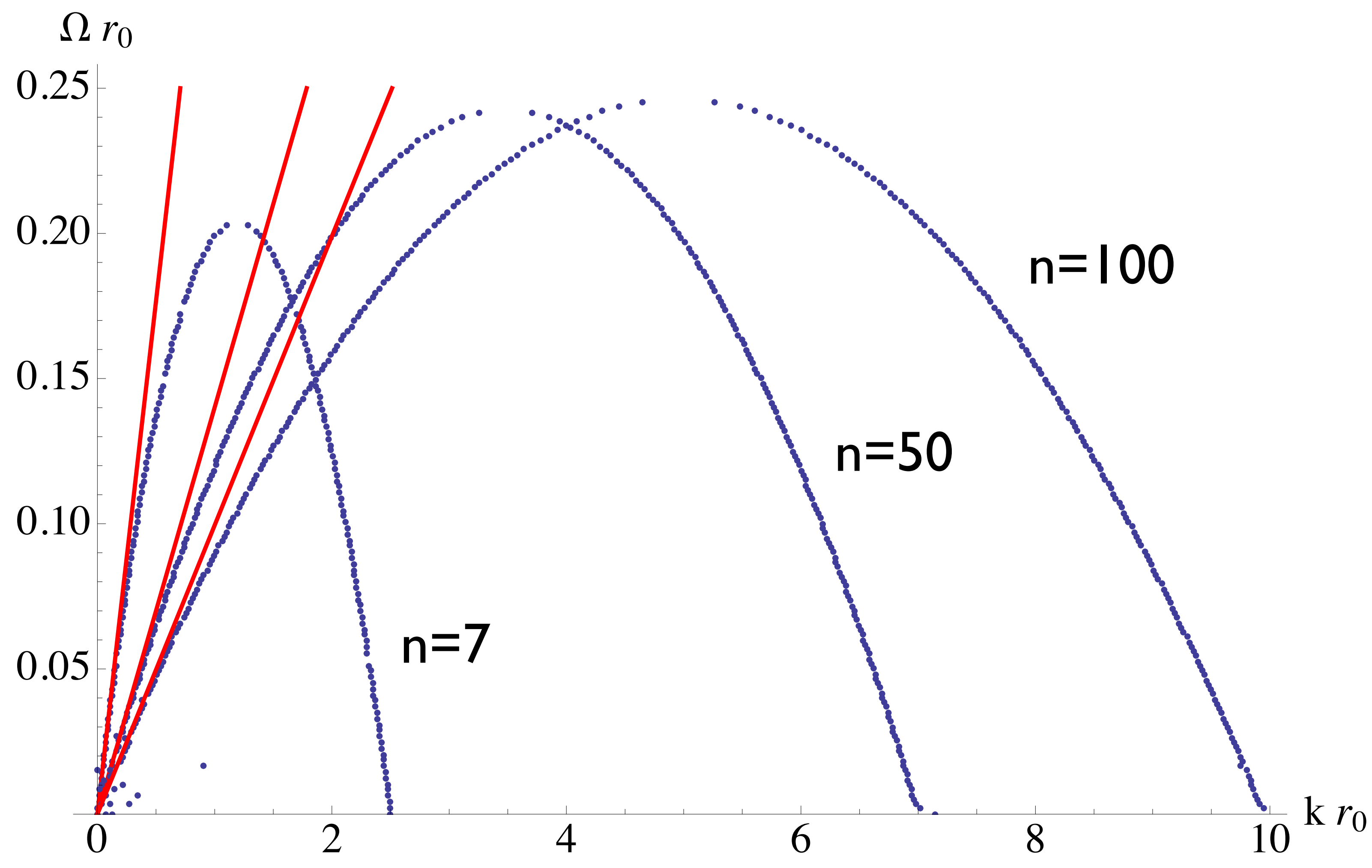
$$v_s^2 = \frac{dP}{d\epsilon} = -\frac{1}{n+1} < 0$$

➡ **unstable modes: the inhomogeneities tend to grow**

$$\delta r_0 \sim e^{\Omega t + ikz}$$

$$\Omega = \frac{k}{\sqrt{n+1}} + O\left(k^2\right)$$

captures the slope of the curve near the origin



Viscosity corrections to the blackfold stress tensor

Camps, Emparan & Haddad '10

$$T_{\mu\nu} = p\,(\eta_{\mu\nu} - (n+1)u_\mu u_\nu) - 2\eta\,\sigma_{\mu\nu} - \zeta\theta\,\Pi_{\mu\nu}$$

Shear viscosity

$$\eta = \frac{s}{4\pi} = \frac{\Omega_{n+1} r_0^{n+1}}{16\pi G_D}$$

Universal, saturates KSS bound

Bulk viscosity

$$\zeta = 2\eta\left(\frac{1}{p} + \frac{1}{n+1}\right) = 2\eta\left(\frac{1}{p} - c_s^2\right)$$

Saturates Buchel bound

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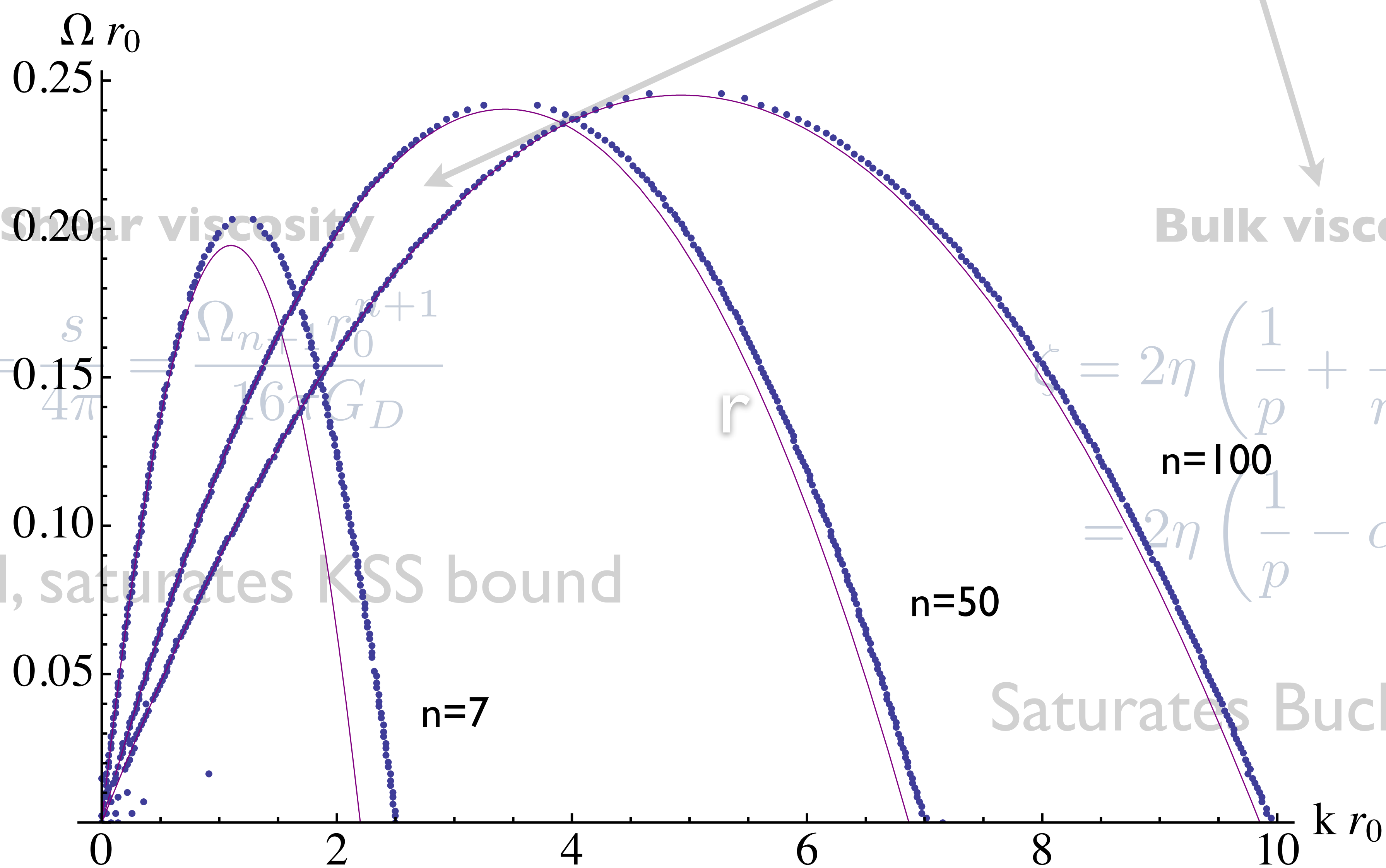


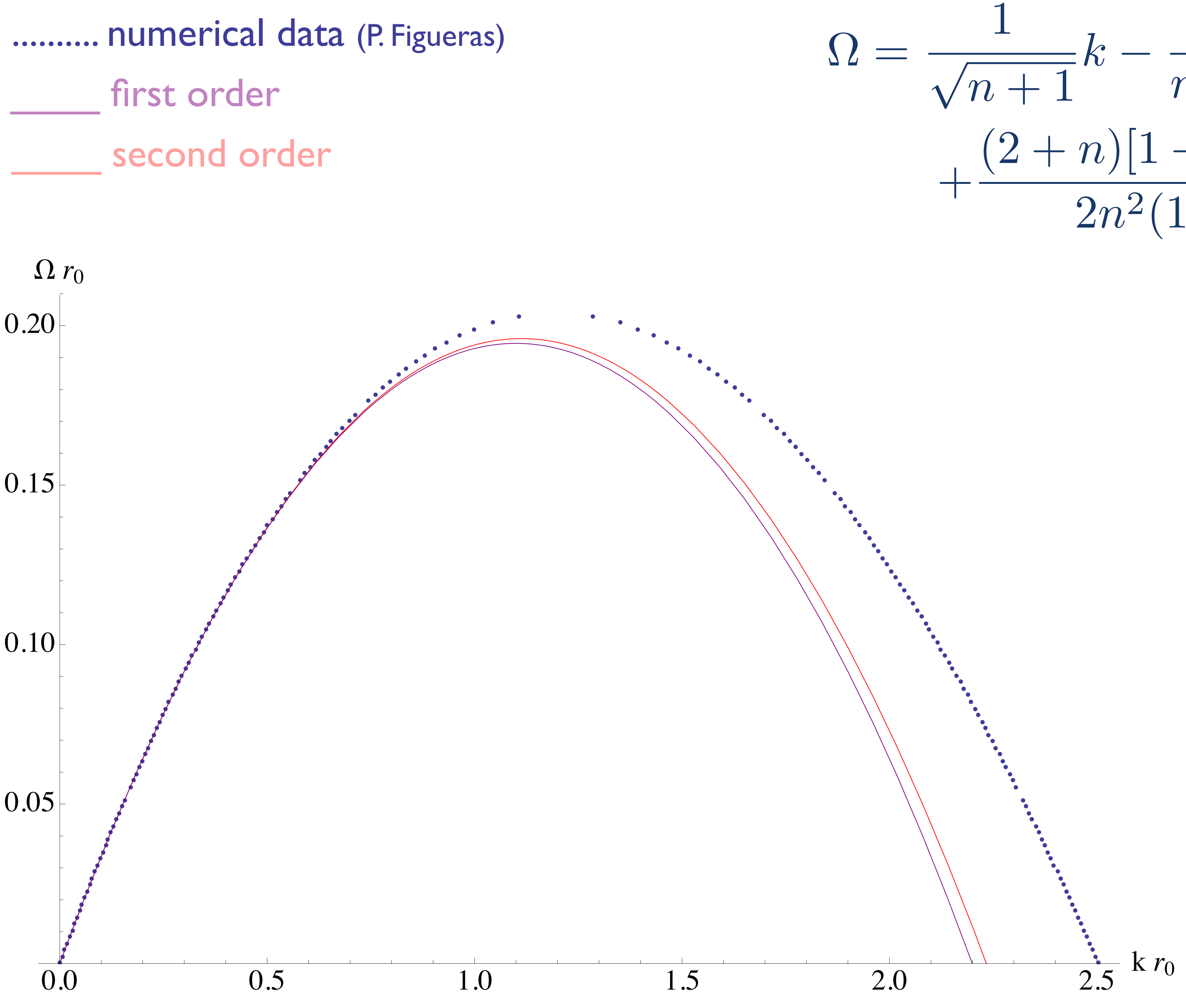
Blackfold stress tensor to second order in the derivative expansion

MC, Camps, Goutéraux & Skenderis '12

$$
\begin{aligned}
T_{\mu\nu} = \; & p\,(\eta_{\mu\nu} - (n+1) u_\mu u_\nu) - 2\eta\,\sigma_{\mu\nu} - \zeta\theta\,\Pi_{\mu\nu} \\
& + 2\eta\tau_\omega \left[\Pi_\mu{}^\alpha \Pi_\nu{}^\beta u^\lambda \partial_\lambda \sigma_{\alpha\beta} - \frac{\theta}{n+1}\sigma_{\mu\nu} + \omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\mu\lambda} \right] \\
& + \zeta\tau_\omega \left[\Pi_{\mu\nu} u^\lambda \partial_\lambda \theta - \frac{1}{n+1}\theta^2\,\Pi_{\mu\nu} \right] \\
& - 2\eta r_0 \left[\Pi_\mu{}^\alpha \Pi_\nu{}^\beta u^\lambda \partial_\lambda \sigma_{\alpha\beta} + \left(\frac{2}{p} + \frac{1}{n+1} \right) \theta\sigma_{\mu\nu} + \sigma_\mu{}^\lambda \sigma_{\lambda\nu} + \frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{n+1} \Pi_{\mu\nu} \right] \\
& - \zeta r_0 \left[\Pi_{\mu\nu} u^\lambda \partial_\lambda \theta + \left(\frac{1}{p} + \frac{1}{n+1} + \frac{p}{(n+1)^2} \right) \theta^2\,\Pi_{\mu\nu} \right]
\end{aligned}
$$

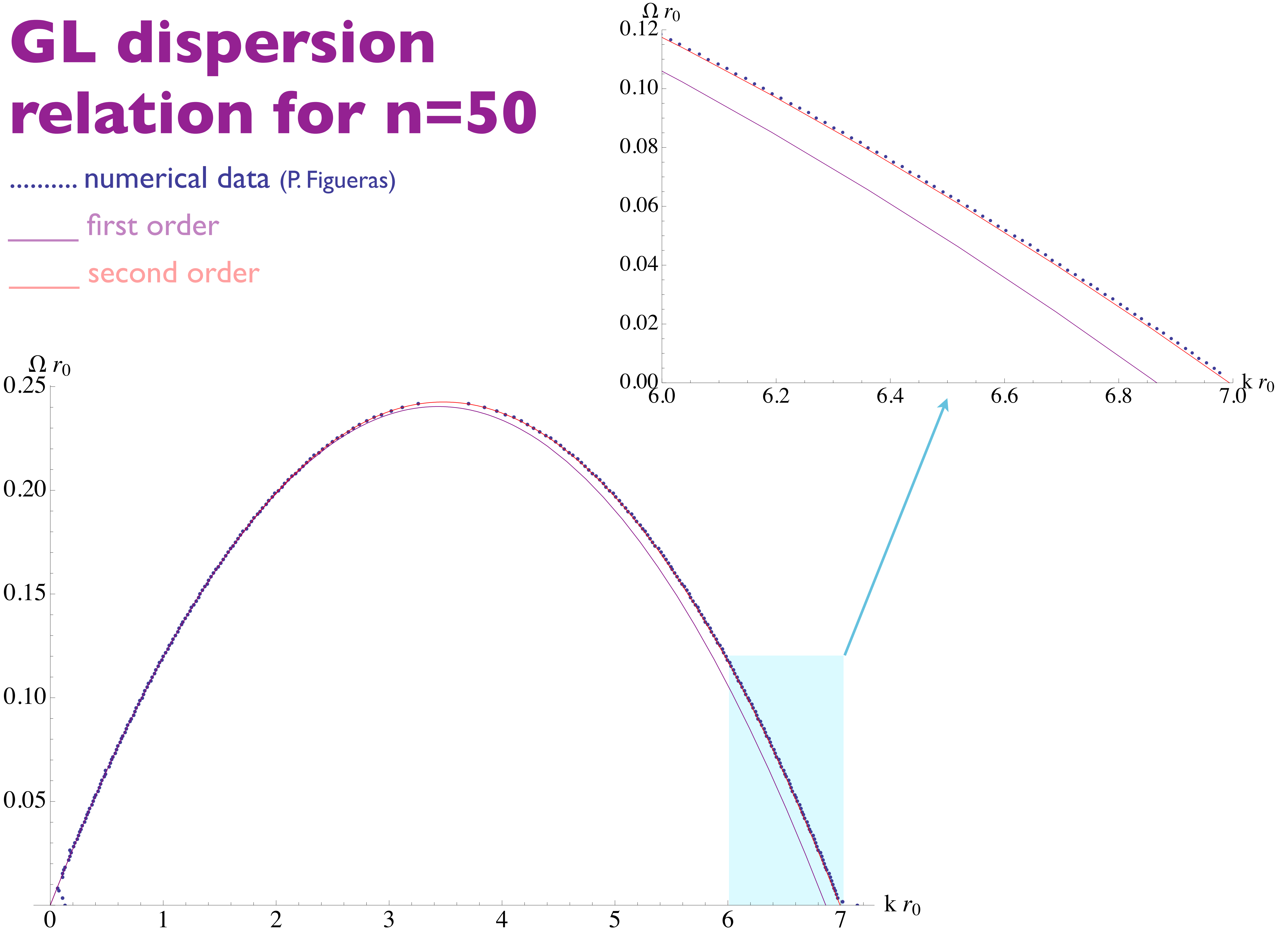
Shear viscosity	Bulk viscosity	Relaxation time
$\eta = \dfrac{s}{4\pi} = \dfrac{\Omega_{n+1} r_0^{n+1}}{16\pi G_D}$	$\zeta = 2\eta \left(\dfrac{1}{p} - c_s^2 \right)$	τ_ω

GL dispersion relation for n=7



$$\Omega = \frac{1}{\sqrt{n+1}}k - \frac{2+n}{n(1+n)}k^2 + \frac{(2+n)[1+n(2\tau_\omega - 1)]}{2n^2(1+n)^{3/2}}k^3 + O(k^4)$$

GL dispersion relation for n=50

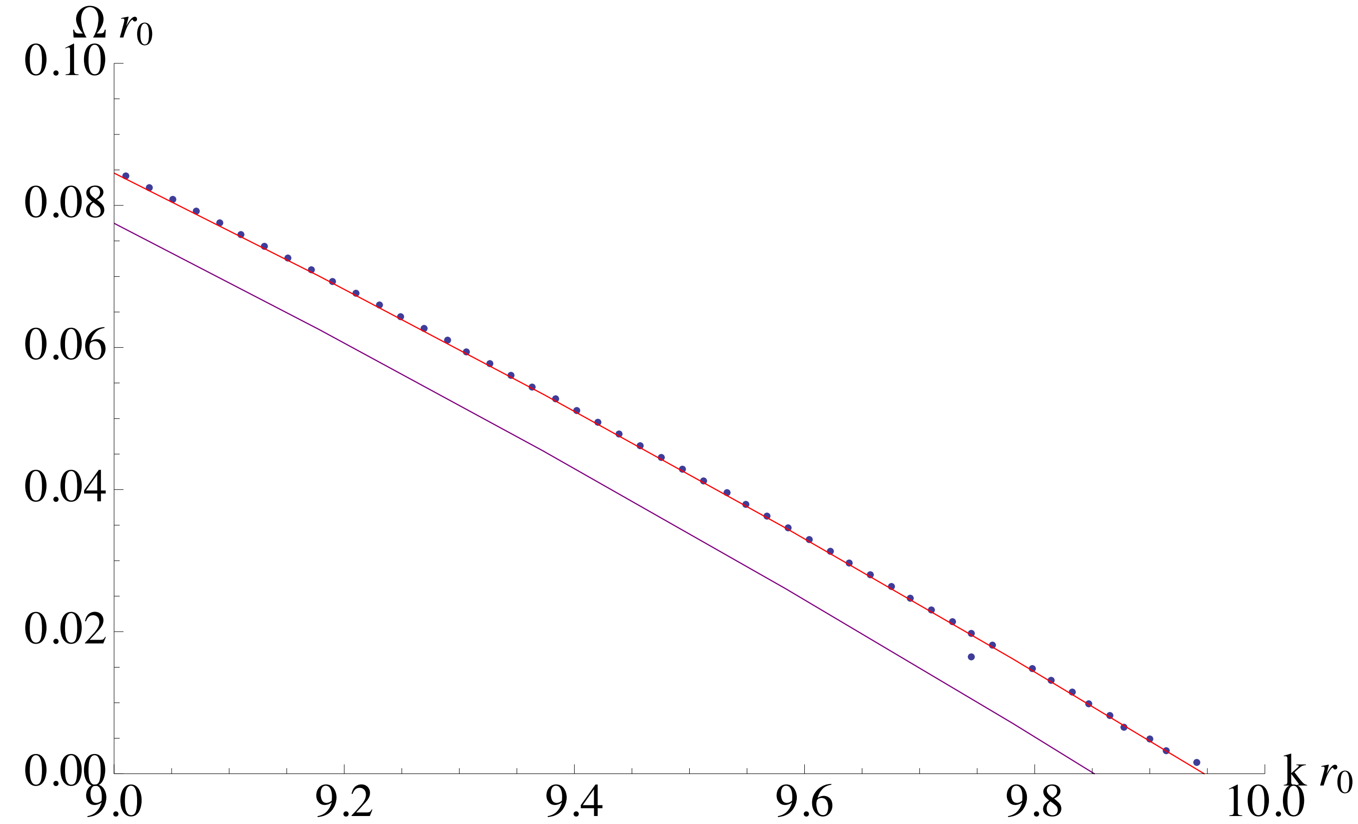


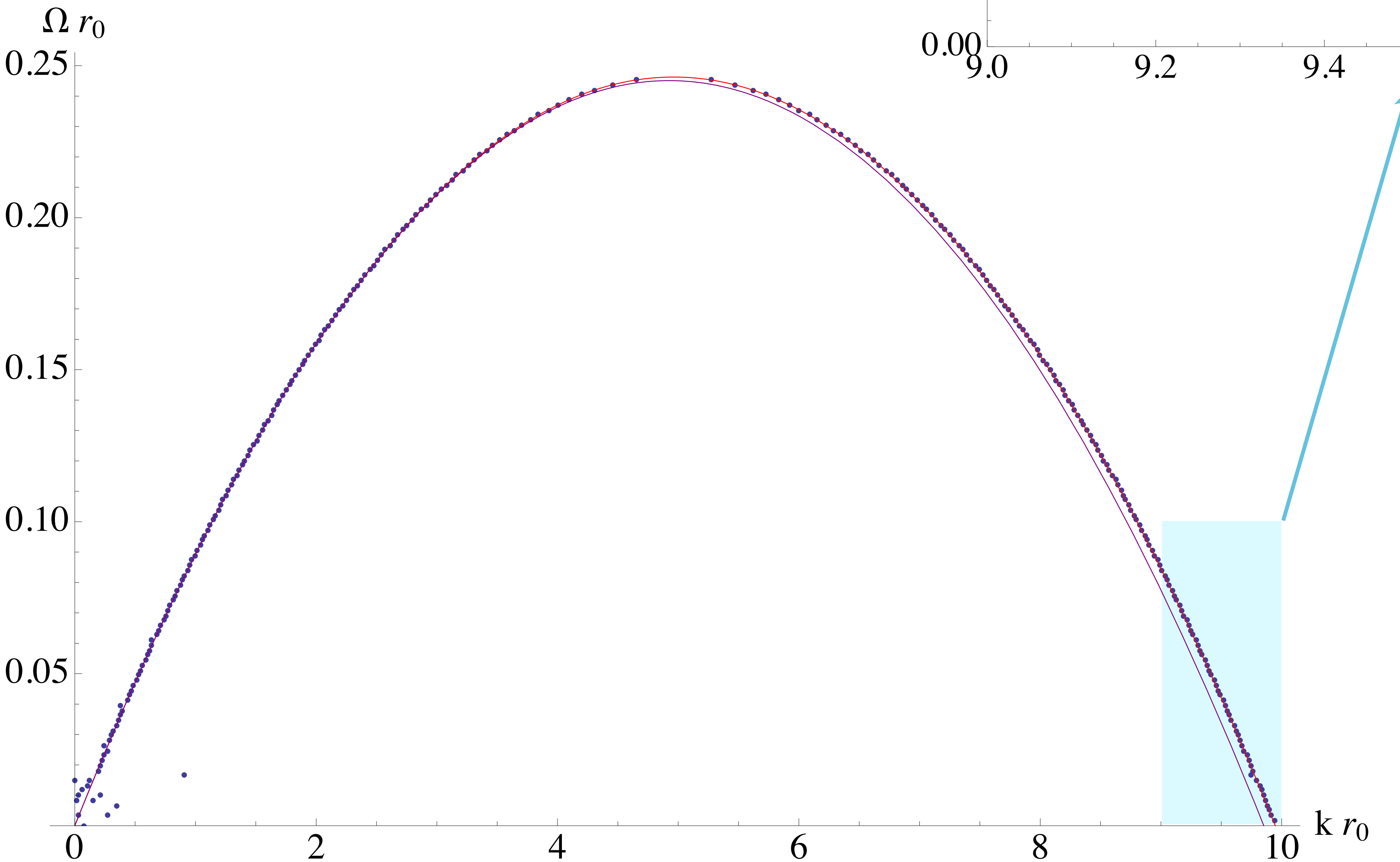
GL dispersion relation for n=100

.... numerical data (P. Figueras)

_____ first order

_____ second order





Conformal origin of the dispersion relation

Anti-de Sitter → Asympt. Flat

Goutéraux, Smolic, Smolic, Skenderis & Taylor '11

$$D = n + p + 3 = d + 1$$

$$\bar{D} = \bar{n} + p + 3$$

$$ds^2_D = ds^2_{p+2} + A(x)\, dy^2_{n+1}$$

$$ds^2_{\bar{D}} = \frac{1}{A(x)} \left(ds^2_{p+2} + \ell^2\, d\Omega^2_{\bar{n}+1} \right)$$

→ **Analytic continuation:** $d = -\bar{n}$

Start with AdS/Fluid metric → **Blackfold metric to second order in derivatives**

Bhattacharyya, Hubeny, Minwalla & Rangamani '07

Bhattacharyya, Loganayagam, Mandal, Minwalla & Sharma '08

MC, Camps, Goutéraux & Skenderis '12

$$v_s^2 = \frac{1}{d-1}$$

$$v_s^2 = -\frac{1}{\bar{n}+1} < 0$$

* Hyperbolic system → time evolution **numerics**

* **Holography** for asymptotically flat spacetimes

* Hyperbolic system → time evolution **numerics**

* **Holography** for asymptotically flat spacetimes

* New stationary black holes

  ~ New topologies, black helices, etc — **Emparan, Harmark, Niarchos & Obers** '09

  ~ Black Rings in anti-de Sitter and de Sitter spaces

  MC, Emparan & Rodriguez '08, **Armas & Obers** '10

* Study of dynamics at long wavelengths

  Camps, Emparan & Haddad '10
  MC, Camps, Goutéraux & Skenderis '12

  ~ Instabilities (GL as sound-mode instability)

  ~ study beyond linear perturbations

* Charged rotating black holes (electric and dipole)

  ~ D>4 Kerr-Newman — **MC, Emparan & Van Pol** '10

  ~ new instabilities — **Emparan Harmark, Niarchos & Obers** '11

* D-brane probes in thermal backgrounds

  Grignani, Harmark, Marini, Obers & Orselli '10 '11 '12

LESSONS TO TAKE BACK HOME

- ✧ Richer BH **phases** & **dynamics** in higher dimensions
  (lack of uniquess, non-spherical topology, instabilities...)

- ✧ Emergence of **widely separated scales** on BH
  (horizons well approximated by black strings & membranes)

- ✧ BH dynamics can be captured by **fluid dynamics**
  (effective theories for black holes, non-linear evolution...)